# Relaxation of 1-D self-gravitating system

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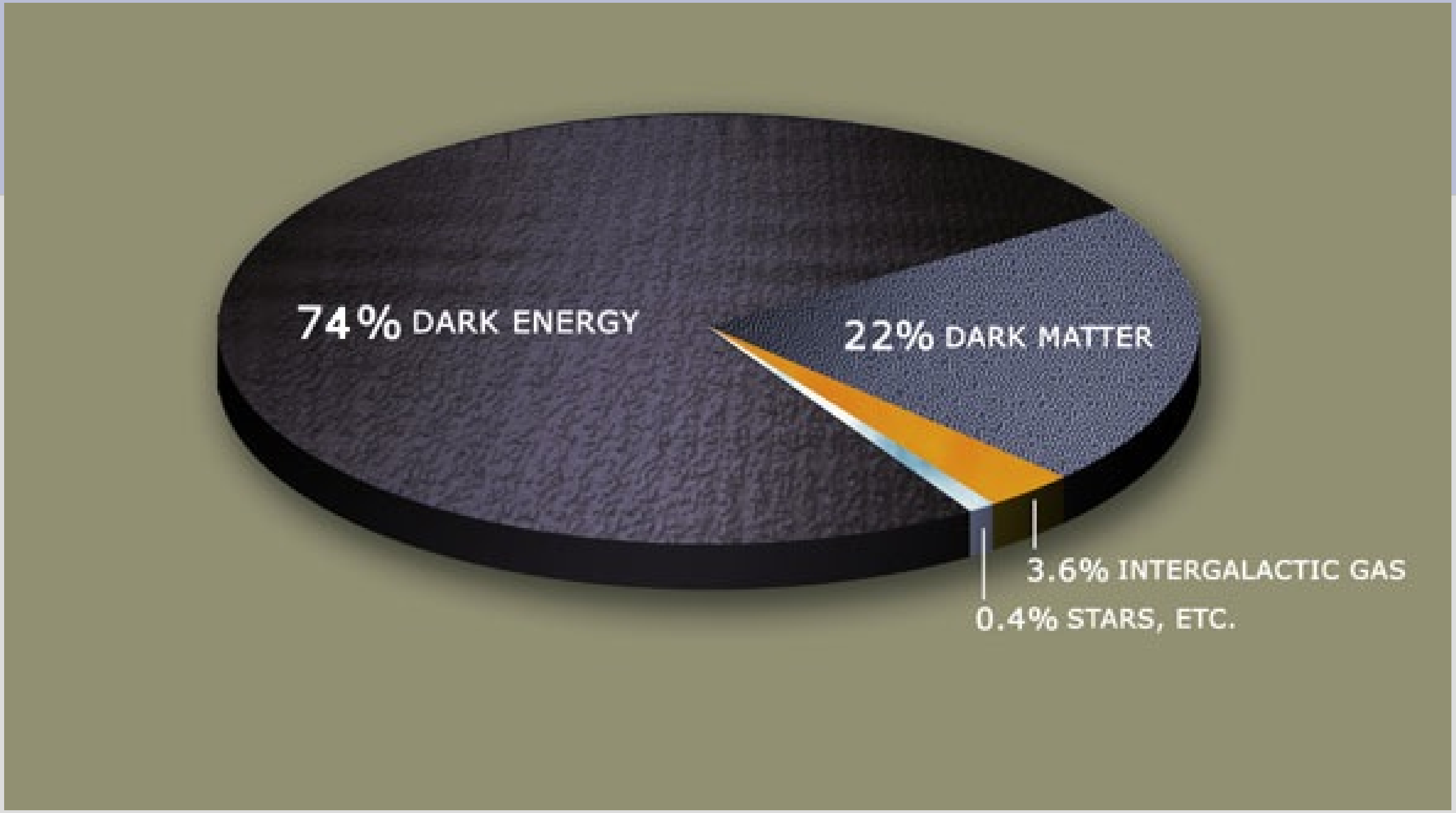
# Outline

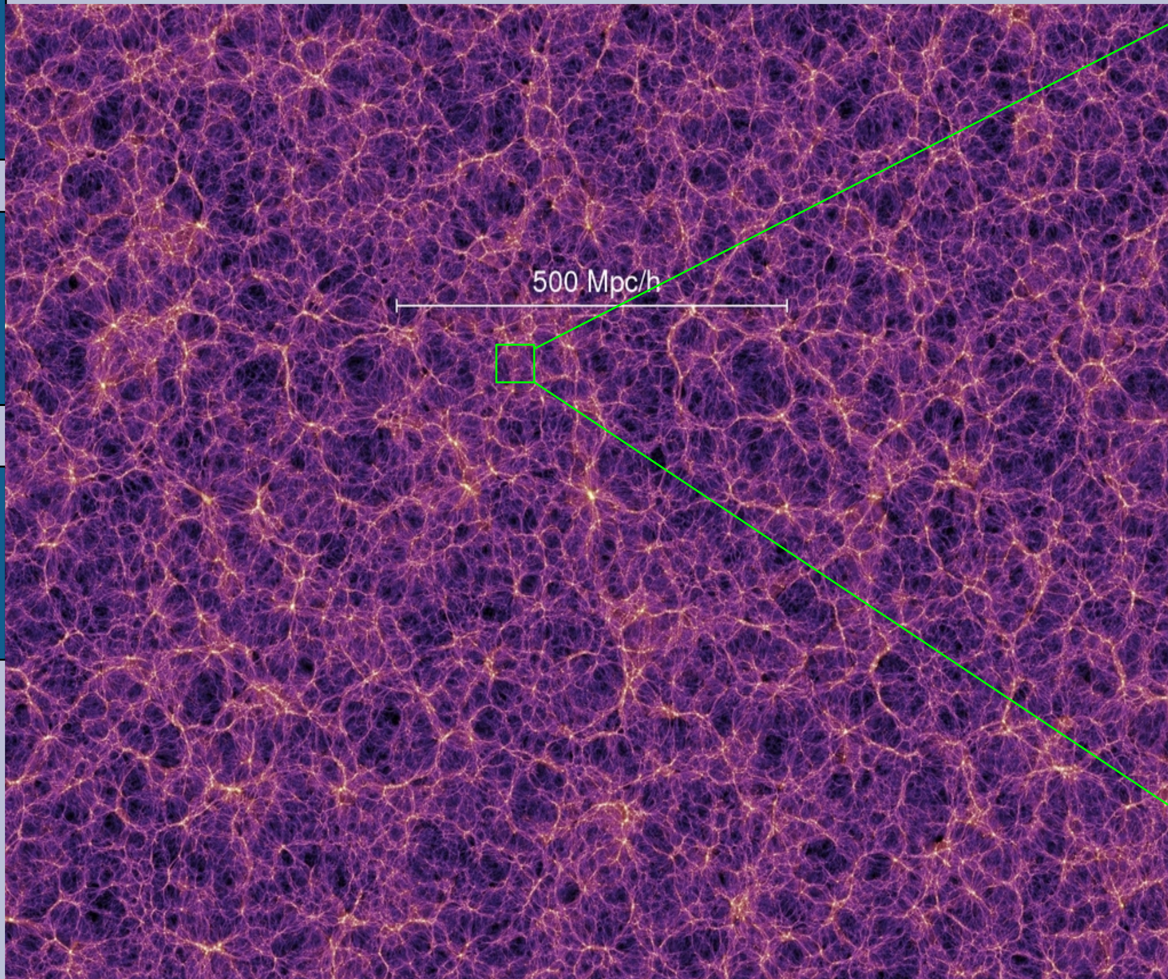
- Motivation
- 3-D numerical investigation
- Simplified toy model : gravity in 1-D
- Theoretical & numerical studies in toy model
- Conclusion & discussion

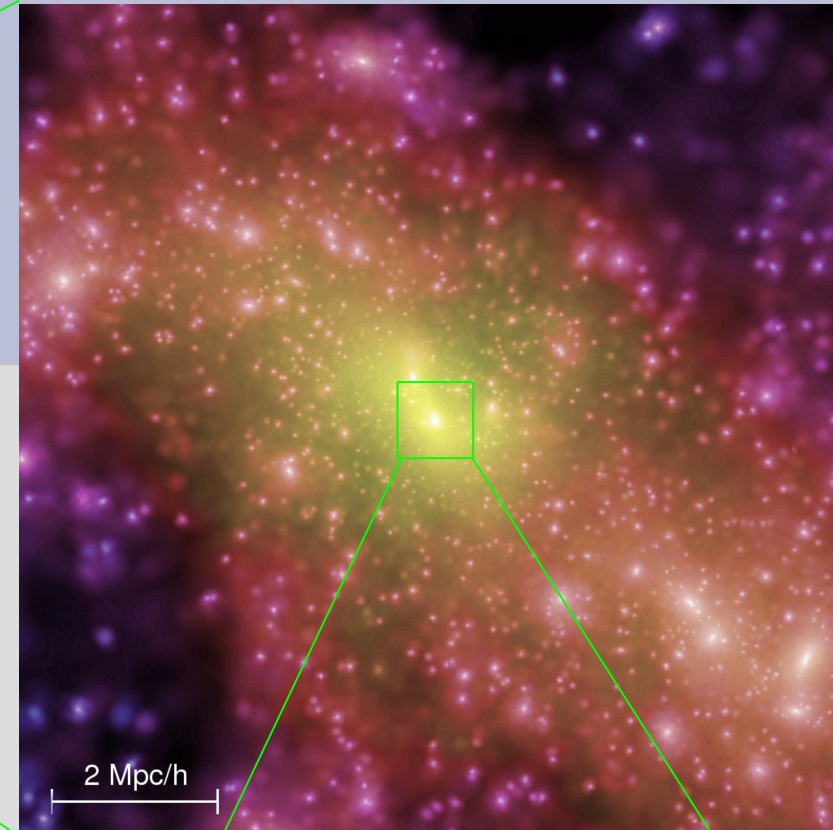
# Motivation




Dark matter accounts for 22% of universe, ~5 times more than visible matter.

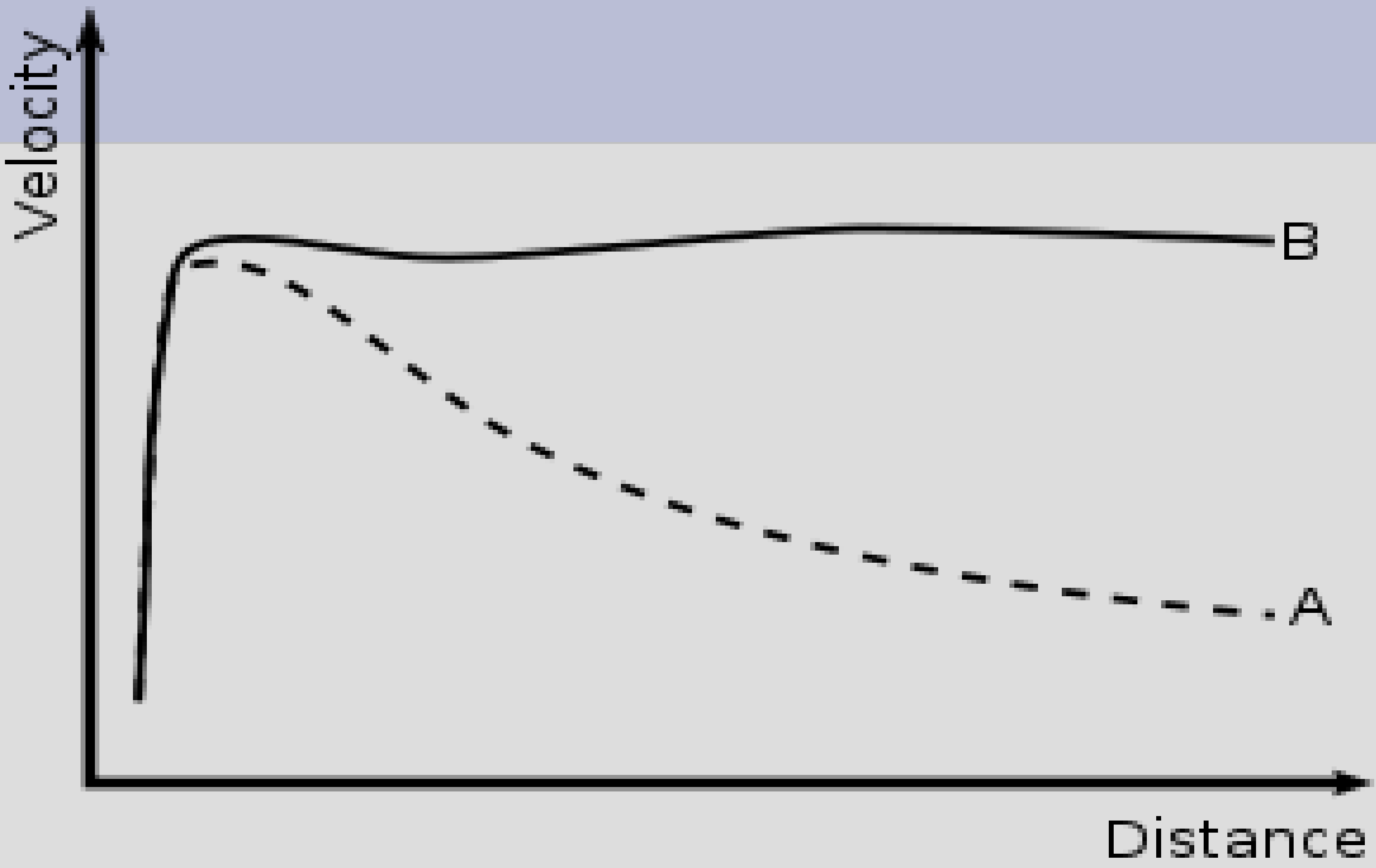
*How and where do they distribute?*




(ref Millennium simulation)

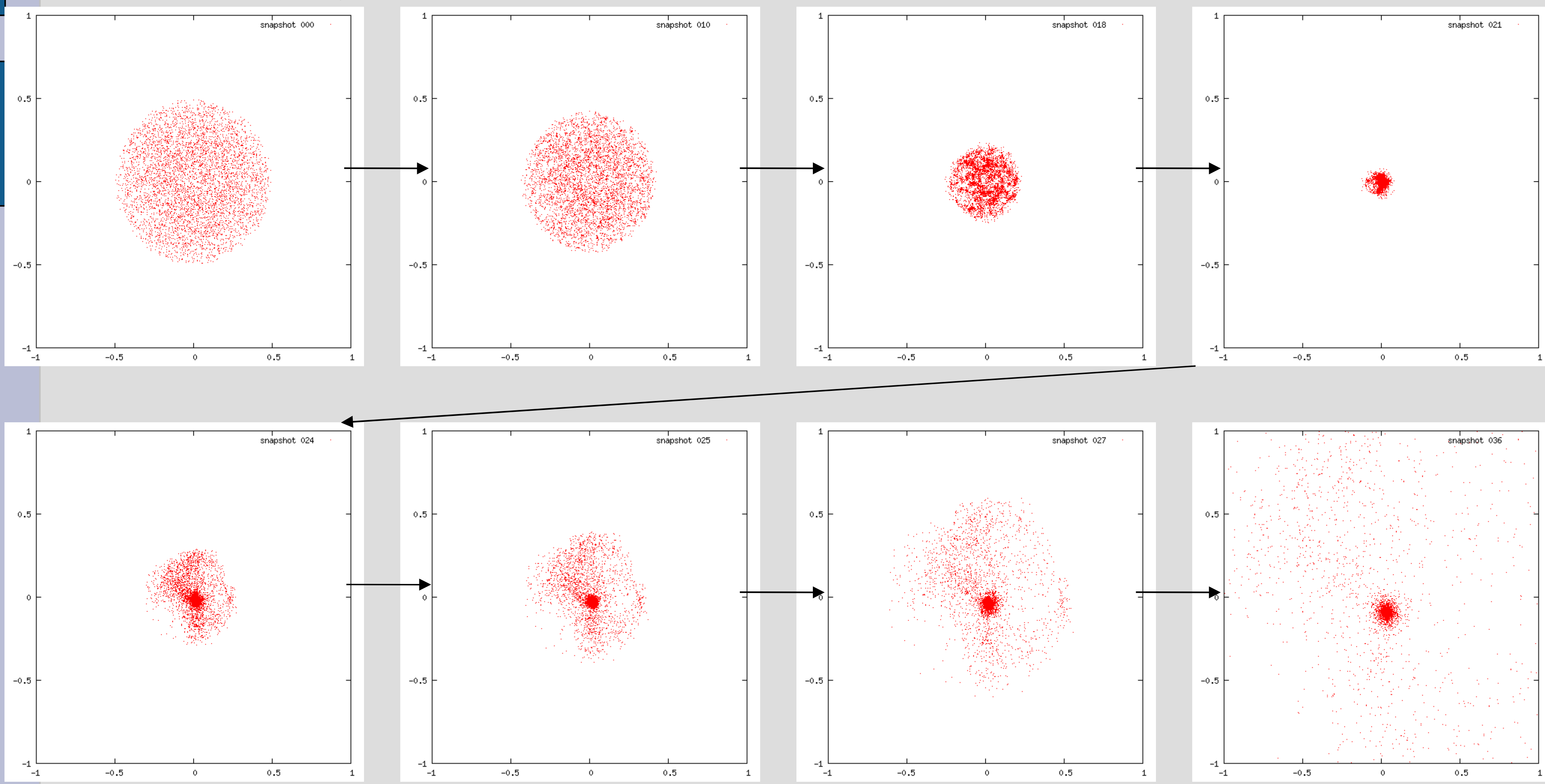
*What is happening in small scale?*




Evidence of galactic dark matter(ref Zwicky 1974)

# 3-D numerical investigation

The isolated 3-D self-gravitating system is employed, without the expansion of universe.



- This model has theoretical obscurity :
  - *Particle ejection*

    → *Non-stationary state*
- And also numerical disadvantages :
  - *Force divergence at small range*

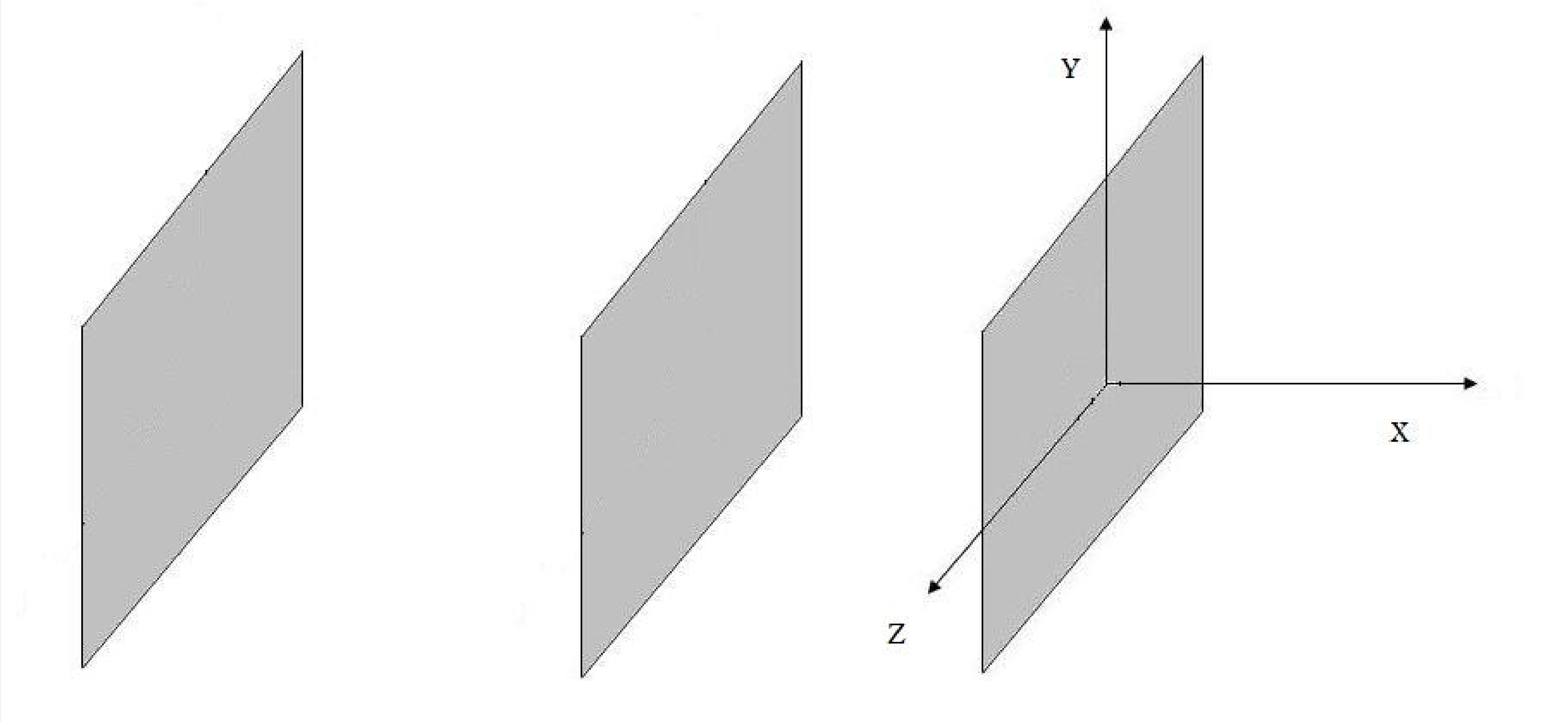
    → *Necessity of smoothing or cut-off range*
  - *Very costly*

    → *Necessity of approximative methods*

*For simplicity, we therefore seek for the toy model*

# Toy model : gravity in 1-D or "sheet model"

- The "sheet model" considers the mass sheets infinitely extended in y-z plane and moving along x axis.

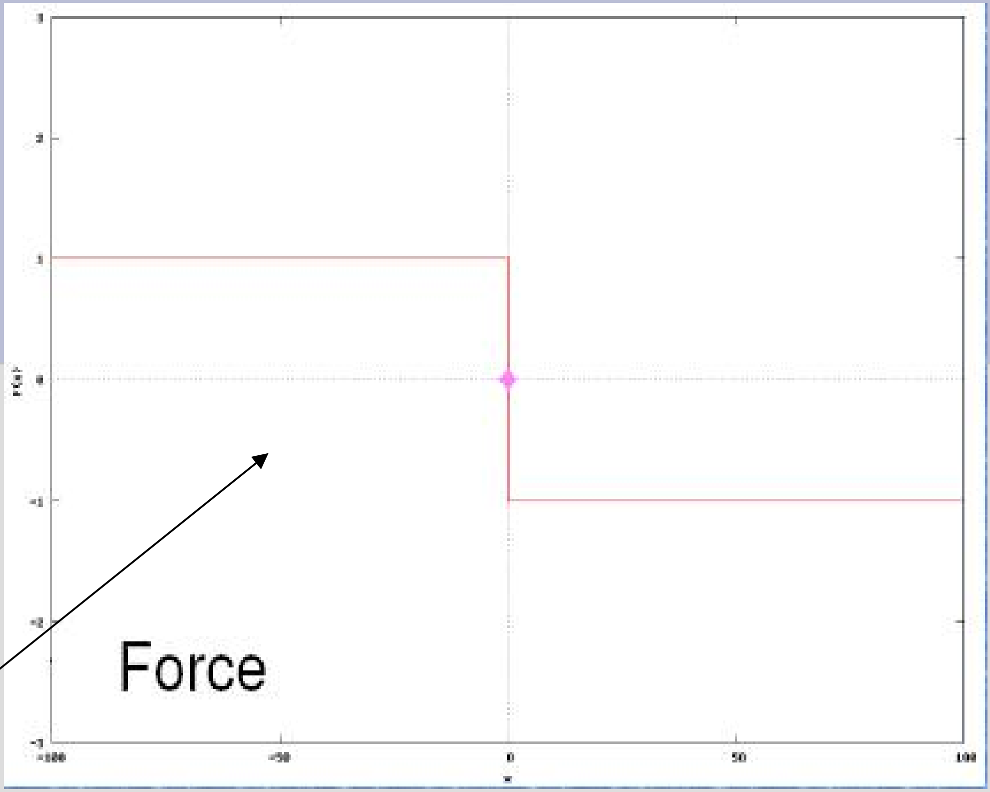


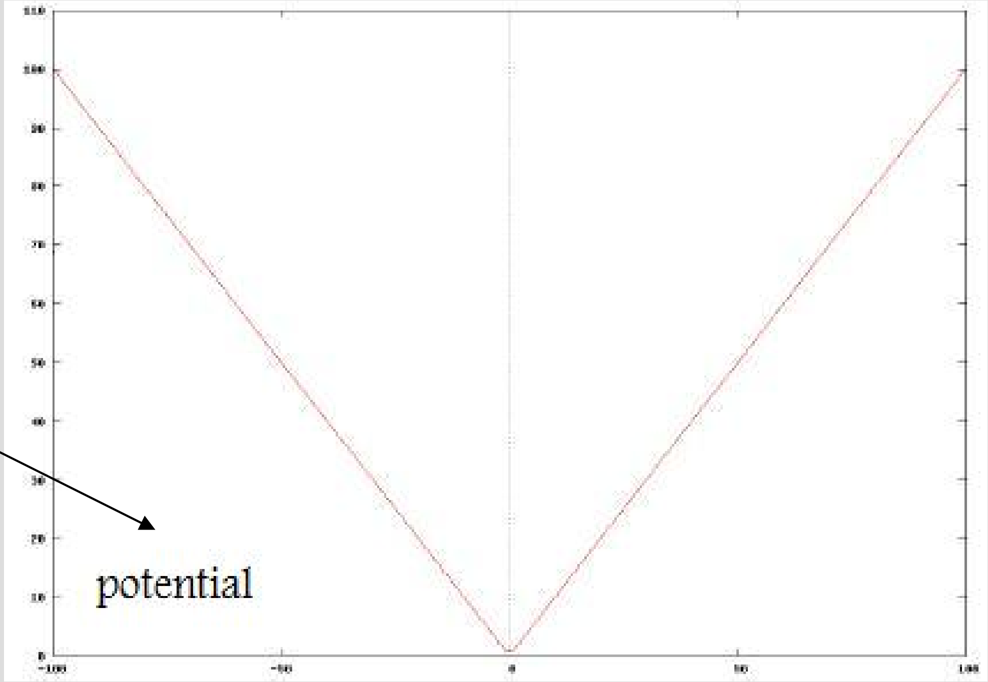
*Constant force independent on distance*

$$\frac{d^2\varphi(x)}{dx^2} = 4\pi G\delta(x)$$

$$F(x) = \frac{d\varphi(x)}{dx} = 2\pi G(1 - 2\Theta(x))$$

$$\varphi(x) = 2\pi G|x|$$



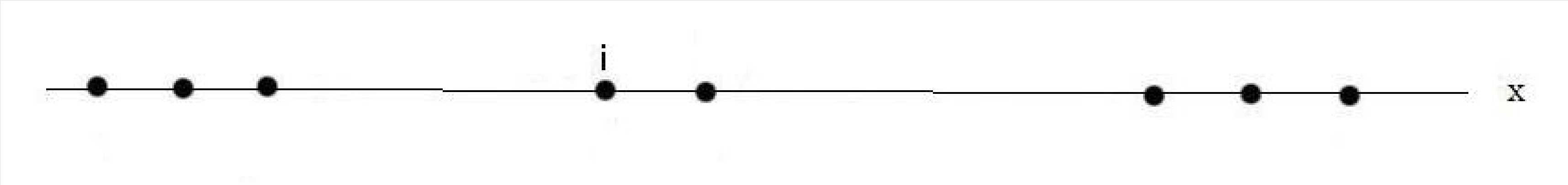

*Infinite escape velocity*

The force exerting on $i^{th}$ particle is then

$$\boxed{F^i \sim N^i_+ - N^i_-}$$

where $N^i_+$ = number of particles on the right

$N^i_-$ = number of particles on the left




*The simulation is done by following the exact trajectories of each particle, subjected to the free-fall motion with constant force.*

# Theoretical & numerical studies in toy model

- We are able to point out two different relaxation processes.

| | | |
|---|---|---|
| (Rapid) | | (Slow) |
| *Initial state --------->* | *Quasi-Stationary State(QSS)* | *---------> Thermal Equilibrium* |

What do we know?

(Rapid) (Slow)

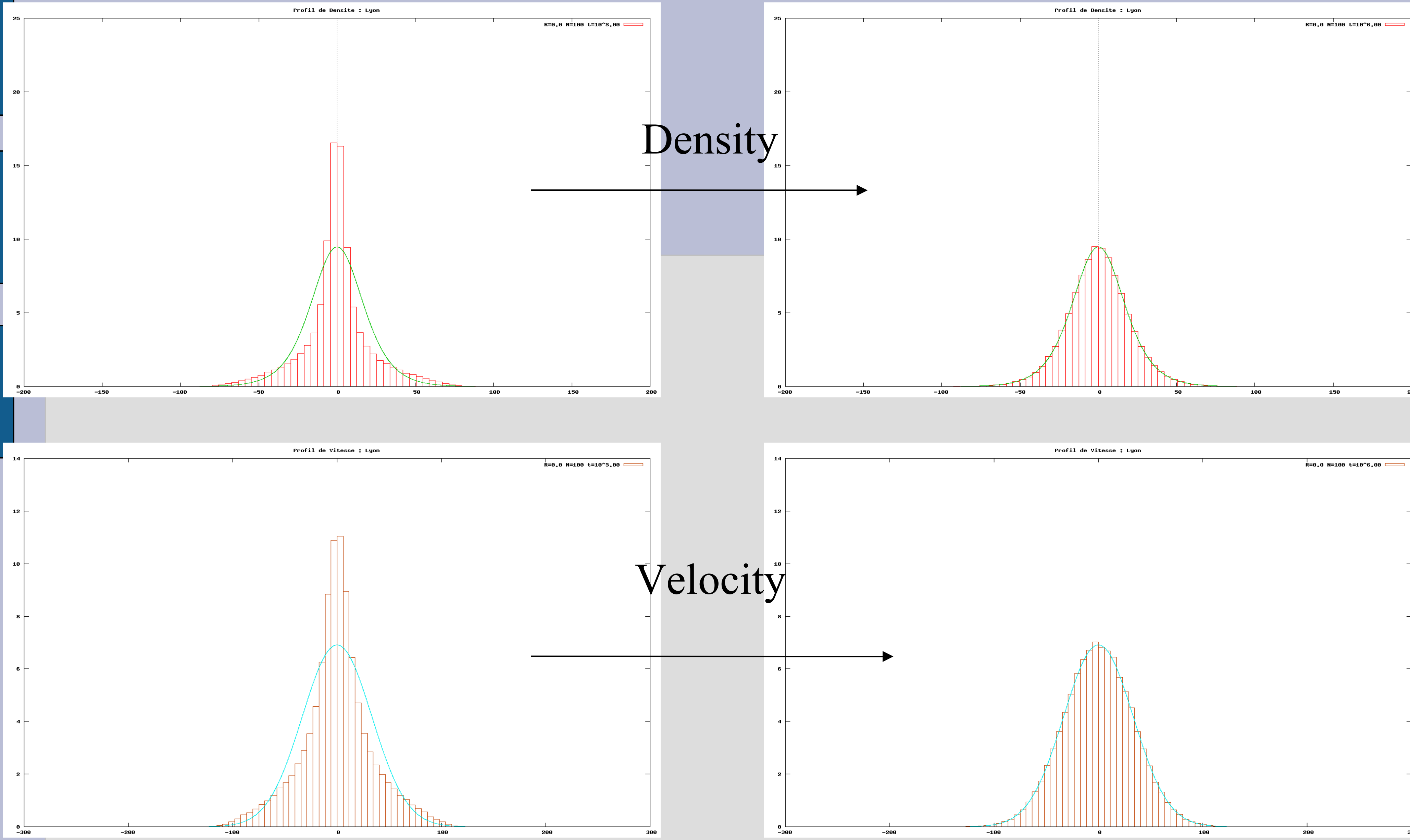
*Initial state --------> Quasi-Stationary State(QSS) ---------> Thermal Equilibrium*

- First, we consider the slow thermalization where the thermal equilibrium is defined by

$$f_V^*(\eta, \xi) = \frac{\pi^{-1/2}}{2} e^{-\eta^2} \operatorname{sech}^2 \xi \, .$$

(ref Rybicki 1971)

*Note that velocity distribution is Maxwellian with inhomogeneous spatial distribution.*




# Thermailzation in density and velocity profiles

What is it?

(Rapid) (Slow)

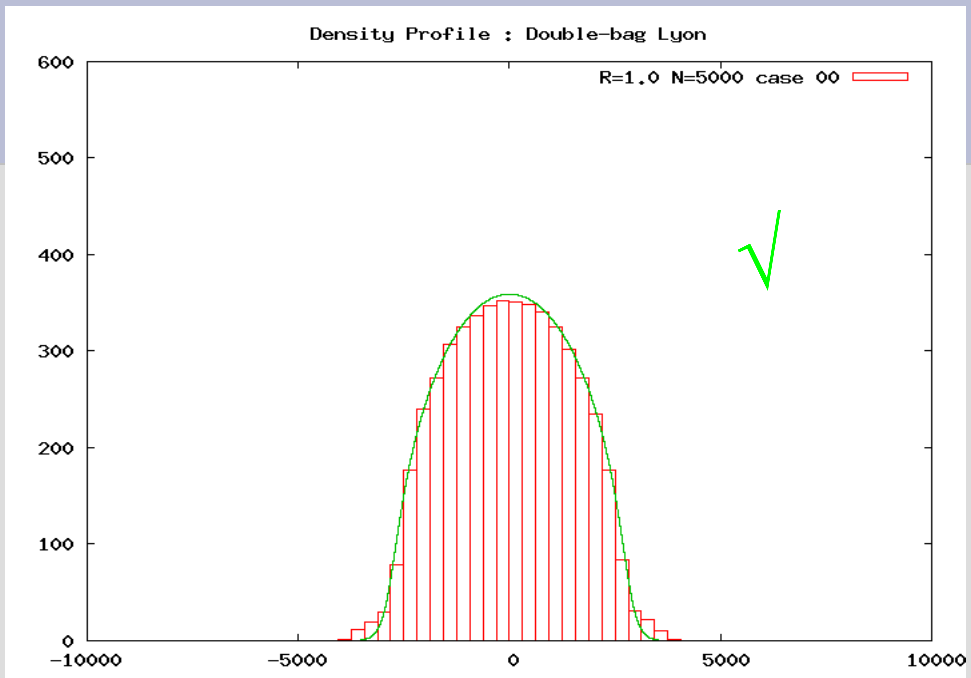
*Initial state* --------> *Quasi-Stationary State(QSS)* ---------> *Thermal Equilibrium*

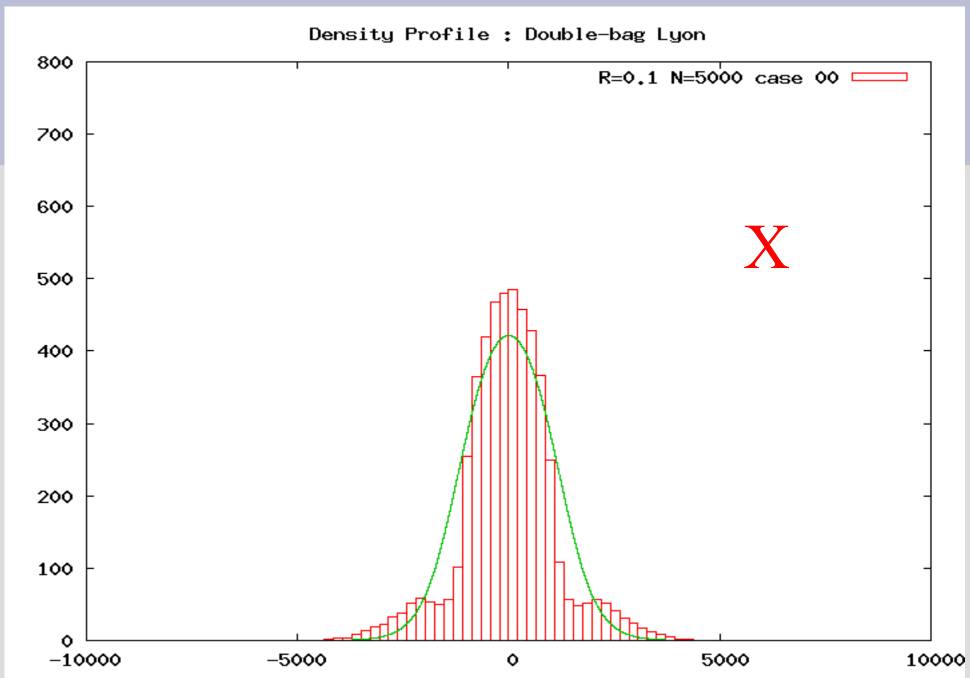
- The self-gravitating system always relaxes to unthermalized Quasi-Stationary State(QSS) at first. The existence and configuration of QSS were theorized in 1967, later known as "violent relaxation theory",

$$\bar{f} = \eta \frac{\exp\{-\beta(\epsilon - \mu)\}}{1 + \exp\{-\beta(\epsilon - \mu)\}}.$$

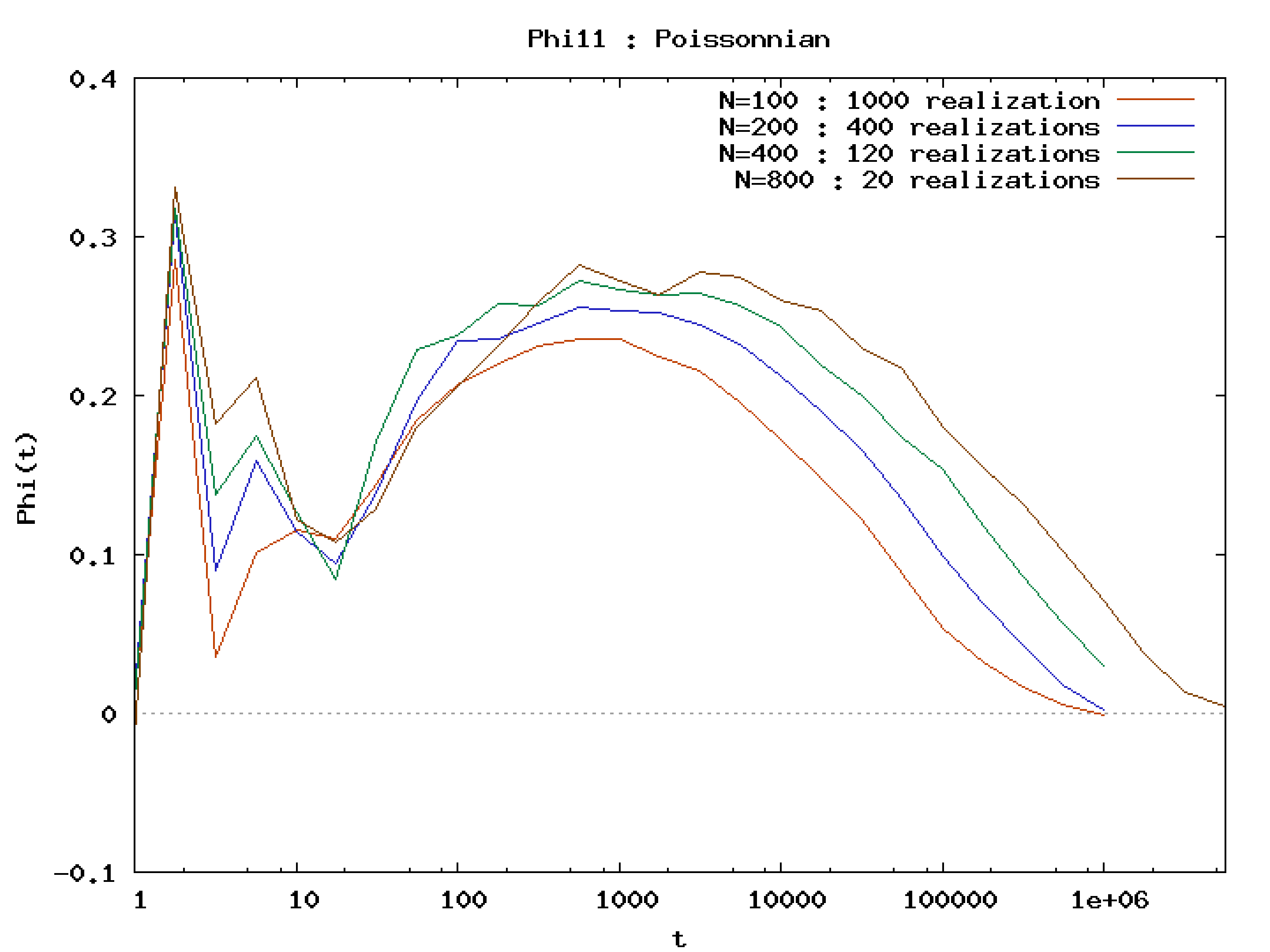
(ref Lynden-Bell 1967)

*This relaxation time does not depend on N, nor initial condition.*



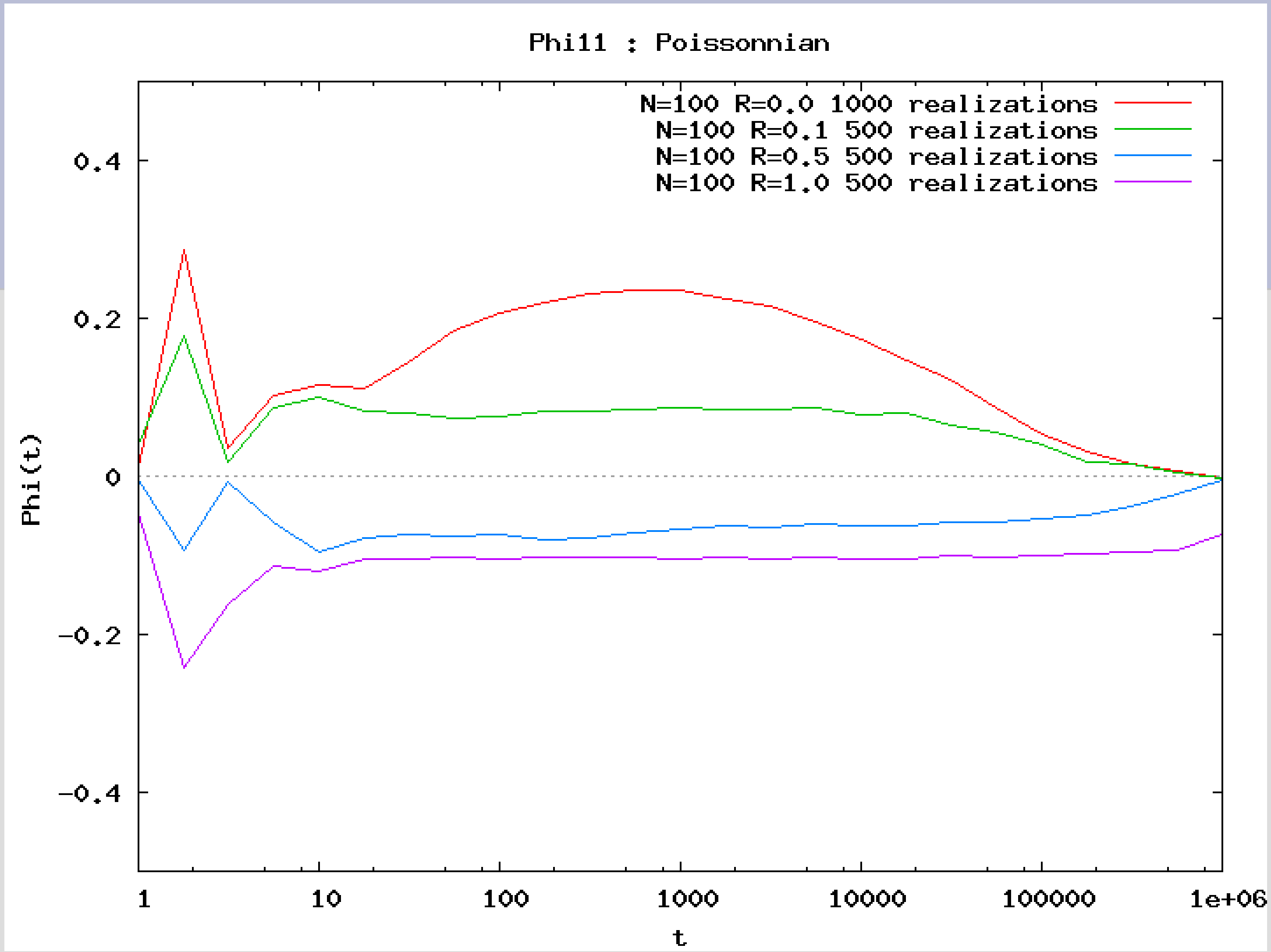




*Theory does not work in every case*




***Dependence on particle number, N?***

***Thermalization time scale increases with N, while violent relaxation does not.***




***Dependence on initial condition?***

***Thermalization time scale changes with IC, while violent relaxation does not.***

# Conclusion & Discussion

- Well defined thermalization
- Not well defined QSS

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- Analogies to 3-D?