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# KINETIC EFFECTS ON THE THERMODYNAMICS OF IRRADIATED MATERIALS

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Université Paris-Saclay, F-91191 Gif-sur-Yvette, France*

## Materials under irradiation: a variety of phenomena

- Point defects supersaturation
- Phase diagrams under irradiation
- Ballistic mixing and patterning
- Radiation induced segregation and precipitation

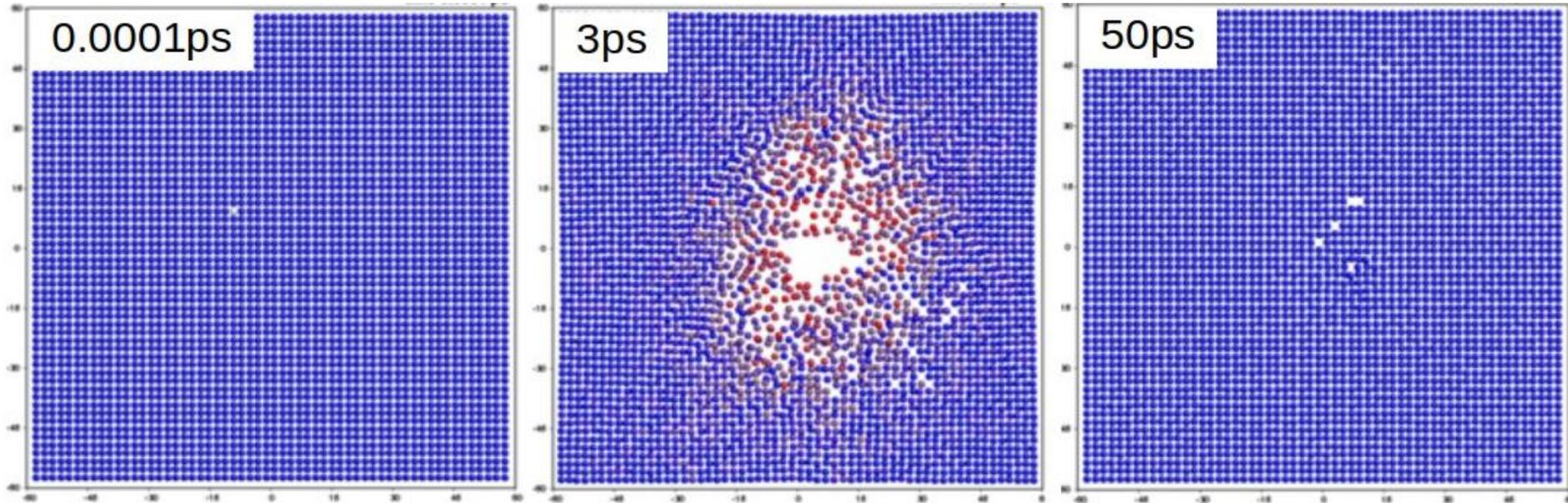
## Computation of transport properties from the atomic scale

- Multi-scale modeling of materials
- The kinetic cluster expansion formalism for dilute systems
- Self-consistent mean field formulation of transport coefficients
- Example calculations
- Changing the reference state of a system with ballistic relocations

## Current questions

# POINT DEFECTS SUPERSATURATION UNDER IRRADIATION

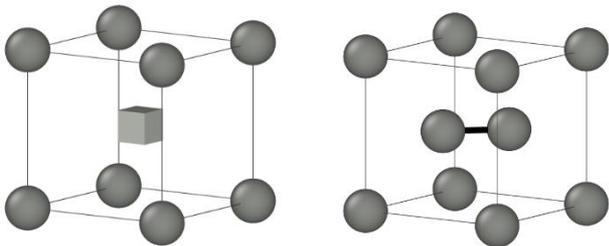
Primary damage in irradiated materials: displacement cascade



*Nordlund et al. (2015) OECD / Nuclear Science, NEA/NSC/DOC(2015)9.*

Driving force : supersaturation with respect to equilibrium concentration  $[PD]^{eq} \simeq \exp\left(-\frac{E_{PD}^f}{k_B T}\right)$

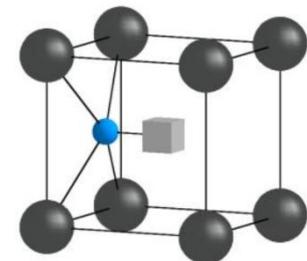
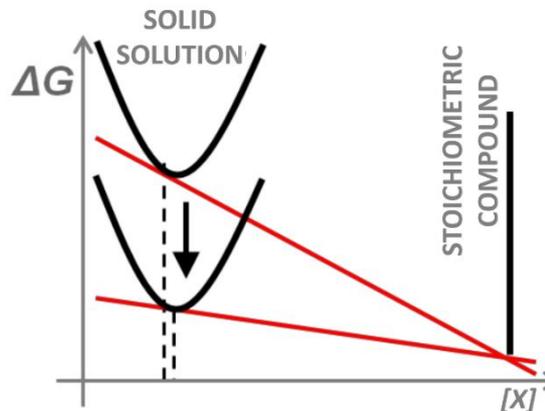
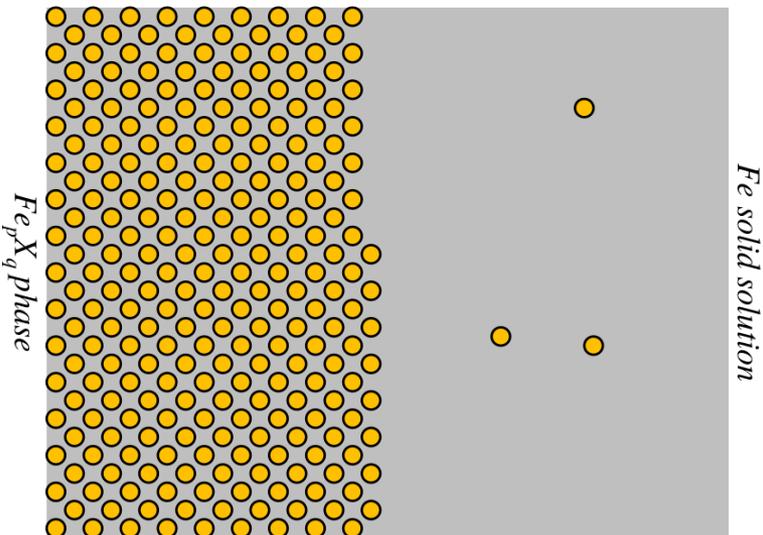
- Vacancy-self-interstitial atom recombination
- Elimination at point defects sinks (interfaces, dislocations, point defect clusters)



*Point-defect mediated solute diffusion introduces two diffusion time scales*

# PHASE DIAGRAMS UNDER IRRADIATION

## EQUILIBRIUM

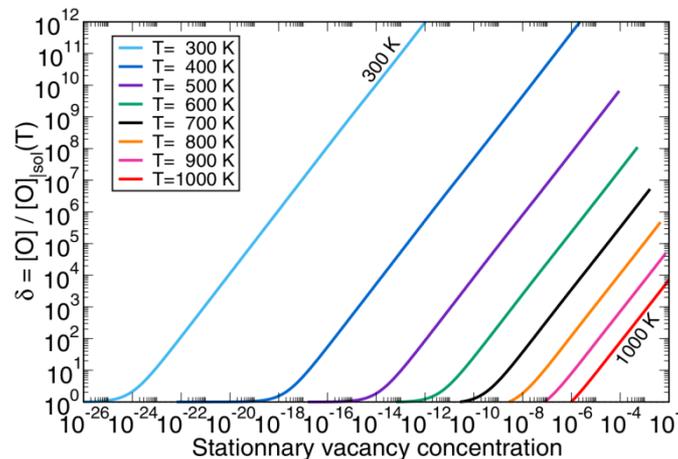
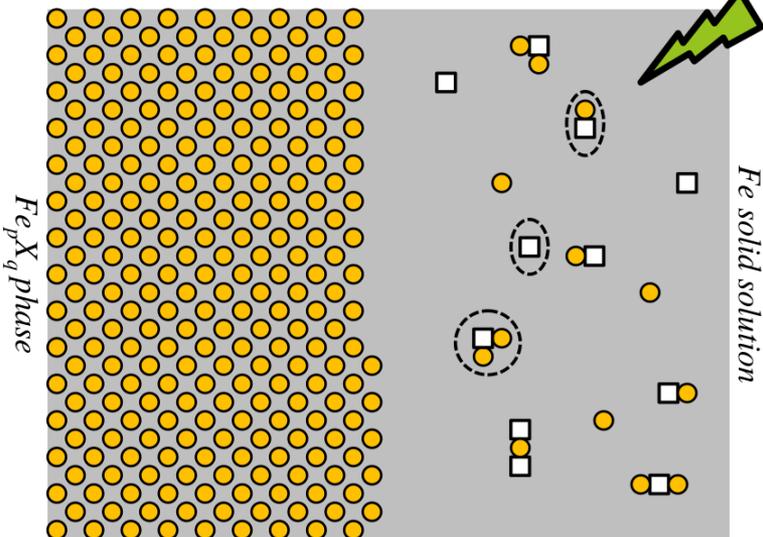


Vacancy-oxygen binding energy in Fe : 1.43 eV

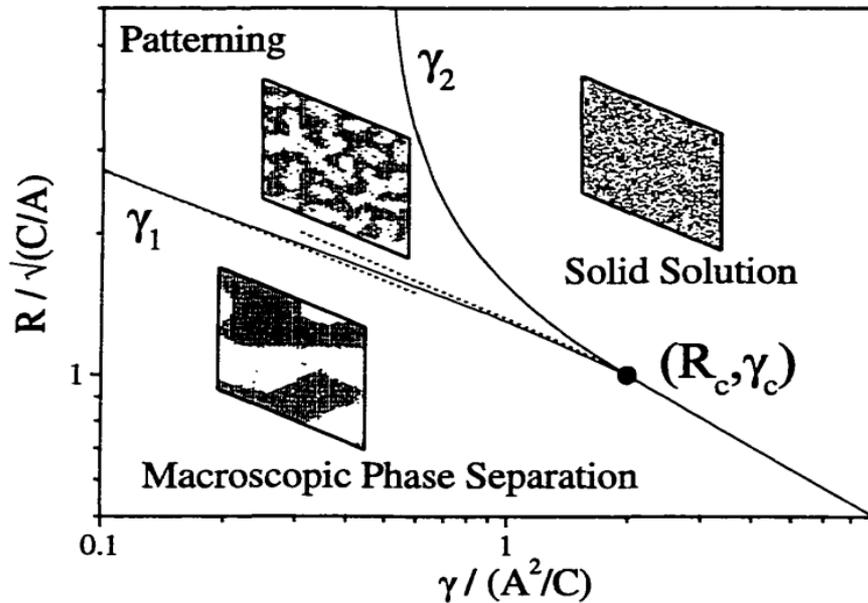
Computing the **point-defect induced solubility limit increase** with a **low-temperature expansion of the grand-canonical partition function**

*Schuler et al. Phys. Rev. B 95 014113 (2017)*

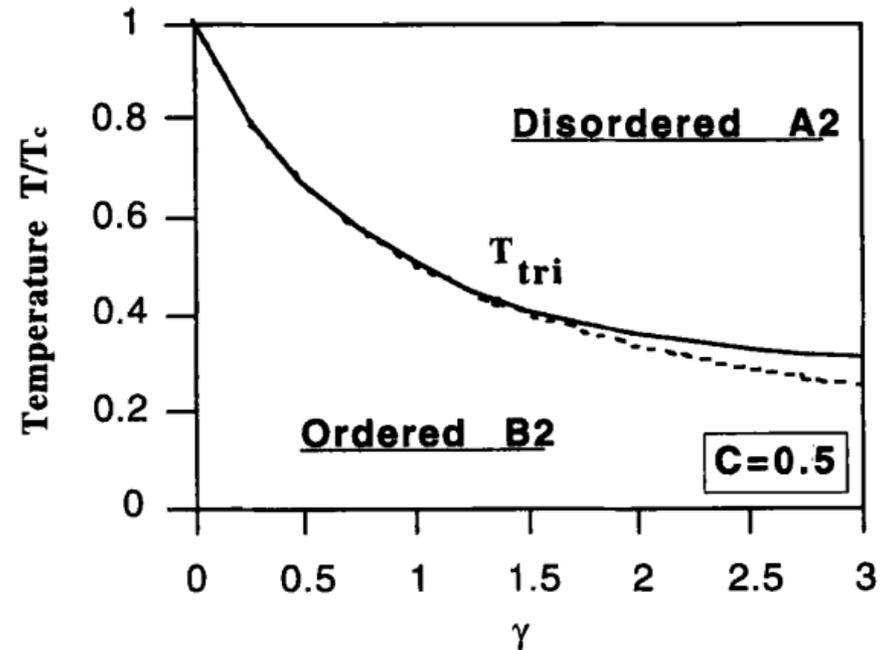
## IRRADIATION



# BALLISTIC MIXING AND PATTERNING UNDER IRRADIATION

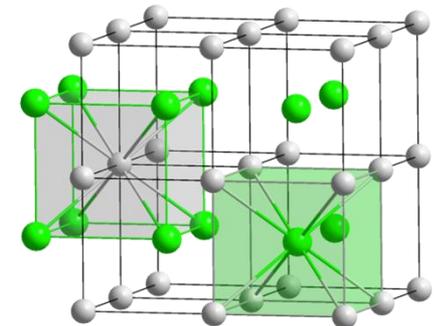


Enrique, Bellon *Phys. Rev. Lett.* **84** 2885 (2000)



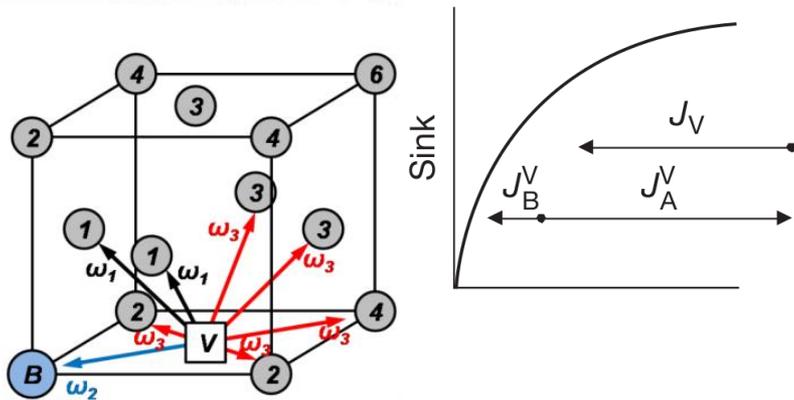
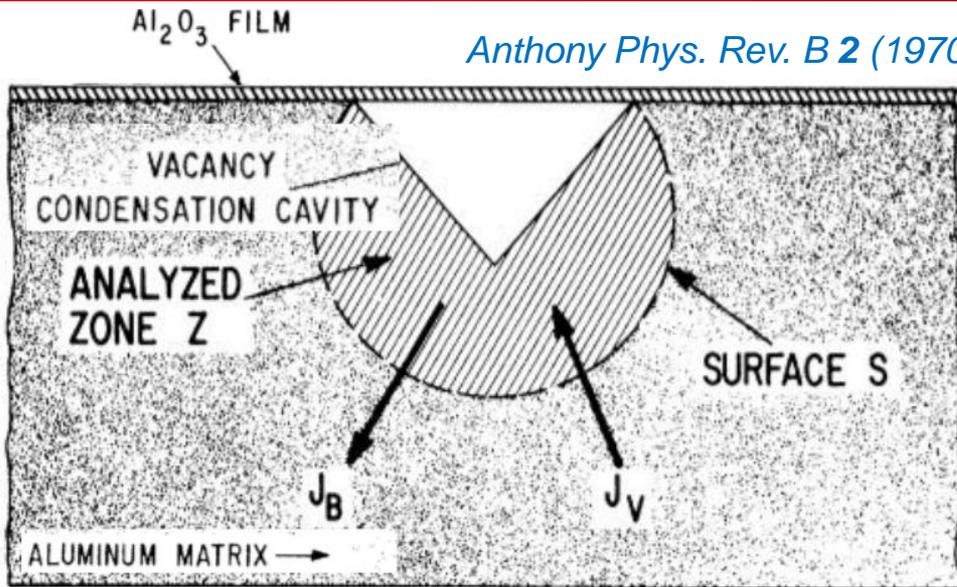
Martin, Bellon *Solid State Phys.* **50** 189 (1997)

Competition between athermal relocation events and thermally-activated jumps



# RADIATION-INDUCED SEGREGATION AND PRECIPITATION

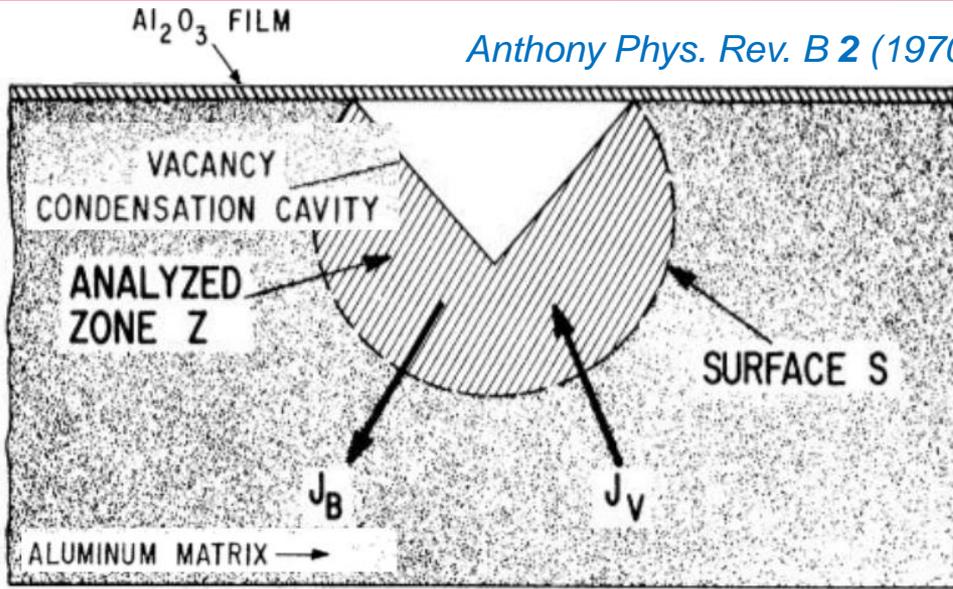
Anthony Phys. Rev. B 2 (1970)



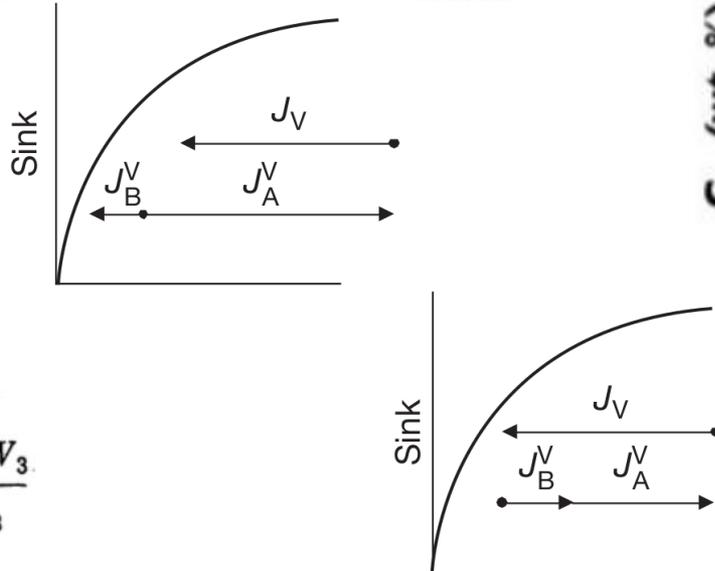
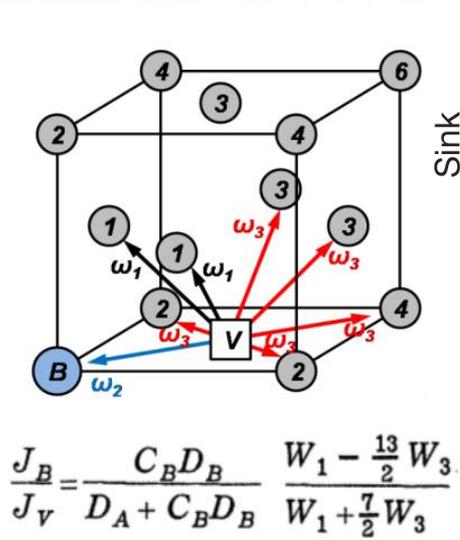
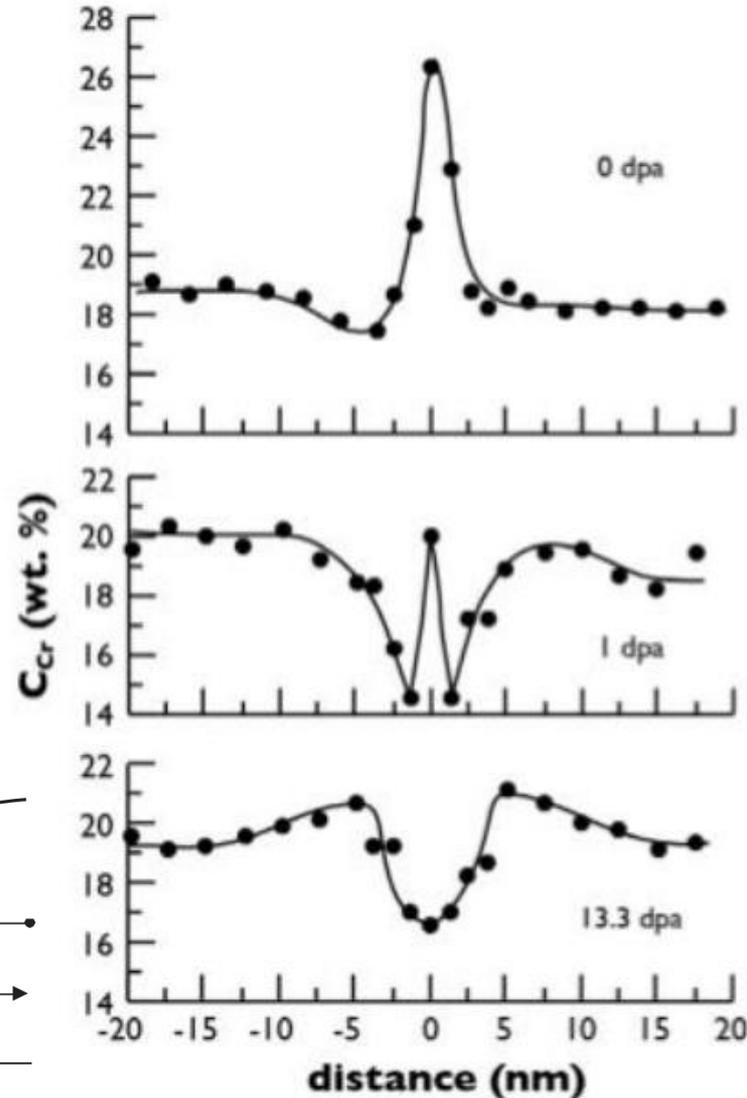
$$\frac{J_B}{J_V} = \frac{C_B D_B}{D_A + C_B D_B} \frac{W_1 - \frac{13}{2} W_3}{W_1 + \frac{7}{2} W_3}$$

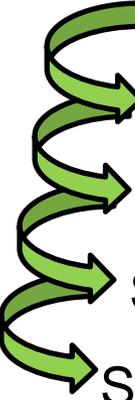
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Anthony Phys. Rev. B 2 (1970)



Ardell and Bellon, Current Opinion in Solid State and Materials Science 20 115 (2016)



- 
- Irradiation by energetic particles
  - Super-saturation of point defects wrt respect to equilibrium
  - Flux of point defects towards point defect sinks
  - Solute redistribution due to flux coupling
  - Steady-state depending on both thermodynamic and kinetic properties

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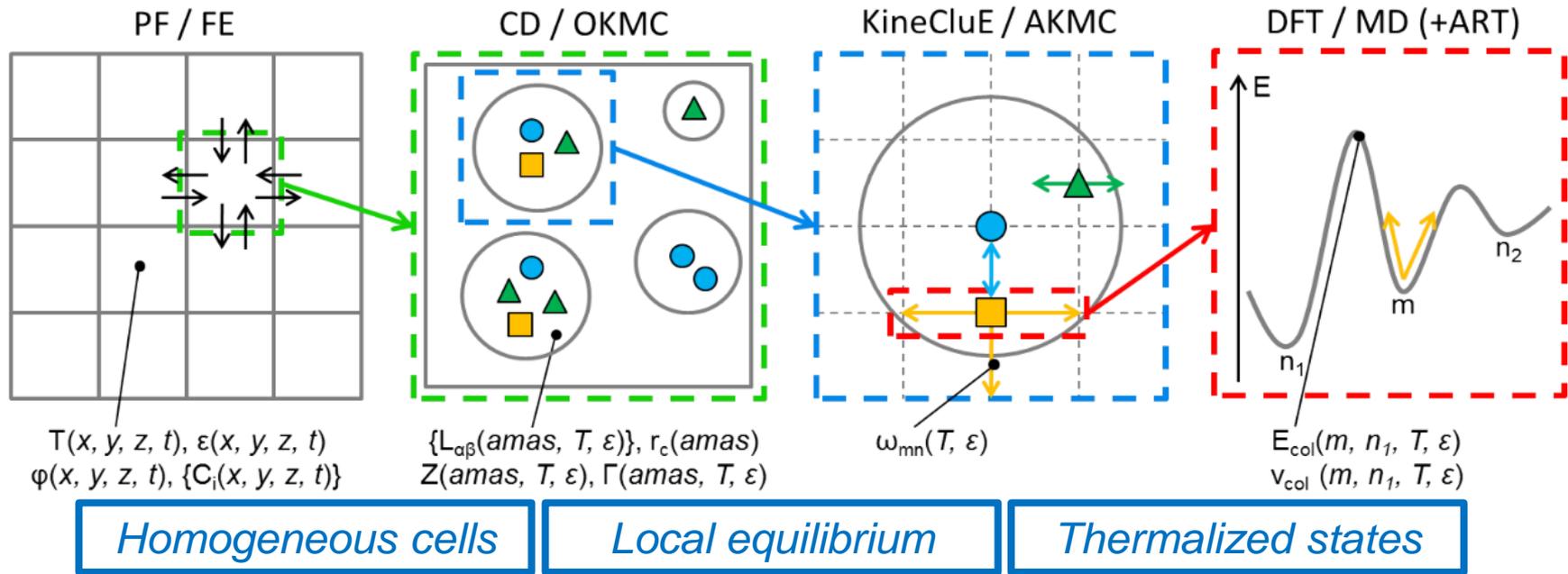
## We need transport coefficients !

Thermodynamics of irreversible processes :

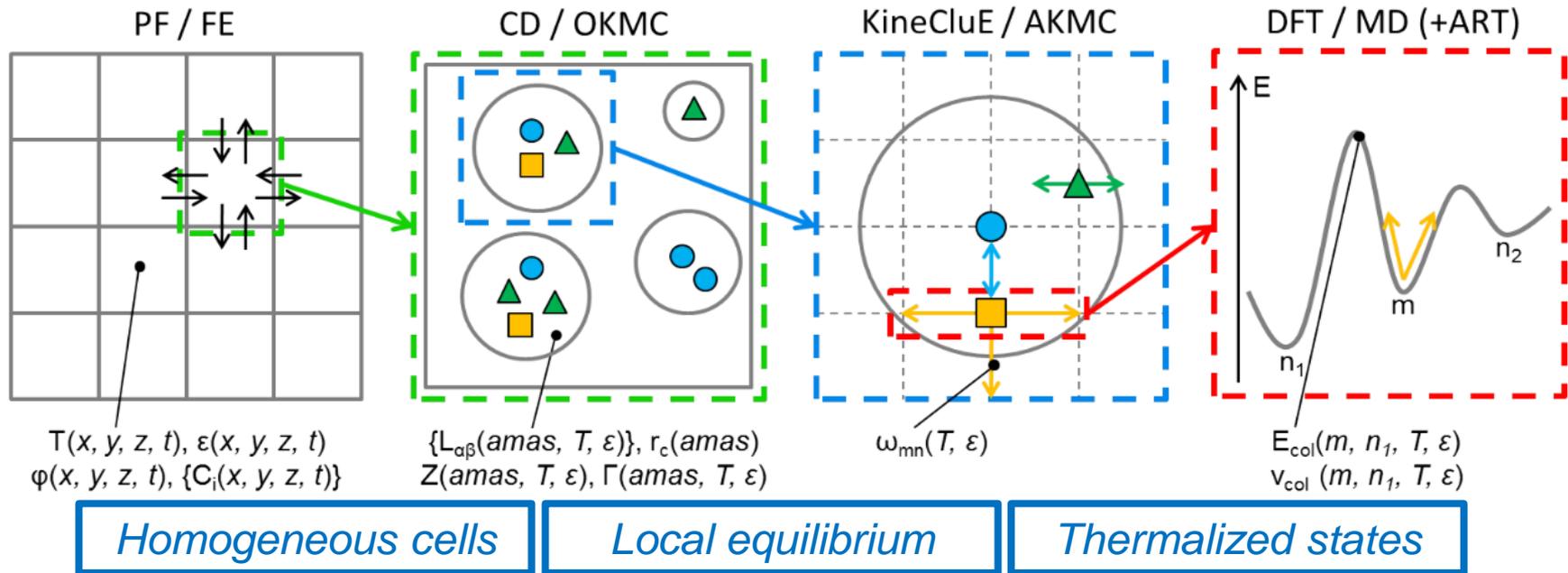
*Onsager Phys. Rev. B* **38** 2265 (1931)

$$\vec{J}_\alpha = - \sum_\beta L_{\alpha\beta} \vec{\nabla} \frac{\mu_\beta}{k_B T}$$

# MULTI-SCALE MODELING OF MATERIALS



# MULTI-SCALE MODELING OF MATERIALS



## Several issues with atomic kinetic Monte Carlo simulations:

- Kinetic trapping
- Result is concentration and temperature dependent
- Off-diagonal coefficients are often difficult to measure precisely

## Analytical models ?

- Dilute models *Lidiard Phil. Mag.* **46** 1218 (1955)
- Concentrated models *Kikuchi J. Chem. Phys.* **51** 161 (1969), *Manning Phys. Rev. B* **4** 1111 (1971)

Thermodynamic cluster expansion:

*Sanchez Phys. A 128 334 (1984)*

$$H(\mathbf{n}) = \sum_i \sum_{\alpha} n_i^{\alpha} \varepsilon_i^{\alpha} + \frac{1}{2!} \sum_{i,j} \sum_{\alpha,\beta} n_i^{\alpha} n_j^{\beta} \varepsilon_{\lambda(ij)}^{\alpha\beta} + \frac{1}{3!} \sum_{i,j,k} \sum_{\alpha,\beta,\gamma} n_i^{\alpha} n_j^{\beta} n_k^{\gamma} \varepsilon_{\lambda(ijk)}^{\alpha\beta\gamma} + \dots,$$

# KINETIC CLUSTER EXPANSION FORMALISM FOR DILUTE SYSTEMS

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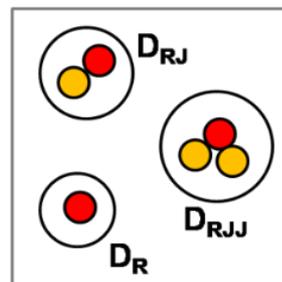
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Kinetic cluster expansion:

*Schuler Comp. Phys. Comm. submitted (2018)*

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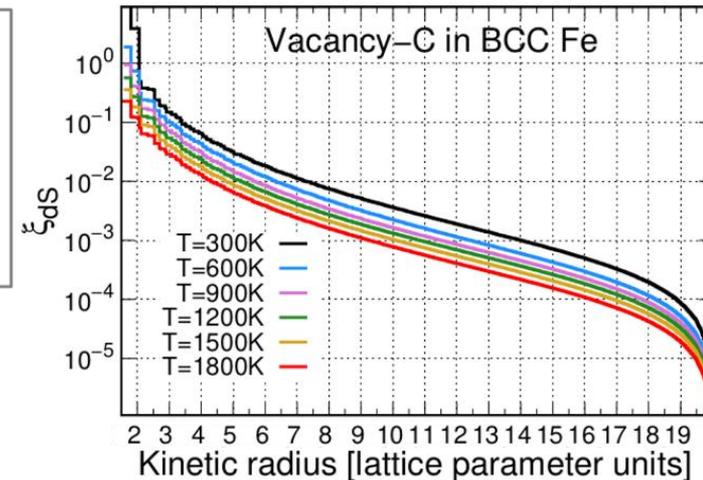
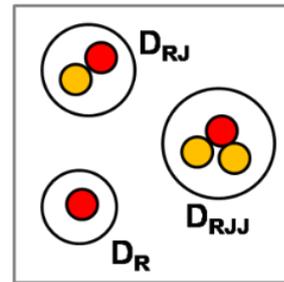
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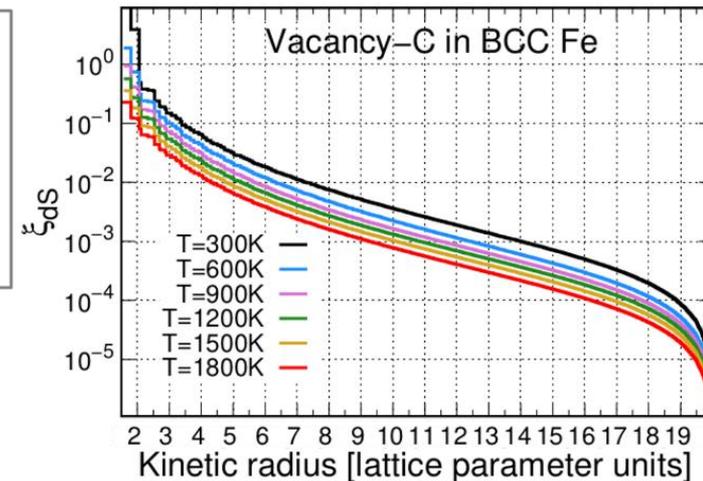
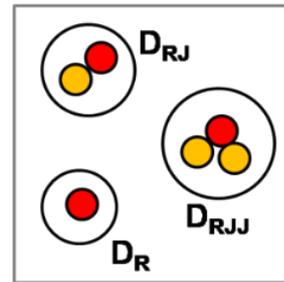
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- Configuration space is split into **independent sub-spaces called cluster**, defined by a kinetic radius
- Transport coefficients converge well with increasing kinetic radius
- This way, cluster transport coefficients are **intrinsic equilibrium properties** of each cluster
- We are thus able to define **transport coefficients for out of equilibrium systems** (not feasible with AKMC)



$$L_{\alpha\beta} = \sum_c [c]^{oe} L_{\alpha\beta}^{eq}(c)$$

# KINETIC CLUSTER EXPANSION FORMALISM FOR DILUTE SYSTEMS

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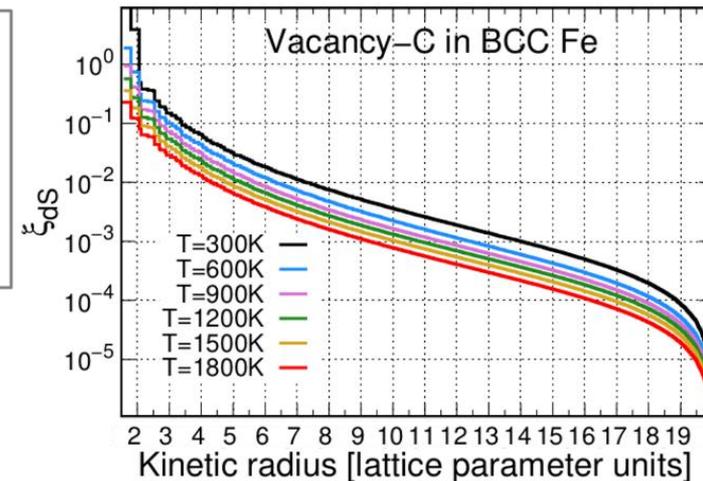
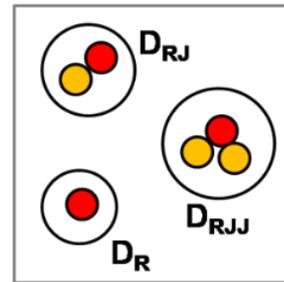
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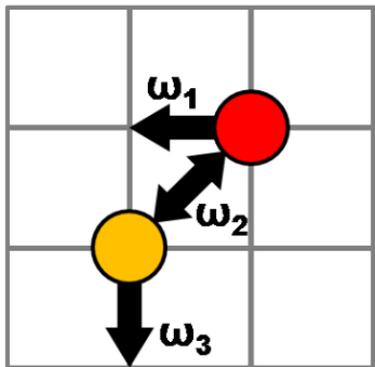
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Open-source code **KineCluE**

## COARSE-GRAINING PROCESS

$$\{\omega_{p \rightarrow q}\}$$

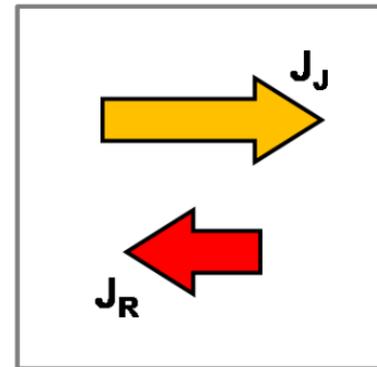
Atomic jump  
frequencies



*DFT+SCMF*

$$\begin{pmatrix} L_{VV} & L_{VX} \\ L_{XV} & L_{XX} \end{pmatrix}$$

Continuous  
description  
(TIP)



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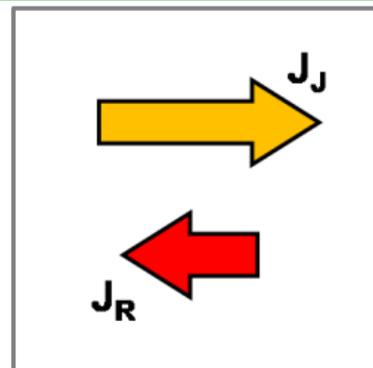
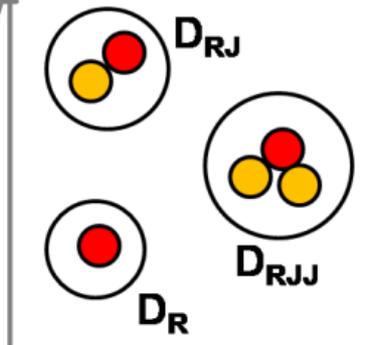
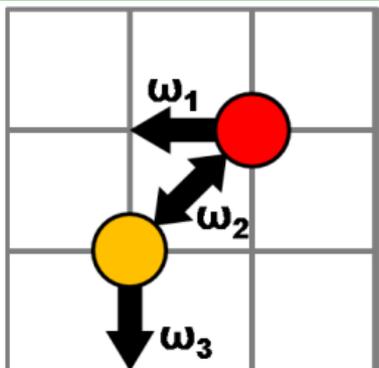
$$\begin{pmatrix} L_{VV}^{eq}(c_i) & L_{VX}^{eq}(c_i) \\ L_{XV}^{eq}(c_i) & L_{XX}^{eq}(c_i) \end{pmatrix}$$

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Atomic jump frequencies

Cluster transport coefficients

Continuous description (TIP)



**DFT+SCMF**

$$L_{\alpha\beta} = \sum_{c_i} [c_i] L_{\alpha\beta}^{eq}(c_i)$$

Local equilibrium within a cluster is assumed

# TRANSPORT COEFFICIENTS

## Fluctuations point of view

*Kinetic Monte Carlo, Molecular Dynamics*

*Allnatt J. Phys. C: Solid State Phys. 15 5605 (1982)*

$$L_{\alpha\beta} = \lim_{\tau \rightarrow \infty} \frac{\langle \Delta \vec{R}_{\alpha}(\tau) \Delta \vec{R}_{\beta}(\tau) \rangle}{6V\tau}$$

## Thermodynamics of irreversible processes

*Analytical models*

*Onsager Phys. Rev. B 38 2265 (1931)*

$$\vec{J}_{\alpha} = - \sum_{\beta} L_{\alpha\beta} \vec{\nabla} \frac{\mu_{\beta}}{k_{\text{B}}T}$$

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### SCMF in brief:

- Rigid lattice
- Atomic jump mechanisms
- Application of a driving force (small)
- Computation of the resulting flux
- Identification of transport coefficients

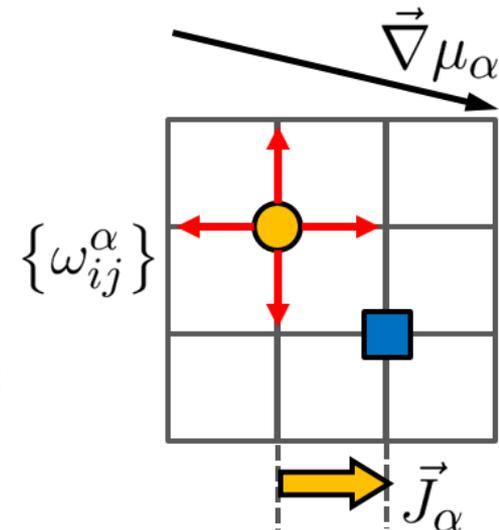
*Nastar et al. Phil. Mag. A, 80 155 (2000)*

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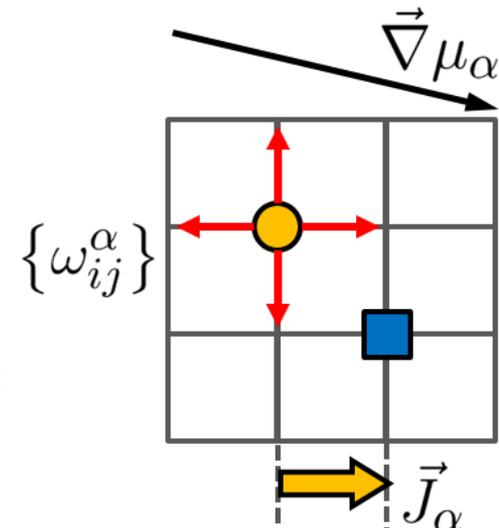
Can we use our transport coefficient formalism to obtain equilibrium fluctuations, and hence study nucleation phenomena ?

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*Analytical models*

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*Microscopic master equation*

$$\frac{dP(\mathbf{n}, t)}{dt} = \sum_{\tilde{\mathbf{n}}} [W(\tilde{\mathbf{n}}, \mathbf{n}) P(\tilde{\mathbf{n}}, t) - W(\mathbf{n}, \tilde{\mathbf{n}}) P(\mathbf{n}, t)]$$

*Out-of-equilibrium probability distribution*  $P(\mathbf{n}, t) = P_0(\mathbf{n}) \delta P(\mathbf{n}, t)$

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*Continuity equation per crystallographic site*

$$\frac{d[\alpha]_i}{dt} = - \oiint_S \vec{J}_i^\alpha \cdot d\vec{S} \quad [\alpha]_i = \langle n_i^\alpha \rangle^{\text{oe}} = \sum_{\mathbf{n}} n_i^\alpha P(\mathbf{n}, t)$$

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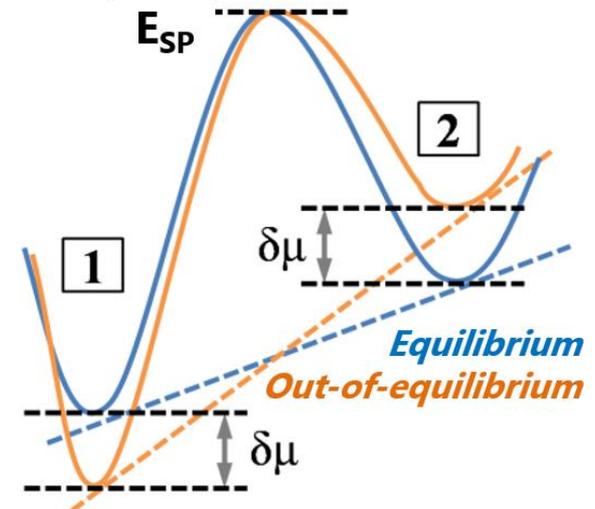
*Assumption : microscopic detailed balance*

$$\frac{d \langle n_j^\beta \rangle^{\text{oe}}}{dt} = \left\langle \sum_{\tilde{\mathbf{n}}} n_j^\beta W(\mathbf{n}, \tilde{\mathbf{n}}) [\delta P(\tilde{\mathbf{n}}) - \delta P(\mathbf{n})] \right\rangle$$

*Deviation from equilibrium probability distribution*

$$\delta P(\mathbf{n}, t) = \exp \left( \delta \Omega + \sum_{i,\alpha} n_i^\alpha \frac{\delta \mu_i^\alpha}{k_B T} - \sum_{\sigma} n_{\sigma} \frac{v_{\sigma}}{k_B T} \right)$$

*Assumption : small deviations from equilibrium*



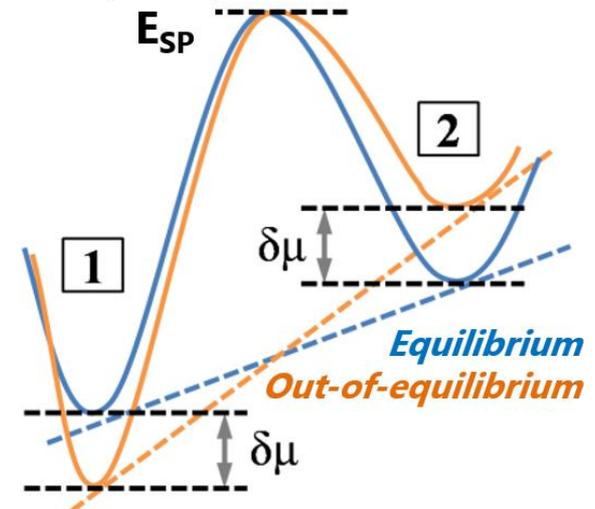
# SELF-CONSISTENT MEAN FIELD FORMULATION OF TRANSPORT COEFFICIENTS (2/2)

*Deviation from equilibrium probability distribution*

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*Assumption : small deviations from equilibrium*

Effective interactions do not obey the same symmetries as thermodynamic interactions !



*Deviation from equilibrium probability distribution*

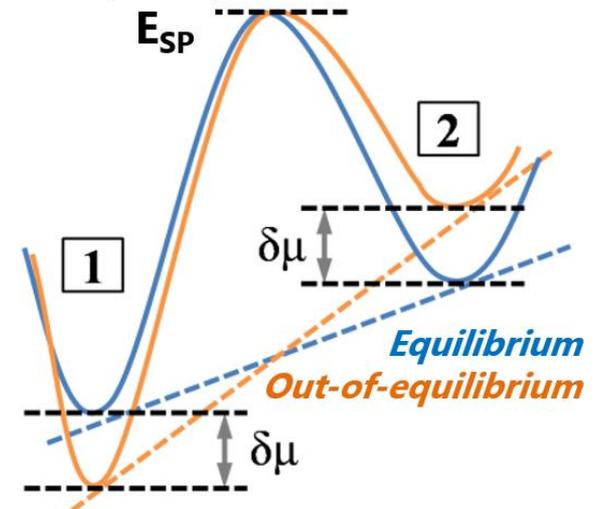
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*Imposing all moments of the probability distribution to be stationary, we find the values of effective interactions*

$$\frac{d}{dt} \left\langle n_{i_1}^{\alpha_1} n_{i_2}^{\alpha_2} \dots n_{i_n}^{\alpha_n} \right\rangle^{oe} = 0$$

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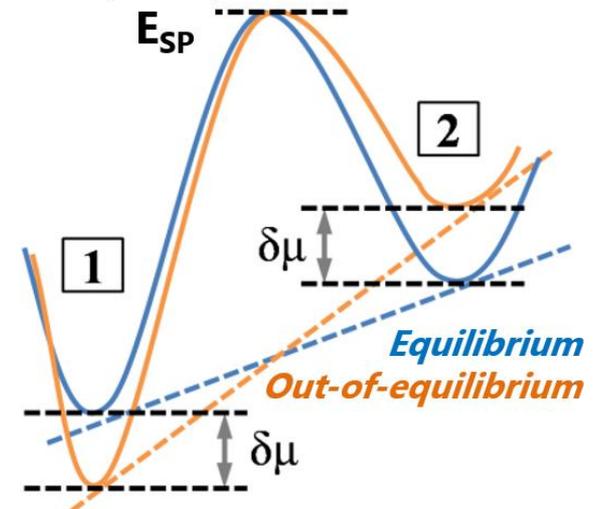
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- Can we use these effective interactions to compute out-of-equilibrium phase diagrams ?
- Can we go beyond small deviations from equilibrium ?
- Boundary conditions for subcluster moments ?

# EXAMPLE CALCULATION : CONCENTRATION EFFECTS IN Fe-C ALLOYS

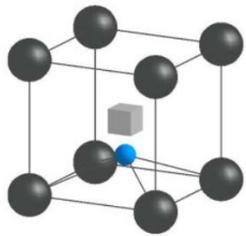
Flux coupling  
between vacancy  
and carbon:

$$\vec{J}_V = -L_{VV} \vec{\nabla} \mu_V$$

$$\vec{J}_C = \frac{L_{CV}}{L_{VV}} \vec{J}_V$$

1. Classical « dilute » approximation : at most one vacancy interacting with one carbon atom => **negative flux coupling**

*Schuler Phys. Rev. B 93 224101 (2016)*



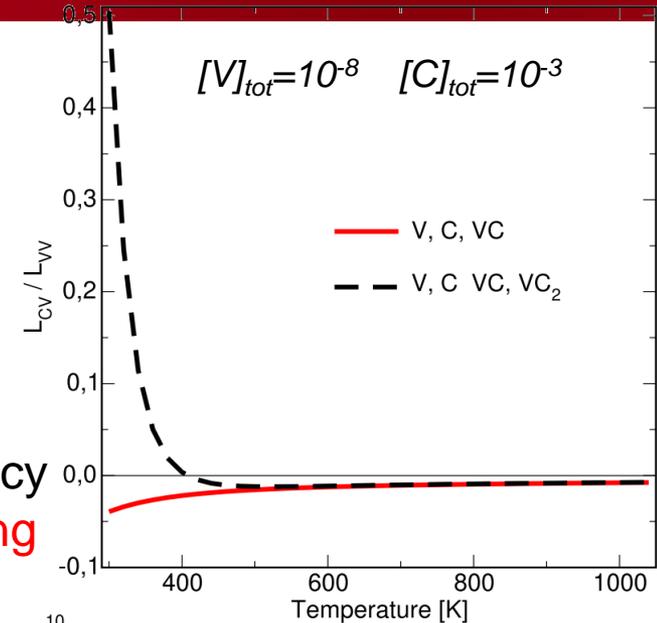
$$\frac{L_{CV}}{L_{VV}} = \frac{[VC] L_{CV} (VC)}{[V] L_{VV} (V) + [VC] L_{VV} (VC)}$$

# EXAMPLE CALCULATION : CONCENTRATION EFFECTS IN Fe-C ALLOYS

Flux coupling  
between vacancy  
and carbon:

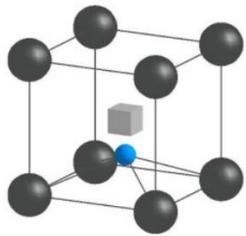
$$\vec{J}_V = -L_{VV} \vec{\nabla} \mu_V$$

$$\vec{J}_C = \frac{L_{CV}}{L_{VV}} \vec{J}_V$$



1. Classical « dilute » approximation : at most one vacancy interacting with one carbon atom => **negative flux coupling**

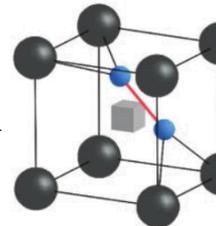
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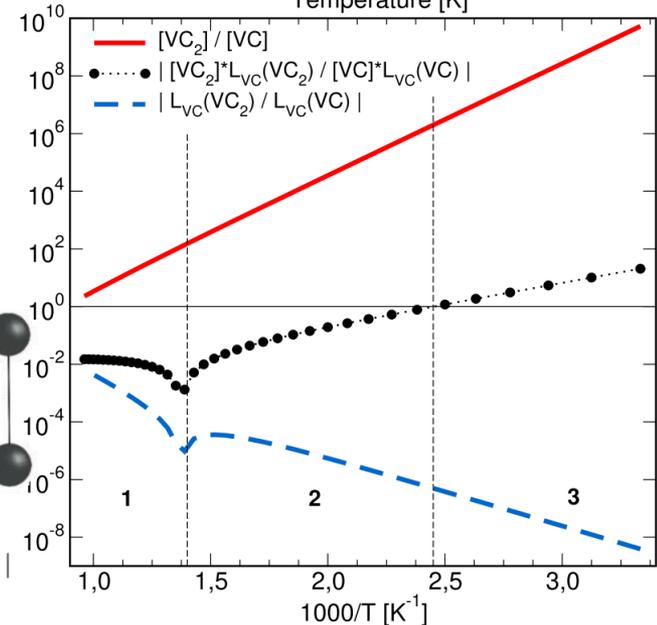
$$\frac{L_{CV}}{L_{VV}} = \frac{[VC] L_{CV}(VC)}{[V] L_{VV}(V) + [VC] L_{VV}(VC)}$$

2. First order concentration effect : allowing a second C atom around V => **qualitatively different result !**

$$\frac{L_{CV}}{L_{VV}} = \frac{[VC] L_{CV}(VC) + [VC_2] L_{CV}(VC_2)}{[V] L_{VV}(V) + [VC] L_{VV}(VC) + [VC_2] L_{VV}(VC_2)}$$



Thomas Schuler |



DFT data from *Barouh Phys. Rev. B 92 104102 (2015)*

# EXAMPLE CALCULATION: JUMP FREQUENCY SENSITIVITY STUDY

*Goal: identification of key input parameters*

We compute the gradient of cluster transport coefficients in jump frequency space :

$$\nabla \mathbf{L}_d = \left( \frac{\partial \mathbf{L}_d}{\partial \omega_i} \Big|_{\mathbf{w}_0} \right)$$

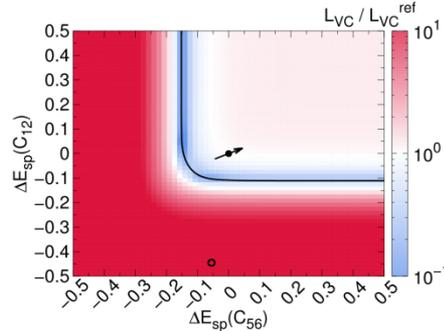
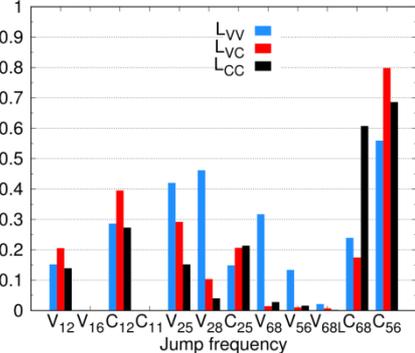
We have analytic expressions for this gradient => does not cost much computation time

$$\mathbf{L}_d = \Lambda_d^0 - \Lambda_d \tilde{\mathbf{T}}^{-1} \Lambda_\mu^t$$

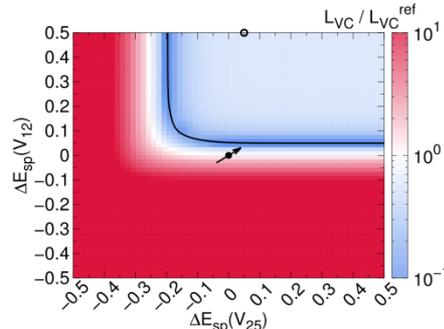
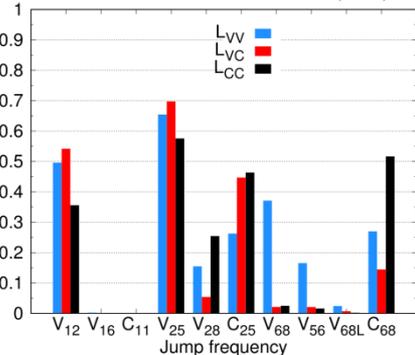
$$\frac{\partial \mathbf{L}_d}{\partial \omega_i} = \frac{\partial \Lambda_d^0}{\partial \omega_i} + \Lambda_d \tilde{\mathbf{T}}^{-1} \frac{\partial \tilde{\mathbf{T}}}{\partial \omega_i} \tilde{\mathbf{T}}^{-1} \Lambda_\mu - \frac{\partial \Lambda_d}{\partial \omega_i} \tilde{\mathbf{T}}^{-1} \Lambda_\mu - \Lambda_d \tilde{\mathbf{T}}^{-1} \frac{\partial \Lambda_\mu}{\partial \omega_i}.$$

Iterative process because the analysis is only local

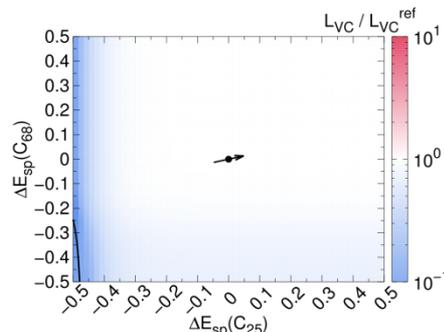
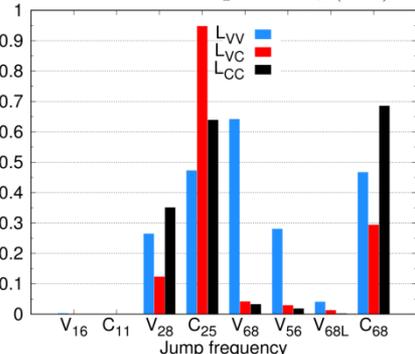
Initial:  $L_{VC}(VC) = -2.345 \times 10^{-15} \text{ m}^2 \cdot \text{s}^{-1}$



Step 1:  $L_{VC}(VC) = +3.750 \times 10^{-14} \text{ m}^2 \cdot \text{s}^{-1}$



Step 2:  $L_{VC}(VC) = -2.263 \times 10^{-14} \text{ m}^2 \cdot \text{s}^{-1}$



Final:  $L_{VC}(VC) = -2.388 \times 10^{-14} \text{ m}^2 \cdot \text{s}^{-1}$

# CHANGING THE REFERENCE STATE OF A SYSTEM WITH BALLISTIC RELOCATIONS

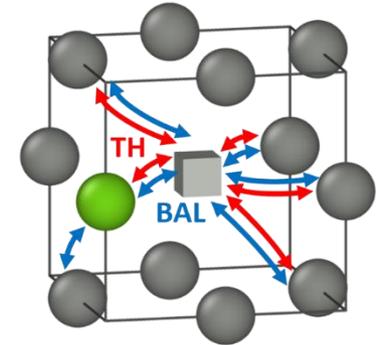
Ballistic relocations introduce **new jump mechanisms** at frequencies that are not thermally activated and hence do not obey detailed balance.

Three things we need to do beforehand :

- Add ballistic relocation mechanisms
- Find the **steady state** (global detailed balance)

$$\frac{dP_i}{dt} = 0 = \sum_j \tilde{P}_j (\omega_{ji}^{th} + \Gamma_{ij}^{bal}) - \tilde{P}_i (\omega_{ij}^{th} + \Gamma_{ij}^{bal})$$

- Compute effective jump frequencies to obey microscopic detailed balance  $\tilde{P}_j \tilde{W}_{ji} = \tilde{P}_i \tilde{W}_{ij}$



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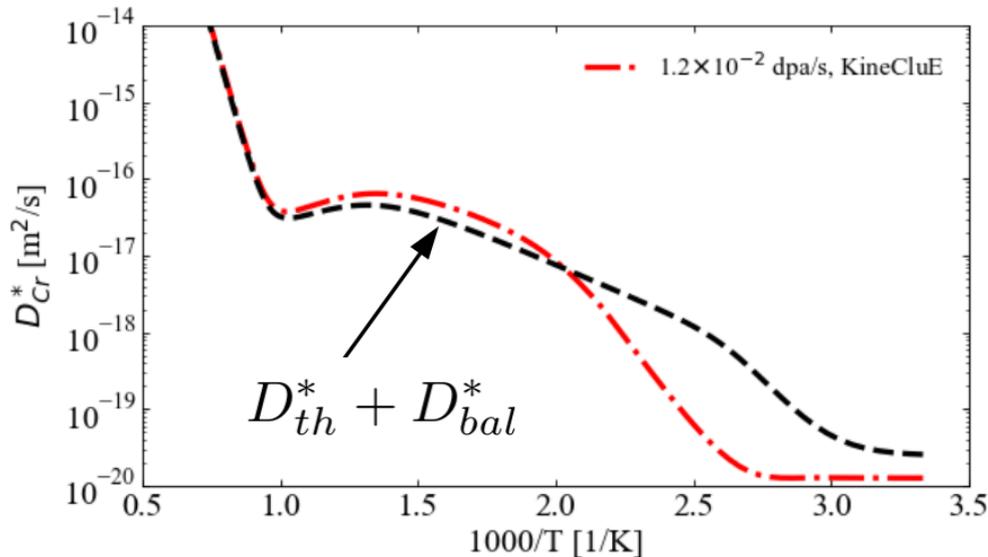
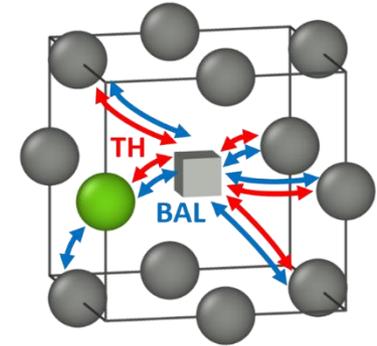
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*Example:* Cr tracer diffusion coefficient in FCC Ni. Because of correlations between thermal and ballistic jumps,

$$D^* \neq D_{th}^* + D_{bal}^*$$

Huang Phys. Rev. B submitted (2018)

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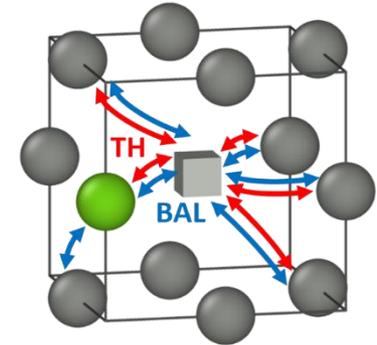
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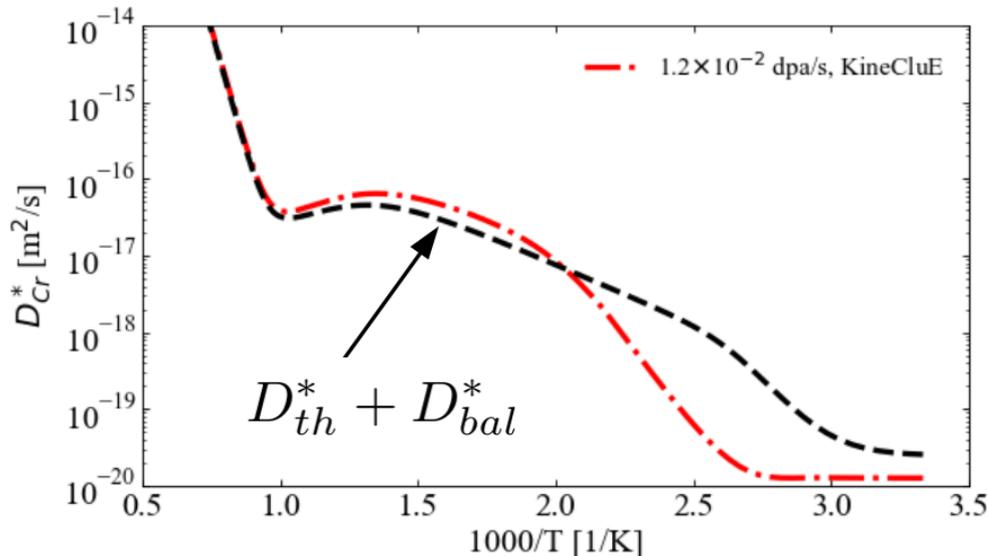
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Is this a good way to approach systems far from equilibrium ?



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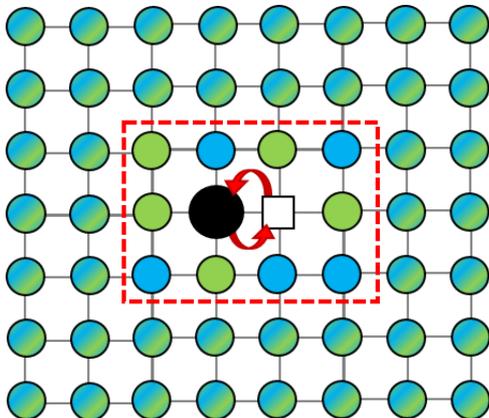
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# OTHER OPEN QUESTIONS

1. **Computational load for clusters larger than 4 elements**
  - Parallelization
  - Thermodynamic sampling
2. **Concentrated alloys**
  - Matrix effect on jump frequencies and kinetic correlations
  - Embedding the cluster in some effective matrix (mean-field)
  - Boundary condition ?



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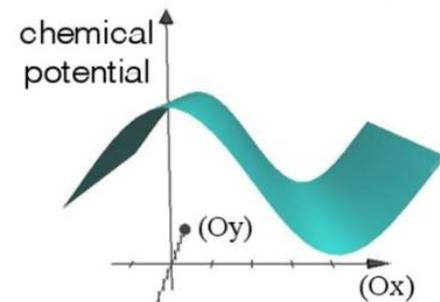
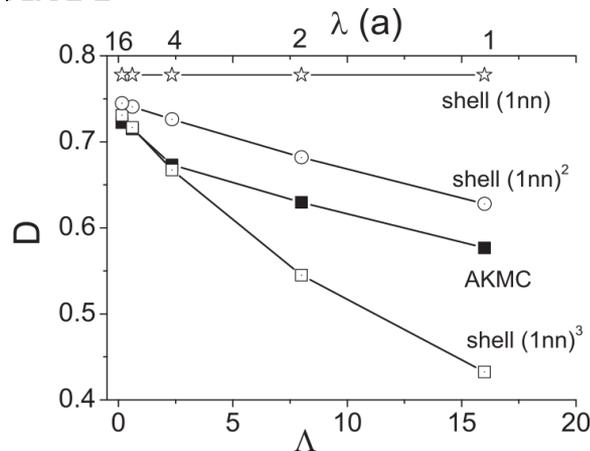
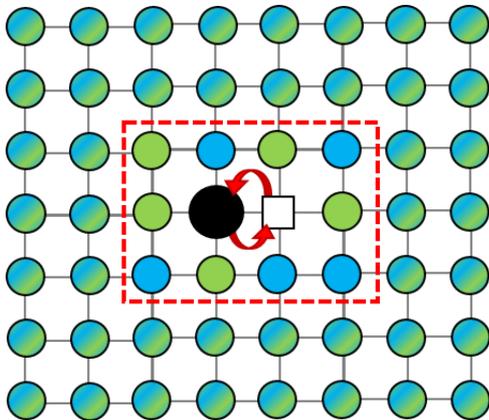
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## 2. Concentrated alloys

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## 3. Inhomogeneous driving forces

- Beyond the thermodynamic formulation of Onsager
- Importance of the different diffusion time scales between point defects and solutes



*star Phys. Rev. B 90 144101 (2014)*

*Thank you for your attention  
Any questions ?*

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