Quantum spacetime imprints in gamma-ray propagation

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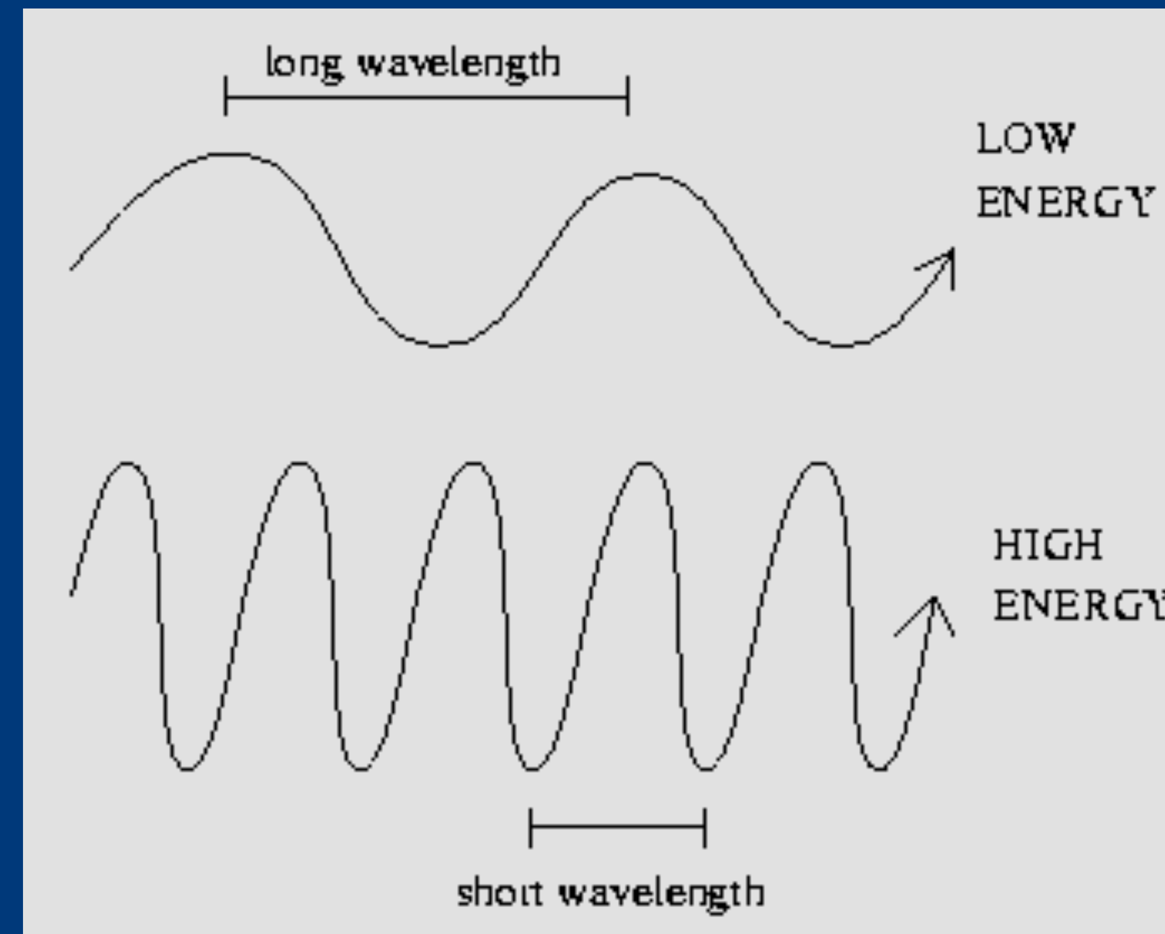
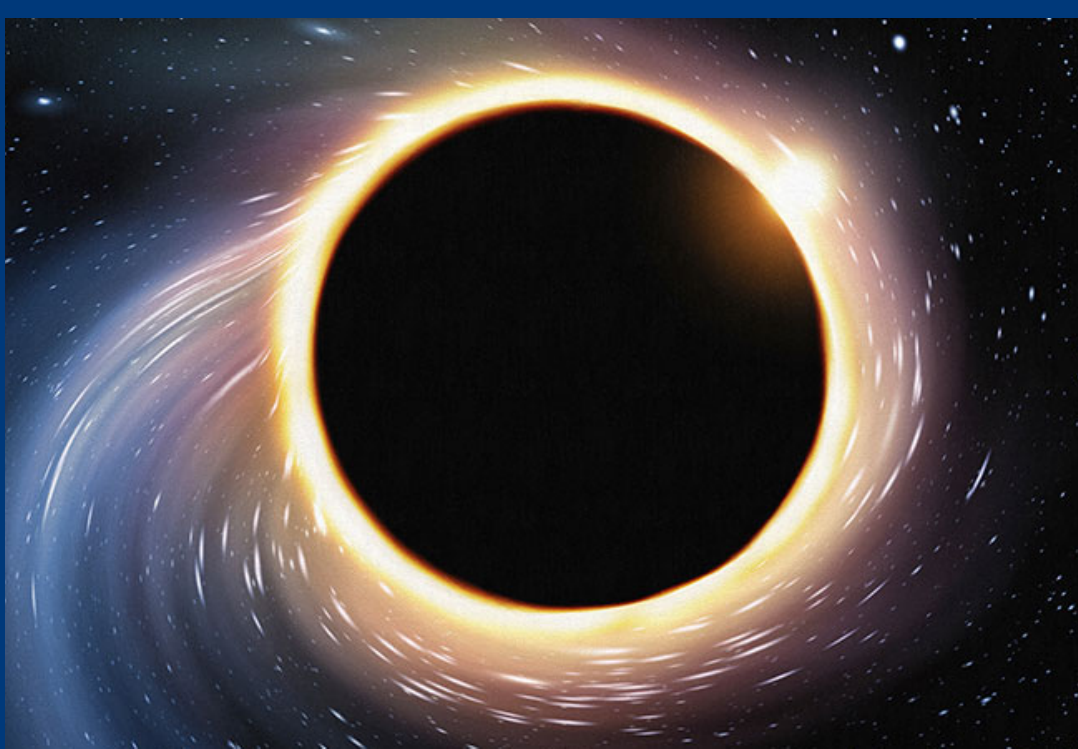
"QM + GR"

$$\lambda_C \sim \frac{\hbar c}{E}$$



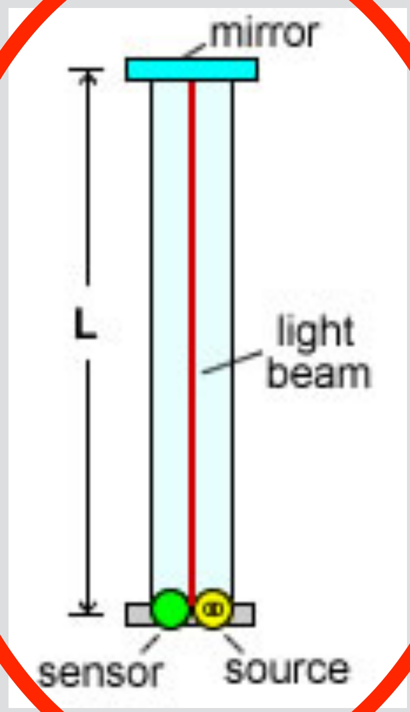
$$\lambda_C \sim \lambda_S \sim \frac{GE}{c^4}$$



$$\lambda \gtrsim \sqrt{\frac{G\hbar}{c^3}} = \ell_{\text{Pl}} \approx 10^{-35} \text{ m}$$

....minimum allowed length??

Quantum measurements with gravitational interactions



$$\delta v(0) = \frac{\delta p(0)}{M} \geq \frac{\hbar}{2M\delta L(0)}$$

Quantum device for distance measurements

$$\delta L_{QM}(T) \geq \delta L(0) + \frac{\hbar L}{cM\delta L(0)} \to [\delta L_{QM}(T)]^2 \geq \frac{\hbar L}{Mc}$$

+ gravitational interactions:

$$\widetilde{L} = L + \frac{r_S}{2}\ln\left|\frac{rc+L-r_S}{rc-r_S}\right| \to \delta L_{GR} \simeq \frac{r_S}{2}\ln\left|\frac{rc+L}{rc}\right| \geq \frac{GM}{c^2}$$

"QM + GR"

$$\delta L_{QG} \geq \left(\ell_{Pl}^2 L\right)^{1/3} \qquad \ell_{Pl} = \sqrt{\frac{G\hbar}{c^3}}$$

....minimum allowed length??


Lorentz invariance violation
Deformed Lorentz invariance

Standard Model extensions

Kostelecky, Mewes, arXiv:0905.0031

General Relativity and Standard model as effective field theories → Lorentz and CPT violations!!

$$S_{(d)} = \int d^4x\, \mathcal{K}_{(d)}^{\alpha_1\alpha_2\alpha_3\ldots\alpha_d} \underbrace{A_{\alpha_1}\partial_{\alpha_3}\ldots\partial_{\alpha_d}A_{\alpha_2}}_{}\,,$$

higher dimensions operators

$$\mathcal{K}_{(d)} \sim \zeta M_{Pl}^{4-d}$$

Symmetry constraints restrict allowed coefficients:
U(1) + space-time translational invariance

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \delta\mathcal{L},$$

$$\delta\mathcal{L}_{\rm photon} \supset -\frac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu} + \frac{1}{2}(k_{AF})_\kappa \epsilon^{\kappa\lambda\mu\nu}A_\lambda F_{\mu\nu}\,,$$

$$(\hat{k}_{AF})_\kappa = \sum_{d=\rm odd} (k_{AF}^{(d)})_\kappa{}^{\alpha_1\ldots\alpha_{(d-3)}}\partial_{\alpha_1}\ldots\partial_{\alpha_{(d-3)}}\,,$$

$$(\hat{k}_F)^{\kappa\lambda\mu\nu} = \sum_{d=\rm even} (k_F^{(d)})^{\kappa\lambda\mu\nu\alpha_1\ldots\alpha_{(d-4)}}\partial_{\alpha_1}\ldots\partial_{\alpha_{(d-4)}}\,,$$

Modified Dispersion Relation

Kostelecky, Mewes, arXiv:0905.0031

$$0 = -\tfrac{1}{3}\hat{\chi}^{\mu\alpha\nu\beta}\left((\widetilde{\wedge^2 M_e})_{\mu\alpha\nu\beta} - 3(\widetilde{\wedge^2 M_o})_{\mu\alpha\nu\beta}\right)$$
$$= -\tfrac{1}{3}\epsilon_{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\nu_1\nu_2\nu_3\nu_4}p_{\rho_1}p_{\rho_2}p_{\rho_3}p_{\rho_4}\hat{\chi}^{\mu_1\mu_2\nu_1\rho_1}\hat{\chi}^{\nu_2\rho_2\rho_3\mu_3}\hat{\chi}^{\rho_4\mu_4\nu_3\nu_4} + 8p_\alpha p_\beta(\hat{k}_{AF})_\mu(\hat{k}_{AF})_\nu\hat{\chi}^{\alpha\mu\beta\nu}.$$

$$\hat{\chi}^{\mu\nu\rho\sigma} = \tfrac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\nu\rho}\eta^{\mu\sigma}) + (\hat{k}_F)^{\mu\nu\rho\sigma},$$

$$M^{\mu\nu} = (\eta^{\mu\nu}\eta^{\alpha\beta} - \eta^{\mu\alpha}\eta^{\nu\beta} + 2(\hat{k}_F)^{\mu\alpha\nu\beta})p_\alpha p_\beta - 2i\epsilon^{\mu\nu\alpha\beta}(\hat{k}_{AF})_\alpha p_\beta$$

EFT scale $\neq$ Lorentz breaking scale

$$E^2 = p^2 + m^2 + \Delta(p)$$

$$\Delta(p) = \tilde{\eta}_1 p^1 + \tilde{\eta}_2 p^2 + \tilde{\eta}_3 p^3 + \tilde{\eta}_4 p^4 + \dots$$

$$\tilde{\eta}_1 = \eta_1\frac{\mu^2}{M}, \quad \tilde{\eta}_2 = \eta_1\frac{\mu}{M}, \quad \tilde{\eta}_3 = \eta_3\frac{1}{M}, \quad \tilde{\eta}_4 = \eta_4\frac{1}{M^2}$$

Tight bounds on LIV effects

Liberati, arXiv:1304.5795

$$E^2 = p^2 + m^2 + \sum_{n=1}^{\infty} \tilde{\eta}_{(n)} p^n \ ,$$

Order	photon	e^-/e^+	Protrons	Neutrinos[a]
n=2	N.A.	$O(10^{-16})$	$O(10^{-20})$ (CR)	$O(10^{-8} \div 10^{-10})$
n=3	$O(10^{-16})$ (GRB)	$O(10^{-16})$ (CR)	$O(10^{-14})$ (CR)	$O(40)$
n=4	$O(10^{-8})$ (CR)	$O(10^{-8})$ (CR)	$O(10^{-6})$ (CR)	$O(10^{-7})^{*}$ (CR)

Deformed Lorentz invariance??!

Doubly or Deformed Special Relativity

Amelino-Camelia, arXiv:gr-qc/0012051

Planck scale with respect to whom?!

1) **The laws of physics take the same form in all inertial frames;**

2) **The laws of physics involve a fundamental velocity scale c and a fundamental length scale Lp**

Modified dispersion relations invariant under some "new" symmetries

DSR1

$$2E_p^2\left[\cosh\left(\frac{E}{E_p}\right)-\cosh\left(\frac{m}{E_p}\right)\right]=\vec{p}^2 e^{E/E_p}\ .$$

Majid, Ruegg, arXiv:hep-th/9405107

DSR2

$$\frac{m^2}{(1-m/E_p)^2}=\frac{E^2-p^2}{(1-E/E_p)^2}\ ,$$

Magueijo, Smolin, arXiv:hep-th/0112090

DSR1 or kappa-Poincaré

Majid, Ruegg, arXiv:hep-th/9405107, Lukierski, Ruegg, Nowicki, Tolstoi, PLB 264 (1991)

deformed mass Casimir

$$C_\lambda(\mathcal{P}) = \frac{e^{\lambda P_0} + e^{-\lambda P_0} - 2}{\lambda^2} - \vec{\mathcal{P}}^2 e^{\lambda P_0} \xrightarrow{\lambda=0} P_0^2 - \vec{P}^2$$

invariant under non-linear deformations of the Poincaré commutators

deformed commutators:

$$\begin{aligned}
[\mathcal{P}_\mu, \mathcal{P}_\nu] &= 0 \\
[M_j, \mathcal{P}_k] &= i\epsilon_{jkl}\mathcal{P}_l \\
[M_j, P_0] &= 0 \\
[\mathcal{N}_j, \mathcal{P}_k] &= i\delta_{jk}\left(\frac{1}{2\lambda}(1 - e^{-2\lambda P_0}) + \frac{\lambda}{2}\mathcal{P}^2\right) - i\lambda\mathcal{P}_j\mathcal{P}_k \\
[\mathcal{N}_j, P_0] &= i\mathcal{P}_j
\end{aligned}$$

deformed coproducts:

$$\begin{aligned}
\Delta P_0 &= P_0 \otimes 1 + 1 \otimes P_0 \\
\Delta M_j &= M_j \otimes 1 + 1 \otimes M_j \\
\Delta \mathcal{P}_j &= \mathcal{P}_j \otimes 1 + e^{-\lambda P_0} \otimes \mathcal{P}_j \\
\Delta \mathcal{N}_j &= \mathcal{N}_j \otimes 1 + e^{-\lambda P_0} \otimes \mathcal{N}_j - \lambda\varepsilon_{jkl}\mathcal{P}_k \otimes M_l
\end{aligned}$$

requiring duality relations

$$< \mathrm{x}_\mu, P_\nu > = -i\eta_{\mu\nu}$$

kappa-Minkowski non-commutative space

$$[\mathrm{x}_0, \mathrm{x}_j] = -i\lambda x_j$$

DSR limit of QG?

$$E_{Pl} = \sqrt{\frac{\hbar c^5}{G}} \qquad \ell_{Pl} = \sqrt{\frac{\hbar G}{c^3}}$$

$$t_{Pl} = \sqrt{\frac{\hbar G}{c^5}} \qquad M_{Pl} = \sqrt{\frac{\hbar c}{G}}$$

$$E_{Pl} = M_{Pl}c^2 = const$$

$$\downarrow$$

$$\boxed{c = const \qquad \hbar \to 0 \qquad G \to 0}$$

Gravity???!

DSR in curved spacetime

Amelino-Camelia, Marciano, Matassa, Rosati, arXiv:1206.5315

Interplay between DSR-deformation scale and expansion-rate scale?!

ansatz for the mass Casimir

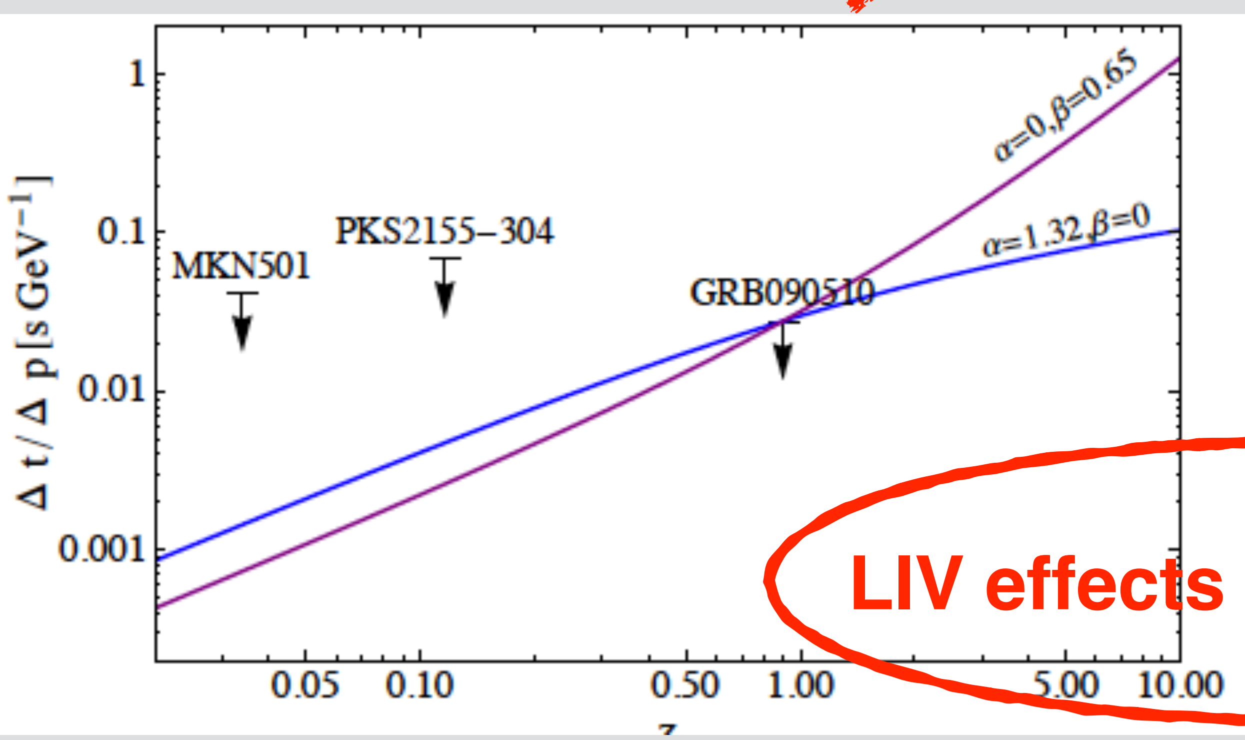
$$\boxed{\mathcal{C}_{H,\alpha,\beta} = E^2 - p^2 + 2H\mathcal{N}p + \ell\left(\alpha E^3 + \beta E p^2\right)}$$

Invariant under a Planck-scale deformed version of de Sitter symmetry group

$$\{E,p\} = Hp - \ell\alpha H E p\ ,$$
$$\{E,\mathcal{N}\} = p - H\mathcal{N} - \ell\alpha E(p - H\mathcal{N}) - \ell\beta E p\ ,$$
$$\{p,\mathcal{N}\} = E + \frac{1}{2}\ell\alpha E^2 + \frac{1}{2}\ell\beta p^2\ .$$

$$\boxed{x_{m=0,p}\left(\eta\right) = x_0 + \left(\eta - \eta_0\right)\left(1 - \ell|p|\right)}$$

derive DSR phenomenology



LIV effects $\neq$ DSR effects

Covariant approach: the Casimir generates worldline evolution in the affine parameter, particles' wordiness with 3 relativistic invariants: c, Lp, H

Rainbow gravity. 1

Magueijo, Smolin, arXiv:gr-qc/0305055

General form for MDR

$$p_0^2 f^2\left(\frac{p_0}{M_P}\right) - (p\cdot p) g^2\left(\frac{p_0}{M_P}\right) = m^2,$$

realised by momentum space maps

$$U\cdot(p_0, p_i) = (U_0, U_i) = \left(f\left(\frac{p_0}{M_P}\right)p_0, g\left(\frac{p_0}{M_P}\right)p_i\right),$$

implies non-linear norm for momenta

$$|p|^2 = \eta^{ab} U_a(p) U_b(p).$$

momenta-dependent metric!!

$$\boxed{ds^2 = \tilde{g}_{\gamma\beta}(p) dx^\gamma dx^\beta = \frac{(dx^0)^2}{f^2(p)} - \frac{(dx^i)^2}{g^2(p)}}$$

PROBLEM: Do not enjoy same symmetries!!!

Rainbow gravity. 2: Hamiltonian and Finsler approach

Amelino-Camelia, Barcaroli, Gubitosi, Liberati, Loret, arXiv:1407.8143

Hamiltonian approach

$$H(x,p) = g^{ab}(x)p_a p_b = m^2 \,.$$

MDR as phase-space Hamiltonian

$$H_{qDS}(x,p) = g^{ab}(x)p_a p_b + \ell G^{abc} p_a p_b p_c = p_0^2 - p_1^2(1-2hx^0) - \ell p_0 p_1^2 (1-2hx^0)$$

q-deSitter case

$$g^{H\,ab} = \frac{1}{2}\bar\partial^a\bar\partial^b H_{qdS} = g^{ab} + \ell 3 G^{abc} p_c,$$
$$= \begin{pmatrix} 1 & -\ell p_1(1+2hx^0) \\ -\ell p_1(1+2hx^0) & -(1+2hx^0)(1+\ell p_0) \end{pmatrix},$$
$$g^{H}{}_{ab} = g_{ab} - \ell 3 g_{aj} g_{bi} G^{ijc} p_c$$
$$= \begin{pmatrix} 1 & -\ell p_1 \\ -\ell p_1 & -(1-2hx^0)(1-\ell p_0) \end{pmatrix}.$$

rainbow-like metrics

Finsler approach

kappa-Poncaré case

$$\mathcal{S}[q,p,\lambda] = \int d\tau \left[\dot\eta\, \Omega + \dot x\, \Pi - \lambda(\mathcal{H} - m^2)\right],$$

$$g_{\mu\nu}(x,\dot x) = \begin{pmatrix} 1 + \frac{3}{2}\ell m \frac{\dot x^0 (\dot x^1)^4}{((\dot x^0)^2 - (\dot x^1)^2)^{5/2}} & \ell\frac{m}{2}\frac{-4(\dot x^0)^2(\dot x^1)^3 + (\dot x^1)^5}{((\dot x^0)^2-(\dot x^1)^2)^{5/2}} \\ \ell\frac{m}{2}\frac{-4(\dot x^0)^2(\dot x^1)^3 + (\dot x^1)^5}{((\dot x^0)^2-(\dot x^1)^2)^{5/2}} & -1 + \frac{1}{2}\ell m (\dot x^0)^3 \frac{2(\dot x^0)^2 + (\dot x^1)^2}{((\dot x^0)^2-(\dot x^1)^2)^{5/2}} \end{pmatrix}$$

Finsler norm

$$\mathcal{L}(q,\dot q) = mF(\dot q)\,.$$

$$g^{\mathrm{F}}_{\mu\nu} = \frac{1}{2}\frac{\partial^2 F^2}{\partial \dot q^\mu \partial \dot q^\nu}$$

Finsler metric

$$F(\dot x) = \left(\sqrt{(\dot x^0)^2 - (\dot x^1)^2} + \frac{\ell}{2} m \frac{\dot x^0 (\dot x^1)^2}{(\dot x^0)^2 - (\dot x^1)^2}\right)$$

$$\begin{cases} F(\dot q) \neq 0 \quad if \quad \dot q \neq 0 \\ F(\epsilon \dot q) = |\epsilon| F(\dot q) \end{cases}$$

Rainbow gravity. 3: modified gravity approach

Olmo, arXiv:1101.2841

Brans-Dicke-like action

$$S[g_{\mu\nu},\varphi,\psi] = \frac{\hbar}{16\pi l_P^2}\int d^4x\sqrt{-g}\left[(1+\varphi)R - \frac{1}{4l_P^2}\varphi^2\right] + S_m[g_{\mu\nu},\psi] \ ,$$

$$\Gamma \longrightarrow \quad f_R R_{\mu\nu}(\Gamma) - \frac{1}{2} f g_{\mu\nu} = \kappa^2 T_{\mu\nu}$$

$$g \longrightarrow \quad \nabla_\alpha\left(\sqrt{-g} f_R g^{\beta\gamma}\right) = 0 \ ,$$

Palatini or metric-affine formalism: connection and metric treated independently

$$G_{\mu\nu}(h) = \frac{\kappa^2}{f_R(T)} T_{\mu\nu} + \Lambda(T) h_{\mu\nu}$$

deformed Einstein equation, metric affected by: total energy momentum + energy momentum density

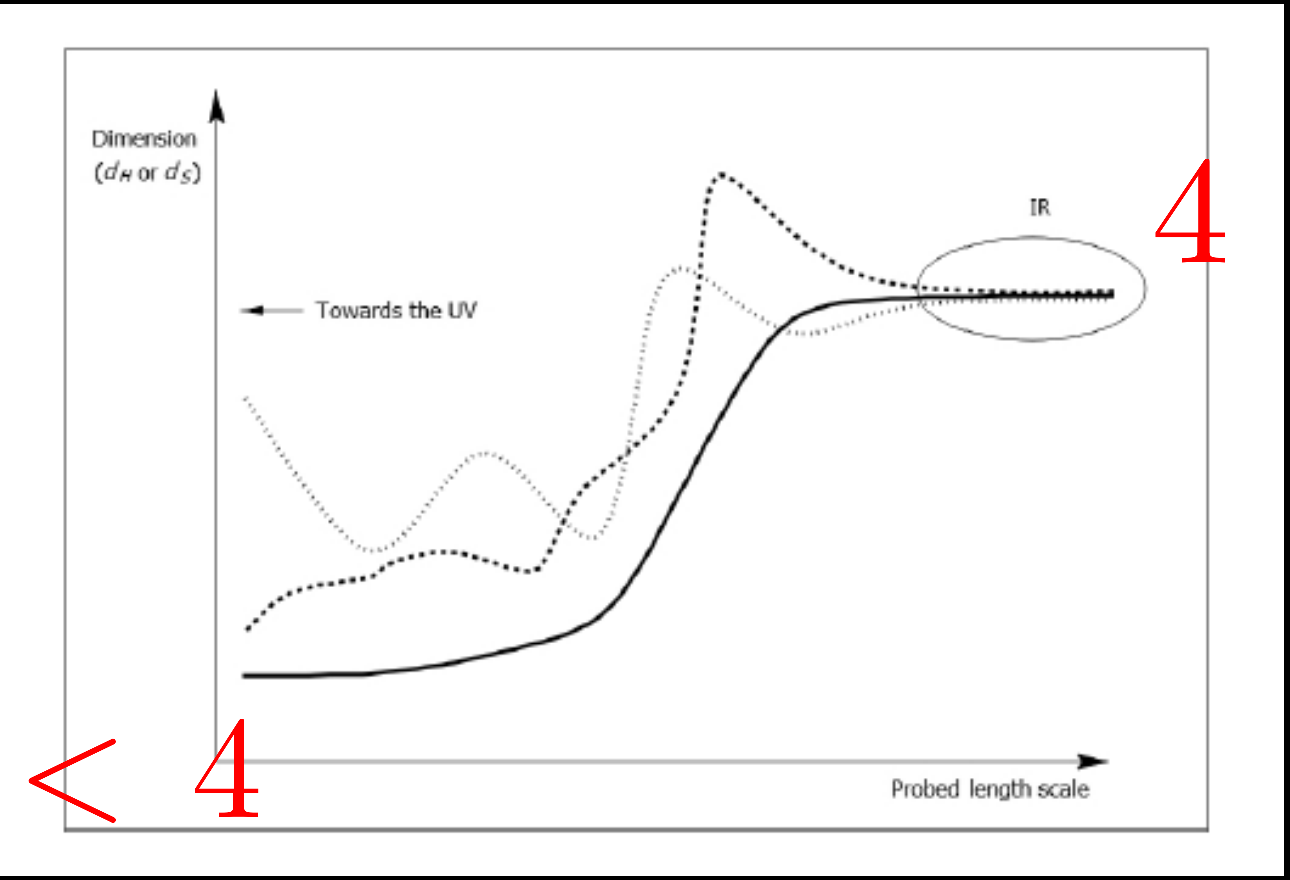
Rainbow-like metric: deformations depend on particle density

$$\boxed{ds^2 = -\frac{\left(1-\frac{2GM_\odot}{r}\right)}{\left(1+\frac{2\rho_{test}}{\rho_P}\right)}dt^2 + \frac{\left(1+\frac{2}{\rho_P}\frac{d(r\rho_{test})}{dr}\right)^2}{\left(1+\frac{2\rho_{test}}{\rho_P}\right)\left(1-\frac{2GM_\odot}{r}\right)}dr^2 + r^2 d\Omega^2 \ .}$$

Multi-fractional spaces

Calcagni, PRL 104 (2010)

dimensional reduction



multi-scale measure

$$\mathrm{d}^4 q(x) = \mathrm{d}q^0(t)\,\mathrm{d}q^1(x^1)\cdots \mathrm{d}q^3(x^3)$$

geometrical and physical coordinates

$$q^i(x^i) = x^i + \frac{\ell_*}{\alpha}\left|\frac{x^i}{\ell_*}\right|^{\alpha} F_\omega(x^i)\,,$$
$$q^0(t) = t + \frac{t_*}{\alpha_0}\left|\frac{t}{t_*}\right|^{\alpha_0} F_\omega(t)\,,$$

$$0 < \alpha < 1$$

geometrical and physical momenta

$$p^\mu(k^\mu) := \frac{1}{q^\mu(1/k^\mu)}$$

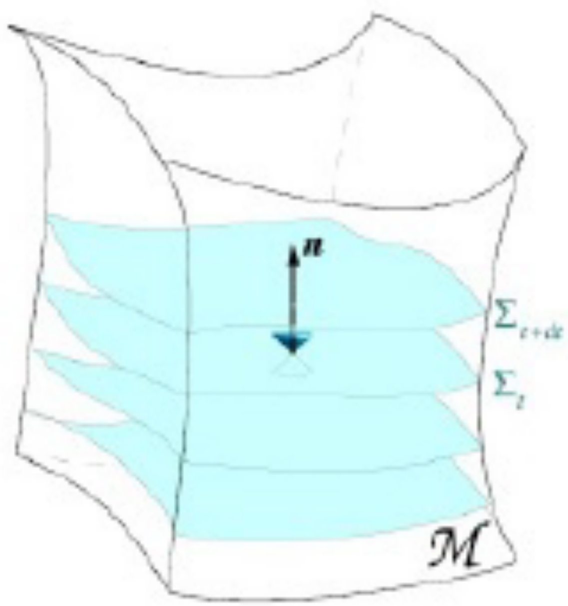
$$[p^0(E)]^2 = |\mathrm{p}|^2 + m^2 = \sum [p^i(k^i)]^2 + m^2$$

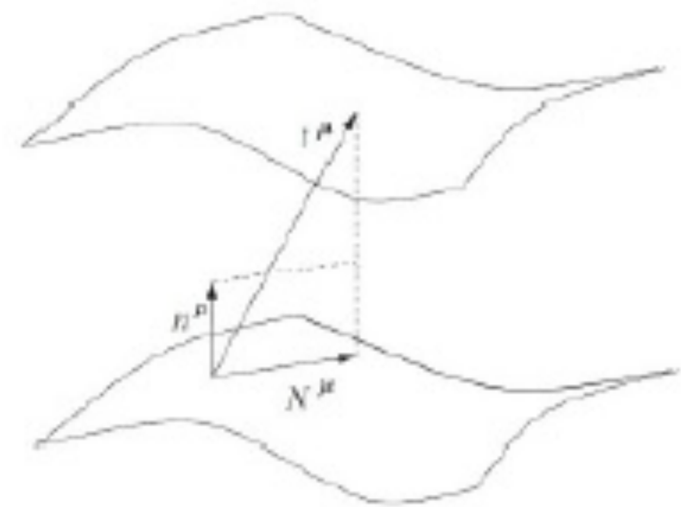
$$|\mathrm{p}|^2 \simeq \sum_i k_i^2 \left[1 - \frac{2}{\alpha}\left|\frac{k_i}{k_*}\right|^{1-\alpha} F_\omega(k_i)\right]$$

modified dispersion!!

A new perspective: HDA

Based on the 3+1 foliation of spacetime[8]





- $g_{\mu\nu}(x) \leftrightarrow h_{ij}(x), N^k(x), N(x)$
- Phase-space variables:
  $\{\pi^{ij}(x), h_{kl}(y)\} = -\frac{1}{2}(\delta^i_k \delta^j_l + \delta^i_l \delta^j_k)\delta^{(3)}(x-y)$

Does it break general covariance?..of course NOT!

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[8] R. L. Arnowitt, S. Deser, C. W. Misner, Gen. Rel. Grav. 40, 1997 (2008).

Hypersurface Deformation Algebra

Dirac, Proc. Roy. Soc. Lond. A246 (1958)

Diffeomorphism invariance is implemented by means of constraints:

$$H[N] = \int d^3x \quad N(x)\left(\frac{\pi_{lk}\pi^{lk}}{\sqrt{-h}} - \frac{\pi^2}{2\sqrt{-h}} - {}^3R\sqrt{-h}\right)$$

$$D[N^k] = -2\int d^3x \quad N^k(x)h_{kj}(x)D_l\pi^{lj}(x)$$

which close the hypersurface-deformation algebra (HDA) [9]:

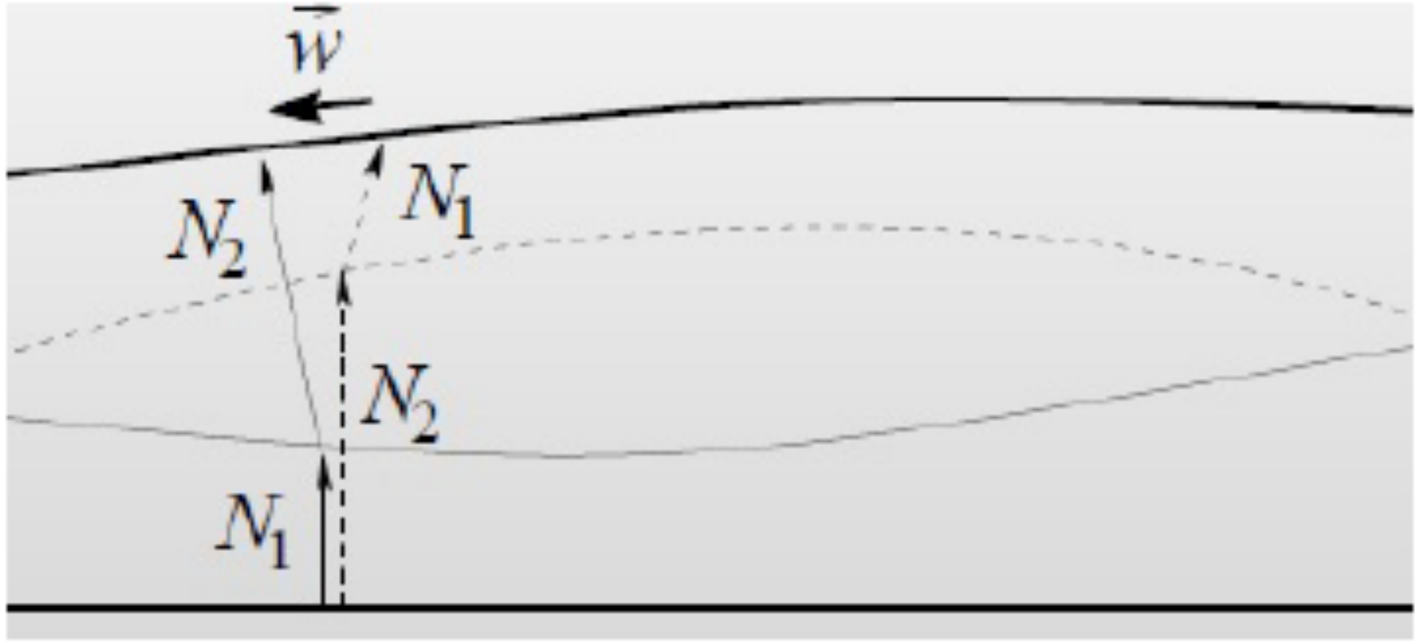
$$\{D[N^i], D[N'^j]\} = D[N'^j\partial_j N^i - N^j\partial_j N'^i]$$
$$\{D[N^i], H[N']\} = H[N^j\partial_j N']$$
$$\{H[N], H[N']\} = D[h^{ij}(N\partial_j N' - N'\partial_j N)]$$

Diffeomorphism transformations

Hypersurface-deformation algebra ensures that theory respects:

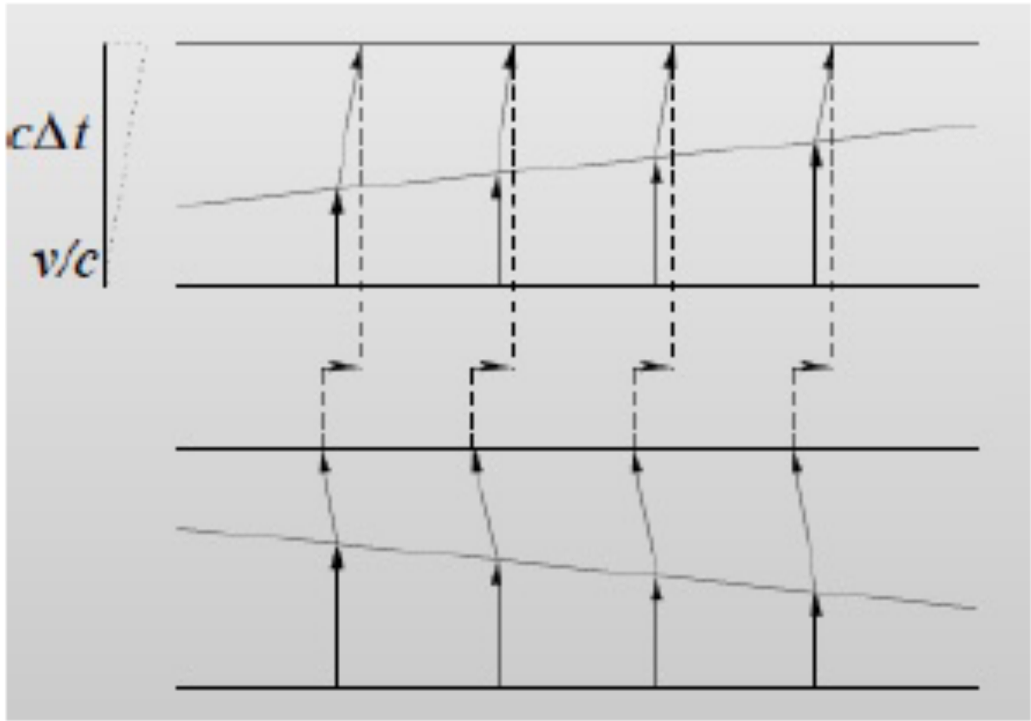
1. gauge transformations for coordinate changes: $\{f, H[N] + D[N^k]\} = \delta f$
2. slicing independence: algebra amounts to deformations of hypersurfaces

Non-linear coordinate changes translates into non-linear deformations of space (example: $\{H, H\} = D$).



Minkowski limit

If we restrict to linear coordinate changes of flat slices $h_{ij} = \delta_{ij}$:



that means choosing:

$$N^k(x) = \delta^k + \epsilon^{kij}\varphi_i x_j, \quad N(x) = \delta + \alpha_i x^i$$

which are the Killing vectors of the Minkowski spacetime.
Then we show that the Poincaré symmetries are contained into the HDA.

From DSR to DGR?

Bojowald, Paily, arXiv:1112.1899

HDA

$$\{D[N^i], D[N'^j]\} = D[N'^j\partial_j N^i - N^j\partial_j N'^i]$$
$$\{D[N^i], H[N']\} = H[N^j\partial_j N']$$
$$\{H[N], H[N']\} = D[h^{ij}(N\partial_j N' - N'\partial_j N)]$$

PA

$$\{P_\mu, P_\nu\} = 0 \quad \{M_{\mu\nu}, P_\rho\} = \eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu$$
$$\{M_{\mu\nu}, M_{\rho\sigma}\} = \eta_{\mu\rho}M_{\nu\sigma} - \eta_{\nu\sigma}M_{\nu\rho} - \eta_{\nu\rho}M_{\mu\sigma} + \eta_{\nu\sigma}M_{\mu\rho}$$

QG models could provide modified HDAs then it should be possible to reduce to corresponding modified PAs

Case of study 1: spherically symmetric (effective) LQG

Brahma, MR, arXiv:1801.09417, Brahma, MR, Amelino-Camelia, Marciano, arXiv:1610.07865, Amelino-Camelia, da Silva, MR, Cesarini, Lecian, arXiv:1605.00497

classical theory

$$\{K_r(r), E^r(r')\} = 2G\delta(r-r'), \quad \{K_\phi(r), E^\phi(r')\} = G\delta(r-r')$$

$$H[N] = -\frac{1}{2G}\int_B drN[K_\phi^2 E^\phi + 2K_r K_\phi E^r + (1-\Gamma_\phi^2)E^\phi + 2\Gamma'_\phi E^r]$$

add (quantum) holonomy corrections

$$K_\phi \to f(K_\phi)$$

$$\{H^Q[N], H^Q[M]\} = D[\beta(\delta K_\phi)h^{rr}(N\partial_r M - M\partial_r N)]$$

$$K_\phi \propto P_r$$

flat Minkowski limit

DSR-like effects from LQG: deformed PA and associated MDR

$$\{B_r, P_0\} = \beta(\lambda P_r)P_r$$

$$P_0^2 = f(P_r) = 2\int \beta(\lambda P_r)P_r\, dP_r$$

Deformation function β depends on formal choices

Case of study 1: spherically symmetric (effective) LQG

Brahma, MR, arXiv:1801.09417, Brahma, MR, Amelino-Camelia, Marciano, arXiv:1610.07865, Amelino-Camelia, da Silva, MR, Cesarini, Lecian, arXiv:1605.00497

Special Relativity

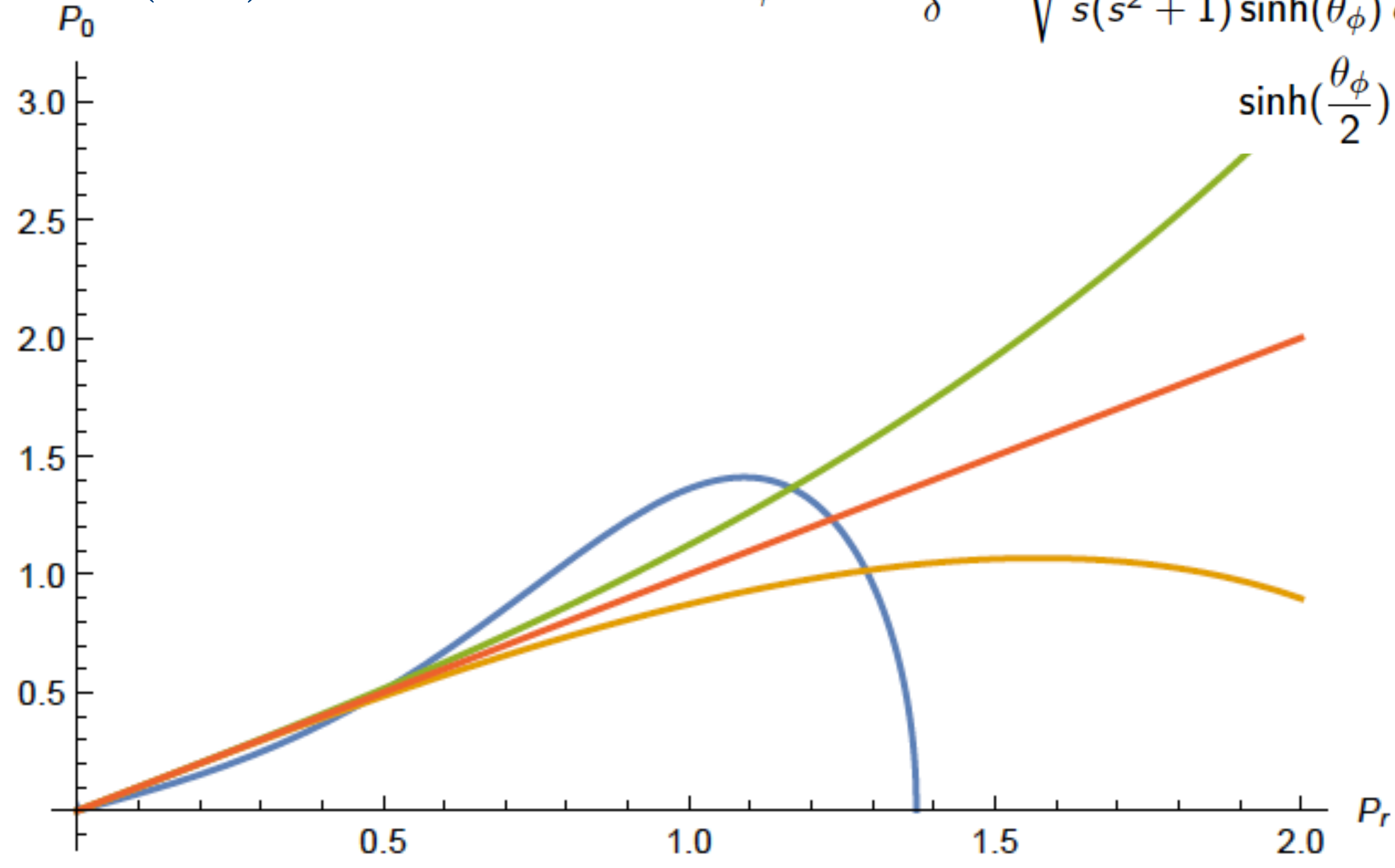
Real SU(2) holonomies: $K_\phi \to \frac{\sin(\delta K_\phi)}{\delta}$

Complex SL(2,C) holonomies: $h_\phi(r,\mu) = \exp(\mu\gamma K_\phi \Lambda^A_\phi) = \cosh(\mu K_\phi)\mathbb{I} - 2\sinh(\mu K_\phi)\overline{\Lambda}\,,$

Complex generalised holonomies: $h_\phi(r,\mu) = \cosh(\mu i K_\phi)\mathbb{I} + \sinh(\mu i K_\phi)\sigma_\phi = \cos(\mu K_\phi)\mathbb{I} + \sin(\mu K_\phi)\Lambda$

Complex SU(1,1) holonomies: $K_\phi \to \frac{\sinh(\delta K_\phi)}{\delta}\sqrt{\frac{-3}{s(s^2+1)\sinh(\theta_\phi)}\frac{\partial}{\partial\theta_\phi}\left(\frac{\sin(s\theta_\phi)}{\sinh(\theta_\phi)}\right)}$

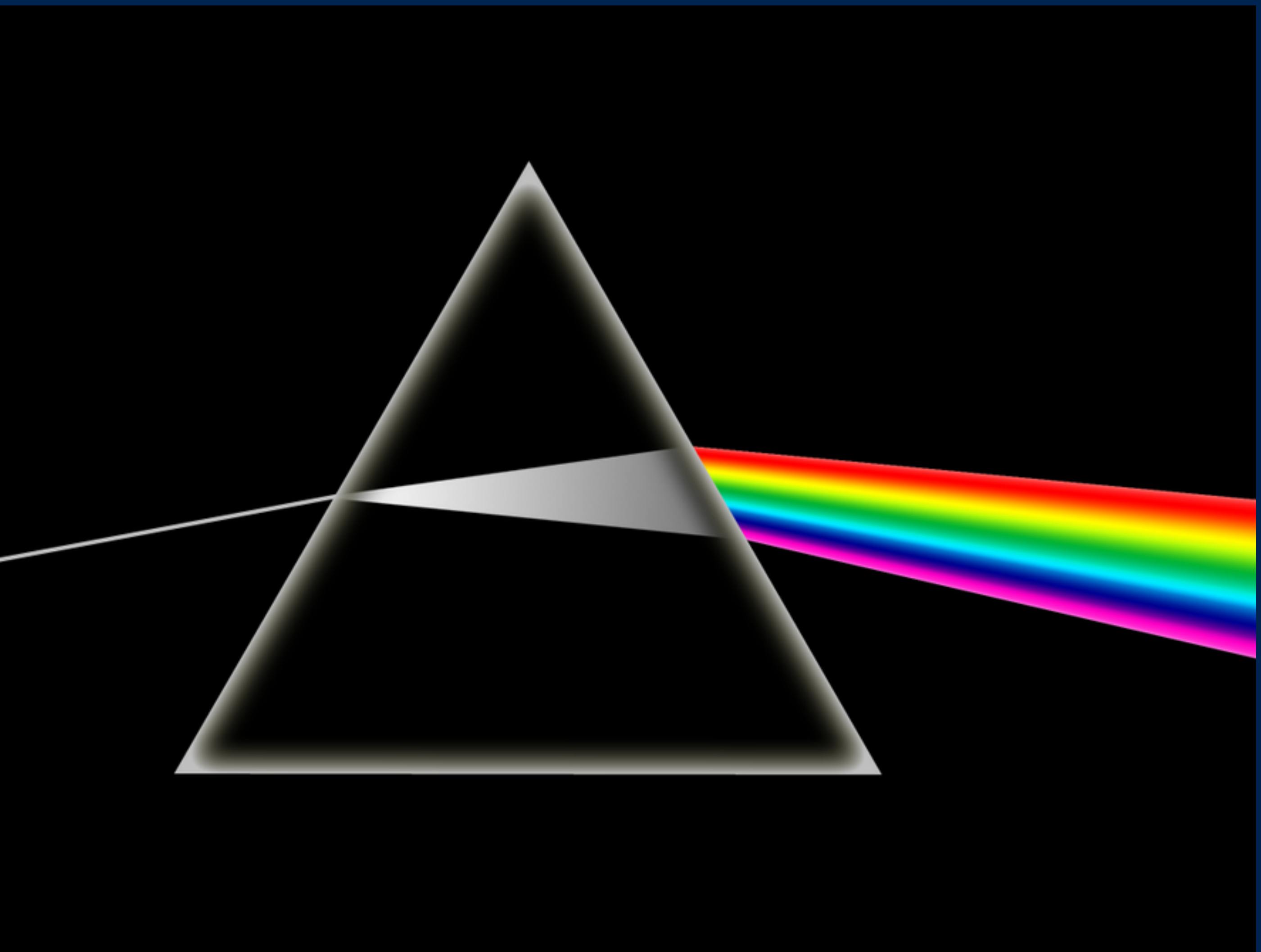
$\sinh\left(\frac{\theta_\phi}{2}\right) = \sinh^2\left(\frac{\delta K_\phi}{2}\right)$



Heuristic simplified approach to MDRs

$$E_P = \frac{1}{\ell_{\mathrm{Pl}}}\frac{\hbar}{c} \simeq 10^{19}\ \ \mathrm{GeV}$$

$$H^2 = m^2c^4 + p^2c^2 \to m^2c^4 + p^2c^2\left(1 + \eta\frac{E}{E_P} + \dots\right)$$



$$\boxed{v = \frac{\partial H}{\partial p} \simeq c\left(1 + \eta\frac{E}{E_P} + \dots\right)}$$

$$\frac{E}{E_P} \approx 10^{-15} - 10^{-6}$$

...extremely small effect, how can we detect it?

Cumulative time lags over cosmological distances

$$\Delta t = \eta \frac{\Delta E}{E_P} D(z)$$

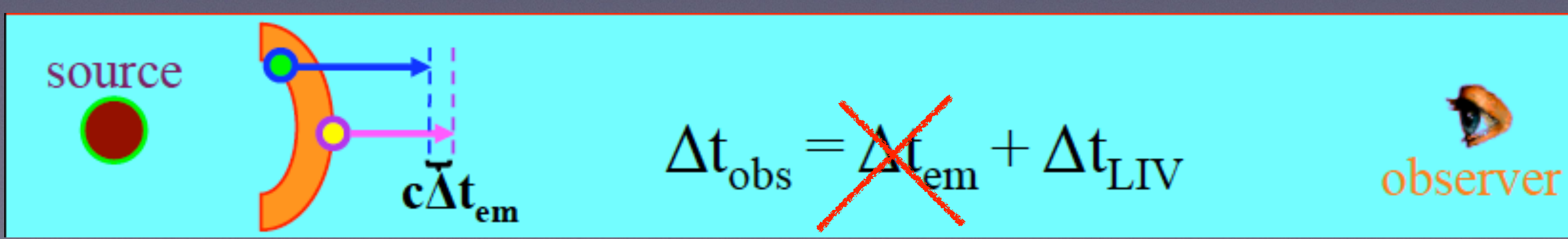
$$D(z) = \int_0^z d\zeta \frac{(1+\zeta)}{H_0\sqrt{\Omega_\Lambda + (1+\zeta)^3\Omega_m}}$$

Jacob, Piran, arXiv:0712.2170

Gamma ray bursts

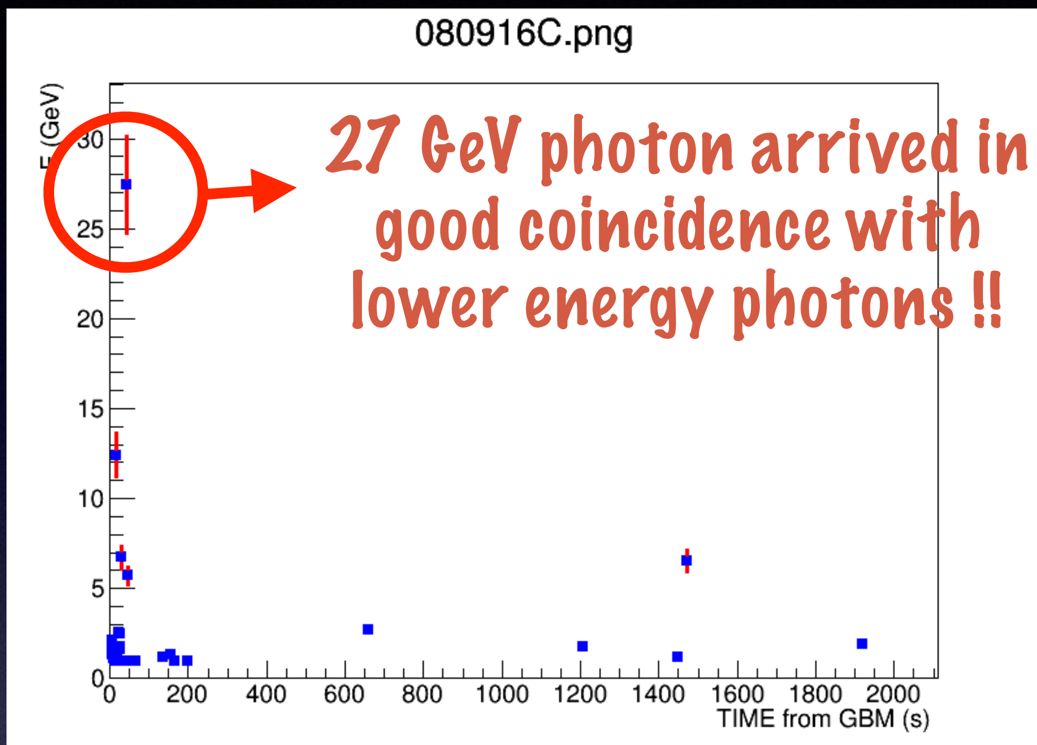
- High energy emission
- Time variability
- Cosmological distances (z>0)

Emission mechanism??



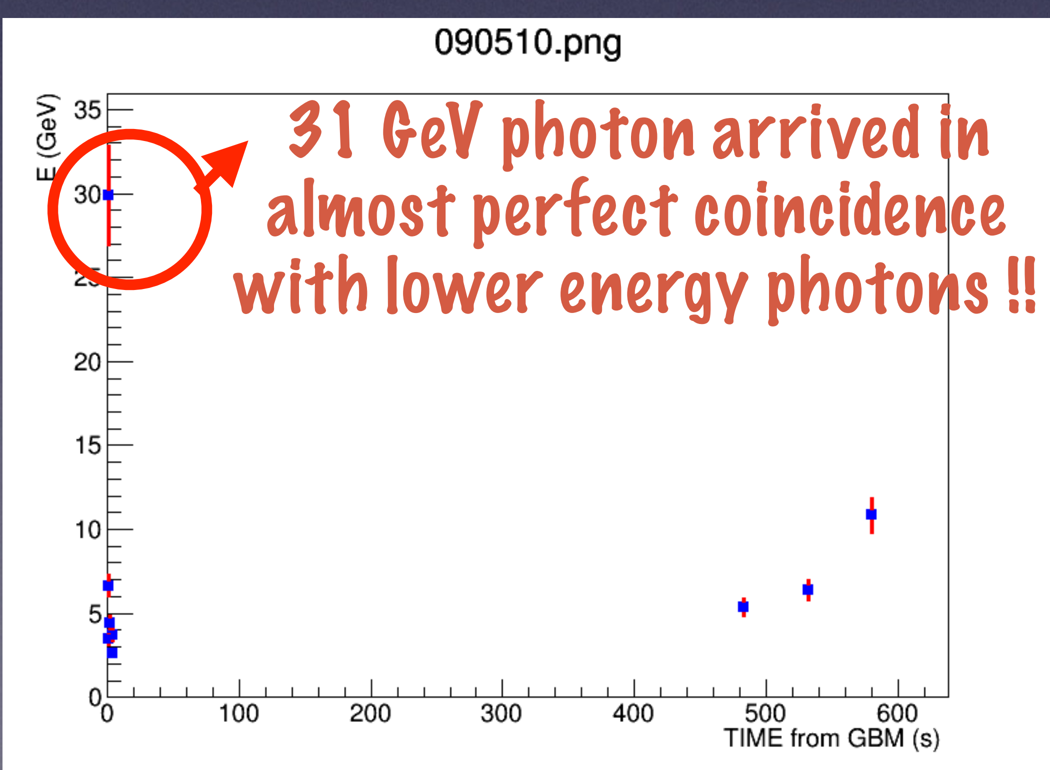
Single-burst analyses: bounds

GRB080916C: Fermi-LAT and GBM Collaborations, Science 323 (2009)



$$M_{QG} \gtrsim 10^{-1} M_P$$

GRB090510: Fermi-LAT and GBM Collaborations, Nature 462 (2009)



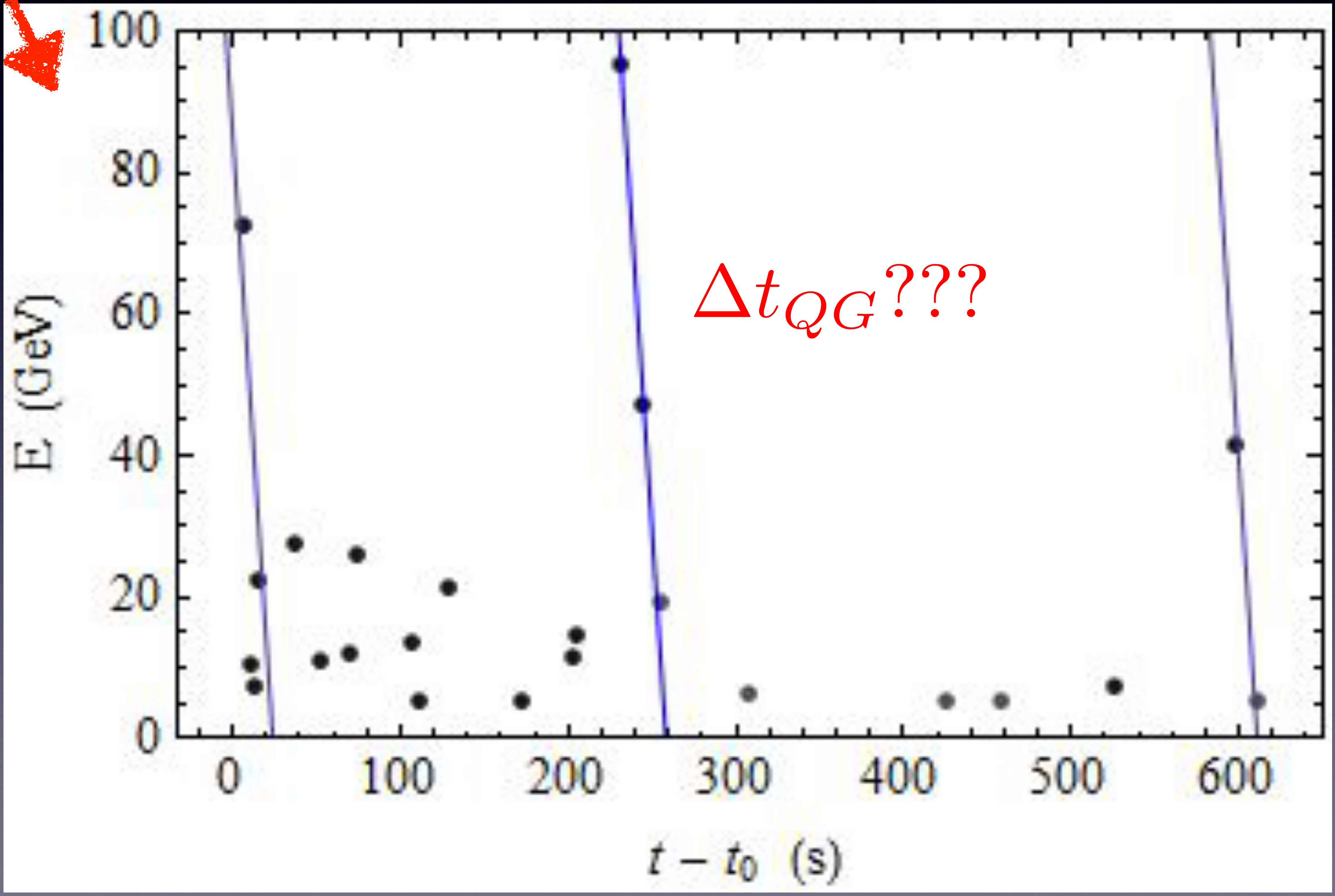
$$M_{QG} \gtrsim M_P$$

Experimental analyses for n=1 LIV reached the Planck scale!!!

Single-burst analyses: searches

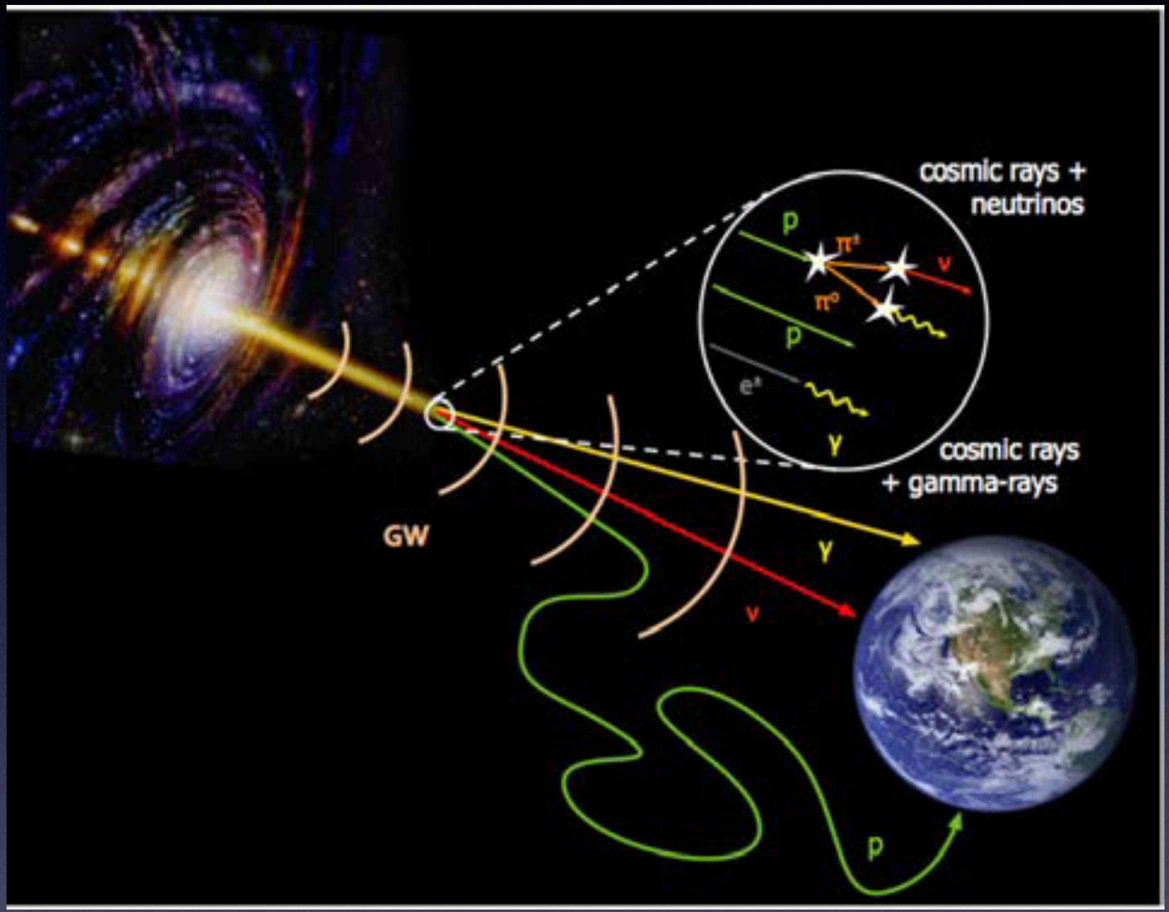
Amelino-Camelia, Fiore, Guetta, Puccetti, Adv.High Energy Phys. (2014)

GRB130427A



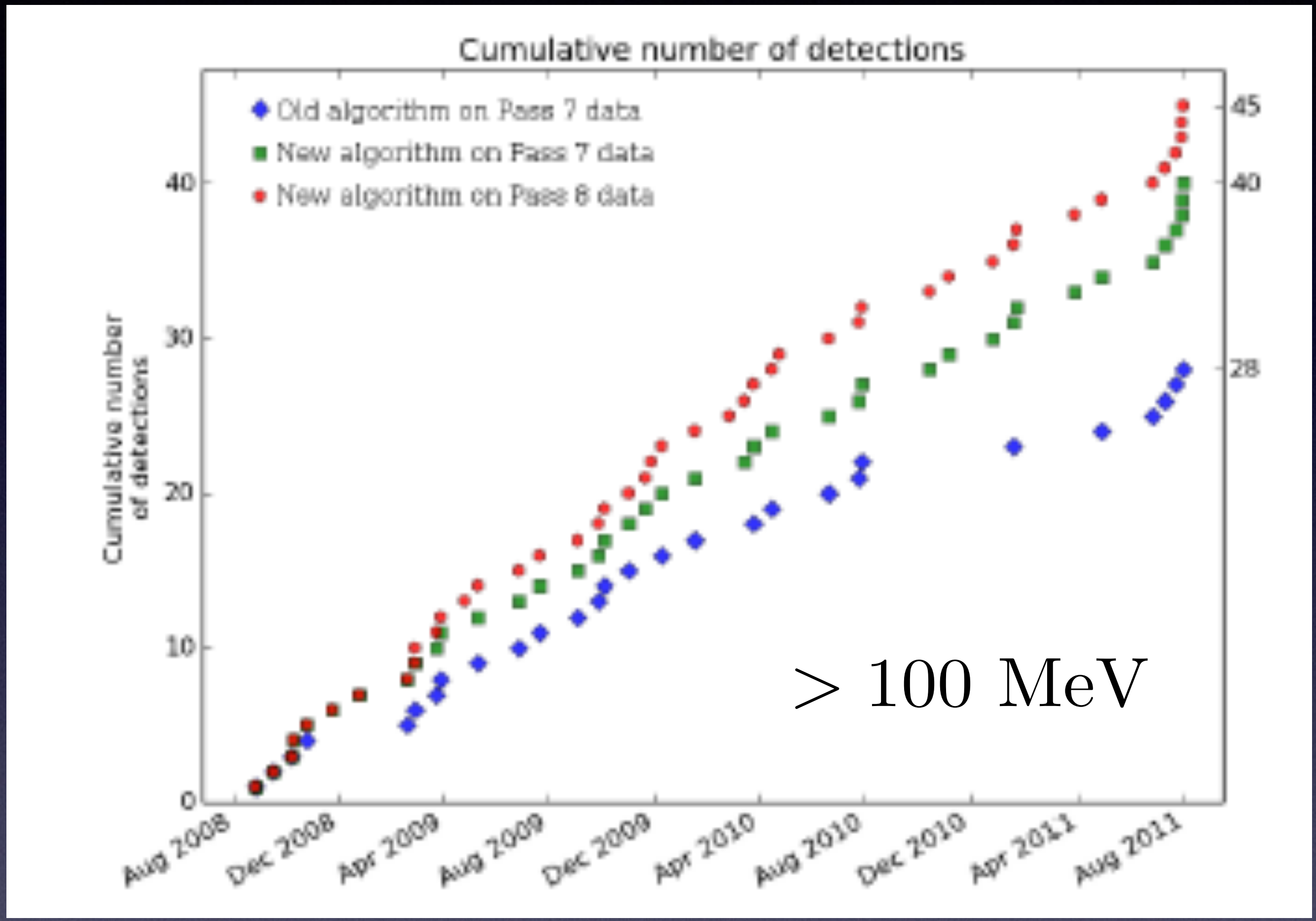
Forthcoming improvements

Multi-messenger



Amelino-Camelia, D’Amico, Rosati, Loret, Nature Astron. 1 (2017)

More statistics



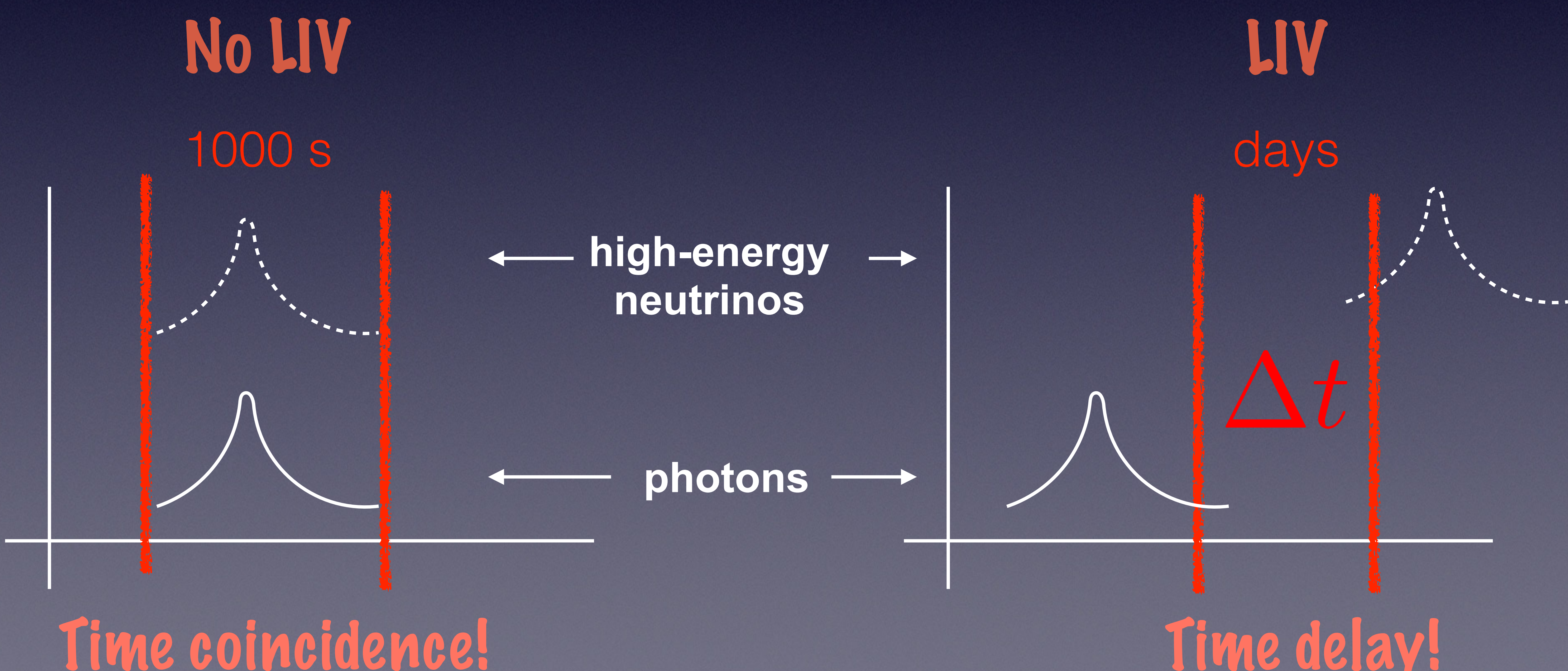
Nava, arXiv:1804.01524

Amelino-Camelia, D’Amico, Fiore, Puccetti, MR, arXiv: 1707.02413

Neutrinos from GRBs?

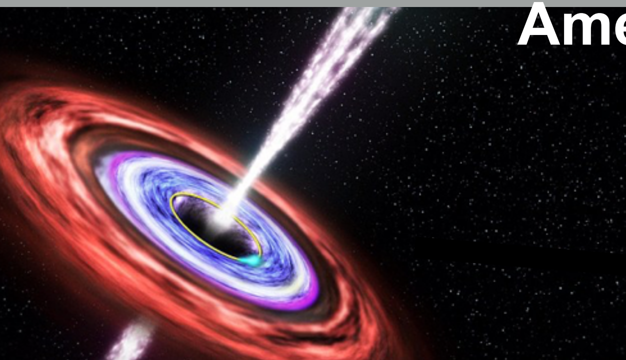
All proposed models for GRB-emission mechanism (e.g. fireball) requires the production of neutrinos: we expect to detect 10 neutrinos per 1000 GRBs in a 1 Km cube detector (IceCube, Km3Net)

IceCube detected no GRB neutrinos so far!!
(2008-2017)



Hints of LIV??

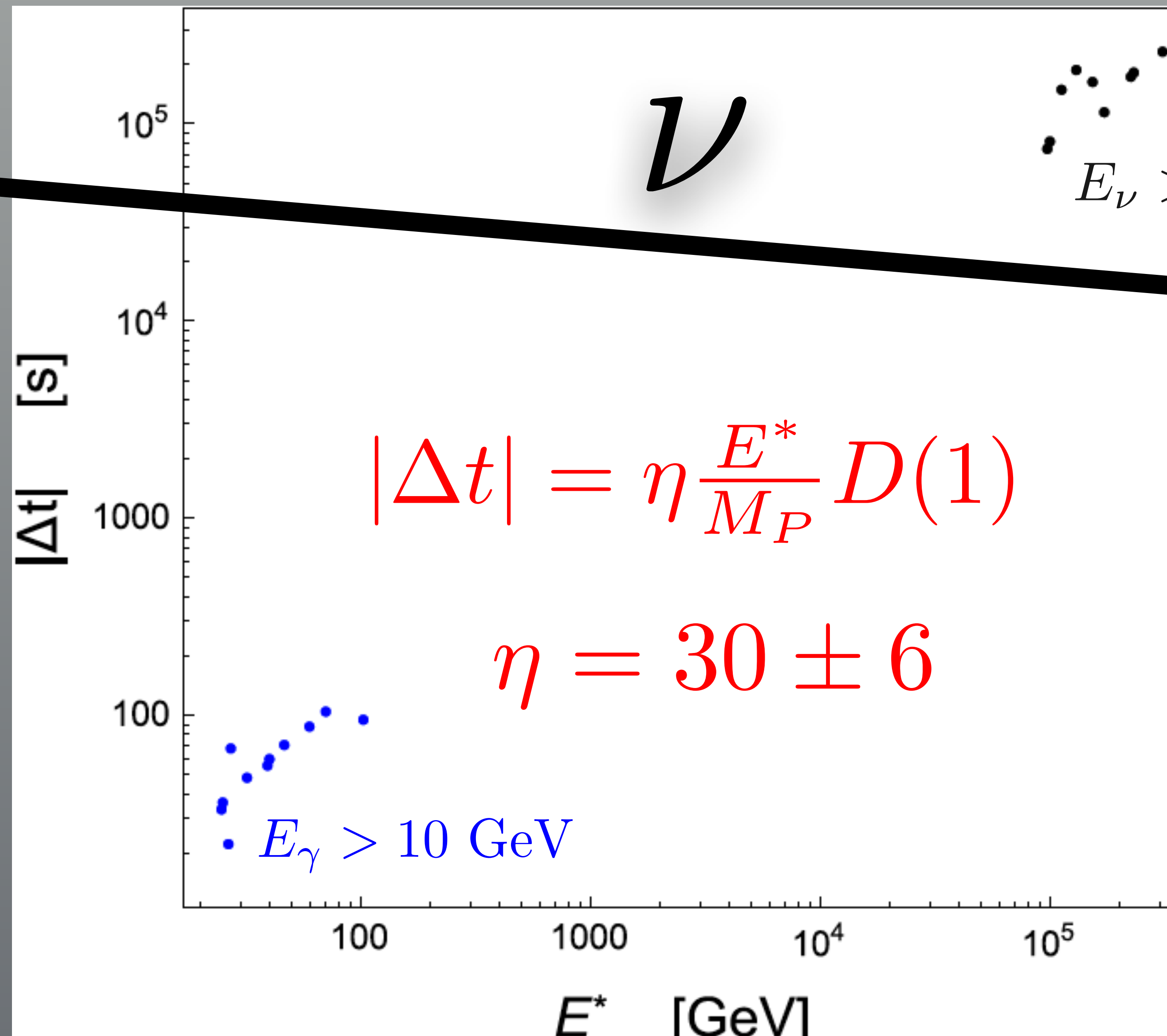
Amelino-Camelia, D'Amico, Rosati, Loret, Nature Astron. 1 (2017)

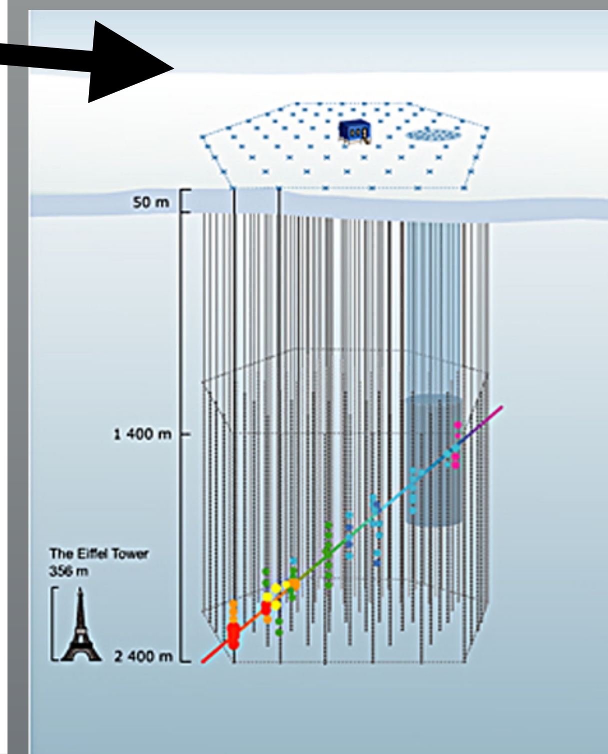


ν

γ

Fermi-LAT





IceCube

in-vacuo dispersion-like feature:

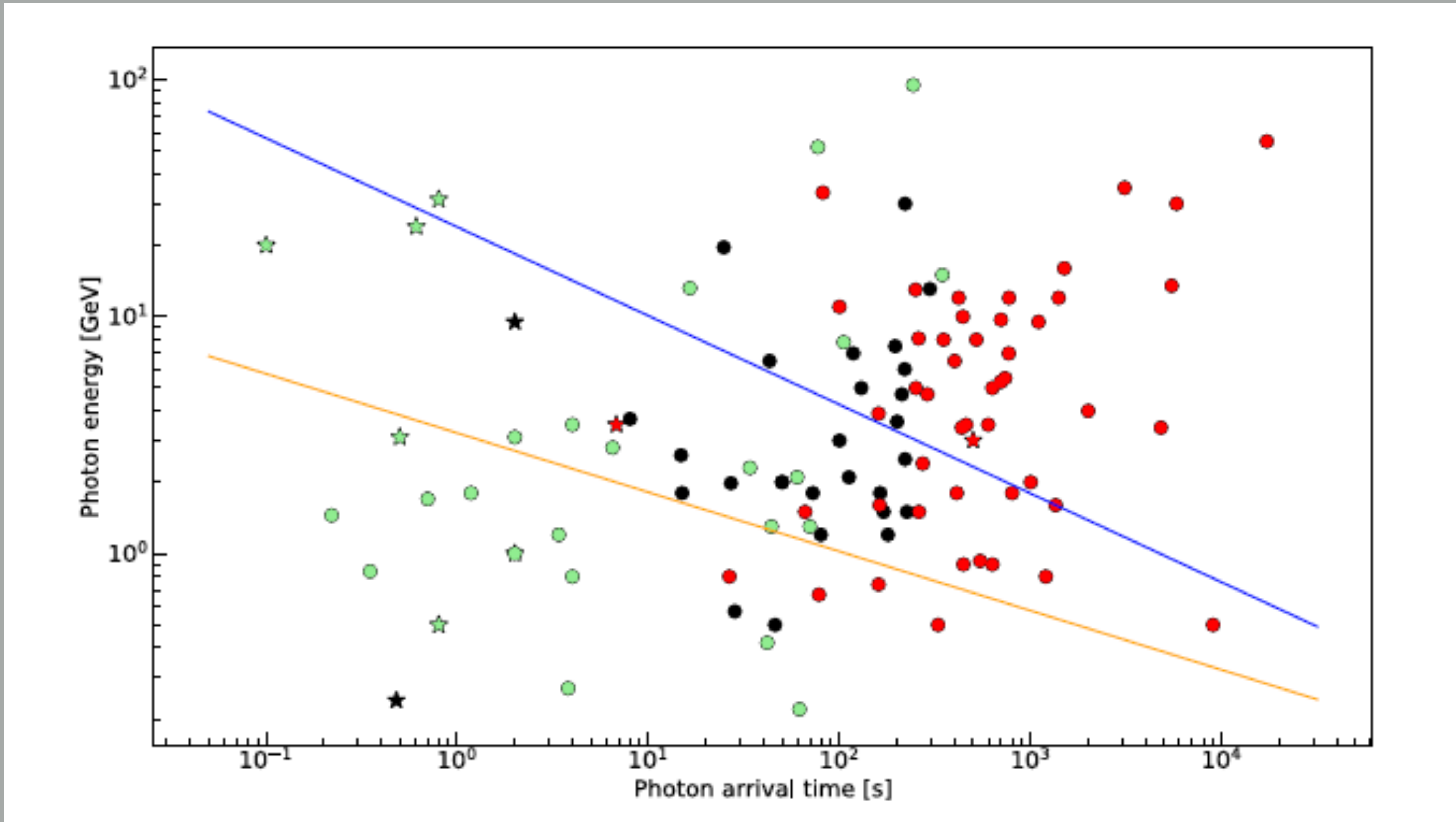
time difference with respect to the first GBM peak

$$|\Delta t| = \eta \frac{E^*}{M_P} D(1) \quad \text{with} \quad E^* = E \frac{D(z)}{D(1)}$$

How to explain delayed GeV component?

Nava, arXiv:1804.01524

- **BLUE LINE:** homogenous medium
- **ORANGE LINE:** wind-like medium



$$E^{syn}_{max} = \frac{50\,\mathrm{MeV} \times \Gamma}{1+z}$$

No simple explanation in terms of synchrotron radiation!!

Increase statistics

Amelino-Camelia, D'Amico, Fiore, Puccetti, MR, arXiv:1707.02413

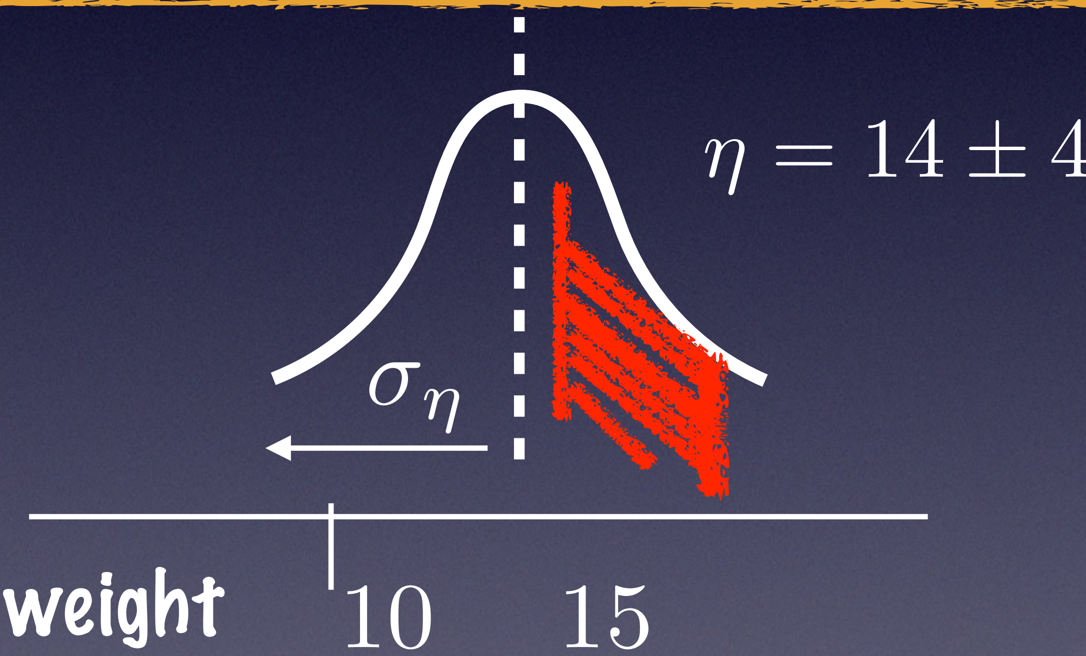
Selection criteria:

- known redshift z
- $E \times (1+z) > 5\ \mathrm{GeV}$

Data sample:
7 GRBs (080916C, 090510, 090902B, 090926A, 100414A, 130427A, 160509A) detected by FermiLAT, 148 photons analysed in total

For each pair of photons we compute:

$$\eta = \frac{\Delta t}{\Delta E}\frac{E_P}{D(z)}$$



10% error on energies considered with Gaussian weight

Frequent non-zero value compatible with 30!?

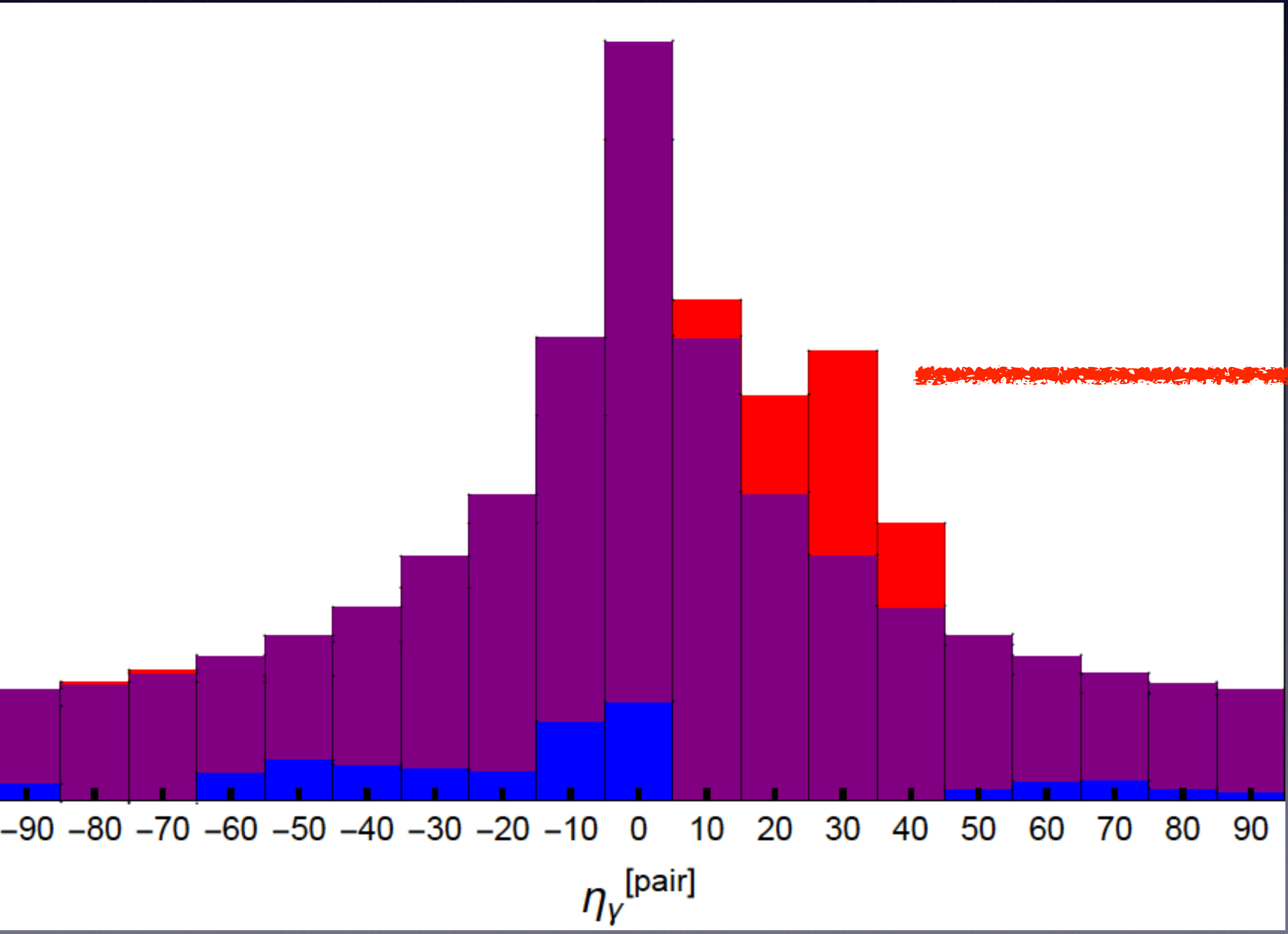
All-pairs analysis

Amelino-Camelia, D'Amico, Fiore, Puccetti, MR, arXiv:1707.02413

Purple bars: simulated data distribution

Blue bars: defect in real data

Red bars: excess in real data

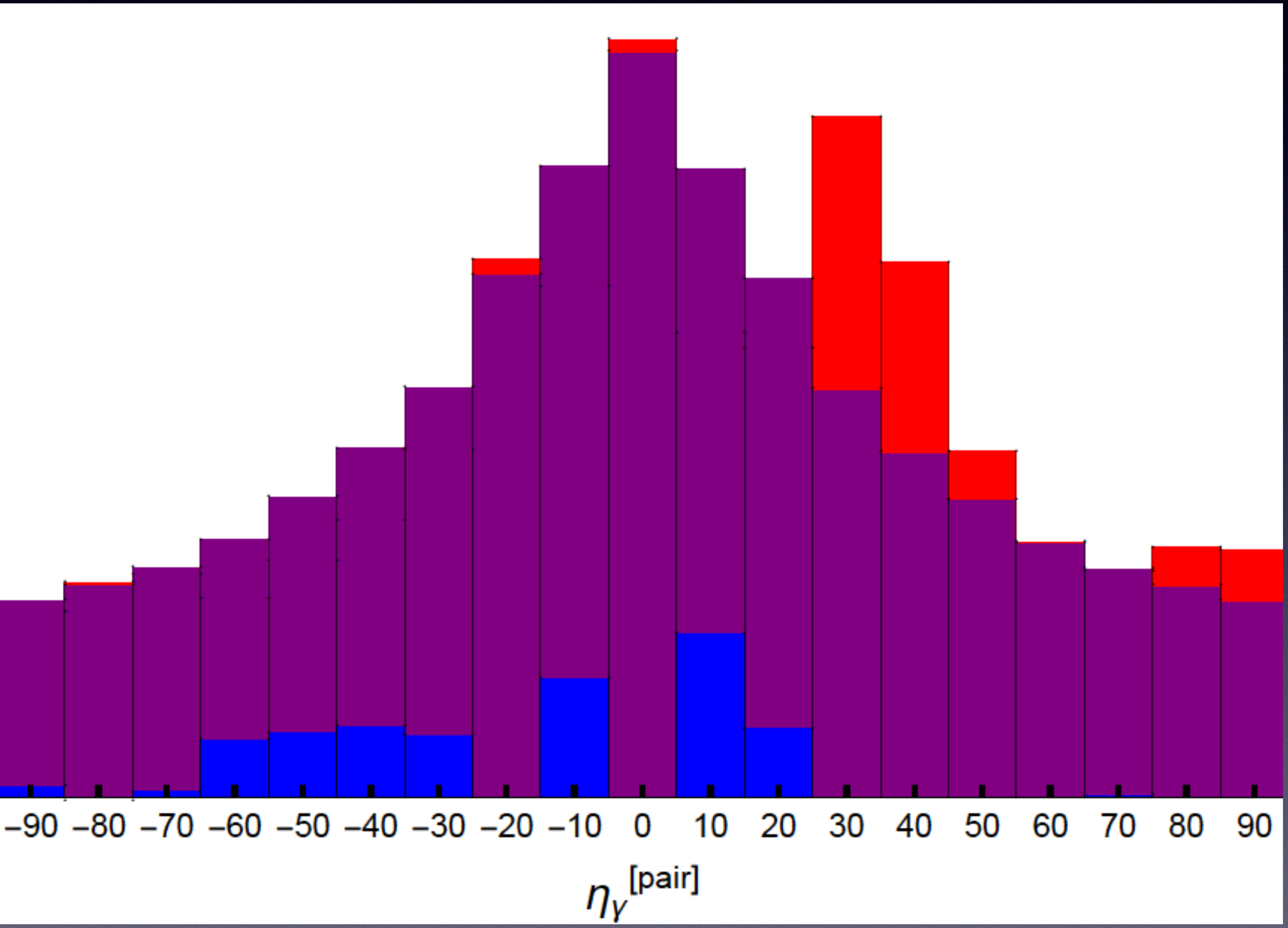


Over 10^5 simulated data (reshuffling times) the observed peak for $25 < \eta < 35$ is reproduced in less than 0.5% of cases!!!

Data samplings

Amelino-Camelia, D'Amico, Fiore, Puccetti, MR, arXiv:1707.02413

no-high pairs



Pairs constituted by all photons with:

$$5 \text{ GeV} < E \times (1+z) < 40 \text{ GeV}$$

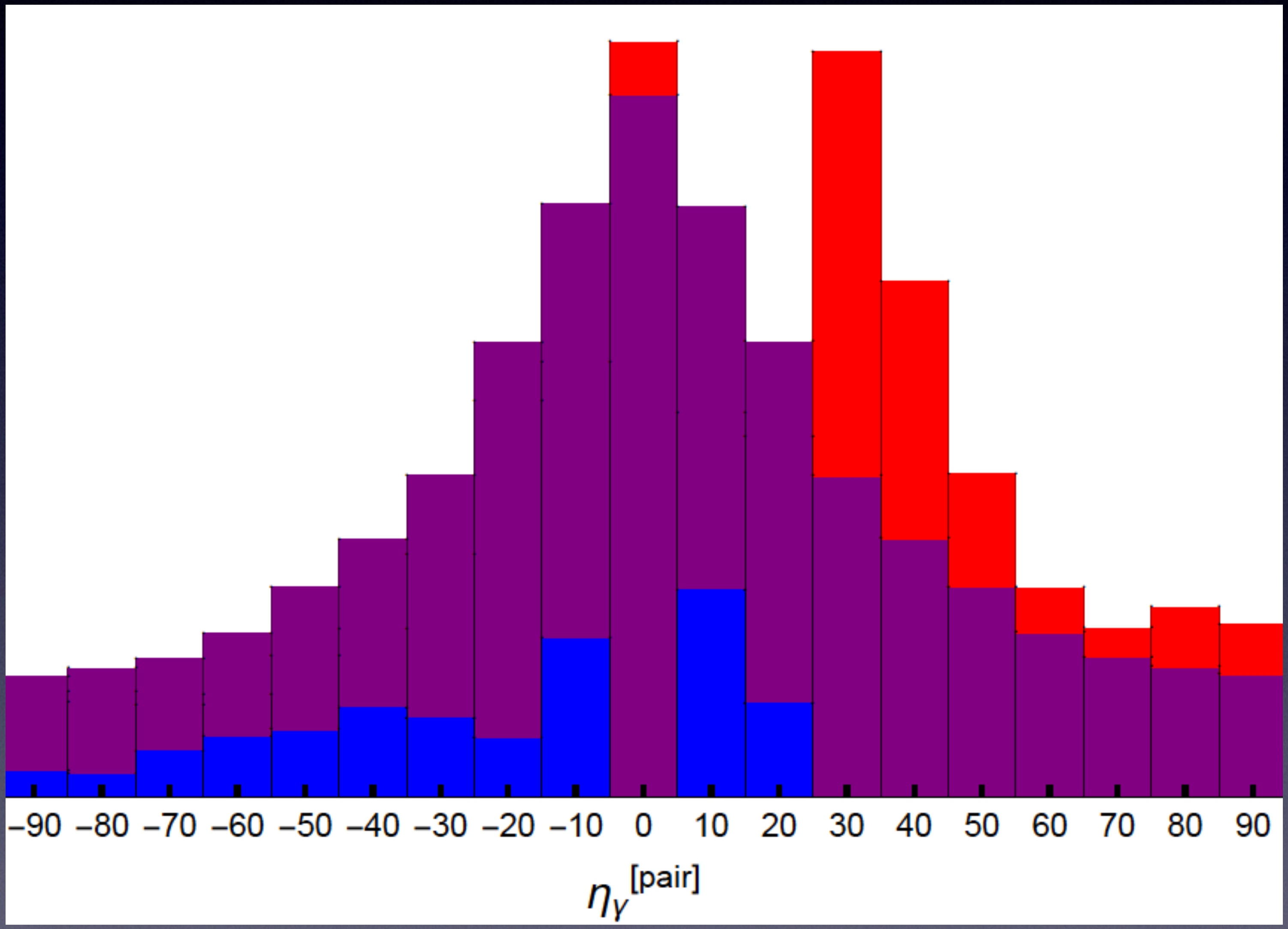
(excluding the energy range analysed by previous analyses)

Peak at $25 < \eta < 35$ appears accidentally only in 0.6 % of cases!

Data samplings

Amelino-Camelia, D'Amico, Fiore, Puccetti, MR, arXiv:1707.02413

medium-low pairs



Pairs constituted by:

1. **"Low" photon with**

$$5\ \text{GeV} < E \times (1+z) < 15\ \text{GeV}$$

2. **"Medium" photon with**

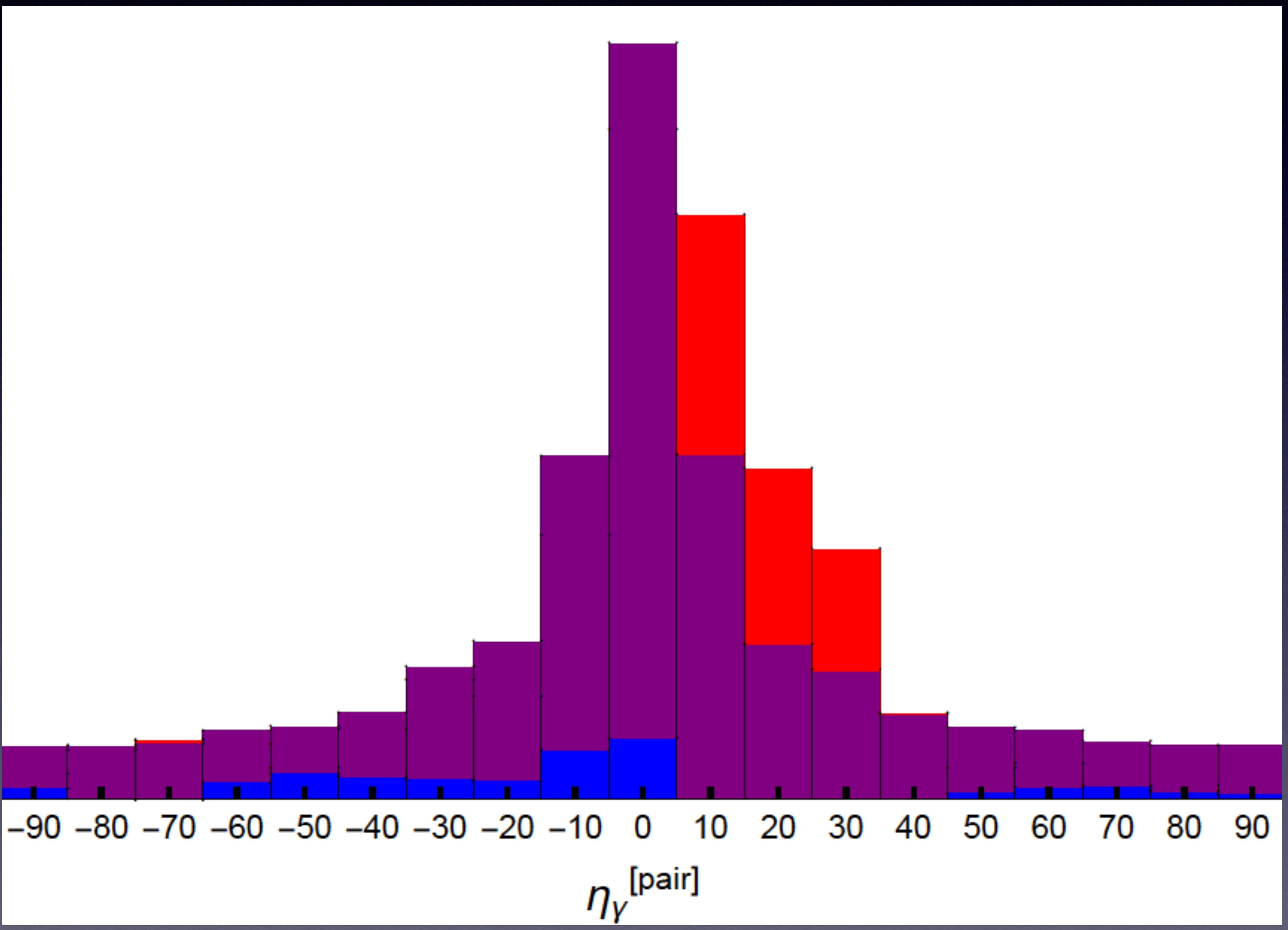
$$15\ \text{GeV} < E \times (1+z) < 40\ \text{GeV}$$

False alarm probability = 0.2 %!

Data samplings

Amelino-Camelia, D'Amico, Fiore, Puccetti, MR, arXiv:1707.02413

high-low pairs



Pairs constituted by:

1. **"Low" photon with**

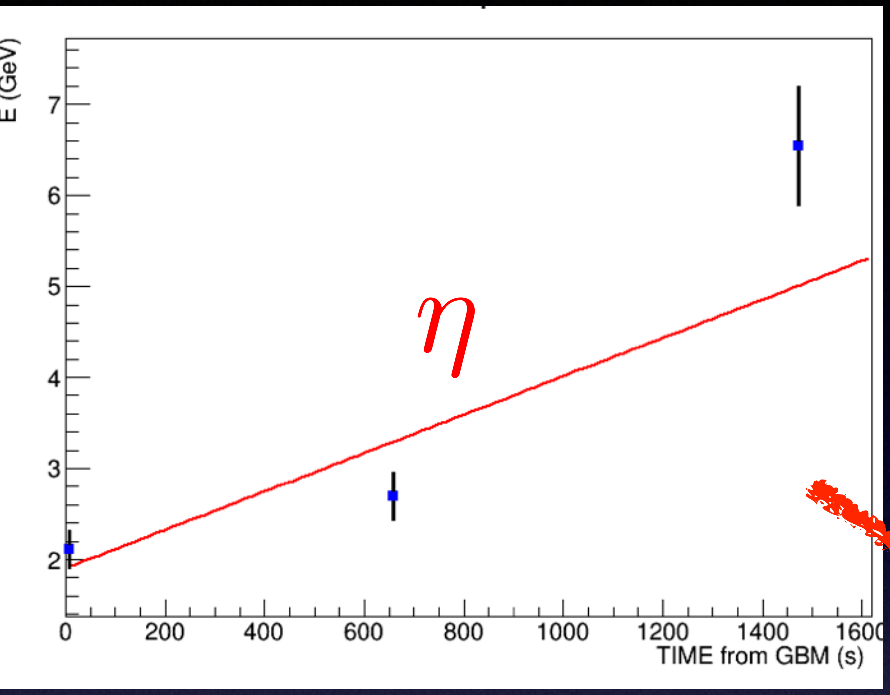
$$5\ \mathrm{GeV} < E \times (1+z) < 15\ \mathrm{GeV}$$

2. **"High" photon with**

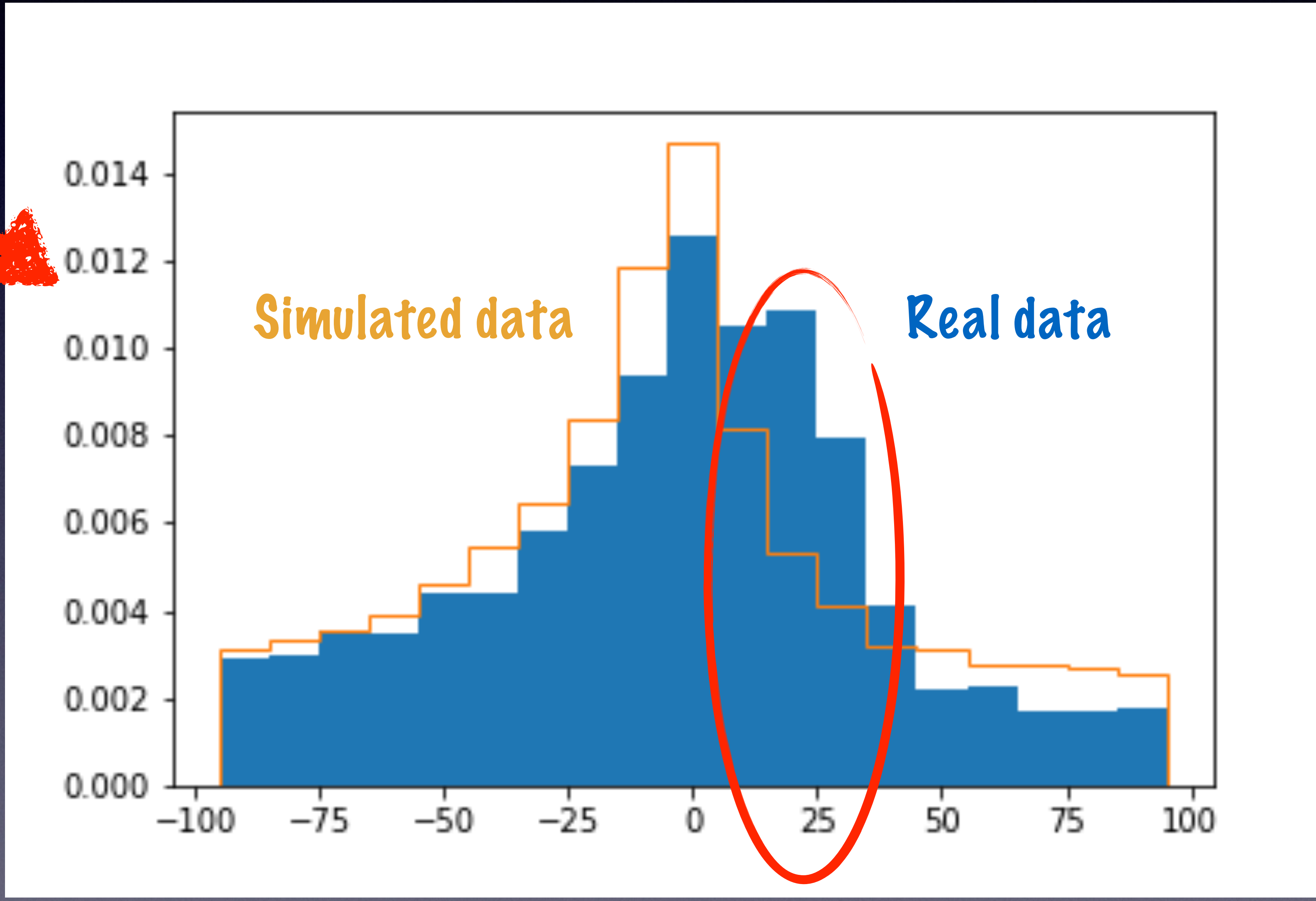
$$E \times (1+z) > 40\ \mathrm{GeV}$$

False alarm probability = 14 %!

Preliminary: all-triplets analysis



η **is computed by performing a linear fit with the constraint:** $\chi^2 < 5$

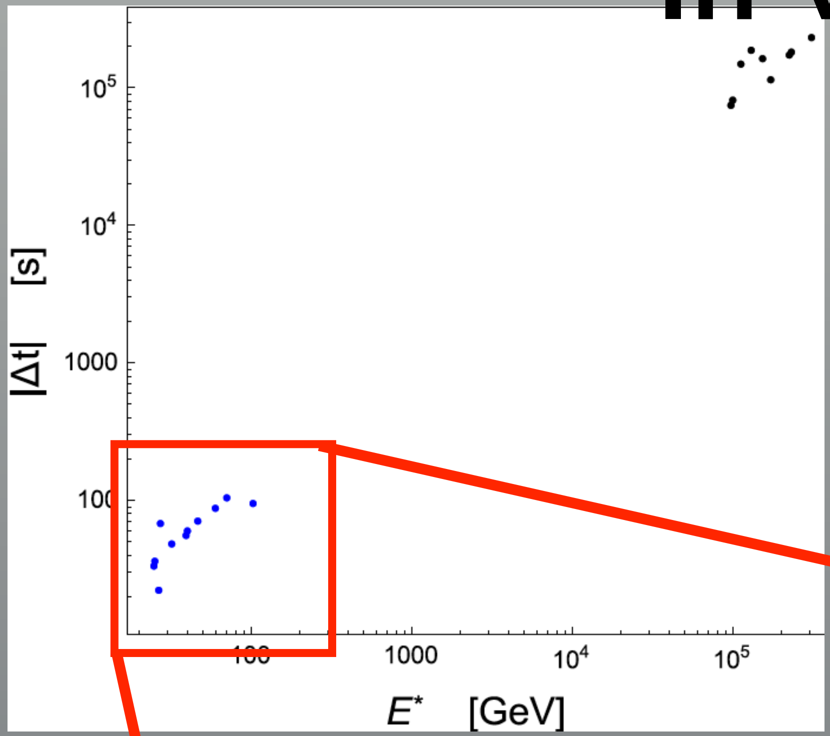


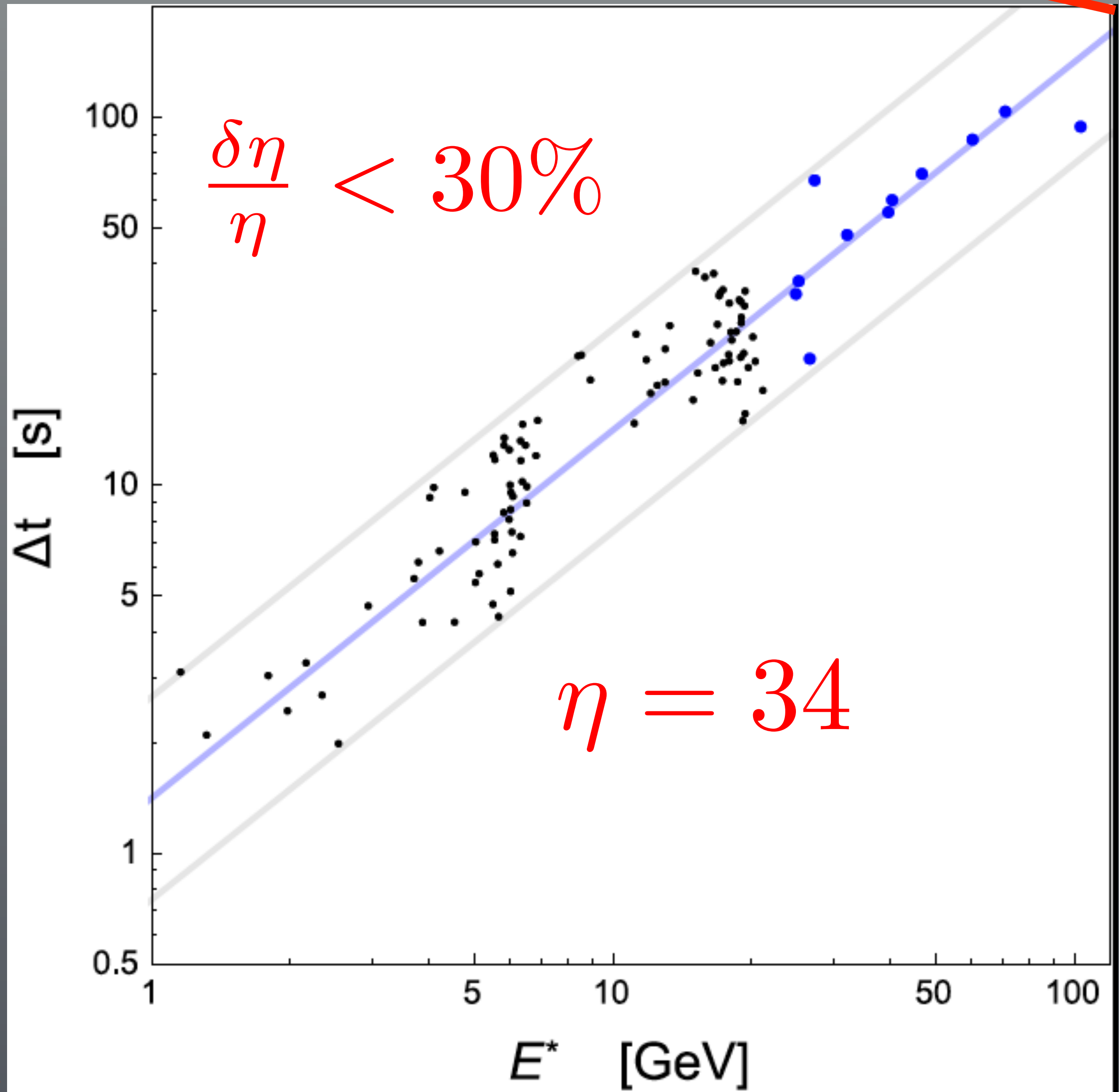
Best fit method: we now consider triplets of photons instead of pairs

Peak at $15 < \eta < 25$ appears accidentally only in 1.5 % of cases!

In-vacuo dispersion-like feature

Amelino-Camelia, D'Amico, Fiore, Puccetti, MR, arXiv:1707.02413





- **BLUE POINTS: GRB photons with energy at the emission greater than 40 GeV**
- **BLACK POINTS: GRB photons with energy at the emission between 5 GeV and 40 GeV**

Consistency between the E>40GeV analysis and the 5GeV<E<40GeV analysis!!!

Summary

Effects on particles propagation are expected due to the "foamy" structure of spacetime near the Planck scale

Two possible theoretical frameworks have been developed: LIV and DSR; both deserve better understanding

Spectral time lags are observed in all GRBs as well as in AGNs

In-vacuo-dispersion like spectral lags in 7 GRBs with energy above 5 GeV at the sourse, but yet they might be manifestations of intrinsic GRB physics

Our results are would not be compatible with single event analyses but are compatible with limits given by population studies (can source effects solve the inconsistency?)

Outlook: (QG) theory side

Still no rigorous derivation of MDR from QG models:

- **What are we testing??!**
- **Work on “DGR”: generalise DSR to gravity and/or introduce gravity effects into DSR models**
- **Missing clear distinction between LIV and DSR signatures**
- **Models for stochastic/fuzzy LIV**

Take input from data

- **Simplest linear LIV models falsified already?**
- **Work on redshift dependence of MDRs**
- **Fractional-order LIV?**

Outlook: experimental side

Quantity and quality of astrophysics data are rapidly improving and allow to pass from single-event analyses to statistical analyses over surveys!

1) Better control over systematics can now be achieved

2) Start testing alternative proposals and suggest phenomenological new models

3) To decouple intrinsic from LIV effects:

- Study the dependence on the redshift (population studies)
- Combine different sources (e.g. AGNs+GRBs)
- Combine different detectors (HESS+MAGIC+FermiLAT)
- Multi-messenger analyses (photons + neutrinos + GWs)
- Model time variability at the emission
- Increase statistics in the GeV range (50 - 100) GeV and above

QG problem originates at the conceptual level: some crude theories can characterize the problem

once the first ultracrude models establish the issue is experimentally relevant better concept-inspired models materialize

Experiment inspired by such crude theories stumbles upon charateristic effect

Until now..

first data-inspired theories

...from now on!!

experimental searches of effects predicted by the new theories

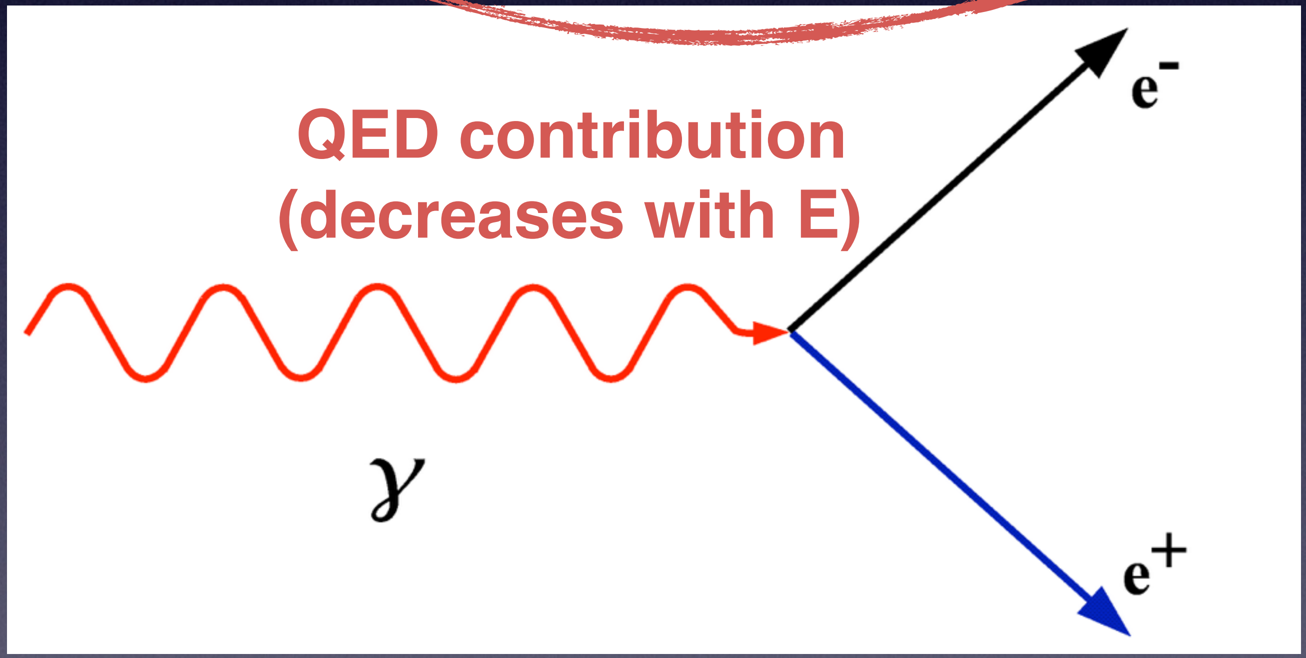
“better” theories are proposed

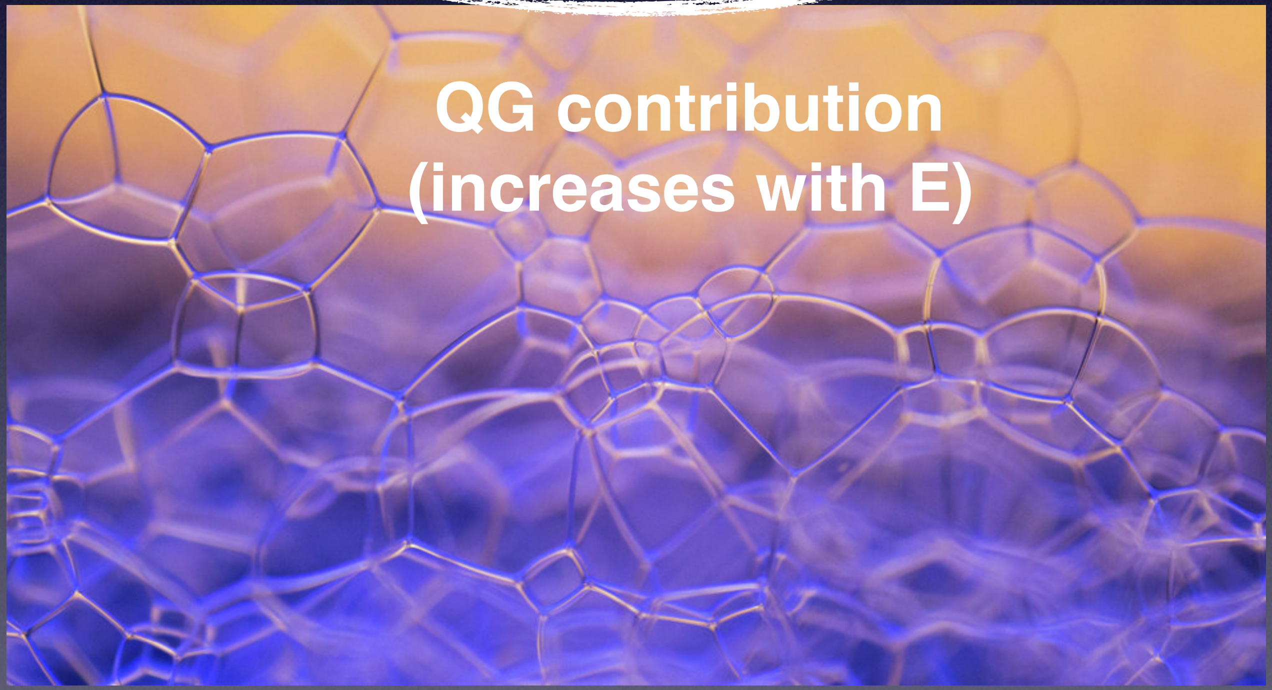

Thanks for your attention!

QG vs Standard physics

$$E \sim 1\text{MeV} \qquad m_e \sim 0.5\text{MeV} \qquad N_b \sim 0.25\ \text{m}^{-3}$$

$$\Delta t \propto \frac{N_b}{m_e E^2} L \simeq 10^{-40} L \quad \ll \quad \frac{E}{M_P} L \simeq 10^{-20} L$$





At high energies (linear) QG-induced delays dominates over conventional in-medium physics effects!!

Matter on (Loop) Quantum Gravity background

Gambini, Pullin, arXiv:9809038; Alfaro, Morales-Tecotl, Urrutia, arXiv:9909079

$$\hat{H}^E_{\text{Maxwell}} = \frac{1}{2}\int d^3x \int d^3y \hat{w}_a(x)\hat{w}_b(y)E^a(x)E^b(y)f_\epsilon(x-y)$$

γ

$$< \Delta|\hat{H}^E_{\text{Maxwell}}|\Delta > = \frac{1}{2}\sum_{v_i,v_j} < \Delta|\hat{w}_a(v_i)\hat{w}_b(v_j)|\Delta > E^a(v_i)E^b(v_j)$$

trace out
gravitational dof

$$\partial_t \vec{E} = -\nabla\times\vec{B} + 2\chi\ell_P\Delta^2\vec{B}$$
$$\partial_t \vec{B} = \nabla\times\vec{E} - 2\chi\ell_P\Delta^2\vec{E}.$$

MDR

$$\boxed{\Omega_\pm = \sqrt{k^2 \mp 4\chi\ell_P k^3} \sim |k|(1\mp 2\chi\ell_P|k|).}$$

ν

$$H^{(1)} := \int d^3x \frac{E_i{}^a}{2\sqrt{det(g)}}\left(i\pi^T\tau_i\mathcal{D}_a\xi + c.c.\right) ,$$

$$\langle W,\xi|\hat{H}^{(1)}|W,\xi\rangle = -\frac{\hbar}{4\ell_P^4}\sum_{v\in V(\gamma)}\frac{8}{E(v)}\epsilon^{ijk}\sum_{s_I\cap s_J\cap s_K=v}\epsilon^{IJK}\times$$
$$\Big\{\langle W,\xi|\hat{\xi}_B(v+s_K(\Delta))\frac{\partial}{\partial\xi^A(v)}(\tau_k h_{s_K(\Delta)})^{AB}\hat{w}_{iI\Delta}(v)\hat{w}_{jJ\Delta}(v)|W,\xi\rangle$$
$$-\langle W,\xi|\tau_k\hat{\xi}(v)\frac{\partial}{\partial\xi(v)}|\hat{w}_{iI\Delta}(v)\hat{w}_{jJ\Delta}(v)|W,\xi\rangle - c.c.\Big\}.$$

trace out
gravitational dof

MDR

$$\boxed{E_\pm(\bar{p}) := E_\pm(p,\mathcal{L})|_{p=\bar{p},\ \mathcal{L}=1/\bar{p}} \approx \bar{p} + \frac{m^2}{2\bar{p}} + \ell_P\left((\theta_2\pm\theta_4)\bar{p}^2 + (\theta_1\pm\theta_3)m^2\right) + (\theta_5\pm\theta_6)\ell_P^2\,\bar{p}^3,}$$

Quantum reference frames

Giacomini, Castro-Ruiz, Brukner, arXiv:0905.0031

Quantum information perspective: physics is all about systems in described in different reference frames

$$|\psi' >= \hat{U}_i|\psi >$$

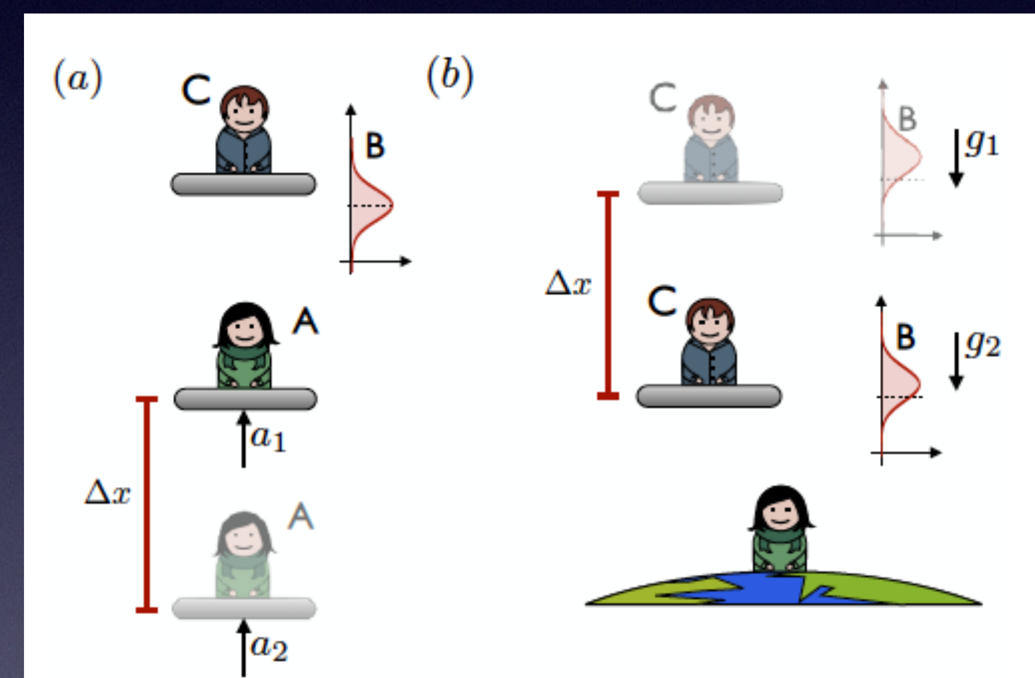
Application: Equivalence Principle for QRFs

System A in a superposition of acceleration

$$-\frac{1}{m_A}\frac{dV(\hat{x}_A)}{d\hat{x}_A}\left|\psi_0(t)\right\rangle_A \approx \frac{1}{\sqrt{2}}\left(a_1\left|\psi_1(t)\right\rangle_A + a_2\left|\psi_2(t)\right\rangle_A\right),$$

A and B described by C

$$\hat{H}^{(C)}_{AB} = \hat{H}_A + \hat{H}_B = \frac{\hat{p}^2_A}{2m_A} + \frac{\hat{p}^2_B}{2m_B} + V(\hat{x}_A)$$



operator transformation from C to A

$$\hat{S}_{EP} = e^{-\frac{i}{\hbar}\left(\frac{\hat{\pi}^2_C}{2m_C}+\frac{m_C}{m_A}V(-\hat{q}_C)\right)t}\hat{\mathcal{P}}^{(v)}_{AC}\hat{Q}_t e^{\frac{i}{\hbar}\left(\frac{\hat{p}^2_A}{2m_A}+V(\hat{x}_A)\right)t}$$

$$\hat{H}^{(A)}_{BC} = \frac{\hat{\pi}^2_B}{2m_B} + \frac{\hat{\pi}^2_C}{2m_C} + \frac{m_C}{m_A}V(-\hat{q}_C) - \frac{m_B}{m_A}\frac{dV}{dx_A}\Big|_{-\hat{q}_C}\hat{q}_B.$$

if the potential is linear everywhere

$$\hat{H}^{(A)}_{BC} = \frac{\hat{\pi}^2_B}{2m_B} + \frac{\hat{\pi}^2_C}{2m_C} - m_C a\hat{q}_C - m_B a\hat{q}_B.$$

diffeomorphisms between QRFs??

Jacob-Piran formula

Jacob, Piran, arXiv:1707.02413

pass to comoving momenta to account for universe expansion

$$H^2 = p^2c^2\left(1+\eta\frac{pc}{E_P}\right) \longrightarrow H = \frac{pc}{a}\sqrt{1+\eta\frac{pc}{aE_P}}$$

$$v = \frac{\partial H}{\partial p} \longrightarrow x(t,p) = \int_0^t \frac{c}{a(t')}\left(1+\eta\left(\frac{pc}{a(t')E_P}\right)\right)dt'$$

turning to redshift variable z:

$$x(z,E_0) = \frac{c}{H_0}\int_0^z\left(1+\eta\left(\frac{E_0}{E_P}(1+z')\right)\right)\frac{dz'}{\sqrt{\Omega_m(1+z')^3+\Omega_\Lambda}}$$

Taking into account that the comoving distance is the same for a high-energy and a low-energy (i.e. no in-vacuo dispersion) photon:

$$\Delta t = \frac{1}{H_0}\frac{\eta E_0}{E_P}\int_0^z\frac{(1+z')dz'}{\sqrt{\Omega_m(1+z')^3+\Omega_\Lambda}} = \eta\frac{E_0}{E_P}D(z)$$

Rainbow gravity. 4: effective QG approach

Assanioussi, Dapor, Lewandowski, arXiv:1412.6000

massive scalar field minimally coupled to gravity in FRW spacetime

$$H_{\vec{k}} = H_0 - \frac{1}{2} H_0^{-1} \left[\pi_{\vec{k}}^2 + (\vec{k}^2 a^4 + m^2 a^6) \phi_{\vec{k}}^2 \right],$$

trace out gravitational dof

$$\hat{H}_{\vec{k}}^{\text{traced}} = \frac{1}{2} \left[\langle \psi_0 | \hat{H}_0^{-1} | \psi_0 \rangle \, \hat{\pi}_{\vec{k}}^2 + \langle \psi_0 | \hat{\Omega}(\vec{k}, m) | \psi_0 \rangle \, \hat{\phi}_{\vec{k}}^2 \right],$$

$$\hat{\Omega}(\vec{k}, m) = \vec{k}^2 \widehat{H_0^{-1} a^4} + m^2 \widehat{H_0^{-1} a^6} .$$

$\approx$

$$\bar{g}_{\mu\nu} dx^\mu dx^\nu = -\bar{N}^2 dt^2 + \bar{a}^2 \left(dx^2 + dy^2 + dz^2 \right) .$$

$$\hat{H}_{\vec{k},m}^{\text{eff}} = \frac{1}{2} \frac{\bar{N}}{\bar{a}^3} \left[\hat{\pi}_{\vec{k}}^2 + (\vec{k}^2 \bar{a}^4 + m^2 \bar{a}^4) \hat{\phi}_{\vec{k}}^2 \right].$$

$$\bar{a}^6 + \frac{\vec{k}^2}{m^2} \bar{a}^4 - \delta = 0, \qquad \frac{\bar{N}}{\bar{a}^3} = \langle \psi_0 | \hat{H}_0^{-1} | \psi_0 \rangle , \qquad \delta := \frac{\langle \hat{\Omega}(\vec{k}, m) \rangle}{m^2 \langle \hat{H}_0^{-1} \rangle} .$$

$\Downarrow$

rainbow-like gravity

$$-m^2 = \bar{g}^{\mu\nu} k_\mu k_\nu = -\frac{k_o^2}{\bar{N}^2} + \frac{k^2}{\bar{a}^2} = -f^2 E^2 + g^2 p^2$$

MDR

$$E^2 = \frac{1}{f^2} \left(g^2 p^2 + m^2 \right) \approx m^2 + (1 + \beta) p^2 + O(p^4)$$

Case of study 2: multifractional gravity

Calcagni, MR, arXiv:1608.01667

q-theory:

$$ {}^q\Gamma^\rho_{\mu\nu} = \tfrac{1}{2} g^{\rho\sigma}\left(\tfrac{1}{v_\mu}\partial_\mu g_{\nu\sigma} + \tfrac{1}{v_\nu}\partial_\nu g_{\mu\sigma} - \tfrac{1}{v_\sigma}\partial_\sigma g_{\mu\nu}\right) $$

$$ {}^q S = \tfrac{1}{2\kappa^2}\int d^D x\, v(x)\sqrt{-g}\left({}^q R - 2\Lambda\right) + S_m $$

$$ \{D^q[M^k], D^q[N^j]\} = D^q\left[\frac{1}{v_j(x^j)}(M^j\partial_j N^k - N^j\partial_j M^k)\right], $$

$$ \{D^q[N^k], H^q[M]\} = H^q\left[\frac{1}{v_j(x^j)} N^j\partial_j M\right], $$

$$ \{H^q[N], H^q[M]\} = D^q\left[\frac{h^{jk}}{v_j(x^j)}(N\partial_j M - M\partial_j N)\right] $$

$$ \beta = \frac{1}{v_i(x^i)} $$

weighted-theory:

$$ S_g = \tfrac{1}{2\kappa^2}\int d^D x\, e^{\Phi/\beta}\sqrt{-g}\left[R - \Omega\partial_\mu\Phi\partial^\mu\Phi - U(v)\right] $$

$$ \Omega := \tfrac{9\omega}{4\beta^2} e^{\frac{2}{\beta}\Phi} + (D-1)\left(\tfrac{1}{2\beta_*} - \tfrac{1}{\beta}\right), \qquad \Phi(x) = \log v(x) $$

$$ H[N] = H_0[N] + H_\phi[N] = \int d^3x\, N(H_0 + \sqrt{h}\mathcal{H}_\phi) $$

$$ \{H[N], H[M]\} = \{H_0[N], H_0[M]\} + \int d^3x\, N(x)\int d^3y\, M(y)\times\{\mathcal{H}_0(x), \sqrt{h}\}\mathcal{H}_\phi(y) + \int d^3x\, N(x)\int d^3y\, M(y)\times\mathcal{H}_\phi(x)\{\sqrt{h}, \mathcal{H}_0(y)\} = D\left[h^{jk}(N\partial_j M - M\partial_j N)\right] $$

$$ \beta = 1 $$

Case of study 3: Moyal noncommutative gravity

Bojowald, Brahma, Buyukam, MR, arXiv:1712.07413

Canonical (Moyal-Weyl) noncommutative spacetime

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} \quad \to \quad F(\hat{x})G(\hat{x}) =: \Omega(f(x) \star g(x))$$

star product $f(x) \star g(x) = f(x) e^{-\frac{1}{2} i \overleftarrow{\partial_\alpha} \theta^{\alpha\beta} \overrightarrow{\partial_\beta}} g(x) =: \overline{f}^\alpha(f(x)) \overline{f}_\alpha(g(x))$

$$R = e^{i\theta^{\alpha\beta}\partial_\alpha \otimes \partial_\beta}, \qquad R^{-1} = e^{-i\theta^{\alpha\beta}\partial_\alpha \otimes \partial_\beta}$$ R-matrix

$$\mathcal{L}_v \triangleright (u \star w) := (\mathcal{L}_v \triangleright u) \star w + \overline{R}(u) \star (\mathcal{L}_{\overline{R}(v)} \triangleright w)$$ deformed Leibniz rule

modified Gaussian condition for the metric

$$\overline{R}(n^\mu) \star (\mathcal{L}_{\overline{R}(v)} \triangleright g_{\mu\nu}) = -\partial_\rho (v^\rho \star n^\nu \star g_{\mu\rho}) \star n^\gamma \star n^\rho \star g_{\gamma\nu}$$

$$[M \star X, n]^a_\star = -\partial_b N \star q^{ab} \qquad -\partial_\nu N \star n^\nu = 0$$

$$[(0, M_1), (0, M_2)]_\star = (0, [M_1 \star X, M_2 \star X]^a_\star),$$
$$[(N_1, 0), (N_2, 0)]_\star = (0, N_1 \star q^{ab} \star \partial_b N_2 - \partial_b N_1 \star N_2 \star q^{ab}),$$
$$[(0, M), (N, 0)]_\star = (\mathcal{L}_{M \star X} \triangleright N, 0)$$

Deformed general covariance!!