Out-of-equilibrium Statistics and Turbulence

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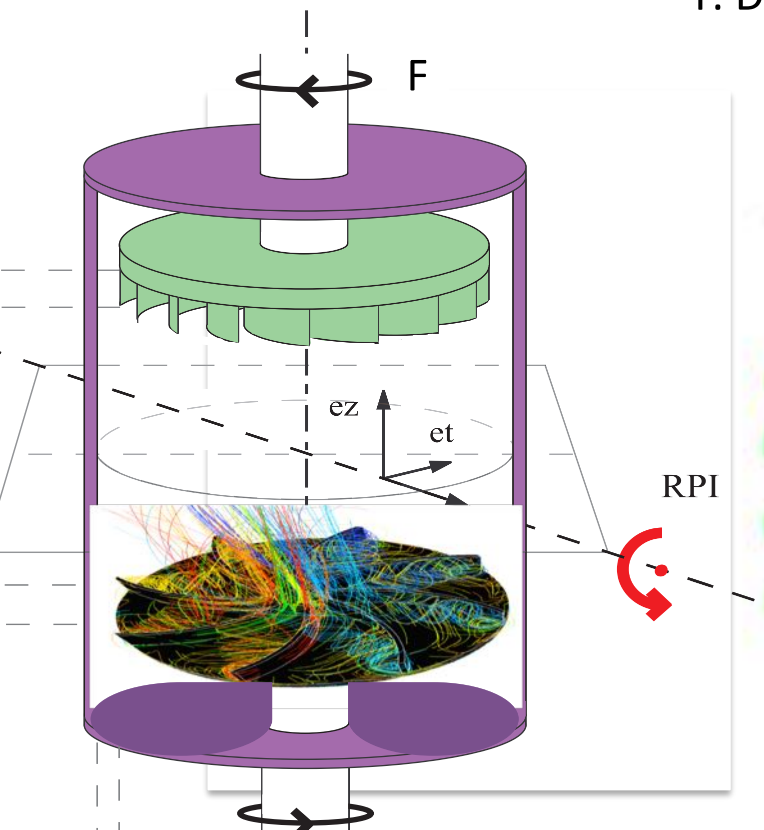
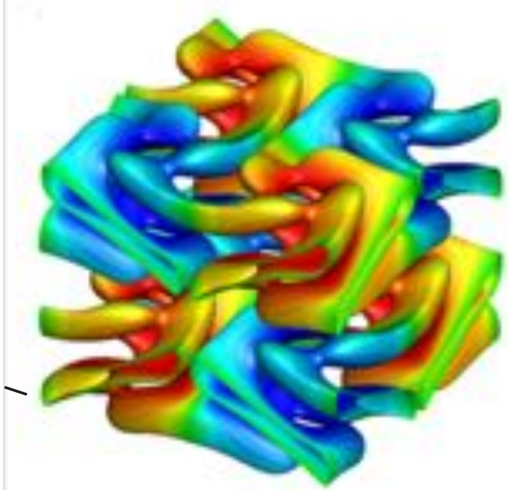
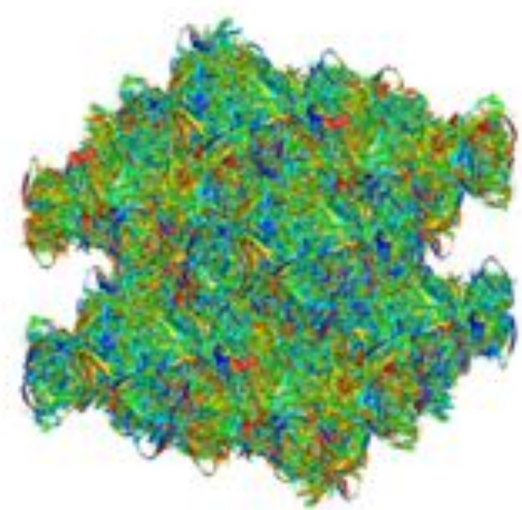






Turbulence

$Re = 10^2$

$Re = 10^7$

$Re = 10^{11}$

$Re = 10^{12}$

$Re = 10^{21}$

Navier-Stokes Equations:

$$\partial_t u + u \bullet \nabla u = -\nabla p + \nu \Delta u + f$$

Control Parameter: $$\mathrm{Re} = \frac{LU}{\nu}$$

Fluids and Navier-Stokes

$$\vec{\nabla} \bullet \vec{u} = \vec{0}$$

$$\partial_t \vec{u} + (\vec{u} \bullet \vec{\nabla}) \vec{u} = -\frac{1}{\rho} \vec{\nabla} p + \nu \Delta \vec{u}$$

Symmetries

Tme-translation	$t \to t + h$
Space translation	$\vec{x} \to \vec{x} + \vec{h}$
Space-reversal	$(\vec{x}, \vec{u}) \to (-\vec{x}, -\vec{u})$
Galilean invariance	$(\vec{x}, \vec{u}) \to (\vec{x} + \vec{U} t, \vec{u} + \vec{U})$
Scaling	$(t, \vec{x}, \vec{u}) \to (\lambda^2 t, \lambda \vec{x}, \lambda^{-1} \vec{u}) \quad \nu \neq 0$
	$(t, \vec{x}, \vec{u}) \to (\lambda^{1-h} t, \lambda \vec{x}, \lambda^h \vec{u}) \quad \nu = 0$

Fluids and Navier-Stokes

$$\vec{\nabla} \bullet \vec{u} = \vec{0}$$

$$\partial_t \vec{u} + (\vec{u} \bullet \vec{\nabla}) \vec{u} = -\frac{1}{\rho} \vec{\nabla} p + \nu \Delta \vec{u}$$

Broken Symmetry

Time-reversal $\quad t -> -t \quad$ Only for $\nu = 0$

$$u -> -u$$

There is entropy production through viscosity:

Unforced Navier-Stokes

the only equilibrium state (state that satisfies all the symmetries) is u=0

Forced Navier-Stokes

Non-equilibrium states that depend on the Reynolds number

Non-equilibrium states of NSE

$$\vec{u} = \vec{C}$$

How can we explain/describe the time-reverasl symmetry breaking in turbulent flow?

Satisfies all the symmetries

All symmetries are broken

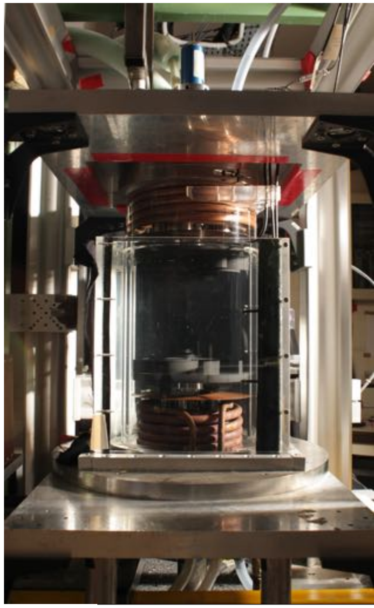
Re<<1

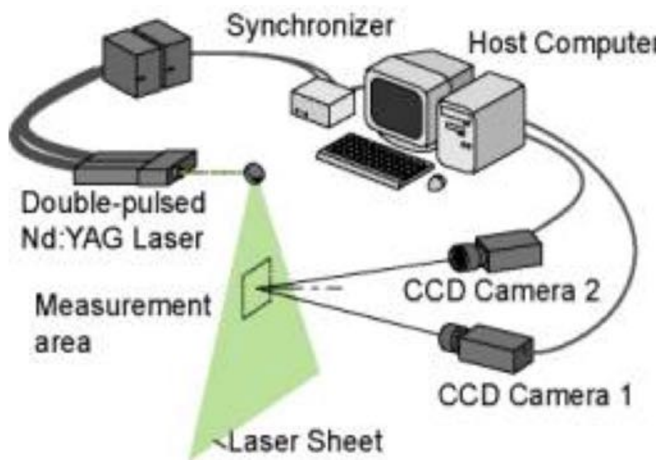
Re>>1

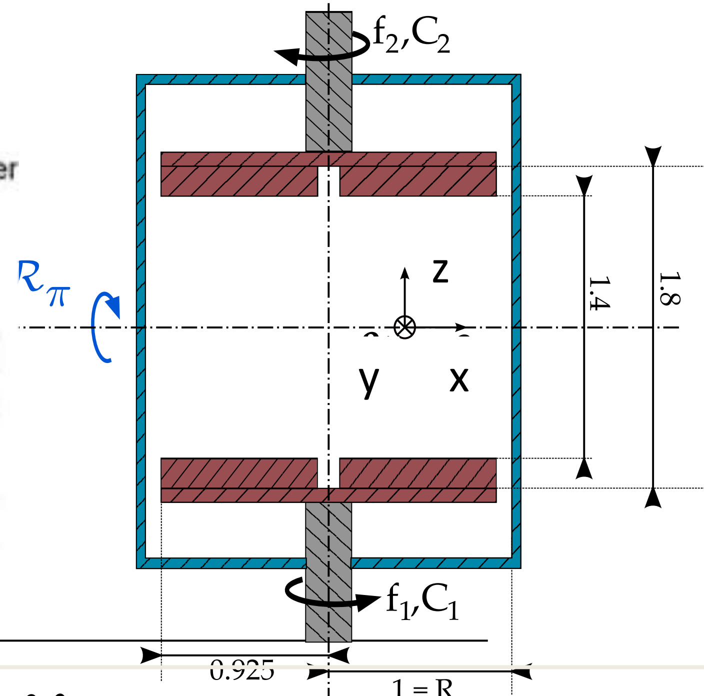
Laminar state

Turbulent state

VKE experiment







Control parameters

Reynolds number

$$Re = \frac{2\pi f R^2}{\nu}$$

$$f = \frac{f_1 + f_2}{2}$$

Measurements

Global energy input via Torque

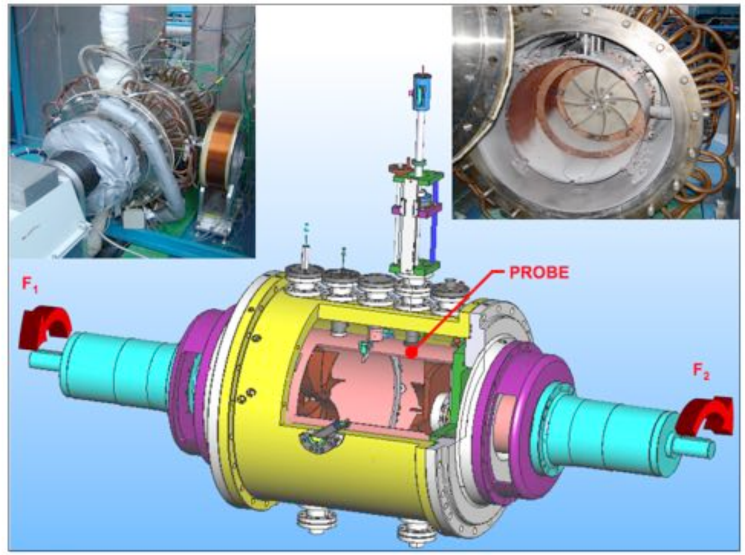
$$K_p = \frac{0.5(C_1 + C_2)}{\rho R^5 f^2}$$

PIV velocity fields

$$V_x(x,z), V_y(x,z)), V_z(x,z)$$

von Karman experiments

VKE



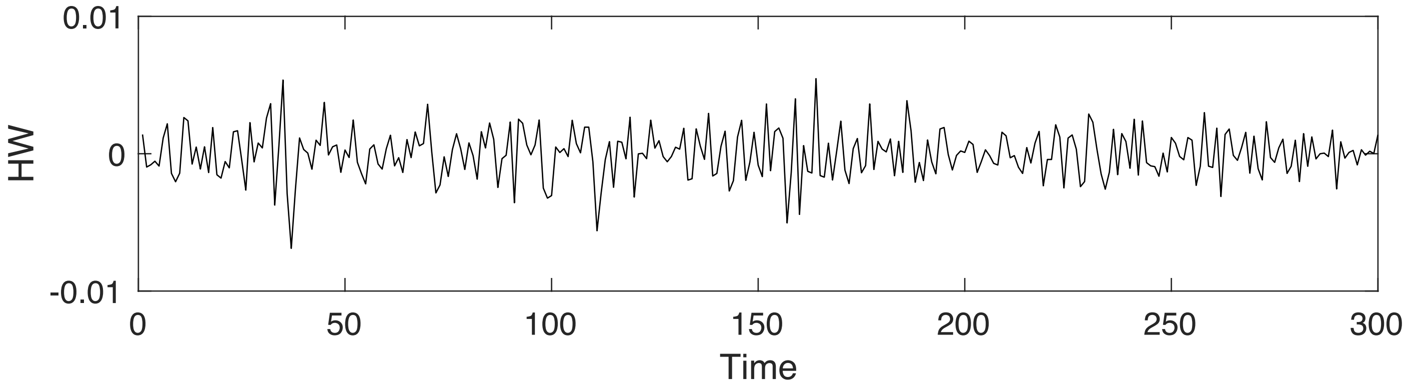
VKS=VKEx2

SHREK=VKEx4

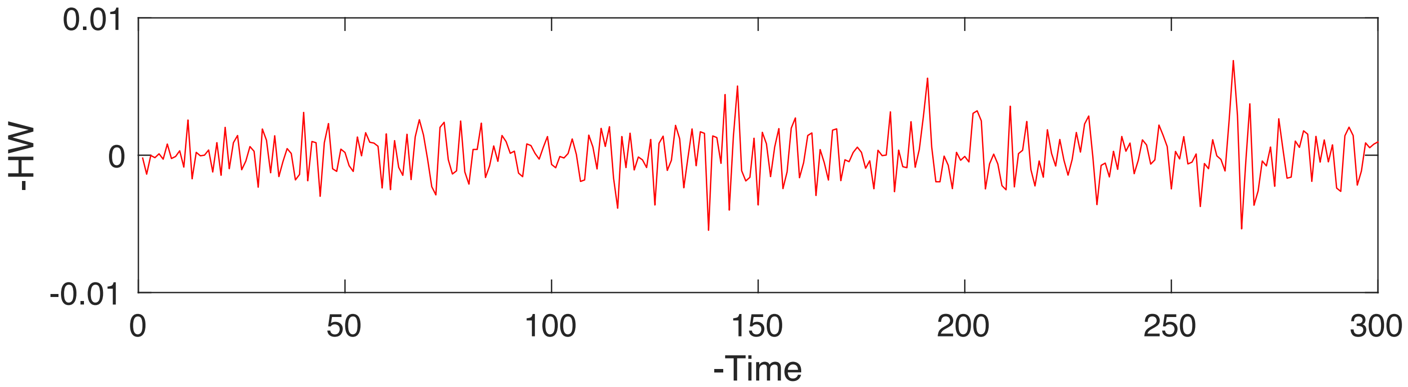
SetUp	Fluid	P(bars)	T(K)	Re
SHREK	HeI	1.1	2.62	10^8
SHREK	N2	1.1	284	10^5
VKS	Na		410	10^7
VKE VKE	H2O Glyc	1.8 1.8	300 300	10^5 10^2

Can you see irreversibility by eye?

Un champ turbulent mesuré avec un fil chaud

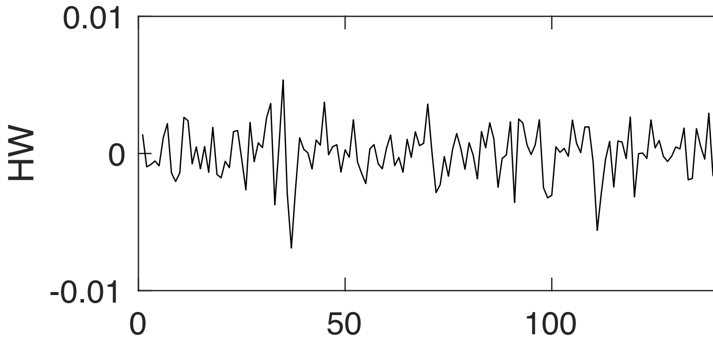


Un champ turbulent renversé   t->-t; u->-u

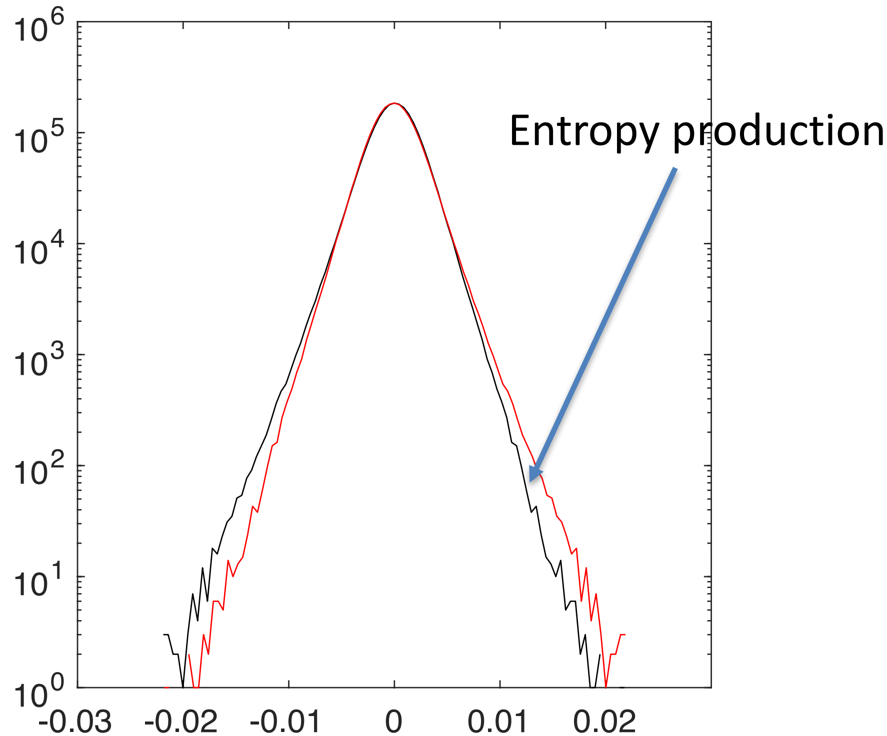


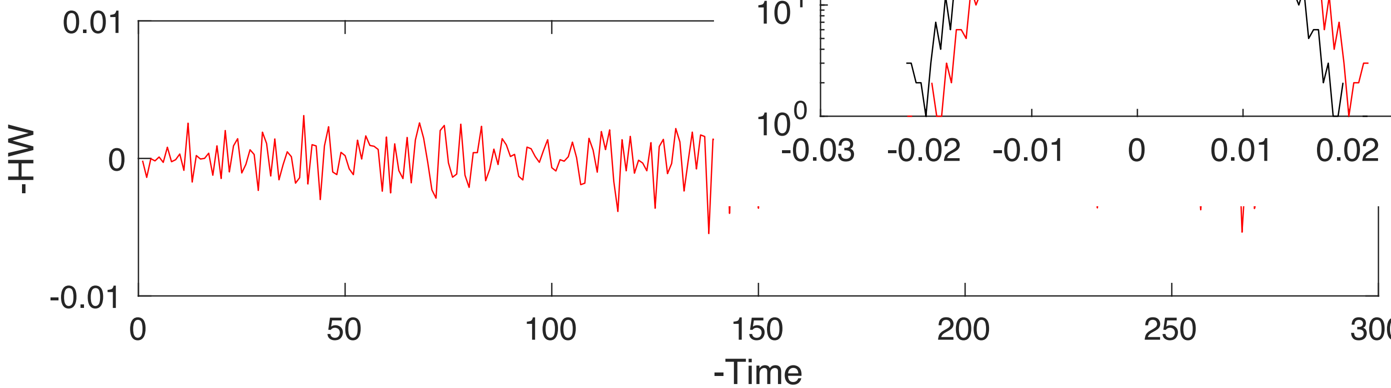
No! Better use something else!

Un champ turbulent mesuré avec un fil chaud



Un champ turbulent renv
t->-t; u->-u



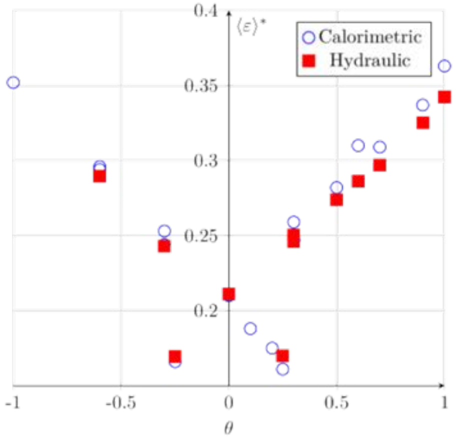


Mystery (i)

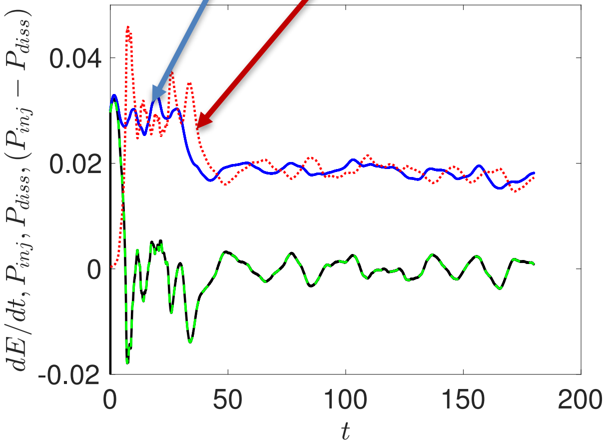
1st law: Energy can be split into work and heat

$$dE = dW + dQ$$

$$\frac{dE}{dt} = Pinj - Pdiss$$



SHREK Collaboration

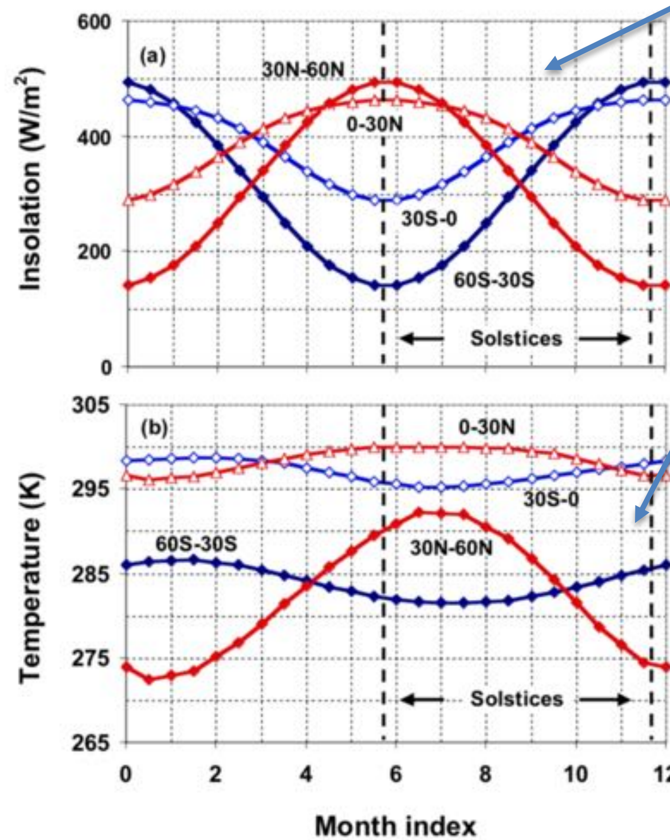


DNS V. Shukla

Work measured by
Torques applied at Shafts
Heat flux measured
By keeping T constant

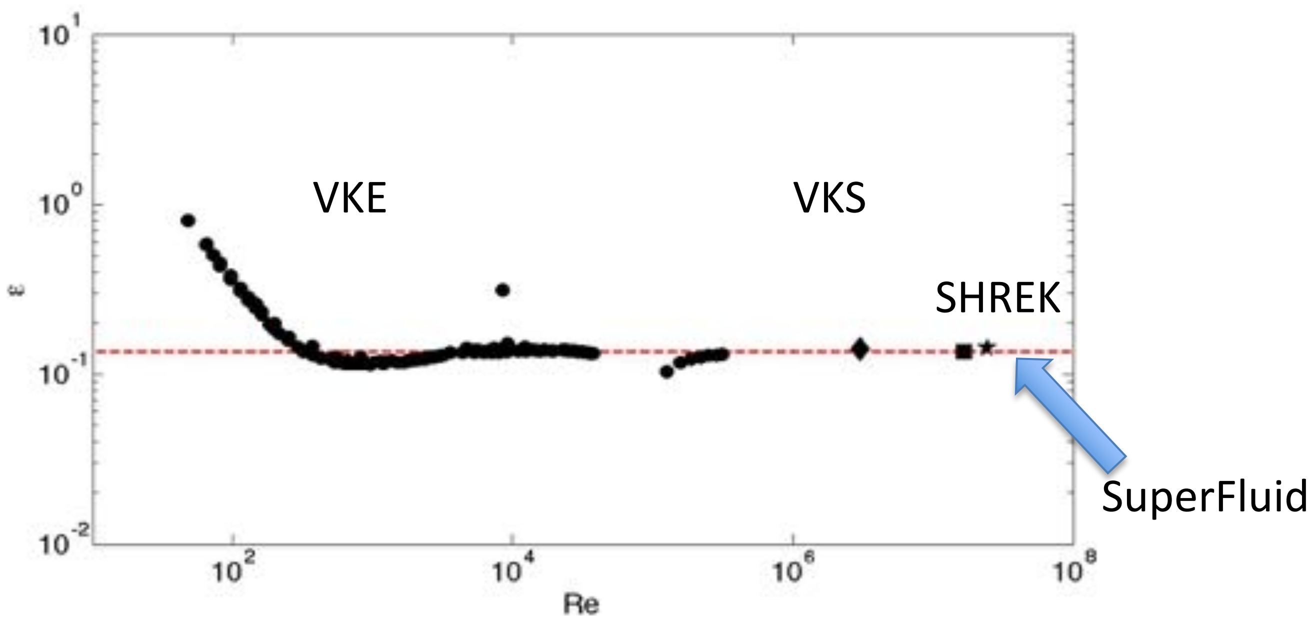
Mystery (i)

$$\frac{dE}{dt} = Pinj - Pdiss$$



Mystery (ii)

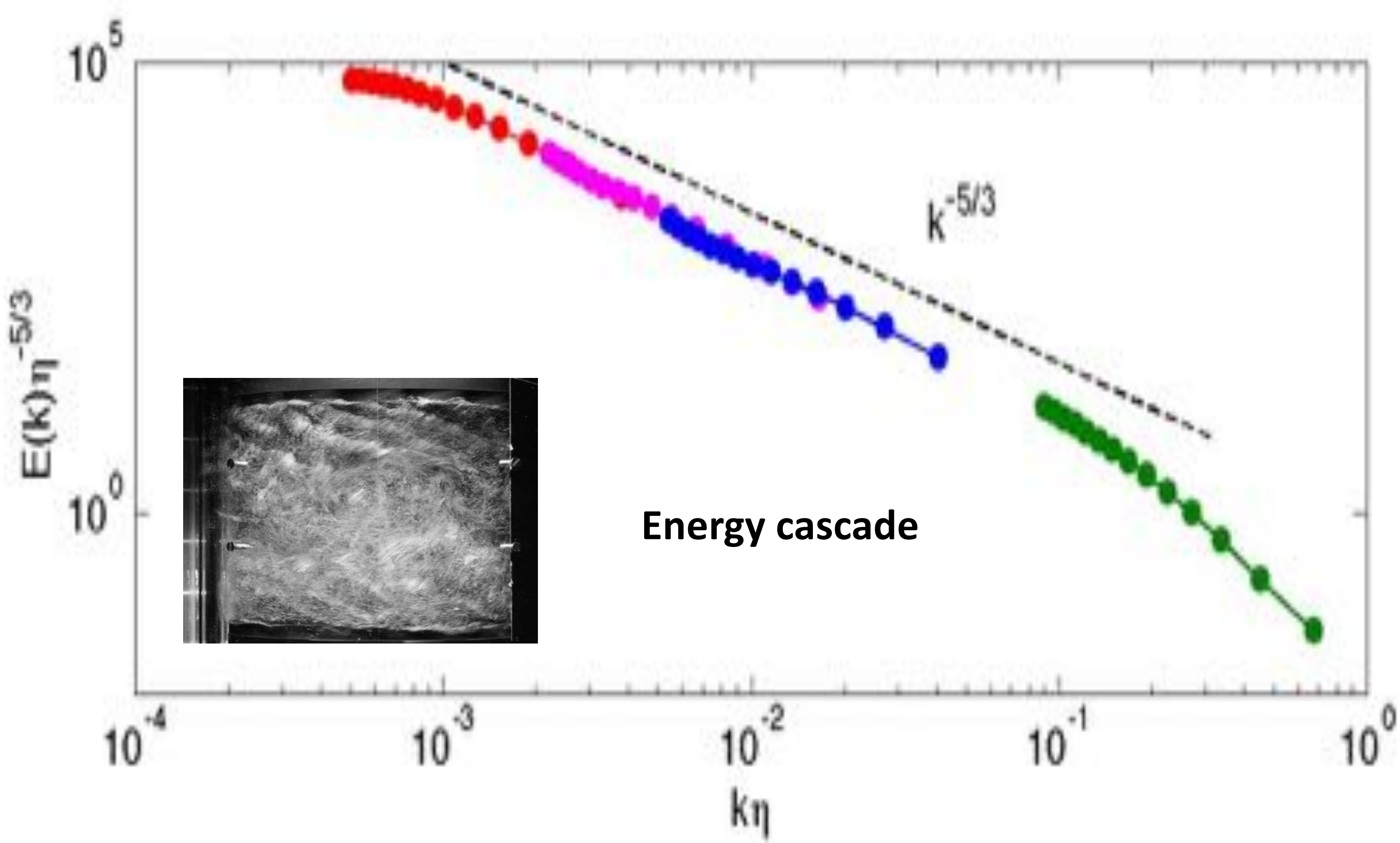
Energy dissipation does not go to zero as $\upsilon \to 0$
Spontaneous symmetry breaking/Dissipation anomaly !!!!!



VKE+VKS collaboration+SHREK collaboration

Saint-Michel et al, POF 26, 125109 (2014);

Observation: power-law spectrum



Saclay team: Debue, Kuzzay, Faranda, Saw, Daviaud, Dubrulle et al, (2016)

Kolmogorov Theory (1)

NSE + homogeneity

$x \longrightarrow x + h$

$$\frac{1}{2}\partial_t \left\langle \left(\delta u_\ell\right)^2 \right\rangle - \varepsilon = +\frac{1}{4}\nabla_\ell \left\langle \left(\delta u_\ell\right)^3 \right\rangle + \nu\Delta_\ell \left\langle \left(\delta u_\ell\right)^2 \right\rangle$$

$$\delta u_\ell = u(x+\ell) - u(x)$$

Karman-Howarth-Monin equation

Karman & Howarth (1938), Monin (1959)

Kolmogorov Theory (2)

KH equation + stationarity + self-similarity $(t, x, u) \longrightarrow (\gamma^{1-h}\, t, \gamma x, \gamma^h u)\ (\nu = 0)$

$$\frac{1}{2}\partial_t \left\langle \left(\delta u_\ell\right)^2 \right\rangle - \varepsilon = \frac{1}{4}\nabla_\ell \left\langle \left(\delta u_\ell\right)^3 \right\rangle + \nu\Delta_\ell \left\langle \left(\delta u_\ell\right)^2 \right\rangle$$

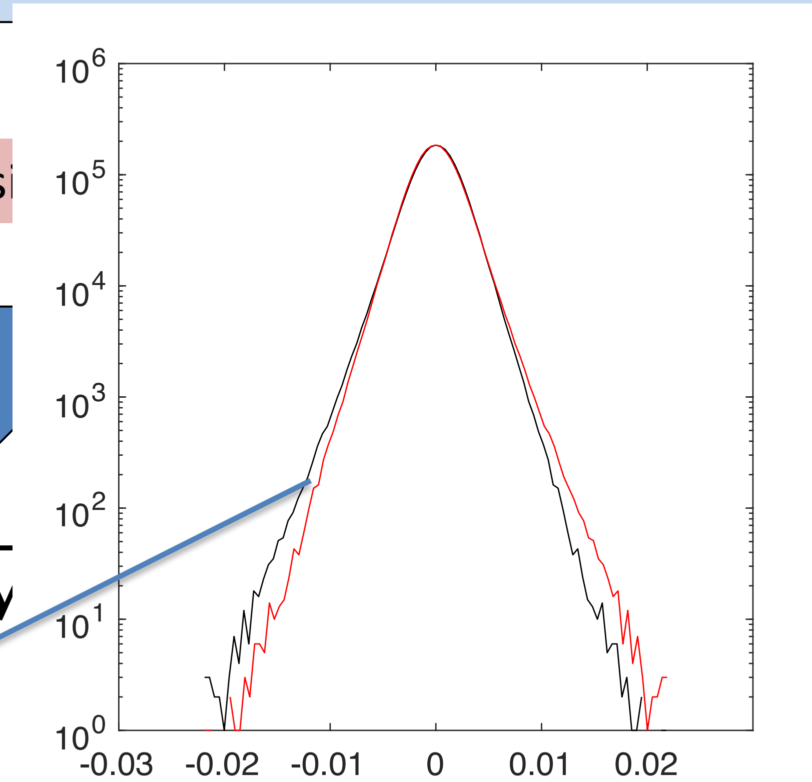
$$\left\langle \left(\delta u_\ell\right)^3 \right\rangle \propto -\frac{4}{3}\varepsilon\ell$$

$$\left\langle \left(\delta u_\ell\right)^2 \right\rangle \propto \left(\varepsilon\ell\right)^{2/3}$$

$$E\left(k\right) = C\,\varepsilon^{2/3} k^{-5/3} \qquad (h = \frac{1}{3})$$

Kolmogorov Theory (2)

KH equation + stationarity + self-si ... $\iota)\ (\nu = 0)$



$$\frac{1}{2}\partial_t \left\langle (\delta u_\ell)^2 \right\rangle - \varepsilon = \frac{1}{4}\nabla \cdots \Big)^2 \Big\rangle$$

$$\left\langle (\delta u_\ell)^3 \right\rangle \propto -\frac{4}{3}\varepsilon\ell$$

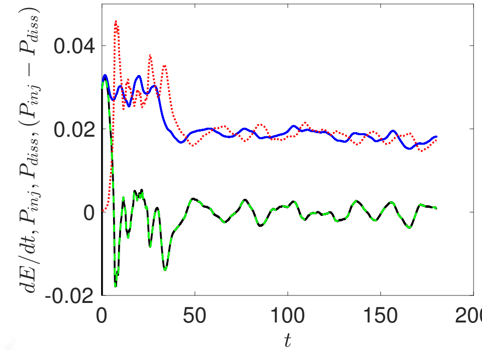
Zero if time reversal symmetry hokds!!!!

$$\left\langle (\delta u_\ell)^2 \right\rangle \propto (\varepsilon\ell)^{2/3}$$

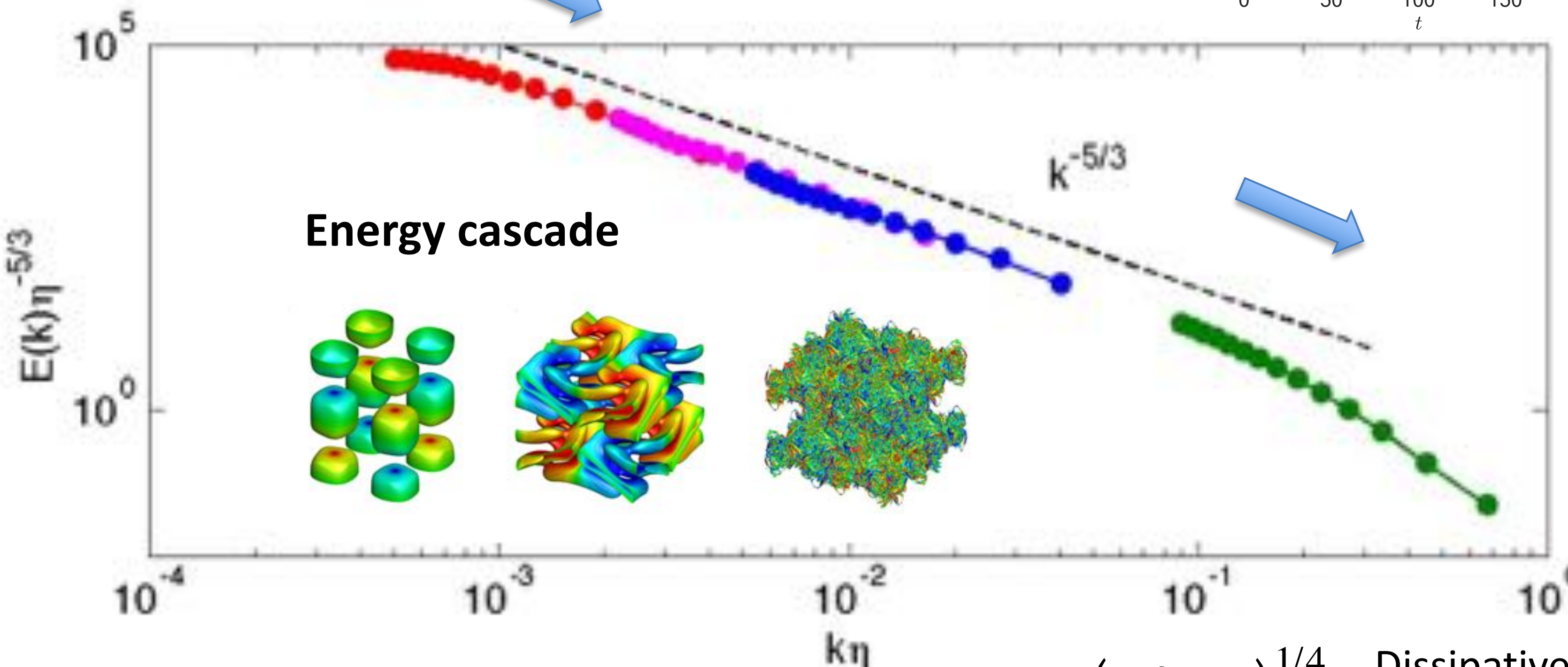
$$E(k) = C\,\varepsilon^{2/3} k^{-5/3}$$

$$(h = \frac{1}{3})$$

Solution to Mystery (i): energy cascade



Work



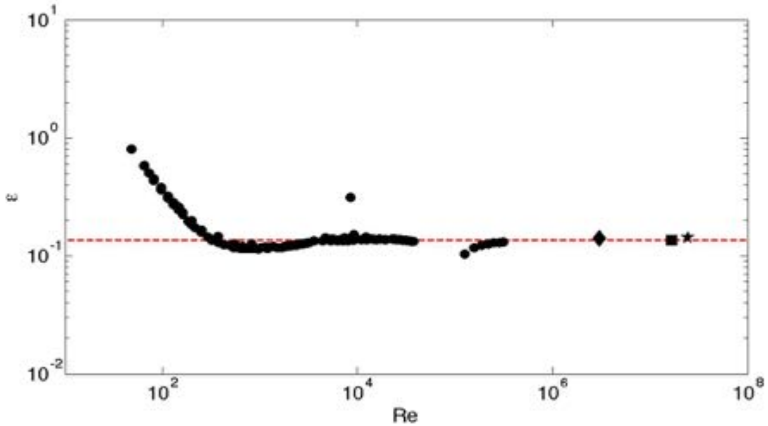
Heat

Injection scale

$$\eta = \left(\nu^3 / \varepsilon\right)^{1/4}$$

Dissipative scale (viscosity)

Mystery (ii) and Onsager's conjecture



"…in three dimensions a mechanism for complete dissipation of all kinetic energy, even without the aid of viscosity, is available."

L. Onsager, 1949

$$\frac{1}{2}\partial_t \mathbf{u}^2 + \mathrm{div}\left(\mathbf{u}\left(\frac{1}{2}\mathbf{u}^2 + p\right) - \nu\nabla\mathbf{u}\right) = D(u) - \nu\ \nabla\mathbf{u})^2$$

Duchon&Robert. Nonlinearity (2000),

Inertial dissipation:

$$D(u) = \lim_{\ell\to 0}\frac{1}{4}\int_{r\leq\ell} d^3r\ \nabla\phi_\ell(r)\cdot\delta u_r|\delta u_r|^2$$

$\delta u(\ell) \sim \ell^{\,h}$ In the limit of $\ell \approx 0$

$$D(u)[x] \propto \lim_{\ell\to 0}\ell^{3h-1}$$

If $h > 1/3$ → Euler equation conserves energy,
Dissipation in Navier-Stokes by viscosity.

If $h \leq 1/3$ → Dissipation through irregularities (singularities)
Without viscosity !

Local Energy Balance at finite scale

Continuous wavelet transform+ Navier-Stokes

$$u^{\ell}(x) = \int dx' \, u(x')\phi_{\ell}(x - x')$$

$$\phi_{\ell}(x) = \frac{1}{\ell^3}\phi\left(\frac{x}{\ell}\right)$$

$$\partial_t u_i^{\ell} + \partial_j (u_i u_j)^{\ell} = -\partial_i p^{\ell} + \nu \partial_k \partial_k u_i^{\ell}.$$

$$\begin{aligned} \partial_t E^{\ell} + \partial_j J_j^{\ell} &= -\frac{1}{4}\int \nabla\phi^{\ell}(\xi)\cdot\delta\mathbf{u}(\delta u)^2 d\xi + \nu\partial^2 E^{\ell} \\ &\equiv -D_{\ell}^{I} - D_{\ell}^{\nu}, \end{aligned}$$

where

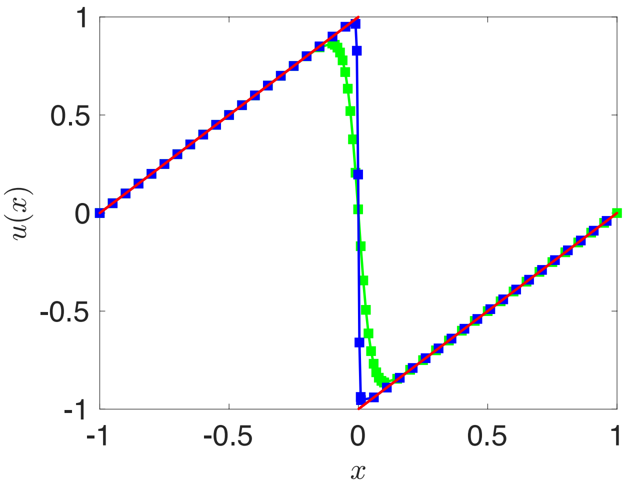
$$J_i^{\ell} = u_i E^{\ell} + (p^{\ell} u_i + p u_i^{\ell})/2 - [(u_i u_j u_j)^{\ell} - (u_i (u_j u_j)^{\ell}]/4 - \nu \partial_i E^{\ell}$$

$$E^{\ell} = \frac{1}{2}\int \phi^{\ell}(\xi) u_i(x) u_i(x+\xi) d\xi$$

$\delta\mathbf{u} = \mathbf{u}(\mathbf{x} + \ell) - \mathbf{u}(\mathbf{x})$ (velocity increment)

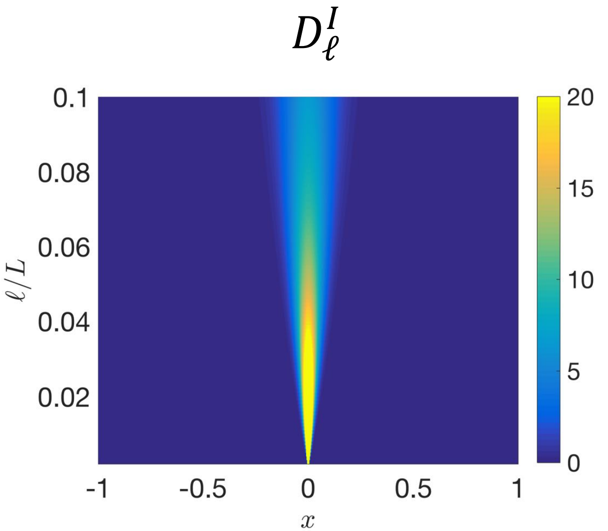
Duchon&Robert (2000), Eyink (2005), Kuzzay et al . (2016)

Example: 1D: Burgers

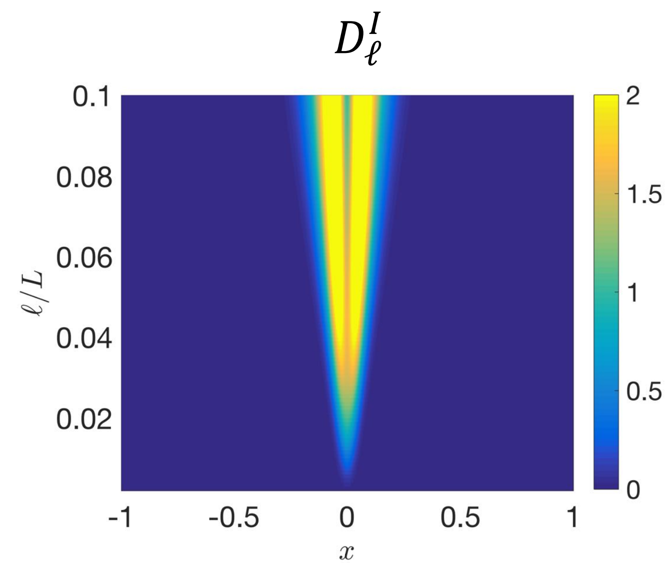


$$\partial_t u + u\partial_x u = \nu\, \partial_{xx} u$$

Turbulence compressible sans pression



Without Viscosity



With Viscosity

Dubrulle 2019 JFM Perspectives

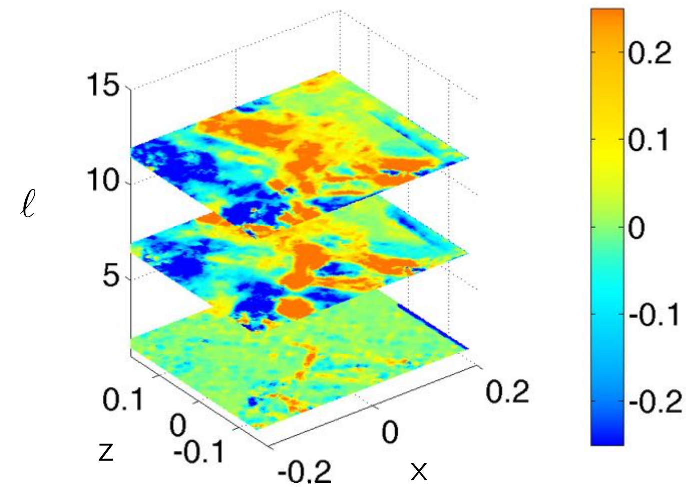
In von Karman 3D flow

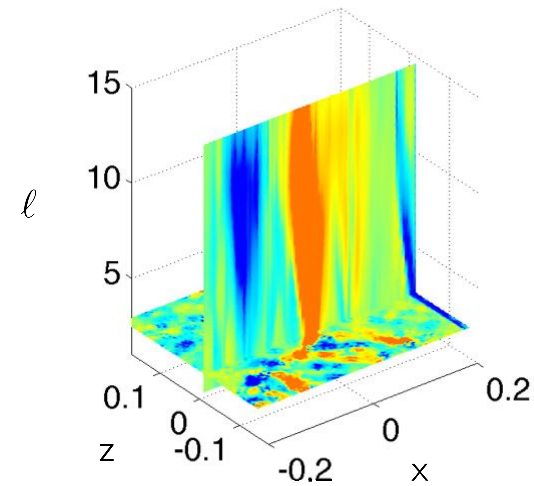
$$D_\ell(\mathbf{u}) = \frac{1}{4}\int_{\mathcal{V}} d^3r\ (\boldsymbol{\nabla} G_\ell)(\mathbf{r})\cdot\delta\mathbf{u}(\mathbf{r})\ |\delta\mathbf{u}(\mathbf{r})|^2,$$

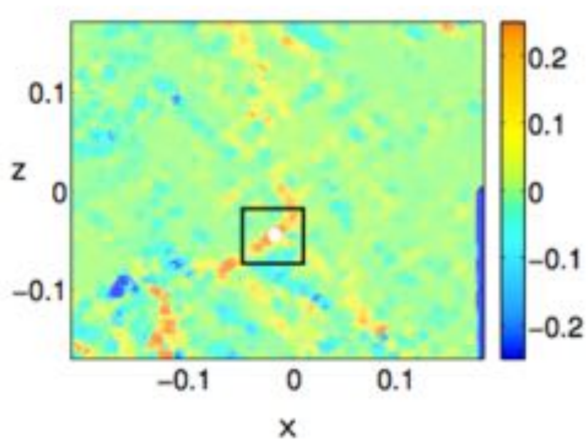
$$G_\ell(r) = \frac{1}{N}\exp(-1/(1-(r/2\ell)^2)),$$

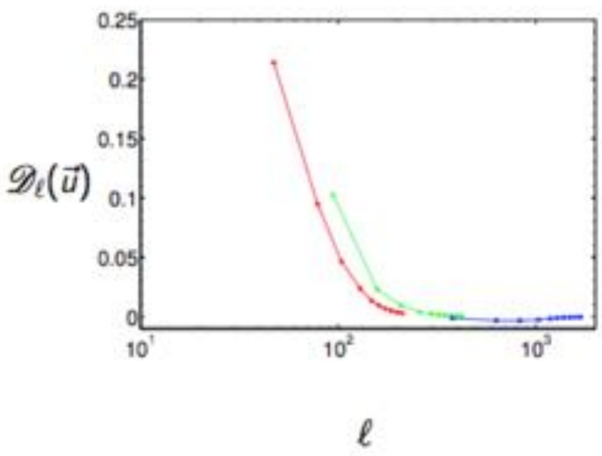
Comparing 2D to 3D data:
Stereo –PIV data can only
Detect events with strong
components laying in the
measurement plane.

- Kuzzay D. et al. (2016), :arXiv:1601.03922.









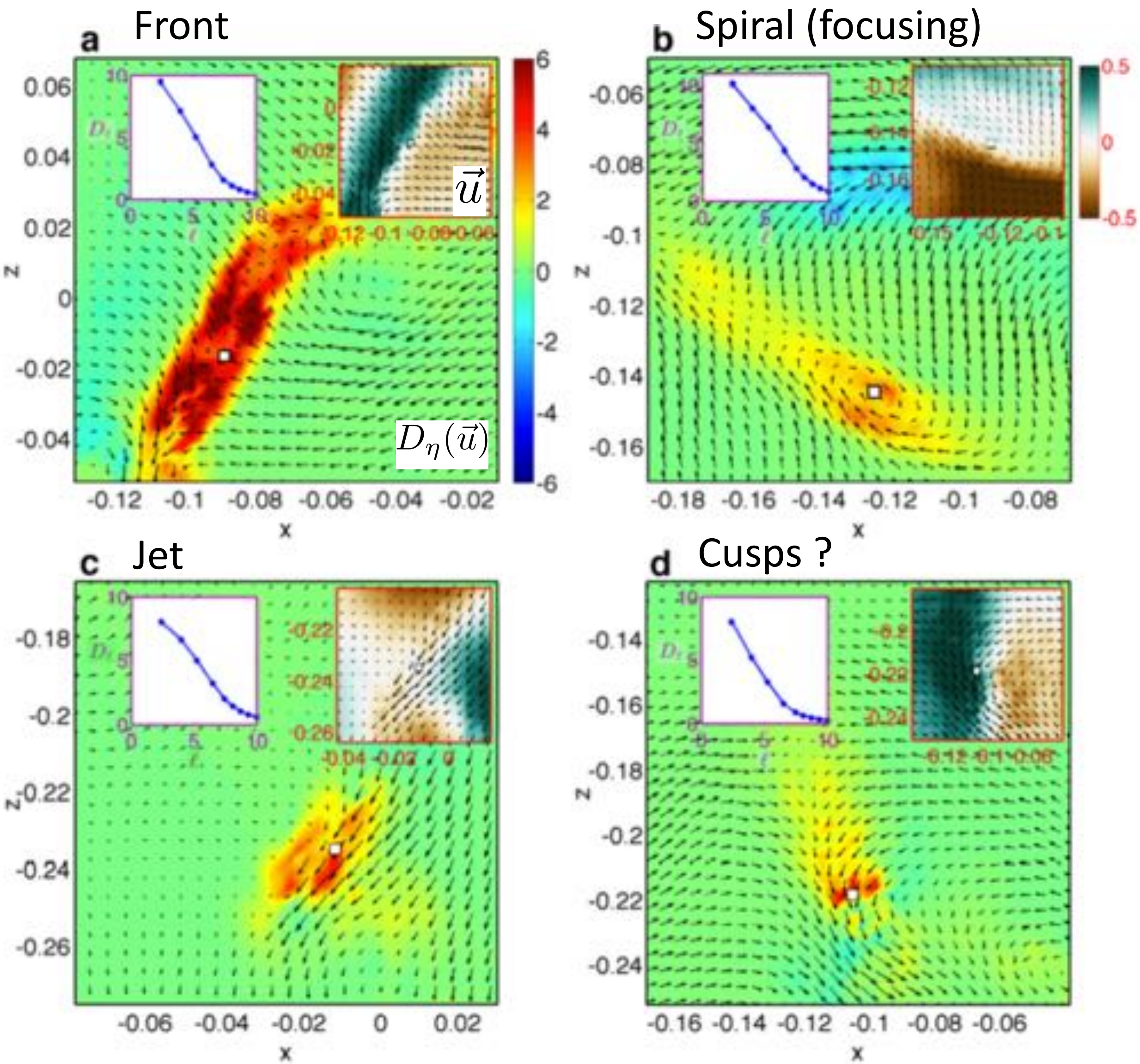
Extreme events of $D_{L=\eta}(u)$

Extreme events : 1000 times of the mean.

Found ~ 30 Events (30,000 frames of 100*100 values)

Can be categorized into 4 geometries (topologies).

75% are fronts.

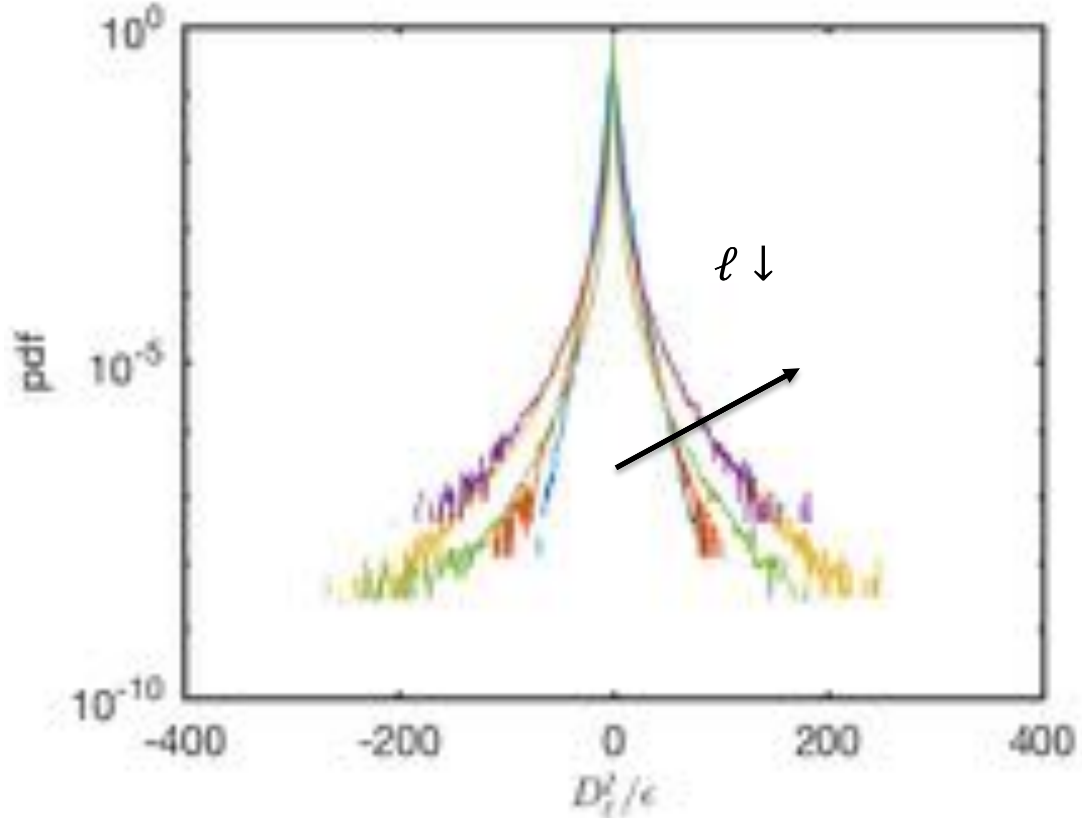


Saw, Kuzzay et al. (2016), Nature-Comm. 7

Statistics of energy transfers

statistics highly non gaussian

PDF of D_ℓ^I

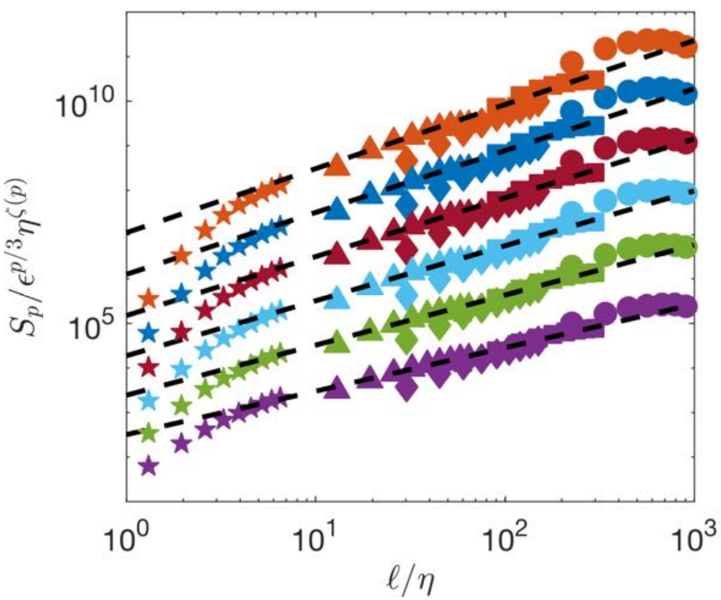


Debue et al et al. (2017), submitted

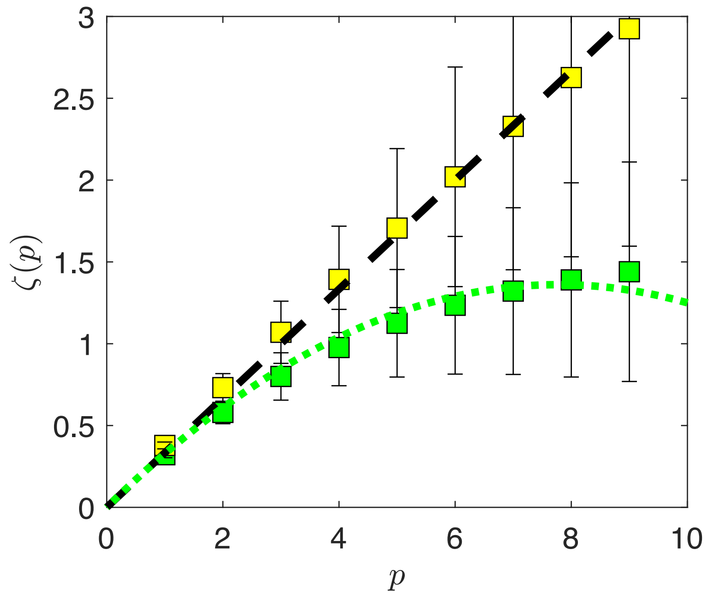
Statistical caracterization of singularities

Large deviation property for velocity
Increments around singularities

$$P\big(ln(\delta u) = hln(\ell)\big) = e^{ln(\ell)C(h)}$$



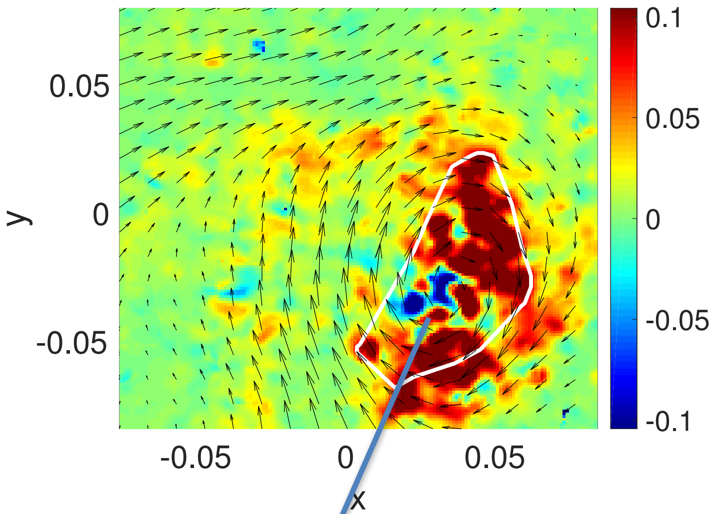
$$\langle e^{p\ ln(\delta u)} \rangle = e^{\zeta(p)\ln(\ell)}$$

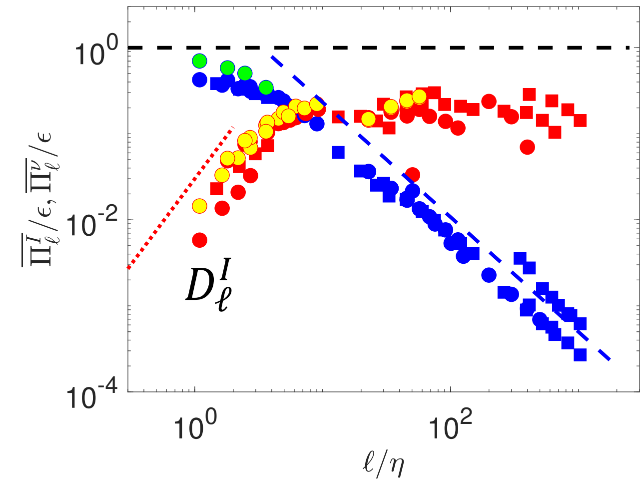


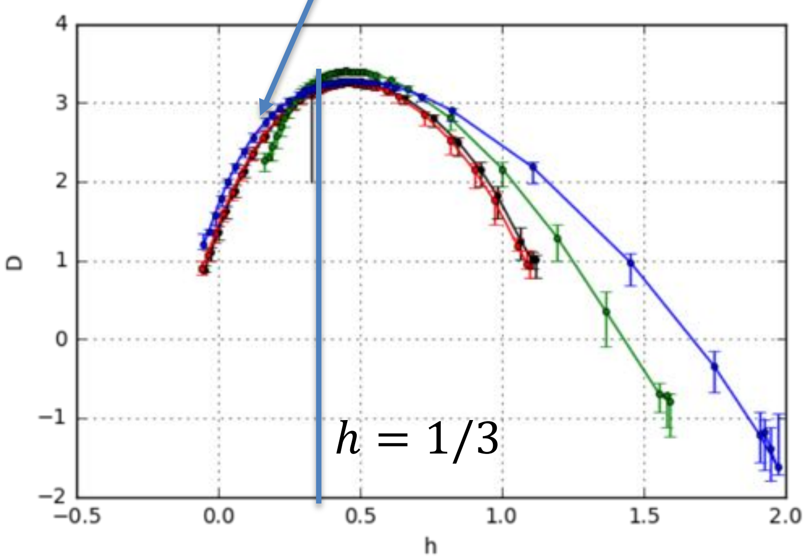
$$\zeta(p) = min_h(ph + C(h))$$

Debue et al. 2018 PRE; Dubrulle 2019 JFM Perspectives

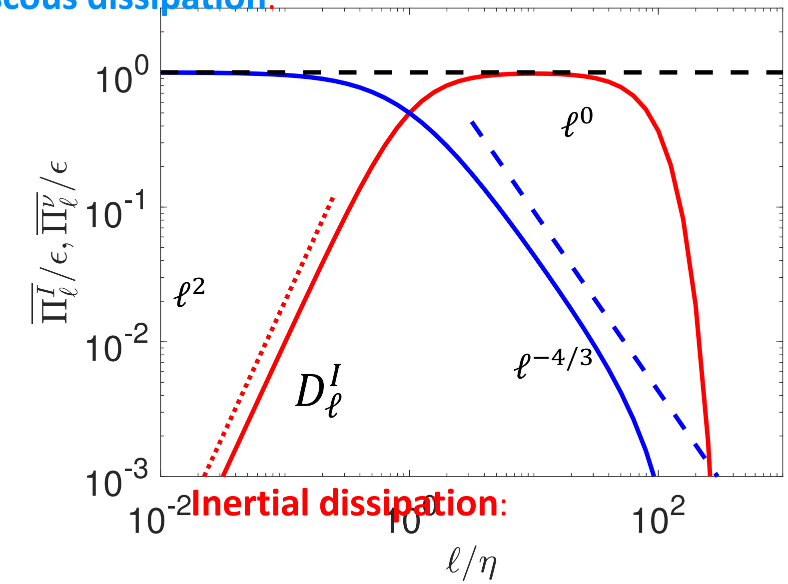
Mean energy budget in von Karman







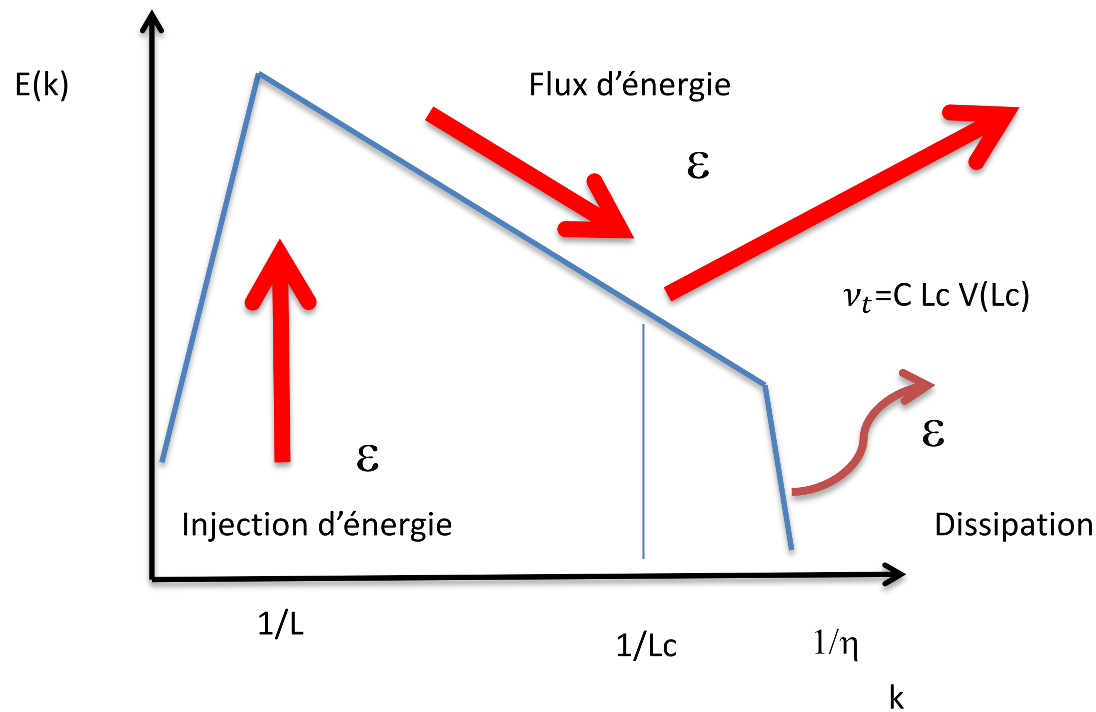
Viscous dissipation:



Inertial dissipation:

Obtained through Legendre transform

Outstanding issues: modelling the flux!



Can we do that using out-of-equilibrium phsics?