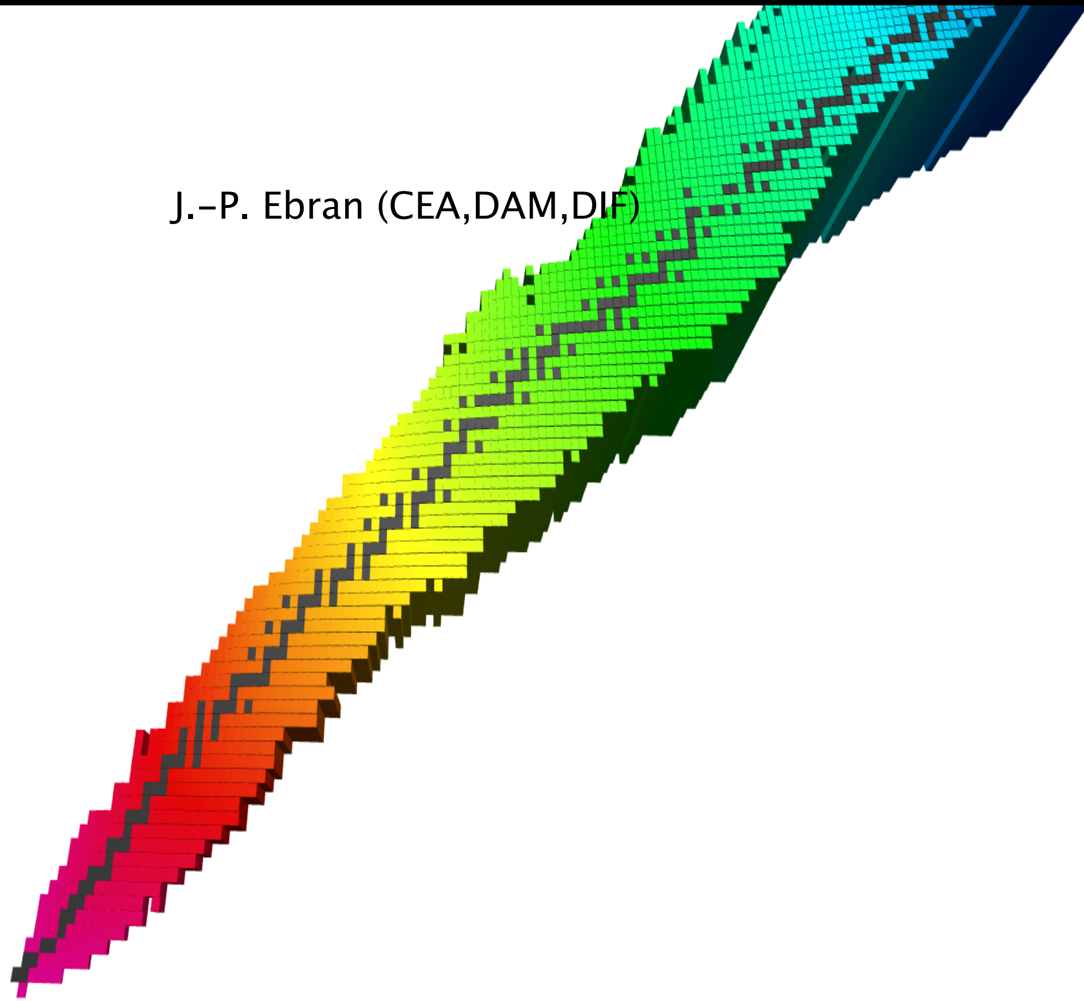
GT3: Quelles sont les nouvelles frontières dans la description microscopique des noyaux ?

-Status of the Nuclear Many-Body Problem & Associated Challenges -

J.–P. Ebran (CEA,DAM,DIF)



✪ *Axes of reflection of the WG3*

➲ Theoretical foundations for the description of nuclear systems

→ **Specificity** of nuclear systems : “How nuclear physics comes to be from QCD ?”

➲ Developing a web of relations with other fields

→ **Universality** transcending nuclear physics : *Many-Body Problem* + *Emergent Phenomena*

✪ Outline

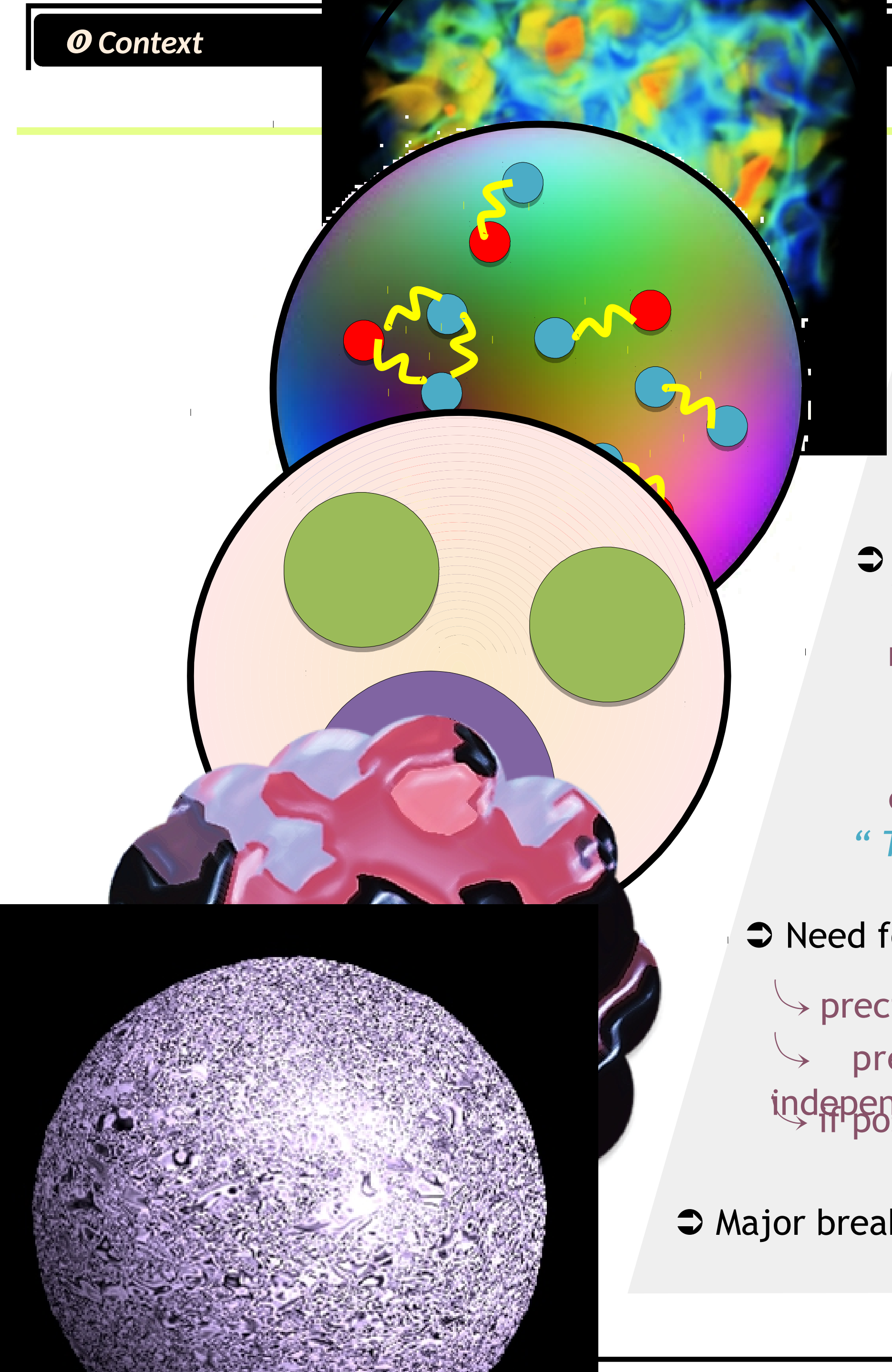
⓿ Context

❶ QCD in a nutshell

❷ Inter-Nucleon Interactions

❸ Many-Body treatment

✪ General considerations



➲ Stunning number of ways to apprehend nuclei

↳ mirror of the richness/complexity of nuclear physics

➲ In general, significant phenomenological component

↳ reachability / accuracy (near experimentally known regions)

↳ invaluable accumulation of knowledge

↳ mysterious success of some approaches / How far can we trust them ?

“ The truth is that we don’t know what we are doing !”

D. L., GT1

➲ Need for anchoring these approaches in sound physics

↳ precise domain of validity

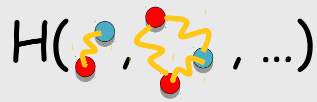
↳ predictive, systematically improvable & model-independent

↳ if possible, without spoiling the reachability

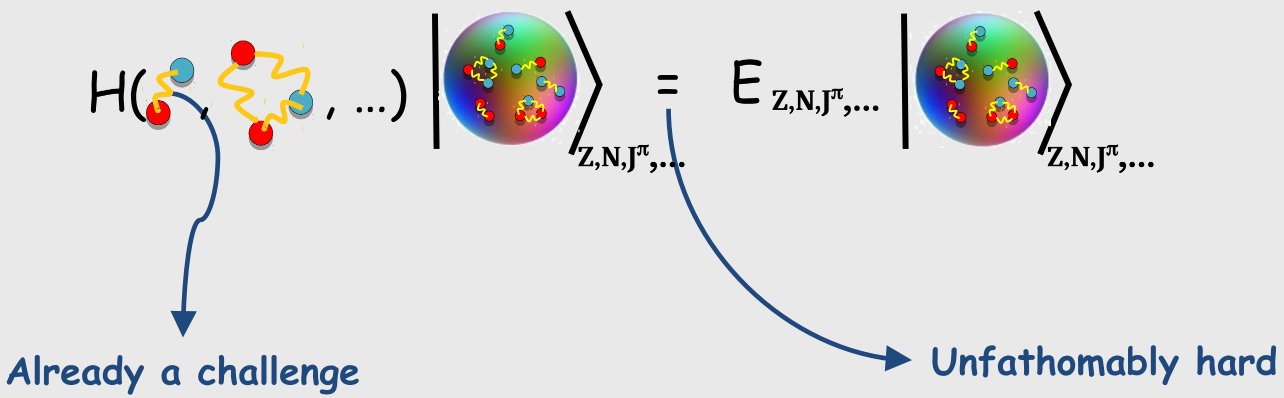
➲ Major breakthrough : introduction of a RG & EFT-based language

✪ Microscopic approach to nuclear systems

➲ Microscopic viewpoint : nucleus = collection of interacting nucleons whose dynamics is encoded in



➲ Challenge : extract the observables of interest from the nuclear Hamiltonian



✪ Outline

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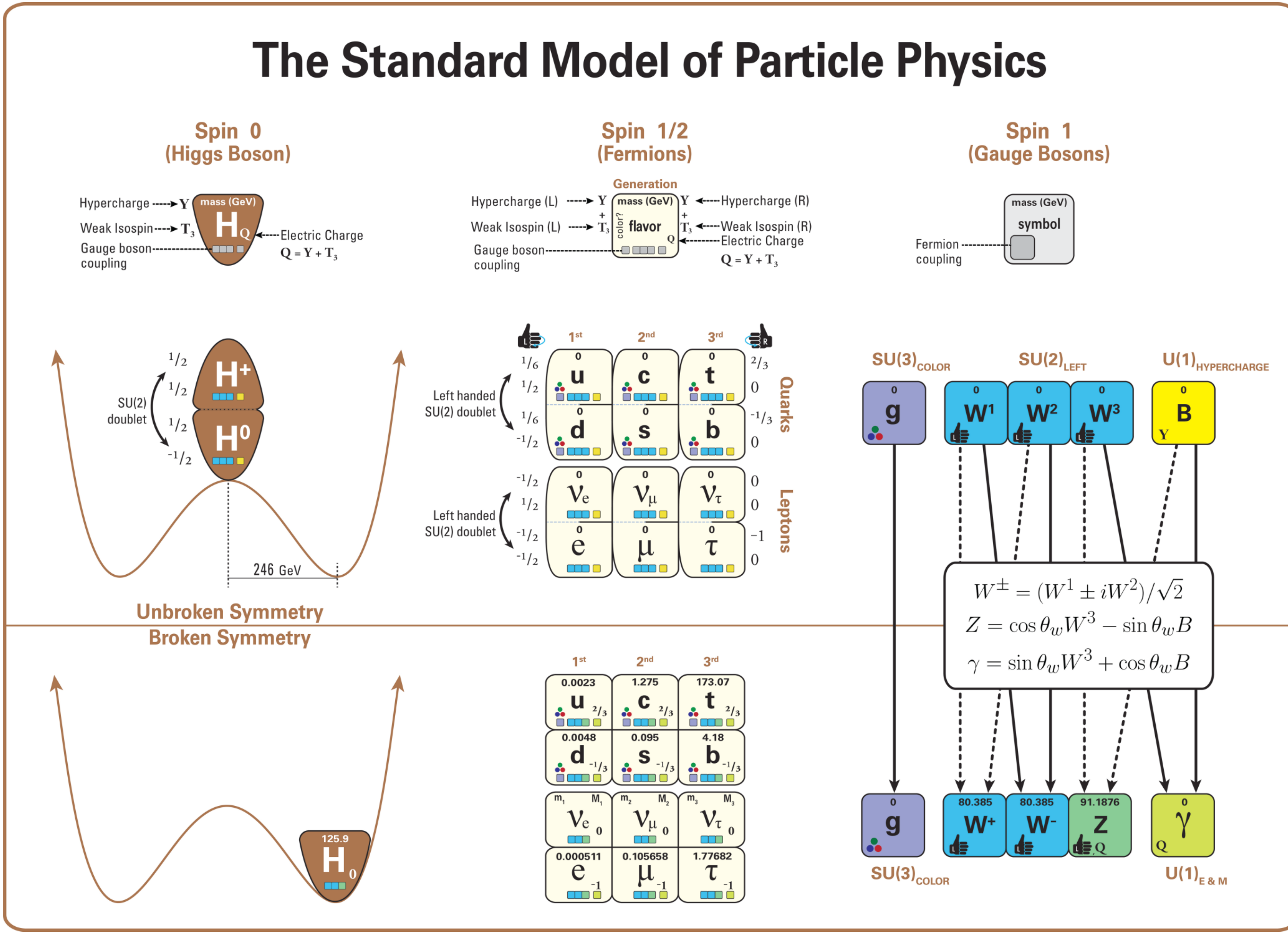
✪ The standard model of particle physics



✪ SU(3) gauge theory

➲ QCD = non-abelian gauge theory of the strong interactions based on $SU(3)_c$

❖ Idealized Lagrangian density of free quarks :

$$\mathcal{L} = \sum_{j=1}^{N_f} \bar{\psi}_j \left(i\partial\!\!\!/\right) \psi_j$$

• flavor

• Quark field

❖ Quark field in the fundamental representation of $SU(3)_c$:

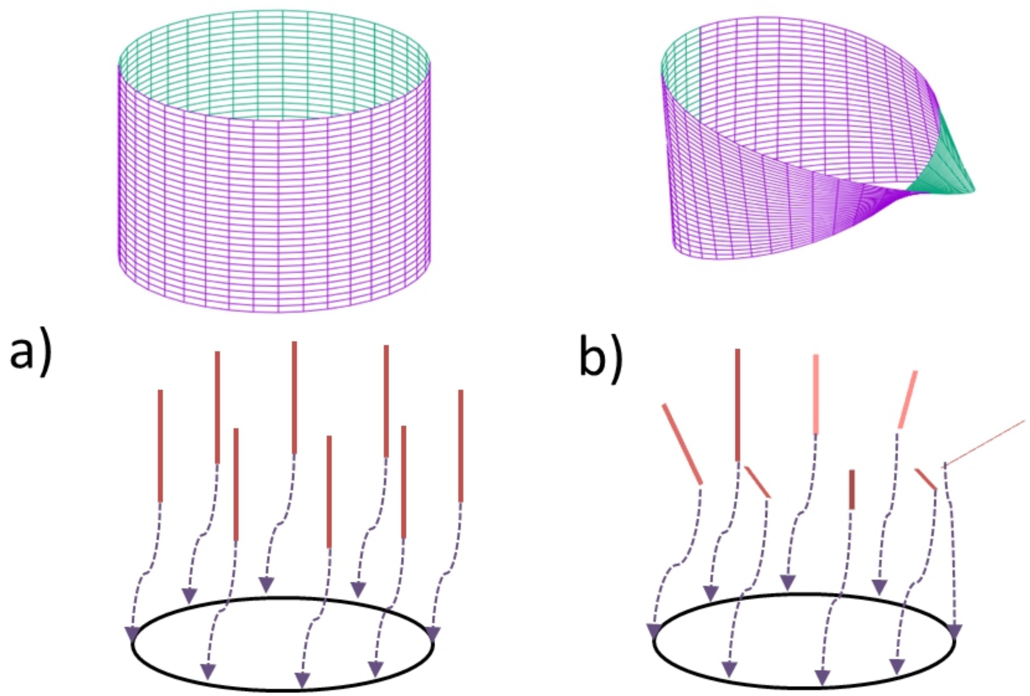
$$\psi_j(x^\mu) \equiv \begin{pmatrix} q_j^{red}(x^\mu) \\ q_j^{green}(x^\mu) \\ q_j^{blue}(x^\mu) \end{pmatrix}$$

❖ Invariance under **global** rotation in gauge space : $\psi_j(x^\mu) \rightarrow e^{i\theta_a T_a} \psi_j(x^\mu)$

➲ Now ask the Lagrangian to be invariant under **local** gauge transformation

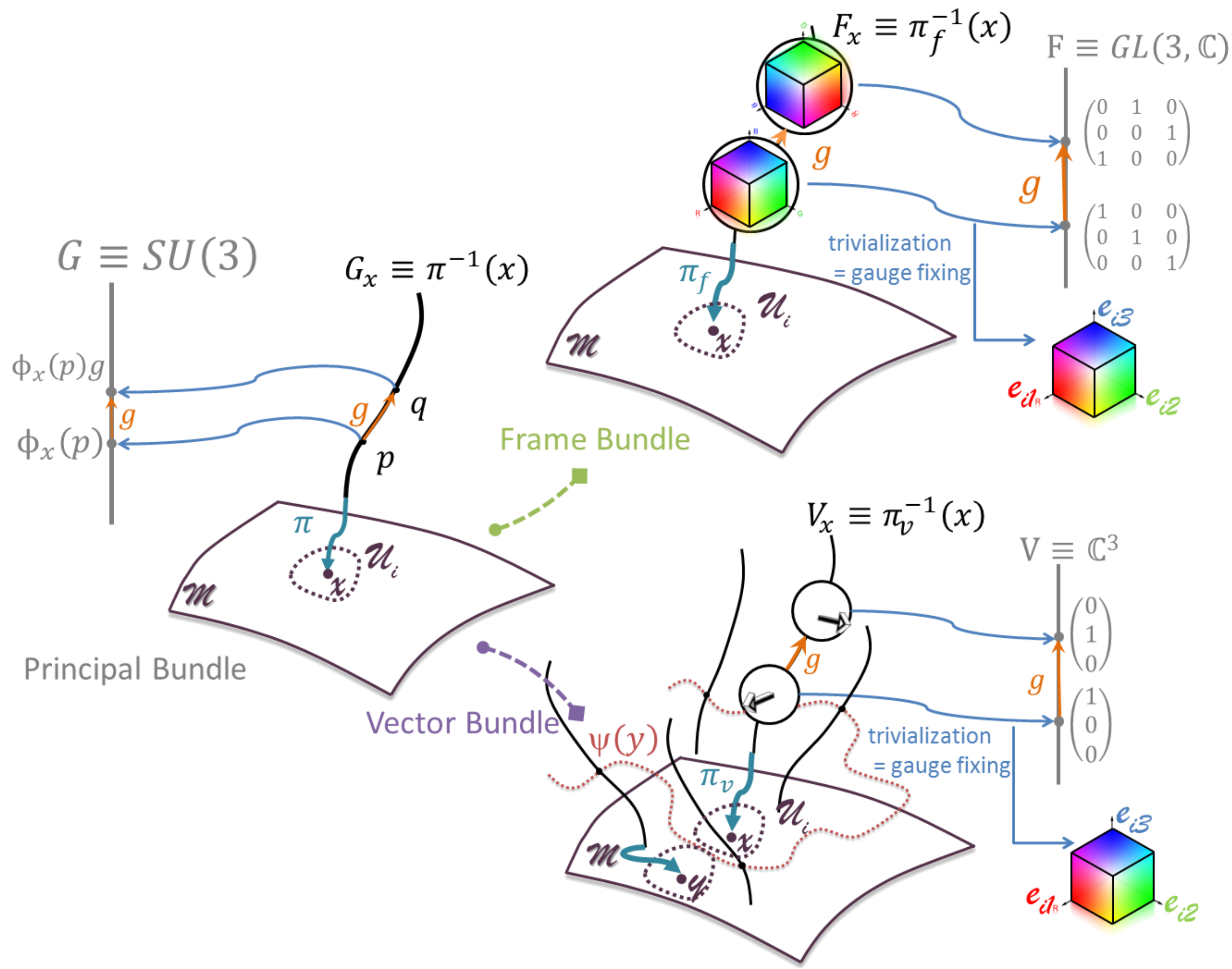
$$\psi_j(x^\mu) \rightarrow e^{i\theta_a(x^\mu) T_a} \psi_j(x^\mu)$$

✪ SU(3) gauge theory



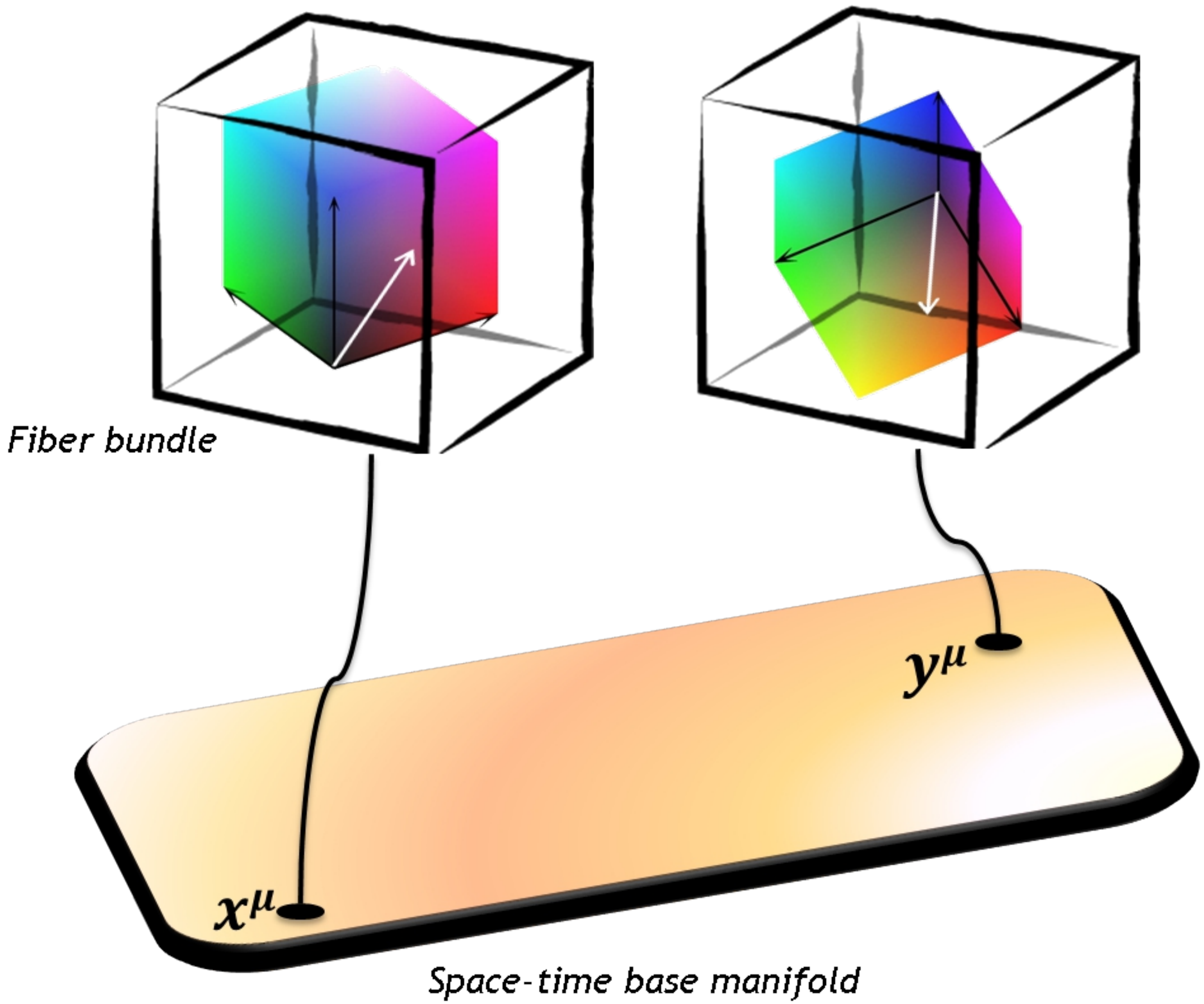
➲ Fiber bundle: generalization of the concept of direct product of two manifolds

✪ SU(3) gauge theory



➲ Fiber bundle formulation of QCD

✪ SU(3) gauge theory



➲ Fiber bundle formulation of QCD

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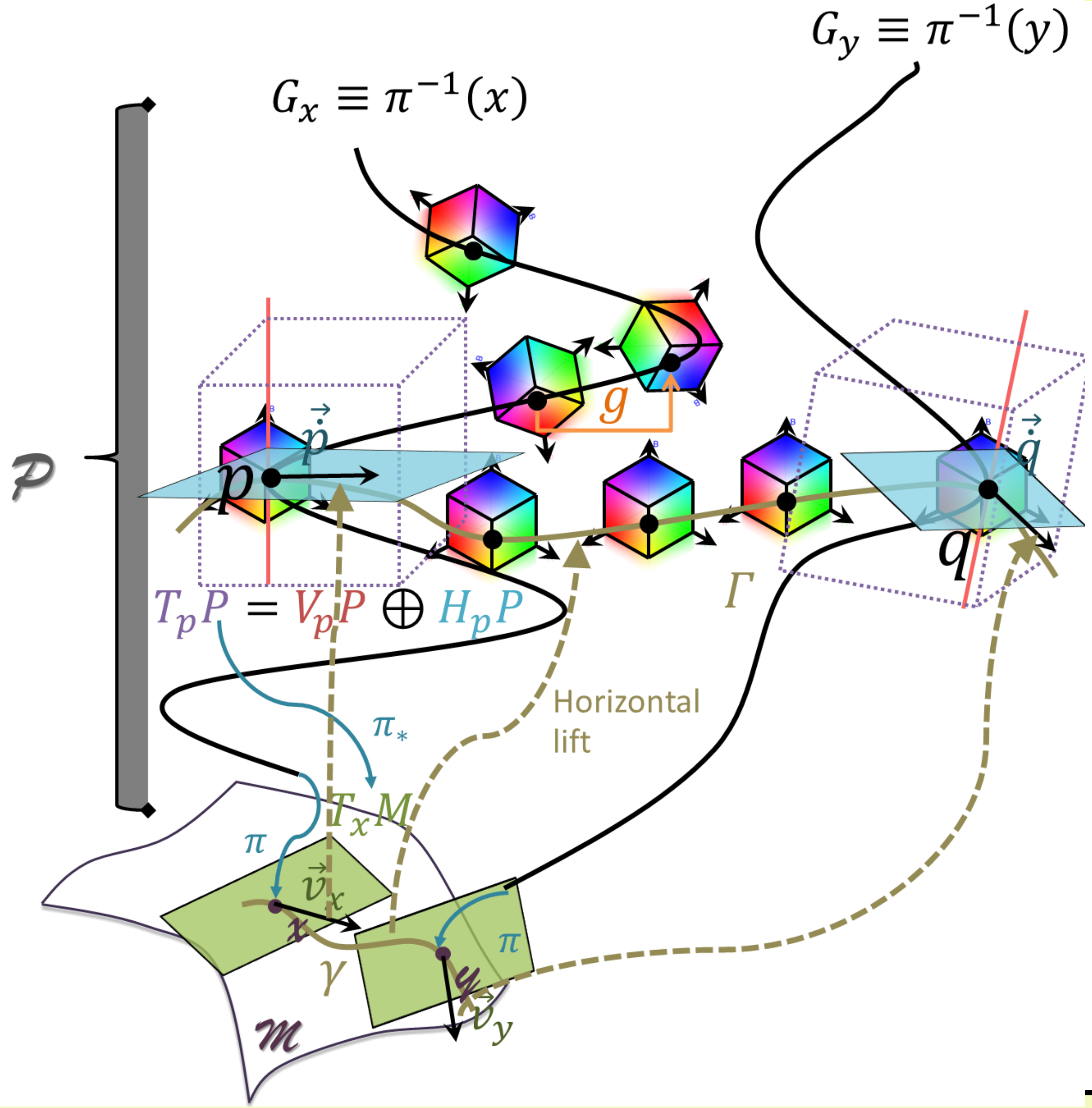
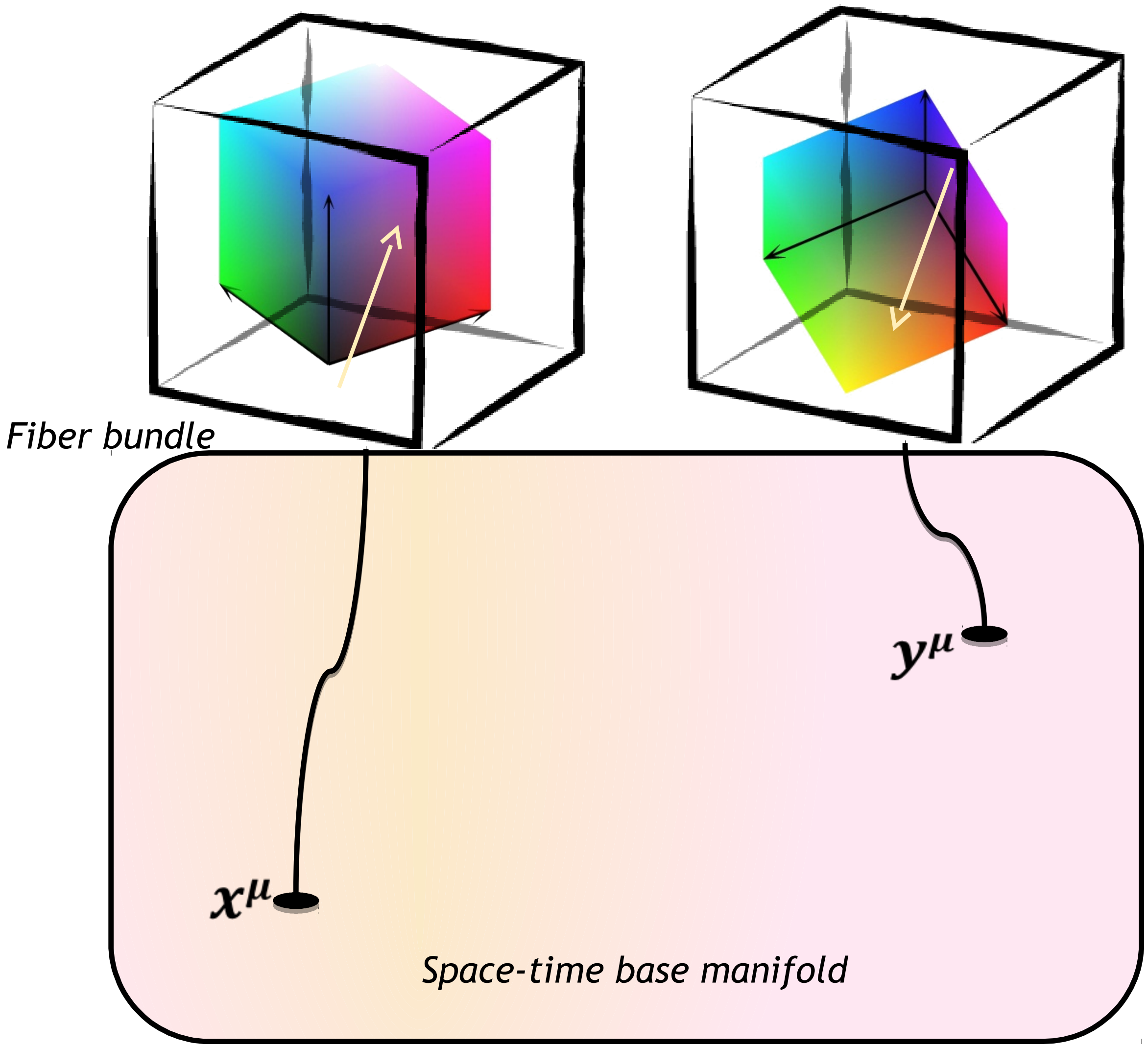


➲ Connection : how to move horizontally in the bundle space = gauge field (its local and perturbative version)

✪ SU(3) gauge theory



➲ Gauge field (connection) : gives a mean to compare internal frames at different space-time points (parallel transport)

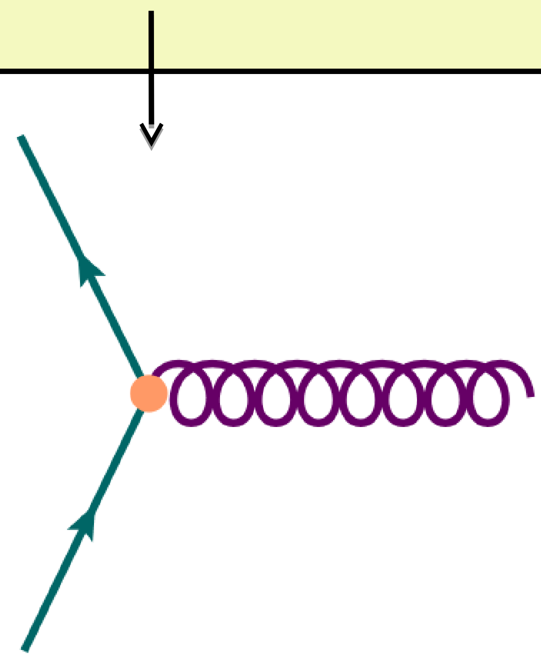
➲ Covariant derivative : measures deviation from parallel transport

$$\partial_\mu \to D_\mu = \partial_\mu - iA_\mu$$

✪ SU(3) gauge theory

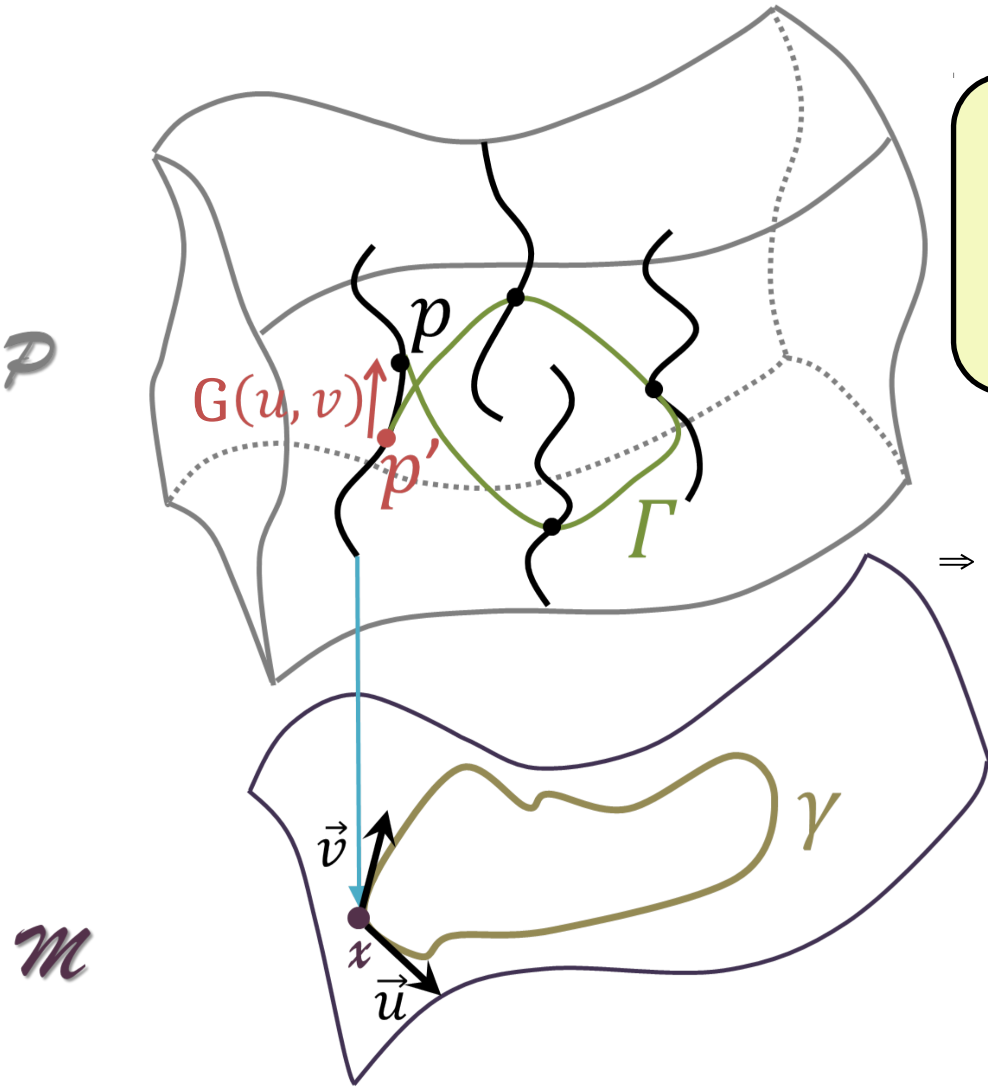
➲ Local symmetry requirement ⇒ interaction :

$$\mathcal{L} = \sum_{j=1}^{N_f} \bar{\psi}_j \left(i \not{D} \right) \psi_j = \sum_{j=1}^{N_f} \bar{\psi}_j \left(i \not{\partial} + \gamma_\mu A^\mu \right) \psi_j$$



$$A_\mu^{here} = g A_\mu^{usual}$$

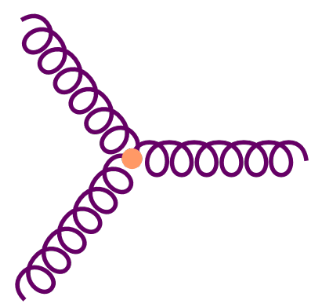
✪ SU(3) gauge theory

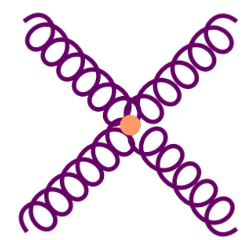


➲ Connection curvature : obstruction to the closure in the fiber bundle ⇒ chromo - electric and magnetic fields

$$G_{\mu\nu} \equiv i\,[D^\mu, D^\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu + i\,[A_\mu, A_\nu]$$

$$\Rightarrow \quad \mathcal{L} = \sum_{j=1}^{N_f} \bar{\psi}_j \left(i\not{D}\right) \psi_j - \frac{1}{4g^2} Tr G^{\mu\nu} G_{\mu\nu}$$



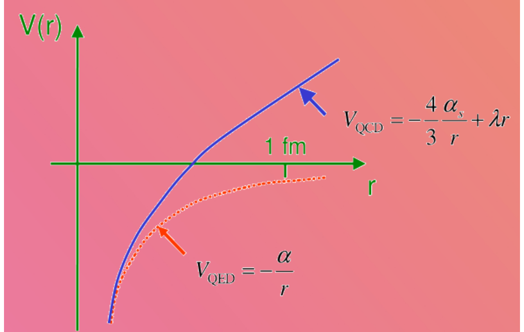


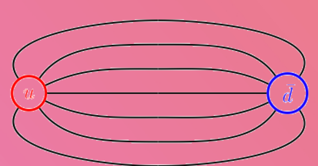
Cubic and quartic gluon self-interaction : makes life interesting

✪ Symmetry pattern

$$G_{\text{class}} \equiv SU(3)_c \times SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_A \times R^{+}_{\text{scale}}$$

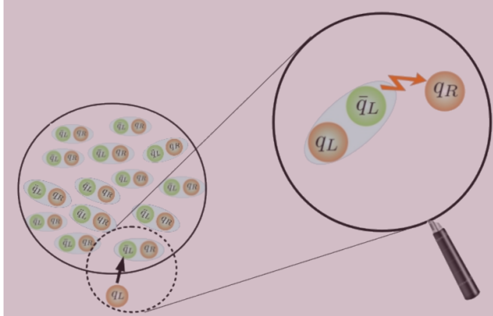
No obvious trace in the low-energy sector because of color confinement





⇒Hadrons = relevant low-energy dofs

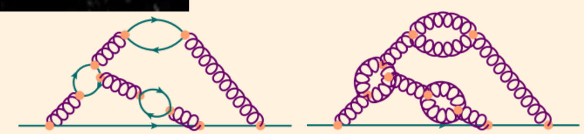
Nambu-Goldstone realization : chiral condensate

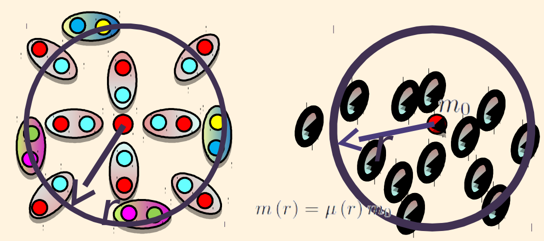


Goldstone physics governs low-energy sector

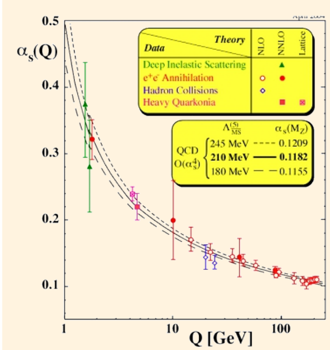
anomalies

$$V(r) = \frac{\mathcal{V}(r)}{\varepsilon(r)} \qquad \varepsilon(r)\,\mu(r) = 1$$





$\mu > 1 \quad \varepsilon < 1$



⇒ Asymptotic freedom

$$\rightarrow \quad SU(N_f)_V \times U(1)_B$$

Symmetry subgroup of QCD matter-free G.S.

➓ Context

❶ QCD in a nutshell

❷ Inter-Nucleon Interactions

❸ Many-Body treatment

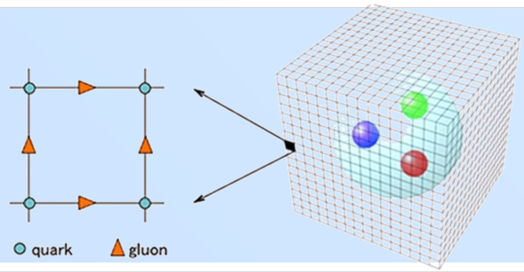
✪ *Model unknown inter-nucleon interactions*

➲ Lattice QCD

❖ Wick rotation ⇒ QFT→ SFT

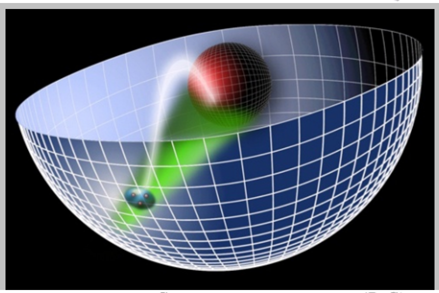
❖ Discretized Euclidian space-time

❖ QCD fields expanded on the lattice, correlation functions evaluated with Monte Carlo techniques



➲ Holographic QCD

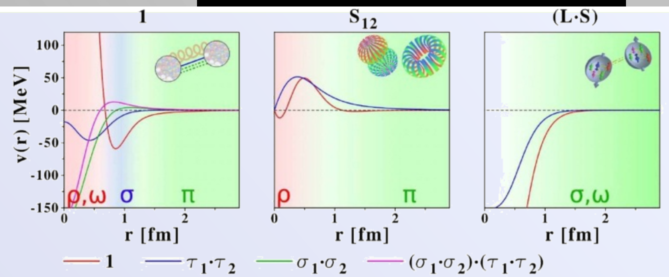
❖ Duality between QCD and a string theory in a higher dimensional AdS space



➲ Realistic interactions

❖ Phenomenological ansatz compatible with symmetry of the 2 (3) nucleons system

❖ Parameters fitted to accurately reproduce phase shifts



QCD Realm
Instanton configuration calculated by Lattice QCD
KEK
$4\pi f_\pi$
…
Realistic interactions (Argone, CD-Bonn, Paris, …)
Hadron Realm
$\Lambda(MeV)$
NN scattering data

✪ *Model unknown inter-nucleon interactions*

➲ Lattice QCD

❖ Wick rotation ⇒ QFT→ SFT

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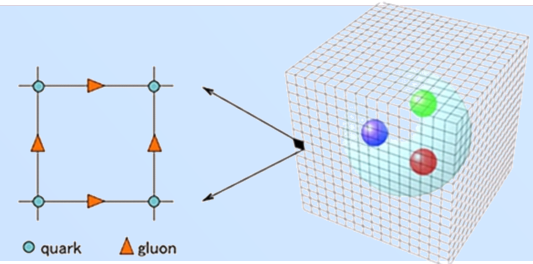

➲ Holographic QCD

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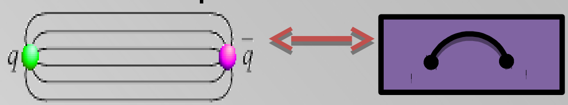
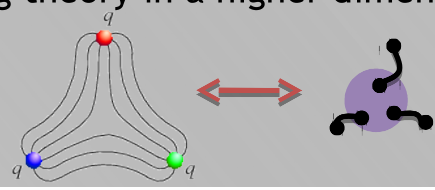
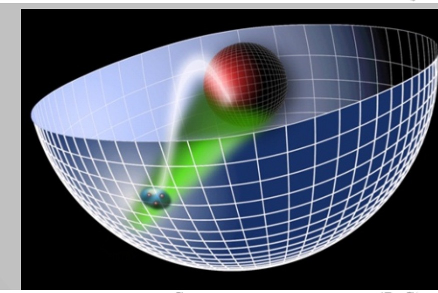
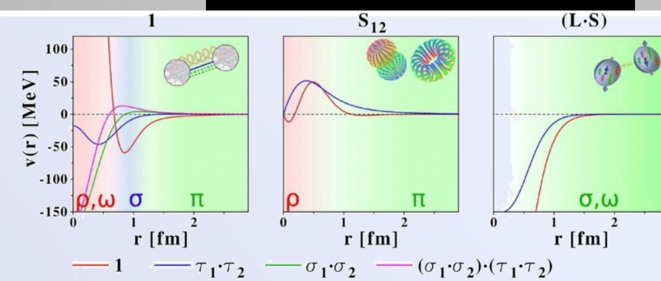




➲ Phenomenological interactions (in free space)

❖ Ansatz compatible with symmetry of the 2 (3) nucleons system

❖ Parameters fitted to accurately reproduce phase shifts

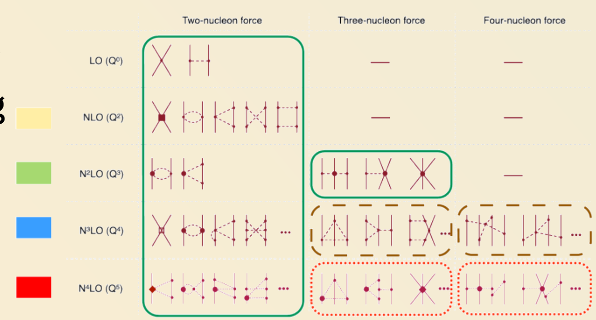


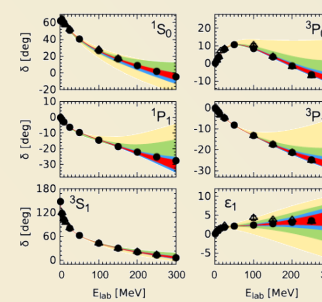
➲ Chiral effective field theory

❖ Based on effective low-energy d.o.f. + constrained by symmetries and symmetry breaking pattern of underlying theory

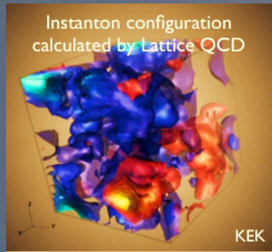
❖ High energy dynamics generically parameterized by contact terms

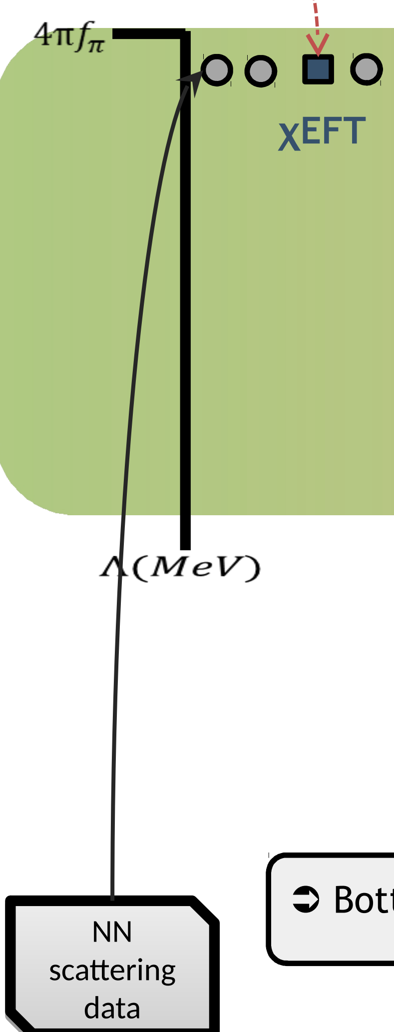
❖ Hierarchy of contributions to inter-nucleonic interactions





[Meißner 2016]





➲ Bottleneck : LECs start becoming underconstrained at high order of the chiral expansion

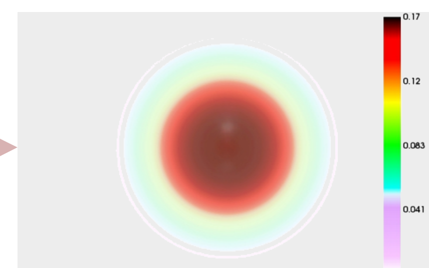
➓ Context

➊ QCD in a nutshell

➋ Inter-Nucleon Interactions

➌ Many-Body treatment

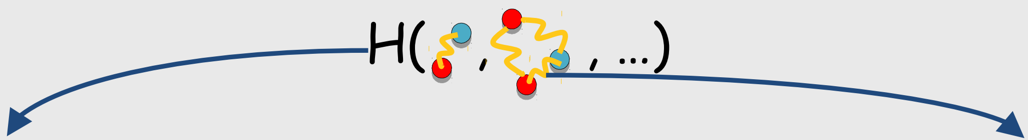
We want to start from here to go there



NN scattering data

✪ Nuclear many-body problem : strategies

➲ Microscopic viewpoint : nucleus = collection of interacting nucleons whose dynamics is encoded in

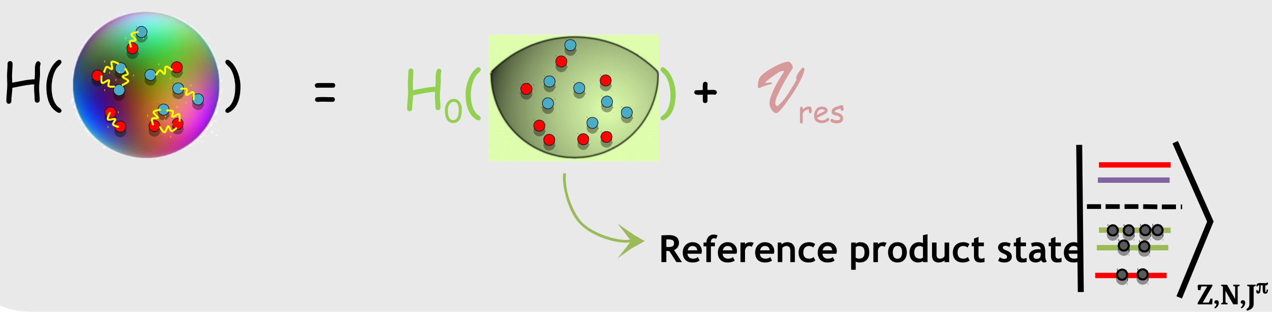


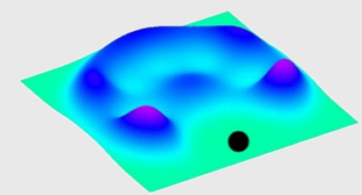
Extraction of the observables from H is unfathomably hard

Description of inter-nucleon interactions is already a challenge

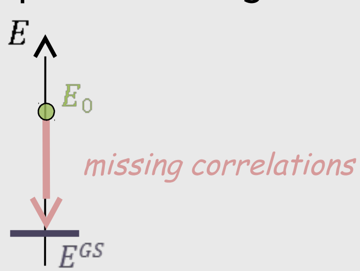
➲ Many-body approaches as implementation of different strategies to apprehend the many-body problem

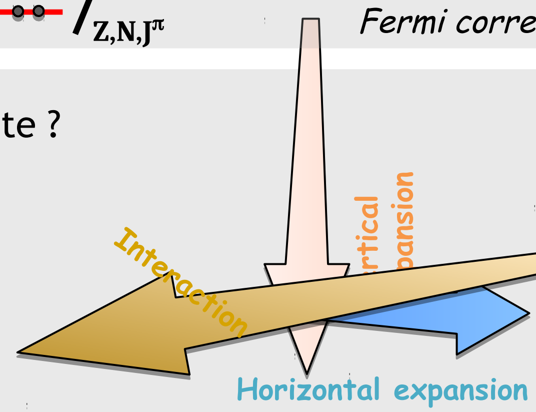
➲ Traditional starting point: split the total Hamiltonian into unperturbed and residual parts



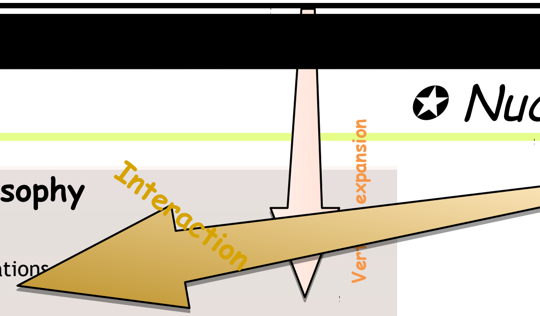


➲ How to incorporate missing correlations on top of a product state ?

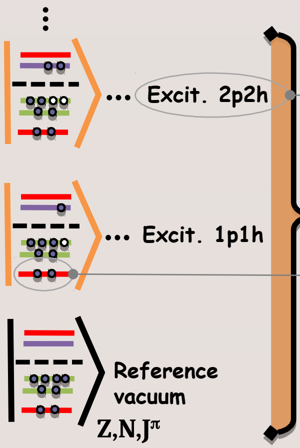




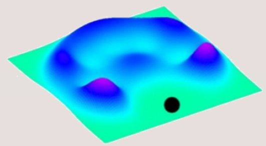
✪ Nuclear many-body problem : strategies



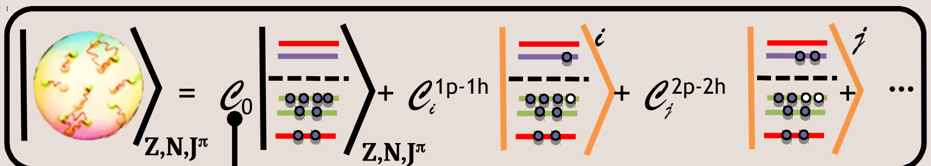
Vertical Philosophy



- ⊙ **non collective** excitations
- ⊙ physics accounted through numerous **rearrangements** of nucleons within the orbitals given by the reference vacuum
- ⊙ **conservation** of symmetries

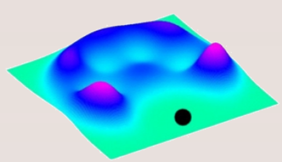


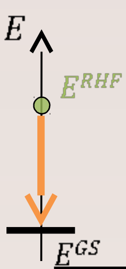
Fermi correlations



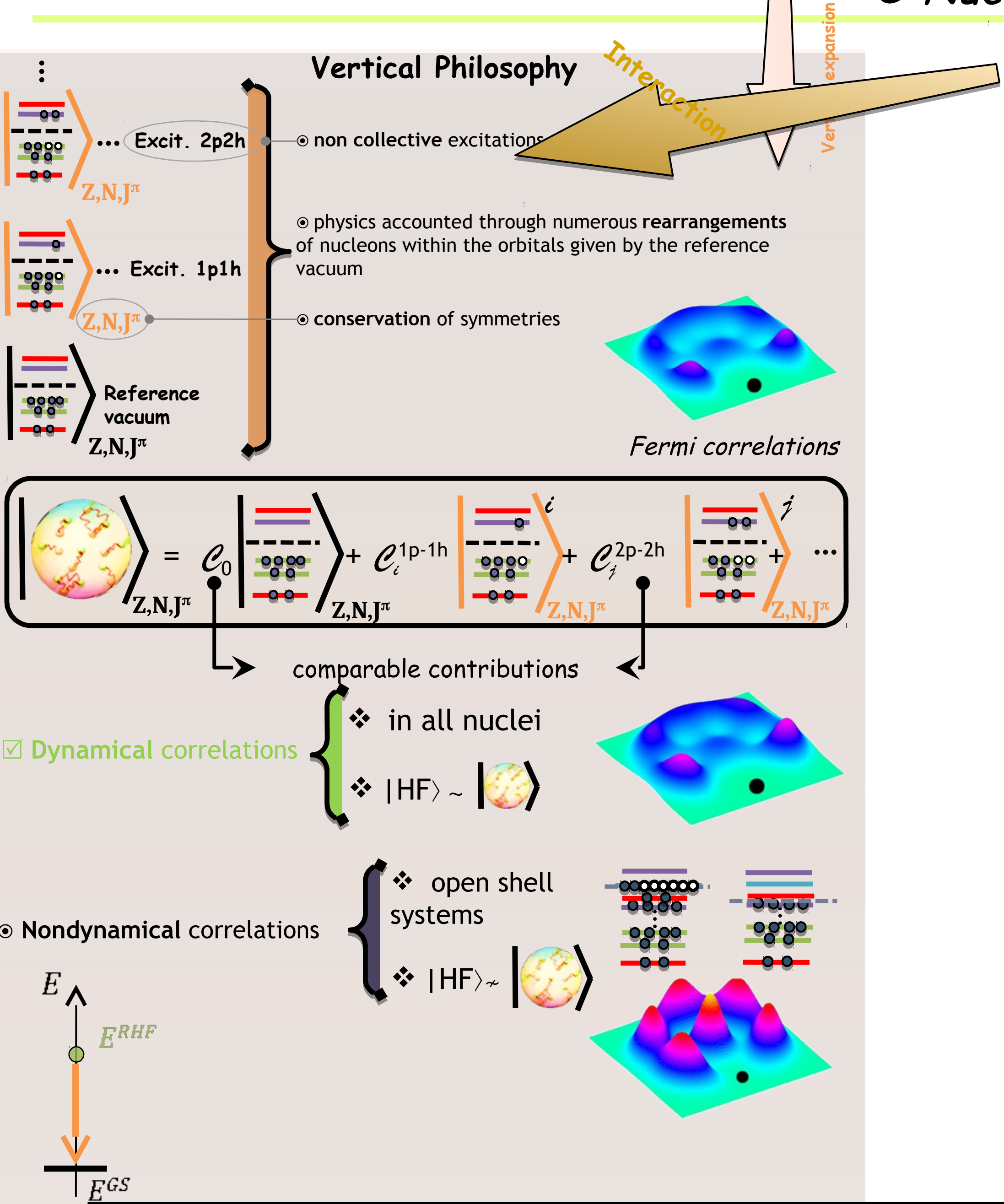
Major contribution

☑ Dynamical correlations
- ❖ in all nuclei
- ❖ |HF⟩ ~ | ⟩





✪ Nuclear many-body problem : strategies



✪ Nuclear many-body problem : strategies

Vertical Philosophy

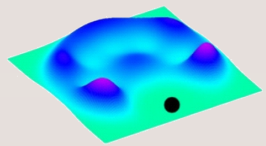
⋮

... Excit. 2p2h — Z,N,J^π

... Excit. 1p1h — Z,N,J^π

Reference vacuum — Z,N,J^π

- ⊙ non collective excitations
- ⊙ physics accounted through numerous **rearrangements** of nucleons within the orbitals given by the reference vacuum
- ⊙ **conservation** of symmetries

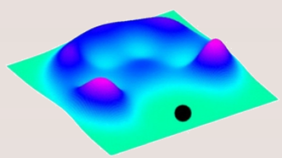


Fermi correlations

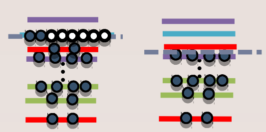
$$|\Psi\rangle_{Z,N,J^\pi} = \mathcal{C}_0 \,|\Phi_0\rangle_{Z,N,J^\pi} + \mathcal{C}_i^{1p\text{-}1h}\,|\Phi_i\rangle_{Z,N,J^\pi} + \mathcal{C}_j^{2p\text{-}2h}\,|\Phi_j\rangle_{Z,N,J^\pi} + \dots$$

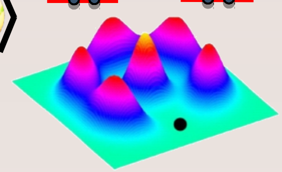
→ comparable contributions ←

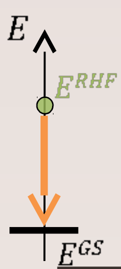
☑ **Dynamical correlations**
- ❖ in all nuclei
- ❖ $|HF\rangle \sim |\Psi\rangle$



⊙ **Nondynamical** correlations
- ❖ open shell systems
- ❖ $|HF\rangle \nsim |\Psi\rangle$





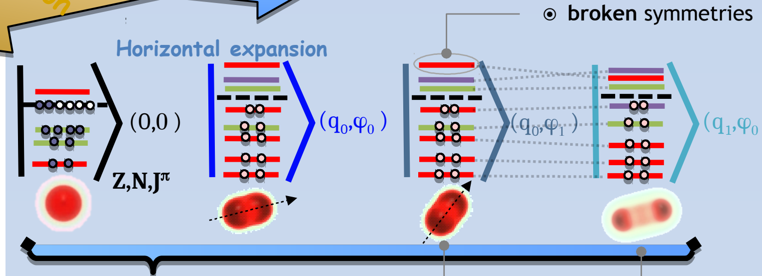


Horizontal philosophy

Interaction

Interaction

expansion

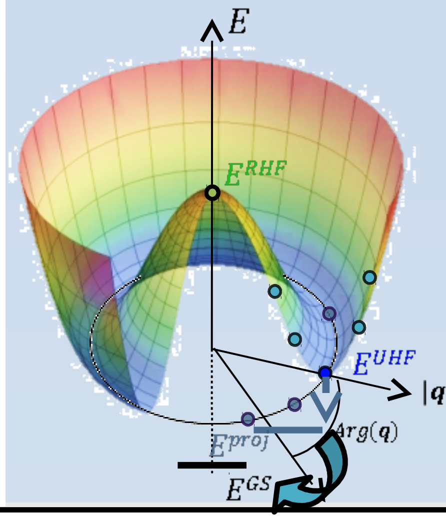


⊙ Physics accounted for through **Collective excitations** spontaneous symmetry breaking **+ fluctuations of the order parameter**

$$|\Psi\rangle = \int dq\, f(q)\, |\Phi(|q|, \arg(q))\rangle$$

➲ Remaining correlations grasped via vertical expansion

or transferred in H_{eff}(...)



ρ/ρ_0

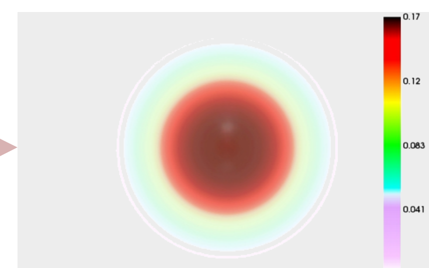
0

1

$4\pi f_\pi$

$\Lambda(MeV)$

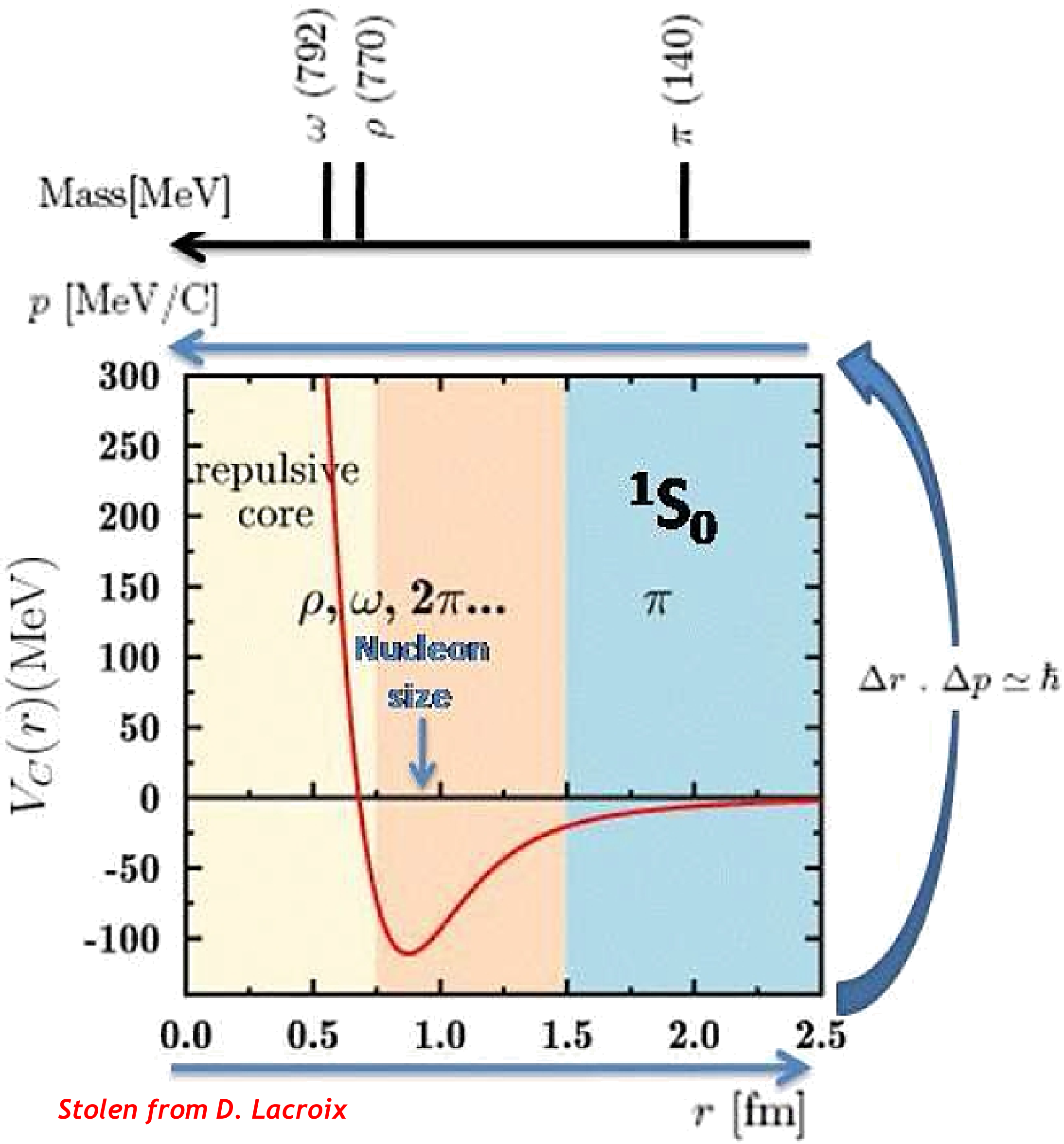
Hartree-Fock based on leads nowhere near



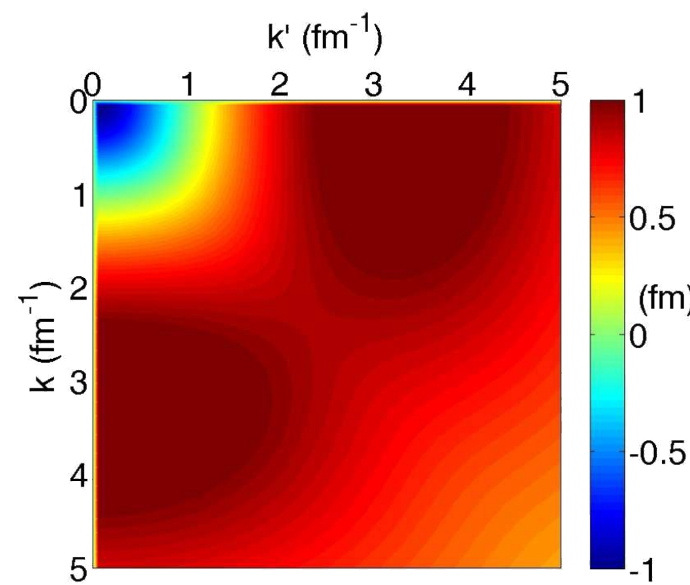
NN scattering data

✪ UV non-perturbativeness

➲ UV divergence (hard core), bound & resonant state ⇒ nonperturbative nuclear many-body problem



Stolen from D. Lacroix

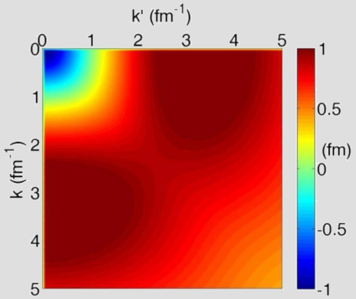


Bogner et al.

✪ Taming down the UV non-perturbativeness

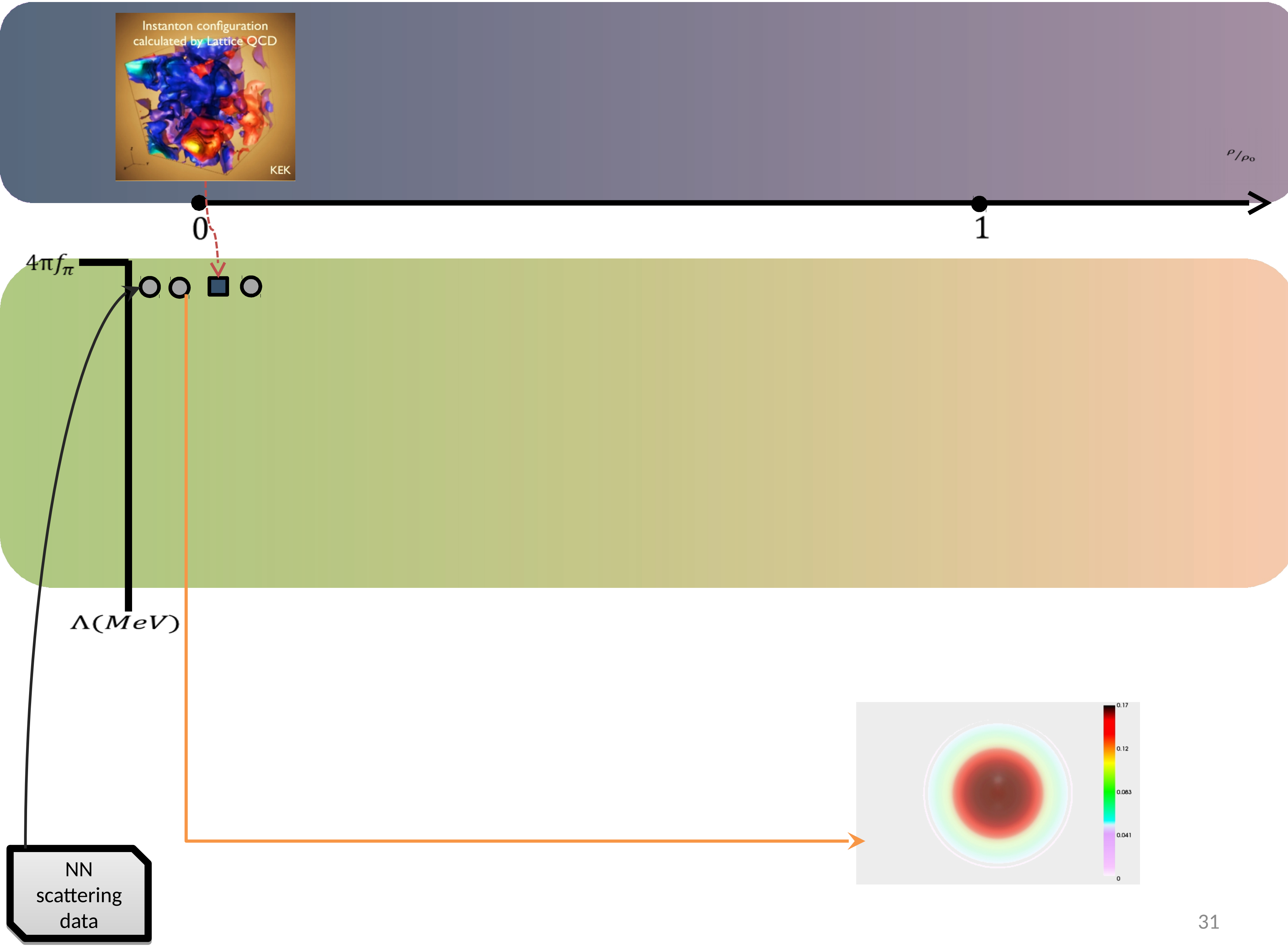
Interaction

There's a hard core
Let's cope with it !



➲ Nonperturbative many-body treatment that scales awfully with the nucleon number

Vertical expansion

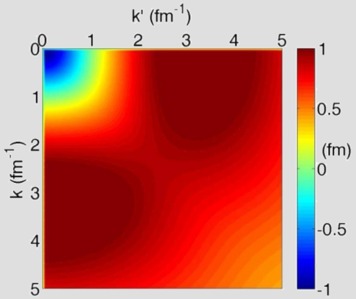


Instanton configuration
calculated by Lattice QCD
KEK
ρ/ρ0
0
1
4πfπ
Λ(MeV)
0.17
0.12
0.083
0.041
0
NN
scattering
data

✪ Taming down the UV non-perturbativeness

Interaction

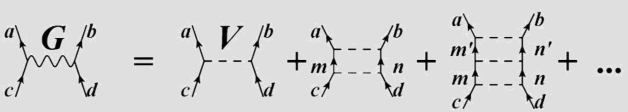
There's a hard core
Let's cope with it !

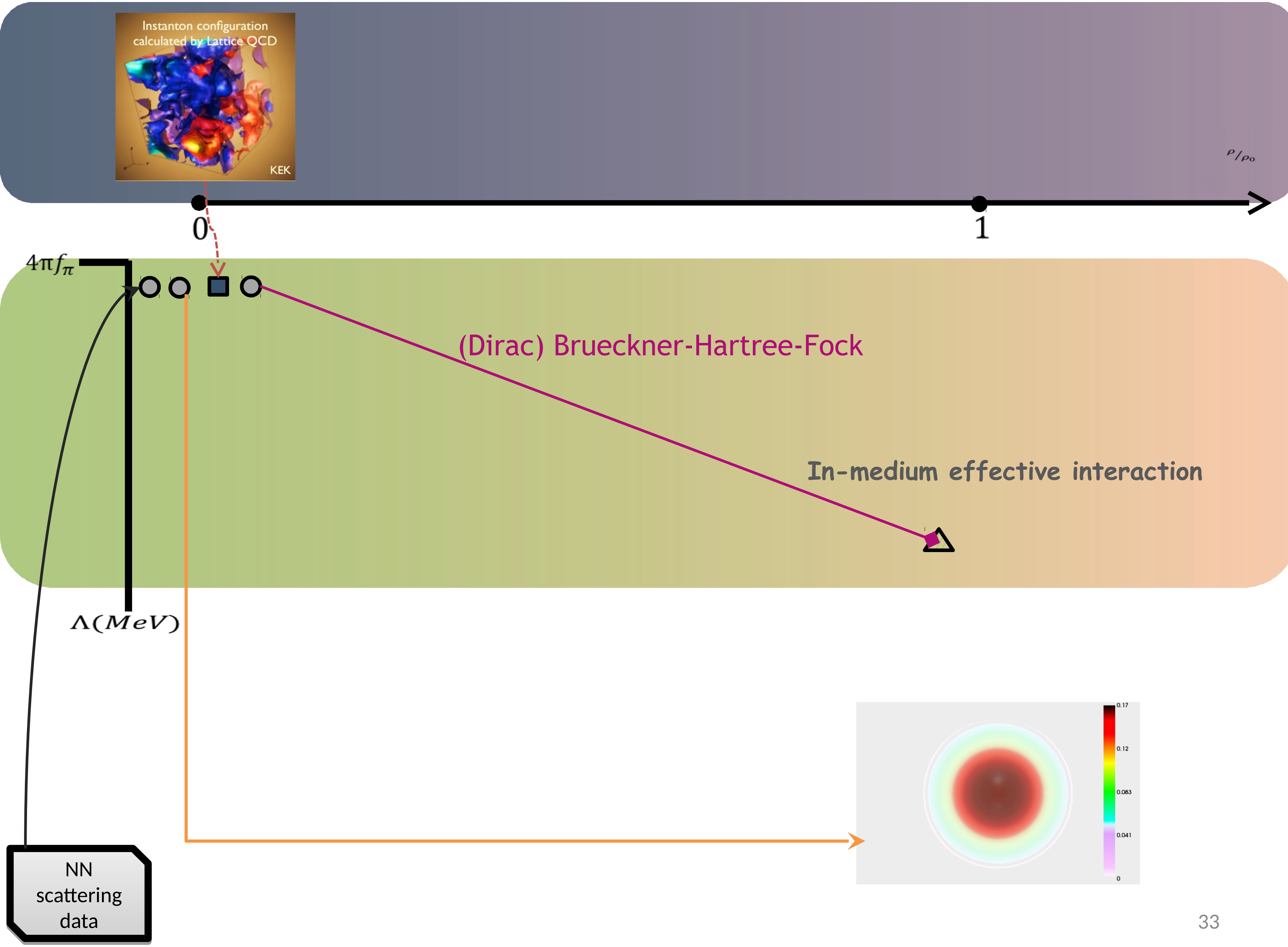


Vertical expansion

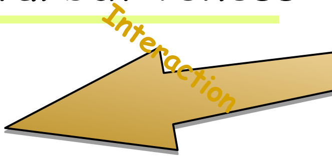
➲ Nonperturbative many-body treatment that scales awfully with the nucleon number

➲ Brueckner resummation of ladder diagrams (no high/low - momentum decoupling though)

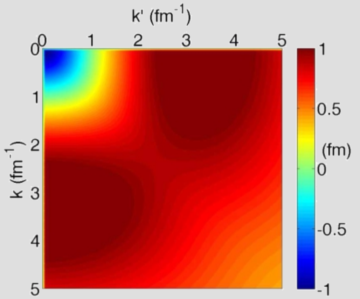


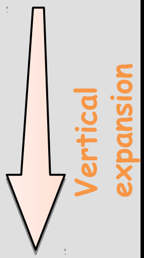

Instanton configuration
calculated by Lattice QCD
KEK
ρ/ρ0
0
1
4πf_π
Λ(MeV)
(Dirac) Brueckner-Hartree-Fock
In-medium effective interaction
0.17
0.12
0.083
0.041
0
NN
scattering
data

✪ Taming down the UV non-perturbativeness



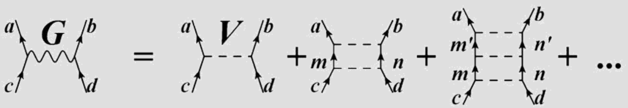
There's a hard core
Let's cope with it !



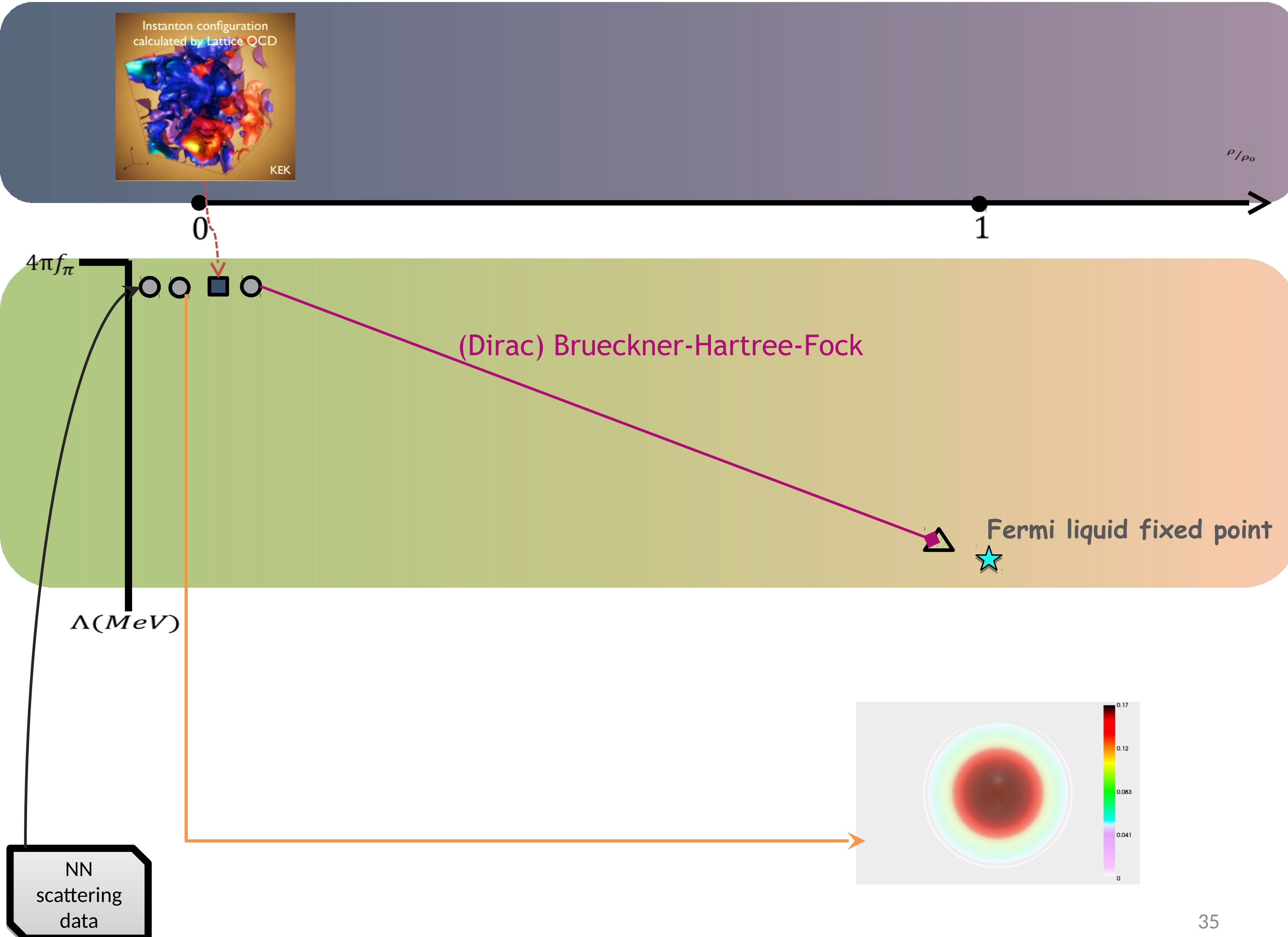


➲ Nonperturbative many-body treatment that scales awfully with the nucleon number

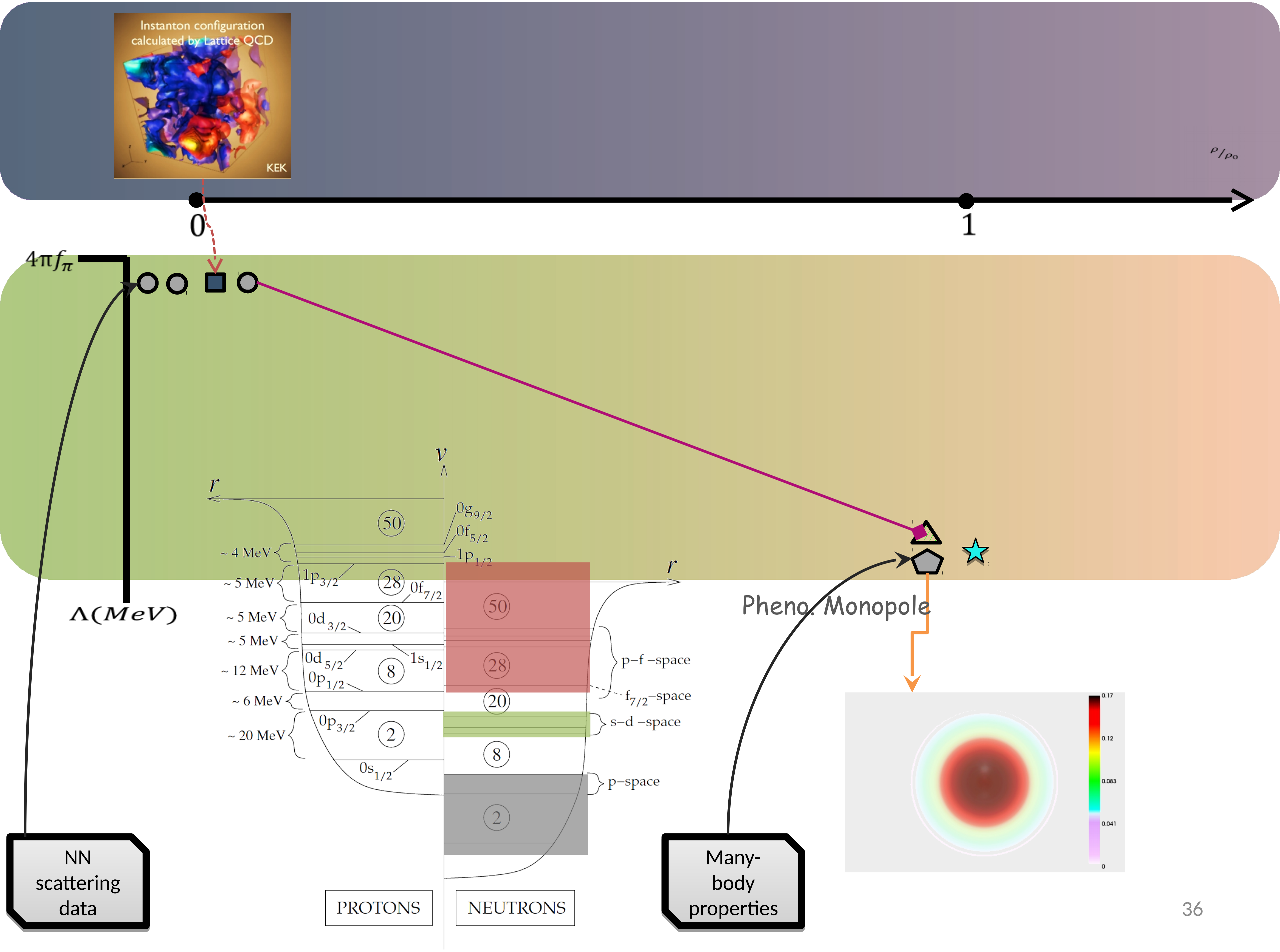
➲ Brueckner resummation of ladder diagrams (no high/low – momentum decoupling though)



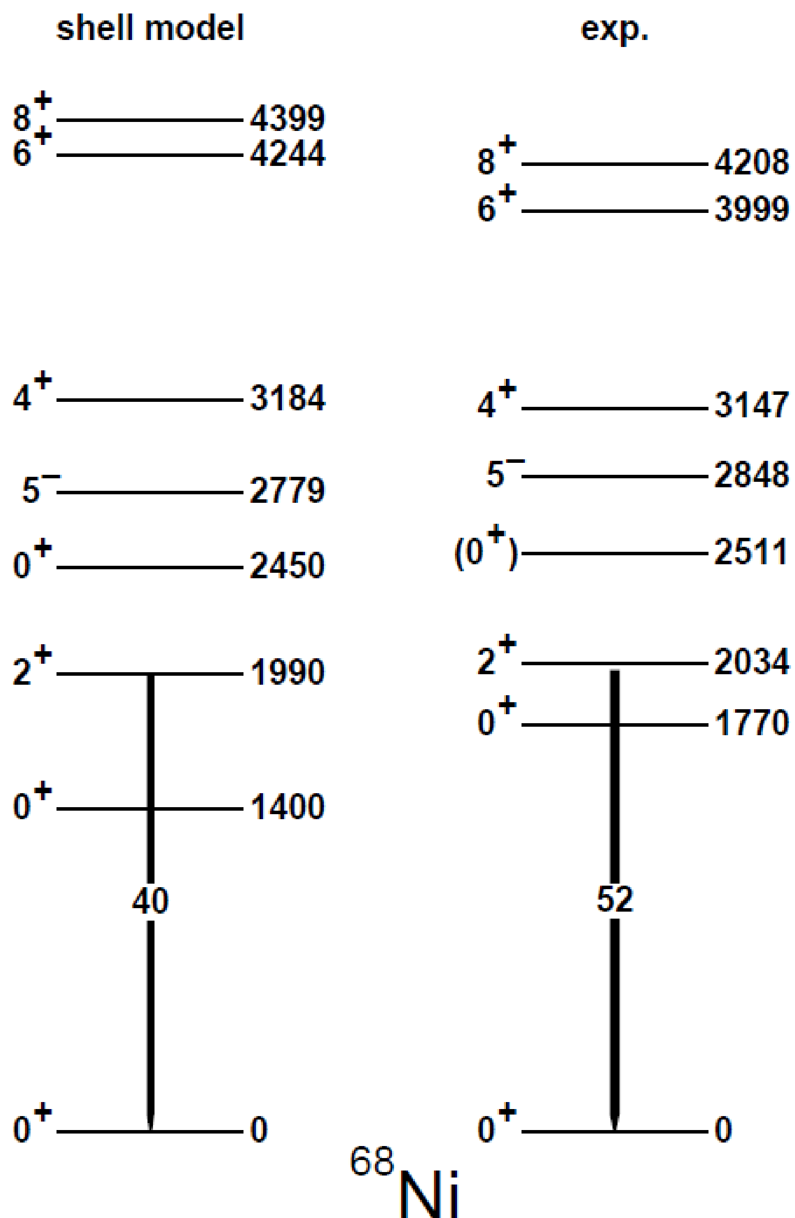
➲ Phenomenological effective in-medium interaction



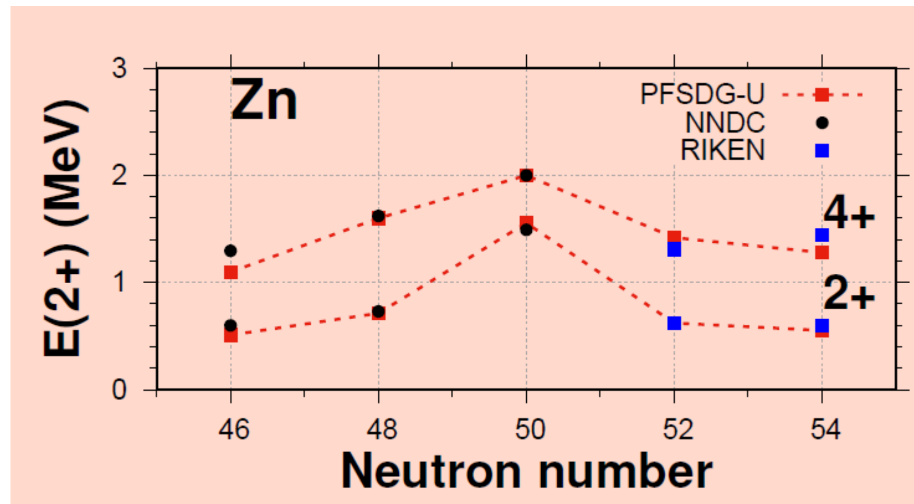
Instanton configuration calculated by Lattice QCD
KEK
ρ/ρo
0
1
$4\pi f_\pi$
(Dirac) Brueckner-Hartree-Fock
Fermi liquid fixed point
$\Lambda(MeV)$
NN scattering data
0.17
0.12
0.083
0.041
0


Instanton configuration calculated by Lattice QCD
KEK
ρ/ρ₀
0
1
4πf_π
Λ(MeV)
NN scattering data
Many-body properties
Pheno. Monopole
PROTONS
NEUTRONS
p−f −space
f_{7/2}−space
s−d −space
p−space
~ 4 MeV
~ 5 MeV
~ 5 MeV
~ 5 MeV
~ 12 MeV
~ 6 MeV
~ 20 MeV
0g_{9/2}
0f_{5/2}
1p_{1/2}
1p_{3/2}
0f_{7/2}
0d_{3/2}
1s_{1/2}
0d_{5/2}
0p_{1/2}
0p_{3/2}
0s_{1/2}
50
28
20
8
2

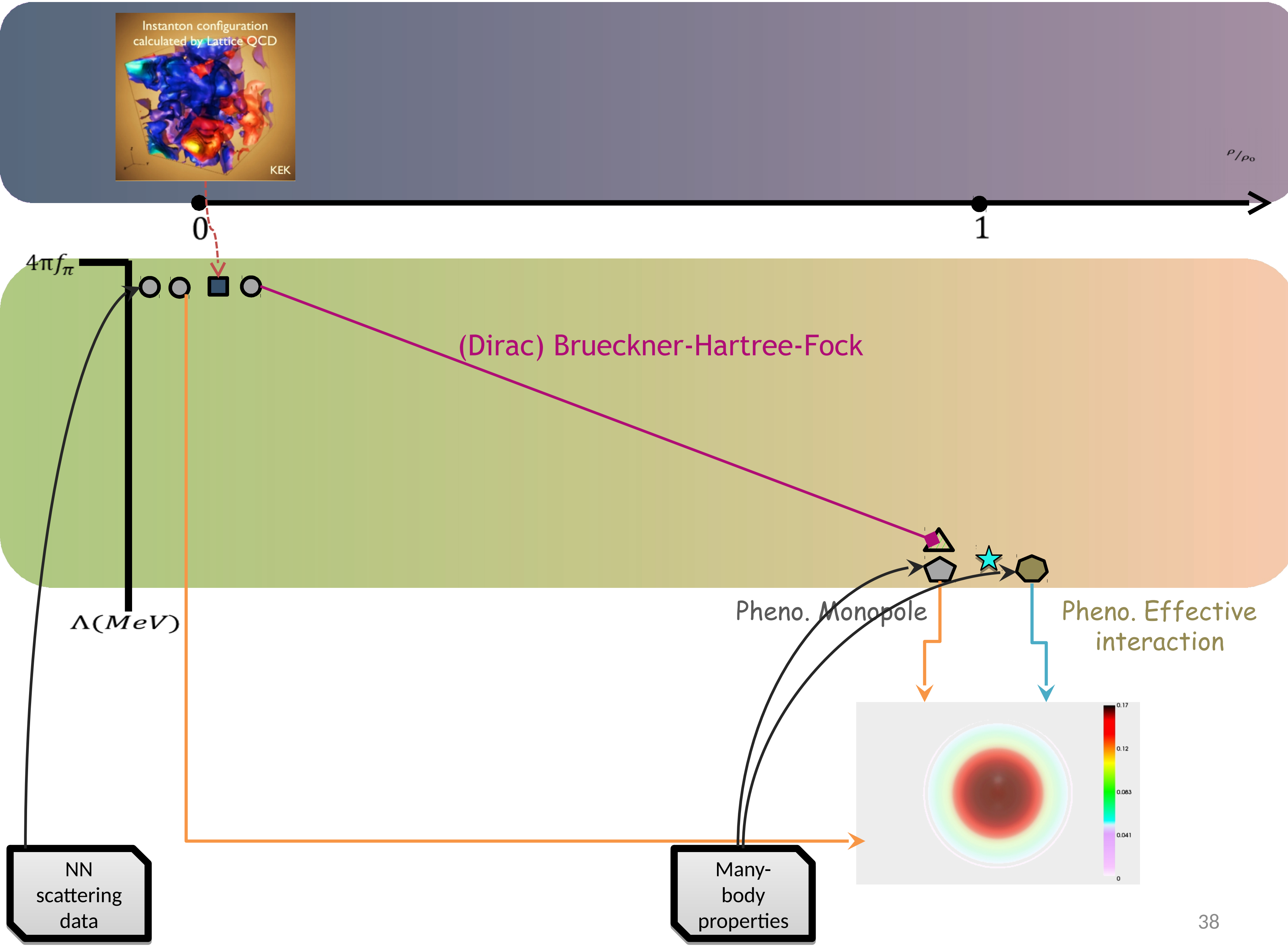
✪ Configuration Interaction Shell Model



Poves et al.

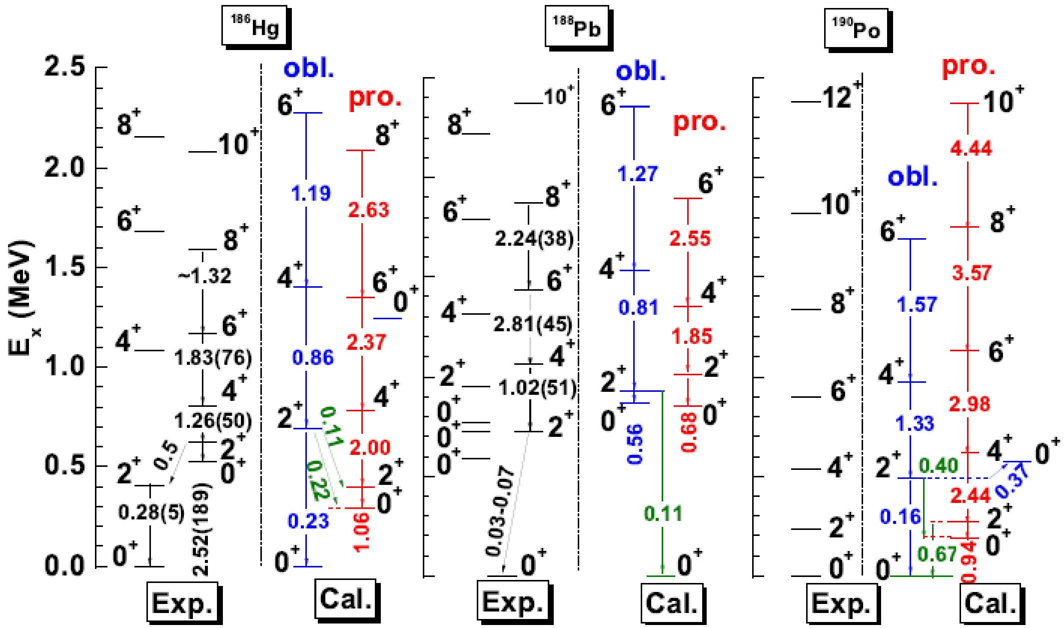


Nowacki et al.

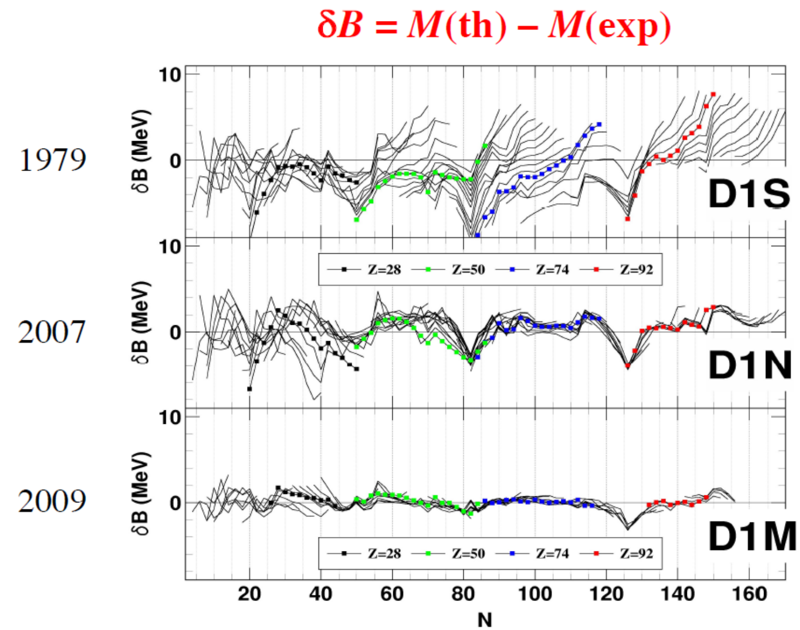


Instanton configuration calculated by Lattice QCD
KEK
ρ/ρo
0
1
4πf_π
(Dirac) Brueckner-Hartree-Fock
Λ(MeV)
Pheno. Monopole
Pheno. Effective interaction
0.17
0.12
0.083
0.041
0
NN scattering data
Many-body properties

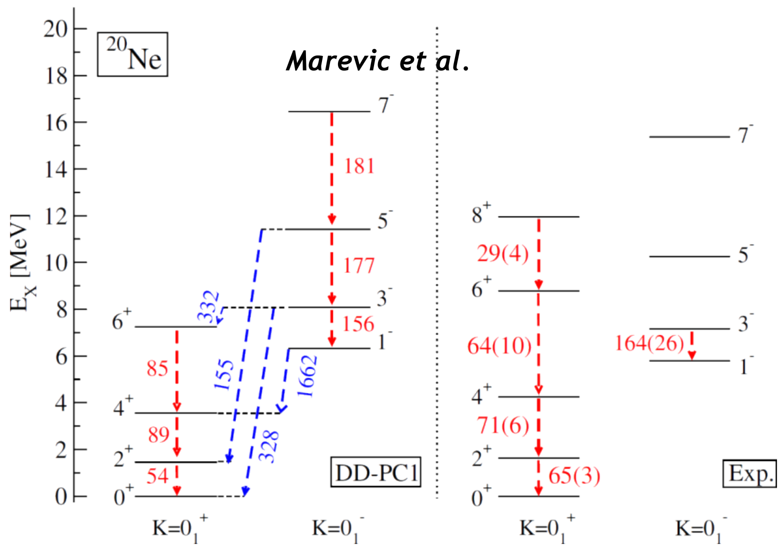
✪ Energy Density Functional

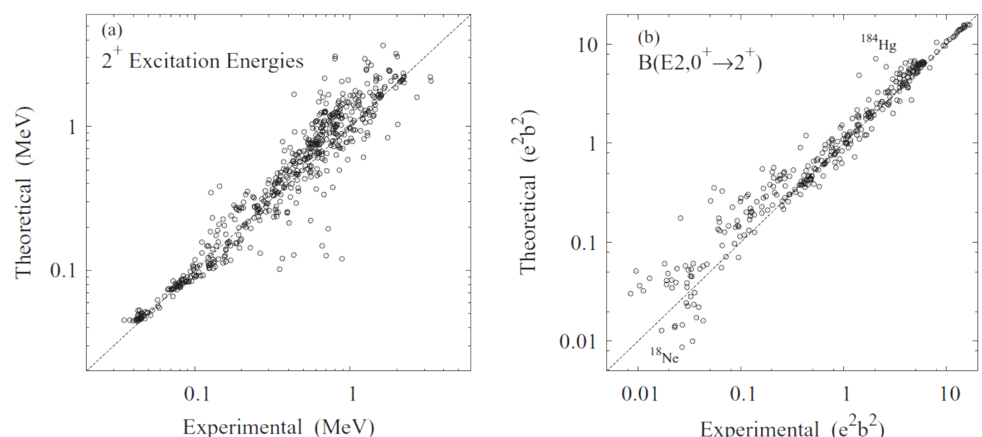


J. Yao, M. Bender, P.-H. Heenen, PRC 87 (2013) 034322



Hilaire & Goriely



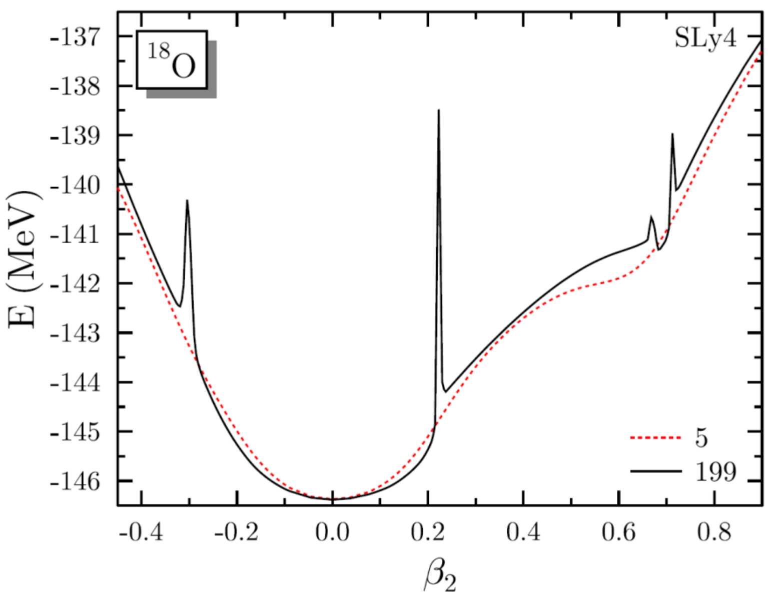


Delaroche et al.

✪ Energy Density Functional and density dependence

➲ Importance of density dependent terms in EDFs for quantitative description of both volume & surface properties : very deep origin

➲ But source of Pauli Violating Contribution that spoils EDF results



Bender, Duguet, Lacroix, …

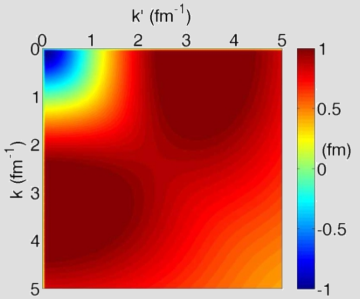
➲ Need for a new generation of nuclear EDFs !

➲ EDF ≡ DFT ? Not so simple …

✪ Taming down the UV non-perturbativeness

Interaction

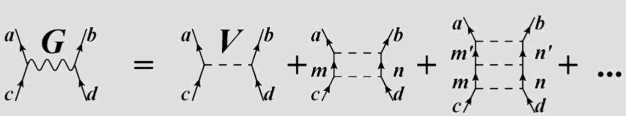
There's a hard core
Let's cope with it !



Vertical expansion

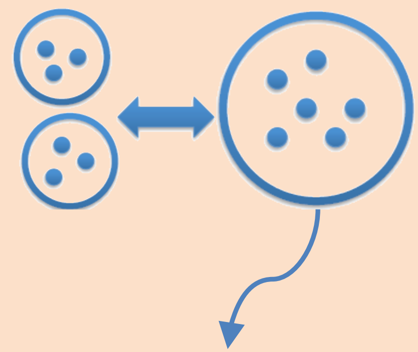
➲ Nonperturbative many-body treatment that scales awfully with the nucleon number

➲ Brueckner resummation of ladder diagrams (no high/low - momentum decoupling though)



➲ Phenomenological effective in-medium interaction

There's no hard core
Just missing dofs !

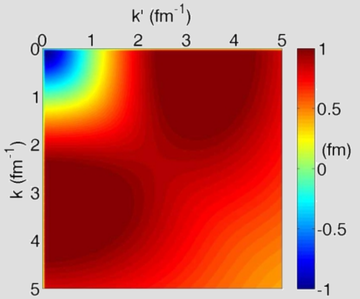


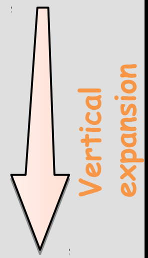
Feshbach state (6-quark cluster) whose energy/form factor comes from NN phase-shift

➲ Composite Bose-Fermi system

✪ Taming down the UV non-perturbativeness

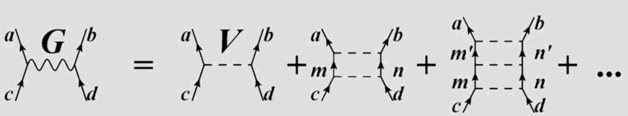
There's a hard core
Let's cope with it !





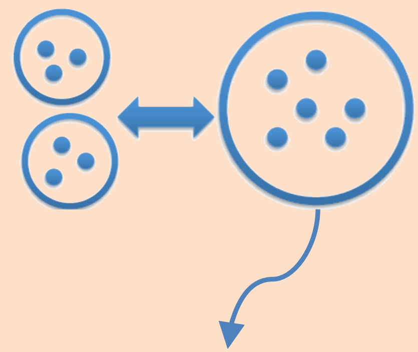
➲ Nonperturbative many-body treatment that scales awfully with the nucleon number

➲ Brueckner resummation of ladder diagrams (no high/low - momentum decoupling though)



➲ Phenomenological effective in-medium interaction

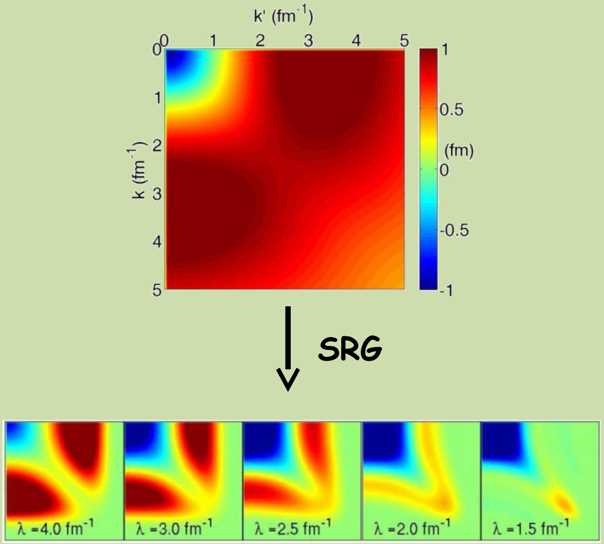
There's no hard core
Just missing dofs !



Feshbach state (6-quark cluster) whose energy/form factor comes from NN phase-shift

➲ Composite Bose-Fermi system

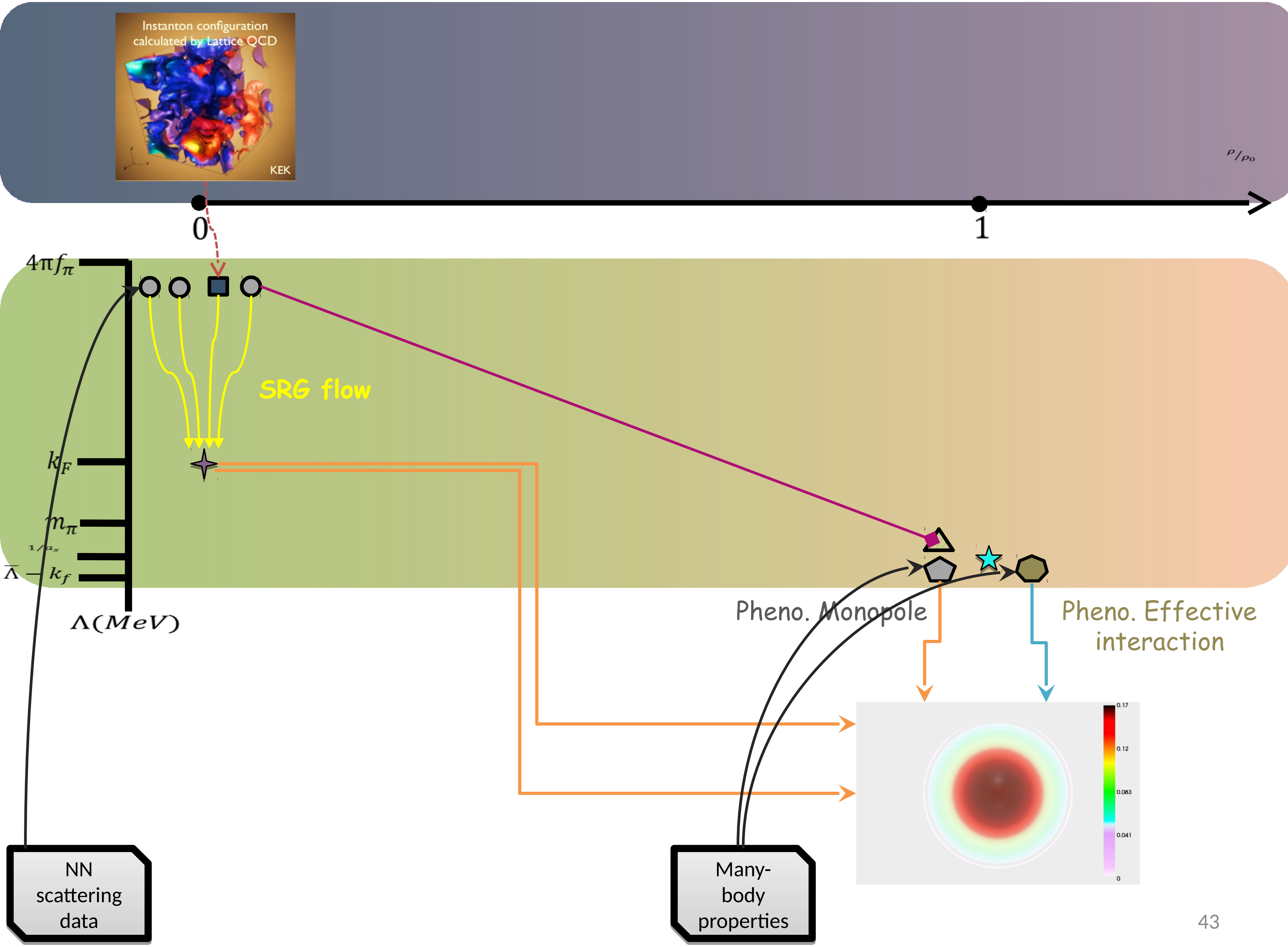
Hard/soft core ...
It's not an observable anyway !

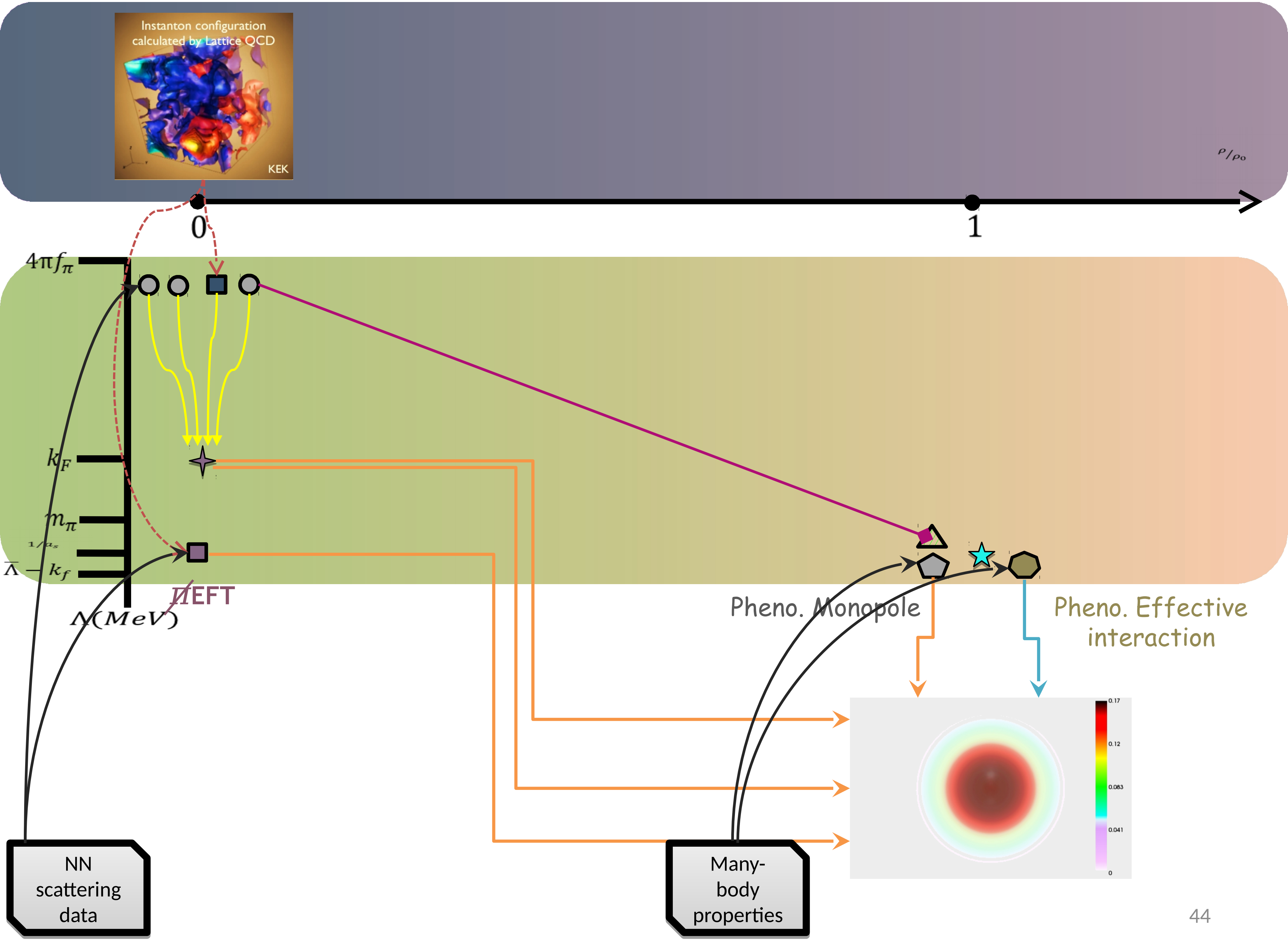


Bogner et al.

➲ Decoupling of high-low momentum modes

➲ Induced xN interations (x>2)

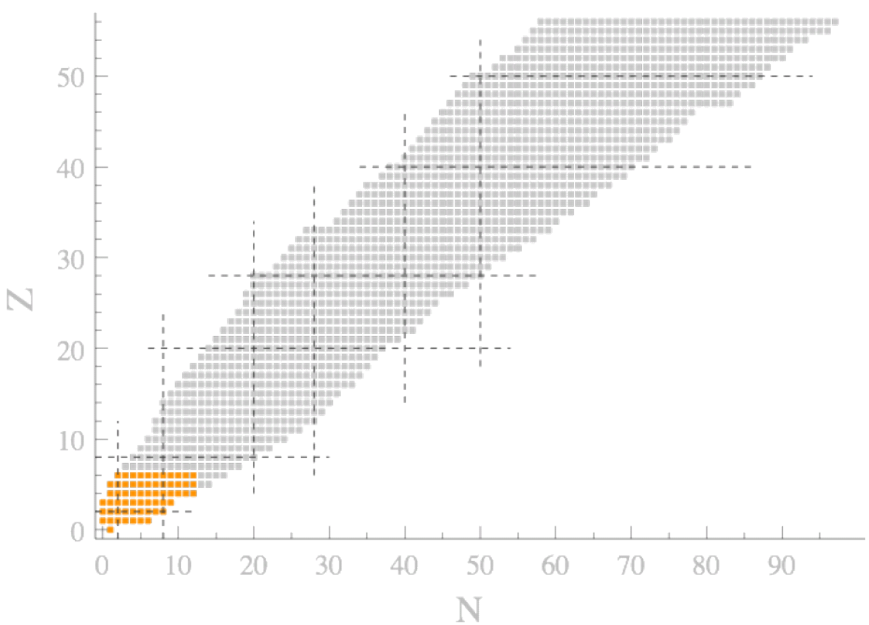

Instanton configuration calculated by Lattice QCD
KEK
ρ/ρ_0
0
1
$4\pi f_\pi$
SRG flow
k_F
m_π
$1/a_s$
$\bar{\Lambda} - k_f$
$\Lambda(MeV)$
Pheno. Monopole
Pheno. Effective interaction
0.17
0.12
0.083
0.041
0
NN scattering data
Many-body properties



Instanton configuration calculated by Lattice QCD
KEK
ρ/ρ₀
0
1
4πf_π
k_F
m_π
1/a_s
Λ̄ − k_f
Λ(MeV)
EFT
Pheno. Monopole
Pheno. Effective interaction
0.17
0.12
0.083
0.041
0
NN scattering data
Many-body properties

✪ Evolution of the ab initio reach

➲ 'Exact' solution (FY, NCSM, GFMC)

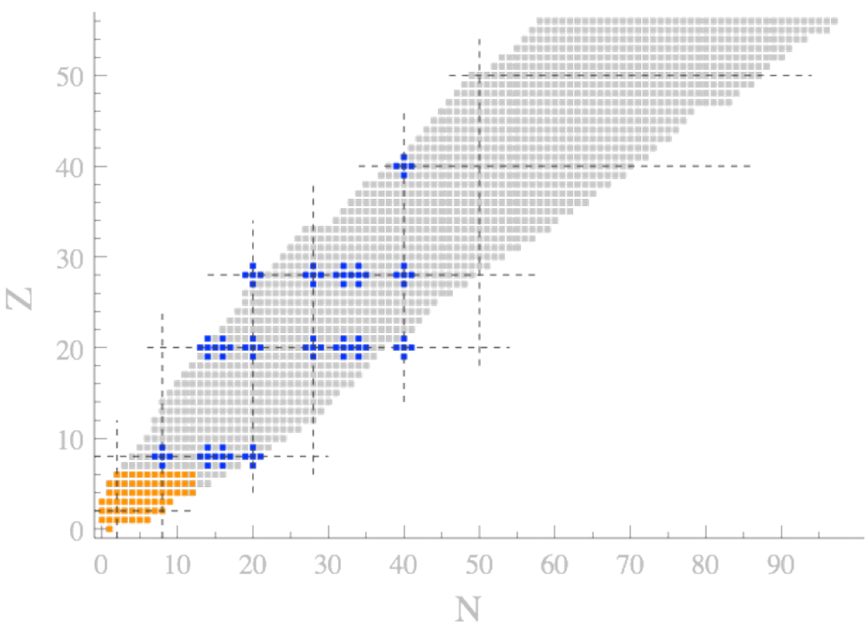


Courtesy of V. Soma, T. Duguet

✪ Evolution of the ab initio reach

➲ ‘Exact’ solution (FY, NCSM, GFMC)

➲ 2005-now: Closed-shell nuclei (A<100) : MBPT, CC, SCGF, IMSRG, …



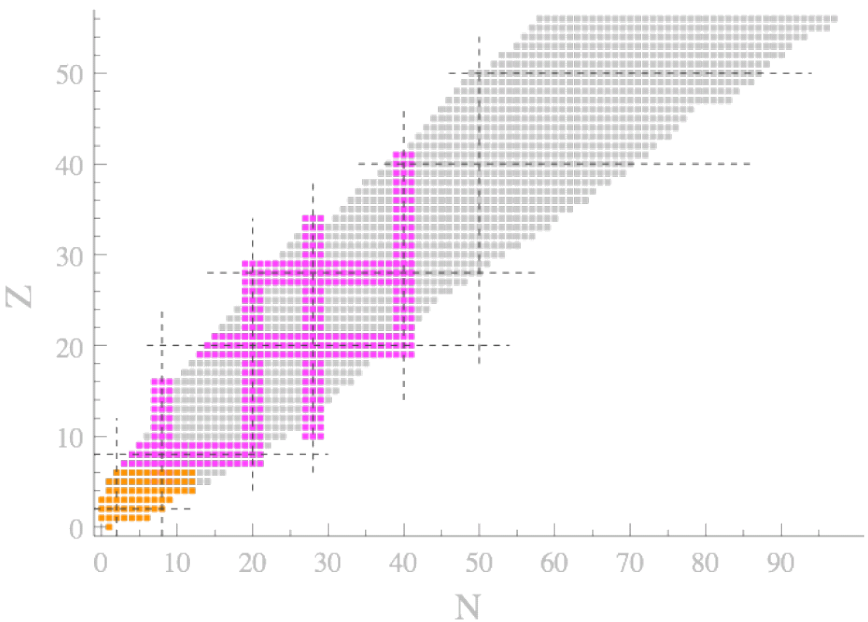
Courtesy of V. Soma, T. Duguet

✪ Evolution of the ab initio reach

➲ 'Exact' solution (FY, NCSM, GFMC)

➲ 2005-now: Closed-shell nuclei (A<100) : MBPT, CC, SCGF, IMSRG, …

➲ 2011-now: Open-shell nuclei (A<100) : multi-reference or symmetry-breaking (MR-IMSRG, IM-NCSM,MCPT / GGF, BCC, BMBPT, …)



Courtesy of V. Soma, T. Duguet

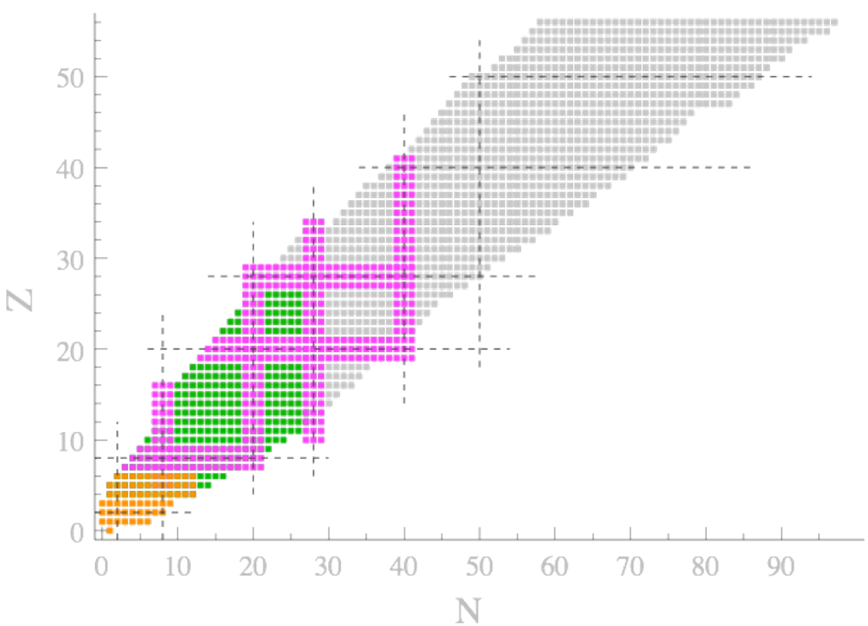
✪ Evolution of the ab initio reach

➲ 'Exact' solution (FY, NCSM, GFMC)

➲ 2005-now: Closed-shell nuclei (A<100) : MBPT, CC, SCGF, IMSRG, ...

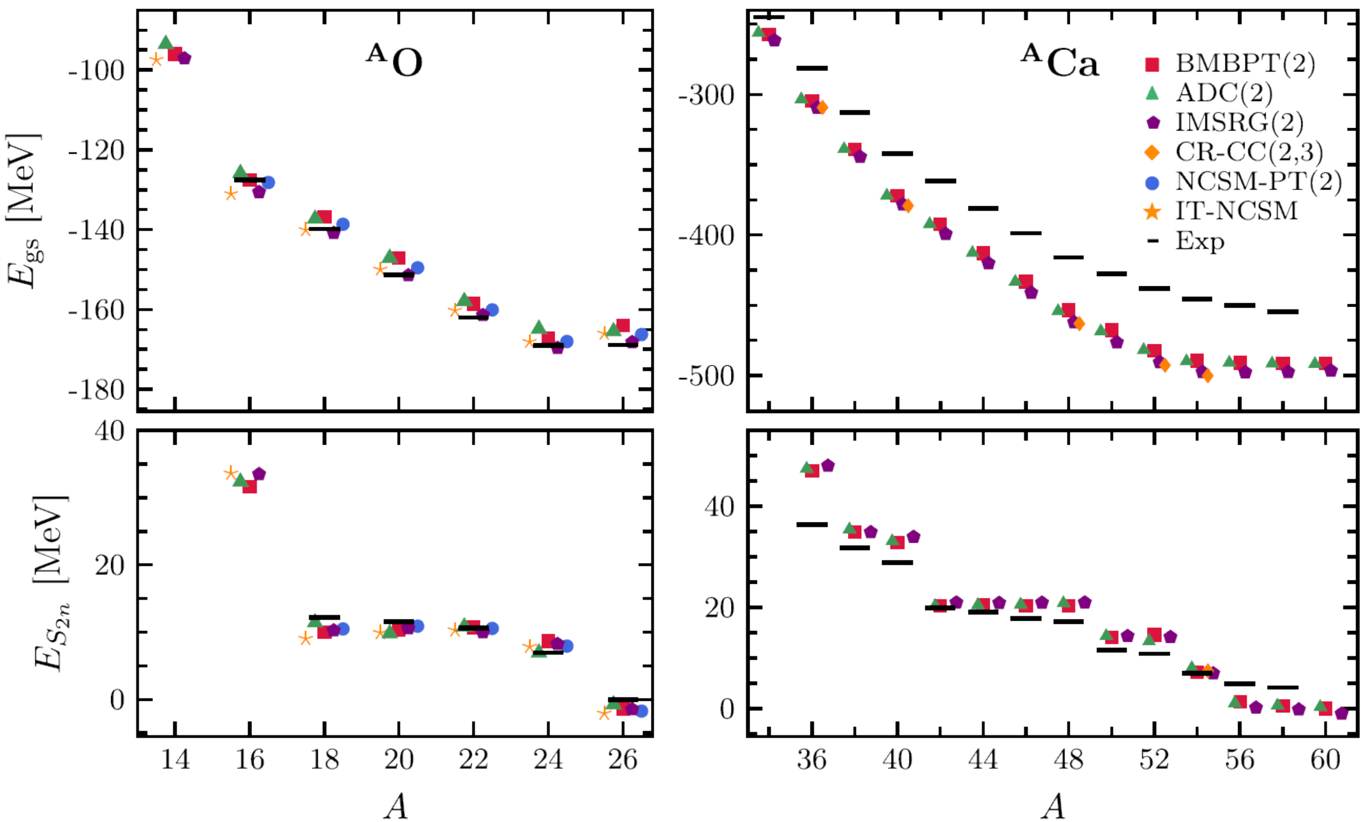
➲ 2011-now: Open-shell nuclei (A<80) : multi-reference or symmetry-breaking (MR-IMSRG, IM-NCSM,MCPT / GGF, BCC, BMBPT, ...)

➲ 2014-now: Ab initio Shell-Model Open-shell (valence-space methods VS-IMSRG, ...)



Courtesy of V. Soma, T. Duguet

✪ Ab initio approaches



Chiral NN+3N Hamiltonian

$\alpha = 0.08\ \text{fm}^4$

1820 basis functions

canonical HFB reference

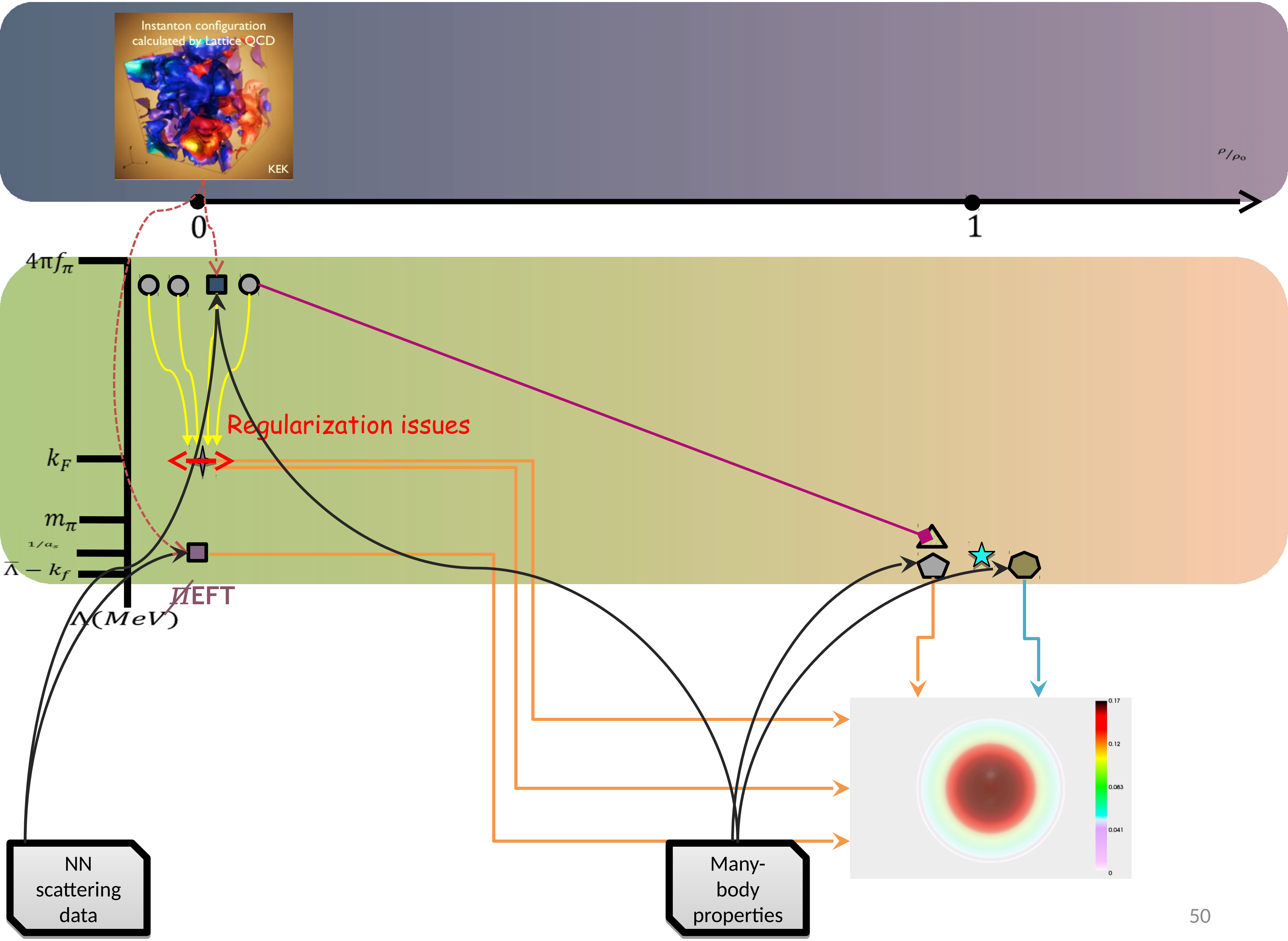
Runtime	
NCSM:	20.000 hours
MCPT:	2.000 hours
IMSRG:	1.500 hours
ADC:	400 hours
BMBPT:	**1 hour**

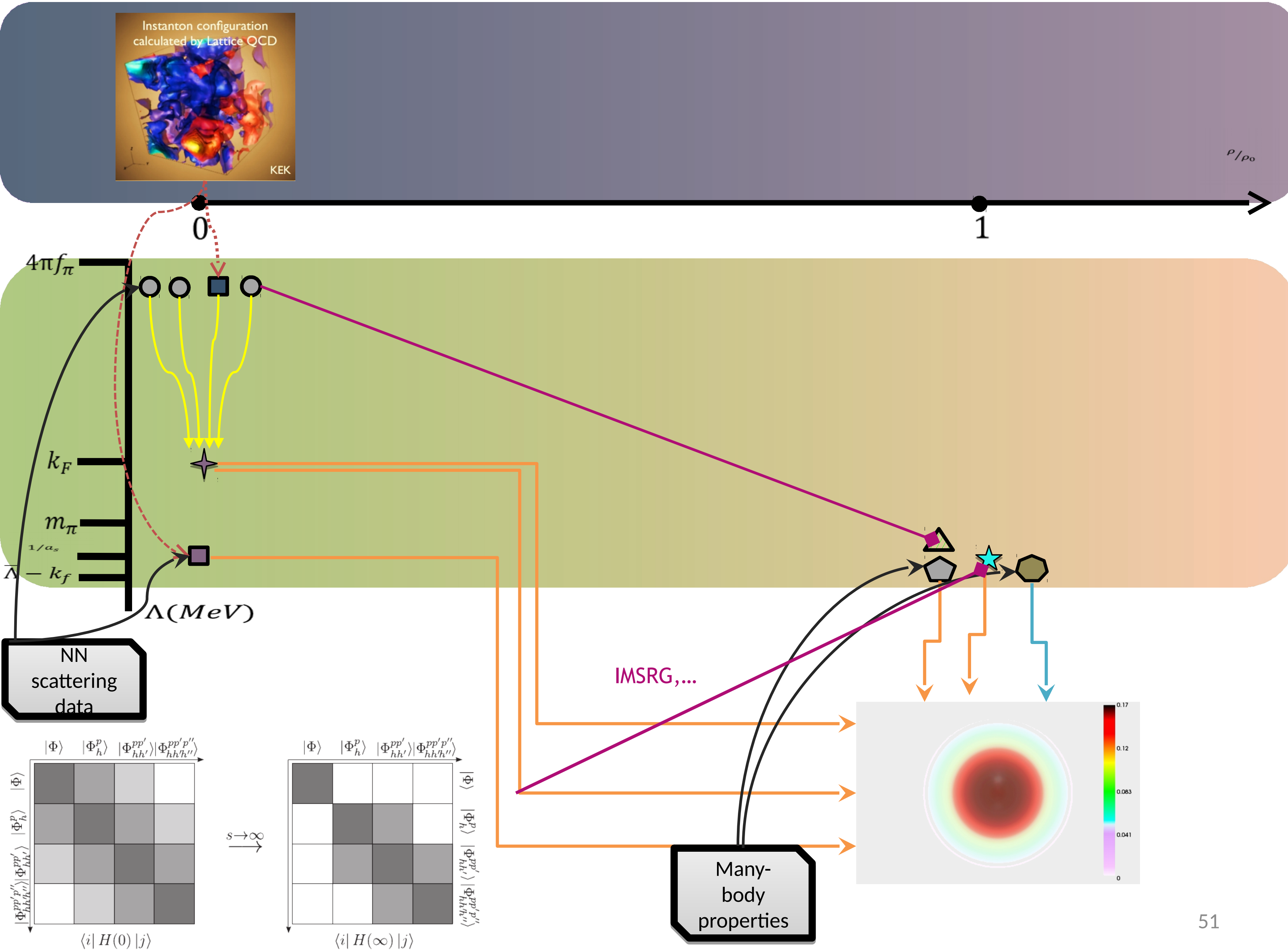
Tichai et al.

➲ Consistency of various ab initio schemes fed by the same input

➲ Many-body method more accurate than the input Hamiltonian

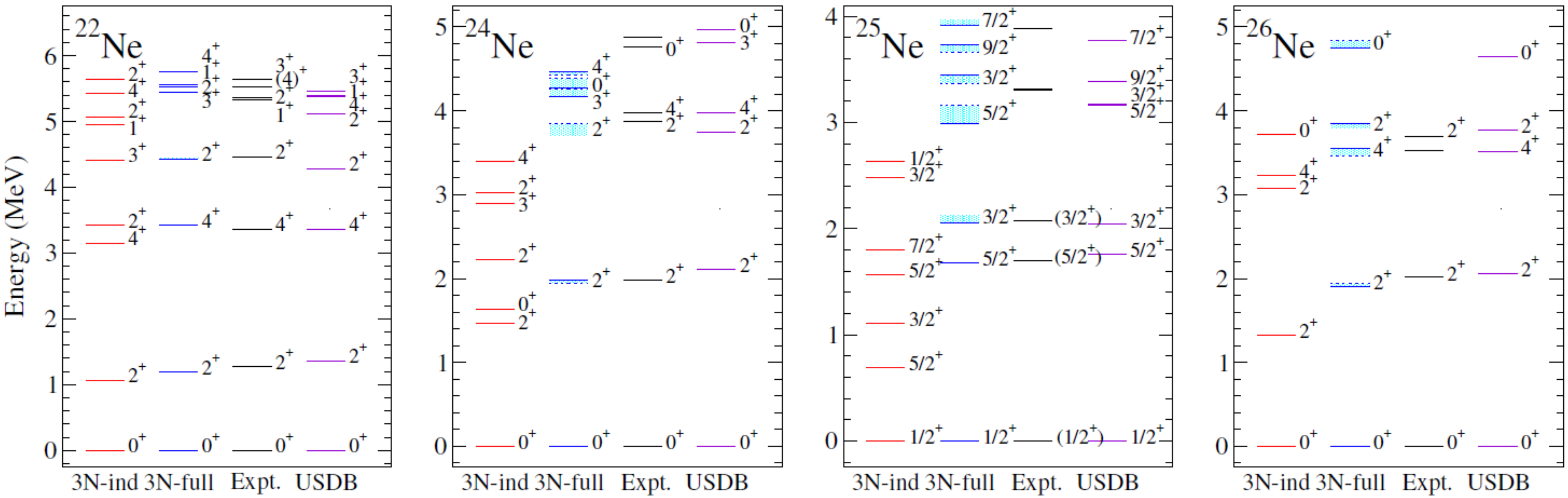
➲ Use also A-body prop. for the fit ($NNLO_{sat}$)


Instanton configuration calculated by Lattice QCD
KEK
ρ/ρ_0
0
1
$4\pi f_\pi$
k_F
m_π
$1/a_s$
$\bar{\Lambda} - k_f$
$\Lambda(MeV)$
Regularization issues
$\not\pi$EFT
0.17
0.12
0.083
0.041
0
NN scattering data
Many-body properties

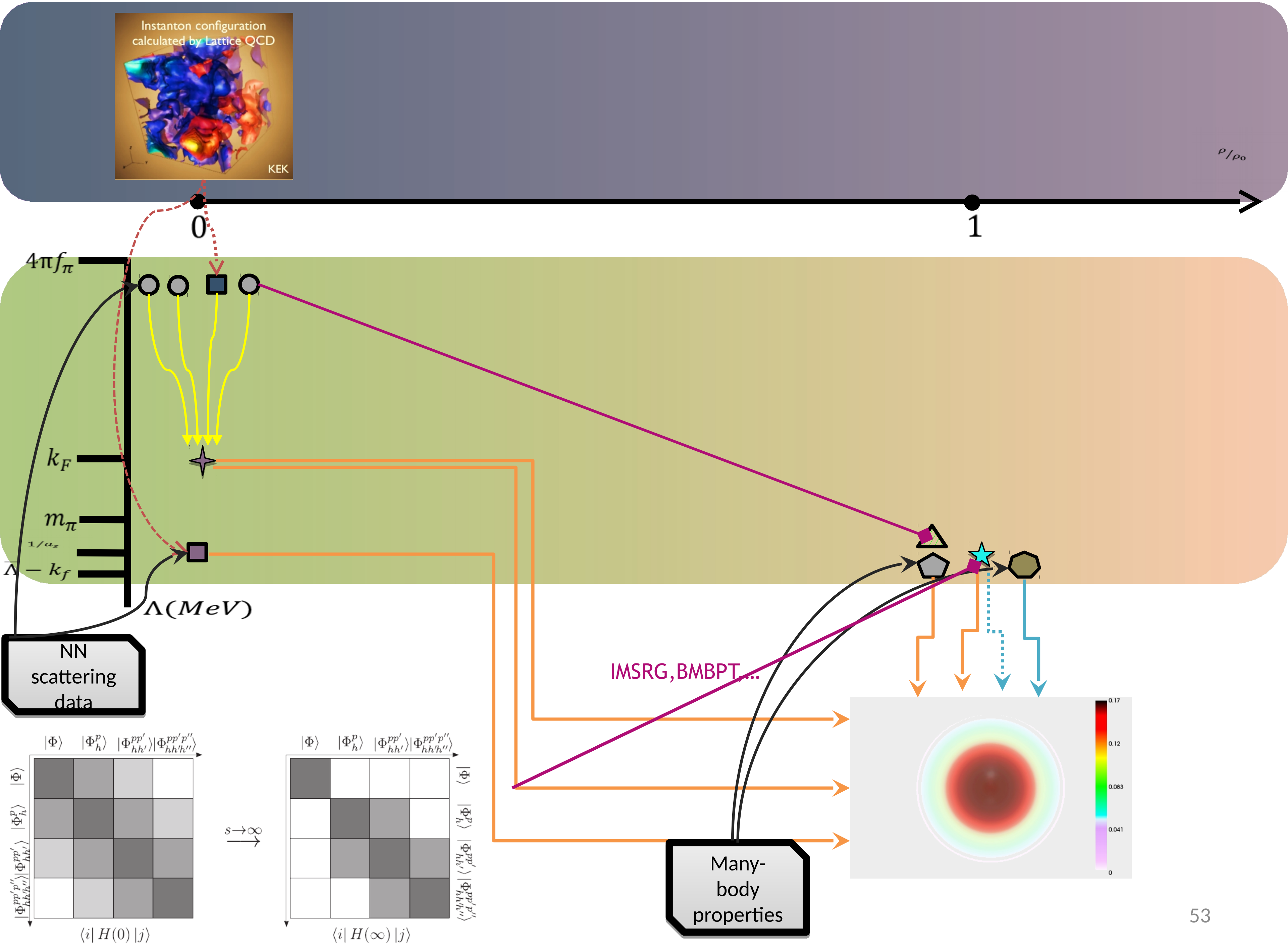


Instanton configuration calculated by Lattice QCD
KEK
ρ/ρ_0
0
1
$4\pi f_\pi$
k_F
m_π
$1/a_s$
$\bar{\Lambda} - k_f$
$\Lambda(MeV)$
NN scattering data
IMSRG,...
Many-body properties
$|\Phi\rangle$ $|\Phi_h^p\rangle$ $|\Phi_{hh'}^{pp'}\rangle$ $|\Phi_{hh'h''}^{pp'p''}\rangle$
$s\to\infty$
$\langle i|\,H(0)\,|j\rangle$
$\langle i|\,H(\infty)\,|j\rangle$
0.17
0.12
0.083
0.041
0

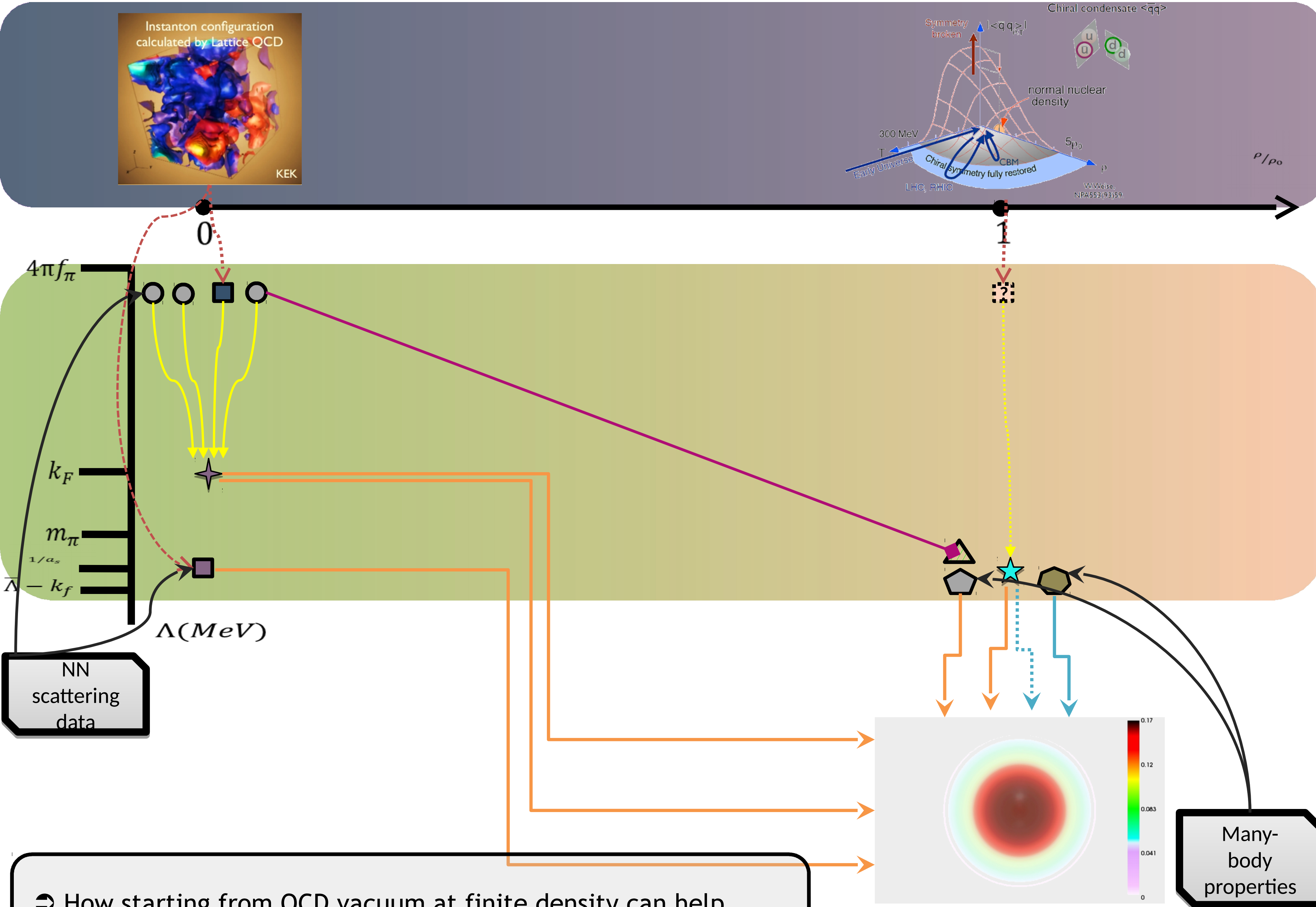
✪ Ab initio Shell Model



Stroberg et al.



Instanton configuration calculated by Lattice QCD
KEK
ρ/ρ_0
0
1
$4\pi f_\pi$
k_F
m_π
$1/a_s$
$\overline{\Lambda} - k_f$
$\Lambda(MeV)$
NN scattering data
IMSRG,BMBPT,...
Many-body properties
$\langle i|\,H(0)\,|j\rangle$
$\langle i|\,H(\infty)\,|j\rangle$
$s\to\infty$



➲ How starting from QCD vacuum at finite density can help treating the nuclear many-body problem ?

Conclusions

- Foundations of microscopic approaches to the nuclear many-body problem : link with QCD

- Inter-Nucleon Interactions : chiral EFT
- Ab initio methods : huge progress. Limitations from the input
- Shell Model : connection with chiral potentials ok
- EDF : How to be as good without density dependences ?

  Link with QCD ?

- Many topics not covered here, e.g. nuclear lattice EFT, scattering & reaction processes, …

✪ Modern status of symmetries

➲ Global vs. Local symmetries : Good criterion is to compute the associated Noether current and check whether the corresponding Noether charge is trivial or not.

➲ Global exact symmetries : incompatible with the fundamental theory of Nature at high energy (yet to be formulated). Rather, emerge as accidental symmetries at low energies

- *Approximate symmetries because explicitly violated by higher order operators (irrelevant in the RG sense)*
- *Non-conservation of L_e-L_μ and L_μ-L_τ, approximate nature of parity & time-reversal symmetries,…*

➲ Gauge symmetries : misnomer. Not symmetries in the first place, and therefore cannot be broken. Rather, property of the description of the system (redundancy that proves very convenient)

- *Exact emergent symmetries useful to make the theory explicitly Lorentz invariant, local, unitary and therefore causal*
- *Also appear in condensed matter physics and nuclear physics*