Emergence of Flow in Relativistic Heavy Ion Collisions

François Gelis

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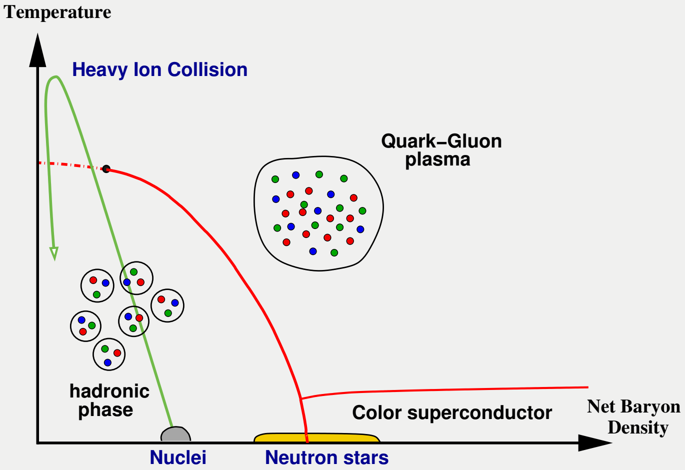
Institut de
Physique Théorique
CEA/DRF Saclay

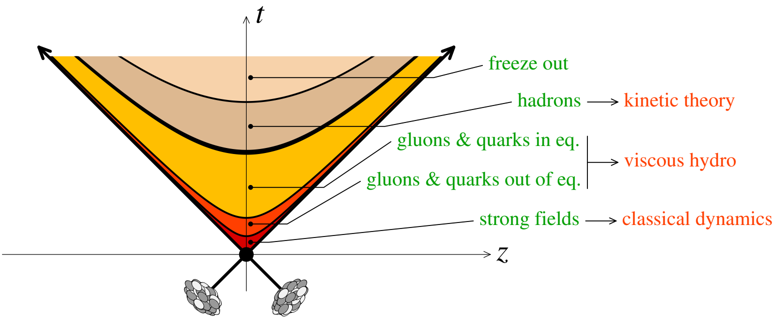
Heavy Ion Collisions

Heavy ion collisions



Experimental facilities : RHIC and LHC

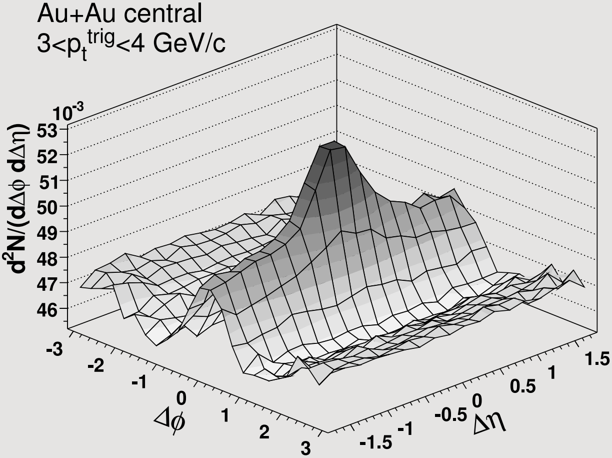
STAGES OF A NUCLEUS-NUCLEUS COLLISION



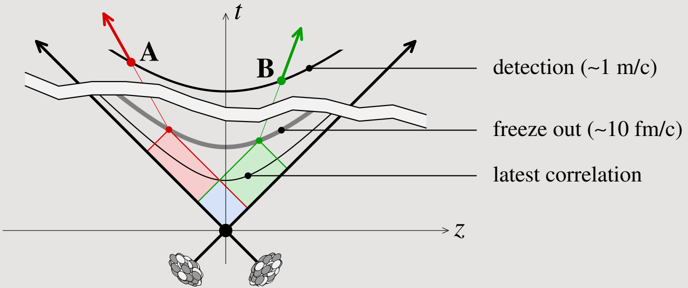
- Hydrodynamics successful at describing the bulk evolution
- In this talk : **Pre-hydrodynamical evolution**

Evidence for hydrodynamical expansion (one out of many...)

Two-particle correlations

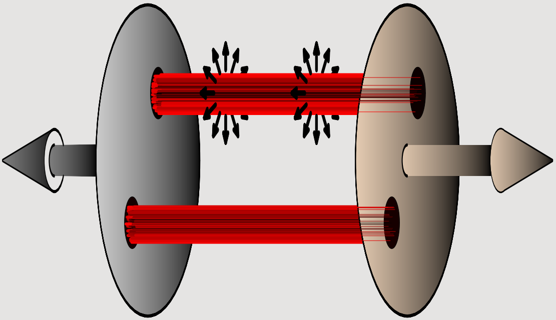


Early origin of correlations in rapidity



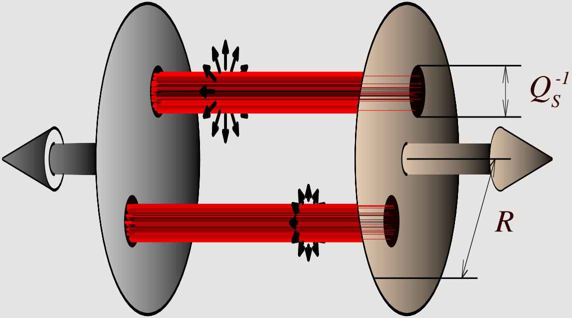
- $t_{\text{correlation}} \lesssim t_{\text{freeze out}} \times e^{-\frac{1}{2}|\Delta\eta|}$
- Correlations in azimuthal anle may be produced much later

Interpretation in terms of flow



- Post-collision color fields organized in “flux tubes”
- Coherent production in a each tube, Isotropic on average

Interpretation in terms of flow



- Post-collision color fields organized in “flux tubes”
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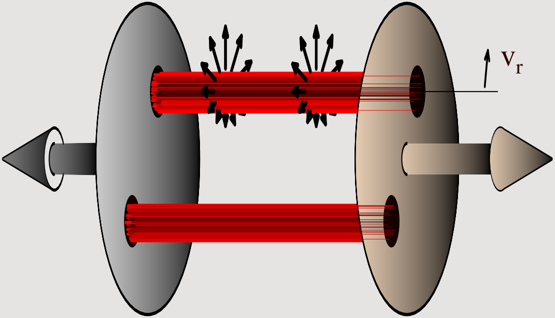
Interpretation in terms of flow



- Post-collision color fields organized in “flux tubes”
- Coherent production in a each tube, Isotropic on average
- No correlations between different tubes
- Radial collective expansion $\Rightarrow$ angular collimation

Equations of hydrodynamics (conservation laws)

$$\partial_\mu T^{\mu\nu} = 0 \qquad , \qquad \partial_\mu J^\mu_{\rm B} = 0$$

Assumptions and inputs

i. Near equilibrium form of $T^{\mu\nu}$:

$$T^{\mu\nu} = \underbrace{(p+\epsilon)\,u^\mu\,u^\nu - p\,g^{\mu\nu}}_{\text{ideal hydro}} \oplus \underbrace{(\eta,\zeta)\partial u}_{\text{viscous terms}} \oplus \cdots$$

ii. Equation of State: $p = f(\epsilon)$

iii. Transport coefficients: $\eta, \zeta, \cdots$

iv. Initial condition

Equations of hydrodynamics (conservation laws)

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu J^\mu = 0$$

A

What makes it work so well?

- Pressure tensor not too far from isotropy
- System not too far from equilibrium
- Low viscosity (compared to entropy density)

Not easy to get in Quantum ChromoDynamics...

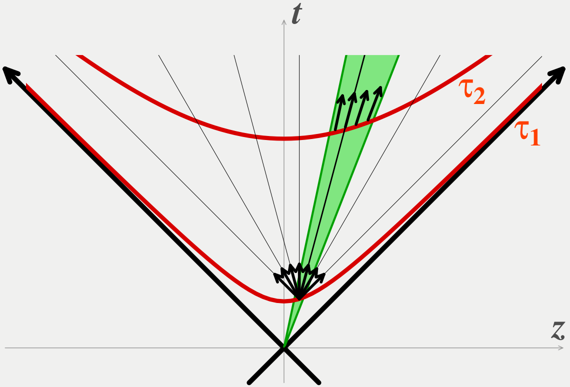
ii. Equation of State: $p = f(\epsilon)$

iii. Transport coefficients: $\eta, \zeta, \cdots$

iv. Initial condition

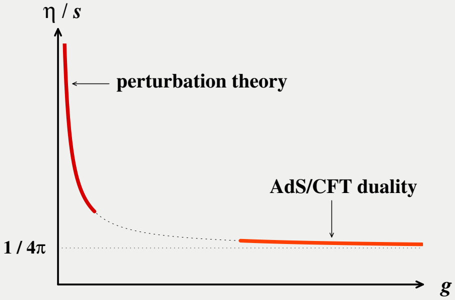
COMPETITION BETWEEN EXPANSION AND INTERACTIONS

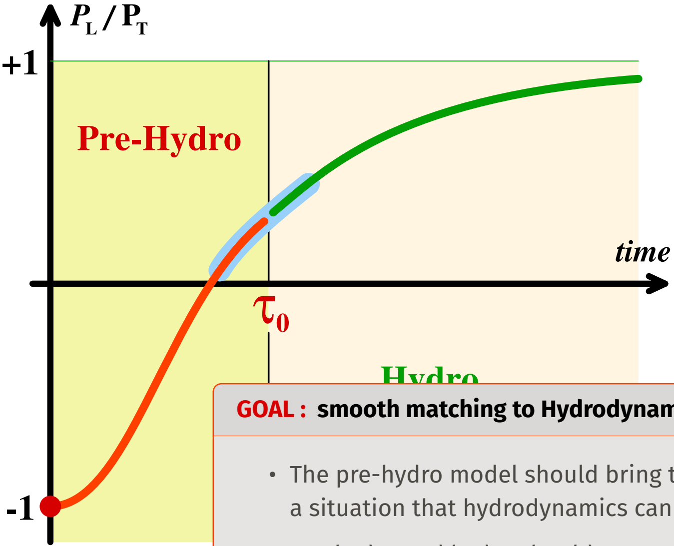
- Very different from isotropization in a box
- Sustained interactions are needed for isotropy to persist despite the expansion



Weak coupling result [Arnold, Moore, Yaffe (2000)]

$$\frac{\eta}{s} \approx \frac{5.1}{g^4 \ln\left(\frac{2.4}{g}\right)}$$

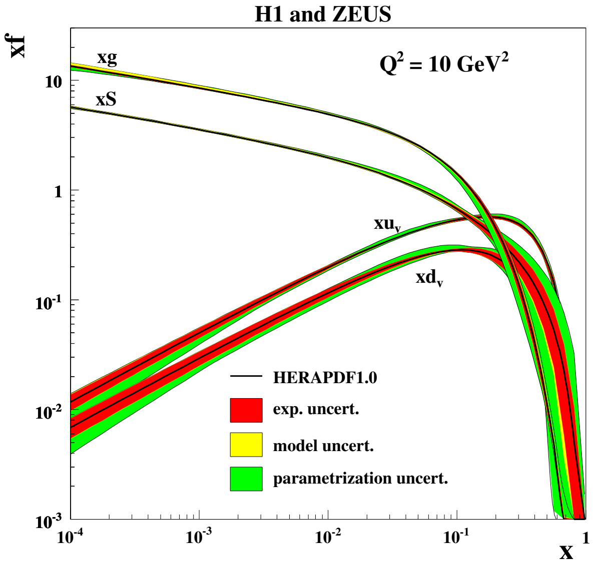




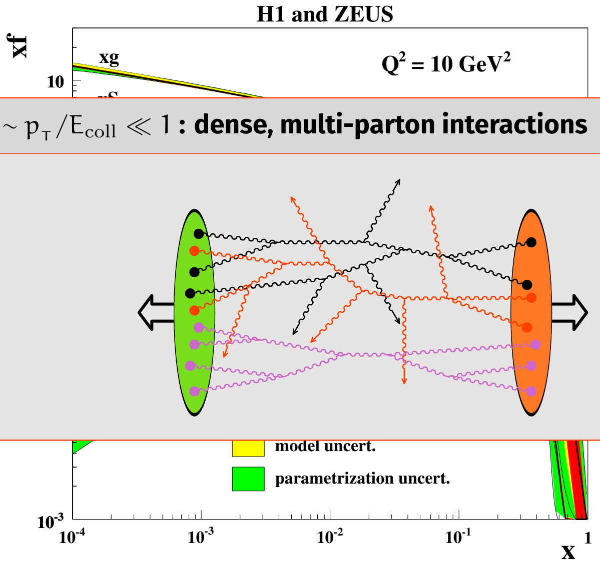
GOAL : smooth matching to Hydrodynamics

- The pre-hydro model should bring the system to a situation that hydrodynamics can handle
- Pre-hydro and hydro should agree over some range of time $\Rightarrow$ no τ_0 dependence
- Description as close as possible to QCD

QCD description

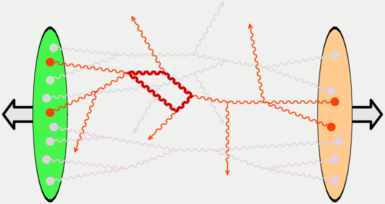

H1 and ZEUS
xf
Q² = 10 GeV²
xg
xS
xu_v
xd_v
HERAPDF1.0
exp. uncert.
model uncert.
parametrization uncert.
10
1
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻³
10⁻²
10⁻¹
1
x

PARTON DISTRIBUTIONS IN A NUCLEON: $x \sim p_T / E_{coll}$



$x \sim p_T / E_{coll} \ll 1$: dense, multi-parton interactions

$$\mathcal{S} = \underbrace{-\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}}_{\text{slow gluons}} + \underbrace{\int (J_1^\mu + J_2^\mu)}_{\text{fast sources}} A_\mu$$



In the dense regime: $J^\mu \sim g^{-1}$, $A^\mu \sim g^{-1}$, $f_k \sim g^{-2}$

$$T^{\mu\nu} \sim \frac{1}{g^2}\left[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\right]$$

$c_0 \equiv$ tree level, $c_1 \equiv$ one loop, etc...

From kinetic theory :

$$\frac{\eta}{s} \sim \frac{\text{mean free path}}{\text{de Broglie wavelength}}$$

- (**de Broglie wavelength**)$^{-1} \sim Q$
- (**mean free path**)$^{-1} \sim \underbrace{g^4 Q^{-2}}_{\text{cross section}} \times \underbrace{\int_k f_k}_{\text{density}} \underbrace{(1+f_k)}_{\text{Bose enhancement}}$

If $g \ll 1$ **but** $f_k \sim g^{-2}$ (**weakly coupled**, **but strongly interacting**)

$$\frac{\eta}{s} \sim g^0$$

Leading Order

- Leading Order = sum of all tree diagrams

  Expressible in terms of classical solutions of Yang-Mills equations :

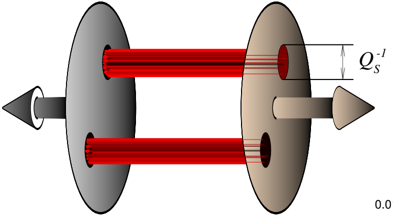
$$\left[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}\right] = J_1^\nu + J_2^\nu$$

- Initial condition : $\lim_{x^0 \to -\infty} \mathcal{A}^\mu(x) = 0$

Components of the energy-momentum tensor

$$T^{00}_{\text{LO}} = \frac{1}{2}\big[\underbrace{\mathbf{E}^2 + \mathbf{B}^2}_{\text{class. fields}}\big] \qquad T^{0i}_{\text{LO}} = \left[\mathbf{E}\times\mathbf{B}\right]^i$$

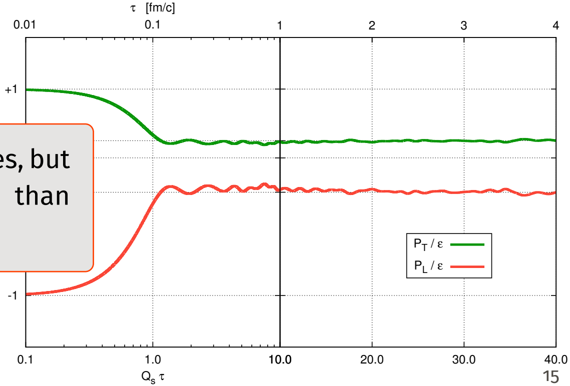
$$T^{ij}_{\text{LO}} = \frac{\delta^{ij}}{2}\left[\mathbf{E}^2 + \mathbf{B}^2\right] - \left[\mathbf{E}^i\mathbf{E}^j + \mathbf{B}^i\mathbf{B}^j\right]$$



At $\tau = 0^+$

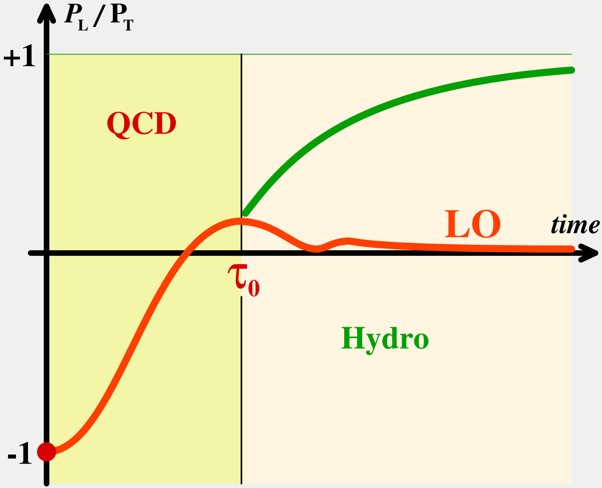
$$\mathbf{E} \parallel \mathbf{B} \parallel \hat{z}$$

$$P_T = \epsilon, P_L = -\epsilon$$

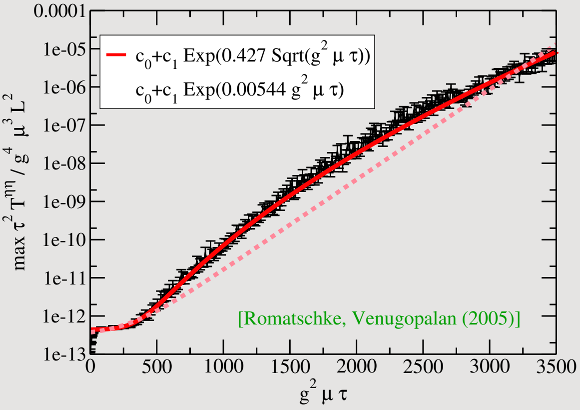


P_L rises to positive values, but remains much smaller than P_T (free streaming)

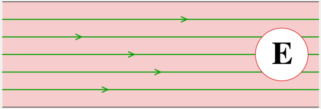
LO : UNSATISFACTORY MATCHING TO HYDRODYNAMICS



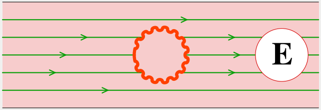
Instabilities, Resummation


0.0001
1e-05
1e-06
1e-07
1e-08
1e-09
1e-10
1e-11
1e-12
1e-13
0
500
1000
1500
2000
2500
3000
3500
c0+c1 Exp(0.427 Sqrt(g2 μ τ))
c0+c1 Exp(0.00544 g2 μ τ)
max τ2 Tηη / g4 μ3 L2
g2 μ τ
[Romatschke, Venugopalan (2005)]

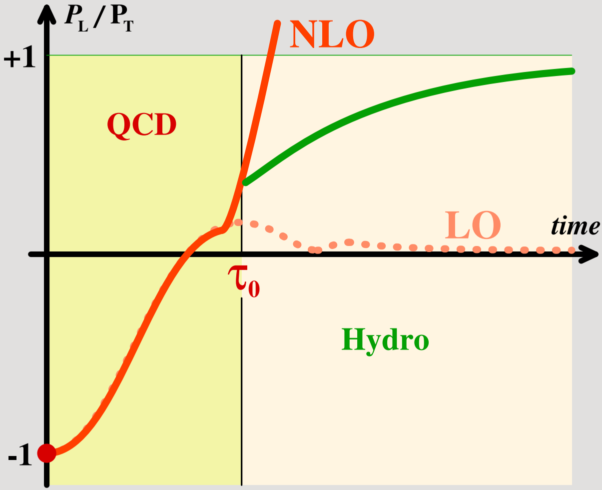
- LO = longitudinal chromo-E and chromo-B fields



- NLO = gluon loop embedded in this field



  - instability $\sim$ imaginary part of the loop $\sim$ gluon pair production
  - BUT : at NLO, no feedback of the produced gluons on the LO field!

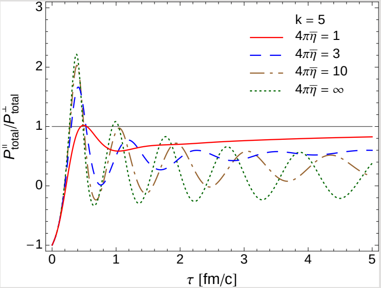


P_L / P_T
+1
NLO
QCD
LO
time
τ_0
Hydro
-1

Color flux tube model : [Ryblewski, Florkowski (2013)]

$$\left(p^\mu \partial_\mu + \underbrace{g F^{\mu\nu} p_\nu}_{\text{Lorentz force}} \partial_p^\mu\right) G = \underbrace{\frac{dN}{d\Gamma}}_{\text{Schwinger}} + \underbrace{C_p[G]}_{\text{collisions}}$$

$$\partial_\mu F^{\mu\nu} = j^\nu \qquad \text{(feedback)}$$



- Field converted into particles by instability
- Nearly constant P_L/P_T
- Ratio depends on $\bar{\eta} \equiv \eta/s$

QCD evolution at NLO + resummation of secular terms

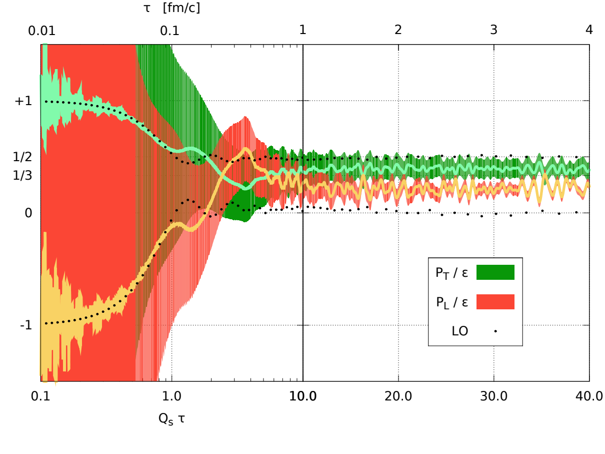
$$\mathcal{O}_{\text{secular}} \equiv \underbrace{\mathcal{O}_{\text{LO}} + \mathcal{O}_{\text{NLO}} + \text{subset of higher orders}}_{\text{terms that have the fastest growth in time}}$$

$$= \int [D\alpha(\mathbf{u})] \; \exp\Big[-\frac{1}{2\hbar} \int_{\mathbf{u},\mathbf{v}} \alpha(\mathbf{u}) \sigma^{-1}(\mathbf{u},\mathbf{v}) \alpha(\mathbf{v}) \Big] \; \mathcal{O}_{\text{LO}}\left[\mathcal{A}_{\text{ini}} + \alpha\right]$$

- In this resummation, the observable is as an average over classical field evolutions with fluctuating initial conditions
- Roughly speaking:
  - the secular resummation promotes a classical initial state to a quantum coherent state
  - fluctuations of $\mathcal{A}_{\text{in}} \sim$ *zero point fluctuations*
- The precise form of the variance ($\hbar\,\sigma$) is obtained from an NLO (analytical) calculation

QCD evolution at NLO + resummation of secular terms

[Epelbaum, FG (2013)]



QCD: coherent initial state

$$\mathcal{A} = \mathcal{A}_{\text{LO}} + \int_{\mathbf{p}} c_{\mathbf{p}}\ \alpha_{\mathbf{p}} \qquad \left\langle c_{\mathbf{p}} c_{\mathbf{p}'} \right\rangle \sim \frac{1}{2}\ \delta_{\mathbf{p}\mathbf{p}'}$$

Occupation number :

$$\left\langle \widetilde{\mathcal{A}}\widetilde{\mathcal{A}}^* \right\rangle_{\tau=0^+} = \underbrace{\widetilde{\mathcal{A}}_{\text{LO}}\widetilde{\mathcal{A}}^*_{\text{LO}}}_{\sim\delta(p_z)f(p_\perp)} + \frac{1}{2}$$

QC

Incoherent initial state:

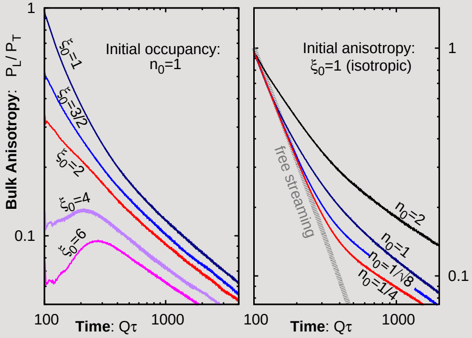
$$A = \int_{\mathbf{p}} c_{\mathbf{p}}\ \alpha_{\mathbf{p}} \qquad \left\langle c_{\mathbf{p}} c_{\mathbf{p}'} \right\rangle \sim \delta_{\mathbf{p}\mathbf{p}'} \left[\frac{1}{2} + f_0(\mathbf{p})\right]$$

$\frac{1}{2} \iff$ zero point fluctuations

$f_0(\mathbf{p}) \iff$ initial particle distribution ($\sim g^{-2}$)

If $f_0(\mathbf{p}) \gg 1$, approximate $\frac{1}{2} + f_0 \ \rightarrow\ f_0$?

How c Incoherent initial state:



- No dependence on the coupling (can be scaled out)
- $P_T/P_L \sim \tau^{-2/3}$

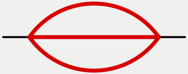
Classical approximation in Kinetic Theory

From QFT to Kinetic Theory

Dyson-Schwinger equations $\longrightarrow$ Kadanoff-Baym equations $\longrightarrow$ Boltzmann : $p^\mu \partial_\mu f = C_{\mathbf{p}}[f]$

- Collision term:

$$C_{\mathbf{p}}[f] = \frac{i}{2}\Big[\Sigma_{11}(p) + \big(\tfrac{1}{2} + f(\mathbf{p})\big)\Big(\Sigma_{21}(p) - \Sigma_{12}(p)\Big)\Big]$$



$$\Longrightarrow \quad C_{\mathbf{p}}[f] = \frac{g^4}{4E_{\mathbf{p}}}\int_{\mathbf{k}}\int_{\mathbf{p}'}\int_{\mathbf{k}'}(2\pi)^4\delta(P+K-P'-K')$$
$$\times\Big[f(\mathbf{p}')f(\mathbf{k}')\big(1+f(\mathbf{p})\big)\big(1+f(\mathbf{k})\big)$$
$$-f(\mathbf{p})f(\mathbf{k})\big(1+f(\mathbf{p}')\big)\big(1+f(\mathbf{k}')\big)\Big]$$

D

Weak classical approximation:

$$C_{\mathbf{p}}[f] \;=\; \frac{g^4}{4E_{\mathbf{p}}}\int_{\mathbf{k}}\int_{\mathbf{p}'}\int_{\mathbf{k}'}(2\pi)^4\delta(P+K-P'-K')$$
$$\times\Big[\big(\tfrac{1}{2}+f(\mathbf{p}')\big)\big(\tfrac{1}{2}+f(\mathbf{k}')\big)\big(\tfrac{1}{2}+f(\mathbf{p})+\tfrac{1}{2}+f(\mathbf{k})\big)$$
$$-\big(\tfrac{1}{2}+f(\mathbf{p})\big)\big(\tfrac{1}{2}+f(\mathbf{k})\big)\big(\tfrac{1}{2}+f(\mathbf{p}')+\tfrac{1}{2}+f(\mathbf{k}')\big)\Big]$$

(f^3 and f^2 correct, but spurious f^1 terms)

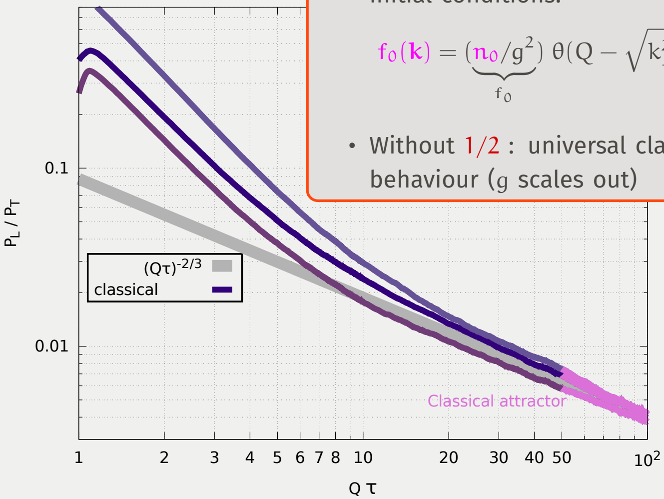
Strong classical approximation: drop the $\frac{1}{2}$ (f^3 correct)

•

— $K')$

$$\times\Big[f(\mathbf{p}')f(\mathbf{k}')\big(1+f(\mathbf{p})\big)\big(1+f(\mathbf{k})\big)$$
$$-f(\mathbf{p})f(\mathbf{k})\big(1+f(\mathbf{p}')\big)\big(1+f(\mathbf{k}')\big)\Big]$$

[Epelbaum, FG, Jeon, Moore, Wu (2015)]

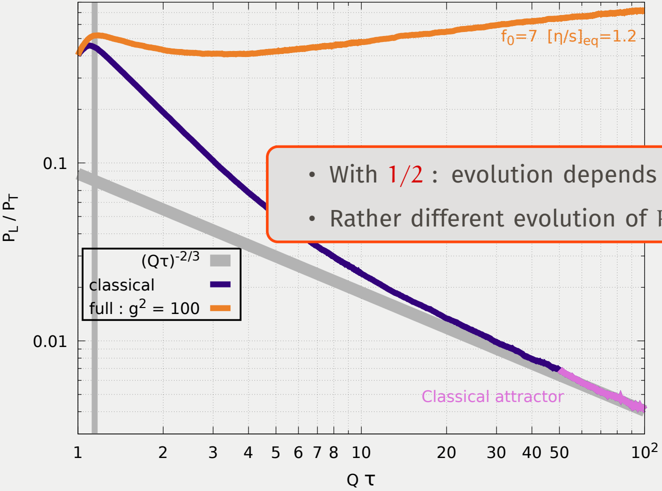


- Initial conditions:

$$f_0(\mathbf{k}) = \underbrace{(n_0/g^2)}_{f_0}\,\theta(Q - \sqrt{k_\perp^2 + \xi_0\,k_z^2})$$

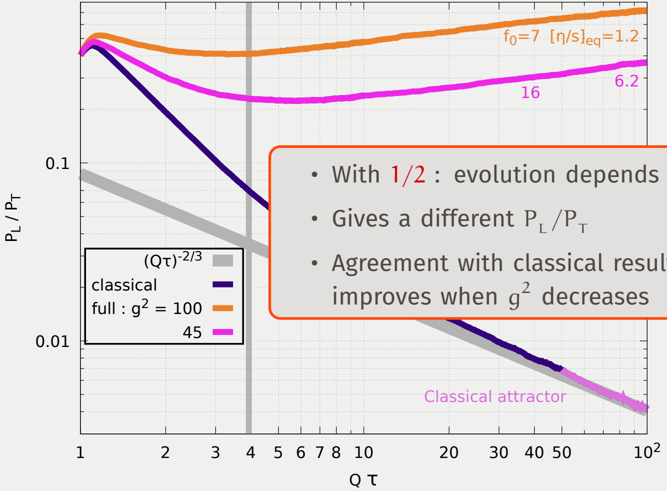
- Without $1/2$: universal classical behaviour (g scales out)

[Epelbaum, FG, Jeon, Moore, Wu (2015)]



- With $1/2$: evolution depends on g^2
- Rather different evolution of P_L/P_T

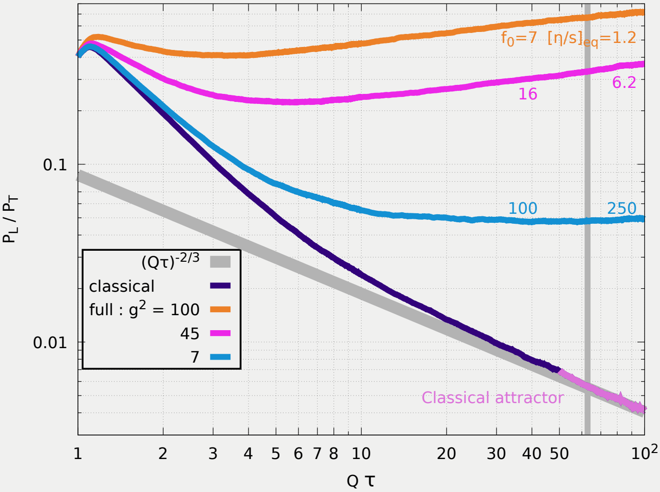
[Epelbaum, FG, Jeon, Moore, Wu (2015)]



- With $1/2$: evolution depends on g^2
- Gives a different P_L/P_T
- Agreement with classical result improves when g^2 decreases

Isotropization in a longitudinally expanding system

[Epelbaum, FG, Jeon, Moore, Wu (2015)]



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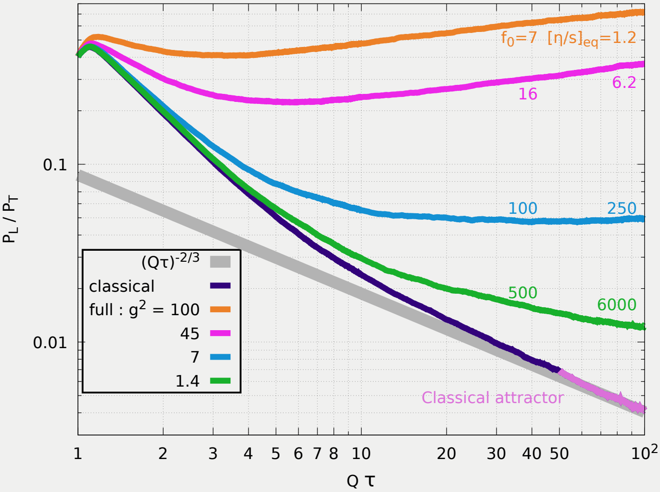
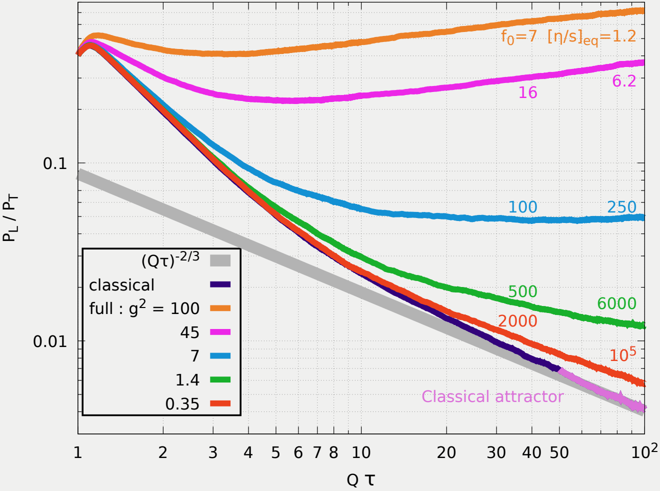


[Epelbaum, FG, Jeon, Moore, Wu (2015)]



WHY IS THE ZERO-POINT $1/2$ IMPORTANT ?

- The $1/2$'s ensure that the terms f^3 and f^2 are correct
- The quadratic terms are important in anisotropic systems

Why is the zero-point 1/2 important ?

- The 1/2's ensure that the terms f^3 and f^2 are correct
- The quadratic terms are important in anisotropic systems

- No 1/2 $\Longrightarrow$ no f^2 terms in Boltzmann eq. :

$$\partial_t f_4 \quad \sim \quad g^4 \int_{123} \cdots \left[f_1 f_2 (f_3 + f_4) - f_3 f_4 (f_1 + f_2)\right] + \cdots \left[\cancel{f_1 f_2 - f_3 f_4}\right]$$

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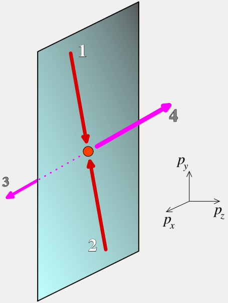

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$$\partial_t f_4 \quad \sim \quad g^4 \int_{123} \cdots \left[f_1 f_2 (f_3 + f_4) - f_3 f_4 (f_1 + f_2) \right] + \cdots \left[f_1 f_2 - f_3 f_4 \right]$$

- When the distribution is very anisotropic, trying to produce the particle 4 at large angle results in $f_3 \approx f_4 \approx 0 \quad \Rightarrow$ nothing left
- Cubic terms $\Leftrightarrow$ stimulated emission : ineffective to produce particles in empty regions of phase-space
- Leading term: $f_1 f_2$

More insights from kinetic theory

Boltzmann in the relaxation time approximation

$$\left(\partial_\tau - \frac{p_z}{\tau}\right) f(\tau, \mathbf{p}) = -\frac{f - f_{eq}}{\tau_R}$$

$\tau_R \equiv$ relaxation time

$f_{eq} \equiv$ local equilibrium dist

- $\tau_R = \infty$: no collisions
- $\tau_R \sim \epsilon^{-1/4}$: conformal; rate scales as inverse temperature

- Define moments :

$$L_n \equiv \int_{\mathbf{p}} \mathbf{p}^2 \; P_{2n}(p_z/p)\; f(\tau, \mathbf{p}) \quad , \quad g_n \equiv \tau\partial_\tau \ln L_n$$

$$L_0 = \epsilon = P_{\rm L} + 2P_{\rm T}, \quad L_1 = P_{\rm L} - P_{\rm T}$$

Boltzmann $\Leftrightarrow$ coupled equations for L_n

$$\begin{aligned} \partial_\tau L_0 &= -\frac{a_o L_0 + c_0 L_1}{\tau} \\ \partial_\tau L_n &= -\frac{a_n L_n + c_n L_{n+1} + b_n L_{n-1}}{\tau} - \frac{L_n}{\tau_{\rm R}} \qquad (n \geq 1) \end{aligned}$$

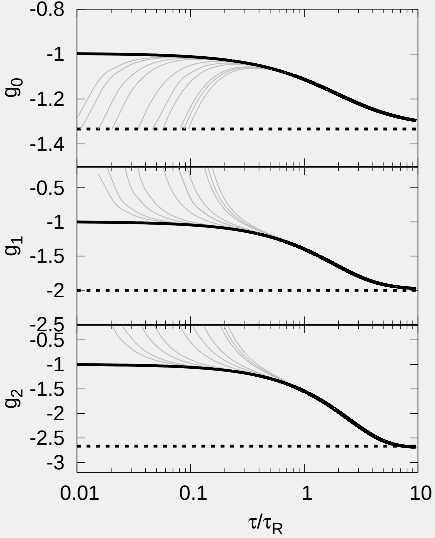
a_n, b_n, c_n = pure numbers, known explicitly (depend only on the free streaming part of Boltzmann eq.)

Free streaming fixed point ($\tau_R = \infty$)

- All the g_n behave as τ^{-1}, with fixed ratios
- $L_1/L_0 \to -\frac{1}{2}$, i.e. $P_L/P_T \to 0$

Interacting fixed point ($\tau_R \sim \epsilon^{-1/4}$)

- $g_0 \to -4/3, \quad g_1 \to -2$
- Locally isotropic distribution



- Universal attractor
- $\tau \lesssim \tau_R$: trajectories first approach free streaming fixed point
- $\tau \gtrsim \tau_R$: trajectories go to the local equilibrium fixed point

Summary

- Strong fields: short mean free path despite weak coupling
- LO: no pressure isotropization, NLO: secular instabilities
- Beyond NLO: **Semi-classical approximation**
  - Weak classical approximation:
    non-renormalizable, sensitive to UV cutoff
  - Strong classical approximation:
    underestimates large angle scatterings
    poor unless η/s very large
- **Kinetic theory:** avoids all these difficulties (but does not cope well with screening effects at long distance)
- Beyond semi-classical in QFT: **Two-PI formalism** (Luttinger-Ward functional)

Analogue in Quantum Mechanics

- Consider the Liouville–von Neumann equation :

$$i\hbar\,\frac{\partial\widehat{\rho}_\tau}{\partial\tau} = \big[\widehat{H},\widehat{\rho}_\tau\big]$$

- Introduce the Wigner transforms :

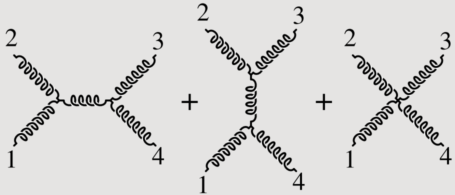
$$\begin{aligned} W_\tau(\boldsymbol{x},\boldsymbol{p}) &\equiv \int d\boldsymbol{s}\; e^{i\boldsymbol{p}\cdot\boldsymbol{s}}\,\big\langle \boldsymbol{x}+\frac{\boldsymbol{s}}{2}\big|\widehat{\rho}_\tau\big|\boldsymbol{x}-\frac{\boldsymbol{s}}{2}\big\rangle \\ \mathcal{H}(\boldsymbol{x},\boldsymbol{p}) &\equiv \int d\boldsymbol{s}\; e^{i\boldsymbol{p}\cdot\boldsymbol{s}}\,\big\langle \boldsymbol{x}+\frac{\boldsymbol{s}}{2}\big|\widehat{H}\big|\boldsymbol{x}-\frac{\boldsymbol{s}}{2}\big\rangle \end{aligned}$$

- LvN equation is equivalent to Moyal-Groenewold equation

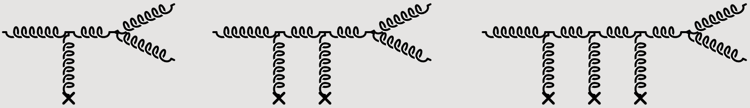
$$\begin{aligned} \frac{\partial W_\tau}{\partial\tau} &= \mathcal{H}(\boldsymbol{x},\boldsymbol{p})\,\frac{2}{i\hbar}\,\sin\left(\frac{i\hbar}{2}\big(\overleftarrow{\partial}_{\boldsymbol{p}}\overrightarrow{\partial}_{\boldsymbol{x}} - \overleftarrow{\partial}_{\boldsymbol{x}}\overrightarrow{\partial}_{\boldsymbol{p}}\big)\right) W_\tau(\boldsymbol{x},\boldsymbol{p}) \\ &= \underbrace{\{\mathcal{H},W_\tau\}}_{\text{Poisson bracket}} + \underbrace{\mathcal{O}(\hbar^2)}_{\text{deviation from classical dynamics}} \end{aligned}$$

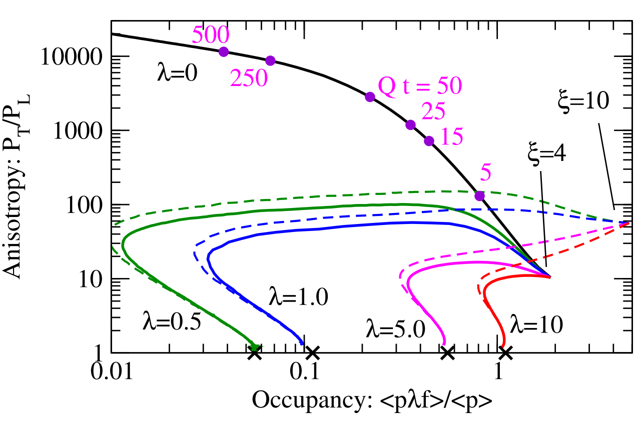
KINETIC THEORY FOR GLUONS [Kurkela, Zhu (2015)]

$2 \to 2$



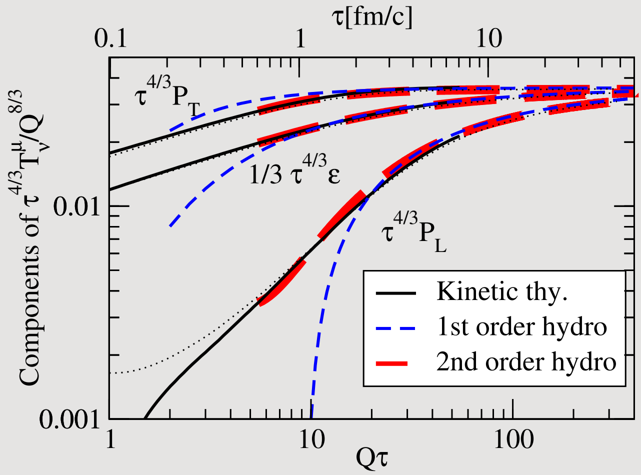
$1 \to 2$, $2 \to 1$ **+ Landau-Pomeranchuk-Migdal resummation**





For $\lambda = 0.5$, the **Strong CSA** breaks down at $Q\tau \approx 2$, while simple estimates suggested that it would be valid up to $Q\tau \approx \alpha_s^{-3/2} \approx 350$

Consistent with hydrodynamics before full isotropization



Energy-momentum tensor

$$
\begin{aligned}
T^{\mu\nu} \quad = \quad & \nabla^\mu\varphi\nabla^\nu\varphi - g^{\mu\nu}\mathcal{L} + \Big[\nabla^\mu_x\nabla^\nu_y G_{xy}\Big]_{x=y} \\
& +\frac{1}{2}g^{\mu\nu}\Big\{V''(\varphi_x)G_{xx} - \Big[\nabla^x_\alpha\nabla^\alpha_y G_{xy}\Big]_{x=y}\Big\} \\
& -g^{\mu\nu}\frac{\delta\Phi}{\delta\sqrt{-g}}
\end{aligned}
$$

Isotropization in a fixed box