Minimal field theories for active systems

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Phénomènes hors d'équilibre
IPhT
02 October 2018

3 ways into non-equilibrium

- **Very slow relaxation towards equilibrium**
  So slow that the transient can be considered a bona fide state of matter
  Ex: glasses, ...

- **Global external forcing of an otherwise equilibrium system**
  imposing T, μ gradients, mechanical stirring ...
  Ex: turbulence, ...

- **Out of equilibrium in the bulk**
  Each microscopic component is out of equilibrium
  Ex: Active matter, ...

Group

H. Chaté

I. Dornic

C. Nardini

A. Patelli
(post-doc, '18-'19)

G. Fausti
(Ph.D., '18-'21)

B. Ventejou
(Ph.D., '18-'21)

Collaborations

Theory/Modeling

- M. Cates (Cambridge, phenomenology of active systems)
- A. Morozov (Edinburgh, hydrodynamic interactions)
- J. Tailleur, F. van Wijland (Paris 6, minimal models)
- B. Delamotte (Paris 7, universality)
- X. Shi (Soochow, large scale numerics)
- E. Bertin (Grenoble, continuum theories)
- K. Mallick (IPhT, Palm funding)
- ...

Experimentalists

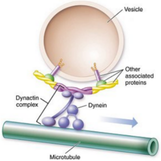
- M. Sano (Tokyo, bacterial thin films)
- H.P Zhang (Shangai, bacterial thin films)
- W. Poon (Edinburgh, 3d bacterial suspensions)
- G. Dumenil (Pasteur, in vivo bacterial behavior)
- G. Theraulaz (Toulouse, self-organization in fish groups)
- L. Blanchoin, M. Théry (Grenoble, cytoskeleton, Impulsion funding)

Active Matter: a brief introduction

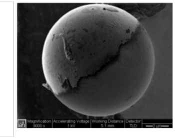
Any active unit is able to extract energy from the environment and dissipate it to self-propel

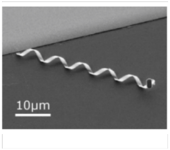
Human scale

Micrometer-scale (biological)



Micrometer-scale (sythetic)





Active matter: a brief introduction

Novel collective phenomena

[Chen et al., Nature, 2017]

- Most of the work on systems where units are μm-sized (bacteria, microtubules, cells – tissues, synthetic active colloids, ...)
- Self-organized structures, collective intelligence, smart materials, ...

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Understanding requires theoretical physics tools

(not only, of course!)

And gives back to theoretical physicists new – intriguing – problems to solve

- 2-d active nematics: topological defects [Dogic et al., Nature 2013, ...]
- +1/2 defects are motile

- In our group [Li et al., PNAS 2018]: quantitative prediction of defects speed and shape; comparison with experiments on 2d bacterial suspensions

2d swarming filamentous bacteria

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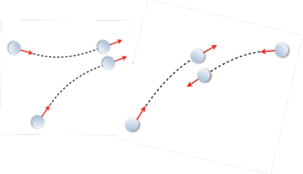

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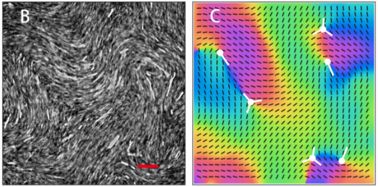
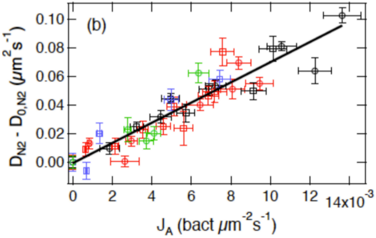


Passive tracer in a bacterial suspension

Tracer diffusivity in 3-d bacterial suspension?

Extensive experimental work

(Wu et al. PRL, 2000; Goldstein et al. PRL, 2009; Poon et al., PRE(R) 2013; Polin et al. Nat. Phys. 2015, ...)



Very low densities (bacteria packing fraction $\phi \sim 0.01 - -0.1\%$)
$\sim$ bacteria don't interact with each other

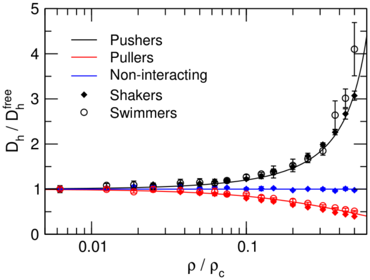
Passive tracer in a bacterial suspension

Can we describe tracer diffusivity at higher densities (but still low $\phi \sim 1\%$)?

- Explicit coarse graining (hydrodynamics $\sim$ long-range)
- Stochastic continuum theory for $f(\mathbf{r}, \mathbf{p}, t) \sim$ Kinetic theory for plasma

$$\partial_t f + \nabla_{\mathbf{r}}(\dot{\mathbf{r}} f) + \partial_{\mathbf{p}}(\dot{\mathbf{p}} f) = \text{noise}$$

- $\dot{\mathbf{r}} = v_s \mathbf{p} + U_{mean-field}\,,\ \dot{\mathbf{p}} = \dots$



- So far: analytical and numerical predictions
- $\rho_c \sim \phi \sim 1\%$
- Experiments in progress in W. Poon group (Edinburgh)

[PRL 028005, 2017]

Minimal field theory for phase separation

M.E. Cates

E. Tjhung

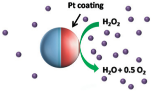
F. Caballero

- Tjhung et al., PRX 031080, 2018
- F. Caballero et al., PRL 020601, 2018
- F. Caballero et al., ArXiv:1809.10433, 2018

Simplest active particles

- Self-propelled spherical particles
- Two-body steric interactions
- No alignment

Ex: Janus colloids



- microscale: persistent Brownian motion(speed v, rotational diffusivity $1/\tau$)
- large-scales: Brownian motion with diffusivity $D \sim v^2\tau$

Can many (so-simple!) particles do anything interesting?

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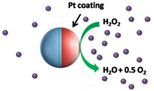

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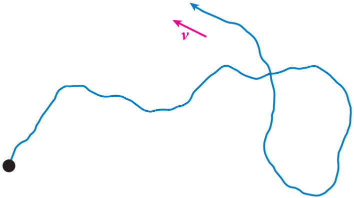
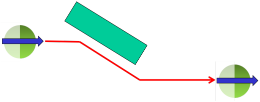
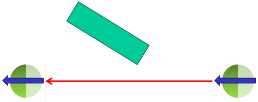


Can many (so-simple!) particles do anything interesting?

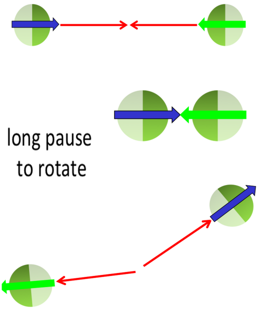
Interactions: TRS breakdown

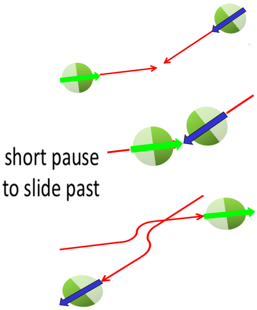
... with obstacles....





... or among particles...



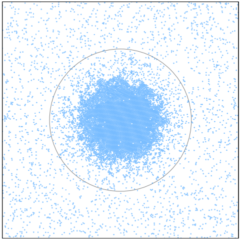


Measuring breakdown of TRS

Entropy production rate: $S = \lim_{T\to\infty} \frac{1}{T} \log\left(\mathcal{P}_F/\mathcal{P}_B\right)$

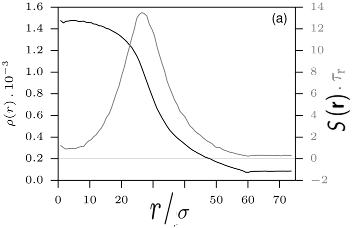
$$S = \frac{\sqrt{\tau}}{2D} \sum_{ij} \left\langle (p_i - p_j)^3 \,:\, \nabla^3 u(r_i - r_j) \right\rangle \qquad U = \frac{1}{2} \sum_{ij} u(r_i - r_j)$$

$S(r)$: contribution to S given by particles in a mesoscopic box centered in r



$D = 1$, $\tau = 10$, $N = 10^4$

$S(|r|)$ for different interface widths



[PRX 021007, 2017]

Part I: Phase separation in active systems

Phase separation of purely repulsive particles

At large scales

$\sim$

liquid-gas phase separation

[Tailleur et al. PRL 2008; Ann. Rev. Cond. Mat. 2015]

Effective equilibrium idea thrived huge number of works

[Marchetti, Speck, Joanny, Di Leonardo, Brader, Redner, Szamel, ...]

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Clusters and bubbly phase separation

Cluster Phases

Bubbly phase separation

Experiments – Janus particles

[J. Palacci et al., Science, 2013; Buttinoni et al, PRL 2013; J Schwarz-Linek et al, PNAS 2012; I Theurkauff et al, PRL 2012]

Simulations – purely repulsive particles

[Stenhammar et al., Soft Matter, 2014]

Cluster phases / bubbly phase separation generic?

Are they related?

Large-scale properties?

[PRX 031080, 2018]

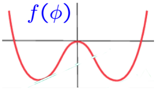
Minimal Field Theory

Minimal modification of Model B to break Time-Reversal Symmetry

$$\dot{\phi} = -\nabla \cdot (\mathbf{J} + \eta) \qquad \mathbf{J} = -\nabla \mu + \zeta(\nabla^2 \phi)\nabla\phi$$

$$\mu(\mathbf{r}) = \frac{\delta \mathcal{F}}{\partial \phi(\mathbf{r})} + \lambda |\nabla \phi|^2 \qquad \mathcal{F} = \int d\mathbf{r} \left[f(\phi) + K|\nabla\phi|^2 \right]$$

$$\langle \eta(\mathbf{r}, t)\eta(\mathbf{r}', t')) \rangle = 2D\,\mathbf{1}\,\delta(\mathbf{r} - \mathbf{r}')\delta(t - t')$$



- J: current
- μ: chemical potential
- ~~μ: chemical potential~~

Obtained from

- symmetries and gradient expansion
- explicit coarse graining of particles models

[PRX 031080, 2018]

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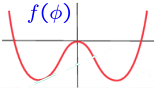

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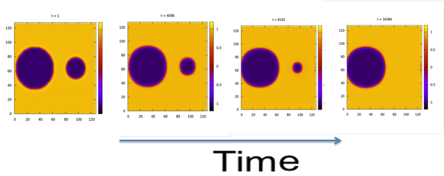
Ostwald Ripening in equilibrium

Model B:
Ostwald Ripening



Reason of:

- Pastis + water $\Longrightarrow$ opaque mixture
- Re-crystallization of water in ice-cream giving crunchy texture

Pressure difference inside two bubbles/clusters of different radii (and no momentum conservation)

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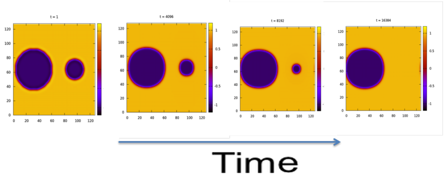

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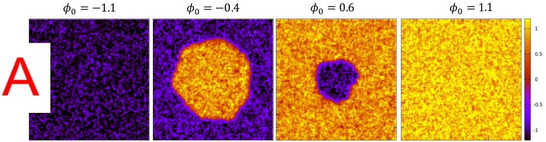


Cesare Nardini

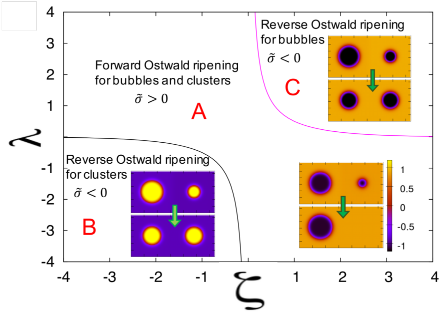
Group

Active Matter

Tracers in a bacterial bath

Incomplete Phase Separation

Mean-field analysis (analytical)



$$\dot{R} \propto \frac{\sigma}{R}\left(\frac{1}{R_c} - \frac{1}{R}\right) + \mathcal{O}\left(\frac{1}{R^3}\right)$$

Formally (but not physically!) equivalent to Ostwald Ripening in equilibrium

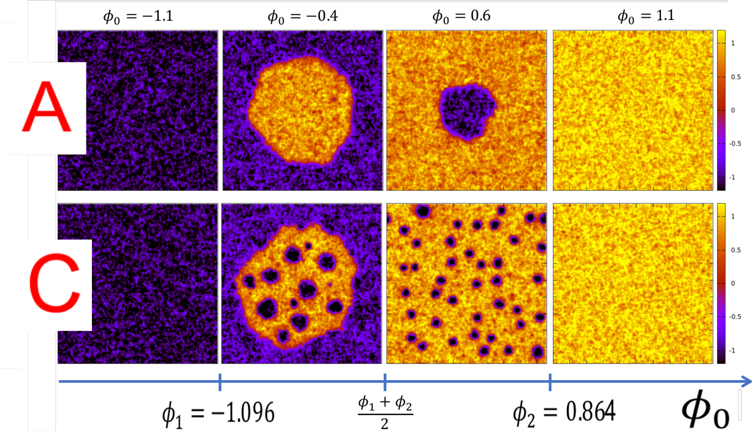
$$\sigma = \frac{K}{\zeta - 2\lambda}\left[\zeta\ \sigma_A\ e^{\frac{\zeta-2\lambda}{K}\phi_+} - 2\,\lambda\,\sigma_B\right]$$

$$\sigma_A = \int_0^\infty \phi'(y)^2\, dy$$

$$\sigma_B = \int_0^\infty \phi'^2(y) e^{\frac{\zeta-2\lambda}{K}y}\, dy\,.$$

[PRX 031080, 2018]

Phase diagram (with noise, numerical)



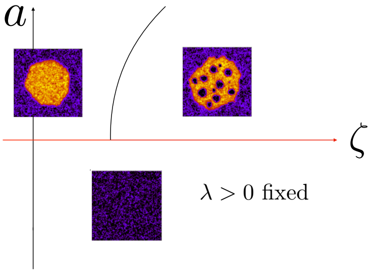
B Symmetry $(\phi, \lambda, \zeta) \to -(\phi, \lambda, \zeta)$

[PRX 031080, 2018]

Manifest TRS breaking

[PRX 031080, 2018]

Does TRS-breaking survives at criticality?



Critical transition to phase separation

(densities of the two phases almost identical)

$$\dot{\phi} = \nabla^2[-a\phi + u\phi^3 - K\nabla^2\phi + \lambda|\nabla\phi|^2] - \zeta\nabla\cdot[\nabla^2\phi\nabla\phi] + \nabla\cdot\eta$$

Renormalisation Group analysis

Dimensional analysis suggest: TRS-breaking terms (λ, ζ) irrelevant

[ArXiv:1809.10433]

Cesare Nardini

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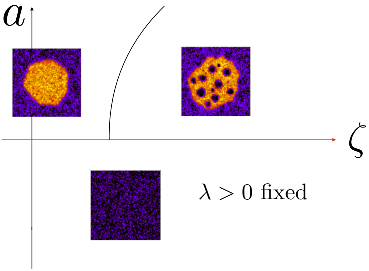

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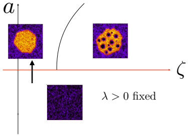
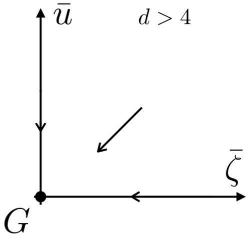
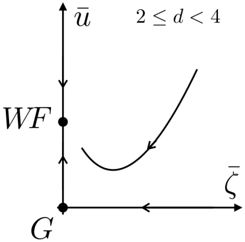
1-loop results: bulk phase-separation

Dimensional regularization close to $d = 4$



Bare parameters: region A (bulk phase separation)



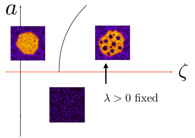


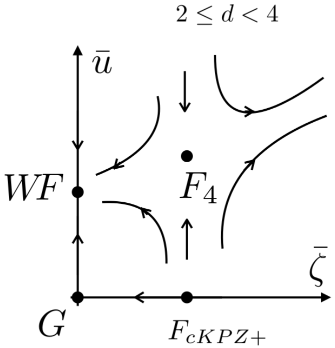
Equilibrium fixed points of the RG flow

- G: Gaussian
- WF: Wilson-Fisher

1-loop results: bubbly phase-separation

Bare parameters: region B,C (micro-/bubbly- phase separation)





- F_{cKPZ+} : transition mounding phase in a surface growth model (PRL 020601 2018)

(Probably) new non-equilibrium universality class describes the transition to bubbly/microphase separation

[ArXiv:1809.10433 (2018)]

Conclusions

Take home message:

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- Theoretical physics tools are crucial for understanding Active matter
- Even the simplest active systems (spherical particles, no alignment): intriguing large scale properties

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Thank you for your attention!

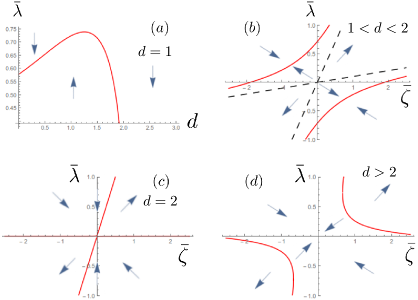
1-loop results

Local free energy absent: $f = 0$: cKPZ+ equation

$$\dot{\phi} = \nabla^2(-K\nabla^2\phi + \lambda|\nabla\phi|^2) - \zeta\nabla\cdot(\nabla^2\phi\nabla\phi) + \nabla\cdot\eta$$

model for growing surfaces with mass conservation

(in the comoving frame)



RG and numerics suggest the presence of a strong coupling regime akin to KPZ equation (rough surfaces)

[PRL 020601, 2018]