

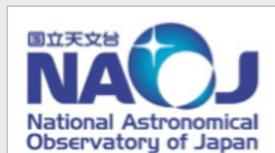


PCCP LECTURES COURSE

The physics of KAGRA Gravitational Wave detector

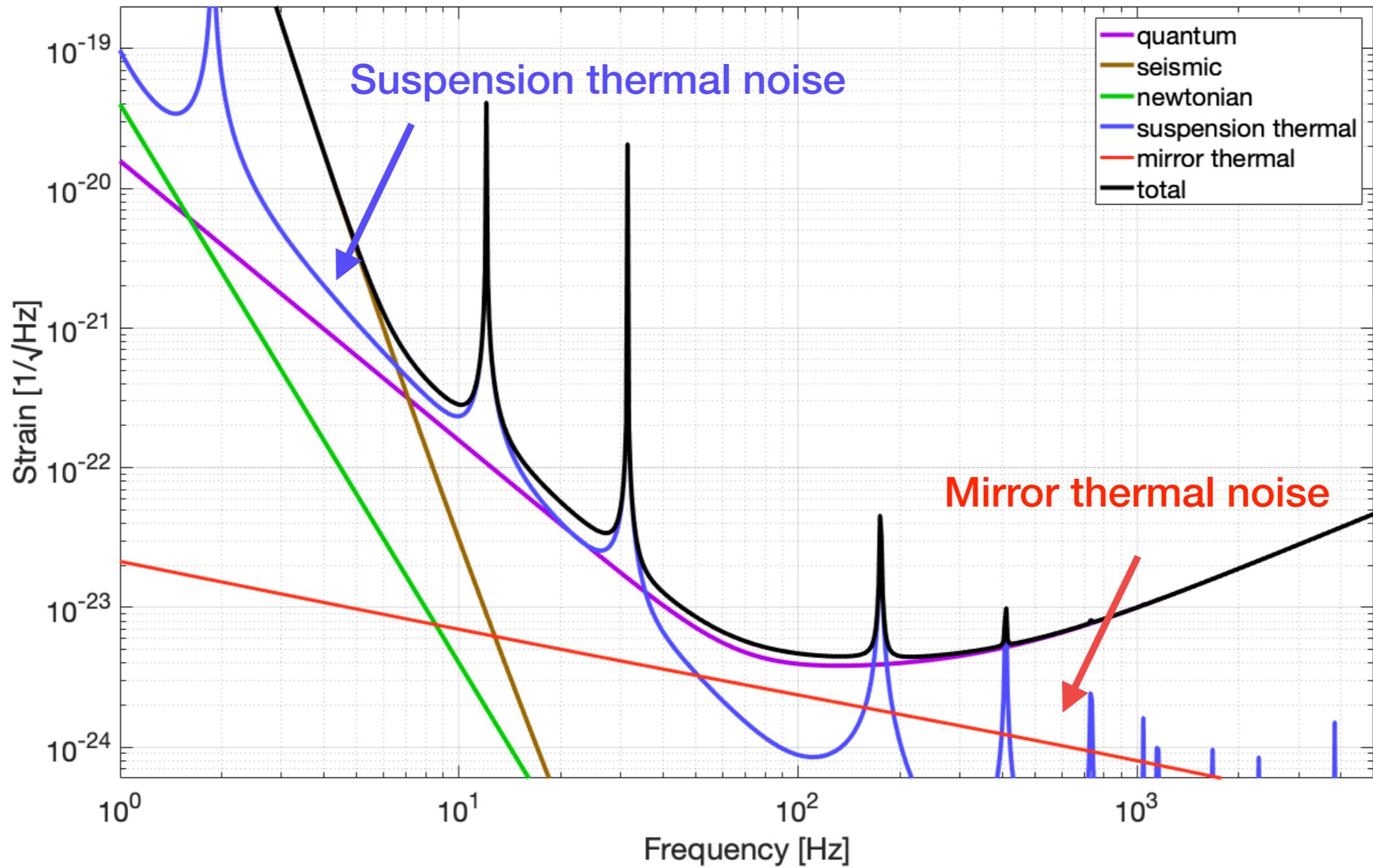
Reducing the thermal noise: a cryogenic detector

Eleonora Capocasa



APC January 2018

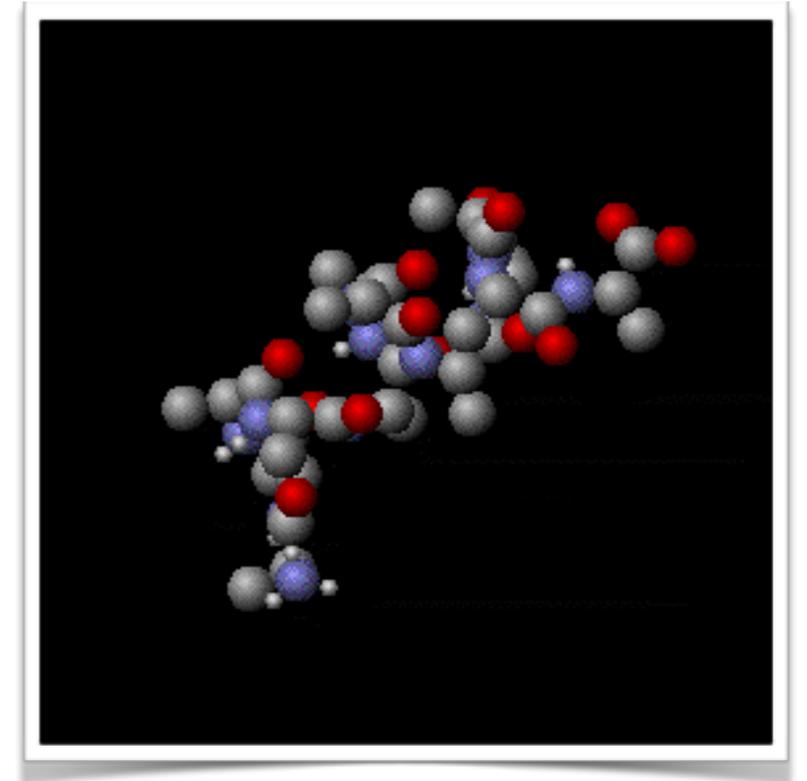
KAGRA: expected sensitivity



Where does thermal noise come from?

- Thermal fluctuation of a atoms and molecules related to their temperature
- **Equipartition theorem:** total thermal energy of each degree of freedom is

$$E = \frac{1}{2}KT$$



- This sets a limit on the rest condition of an harmonic oscillator
- The vibration amplitude of the mechanical mode of a microscopic object (e.g a 20 kg mirror) is $\sim 10^{-15}$ m

**How are these fluctuations distributed in frequency?
What is its spectrum?**

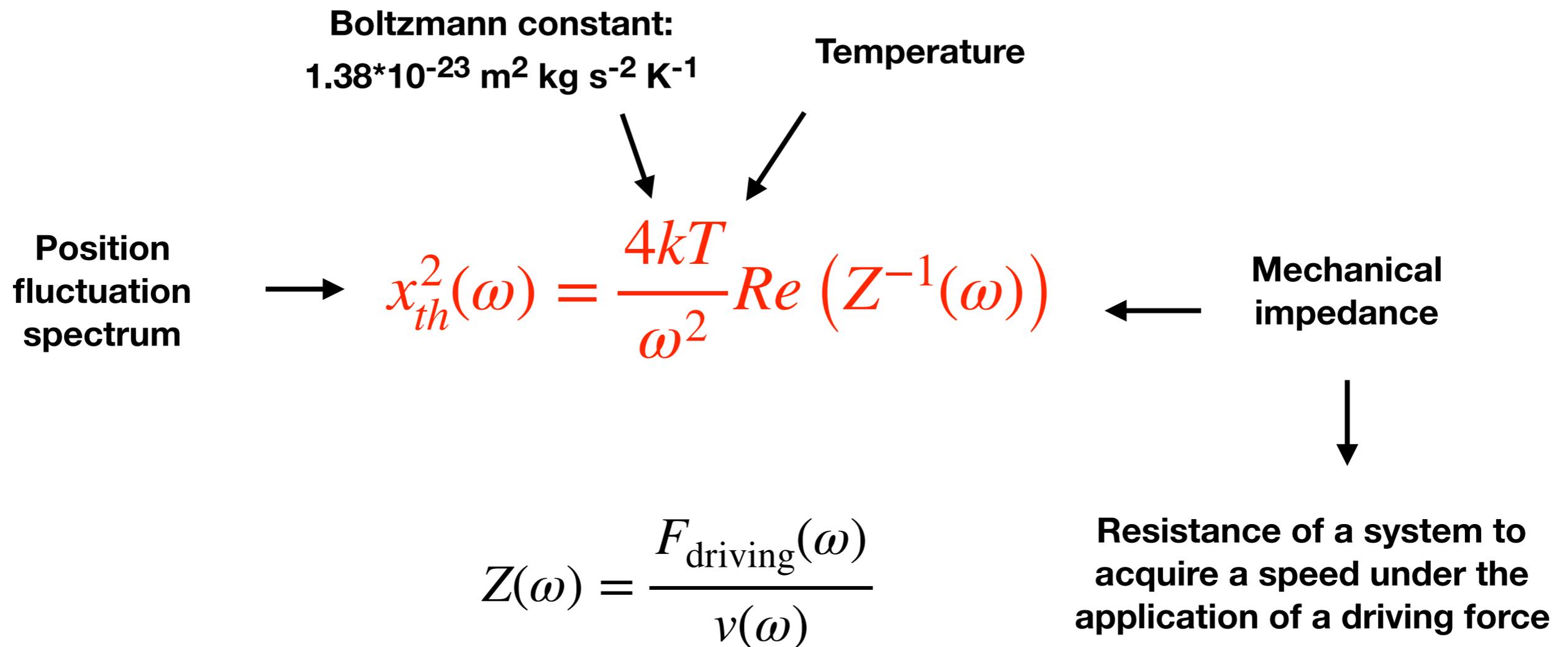
Fluctuation-dissipation theorem

If a system is affected by a dissipation process (i.e energy turned into heat), there is a reverse process which induces a fluctuation of its position, related to the temperature:

- **Brownian noise** and **drag**: objects is moving through a fluid, it experiences drag (kinetic energy into heat). In turn the object does not sit still, but rather moves kicked by molecules in the fluid. (heat energy into kinetic energy)
- **Johnson noise** and **resistance**: Resistance dissipates electrical energy, turning it into heat (Joule heating). In turn a wire loop with a resistor in it has a fluctuating current caused by the thermal fluctuations of the electrons and atoms in the resistor (heat energy into electrical energy)

Fluctuation-dissipation theorem

Relation between the response of the system when it is driven by an external force and the fluctuations of its position when it is not excited by any external force [Callen et al. 1975]



FDT: application to a damped harmonic oscillator

Equation of motion (depends on the type of damping)

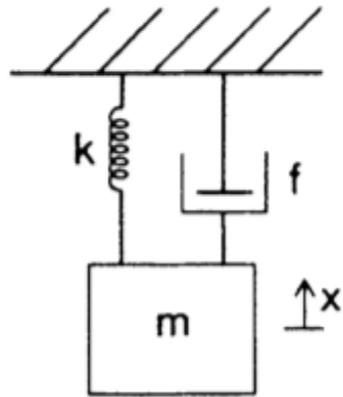


$$Z(\omega) = \frac{F_{\text{driving}}(\omega)}{v(\omega)}$$

$$x_{th}^2(\omega) = \frac{4kT}{\omega^2} \text{Re} (Z^{-1}(\omega))$$

Different types of damping

Viscous damping

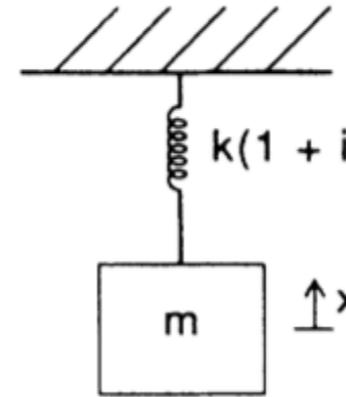


$$m\ddot{x} + f\dot{x} + kx = F$$

- Eg. Residual air, eddy currents

$$x^2(\omega) = \frac{4kTf}{(k - m\omega^2)^2 + \omega^2f^2}$$

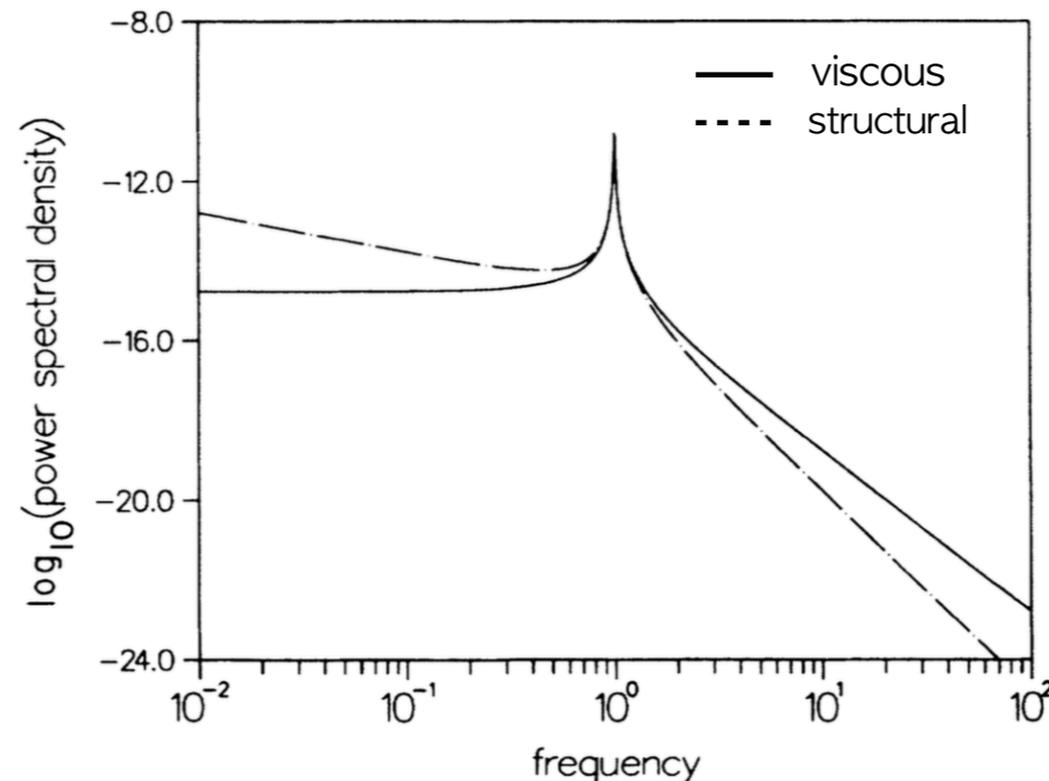
Structural damping



$$m\ddot{x} + k(1 + i\phi)x = F$$

- Due to anelasticity, lag between force and displacement

$$x^2(\omega) = \frac{4kT\phi(\omega)}{\omega[(k - m\omega^2)^2 + k^2\phi^2]}$$



Thermal noise in mechanical experiments

Peter R. Saulson
Phys. Rev. D **42**, 2437 (1990)

Losses vs fluctuations spectrum

Viscous damping

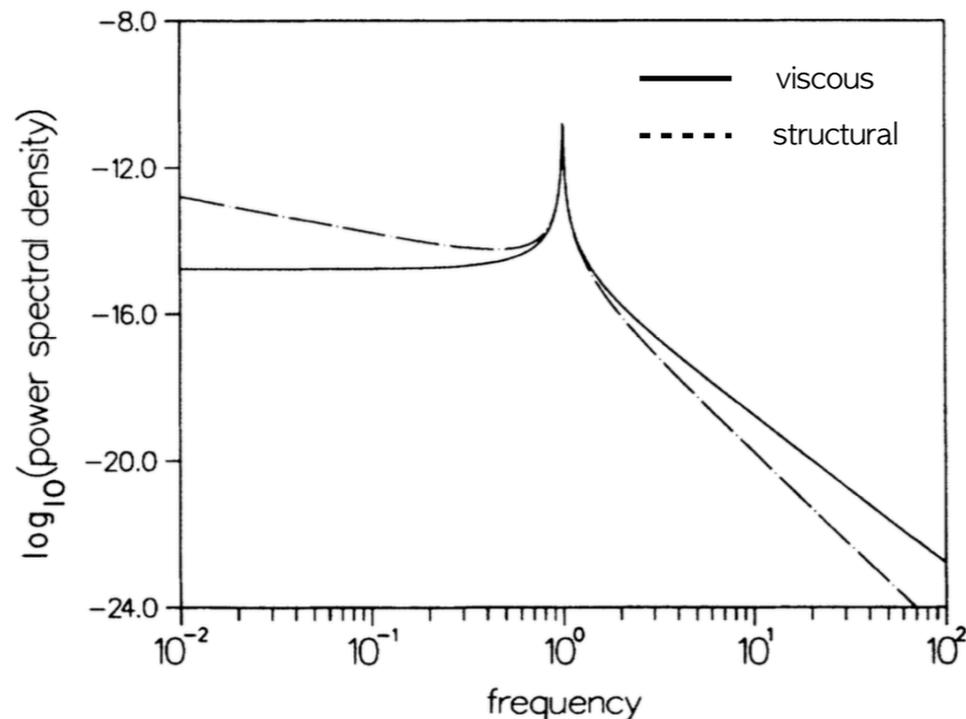
$$x^2(\omega) = \frac{4kT\omega_0/mQ}{(\omega_0 - m\omega^2)^2 + \omega^2\omega_0^2/Q^2}$$
$$\omega_0 = \frac{k}{m} \quad Q = \frac{m\omega_0}{f}$$

- Negligible for our vacuum level ($\sim 10^{-7}$ Pa)

Structural damping

$$x^2(\omega) = \frac{4kT\phi(\omega)}{\omega[(k - m\omega^2)^2 + k^2\phi^2]}$$

- Cut-off at low frequency

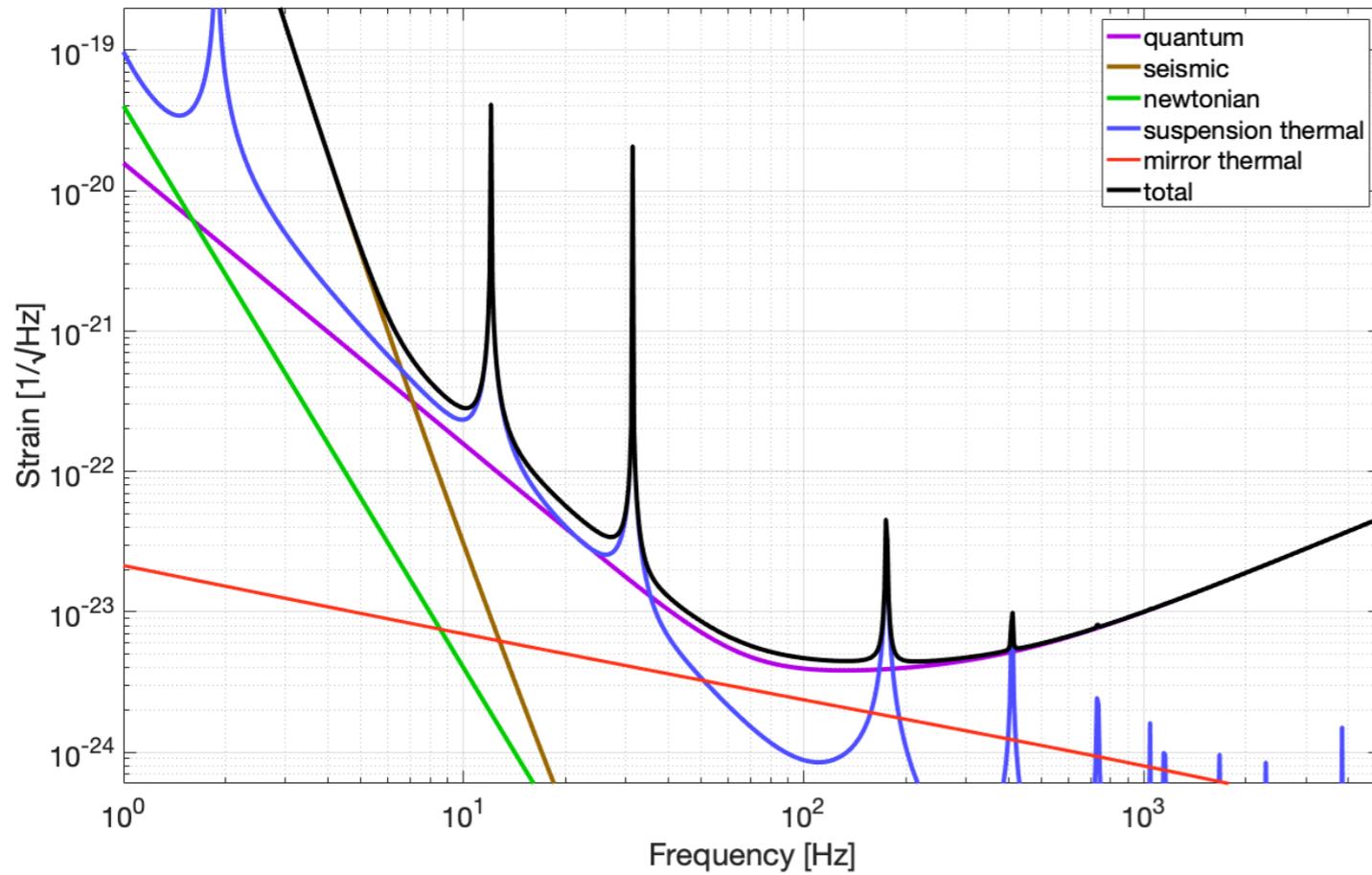


The lower the losses (f , ϕ) the lower the amount of fluctuations out of resonance

What we learnt about the spectrum of thermal fluctuation

- It can be computed using fluctuation-dissipation theorem
- It depends on the damping mechanism
- Structural damping is the dominating effect in our case
- It reduces out of resonance for lower losses
- It reduces at all the frequencies for lower temperature

Thermal noise components in GW detectors

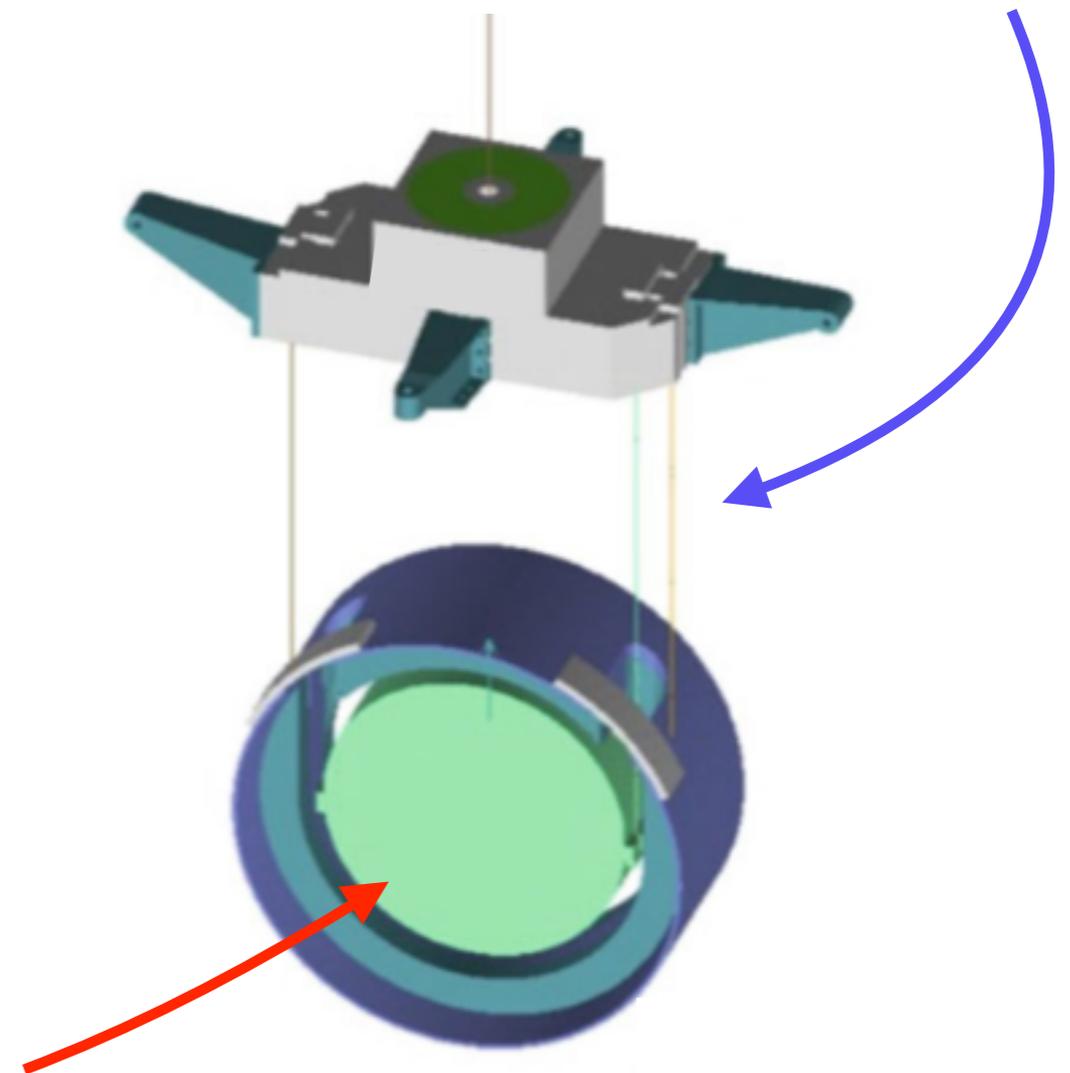


Mirror thermal noise
($\omega_0 \sim \text{few KHz}$)

Relevant fluctuations below resonance

Suspension thermal noise
($\omega_0 \sim \text{few KHz}$)

Relevant fluctuations above resonance



Pendulum thermal noise

- Restoring force mainly provided by **gravity**, which is **lossless**
- Small contribution by **wire elasticity**, which is **affected by losses**
- Total elastic constant of the pendulum is the sum of the two

$$k = k_g + k_{el}(1 + i\phi_w) = \frac{Mg}{L} + \frac{N\sqrt{TEI}}{2L^2}(1 + i\phi_w)$$

N = number of wire
T = wire tension
E = Young modulus
I = momentum of Inertia

Pendulum thermal noise

$$k \simeq k_g \left(1 + \frac{k_{el}}{k_g} i \phi_w \right)$$

- The losses due to the wires are diluted by the factor $\frac{k_{el}}{k_g}$
- The pendulum loss angle is $\phi_p = \frac{N\sqrt{TEI}}{2LMg} \phi_w$
- The associated power spectrum (from dissipation fluctuation theorem)

$$x_{th}^2(\omega) = \frac{4kT\phi_p}{M\omega_0^2} \omega^{-5}$$

We consider only the part above the resonance, which is at ~ 1 Hz

How to reduce pendulum thermal noise?

$$x_{th}^2(\omega) = \frac{4kT\phi_p}{M\omega_0^2}\omega^{-5} \quad \phi_p \sim \frac{\sqrt{M}}{L}\phi_w$$



$$x_{th}^2(\omega) \sim \frac{T\phi_w}{\sqrt{ML}}$$

- Increase mirrors mass
- Increase wires length
- Reduce wire losses
- Reduce temperature

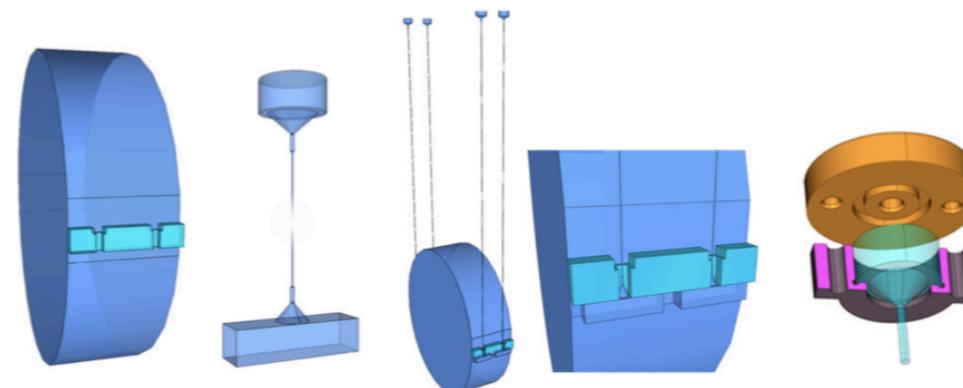
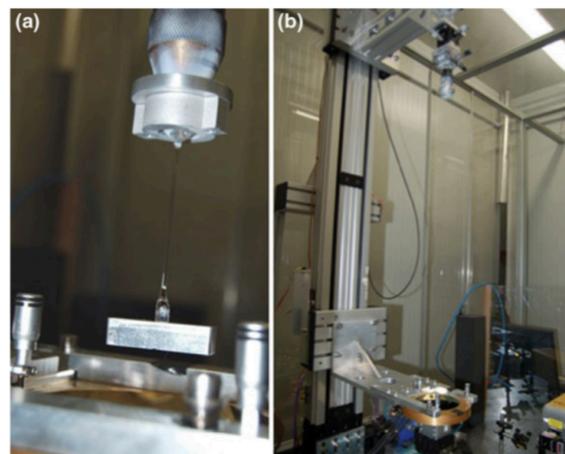
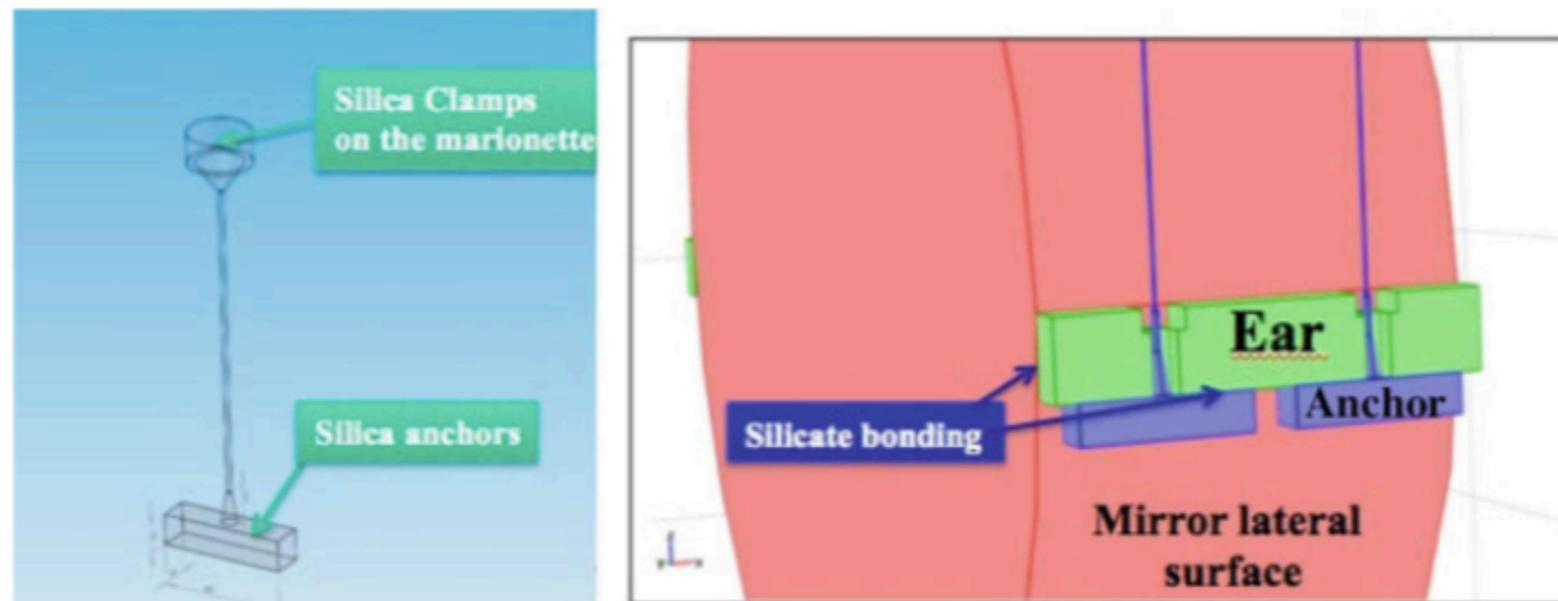
Reduce wire losses

	Steel wires	Silica wires	Sapphire Wires (20 K)
dilution factor	$\sim 10^{-3}$	$\sim 10^{-2}$	$\sim 10^{-2}$
Wire loss angle	$\sim 10^{-3}$	$\sim 10^{-7}$	$\sim 10^{-7}$
Pendulum loss	$\sim 10^{-6}$	$\sim 10^{-9}$	$\sim 10^{-9}$

$$\phi_p = \frac{k_{el}}{k_g} \phi_w$$

Reduce wire losses

- **Monolithic suspension:** fused silica wires, attached to the mirror by welding or using a procedure called silica bonding, which reproduces the connection between surfaces at molecular level



Thermal noise of continuous system

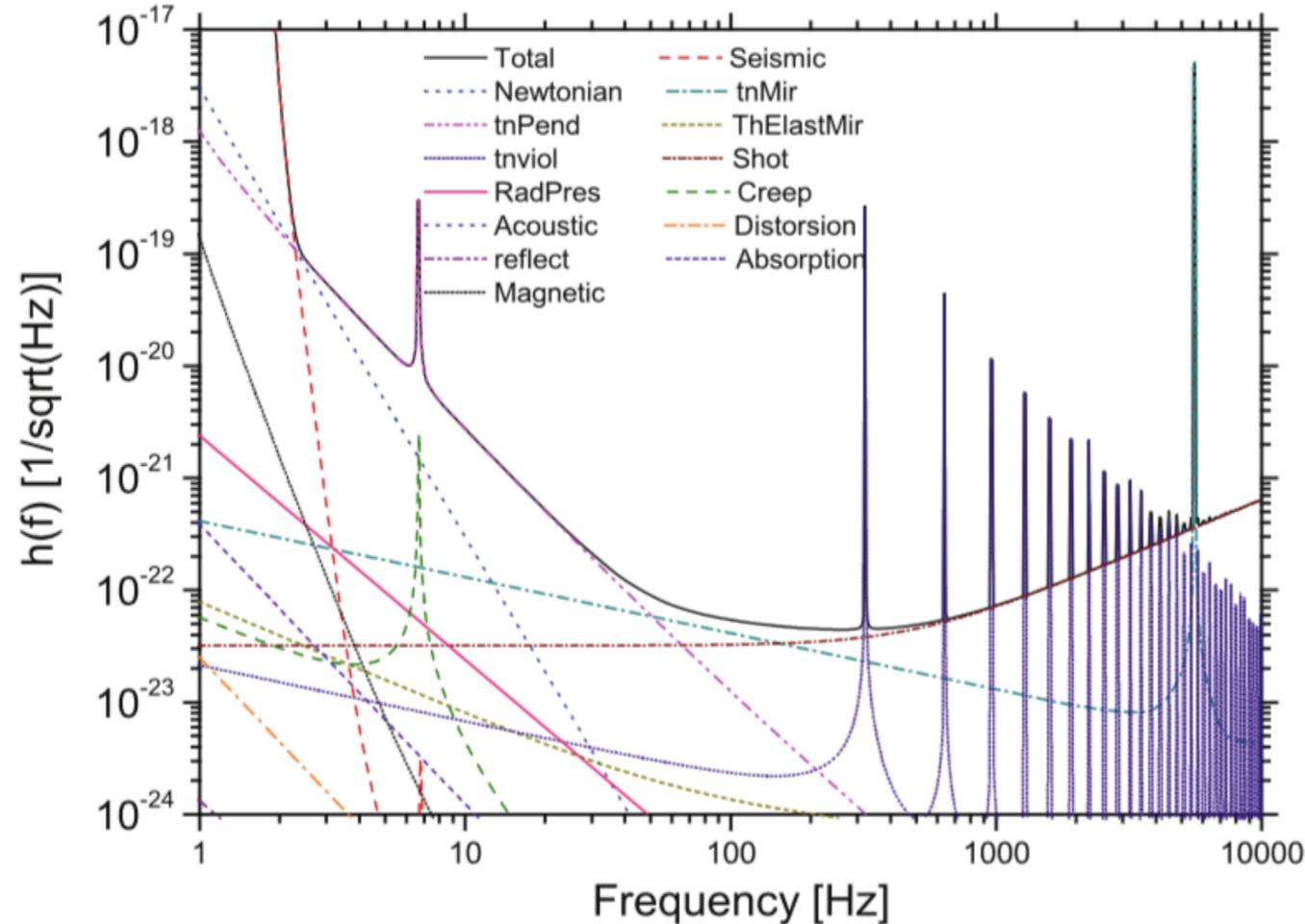
How to compute thermal noise of an observable of a continuous system?

- Consider the system as an infinite number of harmonic oscillators corresponding to all the normal modes. Apply FDT to each of them. Reconstruct the noise of the observable (which is a specific superposition of eigenmodes)
- Apply the FDT directly to the observable (which in the mirror case is the displacement of the mirror surface as sensed by the laser beam) [Y. Levin Phys. Rev. D **57**, 659]

Wire thermal noise (violin modes)

- Thermal noise of the wires caused by losses
- Peaks at the vibration modes

$$x_{th}^2(\omega) = \frac{4k_B T}{\omega} \sum_n \frac{\omega_n^2 \phi_n}{(\omega_n^2 - \omega^2)^2 + \omega_n^4 \phi_n^2} \frac{2\rho L}{\pi^2 m^2 n^2}$$



ρ = wire linear density
 L = wire length

Mirror thermal noise

- Substrate thermal noise

$$x_{th}^2(\omega) = \frac{4k_B T}{\omega^2} \frac{1 - \sigma^2}{\sqrt{\pi} E_0 w} \phi(\omega)$$

σ = poisson modulus

E_0 = mirror Young modulus

w = laser beam radius

- Fused silica shows very good performances at room temperature
 - low loss angle $\sim 10^{-9}$
 - low absorption < 1 ppm
 - high homogeneity

Mirror thermo-optic noise \1

- **Thermo-elastic noise:** fluctuations of the mirror temperature generate a surface displacements in the presence a non null thermal expansion coefficient

$$x_{th}^2(\omega) = \frac{8k_B T}{\omega^2} \frac{2\alpha^2 (1 + \sigma)^2 k T}{\sqrt{\pi} C^2 \rho^2 w^3}$$

σ = poisson modulus

A = thermal expansion coef

w = laser beam radius

C = specific heat per unit vol

k = thermal conductivity

ρ = density

- We want high specific heat, low thermal expansion and low thermal conductivity
- These properties are usually temperature-dependent

Mirror thermo-optic noise \2

- **Thermo-refractive noise:** change in the temperature -> variation of the material index of refraction -> noise induced on the phase of the transmissive optics

$$x_{th}^2(\omega) = \frac{4k_B T}{\omega^2} \frac{4kTl}{\pi(\rho C)^2 w^4} \left(\frac{dn}{dT} \right)^2$$

w = laser beam radius

C = specific heat per unit vol

k = thermal conductivity

Rho = density

↑
Thermo-refractive coefficient

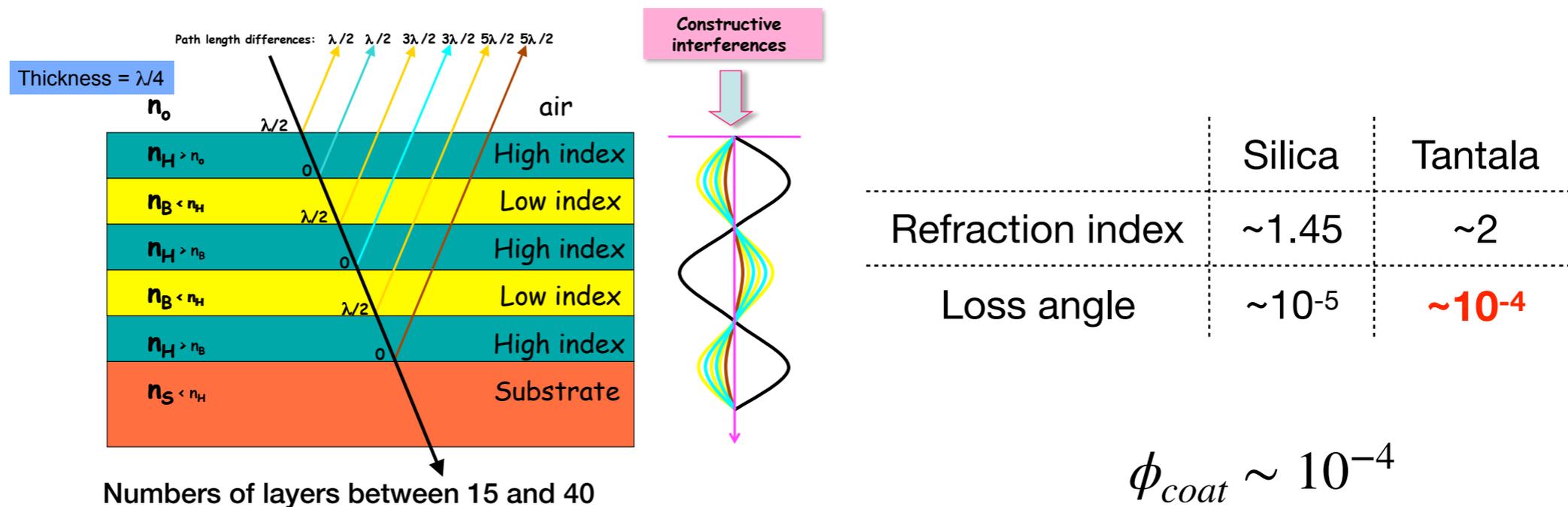
- Expected to be negligible especially for cryogenic sapphire

	Fused Silica (300K)	Fused Silica (20K)	Sapphire (300K)	Sapphire (20K)
α [ppm/cm]	2 - 20	2 - 20?	40 - 140	20 - 90?
κ [W/m•K]	1.4	0.15	46	4.3×10^3
dn/dT [K ⁻¹]	1.4×10^{-5}	1.4×10^{-5}	1.3×10^{-5}	$< 9 \times 10^{-8} $

T.Tomaru- “cryogenics in KAGRA” Joint Gravitational Waves and CERN Meeting- 09/17

Coating thermal noise

- Mirror coating: multilayer of alternating low and high refractive index material in order to obtain the required reflectivity

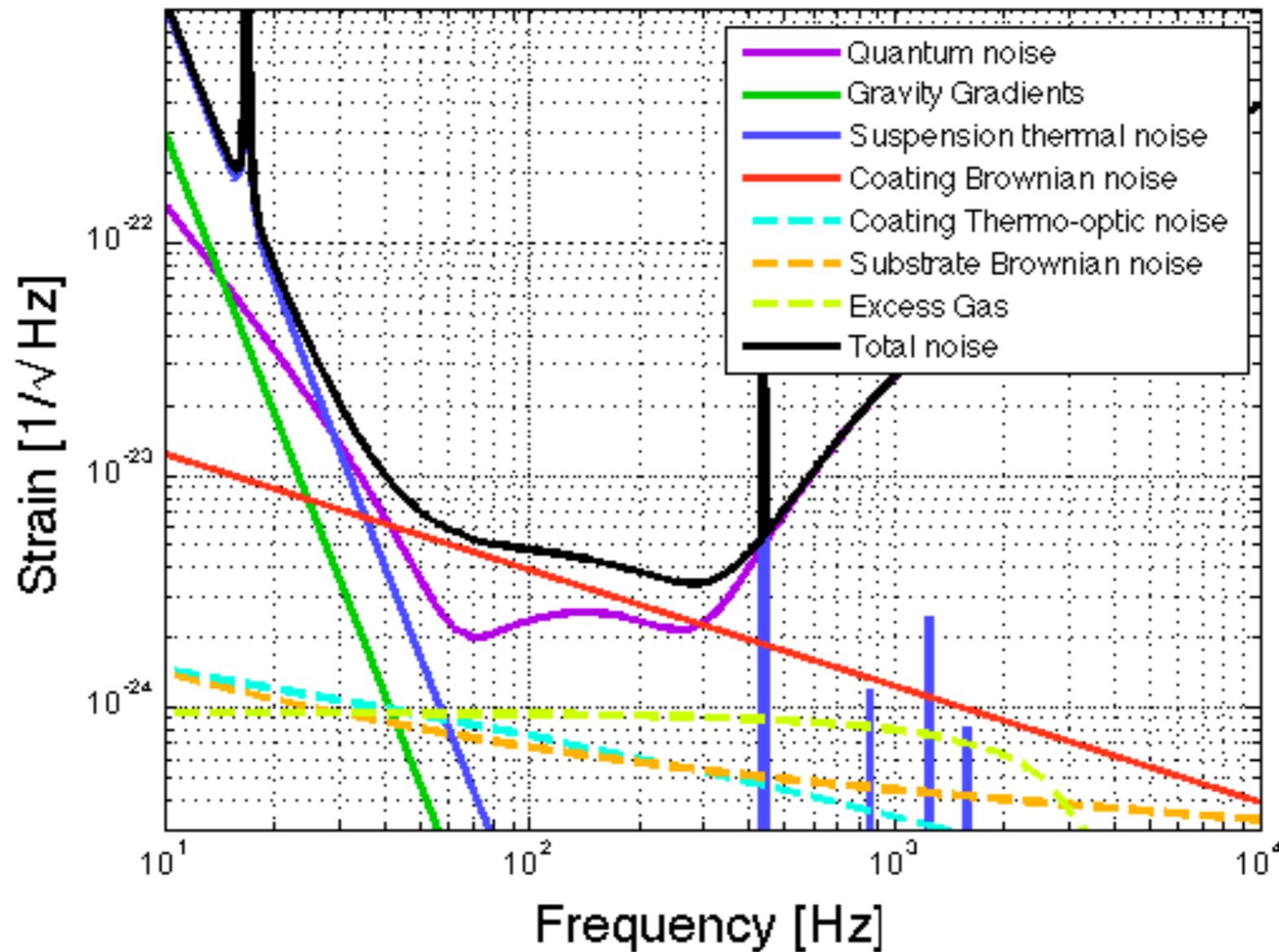


- Total loss angle dominated by Tantalum losses

$$x^2(\omega) = \frac{8k_b T (1 + \sigma)(1 - 2\sigma)t}{\omega \pi E_0 \omega^2} \phi_{coat}$$

Mirror thermal noises: conclusions

- Mirror thermal noise is dominated by coating thermal noise



How to reduce mirror thermal noise/1

- R&D activity to substitute (or dope) Tantalum in amorphous coatings

	SiO ₂	Al ₂ O ₃	Ti : Ta ₂ O ₅	Ta ₂ O ₅	TiO ₂	Nb ₂ O ₅	ZrO ₂
Loss angle	0.5×10^{-4}	2.4×10^{-4}	2×10^{-4}	3.8×10^{-4}	6.3×10^{-3}	6.7×10^{-4}	2.85×10^{-4}
Density (kg m ⁻³)	2200	3700	6425	6850	4230	4590	6000
Thermal conductivity (W m ⁻¹ K ⁻¹)	0.5	3.3	0.6	0.6	0.45	1	1.09
Specific heat (J K ⁻¹ kg ⁻¹)	746	310	269	306	130	590	26
Thermal expansion coefficient (K ⁻¹)	0.51×10^{-6}	8.4×10^{-6}	3.6×10^{-6}	3.6×10^{-6}	5×10^{-5}	5.8×10^{-6}	10.3×10^{-6}
Thermo-optic coefficient (K ⁻¹)	8×10^{-6}	1.3×10^{-5}	14×10^{-6}	2.3×10^{-6}	-1.8×10^{-4}	1.43×10^{-5}	10×10^{-5}
Young's modulus (GPa)	60	210	140	140	290	60	200
Poisson's ratio	0.17	0.22	0.23	0.23	0.28	0.2	0.27
Refractive index	1.45	1.63	2.06	2.03	2.3	2.21	2.15

ET design study

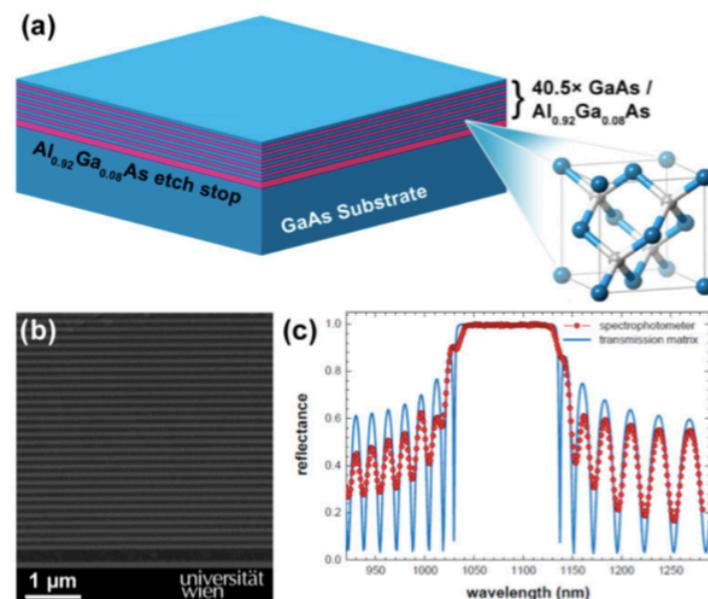
Table 13: List of the optical and mechanical values of different coating materials at 300 K.

- Slight improvement when doping Tantalum with Titanium but still largest loss component

How to reduce mirror thermal noise/2

Use of crystalline coatings

- R&D activity on going
- Much lower losses (factor 10) -> factor 3 TN reduction
- Difficulties to realize mirrors large enough
- Characterization of other optical properties still on going



G.D. Cole et al, Tenfold reduction of Brownian noise in optical interferometry, *Nature Photonics* 7, 644 - 650 (2013)

How to reduce mirror thermal noise/3

Use of larger beam

- Need beam ~ 1 m to get a factor 10 improvement
- Limited by the stability of the cavity (with 3 km, 5 cm radius already very close to the limit)
- Difficulties to realize mirrors large enough

$$x^2(\omega) = \frac{8k_b T}{\omega} \frac{(1 + \sigma)(1 - 2\sigma)t}{\pi E_0 w^2} \phi_{coat}$$

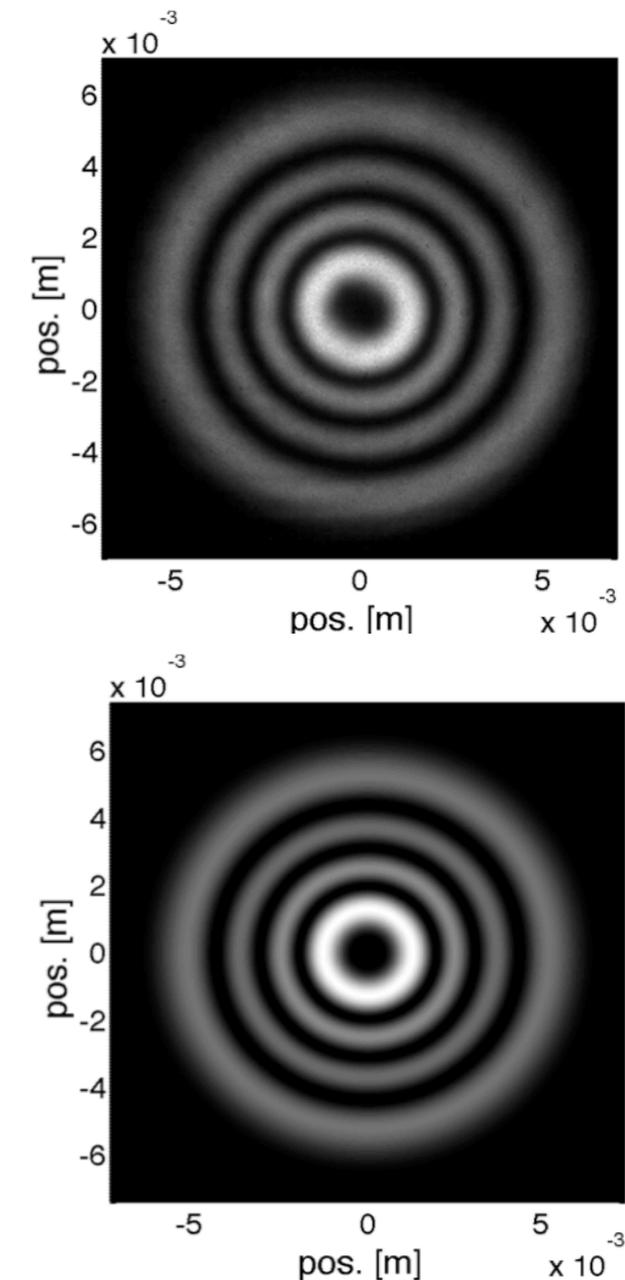
How to reduce mirror thermal noise/3

Use of Laguerre-Gauss modes

- Promising results on table top experiment with LG_{33}
- Problem of degeneracy \rightarrow need very high mirror quality

Fabry-Pérot-Michelson interferometer using higher-order Laguerre-Gauss modes

A. Gatto, M. Tacca, F. Kéfélian, C. Buy, and M. Barsuglia
Phys. Rev. D **90**, 122011 – Published 31 December 2014



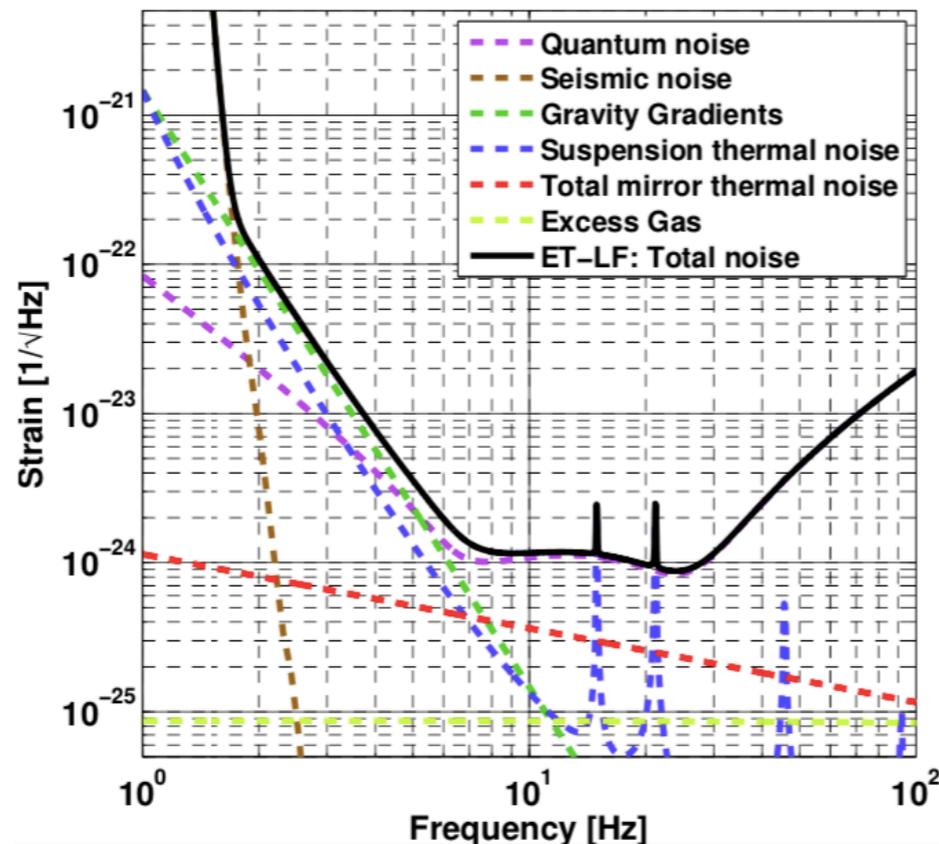
Thermal noises: summary

- Suspension thermal noise dominates up to few tens of Hz
- Mirror thermal noise (due to coatings) dominates up to few hundreds of Hz
- Suspension thermal noise can be improved having
 - Heavier masses
 - Longer wires
 - Lower loss material
- Coating thermal noise can be improved having
 - Lower loss coating material
 - Larger beams
 - Non gaussian beams
- Dependance on temperature
 - Cryogenic operation

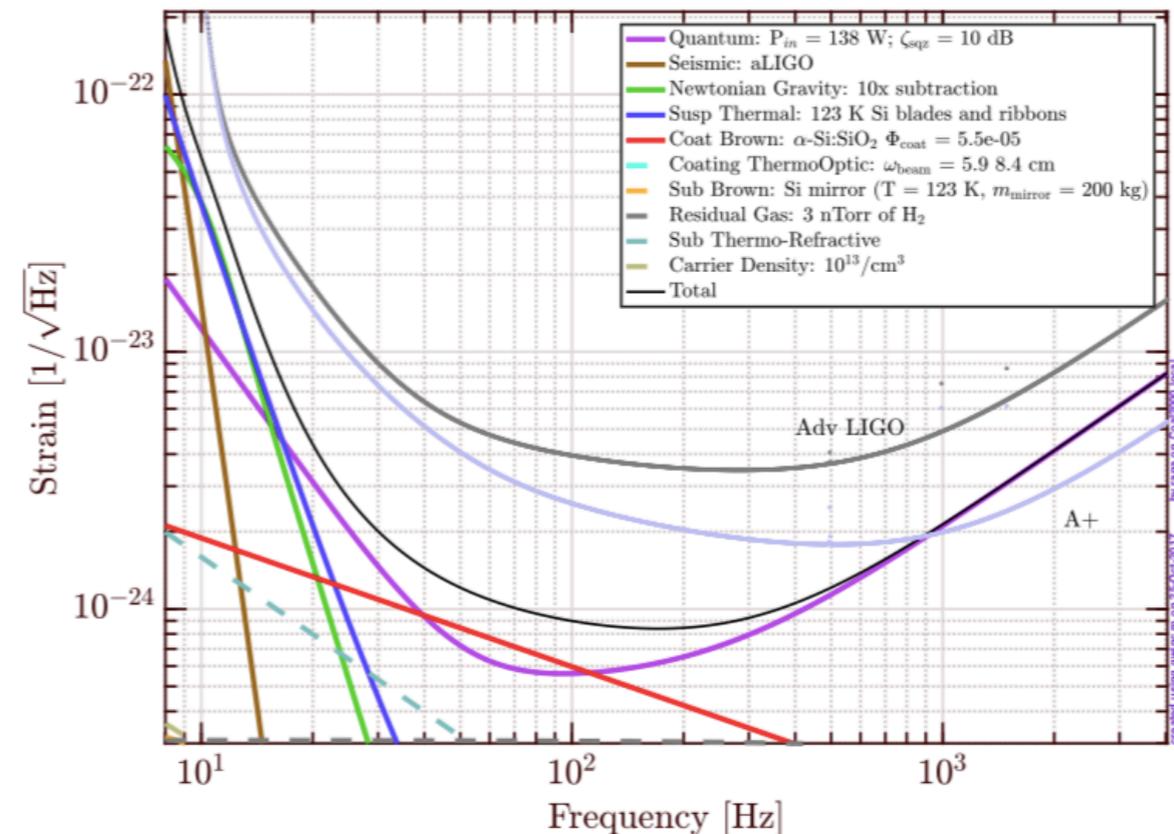
Cryogenics in future detectors

- 3rd generation detectors are designed to operate at cryogenic temperature

ET - LF



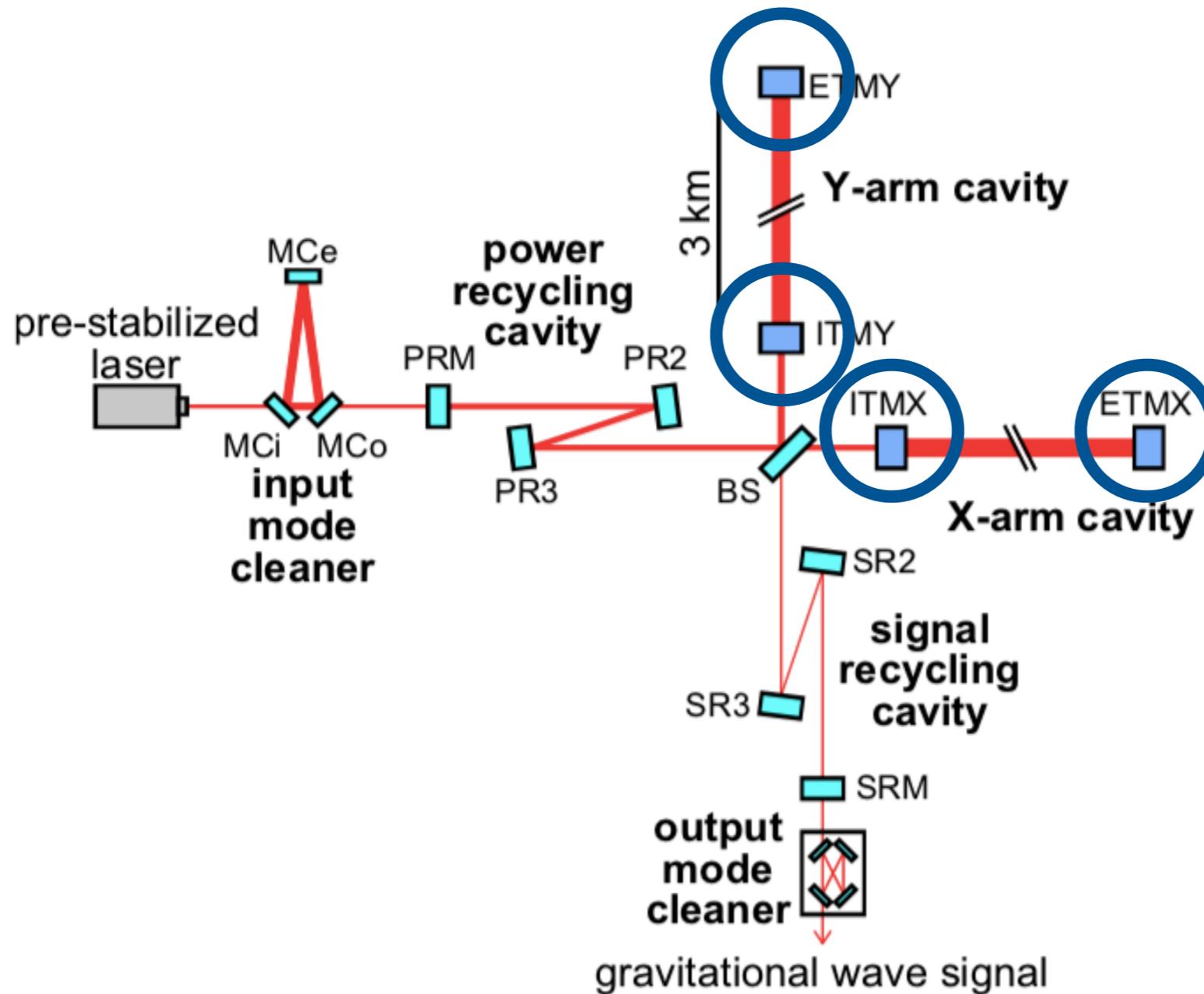
Voyager



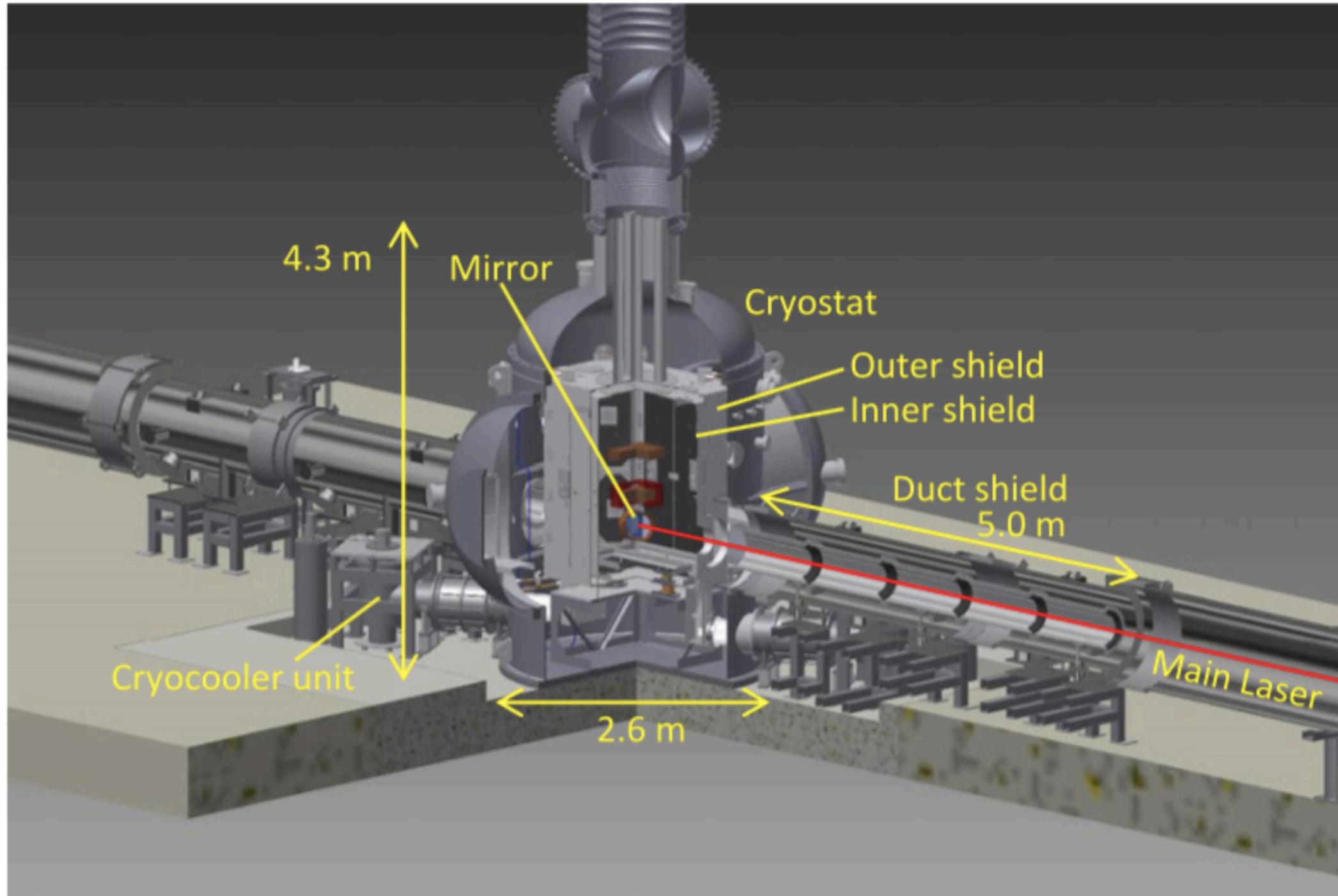
- KAGRA will open the path to test this technique on large scale detectors

KAGRA: cryogenic operation

- **Sapphire** test masses cooled down at **20 K**

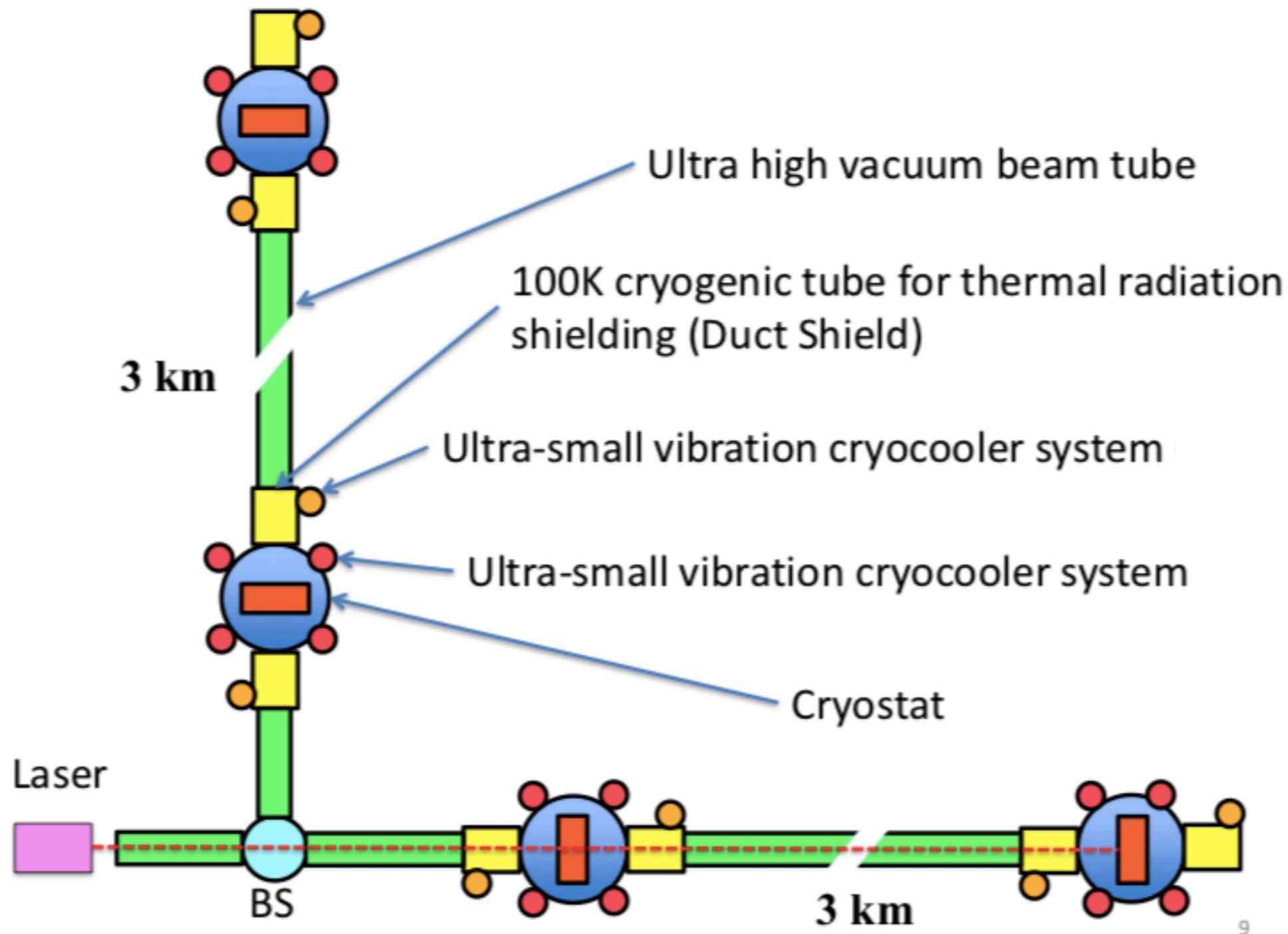


KAGRA cooling system



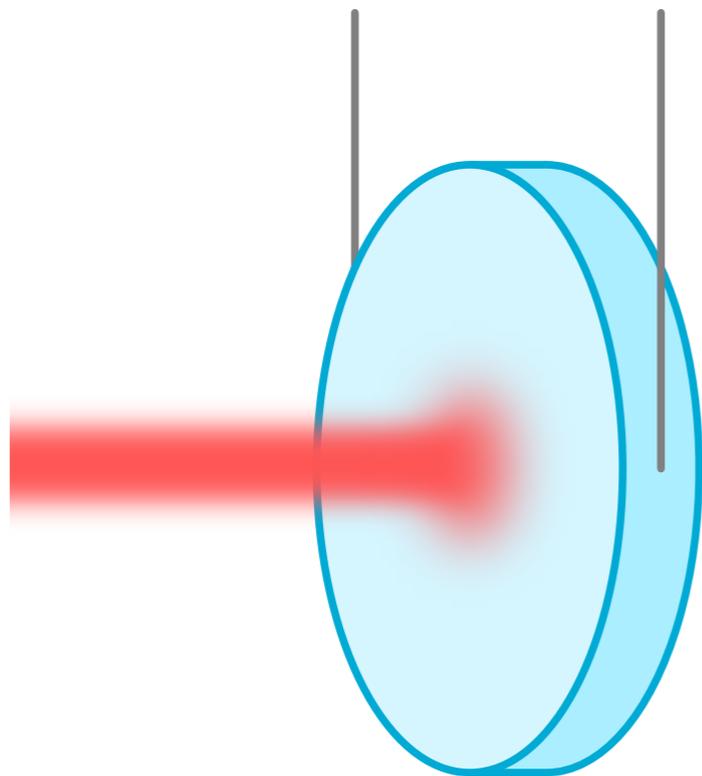
Yusuke Sakakibara *et al* 2014 *Class. Quantum Grav.* **31** 224003

KAGRA cooling system layout



How much heat do we need to extract?

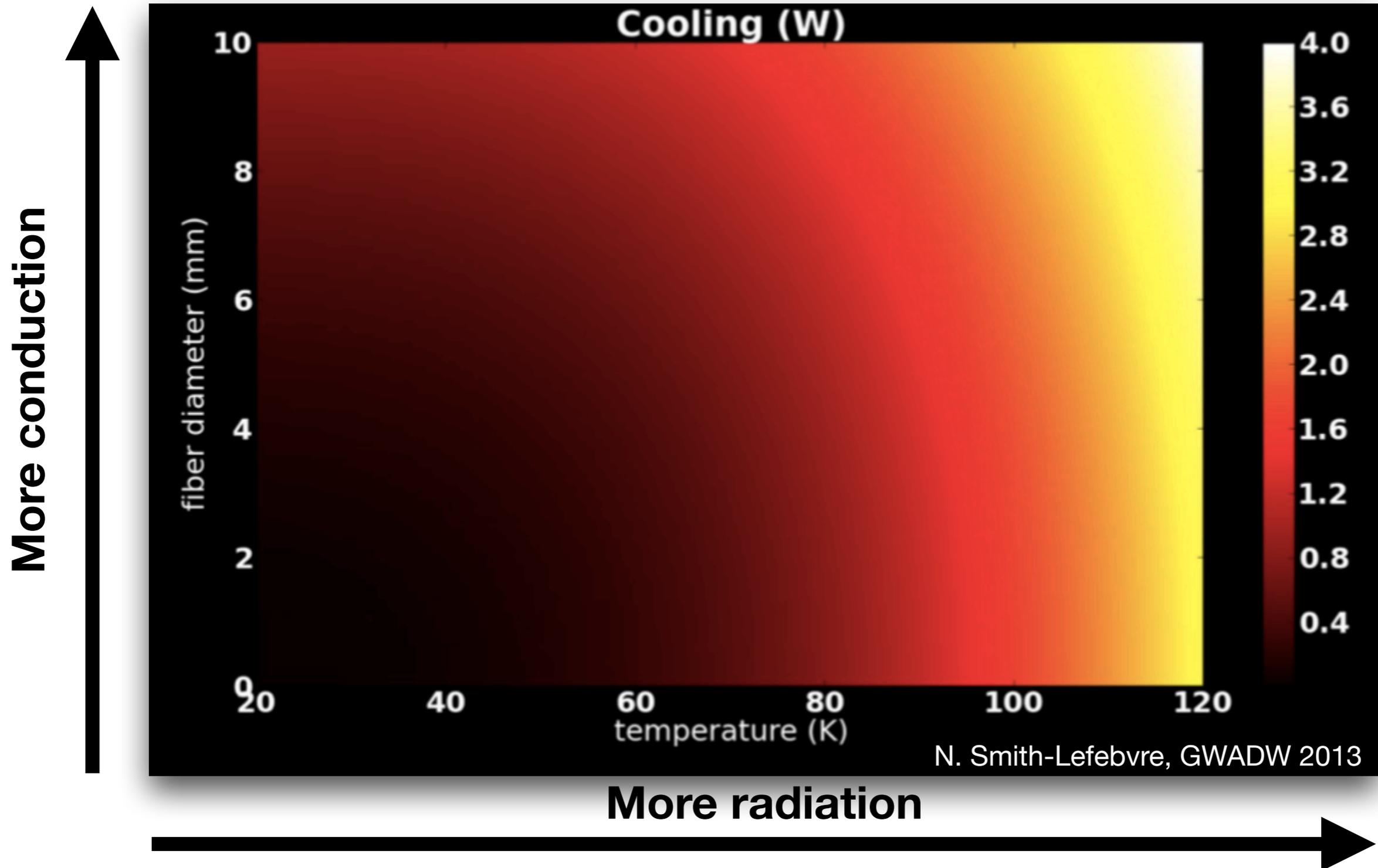
- ~1 W absorbed by sapphire substrate and coating
(~400 kW of circulating power)
- ~ 200 mW introduced through the radiation from the apertures and the view ports



Total = 1.2 W

Cooling mechanism

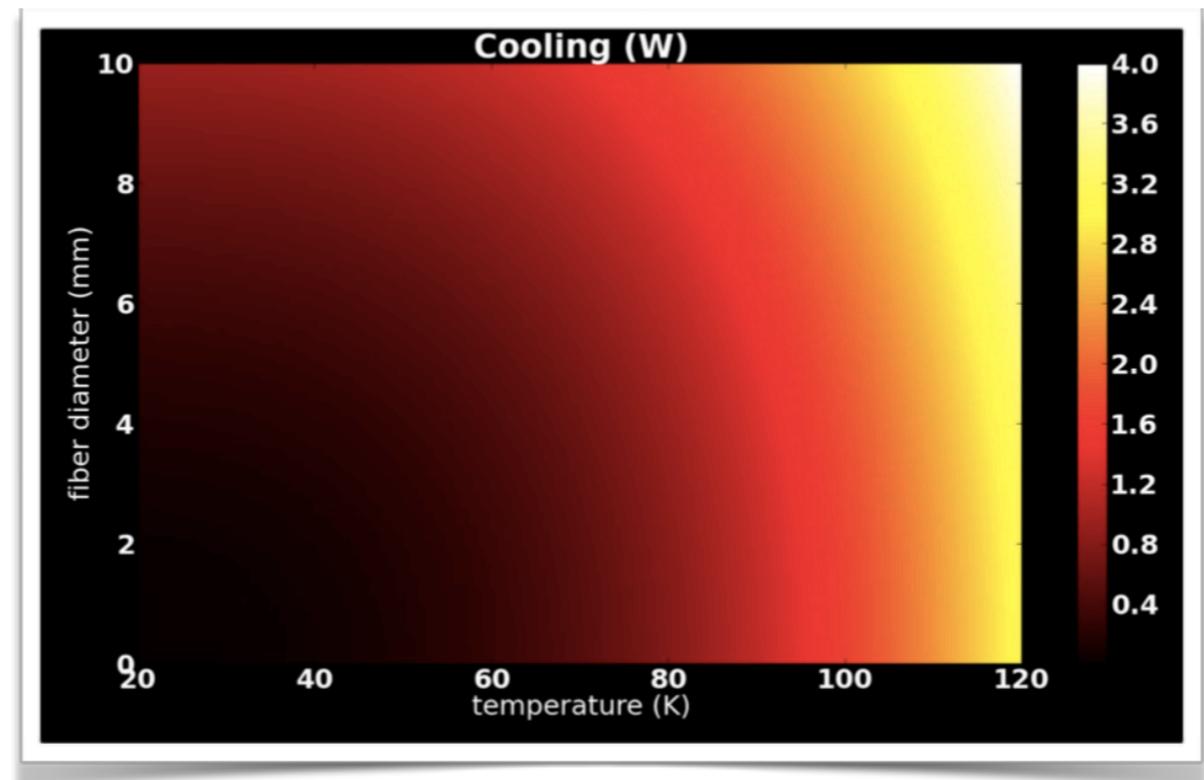
- Conduction
- Radiation



Temperature and cooling trade-off

- Thermal noise increases with temperature and fiber diameter
- Cooling is fixed by the detector configuration (power, mirror absorption)

- Lower temperature requires thicker fibers



Need to compromise between temperature and fiber thickness

Temperature and cooling trade-off

- Material shows temperature dependent properties
- Suitable materials for cryogenic operation can require configuration changing (eg. Silicon not transparent for the current wavelength 1064nm)
- Different compromises can be done:
 - KAGRA sapphire 20 K (thicker fiber)
 - ET Sapphire 20 K (low power → lower cooling)
 - LIGO voyager Silicon 120 K (radiation cooling)

Cryogenic challenges

- Heat evacuation in vacuum
- Vibration isolation
- Choose of appropriate materials for cryogenic operation

History of GW cryogenic in Japan

1998 - First cooling down test on sapphire mirror and fiber suspensions

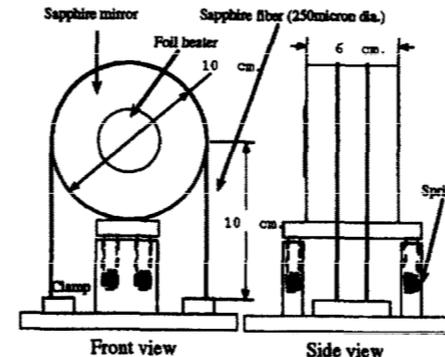
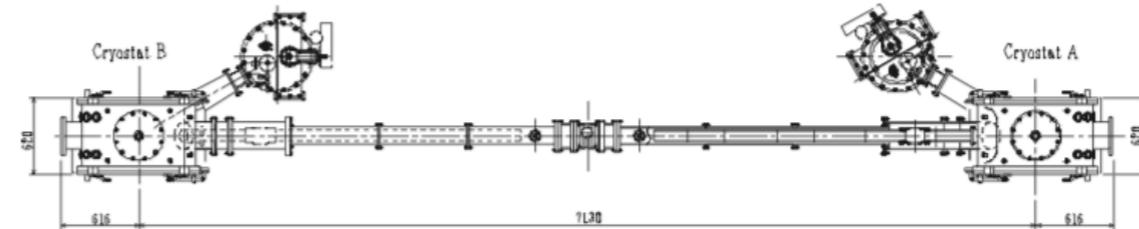


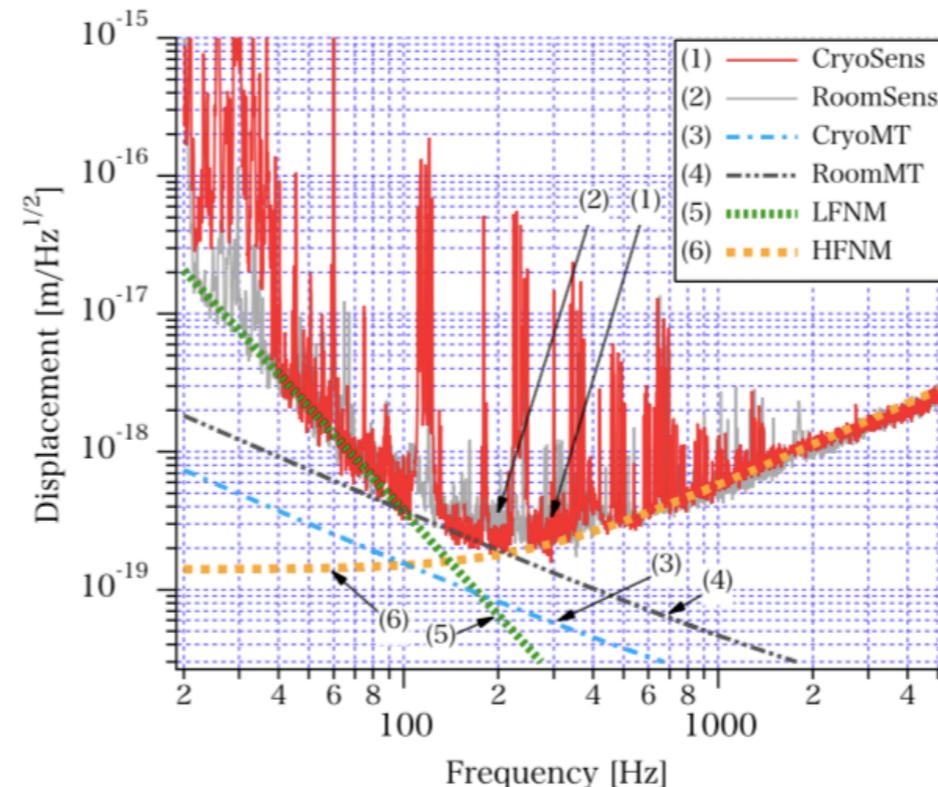
Fig. 1. The experimental setup.

Physics Letters A 242 (1998) 211-214
 Cryogenic cooling of a sapphire mirror-suspension for interferometric gravitational wave detectors
 Uchiyama^a, D. Tatsumi^a, T. Tomaru^a, M.E. Tobar^a, K. Kuroda^a, T. Suzuki^b, N. Sato^b, A. Yamamoto^b, T. Haruyama^b, T. Shintomi^b

2001 - CLIK: 7m cryogenic Fabry-Perot cavity at Kashiwa



2002 - CLIO : 100m cryogenic Fabry-Perot interferometer in Kamioka. Thermal noise reduction demonstration



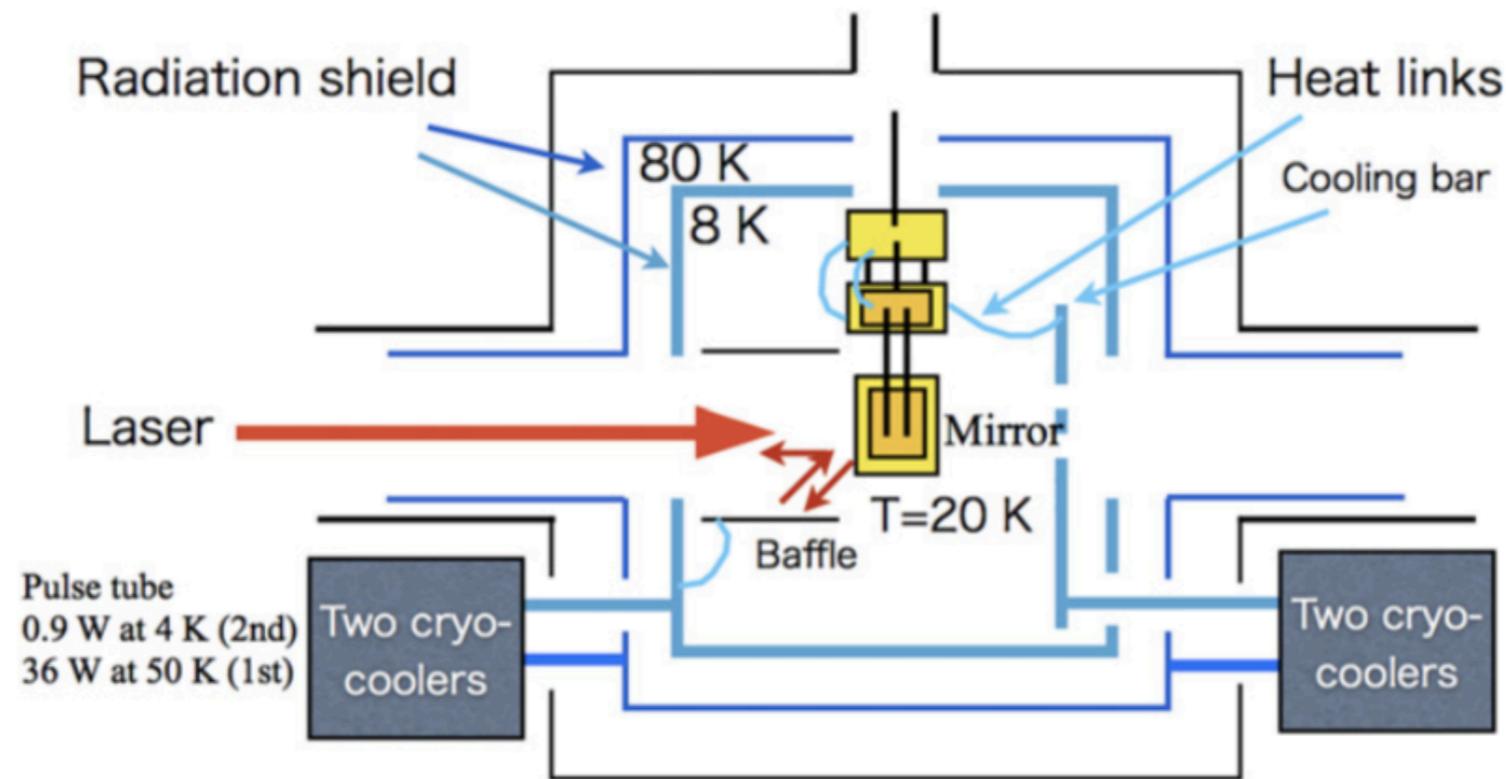
PRL 108, 141101 (2012) PHYSICAL REVIEW LETTERS week ending 6 APRIL 2012

Reduction of Thermal Fluctuations in a Cryogenic Laser Interferometric Gravitational Wave Detector

Takashi Uchiyama,^{1,*} Shinji Miyoki,² Souichi Telada,³ Kazuhiro Yamamoto,^{4,†} Masatake Ohashi,² Kazuhiro Agatsuma,^{2,‡} Koji Arai,^{5,§} Masa-Katsu Fujimoto,⁵ Tomiyoshi Haruyama,⁶ Seiji Kawamura,^{5,†} Osamu Miyakawa,¹ Naoko Ohishi,^{1,‡} Takanori Saito,² Takakazu Shintomi,⁷ Toshikazu Suzuki,⁶ Ryutarō Takahashi,^{5,†} and Daisuke Tatsumi⁵

2017 - KAGRA : First cryogenic operation of a large scale GW interferometer

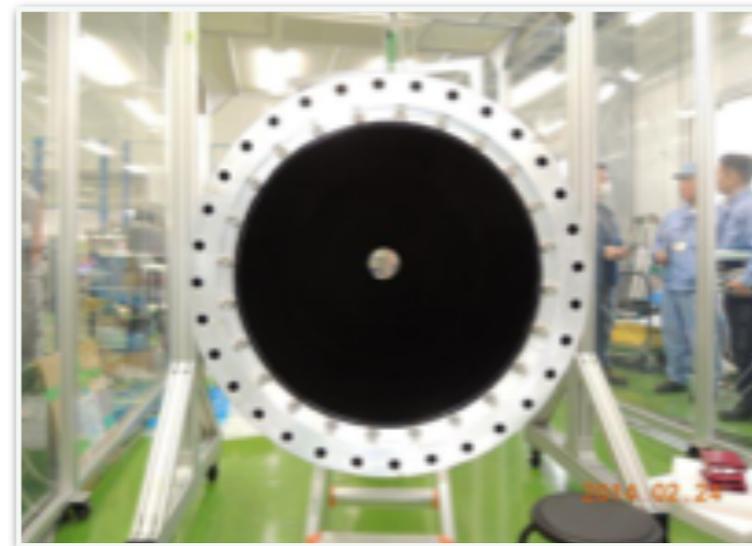
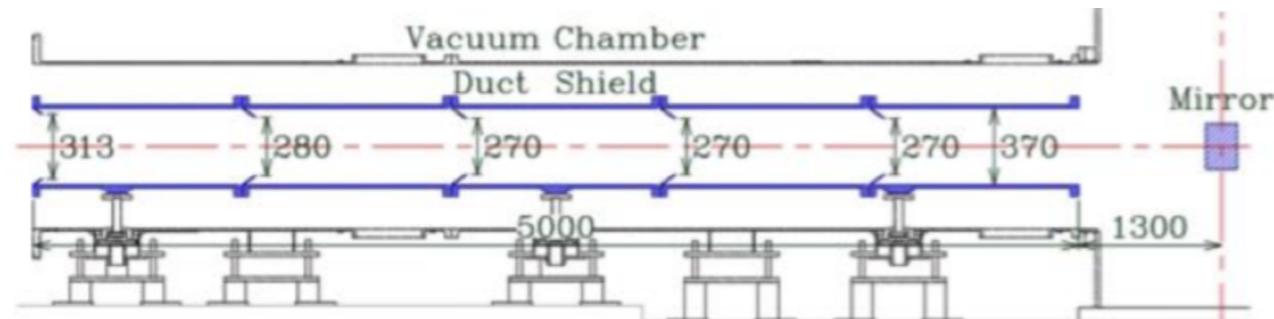
Heat evacuation: the cryostat



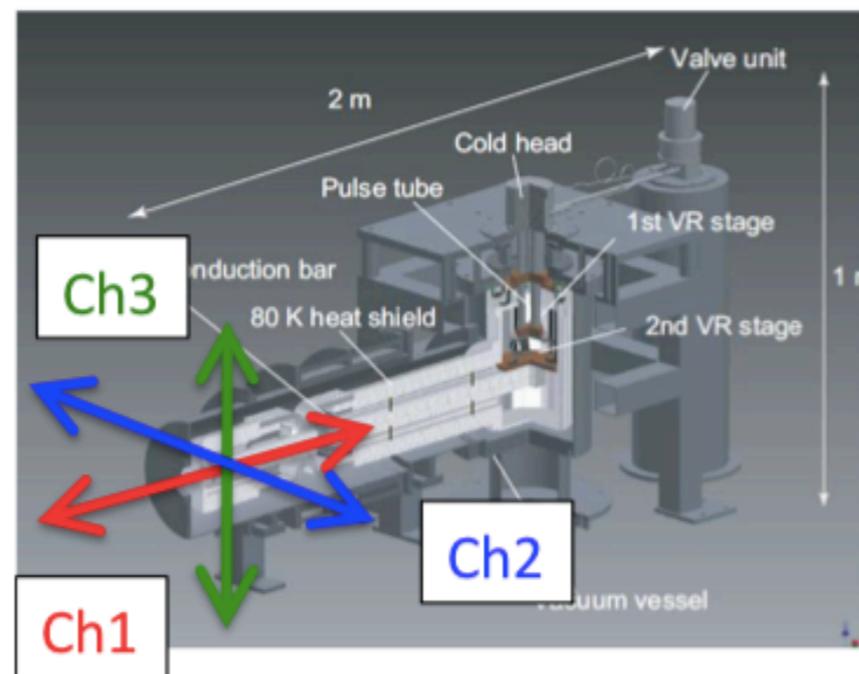
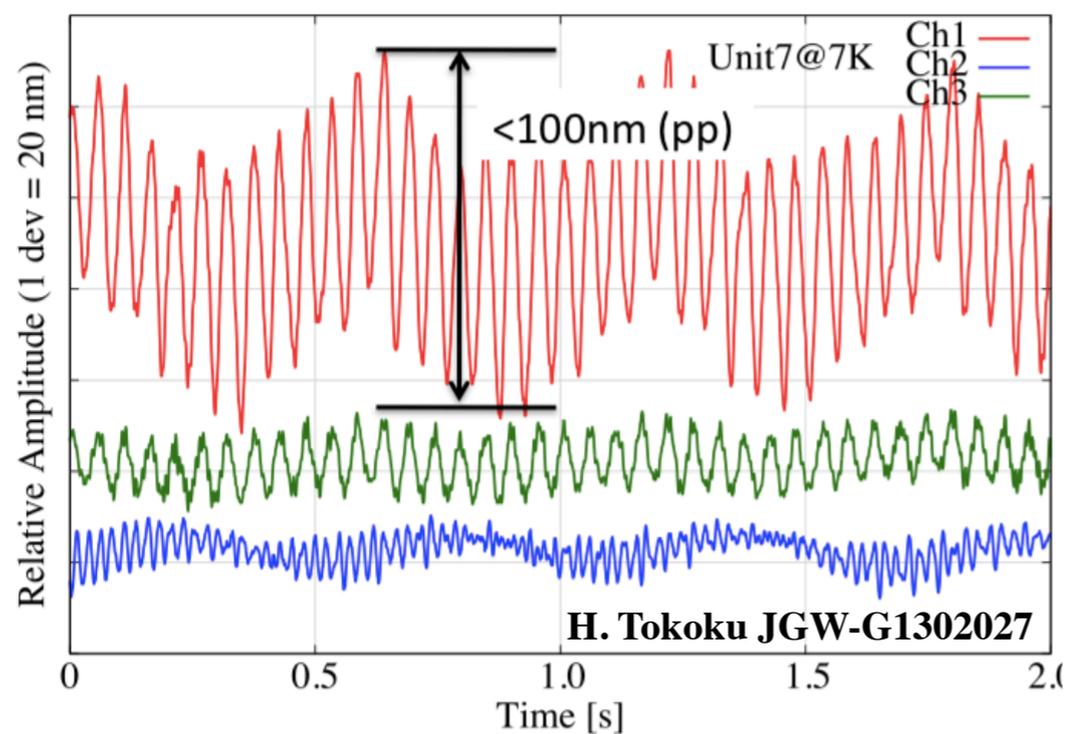
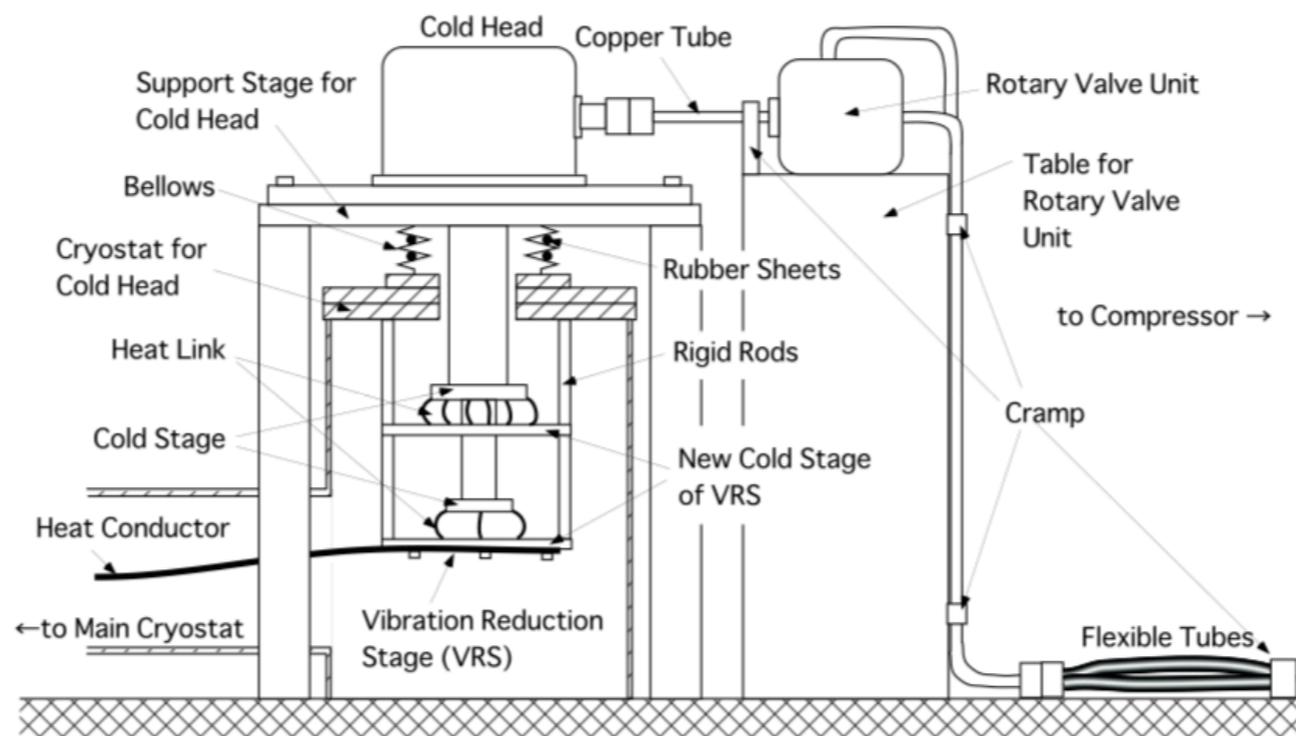
- Payload surrounded by inner (8K) and outer shield (80K)
- Four low vibration double-stage pulse-tube cryocoolers for each cryostat
- Inner shield coated with the diamond-like carbon (DLC) to reduce to cool down time (large absorption of radiated heat)
- Different kinds of baffles to absorb stray light

Duct shield

- Duct shields for absorbing thermal radiation from room temperature ducts
- ~17 m long at 100K, with black coating and baffles
- Two cryocoolers
- Reduce about ~1000 times thermal radiation heat from room temperature pipe



The pulse tube cryocoolers



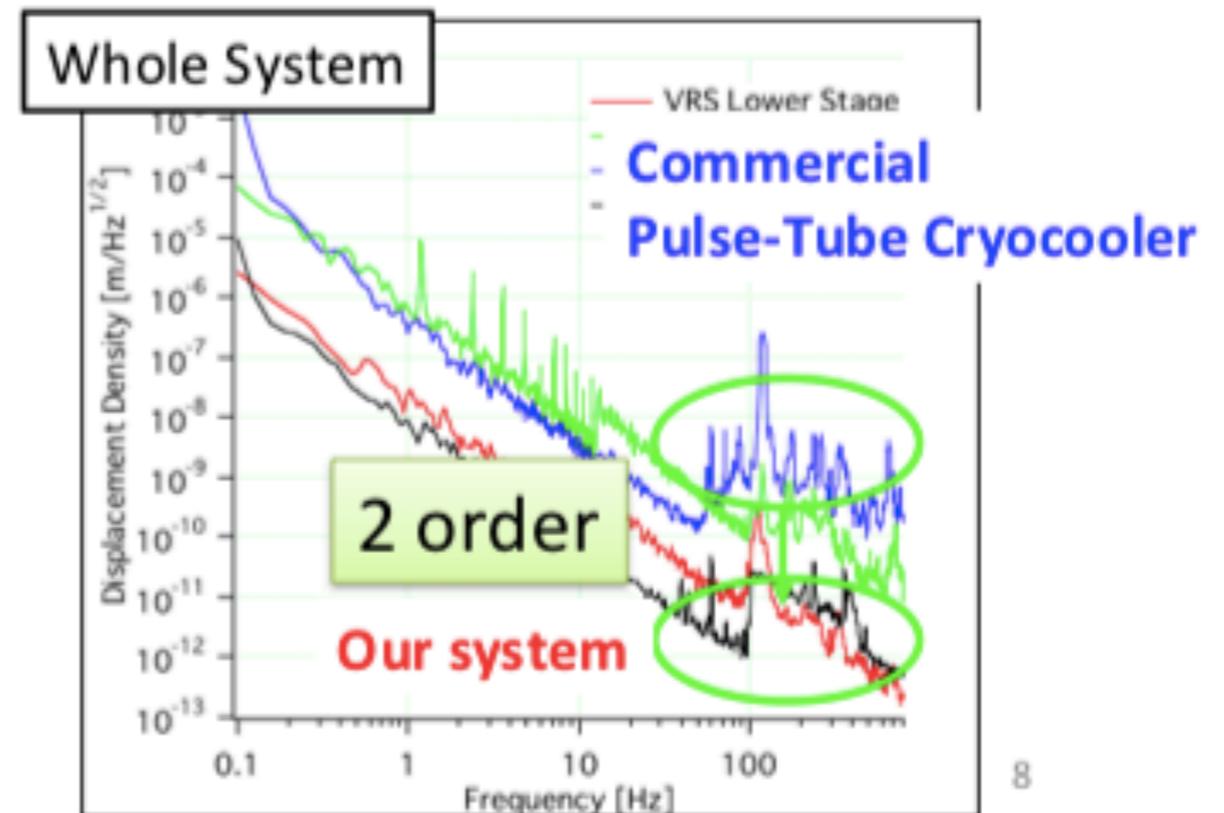
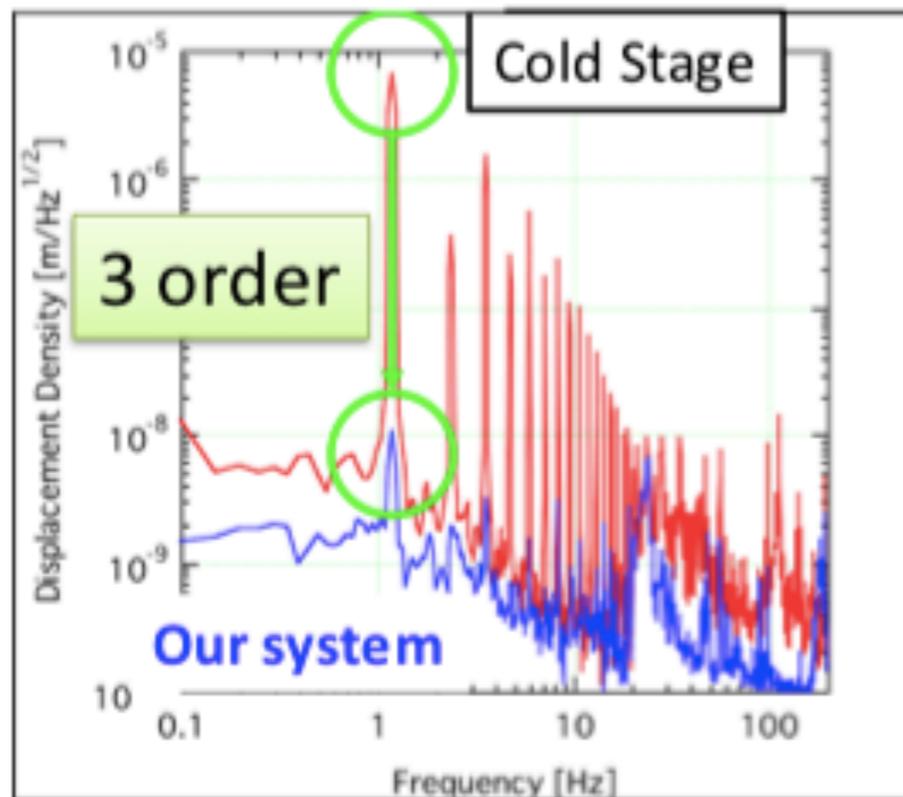
Tomaru T. et al. (2005) Vibration-Free Pulse Tube Cryocooler System for Gravitational Wave Detectors, Part I: Vibration-Reduction Method and Measurement. In: Ross R.G. (eds) Cryocoolers 13. Springer

The pulse tube cryocoolers

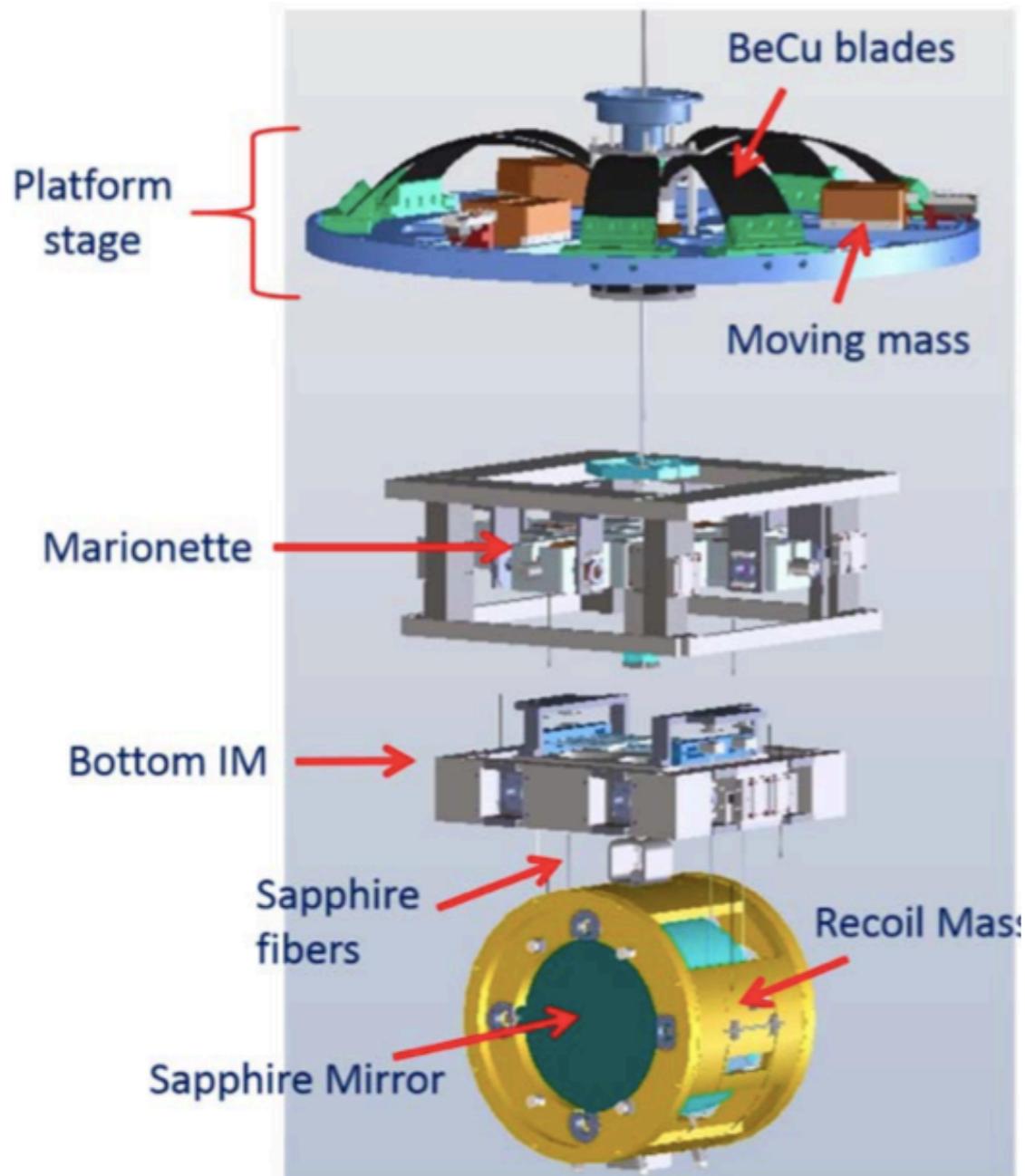
- Developed for CLIO prototype (T.Tomaru et al CQG, Vol 21, N 5)
- ~nm vibration at cold stage
- Vibration level of whole system comparable with Kamioka seismic vibration



Commercial Pulse-Tube Cryocooler



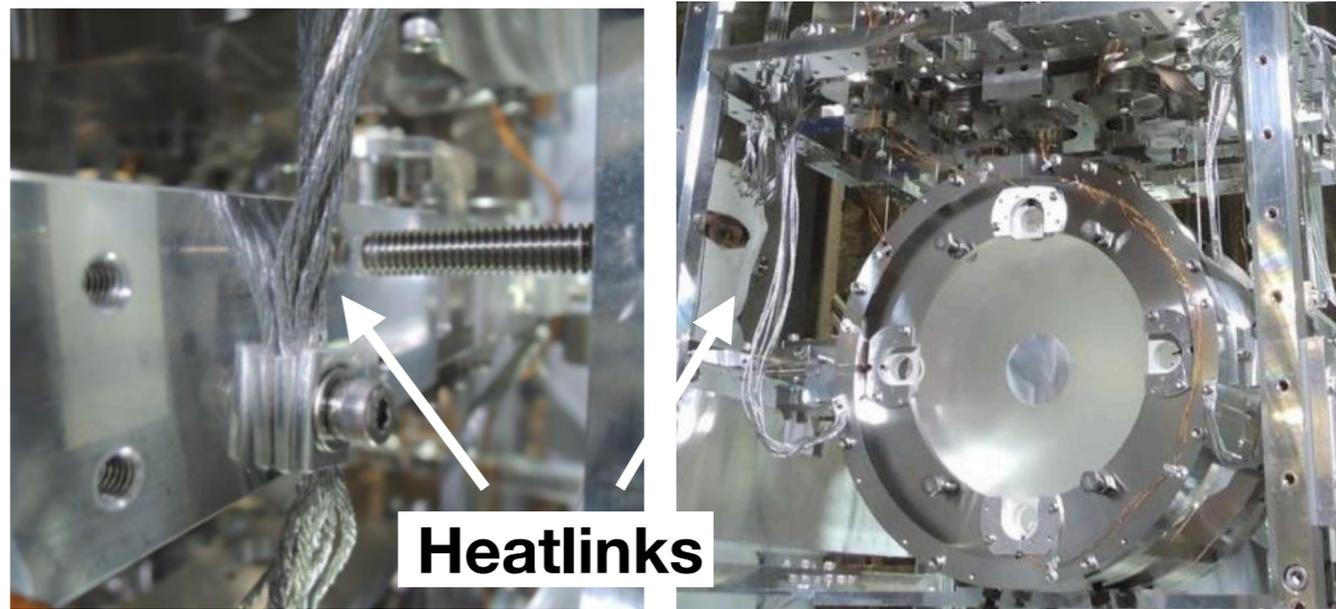
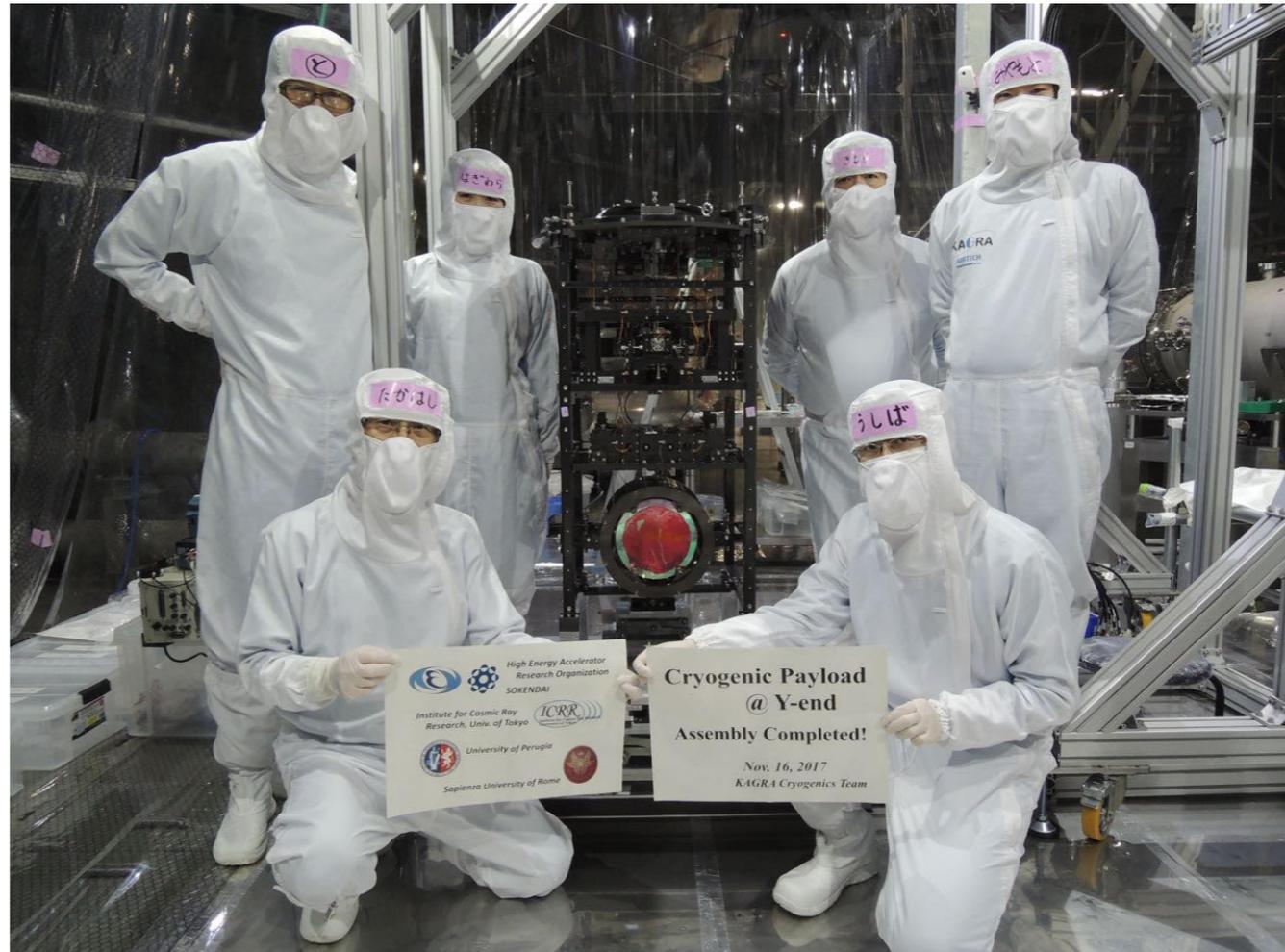
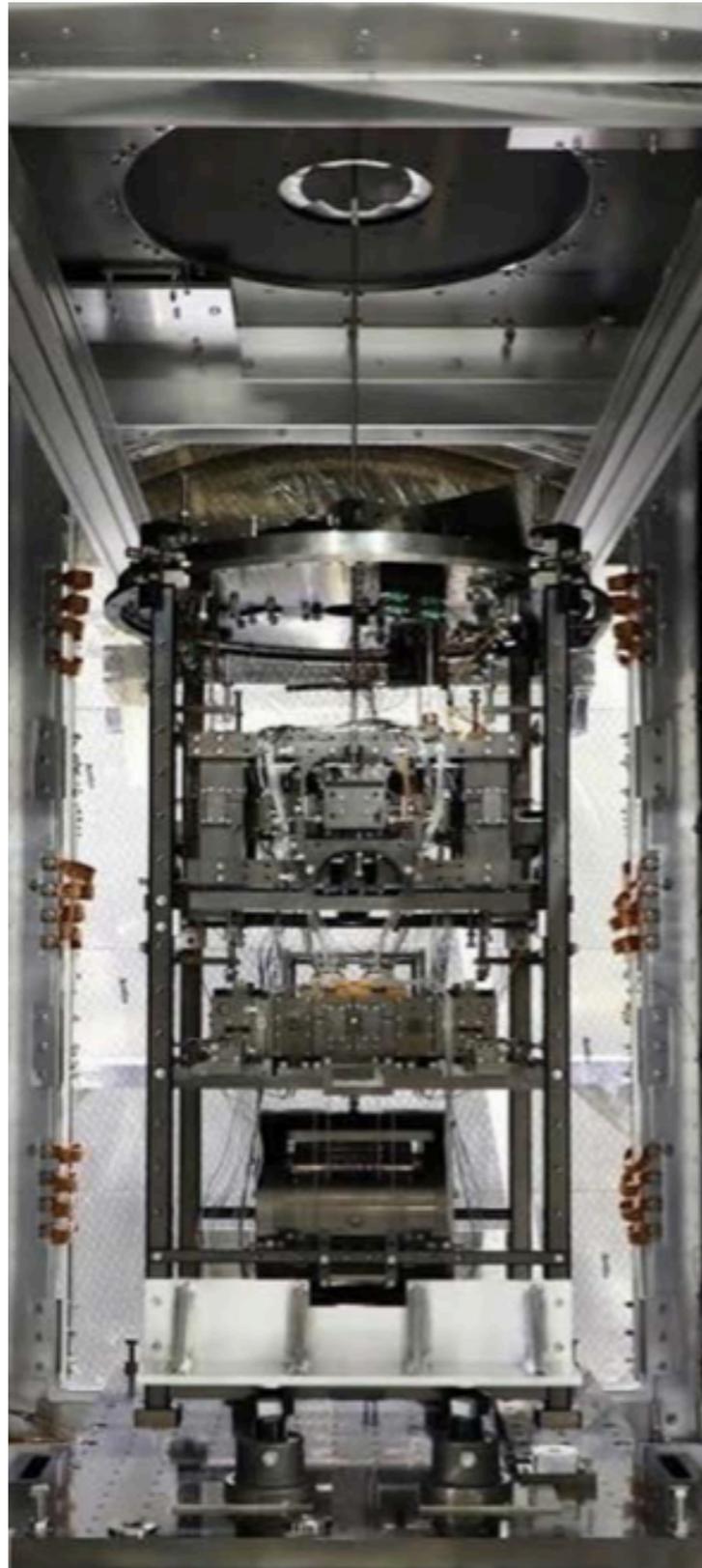
The cryogenic payload



R Kumar et al 2016 J. Phys.: Conf. Ser. 716 012017

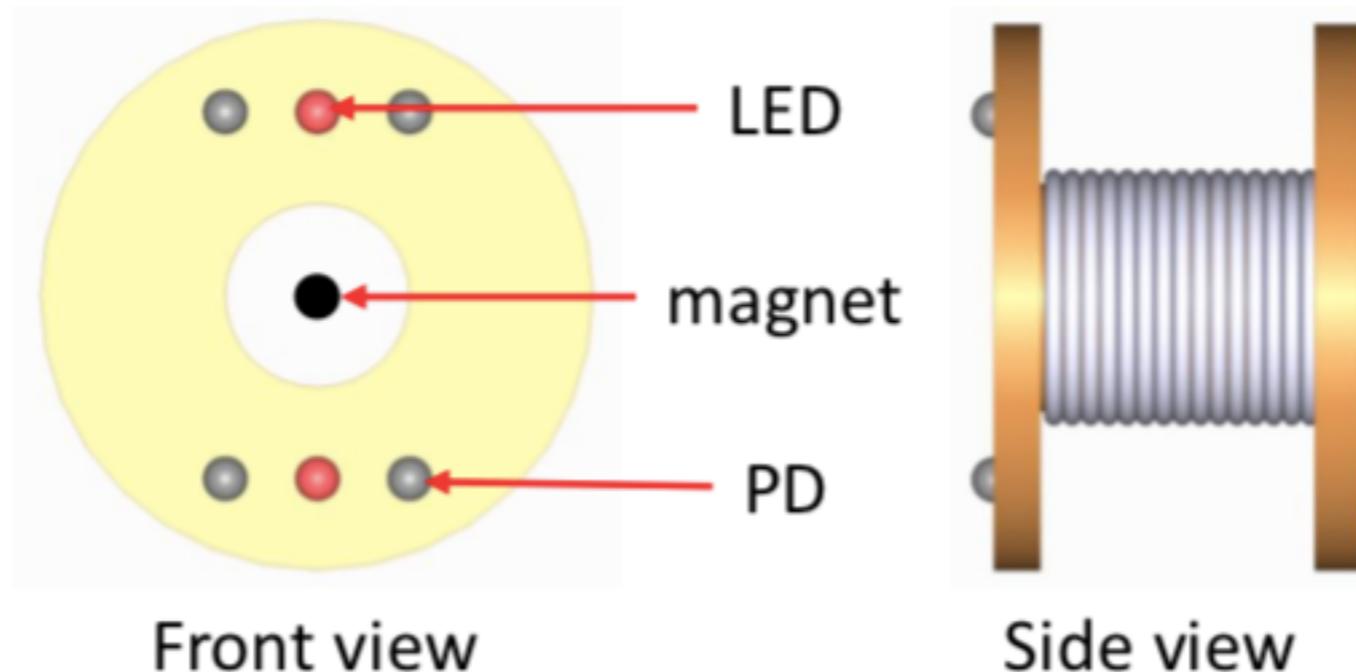
- 4 stages
- Marionette is suspended from the platform with a single maraging steel wire.
- Intermediate mass is suspended from the marionette with four CuBe fibers.
- high purity aluminum heat link to evacuate heat

The cryogenic payload

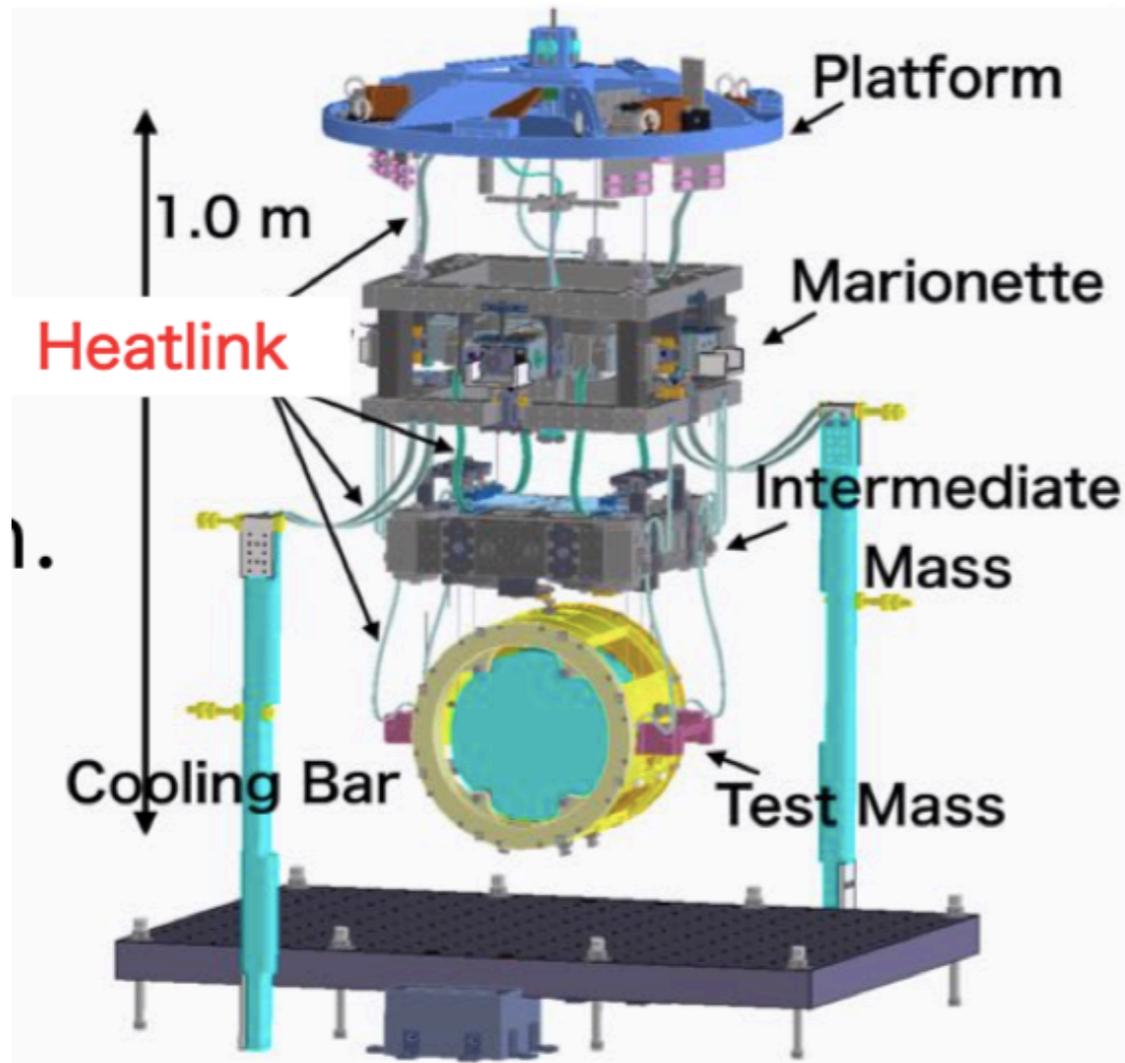


The cryogenic payload: sensing and actuators

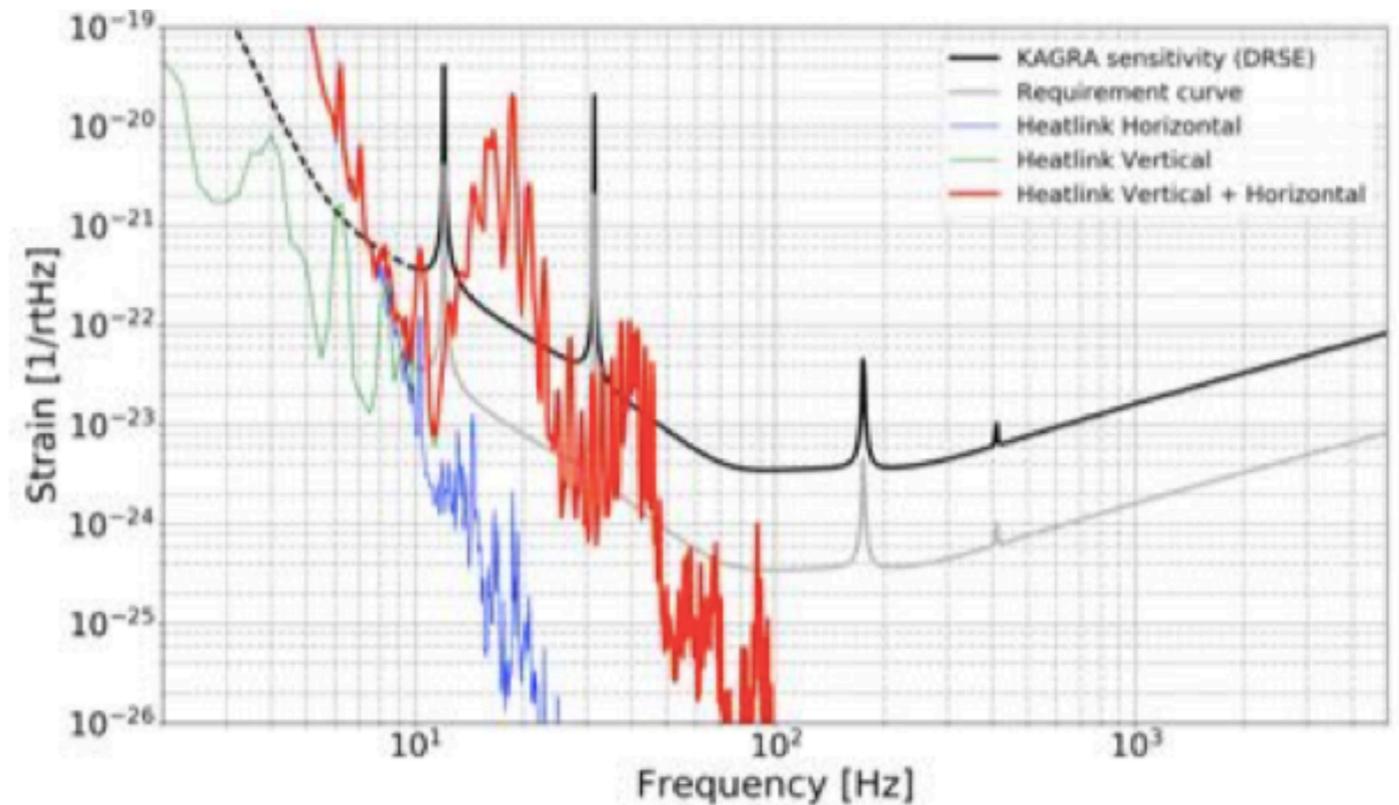
- Optical levers to monitor displacement with respect to ground
- Photosensors for differential displacement monitor between main chain and recoil chain
- Coil-magnet actuators for local control of the payload



The cryogenic payload: heatlinks



- Heatlink couple too much vibrations and can spoil the sensitivity

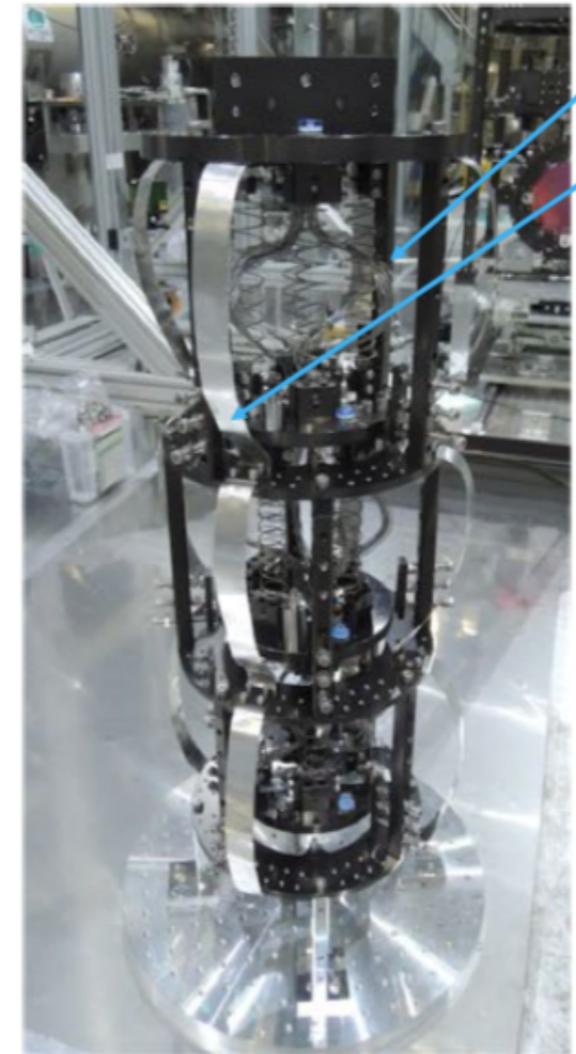
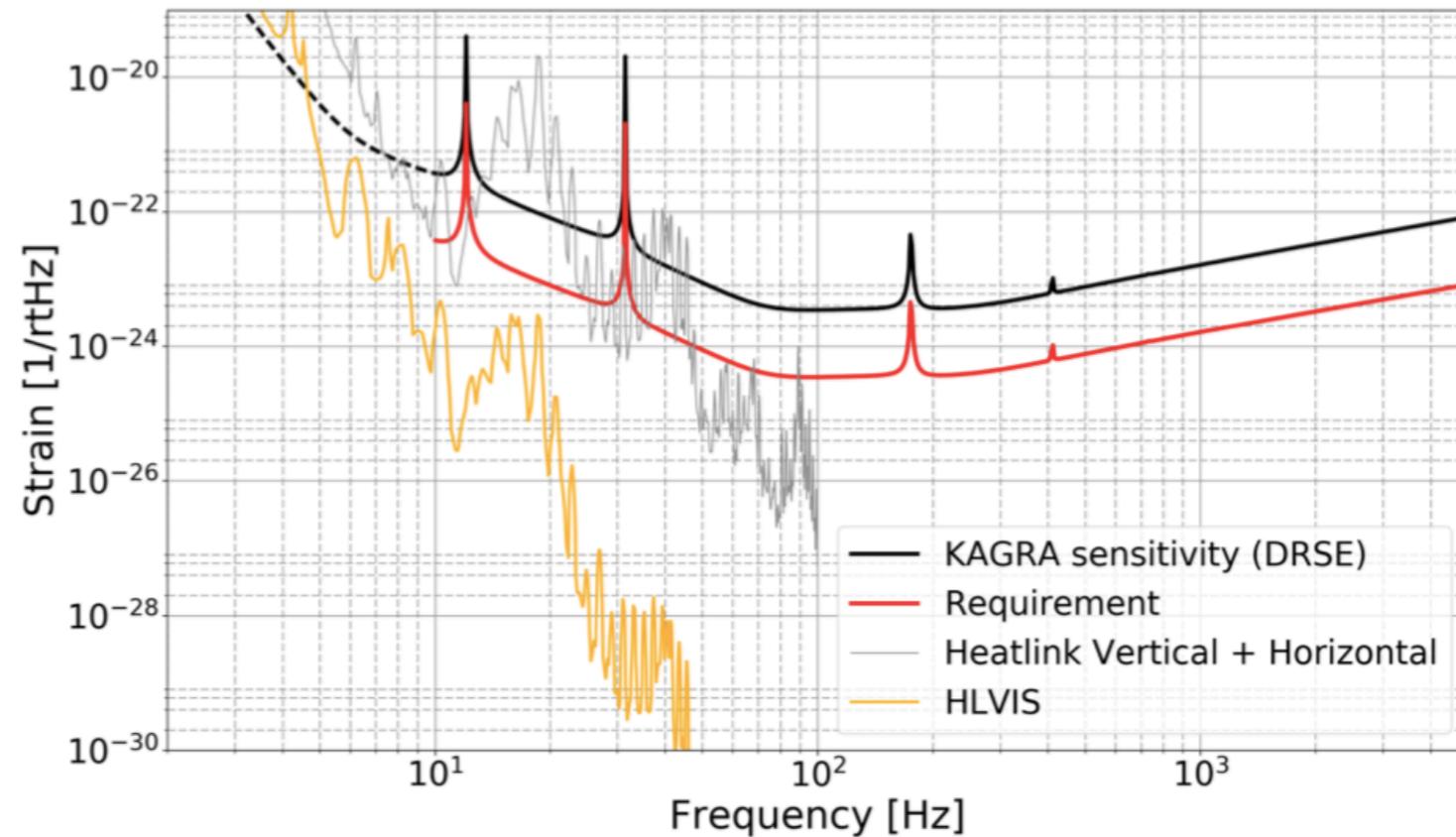


T. Yamada JGW-G1808903
F2F meeting 08-18

- Heatlink-Vibration-Isolation (HLVIS) system was developed and recently installed to attenuate those vibrations

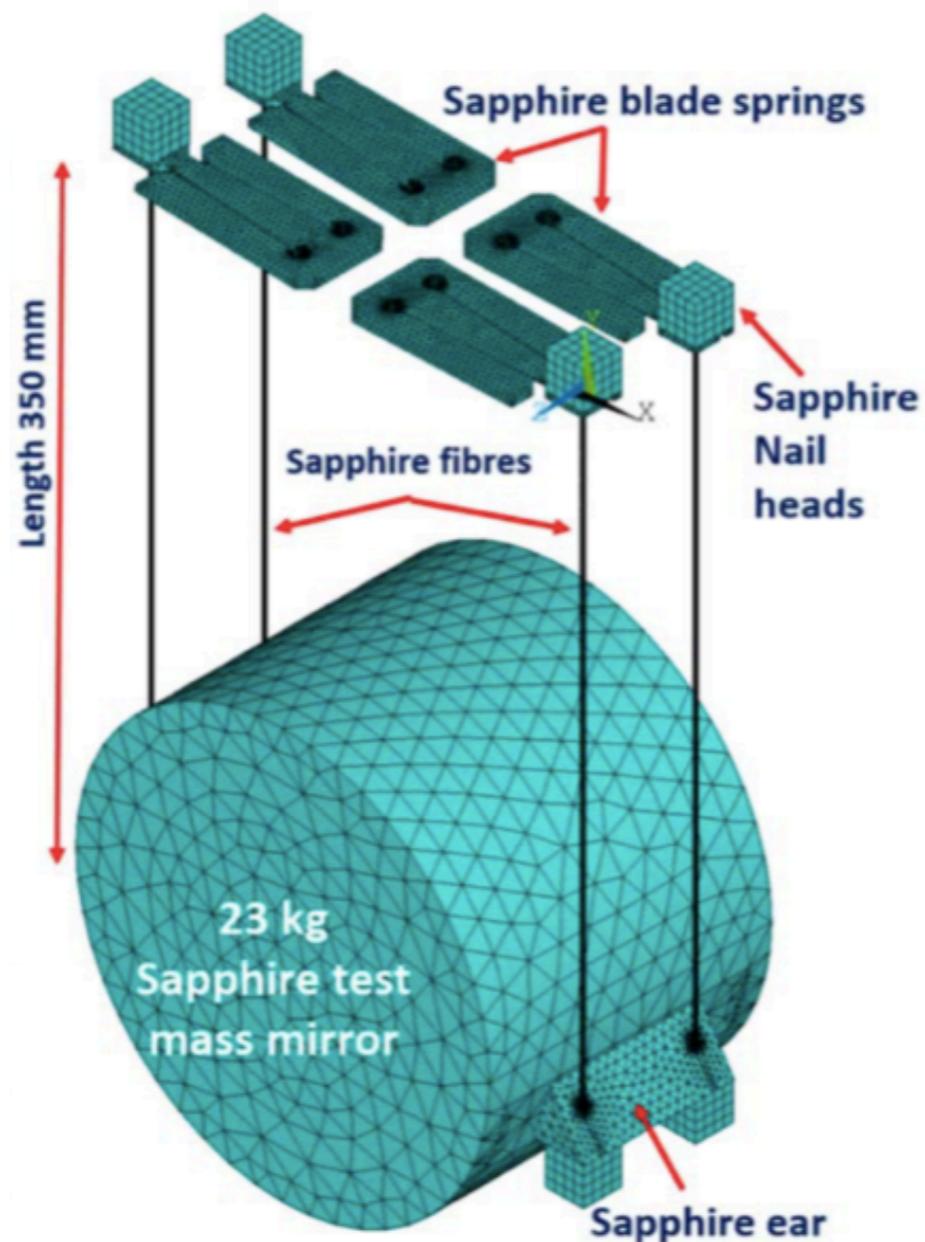
Heatlink vibration isolation

- Heatlink-Vibration-Isolation (HLVIS) system was developed and recently installed to attenuate those vibrations



T. Yamada JGW-G1808903 F2F meeting 08/18

The cryogenic payload: last stage



R Kumar et al 2016 J. Phys.: Conf. Ser. 716 012017

- 4 sapphire fibers of 1.6 mm diameter to extract heat
- Transferable heat

$$K = \int_{T_1}^{T_2} \frac{\pi d_w^2}{4l_{\text{sus}}} N_w \kappa(d_w, T) dT$$

Thermal conductivity

$$\kappa \simeq 5270 d_w (T/1 K)^{2.24} [\text{W}^{-1} \text{m}^{-1} \text{K}^{-1}]$$

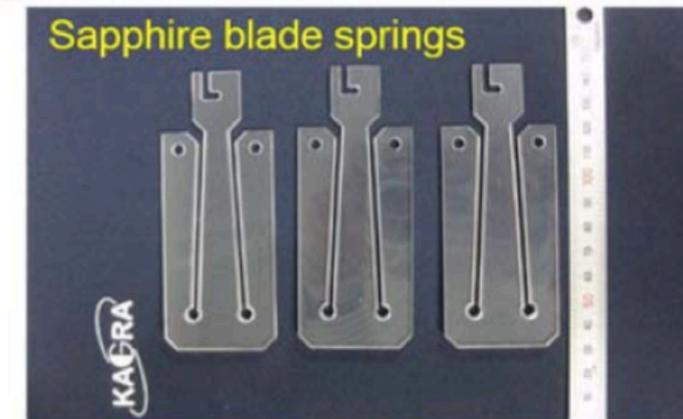
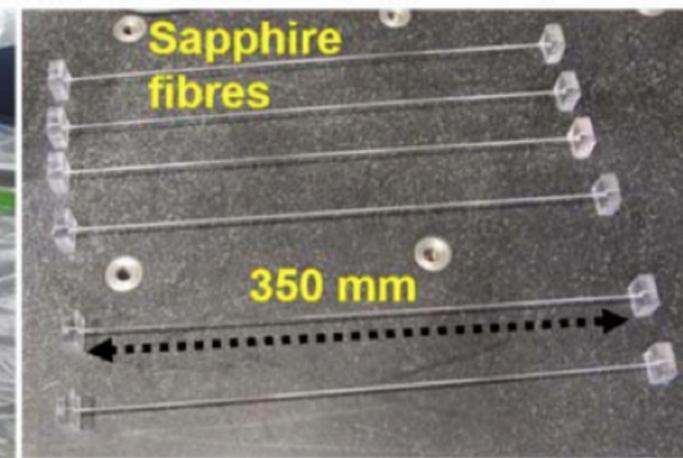
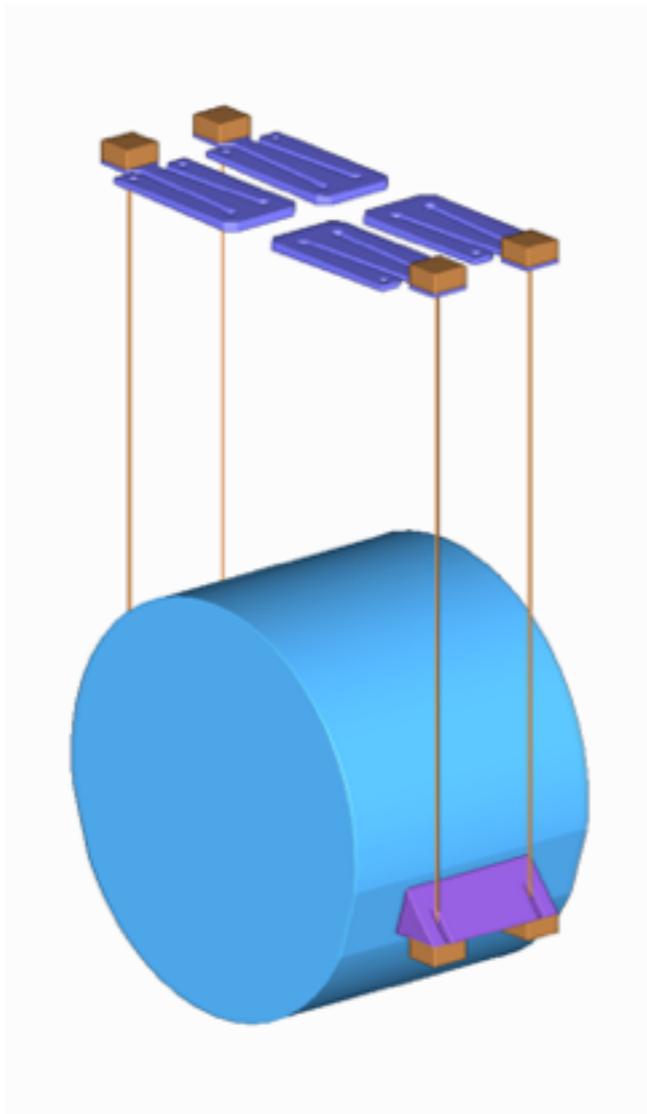
d_w = wire diameter

l_{sus} = wire length

N_{sus} = wire number

Connection between mirror and fibers

- Hydroxide Catalysis Bonding (HCB), based on chemical reaction
- Same used in LIGO and Virgo
- Some R&D to confirm that it can be used with sapphire cryogenic temperature



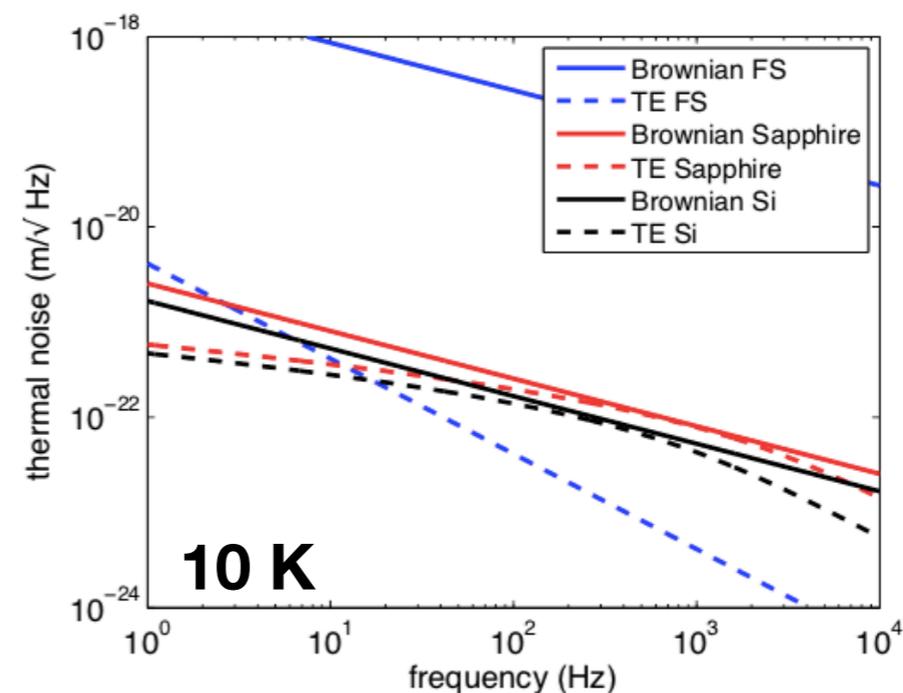
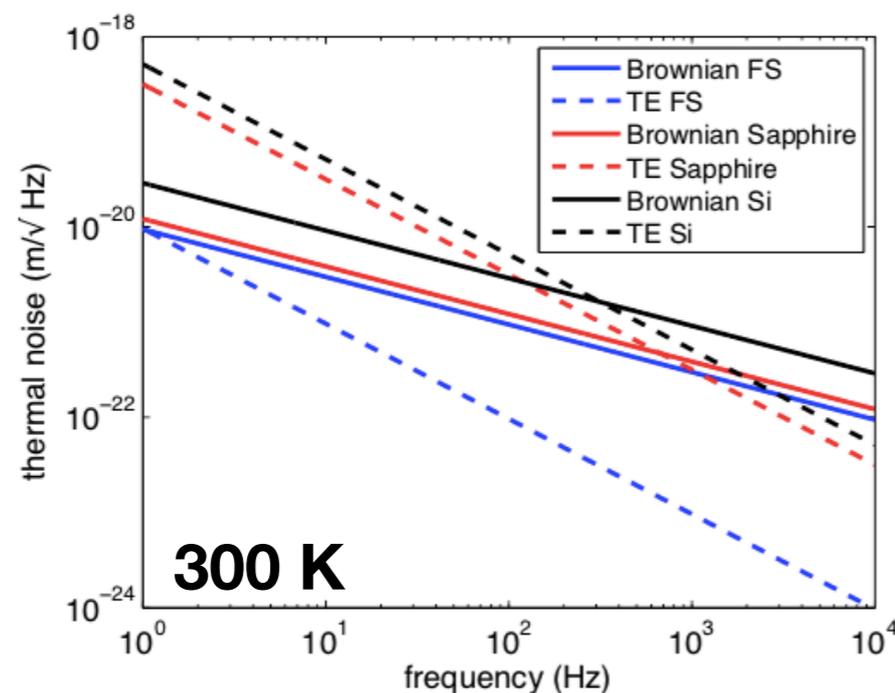
Mechanical loss of a hydroxide catalysis bond between sapphire substrates and its effect on the sensitivity of future gravitational wave detectors

K. Haughian, D. Chen, L. Cunningham, G. Hofmann, J. Hough, P. G. Murray, R. Nawrodt, S. Rowan, A. A. van Veggel, and K. Yamamoto
Phys. Rev. D **94**, 082003 – Published 12 October 2016

Which material for cryogenic operation?

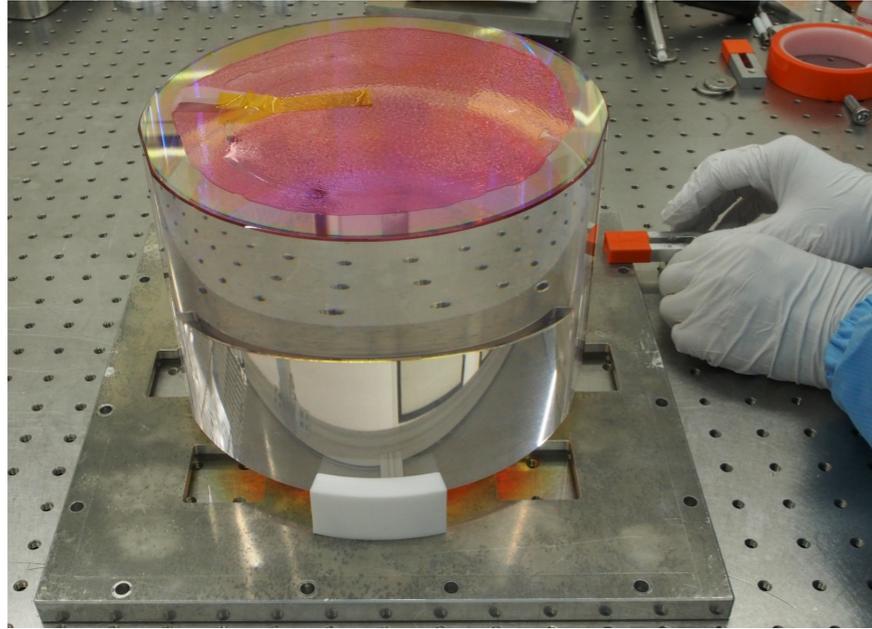
$$S_{TN} \propto \sqrt{T\phi}$$

- Silica has larger losses and low thermal conductivity at low temperature
- Silicon have low losses but its not transparent at 1064
- Sapphire have low losses and high thermal conductivity

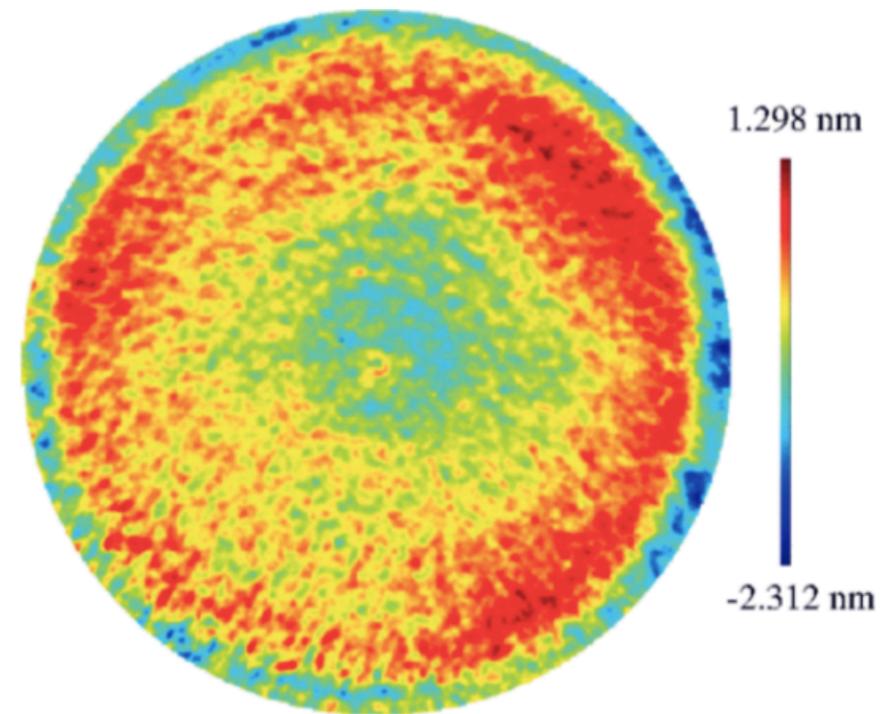
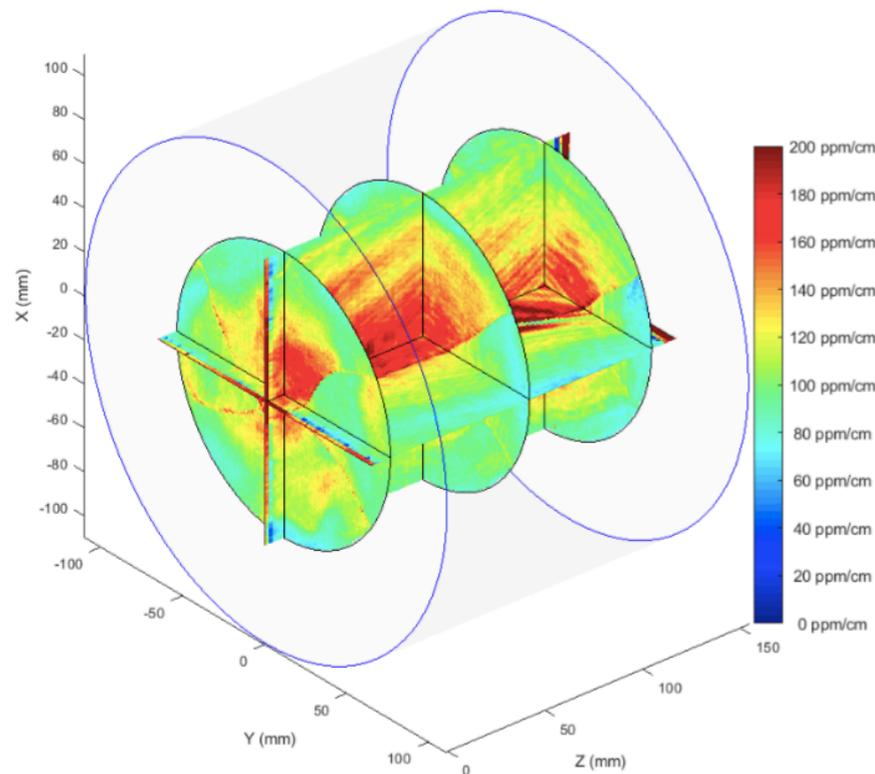


ET design study

KAGRA cryogenic mirrors



- Sapphire, 23 kg, 22 cm in diameter and 15 cm in thickness
- Less experience than fused silica: birifrangency, hardness, absorption

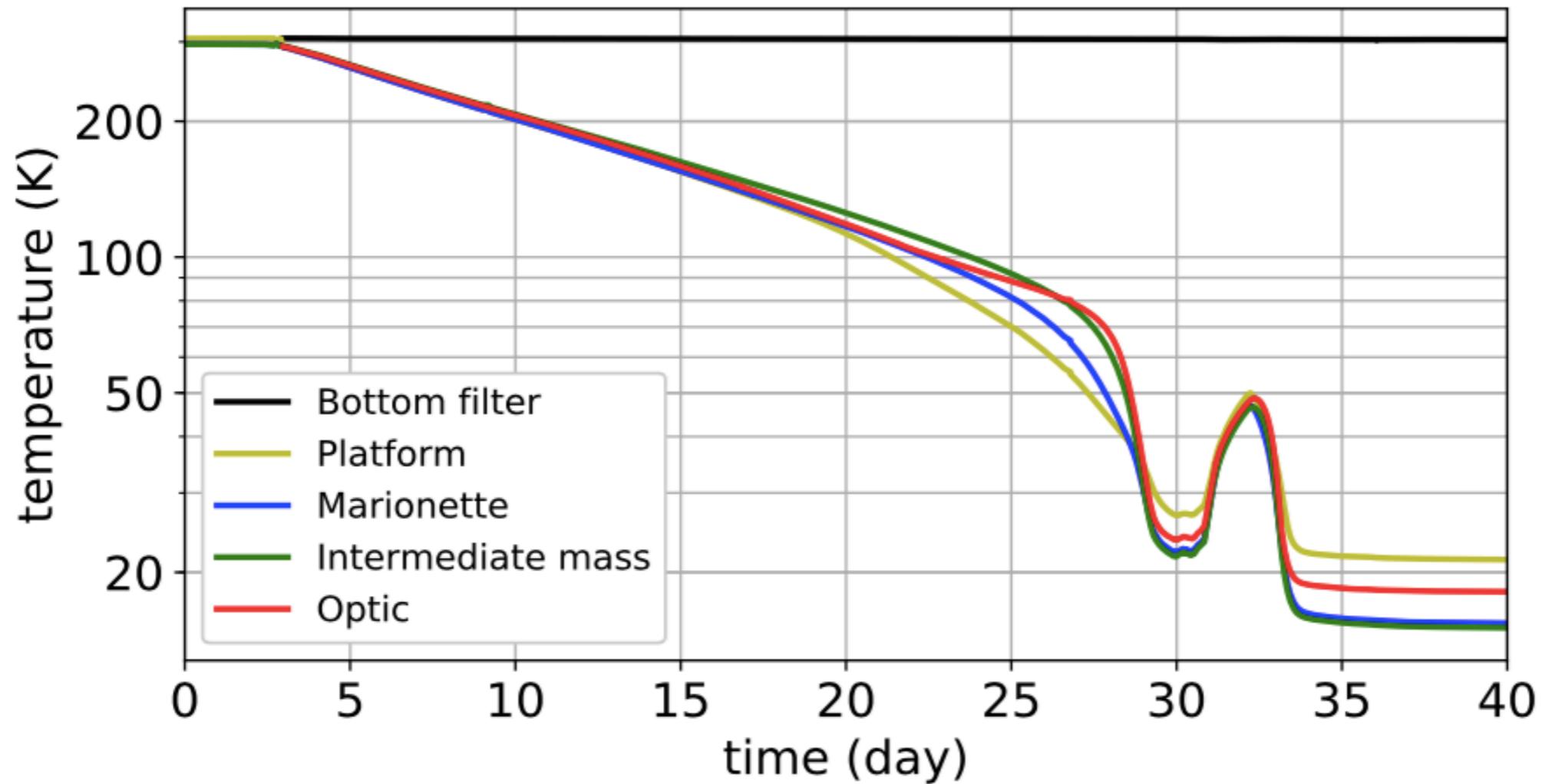


Absorption measured at NAOJ. M. Marchiò.

Hirose et al. Phys. Rev. D **89**, 062003

Cooling time

- Radiation is the dominant process for the first 20 days
- Bump due to cryocoolers restart



arXiv:1901.03569

Conclusions

- Thermal noise of mirror and suspension is one of the main limitation for GW detectors
- Cryogenic operation is a “straightforward” way to reduce thermal noise
- The physics is simple but the technology is challenging
- R&D and prototyping activity for more than 20 year in Japan
- Technology has proven to be mature and it worked well in the preliminary KAGRA operation
- Promising technology to be integrated in 3rd generation detectors