The Importance of Being..
..Far From Thermal Equilibrium

Marco Schiro'
CNRS-IPhT Saclay

IPhT Colloque, Isle-sur-la-Sorgue Oct 2018

ST. JAMES'S THEATRE.

SOLE LESSEE AND PROPRIETOR - - MR. GEORGE ALEXANDER.

PRODUCED THURSDAY, FEBRUARY 14th, 1895.

To-day at 3, and Every Evening at 8.45,

The Importance of being Earnest,

A TRIVIAL COMEDY FOR SERIOUS PEOPLE,

BY OSCAR WILDE.

John Worthing, J.P.	of the Manor House, Woolton, Hertfordshire	Mr. GEORGE ALEXANDER
Algernon Moncrieffe	(his Friend)	Mr. ALLAN AYNESWORTH
Rev. Canon Chasuble, D.D.	(Rector of Woolton)	Mr. H. H. VINCENT
Merriman	(Butler to Mr. Worthing)	Mr. FRANK DYALL
Lane	(Mr. Moncrieffe's Man-servant)	Mr. F. KINSEY PEILE
Lady Bracknell		Miss ROSE LECLERCQ (By permission of Mr. J. COMYNS CARR.)
Hon. Gwendolen Fairfax	(her Daughter)	Miss IRENE VANBRUGH
Cecily Cardew	(John Worthing's Ward)	Miss VIOLET LYSTER
Miss Prism	(her Governess)	Mrs GEORGE CANNINGE

Time - - The Present.

Act I. - Algernon Moncrieffe's Rooms in Piccadilly (*H. P. Hall*)
Act II. - The Garden at the Manor House, Woolton (*H. P. Hall*)
Act III. - Morning-Room at the Manor House, Woolton (*Walter Hann*)

Preceded, (Evenings only) at 8.20, by a Play in One Act, by LANGDON E. MITCHELL, entitled

IN THE SEASON.

Sir Harry Collingwood	Mr. HERBERT WARING
Edward Fairburne	Mr. ARTHUR ROYSTON
Sybil March	Miss ELLIOTT PAGE

Scene - A Room in Sir Harry Collingwood's House. Time - The Present.

Programme of Music.

MARCH	"Cannon"	*Gung'l*
MAZURKA	"La Trigune"	*Louis Ganne*
VALSE	"Enfin Seuls"	*A. M. Fechner*
CANZONETTA		*B. Godard*
VALSE	"Acclamations"	*E. Waldtenfel*
OVERTURE	"Raymond"	*A. Thomas*
DANSE	"Glave"	*L. Gregh*
1ST MAZURKA		*L. Gregh*
WALTZ	"Summer Time"	*Ed. Hesse*

The Furniture by FRANK GILES & Co., High Street, Kensington.
The Wigs by W. CLARKSON.
The Scenery by H. P. HALL and WALTER HANN.

NO FEES. The Theatre is Lighted by Electricity. NO FEES.

The Attendants are strictly forbidden to accept gratuities, and are liable to instant dismissal should they do so.

Visitors to the Theatre are earnestly begged to assist the Management in carrying out a regulation framed for their comfort and convenience.

The Etchings and Engravings in the corridors and vestibule supplied and arranged by I. P. MENDOZA, King Street, St. James's.
The Floral Decorations by REID & Co., King Street, St. James's.
Photographs of the Artistes appearing at this Theatre, can be obtained of ALFRED ELLIS, 20, Upper Baker Street, N.W.

MATINEE EVERY SATURDAY AT 3.

Doors open at 2.30. Carriages at 4.45.

PRICES:—Private Boxes, £1 11s. 6d. to £4 4s. Stalls, 10s. 6d. Dress Circle, 7s.
Upper Boxes, Numbered and Reserved (Bonnets allowed), 4s. Pit, 2s. 6d. Gallery, 1s.
Doors open at 8. Commence at 8.20. Carriages at 10.45.
Box Office (Mr. ARNOLD) open daily from 10 till 5 o'clock, and 8 till 10 p.m.
Seats can be booked one month in advance by Letter or Telegram, or Telephone No. 3903.

The Importance of Being Far From Thermal Equilibrium

*A **non-**trivial **StatMech** Comedy for Serious **Condensed Matter** People*

written by
Marco Schiro'
CNRS-IPhT Saclay

IPhT Colloque, Isle-sur-la-Sorgue Oct 2018

Acknowledgements

- Francesco Peronaci (IPhT 2016-2018 —> Max Planck Dresden)
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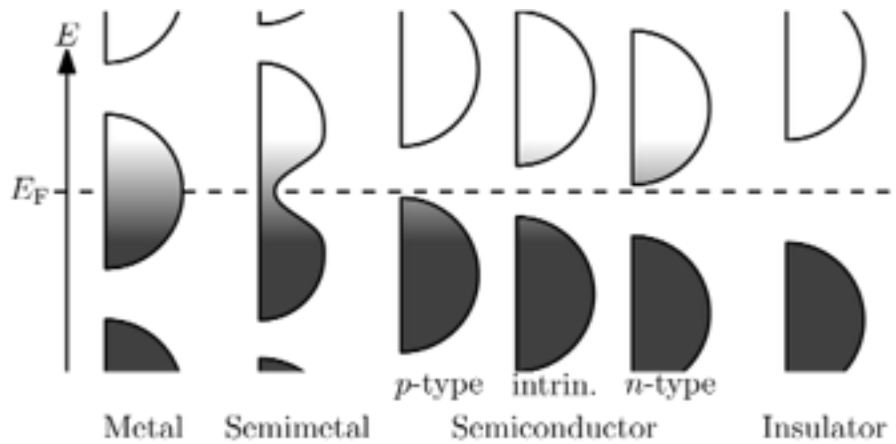
- Olivier Parcollet , Gregoire Misguich, Pierfrancesco Urbani, Giulio Biroli,..

Outline

- Equilibrium vs Non-Equilibrium Quantum Systems
- Theoretical Challenges and Open Questions
- Few Examples

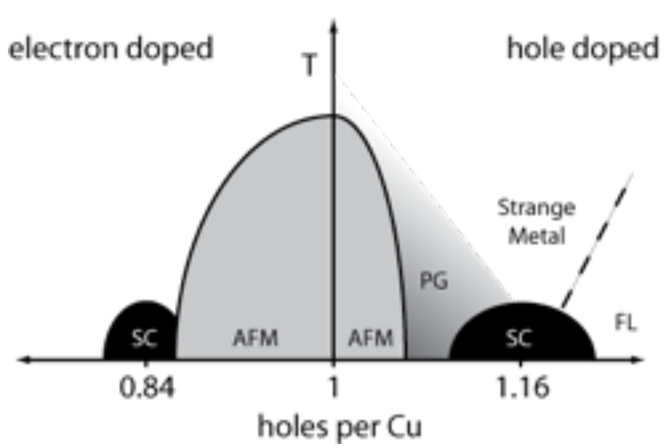
Quantum Mechanics + Statistical Physics



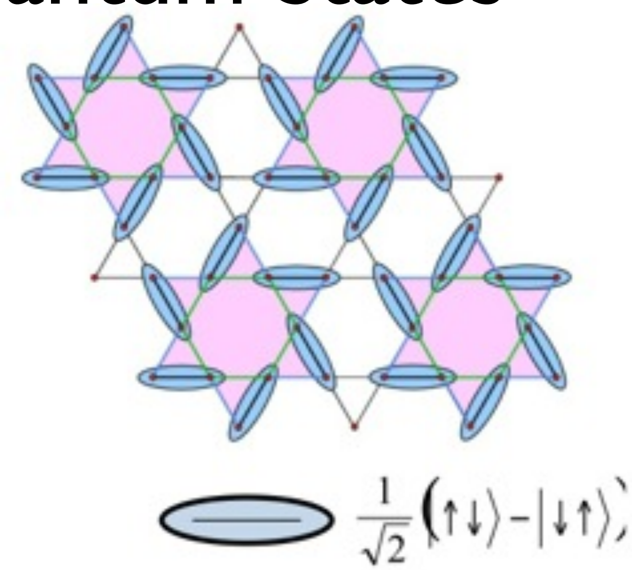
- Weakly Correlated States, independent particle picture

Metals, Insulators,…

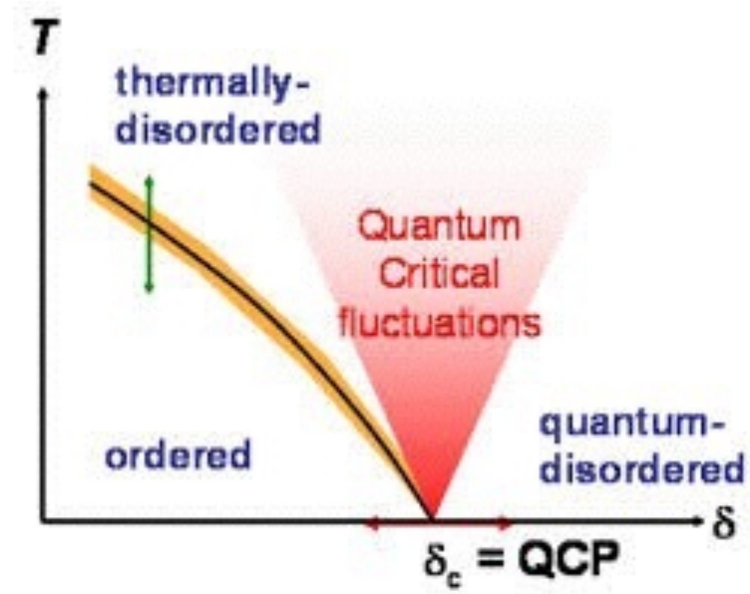
- Strongly Coupled Quantum States



HighTc Superconductivity



Spin Liquid



Quantum Critical States

- Main Focus (so far): Physics at/close-to Thermal Equilibrium

☑ Linear-Response Regime

Linear Response & Fluctuation-Dissipation Theorem

Ryogo Kubo, Boltzmann Medal 1977

- External Perturbation

$$H(t) = H + V_{ext}(t) \qquad V_{ext}(t) = f(t)\,A$$

- Expand to linear order in $V_{ext}(t)$

$$\langle A(t)\rangle = \langle A\rangle_0 + \int_{-\infty}^{\infty} \chi(t-t')f(t')$$

Response Function

$$\chi(t-t') = -\frac{i}{\hbar}\theta(t-t')\langle[A(t), A(t')]\rangle_0$$

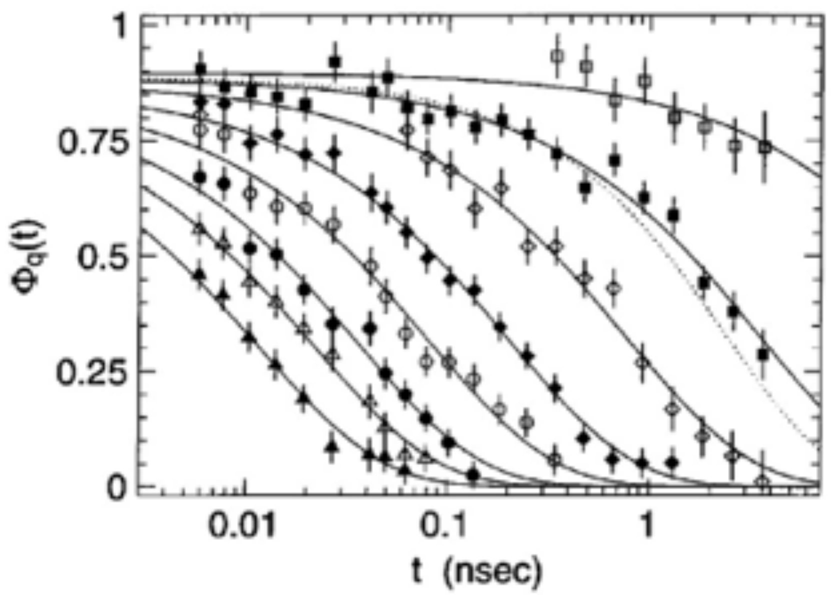
- Fluctuation-Dissipation Theorem

$$F(t) = \frac{1}{2}\langle\{A(t), A(0)\}\rangle_0 = \frac{\hbar}{2}\int dte^{-i\omega t}\,\coth(\beta\hbar\omega/2)\mathrm{Im}\chi(\omega)$$

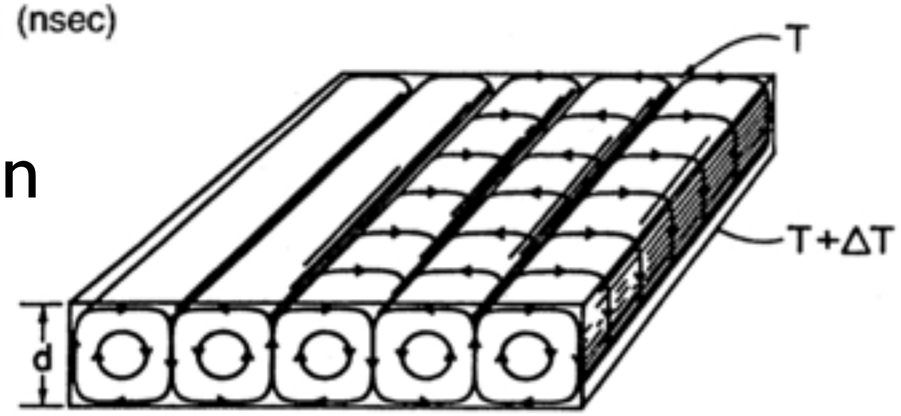
Classical Systems Out of Equilibrium

- Emergent Behavior Far From Equilibrium, Non-Linearities&Noise

Aging and Slow Dynamics in Glasses

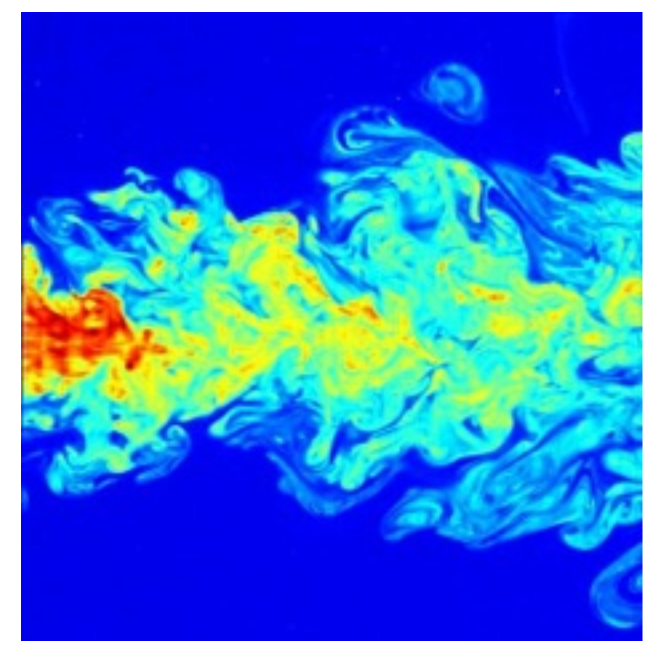


Pattern Formation
Instabilities



Dynamic Critical Phenomena

Turbulent Flow

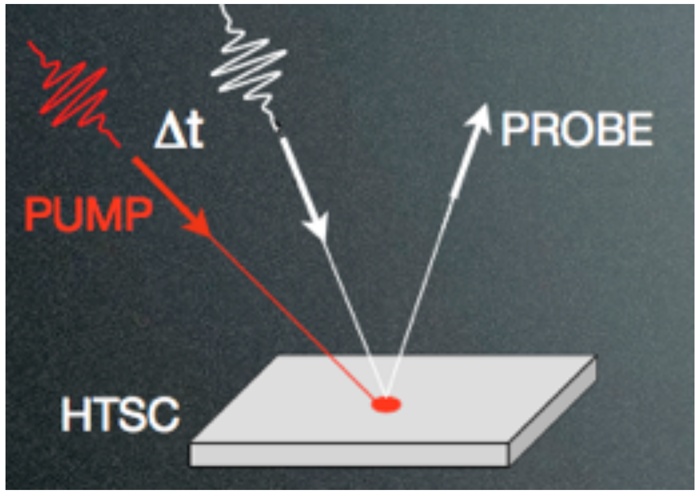


- So what about Quantum Systems far from Equilibrium?

Fresh New Input From Experiments…

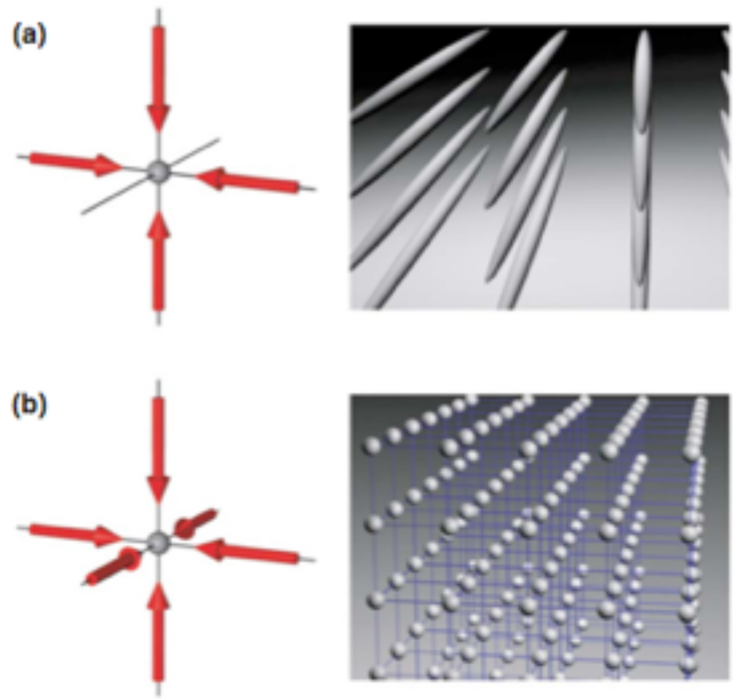
Condensed Matter

Electrons in Solids under ultrafast optical fields



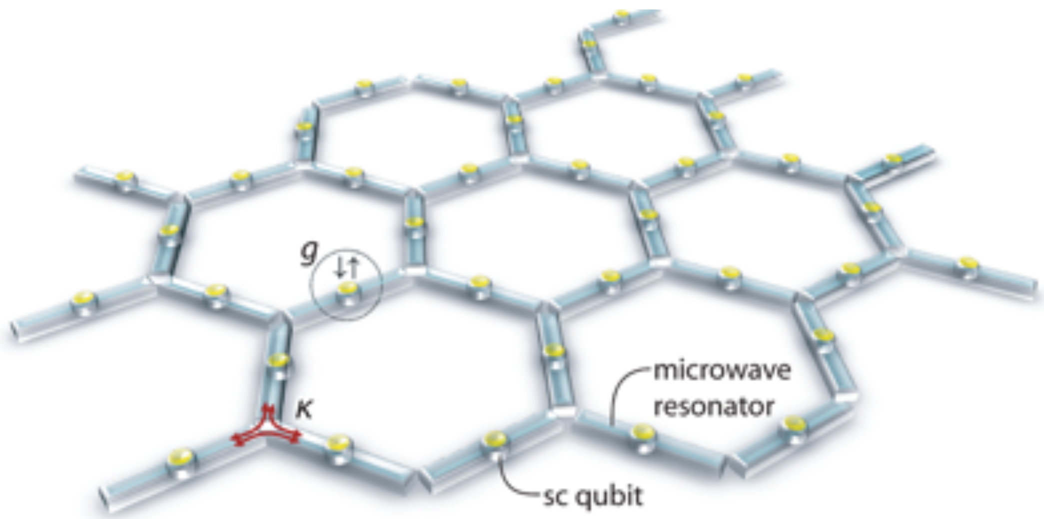
Atomic Physics

Neutral Atoms Trapped in Optical Lattices



Quantum Optics

Arrays of coupled non-linear CQED cavities

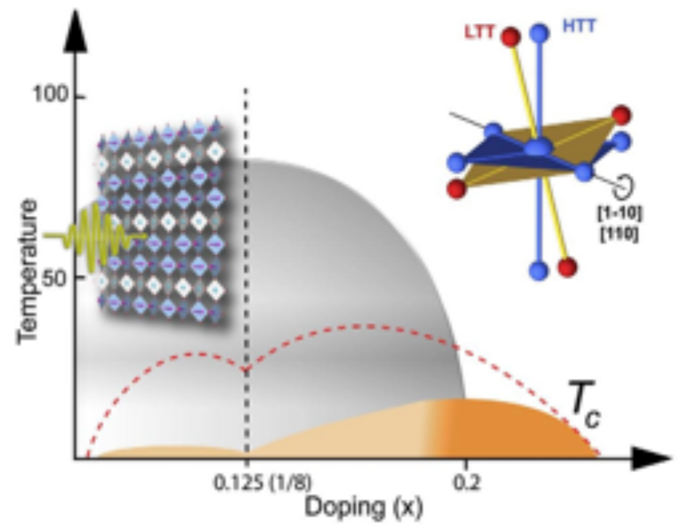


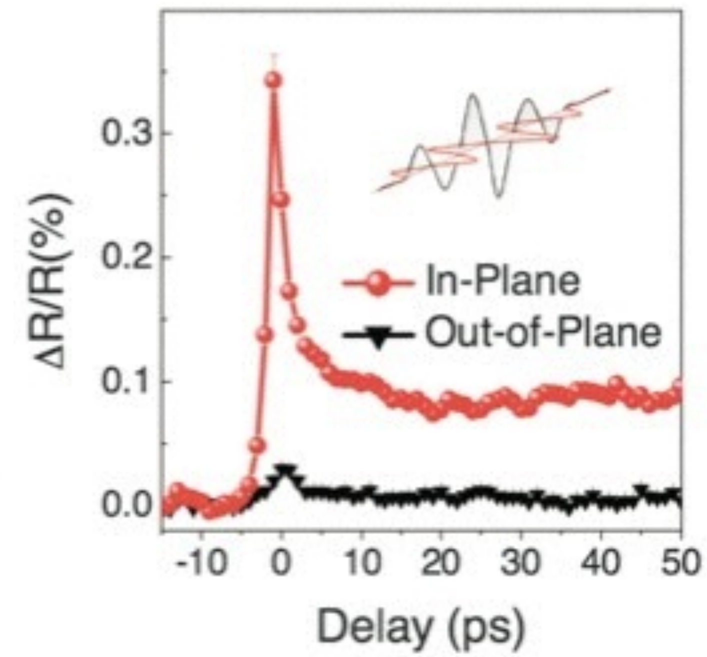
Controlling Phases of Matter in New Ways

Light-Induced Superconductivity in a Stripe-Ordered Cuprate

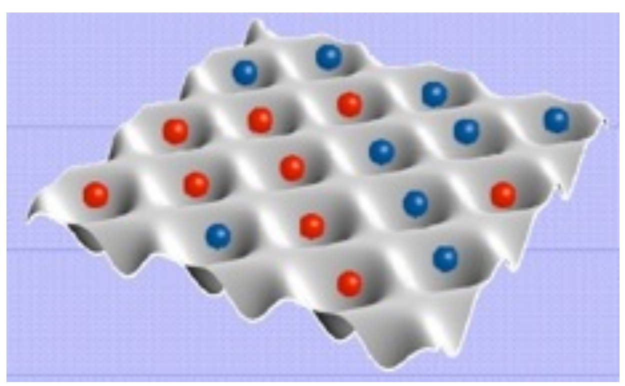
D. Fausti,[1,2*†‡] R. I. Tobey,[2†§] N. Dean,[1,2] S. Kaiser,[1] A. Dienst,[2] M. C. Hoffmann,[1] S. Pyon,[3] T. Takayama,[3] H. Takagi,[3,4] A. Cavalleri[1,2*]

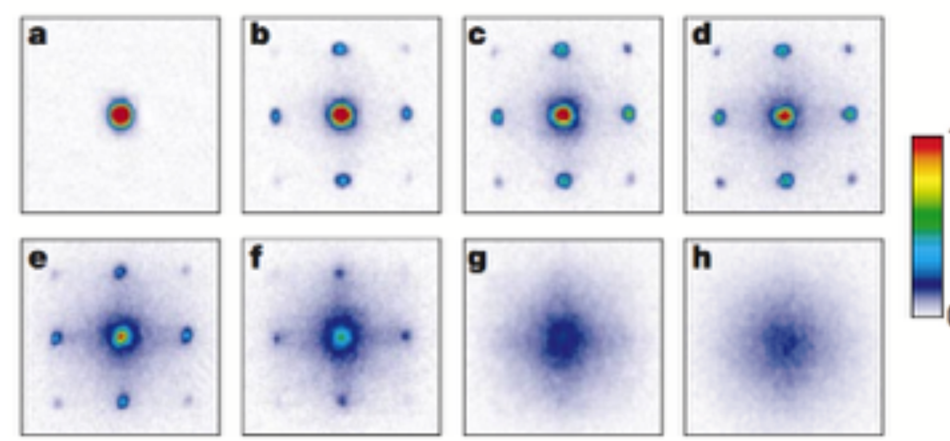
D. Fausti et al, Science (2011)





Quantum Simulation: Engineering New Phases of "Synthetic" Quantum Matter





- M. Greiner et al, Nature (2002)

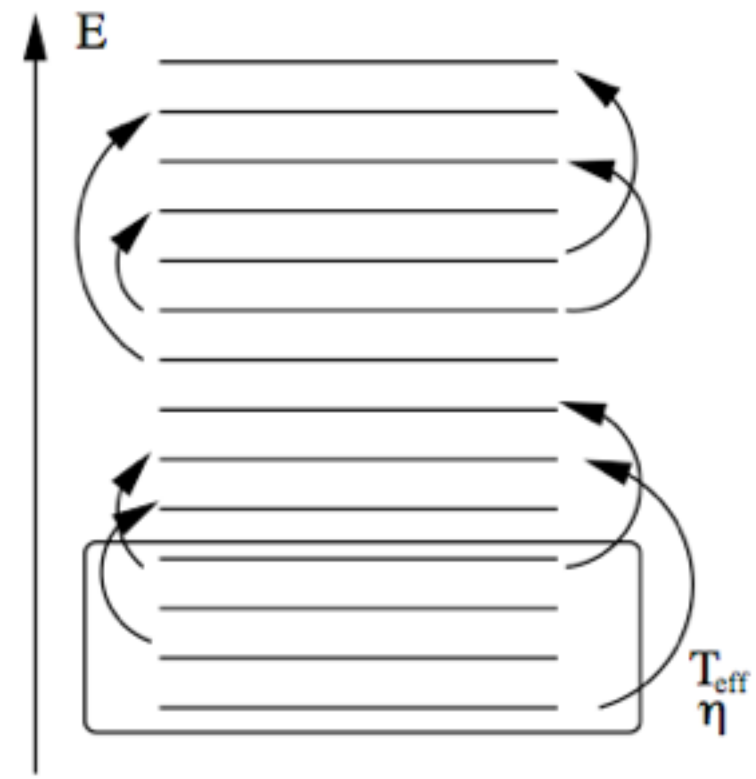
Theorist's Dream

- Use non-equilibrium perturbations as new knob to control/stabilize/create new quantum phases of matter

Theoretical Challenges in Quantum Many-Body Systems Far From Equilibrium

Equilibrium vs Non-Equilibrium

- The "Equilibrium Framework":
  - Physical Intuition: Ground State and Low-Lying Excitations dominates the Physics
  - Universality, Renormalization Group, Critical Phenomena, ...



- Non-equilibrium effects challenge this picture!
  - Perturbation couple low-energy/high-energy sectors
  - Spectrum AND Occupations are unknown

$$n(\omega) \neq \frac{1}{e^{\beta\omega} \pm 1}$$

- Different Flavors of Quantum "Non-Equilibrium"
  - Closed Quantum Systems (Unitary Dynamics)
  - Driven and Dissipative Systems (Open Quantum Systems)

Field Theory Perspective

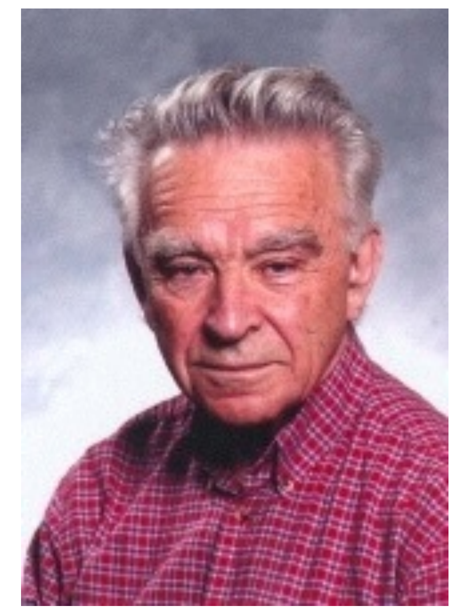
- Crucial in Equilibrium: Gell-Mann&Low Theorem (under adiabatic switching assumption)

$$\langle +\infty | -\infty \rangle = e^{-2i\delta}$$

$$\langle -\infty | S[+\infty, -\infty] | -\infty \rangle = e^{2i\delta}$$

- No longer true away-from-equilibrium!

- QFT on the "Keldysh" Contour, two fields +/- $\psi_\alpha(x)$

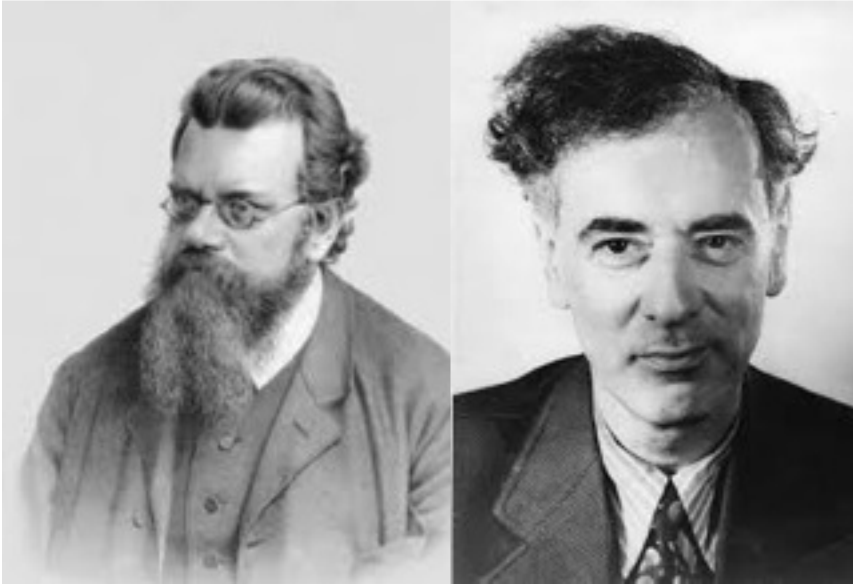


Leonid Keldysh

- No reference to the state at distant future
- Two (independent) Green's functions: Retarded(spectrum) and Keldysh(statistics)

- Recover the "De Dominicis" Field Theory (MSRJD) in the classical limit

Quantum Heating



- Physical intuition (Semi-classics, Quantum Boltzmann Equation):

Interactions induce scattering, loss of memory of initial condition

$$\frac{dn_{\mathbf{k}}}{dt} = I_{coll}[n_{\mathbf{k}}(t)]$$

$$I_{coll}[n_{\mathbf{k}}(t)] = -\sum_{k_1,p,p_1} W^{p,p_1}_{k,k_1}\left[n_k(t)n_{k1}(t)(1-n_p(t))(1-n_{p1}(t)) - (1-n_k(t))(1-n_{k1}(t))n_p(t)n_{p1}(t)\right]$$

- Non-equilibrium perturbation drives the system to another ("effective") equilibrium…

- …usually "hotter"! $E_{exc} = E_0 - E_{gs} \sim \varepsilon_{exc} L^d > 0$

- In modern terms: "Typical" (High-Energy) Excited States are "Thermal" (ETH)

- Quantum Mechanics at Finite Temperature? (Boring)

Can Quantum Many Body Effects Survive Far From-Equilibrium?

Three Examples..

- **Shaking**: Periodically Driven Quantum Many Body Systems

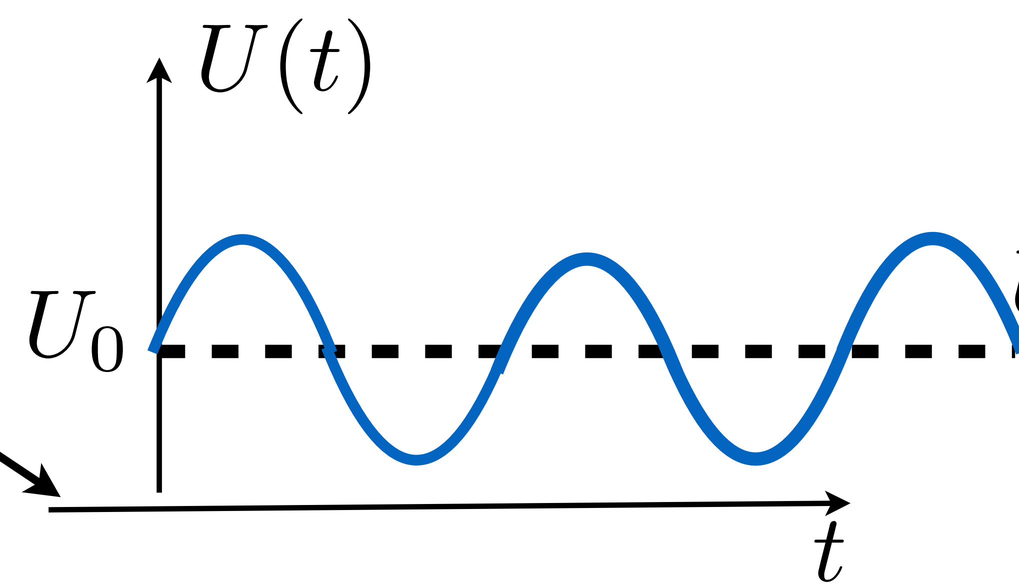
- Introduce **Randomness**: Many Body Localization

- **Take a Cold Bath**: Open-Dissipative Quantum Systems

Periodically Driven Interacting Fermions

$$H(t) = -\sum_{\langle ij \rangle} V_{ij} c^\dagger_{i\sigma} c_{j\sigma} + U(t) \sum_i (n_i - 1)^2$$

$$|\Psi(t=0)\rangle = |\Psi_0\rangle$$



$$U(t) = U_0 + \delta U \sin \Omega t$$

$$i\partial_t |\Psi(t)\rangle = H(t)|\Psi(t)\rangle \qquad O(t) = \langle \Psi(t)|O|\Psi(t)\rangle$$

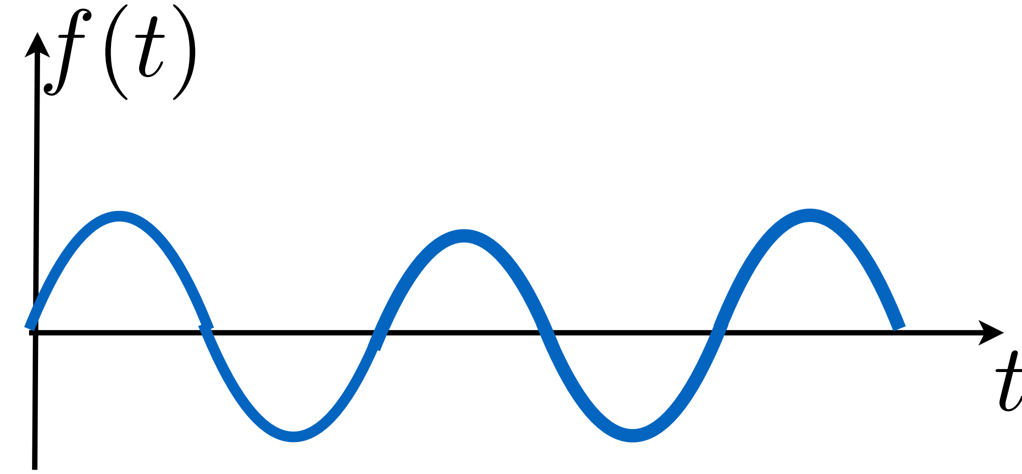
Unitary Quantum Dynamics (**closed system**, no external bath)

- Pure states remain pure! $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$ $\quad \mathrm{Tr}\rho^2(t) = \mathrm{Tr}\rho_0 = 1$
- Quantum Mechanics is linear!
- **Focus:** <u>“Local” Observables</u>

Linear Response To Periodic Drive

$$H = H_0 + f(t)A$$

$$\langle A\rangle_t - \langle A\rangle_{eq} = \int dt'\chi(t-t')f(t')$$



- Periodic Perturbation with sudden switch $f(t) = \theta(t) f_0 \sin\Omega t$

$$\langle A\rangle_t - \langle A\rangle_{eq} \xrightarrow{t\to\infty} f_0\left(\chi^{'}(\Omega)\sin\Omega t - \chi^{''}(\Omega)\cos\Omega t\right)$$

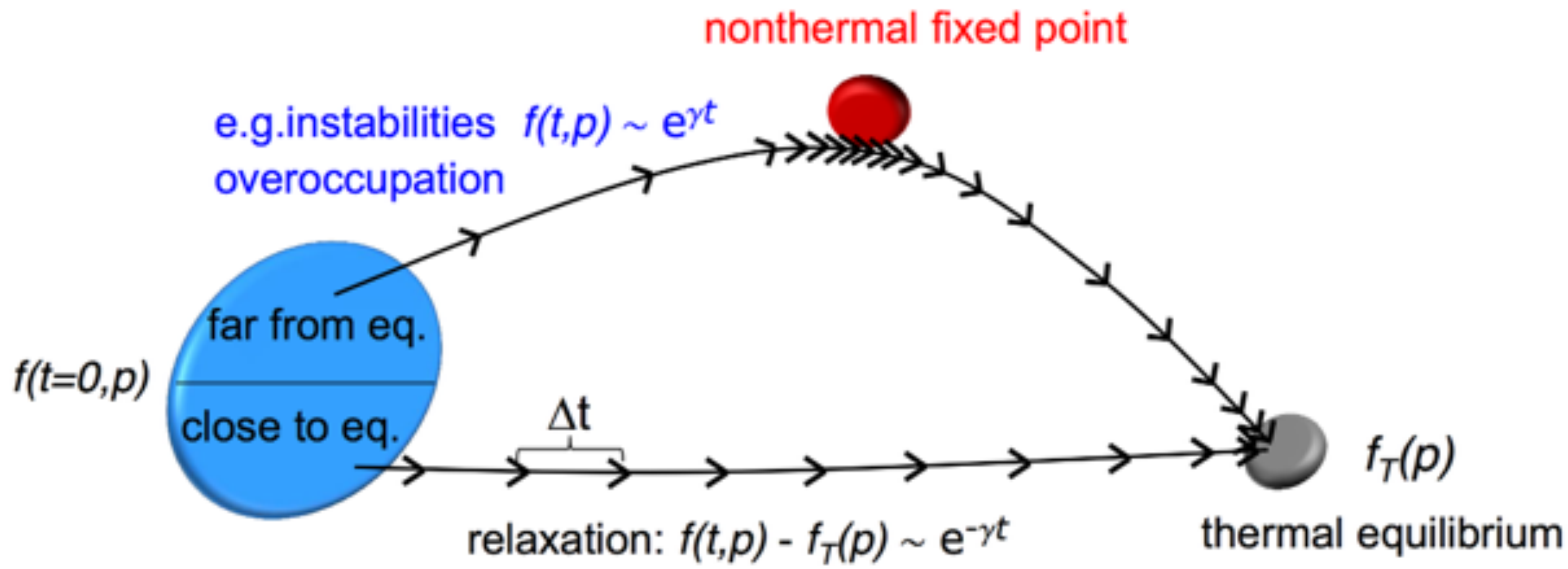
- Rate of Energy Absorption.. $\dot{E} = \langle \frac{dH}{dt} \rangle = \dot{f}\langle A\rangle$

- ..Over a Period

$$E(nT) - E(0) = -\chi^{''}(\Omega)nT > 0$$

☑ Eventually an ergodic/non-integrable system will thermalize to infinite temperature…unless…

(Floquet) Pre-Thermalization



- Originally introduced in the context of high-energy physics (heavy ions collisions) and quantum quenches/thermalization of isolated systems
  - M.Schiro, M. Fabrizio (2010-2012)

Periodic Driving:

- High-frequency driving regime $\Omega \to \infty$
  - Bukov, D'Alessio, Polkovnikov Adv Phys (2015)
  - Abanin, De Roeck, Wei Ho, Huveneers (2015-2017)
- Heating is exponentially slow, energy is quasi conserved for $t \ll \tau_* \sim e^{C\Omega}$
- Floquet Hamiltonian local (non-ergodic) controlling dynamics up to τ_*
- **Here:** A New Mechanism for Floquet Prethermalization: Strong Correlations!

Nonequilibrium Dynamical Mean Field Theory

A.Georges et al, RMP (1996)
Aoki et al, RMP (2014)

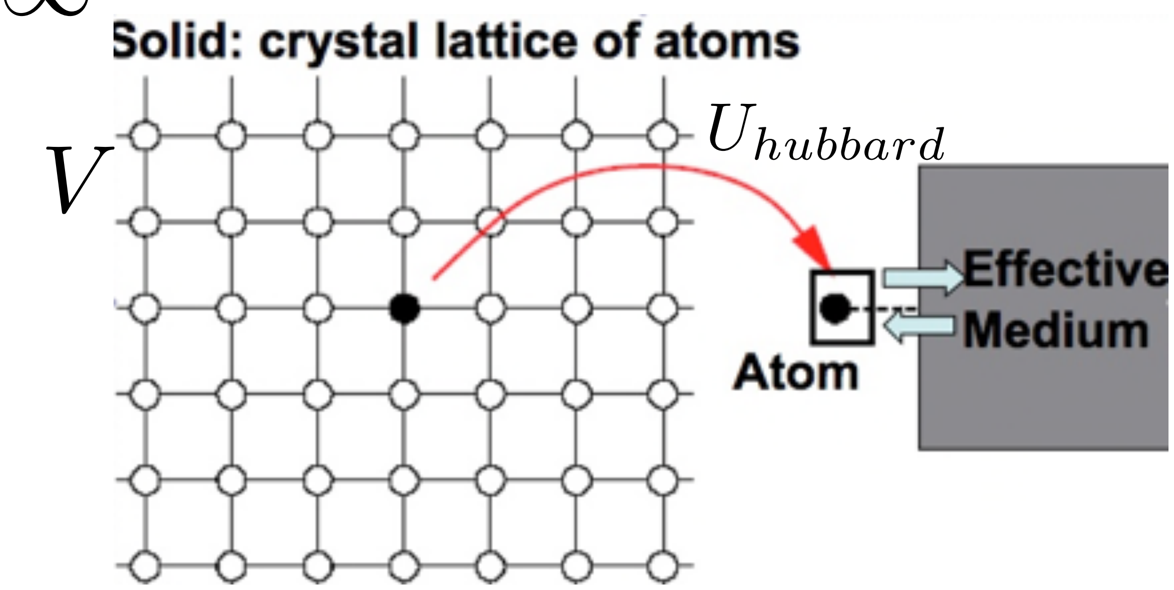
$$H(t) = \sum_i H_i^{loc}(t) + V \sum_{ij} c_i^\dagger c_j$$

$z =$ Coordination Number

- Diagrammatics become "local" in $z \to \infty$

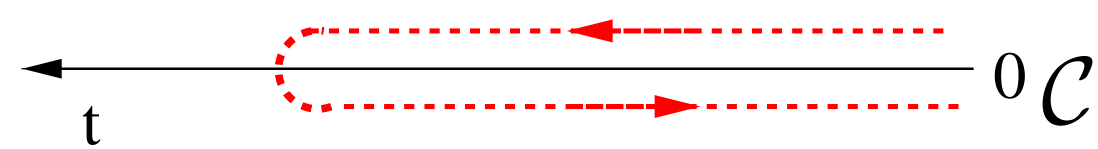
$$V \to V/\sqrt{z}$$

$$\Sigma_{ij}(\omega) \to \delta_{ij}\Sigma(\omega)$$



- Exact Mapping on a Quantum Impurity Model in a self-consistent bath

- Effective (Keldysh) Action



$$i\mathcal{S}_{eff} = i\mathcal{S}_{loc} + \int_C dt dt' \sum_\sigma c_\sigma(t)\Delta_\sigma(t,t')c_\sigma^\dagger(t')$$

$$\Delta_\sigma(t,t') = F[G_\sigma(t,t')] \qquad G_\sigma(t,t') = -i\langle T_C c_\sigma(t) c_\sigma^\dagger(t')\rangle_{\mathcal{S}_{eff}}$$

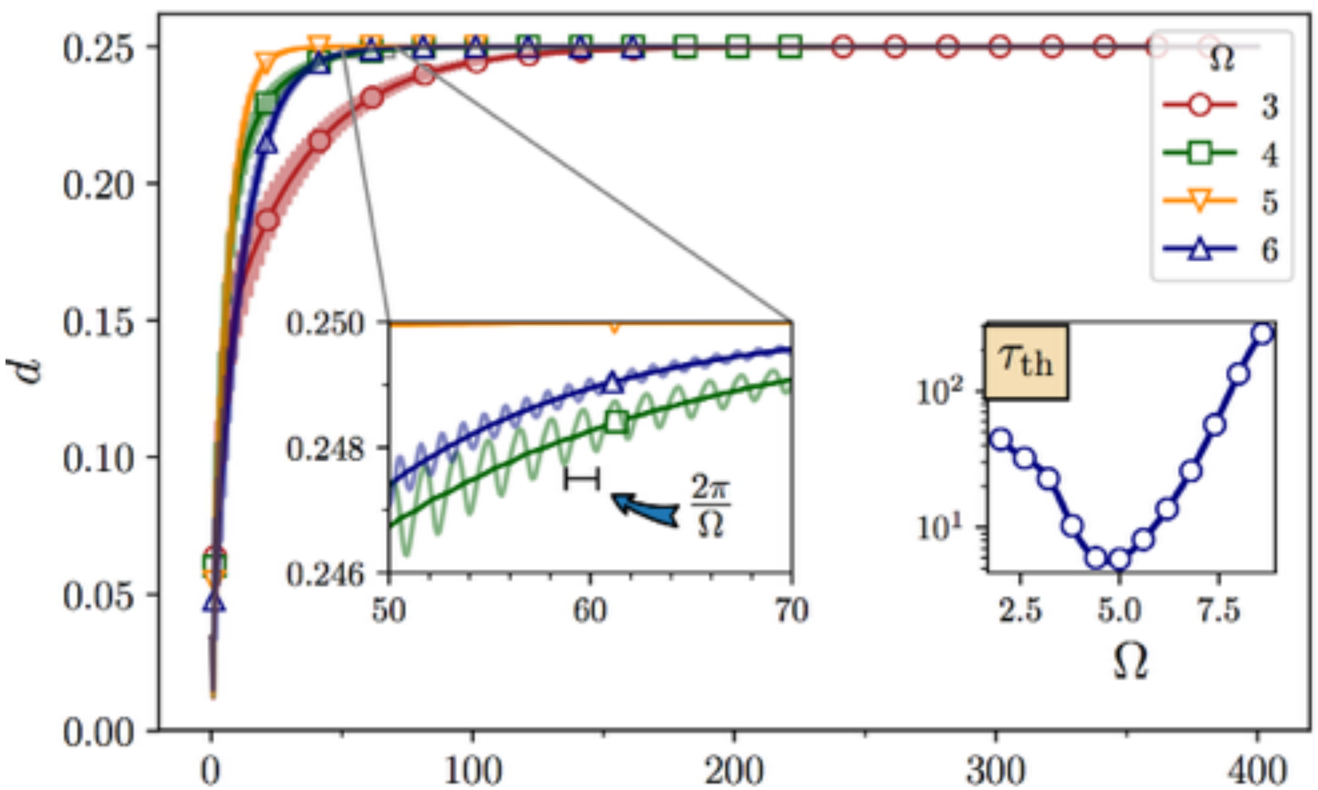
- ... but the bath is out of equilibrium!!

Periodically Driven Fermi Hubbard Model

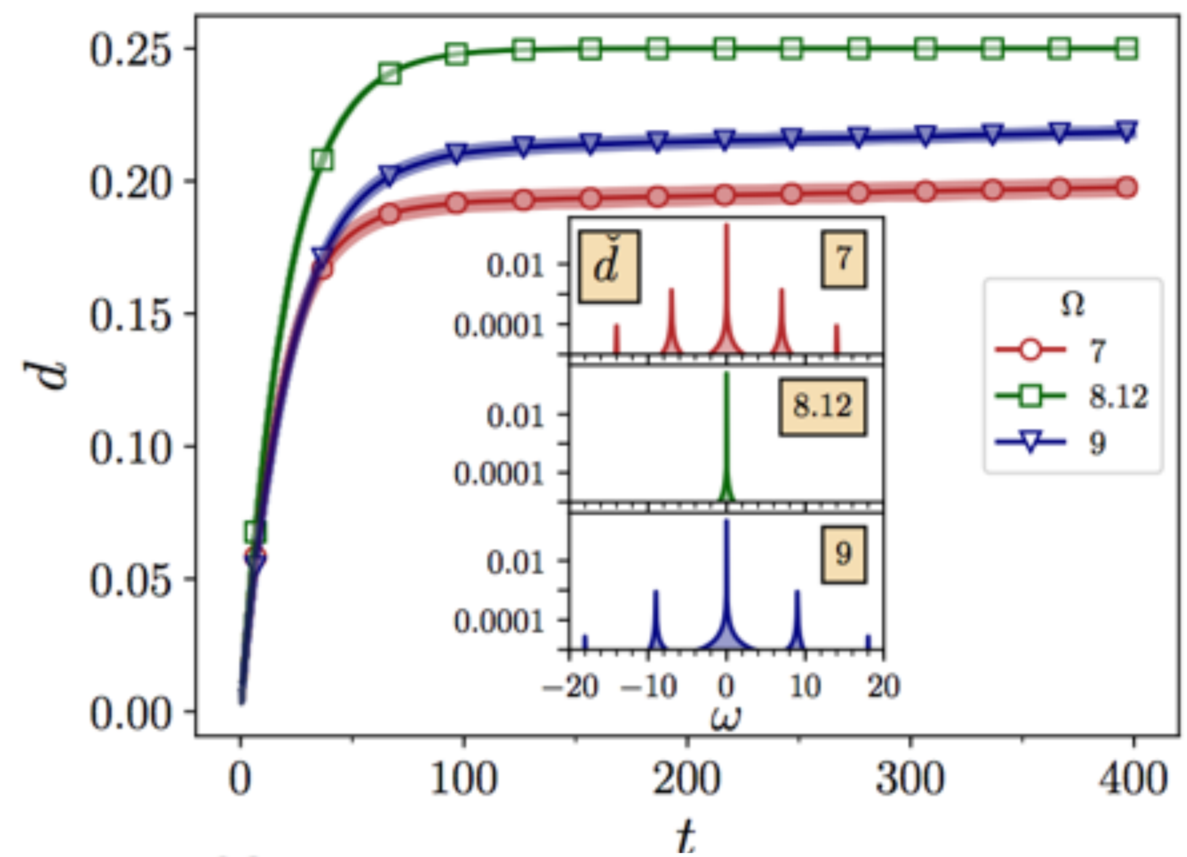
- F. Peronaci, M. Schiro', O. Parcollet, PRL (2018)

- Dynamics of Doubly Occupied Sites $d(t) = \langle n_{i\uparrow}(t) n_{i\downarrow}(t) \rangle$

$U_0 \simeq V$

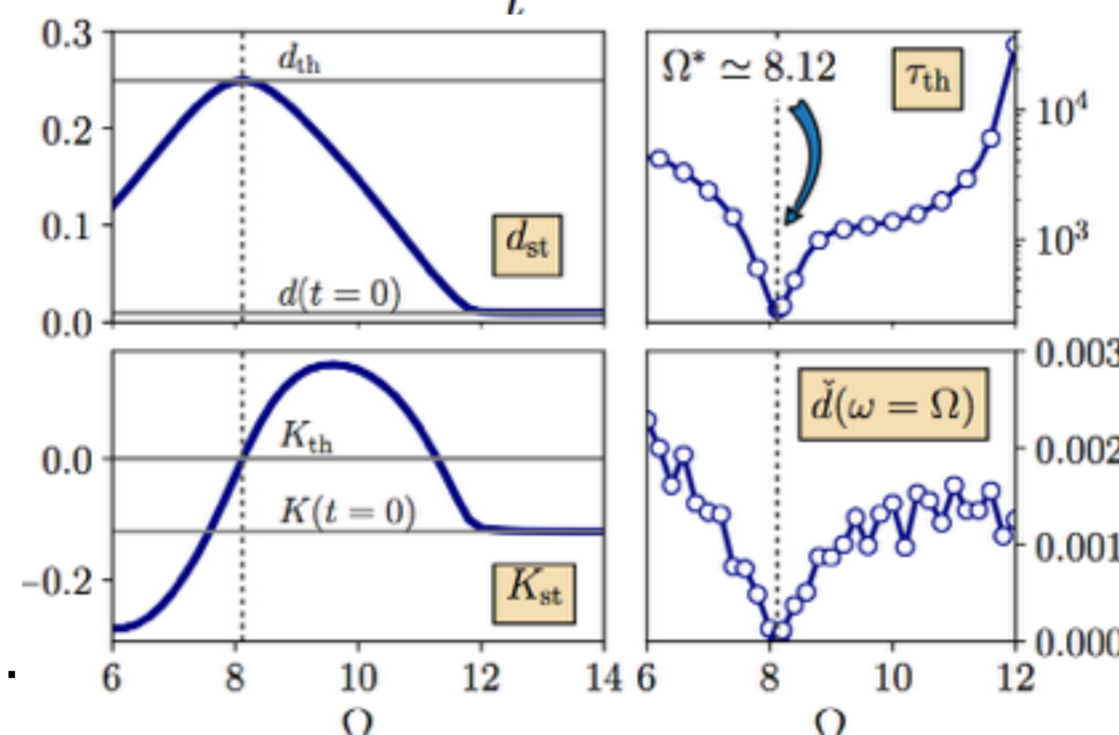


$U_0 \gg V$



- Thermalization strongly depends on interaction and drive frequency/amplitude

- Sharp "Dynamical" Transitions between different Thermalization Regimes

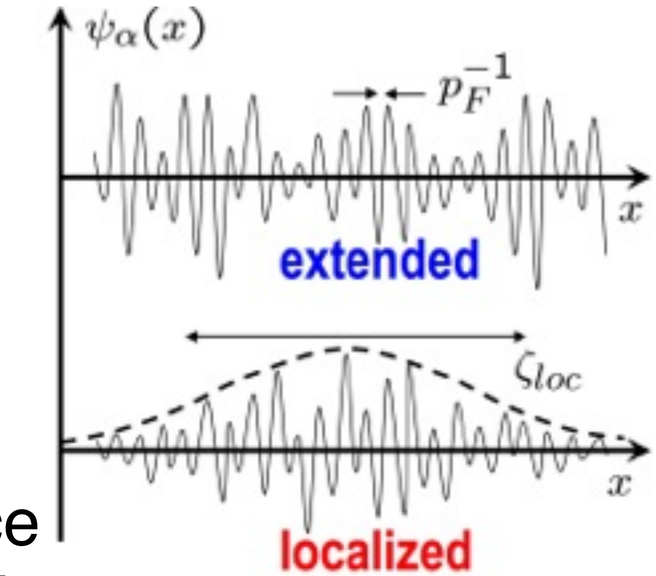


Future Directions:

Couple to a Bath, Include Phonons, Broken Symmetries,..

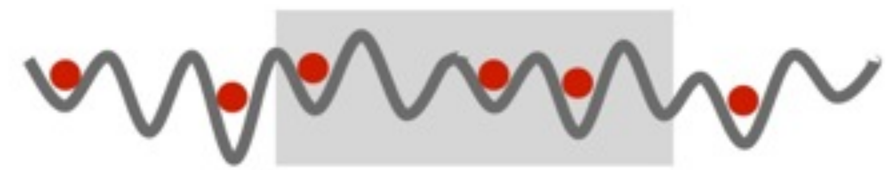
Can Quantum Thermalization **Fully** Break Down?

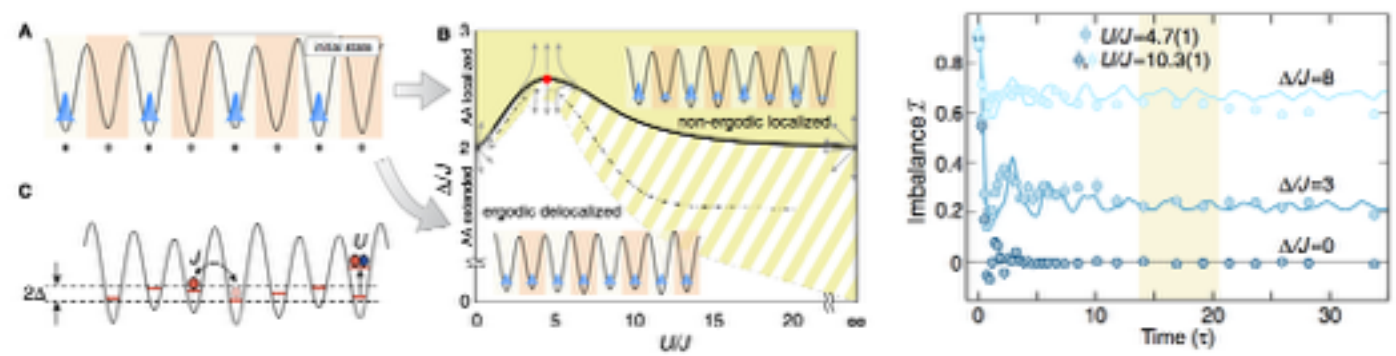
Quenched Disorder and Anderson Localization



- Localized System: No Diffusion, No Transport
- Ergodicity Breaking due to Quantum Interference
- Robust to Interactions and Finite "Temperature": Many Body Localization (MBL)

Basko, Aleiner, Altshuler 2006





Experiments: Cold Atoms (Bloch group), Ions (Monroe group)

> **MBL** is a new quantum phase of matter with unusual properties due to **breakdown of thermalization**

(Few) Consequences of MBL..

- Broken Symmetries in Low Dimensions, Low Eigenstates Entanglement at High Energy
- Persistent Memory of Initial Condition, No Heating Under Perturbations,..

Phenomenology of MBL Systems

- Huse, Oganesyan (2013) & Serbyn, Papic, Abanin (2013)

$$H_{XXZ} = J\sum_i \left(c_i^\dagger c_i + hc\right) + \Delta \sum_i n_i n_{i+1} + \sum_i h_i n_i$$

- Local Integral of Motions $\quad [H_{eff}, n_i] = 0 = [n_i, n_j]$

$$H_{eff}[\{n\}] = \sum_i h_i n_i + \sum_{ij} J_{ij} n_i n_j + \sum_{ijk} J_{ijk} n_i n_j n_k + \ldots$$

$$J_{ij} \sim \exp\left(|i-j|/\xi\right) \qquad J_{ijk} \sim \exp\left(-max(r_{ij}, r_{ik}, r_{jk})/\xi\right)$$

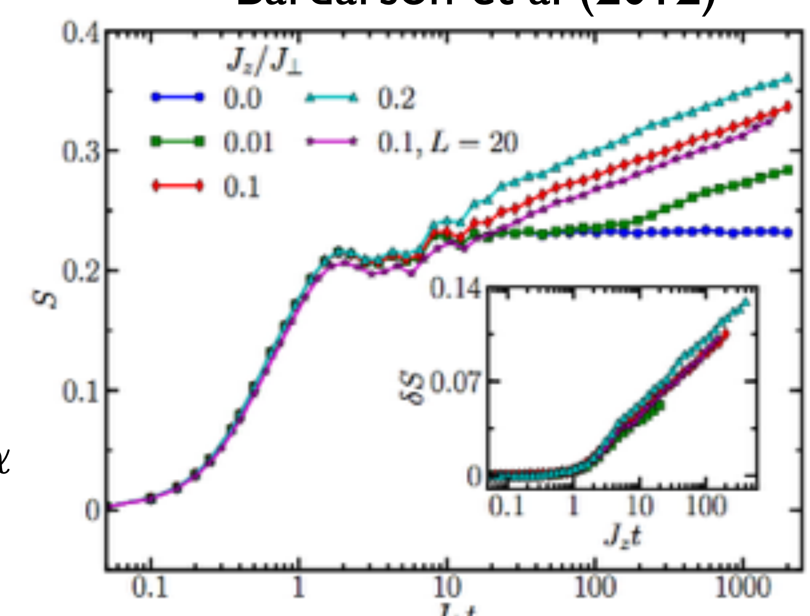
- (Few) Dynamical Consequences of MBL

Log-growth of entanglement $\quad S_{ent}(t) \sim \xi \log(Jt)$

Power laws in quench-dynamics $\quad \langle O(t)\rangle \sim O_{ss} + \delta O/t^\alpha$

Serbyn et al (2013)

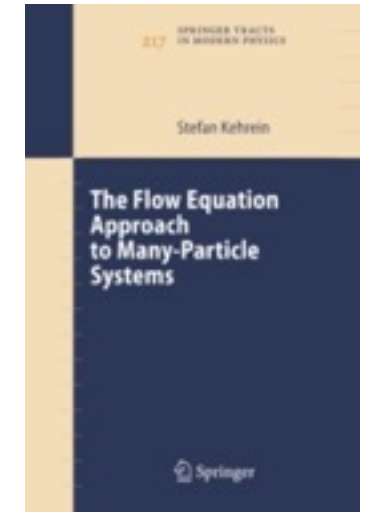
Bardarson et al (2012)



Flow Equation Method

(aka: Continuous Unitary Transformation, Double Bracket Flow, …)

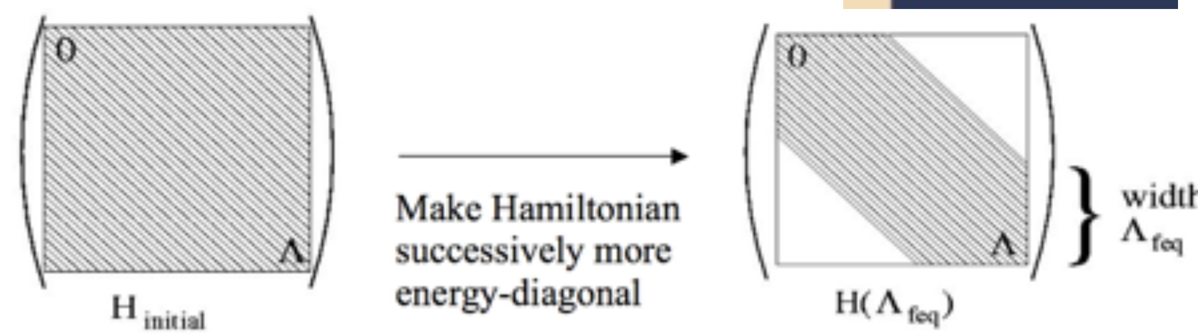
Glazek&Wilson, Wegner ('93-'94)
S. Kehrein ('96-..)
C.Monthus (2016)



- Iterative "Diagonalization" of Hamiltonian

$$l = 0 \longrightarrow l = \infty$$

$$H(0) = H \qquad H(\infty) = H_*$$



- Anti-hermitian generator $\eta^\dagger(l) = -\eta(l)$ $\qquad \partial_l H(l) = [\eta(l), H(l)]$

- "Canonical" (Wegner) Choice

$$H(l) \equiv H_0(l) + V(l)$$

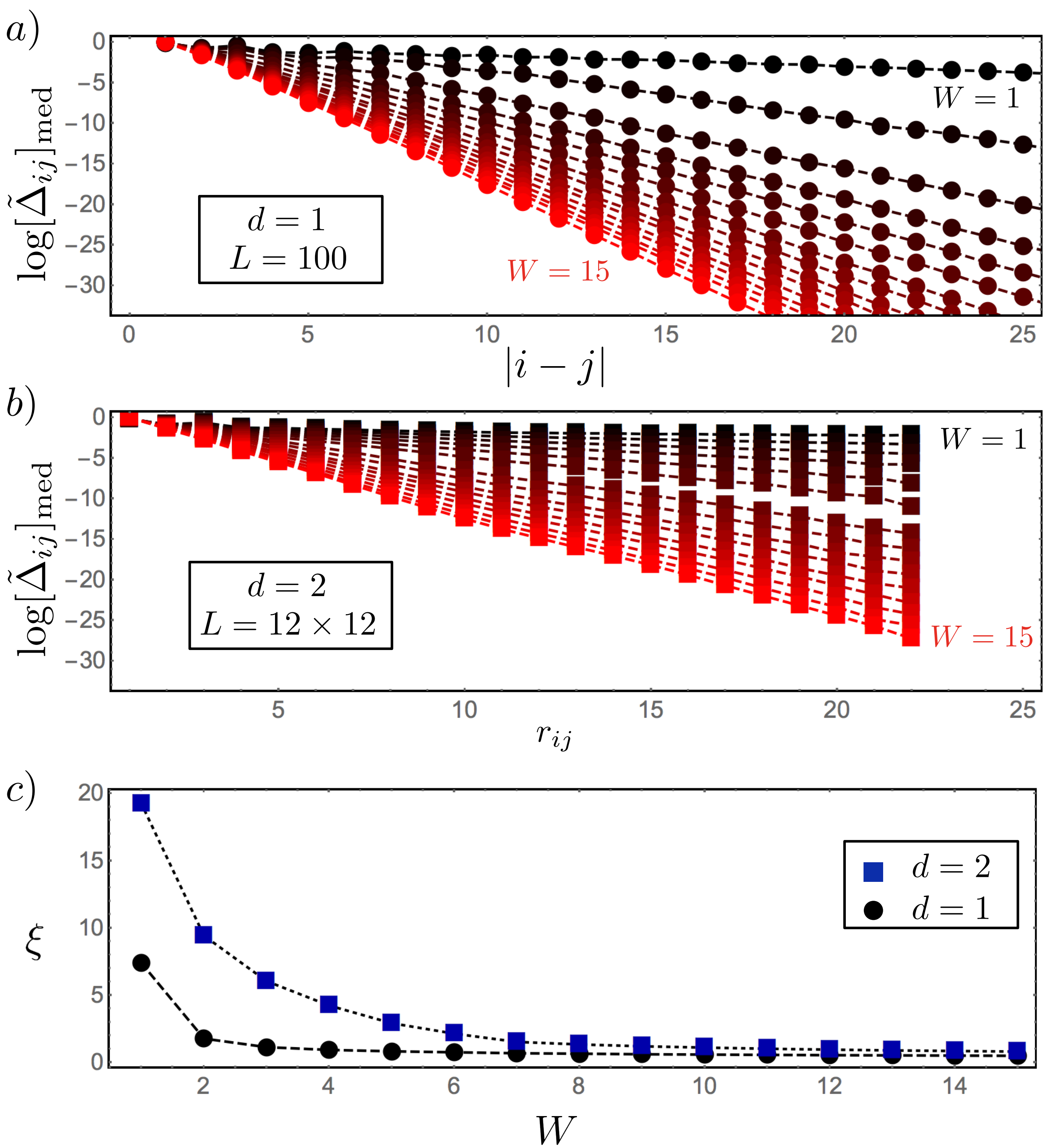
$$\eta(l) = [H_0(l), V(l)]$$

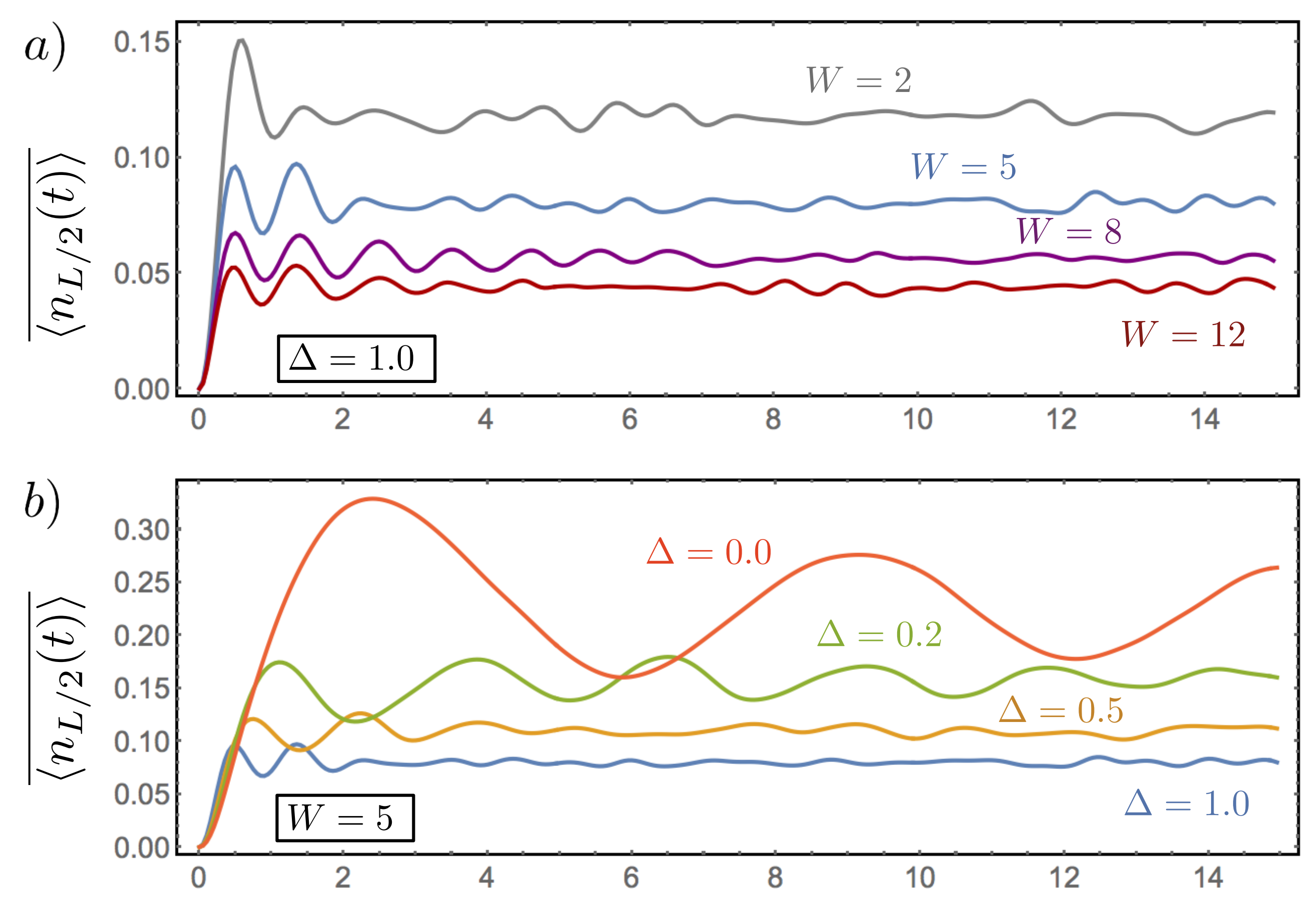
- Flow Equation for the disordered XXZ Chain $\qquad H(l) = H_0(l) + V(l)$

$$H_0(l) = \sum_i h_i(l) n_i + \sum_{ij} \Delta_{ij}(l) n_i n_j \qquad V(l) = \sum_{ij} J_{ij}(l) \left(c_i^\dagger c_j + hc \right)$$

Dynamics of MBL Systems with Flow Equation

- S.Thomson, M. Schiro', PRB(R) 2017

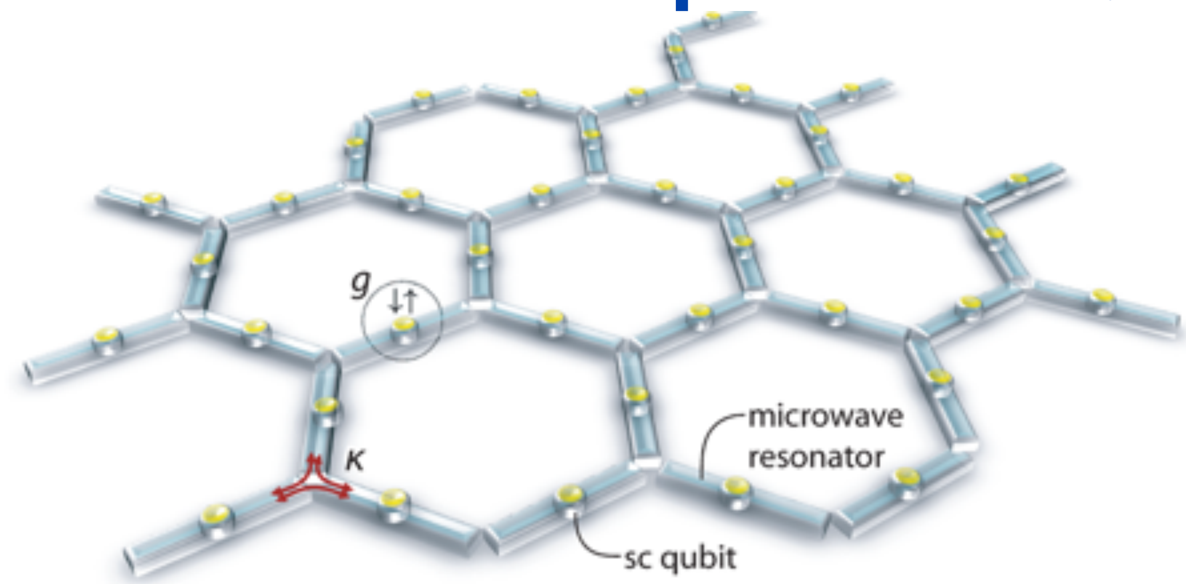


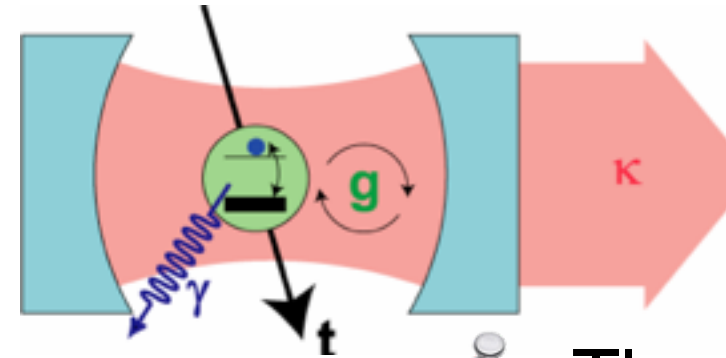


A microscopic approach to MBL phases, their local integral of motion and dynamics

Future Directions: Drive/Dissipation? Mean Field Theory for MBL?

Driven-Dissipative Quantum Many Body Systems





- The Rabi Model

$$H_{Rabi} = \omega_r \, a^\dagger a + \omega_q \sigma^+ \sigma^- + g \left(a^\dagger + a\right) \left(\sigma^+ + \sigma^-\right)$$

- M. Schiro et al, PRL (2012, 2016)

- Open Quantum System: Master Equation

$$\partial_t \rho = -i[H, \rho] + \mathcal{D}[\rho]$$

- Competition between **coherent**(unitary) and **dissipative** evolution

  - Drive/Dissipation described by "Jump" Operators

$$\mathcal{D}[\rho] = \sum_\alpha L_\alpha \rho L_\alpha^\dagger - \frac{1}{2} \left\{ L_\alpha^\dagger L_\alpha , \rho \right\}$$

- **Questions:**

Non-Equilibrium Stationary States?
Dissipative Quantum Phase Transitions? Limit-Cycles?Bistability?

Finite-Frequency Criticality of Driven Lattice Bosons

$$H = \sum_{\langle ij \rangle} J_{ij} \left(a_i^\dagger a_j + hc \right) + \sum_i H_{loc}^i + H_{drive}(t)$$

- Local Hamiltonian:

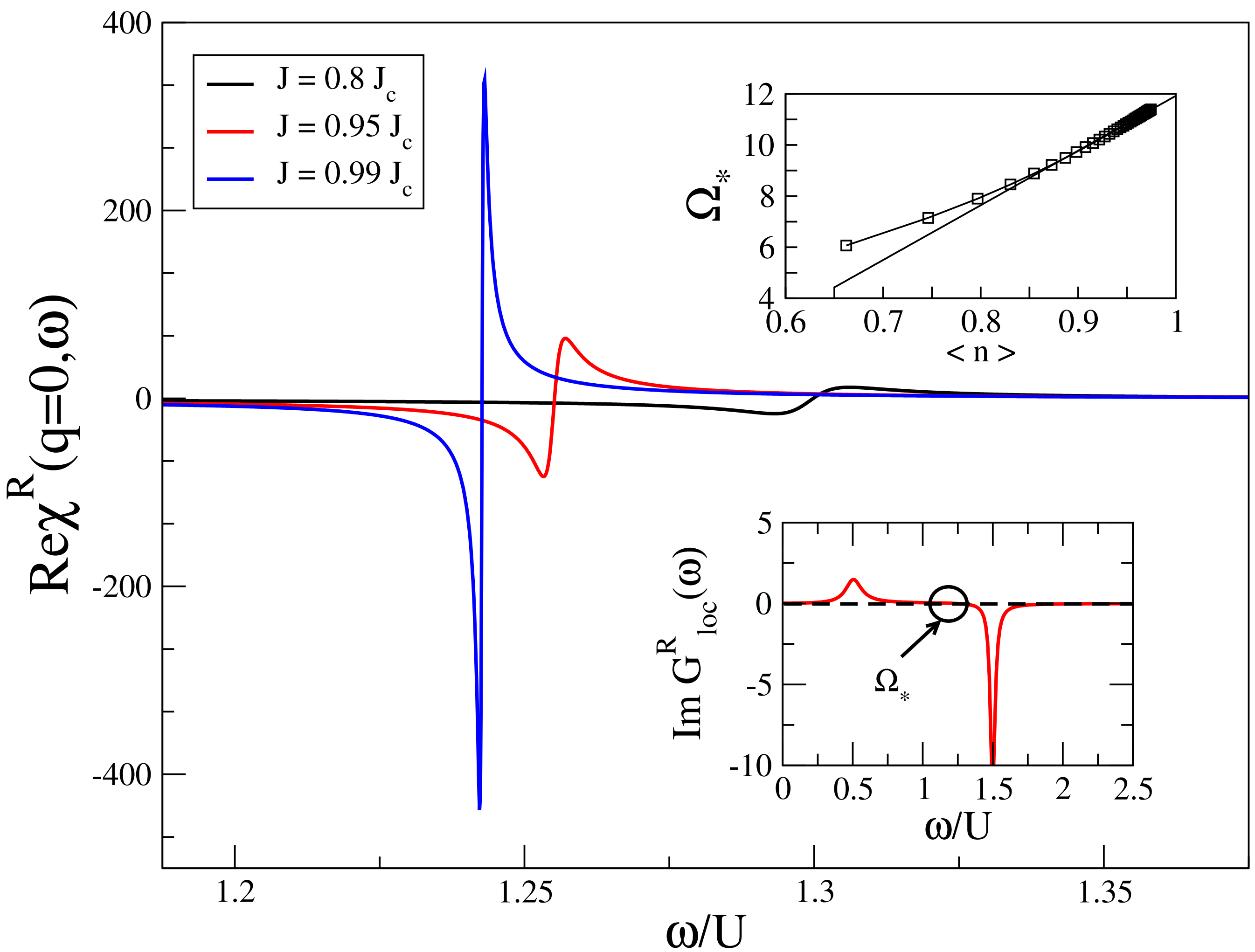
$$H_{loc} = \omega_0 a_i^\dagger a_i + \frac{U}{2} n_i^2$$

- Photon "Losses"

$$\mathcal{D}[\rho] = \kappa \sum_i a_i \rho a_i^\dagger - \frac{1}{2} \left\{ a_i^\dagger a_i, \rho \right\}$$

- Incoherent Drive (inject photons without phase)

- RPA Susceptibility of order parameter



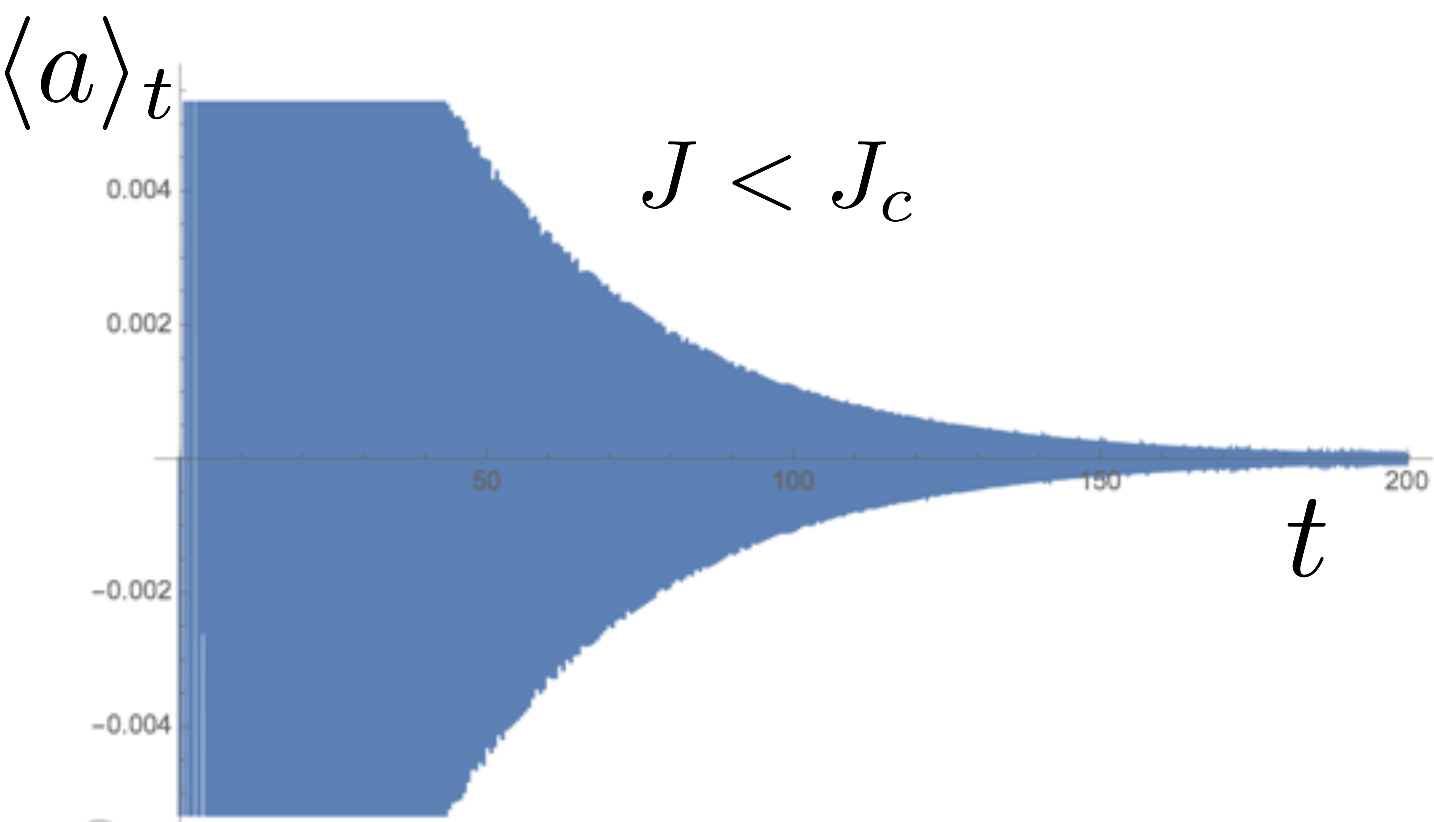
- Divergent Response at **finite frequency**!

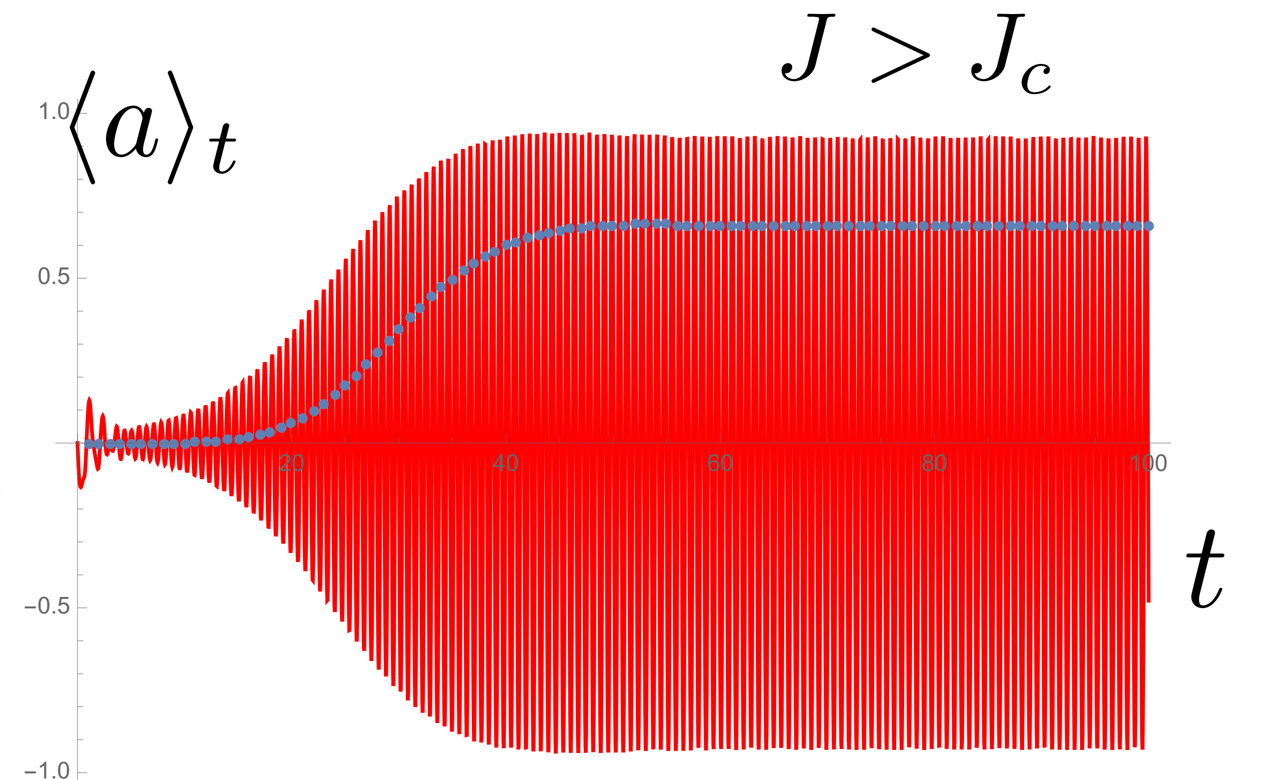
$$\chi_\psi^R(0,\omega) \sim \frac{1}{|\omega - \Omega^*|^\alpha}$$

O. Scarlatella, R. Fazio, M. Schiro', arXiv:1805.02770

Dynamical Consequences of Finite-Frequency Criticality

- O. Scarlatella, R. Fazio, M. Schiro', arXiv:1805.02770





- Relaxation toward stationary state!

- Undamped Oscillations!
- Quantum Limit Cycle aka "Time Crystal"

- **Finite Frequency Criticality**: Paradigm to Study Time-Domain Instabilities in Quantum Systems Far From Equilibrium

Future Directions:

Application to Floquet Time Crystals, Period Doubling and Transition to Chaos (Quantum Dynamical Instabilities), (NP)RG out of equilibrium,..

Bistability: A "zero-dimensional" Example

W. Casteels, R. Fazio, C. Ciuti PRA (2016)

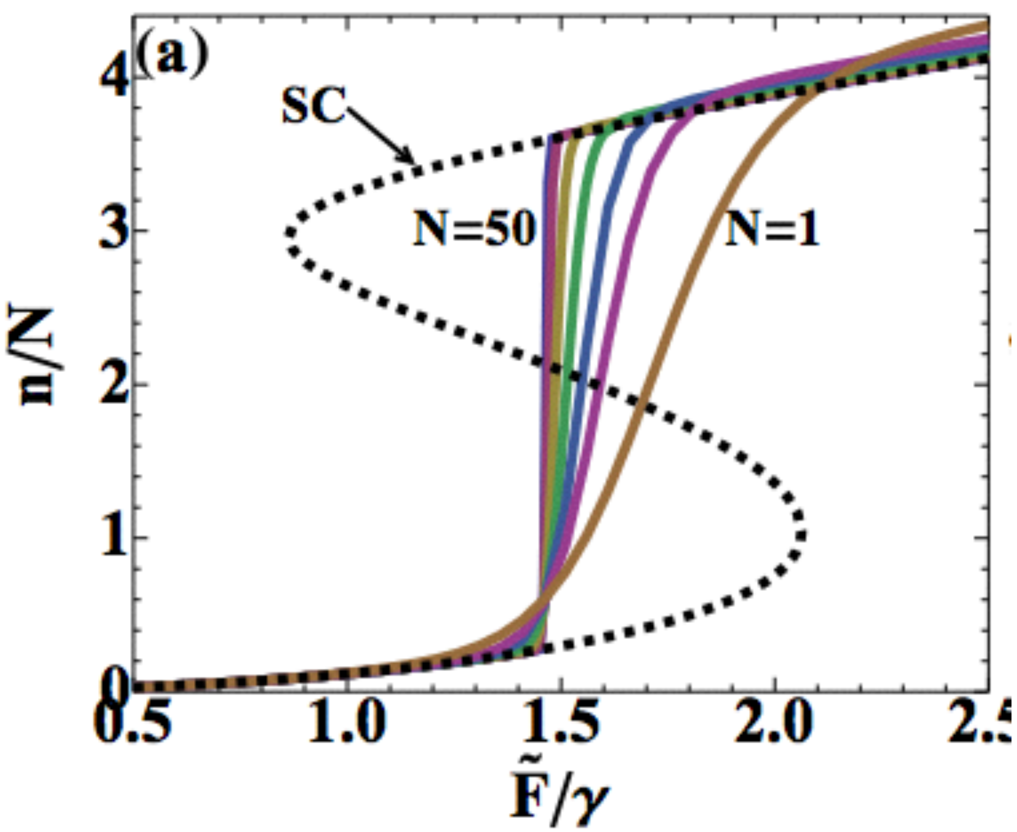
- Driven Cavity with Kerr Non-Linearity

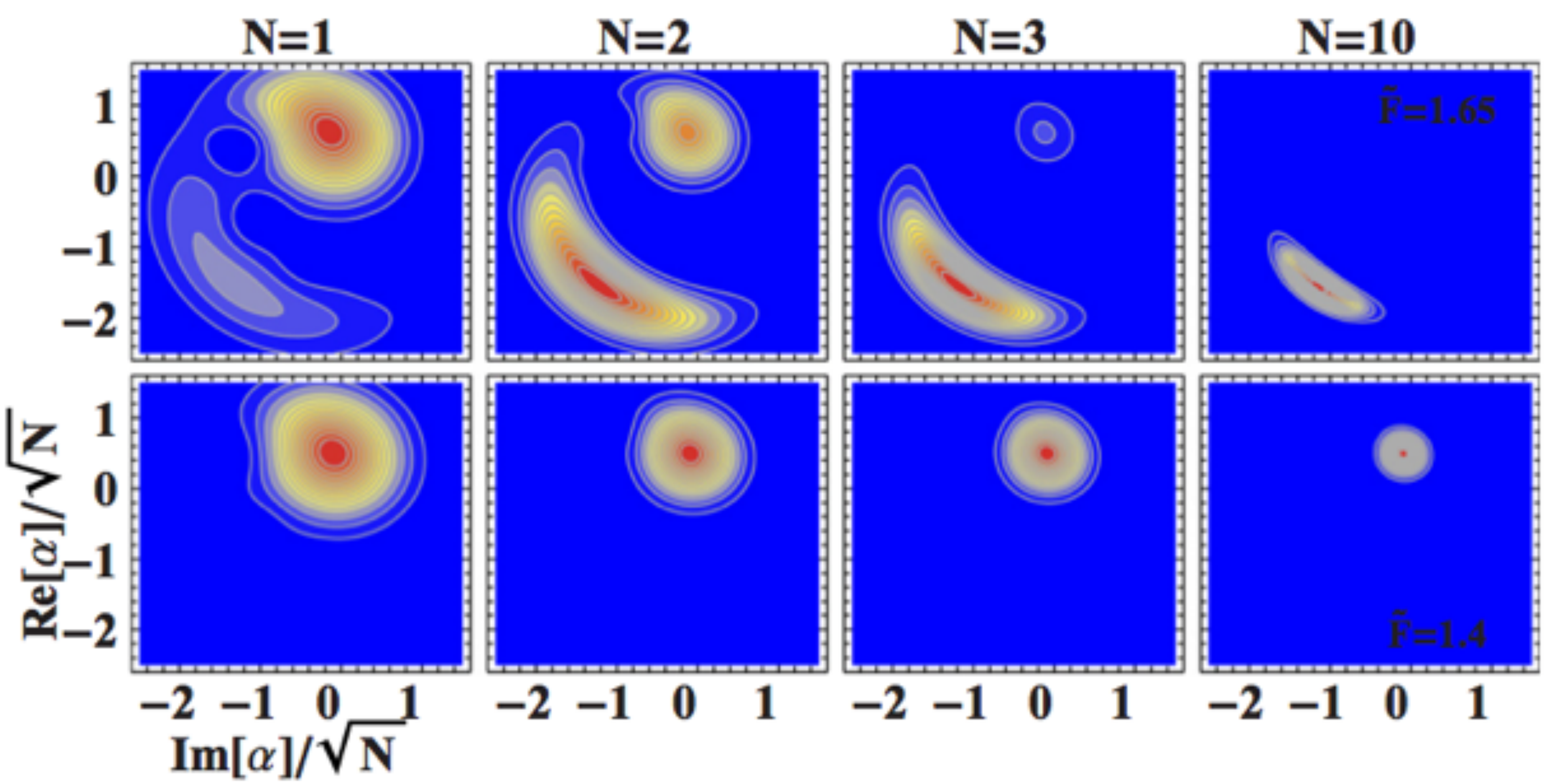
$$H = \omega_c a^\dagger a + U\left(a^\dagger a\right)^2 + F\left(a^{i\omega_p t} + hc\right)$$

$$i\partial_t \rho = -i[H,\rho] + \mathcal{D}[\rho] \qquad \mathcal{D}[\rho] = \frac{\kappa}{2}\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right)$$

- Semiclassics: $$i\partial_t \tilde{\alpha} = \left(-\Delta - \kappa/2 + \tilde{U}|\alpha|^2\right)\tilde{\alpha} + \tilde{F}$$

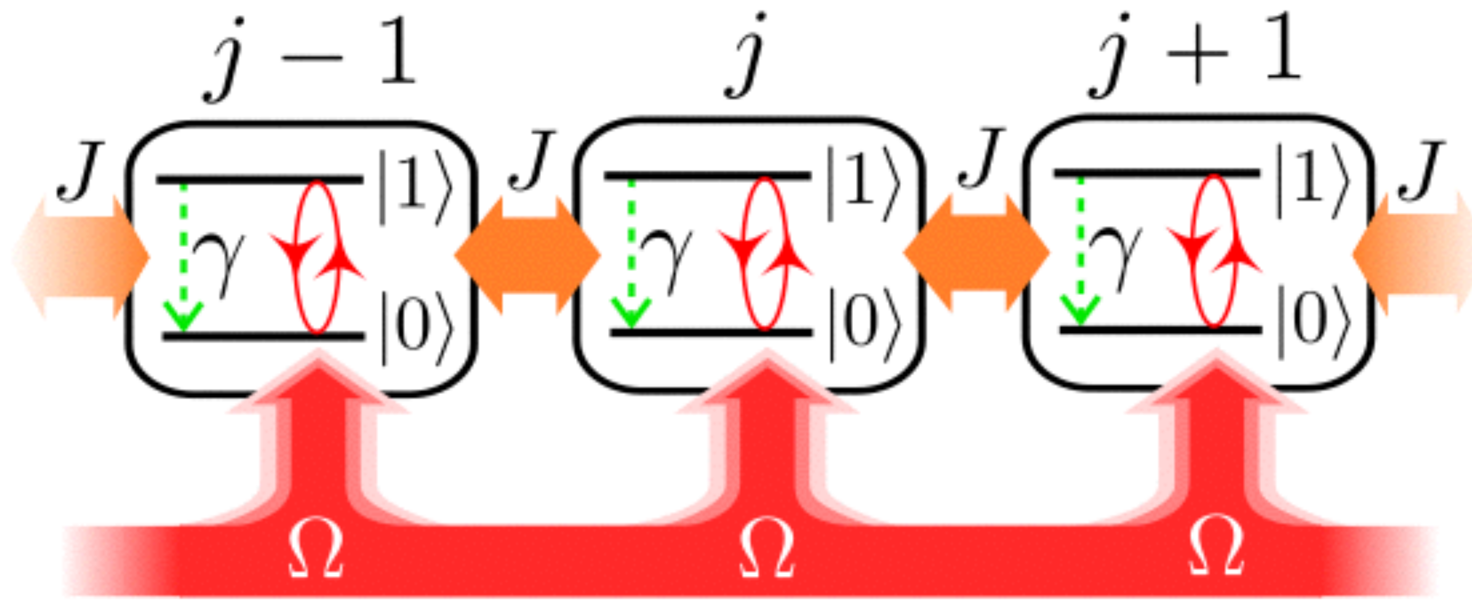
$$\tilde{\alpha} = \langle a\rangle / N$$





Driven-Dissipative Quantum Spin Model

- H. Landa, M. Schiro', G.Misguisch (in progress)



$$i\partial_t \rho = -i[H,\rho] + \mathcal{D}[\rho]$$

$$H = -\sum_{\langle RR'\rangle} J\left(\sigma^+_R \sigma^-_{R'} + hc\right) + \frac{\Delta}{2}\sum_R \sigma^z_R + \Omega \sum_R \sigma^x_R$$

$$\mathcal{D}[\rho] = \frac{\gamma}{2}\sum_R \left(2\sigma^-_R \rho \sigma^+_R - \sigma^+_R \sigma^-_R \rho - \rho \sigma^+_R \sigma^-_R\right)$$

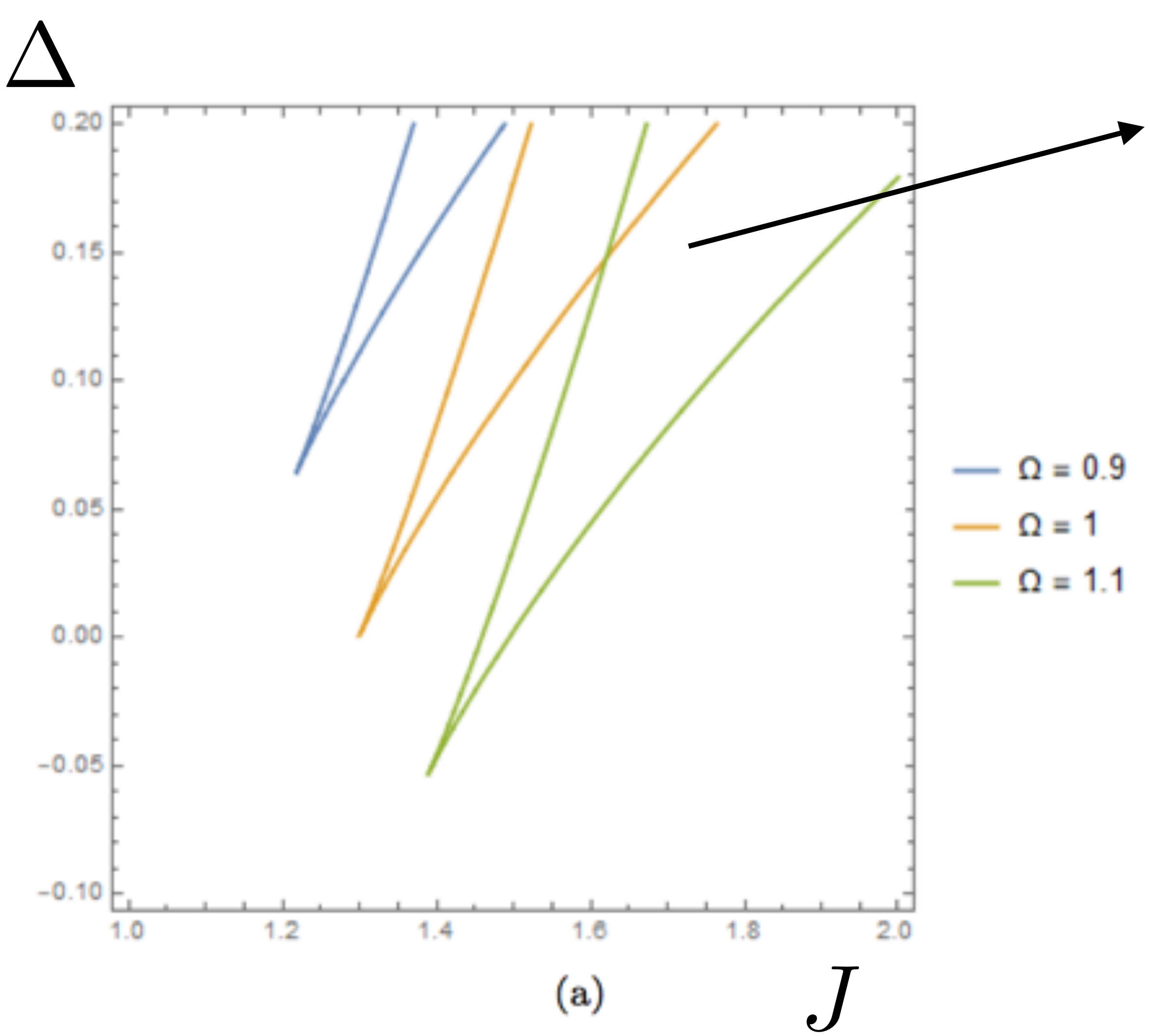
- Stationary State profile of magnetization vs drive/detuning field? Many Body Bistability?

Mean Field Bistability and Beyond

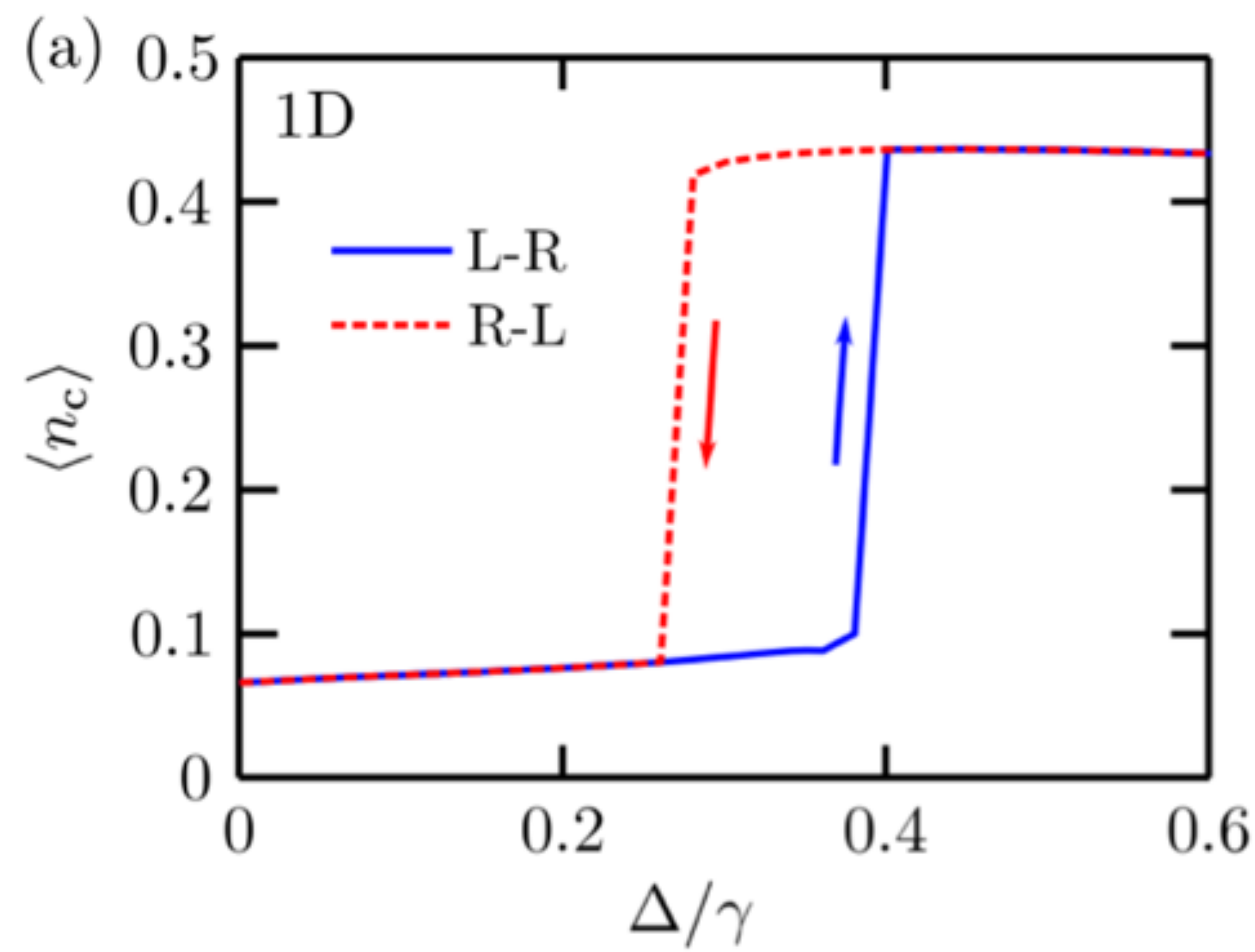
$$\dot{\mu}_x = -J\mu_y\mu_z - \Delta\mu_y - \Gamma\mu_x/2$$

$$\dot{\mu}_y = J\mu_z\mu_x - 2\Omega\mu_z + \Delta\mu_x - \Gamma\mu_y/2$$

$$\dot{\mu}_z = 2\Omega\mu_y - \Gamma\left(1+\mu_z\right)$$



Bistable Region



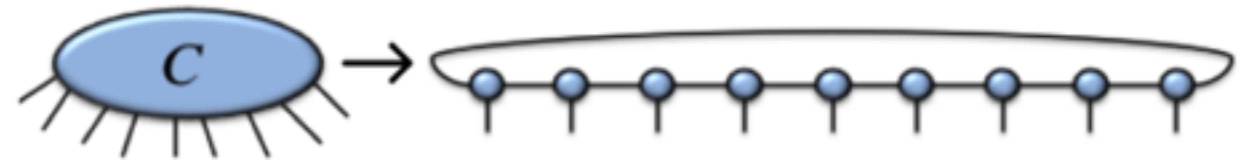
- What is the role of quantum fluctuations beyond mean field?

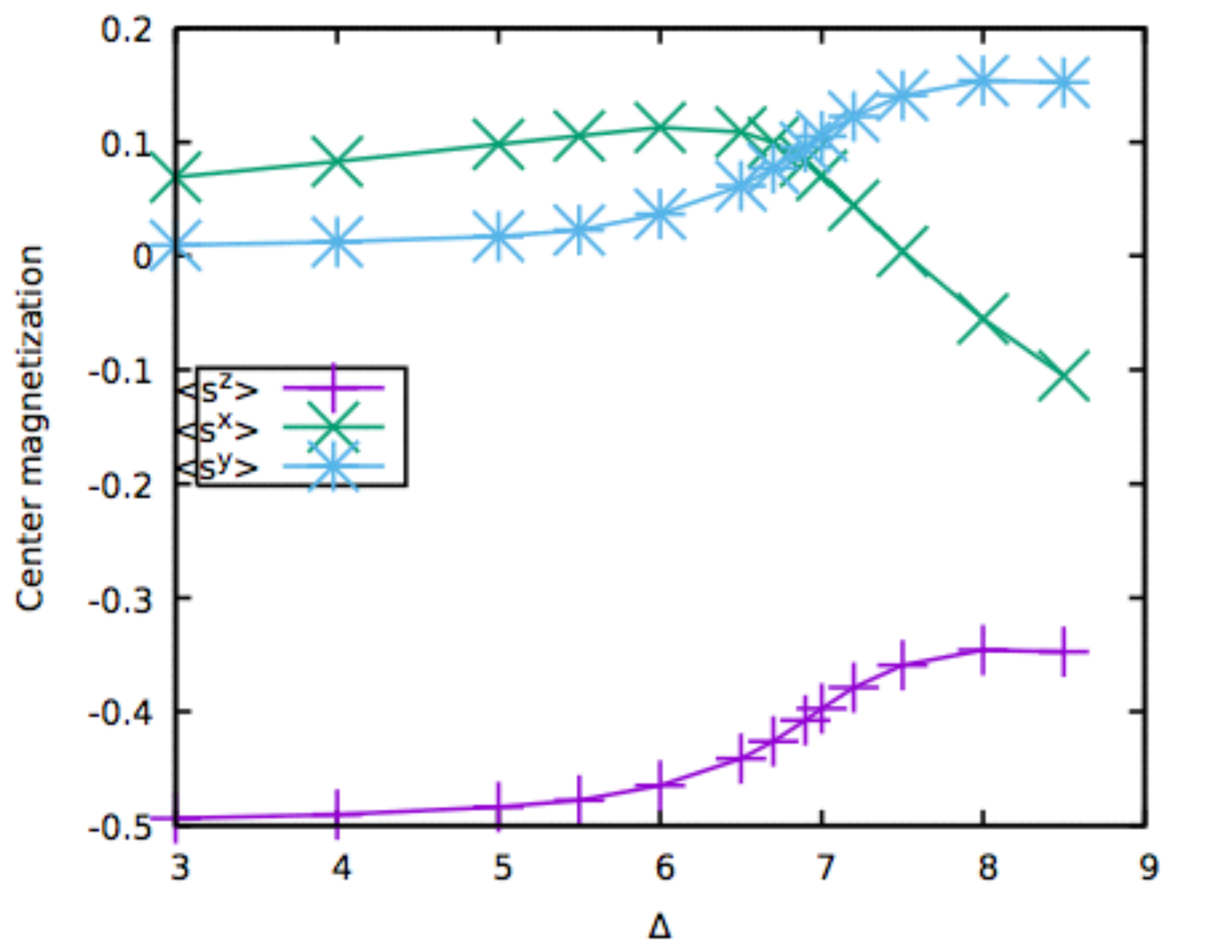
• H. Landa, M. Schiro', G.Misguisch (in progress)

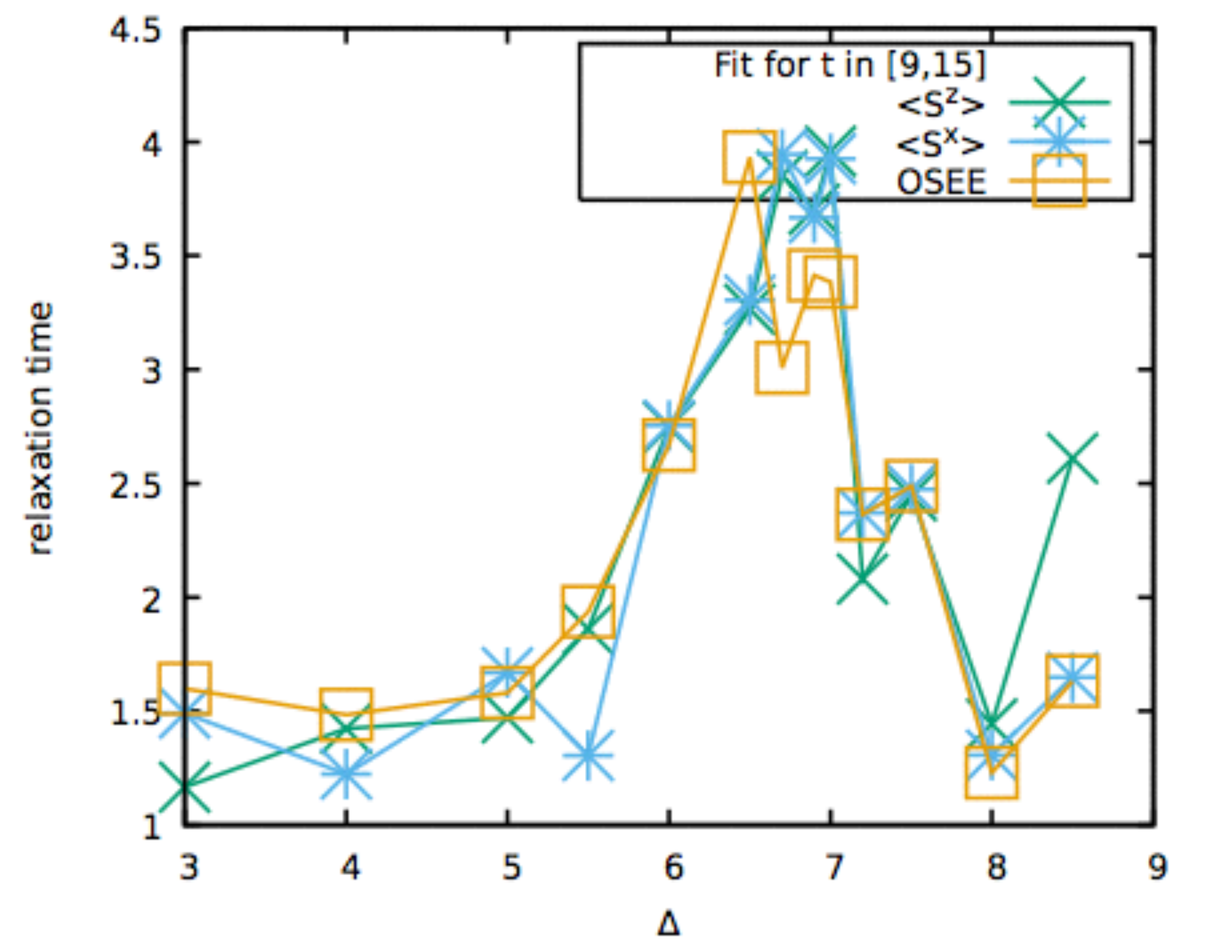
d=1: MPO Simulation for Open Quantum Systems

$$\dot{\rho} = \mathcal{L}\rho$$

$$\rho = \sum_{i_1,\dots,i_N} c_{i_1,\dots,i_N} \otimes_{j=1}^{j=N} \sigma_j^{i_j}$$







- MPO results suggest bistability is washed out in d=1

- H. Landa, M. Schiro', G.Misguisch (in progress)

Conclusions&Perspectives

- [x] Why Quantum Non-Equilibrium Physics?
- [x] Non-Equilibrium Excited States are "Hotter" but not necessarily boring

Future Perspectives:

- [ ] Merge the three directions: disorder, interactions, non-equilibrium/dissipation effects

Thanks!