Turbulence & Auto-organisation dans des Plasmas de Fusion

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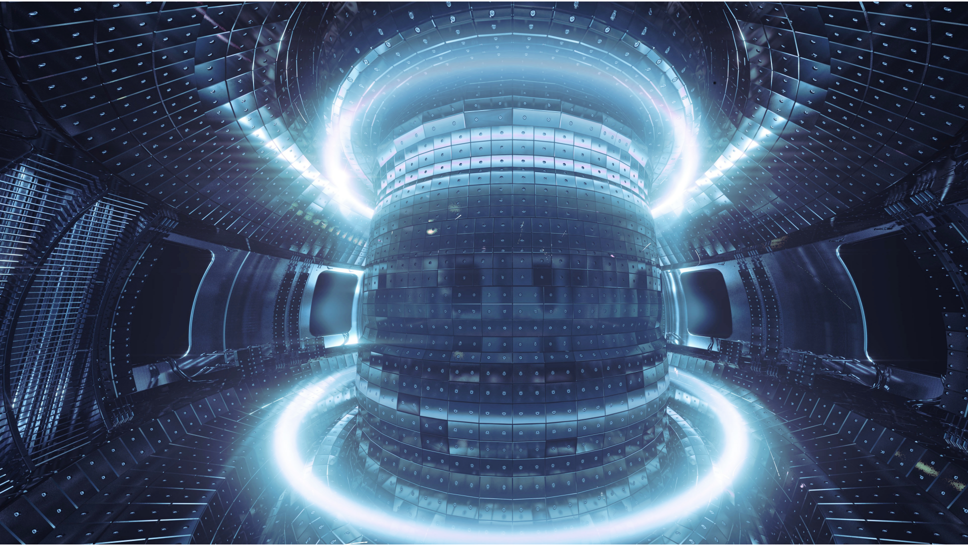
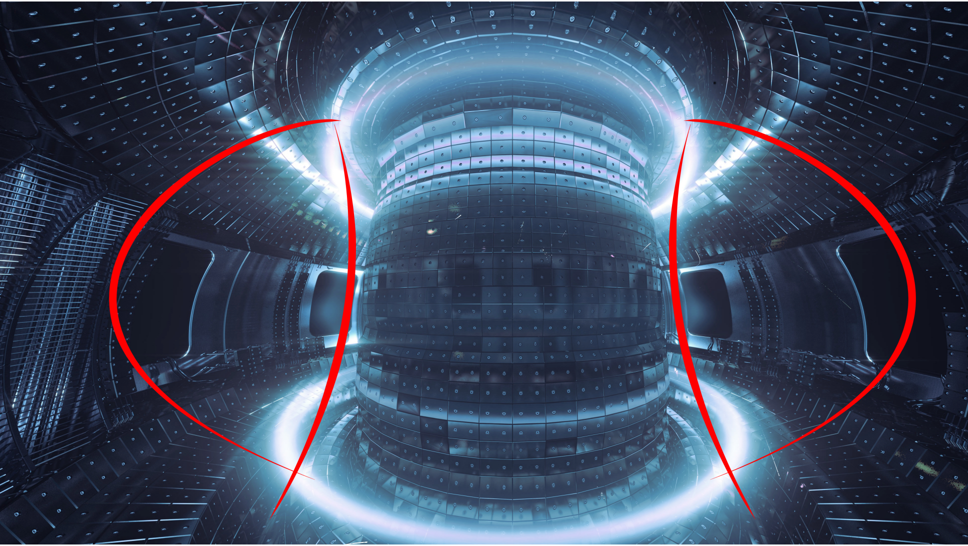




Outline

- Fusion plasmas: out-of-equilibrium by necessity
- Taming the plasma:

  from outside: **B** field;

  from within: turbulence is problem & (partial) solution ➞ transp. barriers
- beautiful lab. for out-of-equil. phenomena ➞ lots to learn from cross-talk

Ackn.: Festival de Théorie, Aix-en-Provence

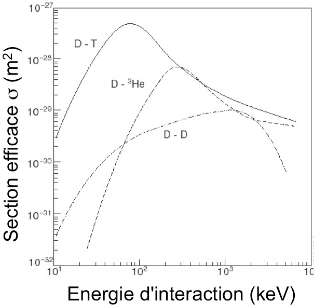
➞ since 2001: 10^{th} anniversary ; $\sim$ 100 participants
plasmas / neutral fluids / GFD / astrophysics / complex systems

A harsh environment, out-of-equilibrium by necessity

$$D + T \longrightarrow {}^4He^{[3.5\ MeV]} + n^{[14.1\ MeV]}$$

$+$ *in situ* tritium generation:

$$\left[\begin{array}{lll} n + {}^6Li & \longrightarrow & {}^4He^{[2.1\ MeV]} + T^{[2.7\ MeV]} \\ n + {}^7Li & \longrightarrow & {}^4He + T + n - 2.5\ MeV \end{array}\right]$$



- requirement: interaction distance $\sim 10^{-13}$ m
  - → threshold reaction: $\langle \sigma v \rangle_{\text{D-T}} = f(T)$
  - → $E_{th} \sim 10 - 20\,\text{keV} \leftrightarrow T \sim 100\,10^6\,\text{K}$
  - → ionised medium ≡ **magnetised plasma**, sensitive to electromagn. field

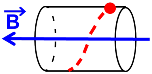
- Lawson criterion: $n_D\, T_D \tau_E = 6\,10^{21}\,\text{m}^{-3}\,\text{keV s}$ →
  - ❶ MHD stability: $\beta = \frac{\sum nT}{B^2/2\mu_0} \ll$ few %
  - ❷ turbulence $\tau_E = \frac{a^2}{D_{turb}} \sim$ few sec.

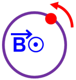
- predict β & τ_E? →
  - T is ∼fixed
  - limit on $n \sim 10^{20}\,\text{m}^{-3}$ $[\sim 10^{-5} n_{air}]$
  - $\tau_E \equiv$ need to understand turbulence & control it

Wager: "build one of the best insulators from one of the best conductors"

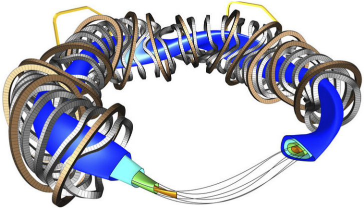
Insulator is the **B** field → taylor geometry to confine & tame transport

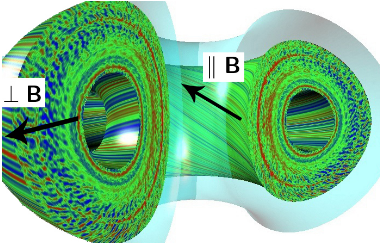
key: strong **B** field ↔ strong insulator





$\rho_\perp = v_\perp/\Omega_c \approx 10^{-2}\,\mathrm{m}$ → | **B** $\approx 5\,\mathrm{T}$
minor radius $a \approx 2\,\mathrm{m}$
$\rho_\star = \rho_\perp/a \approx 10^{-3}$





⊥ B	$\mathcal{P}_{out}$ [MW]	therm.cond. λ [W/m/K]	$\mathcal{R} = a/\lambda$ [K m^2/W]	bât.basse conso.	bât. énergie positive	$\mathcal{R}$
current	< 20	$\sim 10^{-3}$	$250 - 1000$	> 8	> 10	combles
ITER	~ 100	$< 5\,10^{-3}$	> 400	> 4	> 5	mur
reactor	~ 300	$< 10^{-2}$	> 300	> 4	> 5	sol

∥ B $\frac{\lambda_\parallel}{\lambda_\perp} \sim 10^8$ → intrinsic anisotropy → | fluid particle ↔ **B** flux surface analogy $\sim 2D$ turbulence

2D turb.: deep formal analogies between GFD & magn. fusion → robust saturation mechanism: shear flows [jets / zonal flows]

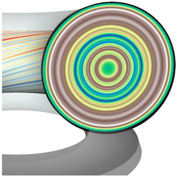
- Charney equation:

$$\left(\frac{\partial}{\partial t} + u_g \cdot \nabla\right)\Big[\underbrace{h - \nabla^2 h + \ln \tfrac{H_0}{f}}_{\text{planetary PV}}\Big] = 0$$

- Hasegawa–Mima eq.:

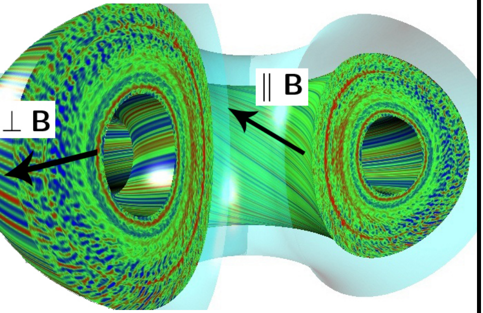
$$\left(\frac{\partial}{\partial t} + u_{\mathbf{E}\times\mathbf{B}} \cdot \nabla\right)\Big[\underbrace{\phi - \nabla^2 \phi + \ln \tfrac{n_0}{\omega_{c,i}}}_{\text{plasma PV}}\Big] = 0$$

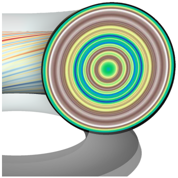
- ZF drive: $\partial_t \langle v_{Ey} \rangle = \langle v_{Ex}\,(\mathrm{PV}) \rangle$
- [Taylor 1915] $\underbrace{\langle v_{Ex}\,(\mathrm{PV}) \rangle}_{\text{flux of PV}} = \underbrace{-\partial_x \langle v_{Ex}\, v_{Ey} \rangle}_{\text{Reynolds force}}$



Plasma	Planetary
Lorentz: $\omega_c \mathbf{z} \times \mathbf{v}$ cyclo. freq. ω_c	Coriolis: $f \mathbf{z} \times \mathbf{v}$ $f = f_0 + \beta y$
dens. $n = n_0 + \tilde{n}$ QN: $\frac{\tilde{n}}{n_0} = \frac{e\phi}{T}$ elect. pot. ϕ	$H = H_0 + h$ SW: $\frac{h}{H_0} = o(1)$ depth h or Ψ
B field	planet rot. $\mathbf{\Omega}$
$\rho_\star = \rho_i / a$	$\mathcal{R}o = U / f_0 L$
H–M	Charney
drift wave	Rossby
2D $\perp$ **B**	2D $\perp$ $(f\mathbf{z})$
adv. **E** $\times$ **B** $\mathbf{v}_{\mathbf{E}\times\mathbf{B}} = -\frac{\nabla\phi\times\mathbf{z}}{\lvert\mathbf{B}\rvert}$	geostrophic vel. $\mathbf{v}_g = -\frac{g\nabla h\times\mathbf{z}}{f_0}$
PV [f or $J_0 f$]	PV

2D turb.: deep formal analogies between GFD & magn. fusion
→ robust saturation mechanism: shear flows [jets / zonal flows]





Zonal flows (jets) are vastly studied:

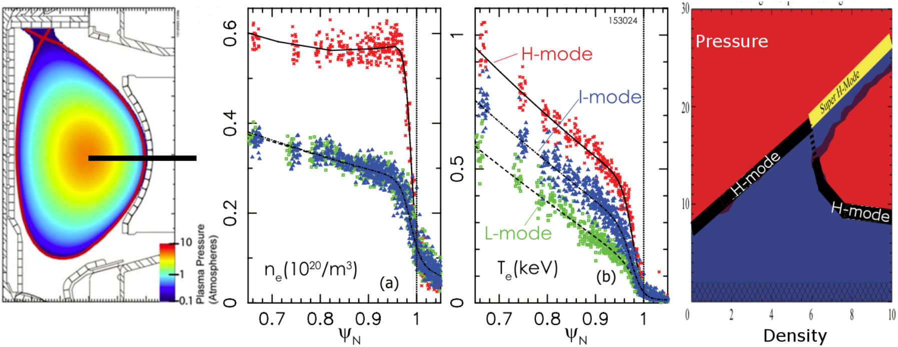
[eg. Diamond PPCF 05]

- robust generation via eg. **modul. instability**
- decorrelate eddies → **reduce transport**
- many modes produce shearing + straining + transport
  ↳ ZF more effective & drive no transport
  → **safe repository** for plasma free energy
- modes of **minimal inertia**
  → easy to excite ZFs
- modes of **minimal damping**
  → once excited, live on

deep analogies ↔ common patterns

ZFs: a seed for transport bifurcations

Transport bifurcations occur: universally measured, moderately understood, not reproduced in modeling



- fast / dithering–cyclic / slow: few μs to fractions of τ_E
- narrow region of change, driven or influenced not locally, impact global
- transport barriers may differently affect heat, momentum or particles
- periodic relaxations & hysteresis
- multiple stability & 'valleys of stability'

How to model such phenomena?

Fluid or kinetic: how to model magnetised plasmas?

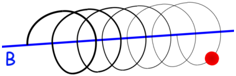
Fluid description appealing. . . yet often incorrect: **3d+time**

$$\rho \mathrm{d}_t \mathbf{v} = -\nabla p + \mathbf{j} \times \mathbf{B} \quad ; \quad \partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) \quad ; \quad + \text{ eqs. } \rho \;\&\; p$$

- Fluid description: $\omega \gg \mathbf{k} \cdot \mathbf{v}$, $\delta_{\text{orbit}} \ll \delta_{\text{mode}}$ fails when wave–particle resonances, finite extension particle orbits
- Especially critical for low freq. phenomena & small scales
  ↳ **turbulence:** dense resonances $\omega \sim \mathbf{k} \cdot \mathbf{v}$ & $\delta_{\text{orbit}} \sim \delta_{\text{mode}} \sim 1\,\text{cm}$

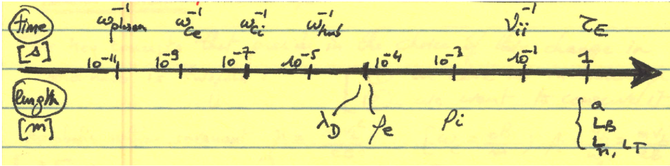
Kinetic description $\partial_t F + \nabla \cdot (\mathbf{v} F) = \mathcal{C}(F) + \mathcal{S}(F)$ + Maxwell **6d+time**

$\exists$ small parameters $\equiv$ focus on **low-freq.** electromagnetic fluctuations



reduction possible

- strong **B** field [~~cyclotron motion~~]
- effective separation of scales



Modern workhorse: 'gyro'-kinetics

- $\partial_t F + \nabla \cdot (\mathbf{v}F) = \mathcal{C}(F) + \mathcal{S}(F) \quad +$ Maxwell

key idea: eliminate high-freq. processes $\omega > \omega_c$

4 gyrocentre coord. + magn. moment

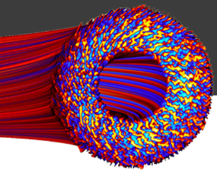
6d $\left\{ \begin{array}{l} \bullet\ \omega_{p,s}\,\Delta t < 1 \\ \bullet\ \Delta x < \lambda_{Ds} \end{array} \right.$ $\Rightarrow$ 5d $\left\{ \begin{array}{l} \bullet\ \omega^{\star}_{c,s}\,\Delta t < 1 \\ \bullet\ \Delta x < \rho_s \end{array} \right.$

- $\partial_t \mathbb{F} + \dot{\mathbf{x}}\,\partial_{\mathbf{x}}\mathbb{F} + \dot{v}_{||}\,\partial_{v_{||}}\mathbb{F} + \underbrace{\dot{\mu}}_{=0}\,\partial_\mu \mathbb{F} = \mathcal{C}(\mathbb{F}) + \mathcal{S}(\mathbb{F}) \quad +$ Maxwell [pol.; magn.]

Magnetised plasma:

- large reservoirs of free energy | $\nabla T,\ \nabla n \sim 4$ decades, $\nabla I \sim 3$ decades → | externally driven & 'weakly' open
- multiple scales | $\{$ions, electrons$\} \sim \{3, 5\}$ decades; time ~ 5 decades
- $\sim$2d turb. with important(?) kinetic features | Landau resonance ↔ dissipation; damping of zonal flows; mom.exchange: collisions, heating

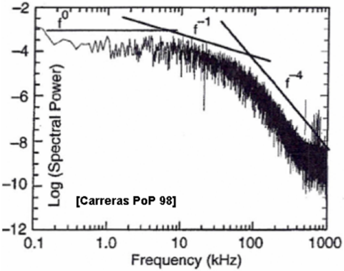
How does the plasma organise? transport?

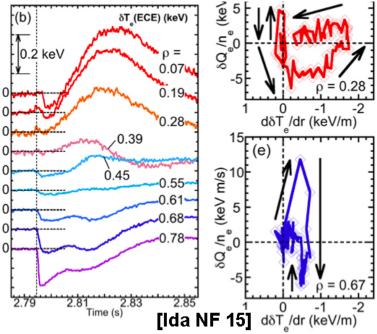


Validity of local transport closure: dubious

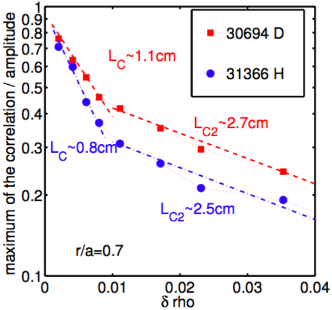
➤ first idea: ∃ local flux–gradient relation

- diffusive closure fails:
  ↳ hysteresis, fronts, not-local mixing, etc.





[Ida NF 15]



Emergence of a non-local & non-diffusive mesoscale

$Q = -n\chi(r)\nabla T$ ➠ $Q = -\int \mathcal{K}(r,r')\nabla T(r')\,\mathrm{d}r'$

➤ $\mathcal{K}(r,r') = \frac{S}{\pi}\frac{\Delta/2}{(\Delta/2)^2+(r-r')^2}$

- scale-invariant dynamics

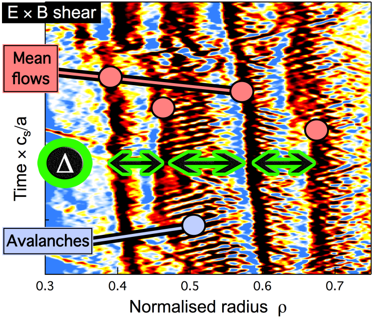
NB: $\int (r-r')^2 \mathcal{K}(r,r')$ *divergent*

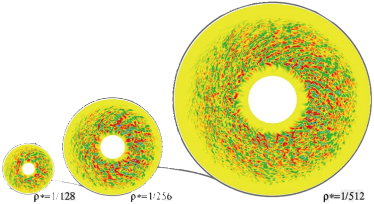
➤ physics of Δ: many aspects of SOC [is it?]

- $\Delta \equiv$ 'contamination' **spreading** length
- stiff, multiple metastable states,

organisation in spatially-separated domains, strong intermittency, behavioural change with source

↳ $\Delta \equiv$ <u>'avalanche'</u> scale $\sim \rho$, NOT $\sim a$





Two interconnected secondary structures: ZMF & 'avalanches' → '$\mathbf{E} \times \mathbf{B}$ staircase' solves the pattern coexistence problem

The conundrum: robust secondary patterning trends. . . yet mutually-exclusive

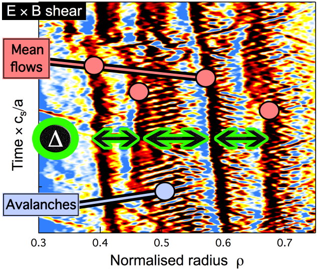
- avalanches correlated over-turnings ➠ extended, intermittent transport
- zonal flows limit the turbulent mixing, regulates transport via shearing

How can they coexist?

❶ they do not both exist in different regions of parameter space

❷ separation in time both exist alternately

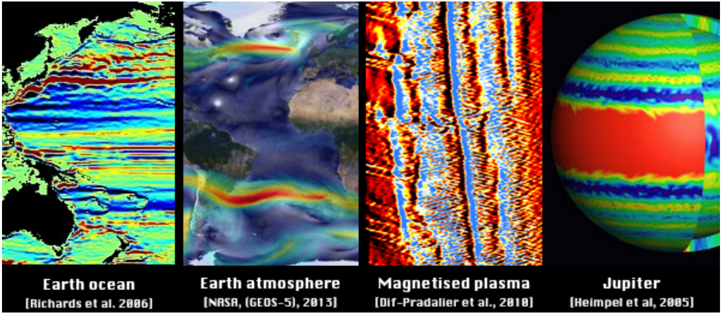
❸ separation in space ➠ the **"E × B staircase"**



the E×B staircase

- ZFs concentrate into thin layers & endure ➠ mean flows (MFs)
- MFs organise into a lattice of quasi-regularly spaced layers
- avalanches propagate in-between

Spontaneous large-scale layering in quasi-2d: a pervasive problem in fluids, understanding still elusive



Oceans: thermohaline staircase, differential diffusion of competing heat & salt gradients leads to alternately sharp/flat gradient zones [**stratification**]

Atmospheric fluids: potential vorticity staircase, ZF structure from Rossby wave dynamics in QG turb. ["veins & arteries of the weather system"] [$\boldsymbol{\Omega}$];

Magnetised plasma: $\mathbf{E} \times \mathbf{B}$ staircase, quasi-regular ZF pattern of weak transport barriers co-located with prof. corrugations interspersed by avalanching [$\mathbf{B}$]

Analogy with traffic jams: heat avalanche dynamics may generate staircase-like patterns

- traffic flow $\equiv$ continuum (fluid) [Whitham 74]

$$\partial_t \rho + \partial_x(\rho u) = 0$$

$$\partial_t u + u\partial_x u + \frac{1}{\rho}\partial_x p = \frac{1}{\tau}(\bar{u} - u)$$

$1/\rho\partial_x p \equiv$ drivers' "anticipation" to traffic

$\tau \equiv$ relax. $\equiv$ drivers' "response" time

- shock-like solutions
  - traffic moves freely;
  - small instabilities amplify ➠ travelling wave arises & moves backwards;
  - jam cluster appears and propagates backward like a solitary wave

Human error is needed to **cause fluctuations in behaviour** $\leftrightarrow$ "response time"

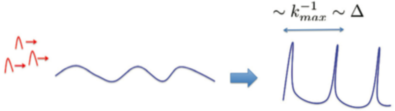
- in plasmas [Kosuga 13 & 14]

human error	pressure fluct.
time delay τ	delay $\delta T \leftrightarrow \delta Q$
traffic flow	heat avalanche
"jamiton"	"staircase"

- time delay btw *flux* $\leftrightarrow$ *gradient*
- modified Burgers ➞ telegraph

$$\tau\partial_t^2\delta T + \partial_t\delta T + \lambda\delta T\partial_x\delta T = \nu\partial_x^2\delta T$$

- clustering instability



Bi-stability may also generate staircase-like patterns [1/2]

- start from H–W eqs [Ashourvan PRE16, PoP 17]

$$d_t \nabla^2 \phi = \eta \nabla_{\parallel}^2 [\log n - \phi] + \mu \nabla_{\perp}^4 \phi$$

$$d_t \log n = \eta \nabla_{\parallel}^2 [\log n - \phi] + D \nabla_{\perp}^2 n$$

- expand vorticity, density & PV$\equiv n - u$ in terms of mean/fluct.

$$\partial_t \bar{\zeta} + \partial_x \underbrace{\langle \tilde{v}_x \tilde{\zeta} \rangle}_{\text{flux}} = \text{dissip.} + (\text{ext.production})$$

- key: mixing of PV ➞ $\zeta = PV = q$
  - assume diffusive closure $\langle \tilde{v}_x \tilde{q} \rangle \equiv -D \partial_x \bar{q}$
  - dimensional args: $D \sim \ell^2 \, \tilde{q}^2 / \alpha_{\parallel}$

- what prescription for ℓ ?
  ∃ at least 2 scales in model:

❶ length scale $\ell_F \equiv$ constant forcing for turb.

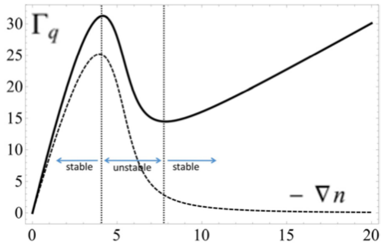
❷ Rhines scale $\ell_R \equiv$ cross-over from:

- eddy-like $\tau_{ov}^{-1} \sim \tilde{v}/\ell_R \sim \tilde{q}$
- wave-like $\omega_{DW} \sim \frac{k}{1+k^2} v_D \sim \frac{k}{1+k^2} \nabla \bar{n} \sim \ell_R \nabla \bar{q}$
- equating both: $\ell_R \sim \frac{\tilde{q}}{\nabla \bar{q}}$

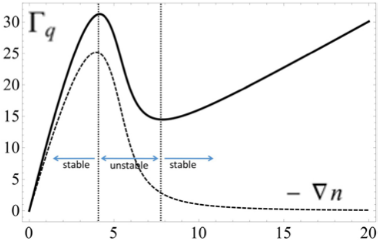
- 'combined' mixing length

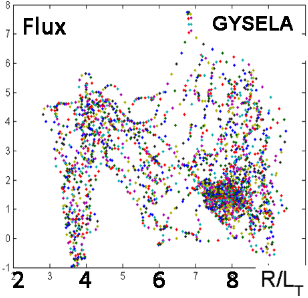
$$\ell \equiv \frac{\ell_F^{1-k}}{\left(\ell_F^{-2} + \ell_R^{-2}\right)^{k/2}}, \quad \text{hence flux:}$$

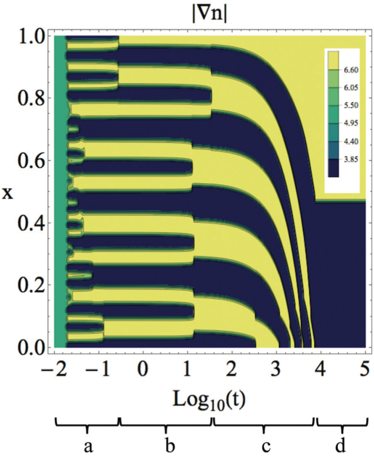
$$\Gamma_q \sim \frac{1}{[1 + \nabla \bar{q}^2 / \tilde{q}^2]^{k/2}} \nabla \bar{q}$$



Bi-stability may also generate staircase-like patterns [2/2]







- stability of shear layers? (log. time scale!)
- Competition mergers/re-formation?

→ what evidence from experiments?

Conclusions

- Large sources of free energy, flux-driven, multiscale, open ➞ out-of-equil. system
- Turbulence is ∼2d, with a kinetic character
  ↳ resonances & dissip., collisions, heating, . . .

Propensity
- ■ concentrate energy & momentum ➞ barriers for transp., bifurcations
  ↳ ubiquitous in 2d ; act $\neq$ heat, momentum or particles
- ■ intermittently 'discharge' free energy contents ➞ 'avalanches'
  ↳ emergent dynamic mesoscale

➞ $\mathbf{E} \times \mathbf{B}$ staircase solves pattern coexistence pb ➞ gal understanding still elusive

- <u>unifying</u> theme(?) patterns of transp. barriers ➞ cross-talk [astro/GFD/fluids]
  ↳ 'natural upshot' of modulation in bistable/multi-stable system
  ↳ mixing scale dependence on gradients leads to multiple/bi stability