Higgs masses of the general Two-Higgs-Doublet Model in the Minkowski-space formalism

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Contents

Higgs mechanism

- Four forces in nature:

Interactions	Gravity	Weak	Electromagnetic	Strong
Carried by	1 Graviton (not yet observed)	W^+, W^-, Z	Photon	8 Gluons
Masses (GeV)	0	80, 80, 91	0	0

- The **standard model** (SM): electromagnetic, weak and strong interactions based on local gauge invariance
- Unification $\rightarrow$ **electroweak interactions**

What is the origin of the particle masses?

- **Higgs mechanism** of spontaneous symmetry breaking
  - gives masses to W^+, W^-, Z bosons and fermions

One Higgs doublet

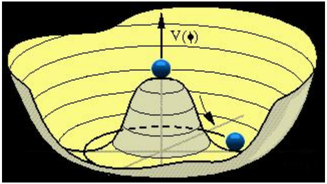
- In the minimal standard model: one Higgs doublet of scalar fields $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \rightarrow <\phi>= \begin{pmatrix} 0 \\ v \end{pmatrix}$ (conservation of em)

→ Introduction of 4 degrees of freedom:

After symmetry breaking:

- 3 absorbed by W^+, W^-, Z
- 1 physical Higgs boson

- Higgs potential has **2 parameters**: $V = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$:



Two Higgs doublets

Motivations to expand the Higgs sector

- Higgs sector still not discovered in experiment
- Requirements of higher scale symmetries (supersymmetry, grand unification theory...)
- New source of CP violation
- Dark matter
- Fermion mass spectrum

Two Higgs doublets

- Various models: one of the simplest is the **Two-Higgs-Doublet Model** (T.D.Lee 1973)

- Two Higgs doublets $\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}$

$$\rightarrow < \phi_1 >= \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad < \phi_2 >= \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

→ Introduction of 8 degrees of freedom

After symmetry breaking:

- 3 absorbed by W^+, W^-, Z
- 5 physical Higgs bosons

- Most general gauge-invariant and renormalizable potential contains **14 parameters** → many variants of 2HDM with different symmetries and rich new phenomenologies

The most general two-Higgs-doublet model (2HDM)

Motivations for the most general 2HDM:

- Various sets of parameters $\longrightarrow$ similar phenomenologies $\longrightarrow$ there must be some structure in the "space of models"
- What is the full spectrum of possible symmetries and their phenomenological consequences offered?
- Recovering all the particular models as limiting cases and establishing relations among them
- Building models with predefined symmetries
- Understanding the future LHC data

Difficulties with the most general two-Higgs-doublet model

The first step is to find the vacuum state ≡ minimum of the most general Higgs potential:

$$\begin{aligned} V_H &= -\frac{1}{2}[m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2) + m_{12}^2(\phi_1^\dagger\phi_2) + m_{12}^{2*}(\phi_2^\dagger\phi_1)] \\ &\quad \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \frac{\lambda_3}{2}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \\ &\quad \frac{\lambda_4}{2}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}[\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_5^*(\phi_2^\dagger\phi_1)^2] + \\ &\quad \{[\lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)](\phi_1^\dagger\phi_2) + h.c.\} \end{aligned} \quad (1)$$

Unfortunately,

- We cannot minimize the most general 2HDM potential with straighforward algebra
- Numerical analysis does not help

⟶ **Other method is needed**

Solution

Method to avoid this computational difficulty was recently suggested by I.Ivanov (2008):

- Establishment of the structure behind 2HDM
- Use of group theory and tensorial algebra

  $\longrightarrow$ description of the vacuum without the need to compute the exact position of the global minimum of the potential
- Explicit description of the space of 2HDM

  $\longrightarrow$ we can study the geometry and symmetries of this space

Structure behind 2HDM

This method is based on two main properties:

- The reparametrization symmetry: any linear transformations between the two doublets preserve the generic form of the potential but with reparametrized coefficients

  $\longrightarrow$ **reparametrization freedom with the group** $GL(2, C)$

- The orbit space: potential depends on 4 combinations $(\phi_i^\dagger \phi_j)$, $i, j = 1, 2$, which can be organized into a 4-vector $r^\mu = (r^0, r^i)$ with:

$$r_0 = (\phi_1^\dagger \phi_1) + (\phi_2^\dagger \phi_2)$$

and

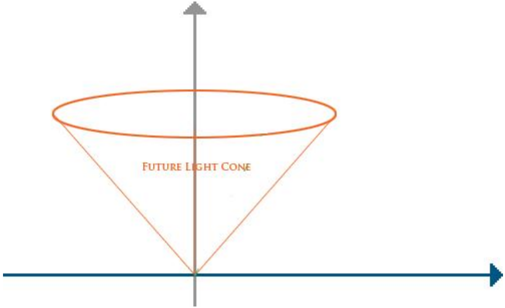
$$r_i = \begin{pmatrix} (\phi_2^\dagger \phi_1) + (\phi_1^\dagger \phi_2) \\ -i((\phi_1^\dagger \phi_2) - (\phi_2^\dagger \phi_1)) \\ (\phi_1^\dagger \phi_1) - (\phi_2^\dagger \phi_2) \end{pmatrix}$$

$\longrightarrow$ **reparametrization freedom with the Lorentz group** $SO(3, 1)$

Structure behind 2HDM

Degrees of freedom	(ϕ_i, ϕ_j)	$\longrightarrow$	r_μ
Reparametrization group	$GL(2, C)$	$\longrightarrow$	$SO(3, 1)$

- Possible r^μ lie on the surface and inside the **future light cone**: $r^\mu r_\mu \geq 0$ and $r^0 \geq 0$



Structure behind 2HDM

- The Higgs potential in the r^μ space can be written as:

$$V_H = -M_\mu r^\mu + \frac{1}{2}\Lambda_{\mu\nu} r^\mu r^\nu$$

- Transformations from the reparametrization group $GL(2, C)$ modify the Higgs kinetic term T $\rightarrow$ **non-diagonal kinetic terms**:

$$T = K_\mu \rho^\mu\,, \quad \rho^\mu = (\partial_\alpha \Phi)^\dagger \sigma^\mu (\partial^\alpha \Phi)\,,$$

  The requirement that the energy density must be positive implies that K^μ lies inside the future lightcone

Structure behind 2HDM

$$B^\mu = \frac{1}{4}(m_{11}^2 + m_{22}^2, -2Re\ m_{12}^2, 2Im\ m_{12}^2, -m_{11}^2 + m_{22}^2),$$

$$\Lambda_{\mu\nu} = \frac{1}{2}\begin{pmatrix} \frac{\lambda_1+\lambda_2}{2} + \lambda_3 & -Re(\lambda_6+\lambda_7) & Im(\lambda_6+\lambda_7) & -\frac{\lambda_1-\lambda_2}{2} \\ -Re(\lambda_6+\lambda_7) & \lambda_4 + Re\lambda_5 & -Im\lambda_5 & Re(\lambda_6-\lambda_7) \\ Im(\lambda_6+\lambda_7) & -Im\lambda_5 & \lambda_4 - Re\lambda_5 & -Im(\lambda_6-\lambda_7) \\ -\frac{\lambda_1-\lambda_2}{2} & Re(\lambda_6-\lambda_7) & -Im(\lambda_6-\lambda_7) & \frac{\lambda_1+\lambda_2}{2} - \lambda_3 \end{pmatrix}$$

The standard K_μ is $(1, 0, 0, 0)$

Vacua

There are three types of **minima** of the potential:

- **Electroweak (EW) symmetry conserving vacuum**: all bosons remain massless

  $\longrightarrow r^{\mu} = 0$

- **Neutral vacuum**: breaks the EW symmetry, conserves the electric charge and the photon remains massless

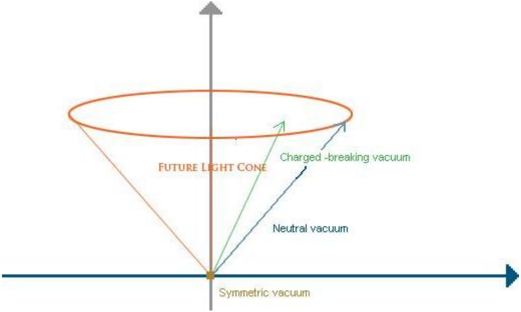
  $\longrightarrow r^{\mu} r_{\mu} = 0, \quad r^{\mu} \neq 0$

- **Charge-breaking vacuum**: breaks the EW symmetry, does not conserve the electric charge and the photon becomes massive

  $\longrightarrow r^{\mu} r_{\mu} \neq 0, \quad r^{\mu} \neq 0$

Vacua

- These three types of vacuum have a geometric meaning: the **singular point, the surface** or **the interior** of the r^μ space



Structure behind 2HDM

With this method, one can understand the **spectrum of the extrema** of the potential **withtout the explicit algebraic solution** of the minimization problem:

- How many minima can the potential have?
- Can the global minimum be degenerated and when does it happen? How is it related to the symmetries of the model?
- What is the phase diagram of the model?

This approach relies on **geometric properties**

Masses of Higgs bosons

Aim of this work

Study of **Higgs boson masses** in all these types of vacua in the most general 2HDM without the explicit solution of the minimization

- Physical quantities = **reparametrization-invariant** quantities
- Mass matrix $\neq$ reparametrization-invariant quantity $\rightarrow \mathcal{M} \neq \mathcal{M}'$
- But the eigenvalues of the mass matrices (masses) are invariant $\rightarrow$ calculated from
  - $Tr(\mathcal{M}) = Tr(\mathcal{M}')$, $Tr(\mathcal{M}^n) = Tr(\mathcal{M}'^n)$, $Det(\mathcal{M}) = Det(\mathcal{M}')$

Masses of Higgs bosons

Steps to get masses

1. Expression of the mass matrix as a function of the parameters
2. $Tr(\mathcal{M})$, $Tr(\mathcal{M}^2)$, $Det(\mathcal{M})$
3. Convert unknown ϕ into known r^μ at the minimum
4. Masses

Masses of Higgs bosons

Constraints on the parameters of the potential from positivity of the Higgs masses

$$\begin{aligned} V_H &= -\frac{1}{2}[m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2) + m_{12}^2(\phi_1^\dagger\phi_2) + m_{12}^{2*}(\phi_2^\dagger\phi_1)] \\ & \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \frac{\lambda_3}{2}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \\ & \frac{\lambda_4}{2}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}[\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_5^*(\phi_2^\dagger\phi_1)^2] + \\ & \{[\lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)](\phi_1^\dagger\phi_2) + h.c.\} \\ &= -M_\mu r^\mu + \frac{1}{2}\Lambda_{\mu\nu} r^\mu r^\nu \end{aligned} \tag{2}$$

Masses of Higgs bosons

Electroweak-symmetric vacuum

- Position:

$$< r_{\mu sym} >= 0$$

- $m^2_{1,2} = -(K^\mu M_\mu) \pm \sqrt{(K^\mu M_\mu)^2 - M^\mu M_\mu}$

  reparametrization invariant expression

  $\rightarrow$ positive if M^μ lies inside the backward lightcone

- $m_1^2, m_2^2 = -\frac{1}{4}(m_{11}^2 + m_{22}^2) \pm \frac{1}{4}(4|m_{12}^2|^2 + m_{11}^4 + m_{22}^4 - 2m_{11}^2 m_{22}^2)^{\frac{1}{2}}$

  $\rightarrow$ positive if $m_{11}^2 < 0\,, \quad m_{22}^2 < 0\,, \quad m_{11}^2 m_{22}^2 > |m_{12}^2|^2$

Masses of Higgs bosons

Charge-breaking vacuum

1. Position:

$$\Lambda^{\mu\nu} < r_{\nu\, ch} >= M^{\mu}$$

2. If $\Lambda^{\mu\nu}$ is not singular, $< r_{\mu\, ch} >= (\Lambda^{-1})_{\mu\nu} M^{\nu}$.
3. $\prod_i m_i^2 = 16\Lambda_0(-\Lambda_1)(-\Lambda_2)(-\Lambda_3)\cdot < r_{\nu\, ch} >^2 (K^{\nu} < r_{\nu\, ch} >)^2$

   $\rightarrow$ minimum only if all $\Lambda_i < 0$, $i = 1, 2, 3$, $< r_{\nu\, ch} >$ and K^{μ} lie inside the future lightcone

Masses of Higgs bosons

Neutral vacuum

1. Positions:

$$\Lambda^{\mu\nu} < r_\nu > -\zeta < r^\mu >= M^\mu \,,$$

   where ζ is a **Lagrange multiplier**

2. **Charged modes** : $m^2_{H^\pm} = 2(K^\mu \zeta_\mu)\,.$

   $\rightarrow \zeta$ must lie on the surface of the future lightcone

3. **Neutral modes** : one golstone mode, no simple solution for the masses,

   a new symmetry: $g_{\mu\nu} r^\mu r^\nu = 0 \longrightarrow \Lambda_{\mu\nu} \rightarrow \Lambda_{\mu\nu} + C g_{\mu\nu}$

Conclusions

- There is now a method to analyze the most general 2HDM
- This method was used to study the vacuum without explicit minimization: the key step is the observation that the space of all 2HDM model has the Minkowski space structure
- This work consisted in developing it further to study the **masses of the Higgs bosons**: we worked out a formalism to compute the traces of any power of the mass matrix in any type of minimum in a general 2HDM

Outlook

What remains to be studied:

- **Dynamics** of the general 2HDM (Feynman rules) as well as Yukawa interactions with fermions in the reparametrization-invariant description.
- **Loop corrections** to the potential and their impact on the geometry
- Extensions to the N Higgs doublets

Thank you for your attention

Thank you for your attention

Is this the real life?

Is this just fantasy?

Caught in a landscape

No escape from our theory

Open your eyes

Look up to the plots and see

We're just theorists

We need your sympathy

coz' It's easy try, easy failed

Little high, little low

Any way the beam blows

Doesn't really matter to her, to me

Mama, they killed my model

Put constraints against its head

Published the paper, now it's dead

Mama, life has just begun

But now I'll have to work as a ...trader

Mama, ooh

Didn't mean to make you cry

If I'm not back again at work tomorrow

Carry on, carry on as if nothing really matters

So late, my time has come

Send paper to arxiv

Parameters fitting all the time

Goodbye, everybody

I've got to go

Gotta leave you all behind and face the truth

Mama, ooooooh (Anyway the beam blows)

I don't want to die

I sometimes dream there'll be no data at all !!!!!!!!!!

I see a little silhouetto of a higgs

Bachacou, Bachacou, will you come to jooooin us

Thunderbolt and lightning, very, very thrilling for me

(Galileo) Galileo (Galileo) Galileo , Galileo Invariance

Magnifico-o-o-o-o

We're just theorists, nobody loves us

We're just theorists, not experimentalists

Please teach us how to express our idea

Easy come, easy go, will you let her taaaalk?

Formula!No, we will not let you talk

Let her talk

Formula! We will not let you talk

Let her talk

Formula! We will not let you talk

Let me talk (Will not let you talk)

Let me talk (Will not let you talk) (Never, never, never, never)

Let me talk, o, o, o, o

No, no, no, no, no, no, no

(Oh mama mia, mama mia) Mama Mia, let me talk

The Formula has the devil put aside for her, for him, for me!

So you think you can leave me and forget the UV

So you think you can mock me and leave me to die

Oh, baby, can't do this to me, baby

Just gotta get out, just gotta get right outta here

(Oooh yeah, Oooh yeah)

Formula really matters

Anyone can see

Formula really matters

Formula really matters to us

Any way the beam blows...