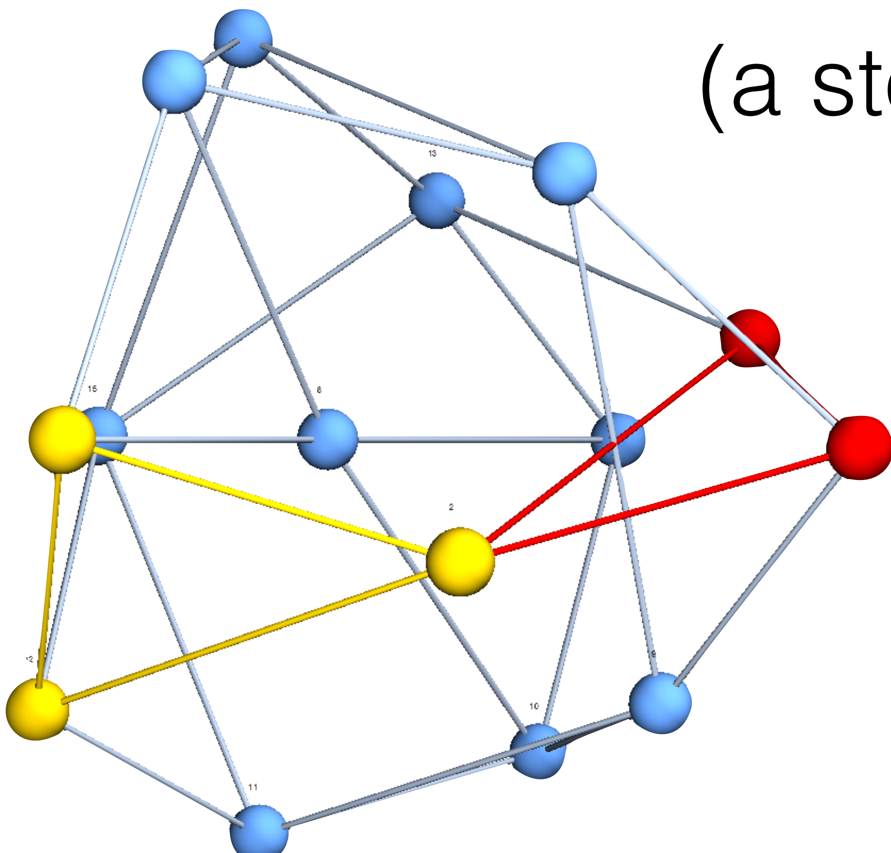
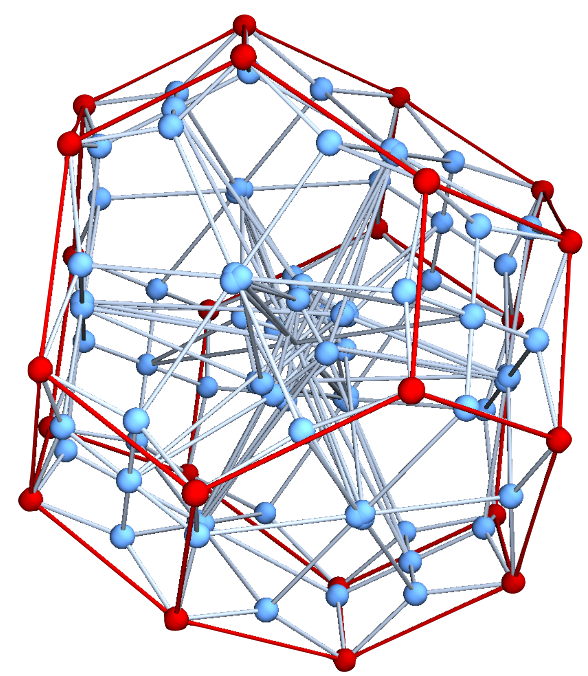
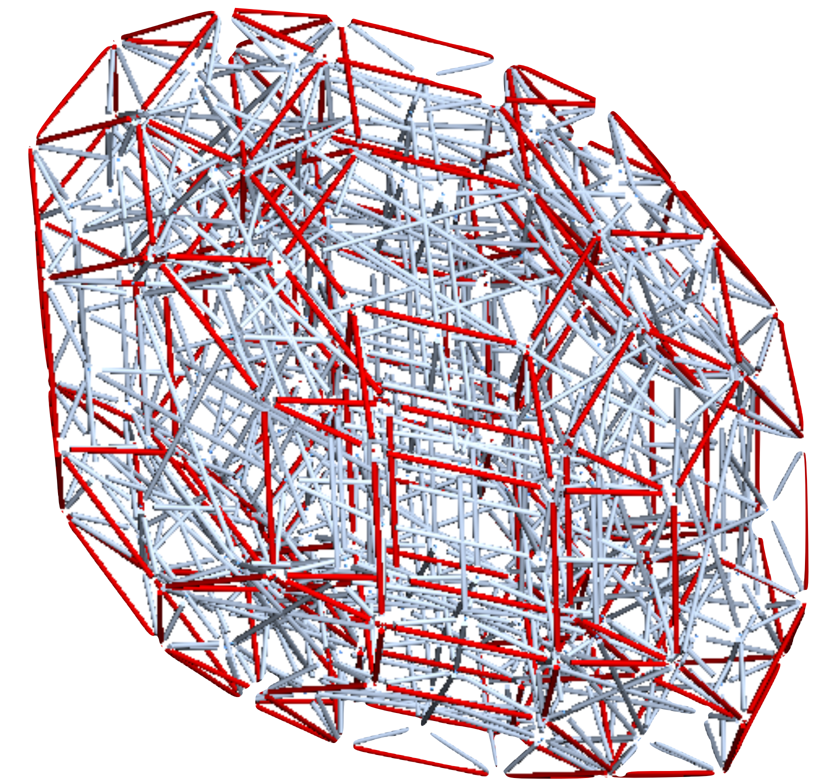
Playful Constructions in Double Copy Predictions

(a story about stories)







Northwestern University

IPhT
cea
saclay

IPhT Colloquium

16 Oct 2018

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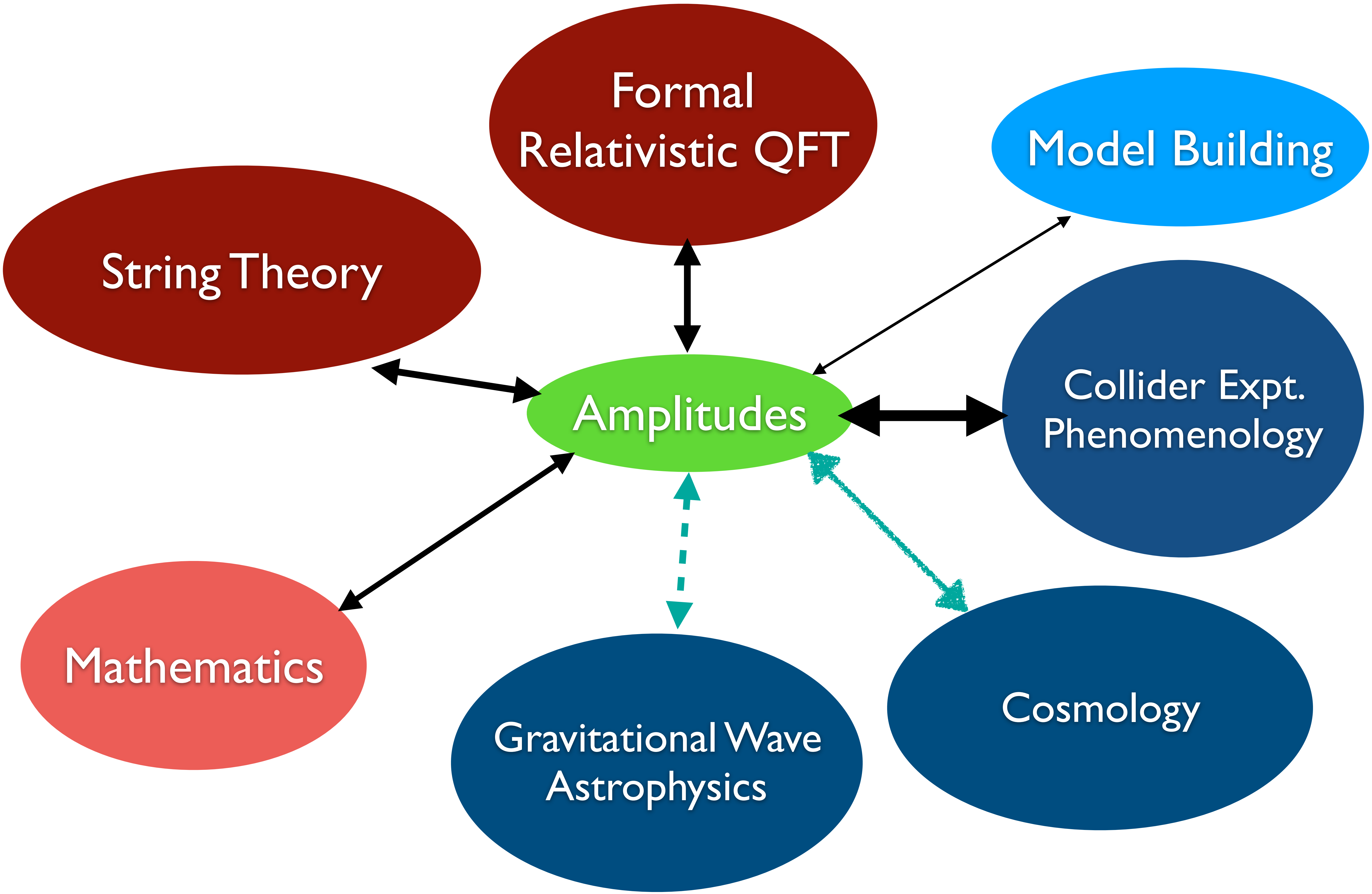
preQFT: Strategic Predictions for Quantum Field Theories

European Commission | Horizon 2020 European Union funding for Research & Innovation

erc
European Research Council
Established by the European Commission

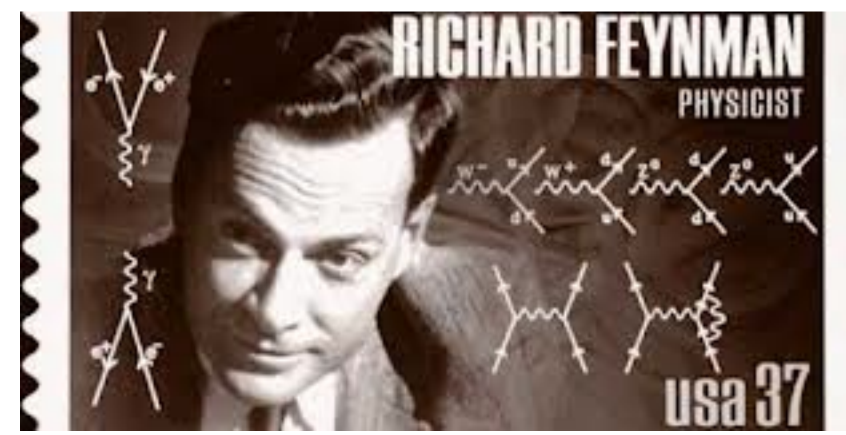
Who we currently connect to:



Stories

Perturbative Quantum stories from Actions?

Use Feynman rules.



$$\mu \quad \frac{4\pi e^2}{q^2} \quad \mu$$

Consider Einstein-Hilbert Action:

$$\mathcal{L}_{\mathrm{gravity}} = -\frac{2}{\kappa^2}\sqrt{g}R$$

Who could complain about this?

Off-shell three-graviton vertex (de Donder/harmonic gauge):

$$
\begin{aligned}
\frac{\delta S^3}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma\tau}\delta\varphi_{\rho\lambda}} \quad \rightarrow \quad & 2\eta^{\mu\tau}\eta^{\nu\sigma}k_1{}^\lambda k_1{}^\rho + 2\eta^{\mu\sigma}\eta^{\nu\tau}k_1{}^\lambda k_1{}^\rho - 2\eta^{\mu\nu}\eta^{\sigma\tau}k_1{}^\lambda k_1{}^\rho + \\
& 2\eta^{\lambda\tau}\eta^{\mu\nu}k_1{}^\sigma k_1{}^\rho + 2\eta^{\lambda\sigma}\eta^{\mu\nu}k_1{}^\tau k_1{}^\rho + \eta^{\mu\tau}\eta^{\nu\sigma}k_2{}^\lambda k_1{}^\rho + \eta^{\mu\sigma}\eta^{\nu\tau}k_2{}^\lambda k_1{}^\rho + \eta^{\lambda\tau}\eta^{\nu\sigma}k_2{}^\mu k_1{}^\rho + \\
& \eta^{\lambda\sigma}\eta^{\nu\tau}k_2{}^\mu k_1{}^\rho + \eta^{\lambda\tau}\eta^{\mu\sigma}k_2{}^\nu k_1{}^\rho + \eta^{\lambda\sigma}\eta^{\mu\tau}k_2{}^\nu k_1{}^\rho + \eta^{\lambda\tau}\eta^{\nu\sigma}k_3{}^\mu k_1{}^\rho + \eta^{\lambda\sigma}\eta^{\nu\tau}k_3{}^\mu k_1{}^\rho - \\
& \eta^{\lambda\nu}\eta^{\sigma\tau}k_3{}^\mu k_1{}^\rho + \eta^{\lambda\tau}\eta^{\mu\sigma}k_3{}^\nu k_1{}^\rho + \eta^{\lambda\sigma}\eta^{\mu\tau}k_3{}^\nu k_1{}^\rho - \eta^{\lambda\mu}\eta^{\sigma\tau}k_3{}^\nu k_1{}^\rho + \eta^{\lambda\nu}\eta^{\mu\tau}k_3{}^\sigma k_1{}^\rho + \\
& \eta^{\lambda\mu}\eta^{\nu\tau}k_3{}^\sigma k_1{}^\rho + \eta^{\lambda\nu}\eta^{\mu\sigma}k_3{}^\tau k_1{}^\rho + \eta^{\lambda\mu}\eta^{\nu\sigma}k_3{}^\tau k_1{}^\rho + 2\eta^{\mu\nu}\eta^{\rho\tau}k_1{}^\lambda k_1{}^\sigma + 2\eta^{\mu\nu}\eta^{\rho\sigma}k_1{}^\lambda k_1{}^\tau - \\
& 2\eta^{\lambda\rho}\eta^{\mu\nu}k_1{}^\sigma k_1{}^\tau + 2\eta^{\lambda\nu}\eta^{\mu\rho}k_1{}^\sigma k_1{}^\tau + 2\eta^{\lambda\mu}\eta^{\nu\rho}k_1{}^\sigma k_1{}^\tau + \eta^{\mu\tau}\eta^{\nu\rho}k_1{}^\sigma k_2{}^\lambda + \eta^{\mu\rho}\eta^{\nu\tau}k_1{}^\sigma k_2{}^\lambda + \\
& \eta^{\mu\sigma}\eta^{\nu\rho}k_1{}^\tau k_2{}^\lambda + \eta^{\mu\rho}\eta^{\nu\sigma}k_1{}^\tau k_2{}^\lambda + \eta^{\nu\tau}\eta^{\rho\sigma}k_1{}^\lambda k_2{}^\mu + \eta^{\nu\sigma}\eta^{\rho\tau}k_1{}^\lambda k_2{}^\mu + \eta^{\lambda\tau}\eta^{\nu\rho}k_1{}^\sigma k_2{}^\mu - \\
& \eta^{\lambda\rho}\eta^{\nu\tau}k_1{}^\sigma k_2{}^\mu + \eta^{\lambda\nu}\eta^{\rho\tau}k_1{}^\sigma k_2{}^\mu + \eta^{\lambda\sigma}\eta^{\nu\rho}k_1{}^\tau k_2{}^\mu - \eta^{\lambda\rho}\eta^{\nu\sigma}k_1{}^\tau k_2{}^\mu + \eta^{\lambda\nu}\eta^{\rho\sigma}k_1{}^\tau k_2{}^\mu + \\
& 2\eta^{\nu\rho}\eta^{\sigma\tau}k_2{}^\lambda k_2{}^\mu + \eta^{\mu\tau}\eta^{\rho\sigma}k_1{}^\lambda k_2{}^\nu + \eta^{\mu\sigma}\eta^{\rho\tau}k_1{}^\lambda k_2{}^\nu + \eta^{\lambda\tau}\eta^{\mu\rho}k_1{}^\sigma k_2{}^\nu - \eta^{\lambda\rho}\eta^{\mu\tau}k_1{}^\sigma k_2{}^\nu + \\
& \eta^{\lambda\mu}\eta^{\rho\tau}k_1{}^\sigma k_2{}^\nu + \eta^{\lambda\sigma}\eta^{\mu\rho}k_1{}^\tau k_2{}^\nu - \eta^{\lambda\rho}\eta^{\mu\sigma}k_1{}^\tau k_2{}^\nu + \eta^{\lambda\mu}\eta^{\rho\sigma}k_1{}^\tau k_2{}^\nu + 2\eta^{\mu\rho}\eta^{\sigma\tau}k_2{}^\lambda k_2{}^\nu + \\
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& \eta^{\nu\sigma}\eta^{\rho\tau}k_1{}^\lambda k_3{}^\mu - \eta^{\nu\rho}\eta^{\sigma\tau}k_1{}^\lambda k_3{}^\mu + \eta^{\lambda\tau}\eta^{\nu\rho}k_1{}^\sigma k_3{}^\mu + \eta^{\lambda\nu}\eta^{\rho\tau}k_1{}^\sigma k_3{}^\mu + \eta^{\lambda\sigma}\eta^{\nu\rho}k_1{}^\tau k_3{}^\mu + \\
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& \eta^{\mu\nu}\eta^{\rho\sigma}k_2{}^\lambda k_3{}^\tau + \eta^{\lambda\sigma}\eta^{\nu\rho}k_2{}^\mu k_3{}^\tau + \eta^{\lambda\nu}\eta^{\rho\sigma}k_2{}^\mu k_3{}^\tau + \eta^{\lambda\sigma}\eta^{\mu\rho}k_2{}^\nu k_3{}^\tau + \eta^{\lambda\mu}\eta^{\rho\sigma}k_2{}^\nu k_3{}^\tau - \\
& \eta^{\lambda\sigma}\eta^{\mu\nu}k_2{}^\rho k_3{}^\tau + \eta^{\lambda\nu}\eta^{\mu\sigma}k_2{}^\rho k_3{}^\tau + \eta^{\lambda\mu}\eta^{\nu\sigma}k_2{}^\rho k_3{}^\tau + 2\eta^{\lambda\rho}\eta^{\nu\sigma}k_3{}^\mu k_3{}^\tau + 2\eta^{\lambda\rho}\eta^{\mu\sigma}k_3{}^\nu k_3{}^\tau - \\
& 2\eta^{\lambda\rho}\eta^{\mu\nu}k_3{}^\sigma k_3{}^\tau + 2\eta^{\lambda\nu}\eta^{\mu\rho}k_3{}^\sigma k_3{}^\tau + 2\eta^{\lambda\mu}\eta^{\nu\rho}k_3{}^\sigma k_3{}^\tau - \eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_1\cdot k_2 - \eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_1\cdot \\
& k_2 - \eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_1\cdot k_2 + \eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_1\cdot k_2 - \eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau}k_1\cdot k_2 + \eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_1\cdot k_2 + \\
& 2\eta^{\lambda\tau}\eta^{\mu\nu}\eta^{\rho\sigma}k_1\cdot k_2 - \eta^{\lambda\nu}\eta^{\mu\tau}\eta^{\rho\sigma}k_1\cdot k_2 - \eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_1\cdot k_2 + 2\eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_1\cdot k_2 - \\
& \eta^{\lambda\nu}\eta^{\mu\sigma}\eta^{\rho\tau}k_1\cdot k_2 - \eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_1\cdot k_2 - 2\eta^{\lambda\rho}\eta^{\mu\nu}\eta^{\sigma\tau}k_1\cdot k_2 + 2\eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_1\cdot k_2 + \\
& 2\eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_1\cdot k_2 - \eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_1\cdot k_3 - \eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_1\cdot k_3 - \eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_1\cdot k_3 + \\
& 2\eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_1\cdot k_3 - \eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau}k_1\cdot k_3 + 2\eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_1\cdot k_3 + 2\eta^{\lambda\tau}\eta^{\mu\nu}\eta^{\rho\sigma}k_1\cdot k_3 - \\
& \eta^{\lambda\nu}\eta^{\mu\tau}\eta^{\rho\sigma}k_1\cdot k_3 - \eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_1\cdot k_3 + 2\eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_1\cdot k_3 - \eta^{\lambda\nu}\eta^{\mu\sigma}\eta^{\rho\tau}k_1\cdot k_3 - \\
& \eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_1\cdot k_3 - 2\eta^{\lambda\rho}\eta^{\mu\nu}\eta^{\sigma\tau}k_1\cdot k_3 + \eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_1\cdot k_3 + \eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_1\cdot k_3 - \\
& \eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_2\cdot k_3 - \eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_2\cdot k_3 - \eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_2\cdot k_3 + 2\eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_2\cdot k_3 - \\
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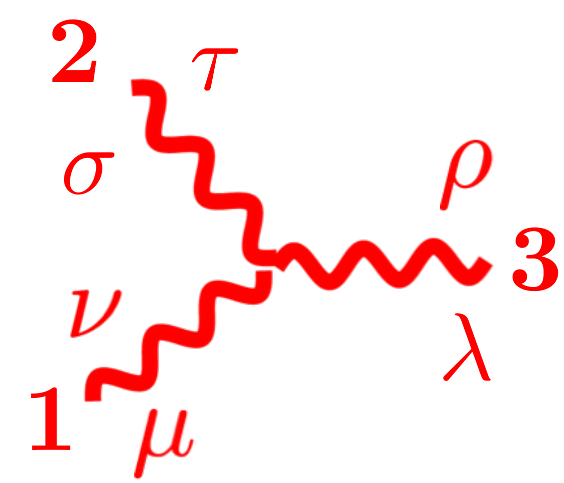
& \eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_2\cdot k_3 + \eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_2\cdot k_3 - \eta^{\lambda\nu}\eta^{\mu\sigma}\eta^{\rho\tau}k_2\cdot k_3 - \eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_2\cdot k_3 - \\
& 2\eta^{\lambda\rho}\eta^{\mu\nu}\eta^{\sigma\tau}k_2\cdot k_3 + 2\eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_2\cdot k_3 + 2\eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_2\cdot k_3
\end{aligned}
$$

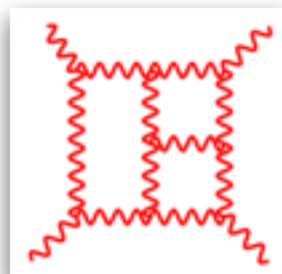
171 terms



[DeWitt, 1967]

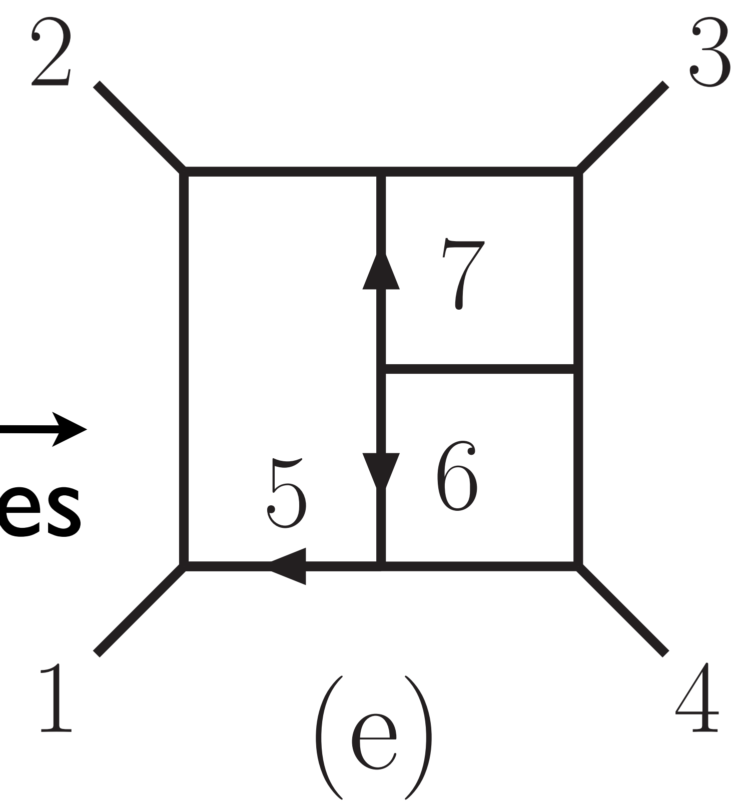
Bern, JJMC, Dixon, Kosower, Roiban '07

MOST SYMMETRIC 4D THEORY, N=8 SUGRA



~10^{20} TERMS

add all other particles



$$\propto \int stu\mathcal{M}_4^{(0)} \frac{\left(s\,(k_4+l_5)^2\right)^2}{d\circ(e) \equiv (l_5^2 l_6^2 l_7^2 (k_1-l_5)^2\cdots)}$$

Some truths obscured by actions:

Calculate with physical (on-shell) quantities: $k_i^2 = 0$

Physical (on-shell) **tree-level amplitudes contain all the information necessary to verify and build *all* loop-level amplitudes**

Bern, Dixon, Dunbar, and **Kosower** (‘94,’95)

Bern, Dixon, and **Kosower** (‘96)

Physical (on-shell) **three-vertices contain all the information necessary to build *all* tree-level amplitudes**

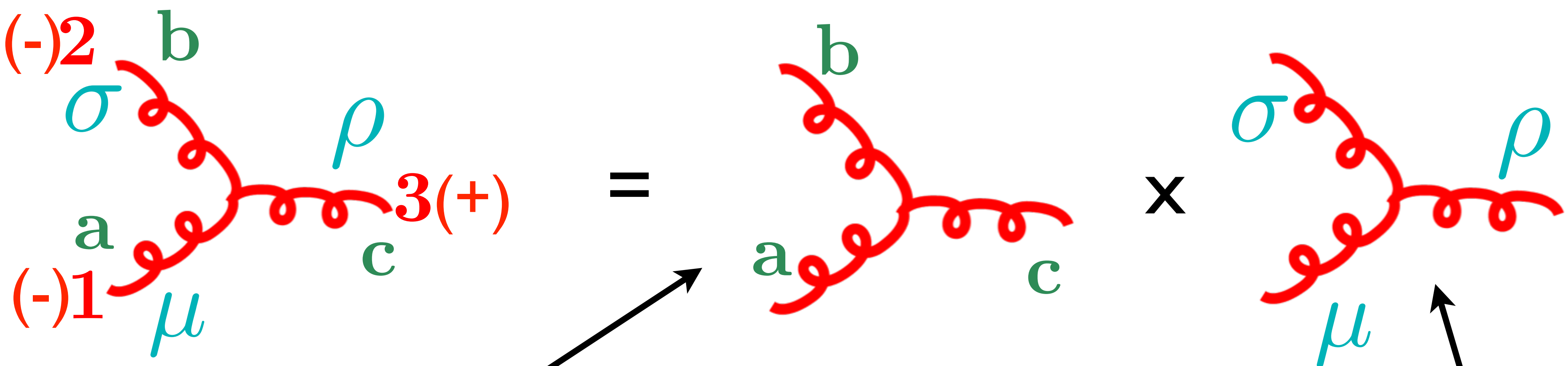
Britto, Cachazo, Feng, and Witten (’05)

Easy verification => Natural construction. **Method of maximal cuts.**

Bern, JJMC, Johansson, Kosower (’07)

$k_i^2 = 0$

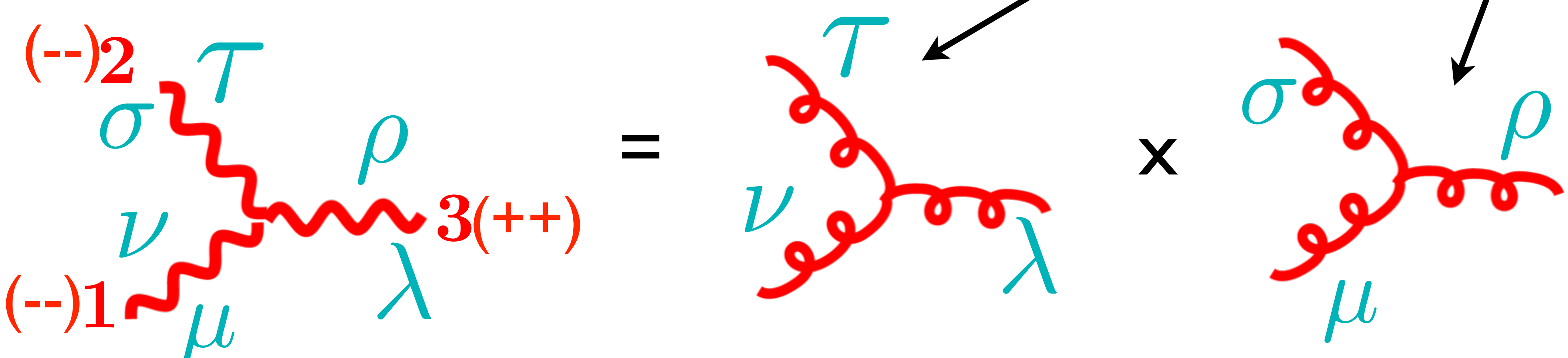
Physical gluon 3-vertex: $f^{abc}\left({k_1}^{\sigma}\eta^{\mu\rho} - {k_2}^{\mu}\eta^{\rho\sigma}\right)$



color weight

kinematic weights

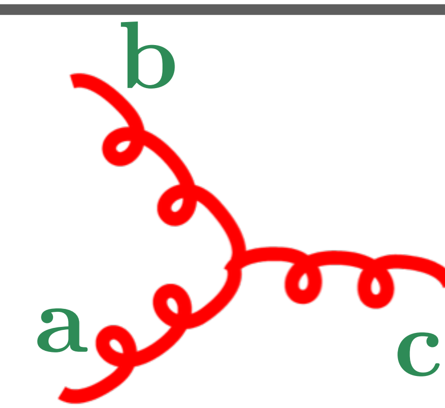
Physical graviton 3-vertex:



$$\left(\mathbf{k_1}^{\sigma}\eta^{\mu\rho} - \mathbf{k_2}^{\mu}\eta^{\rho\sigma}\right)\left(\mathbf{k_1}^{\tau}\eta^{\nu\lambda} - \mathbf{k_2}^{\nu}\eta^{\lambda\tau}\right)$$

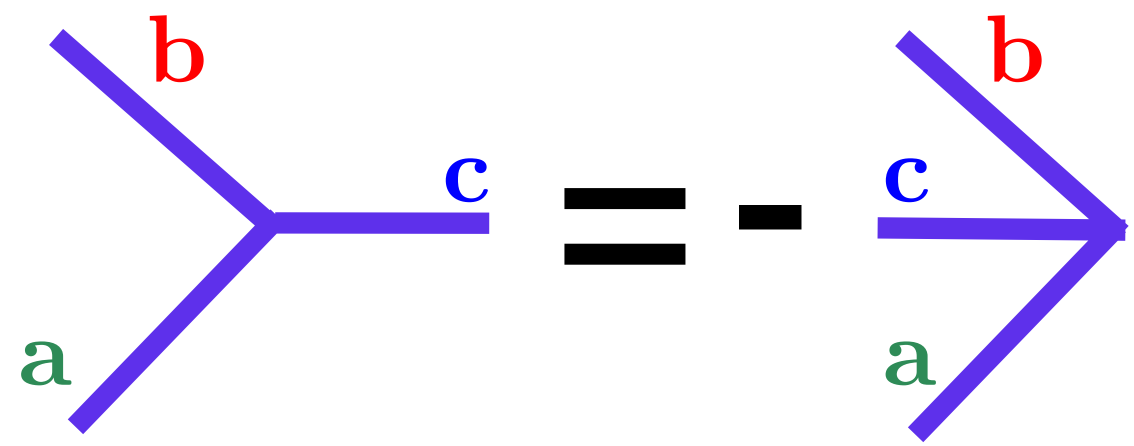
Lie Algebra structure constants:

$$f^{abc}$$



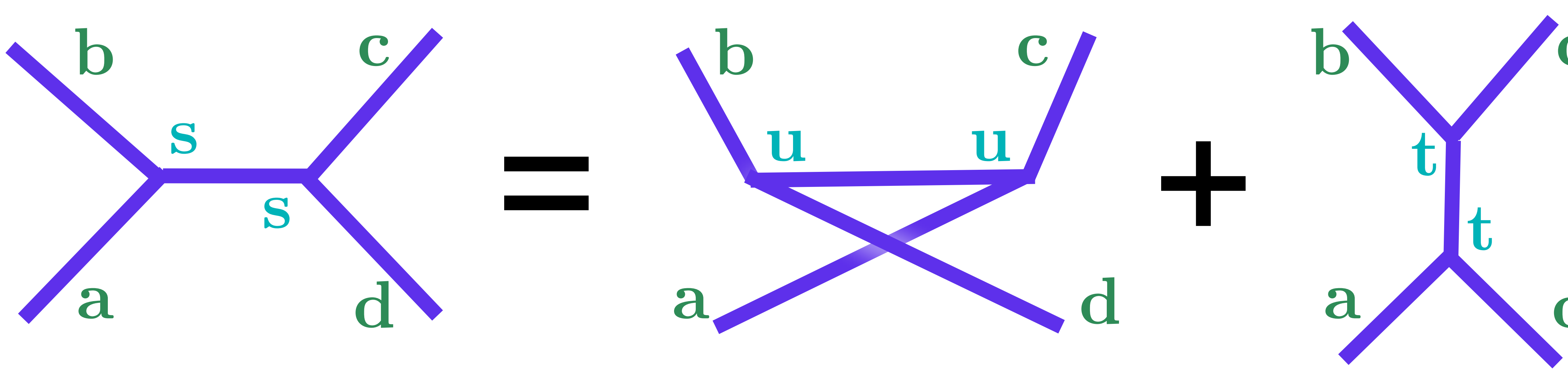
ANTISYMMETRY:

$$f^{abc} = -f^{acb}$$



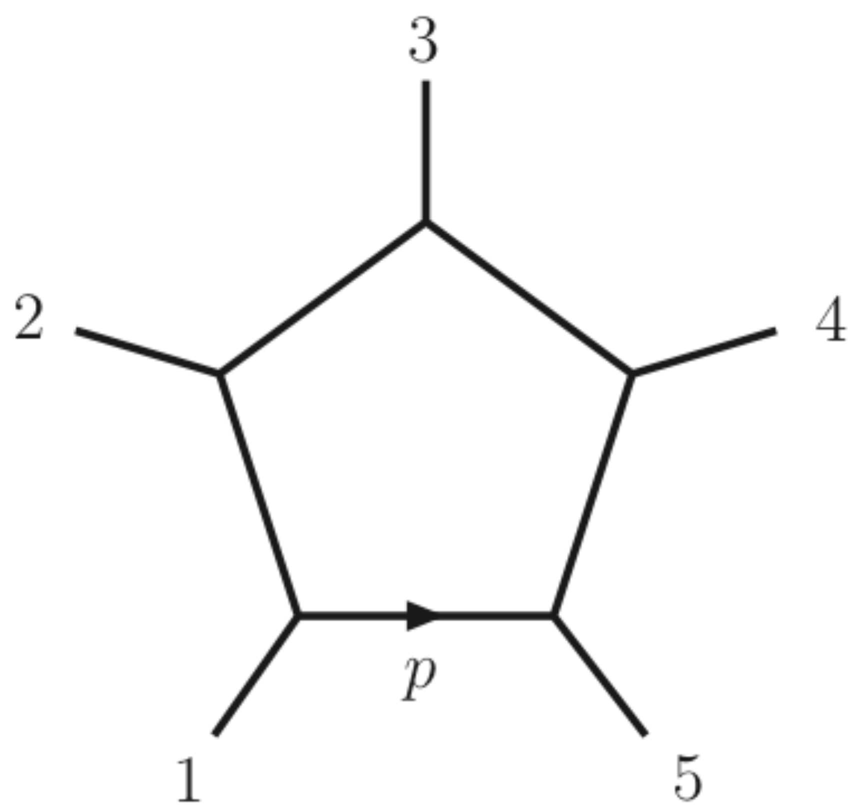
JACOBI:

$$f^{abs}\,f^{scd} = f^{cau}\,f^{udb} + f^{dat}\,f^{tbc}$$

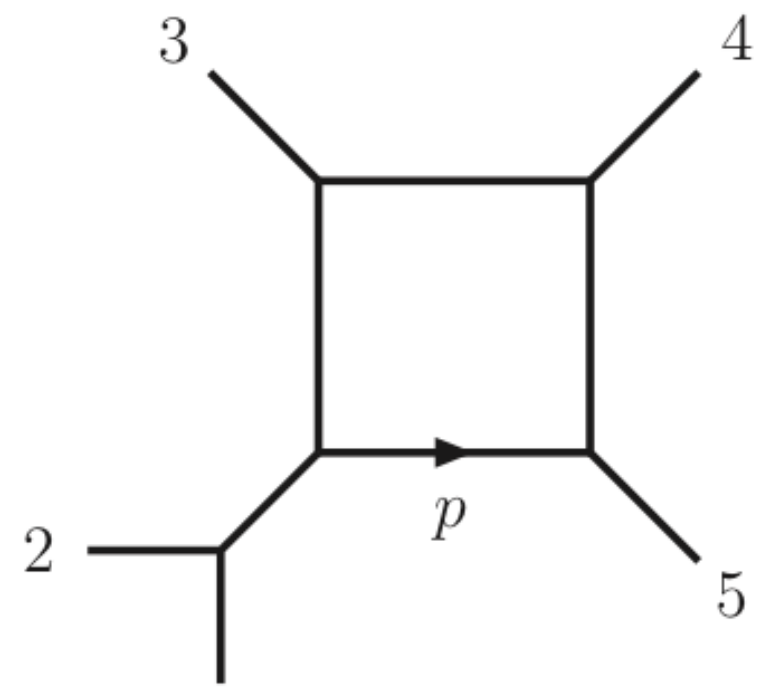


Ready to solve all of life's problems?

Five point 1-loop (no triangles, no bubbles)

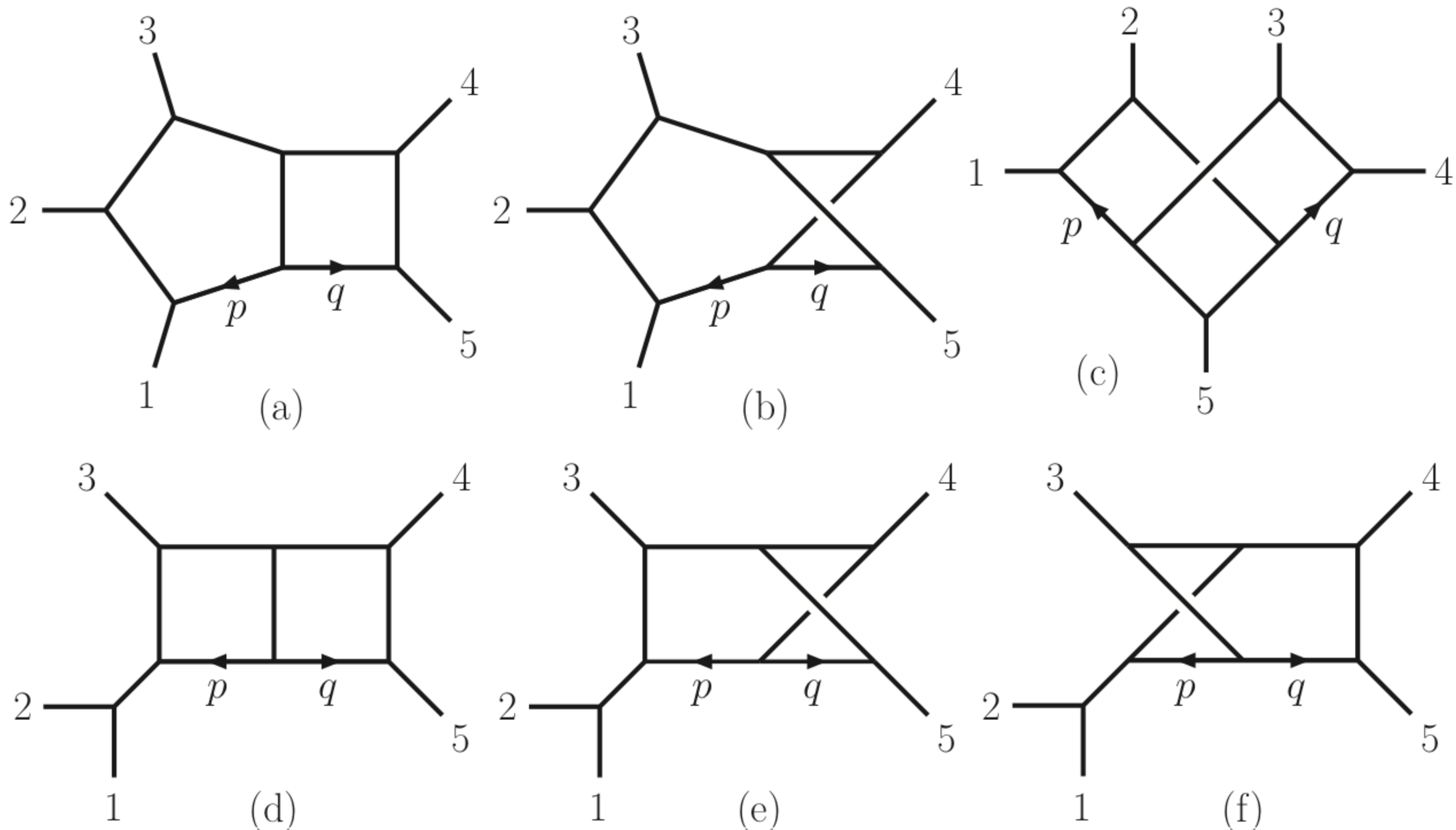


(P)



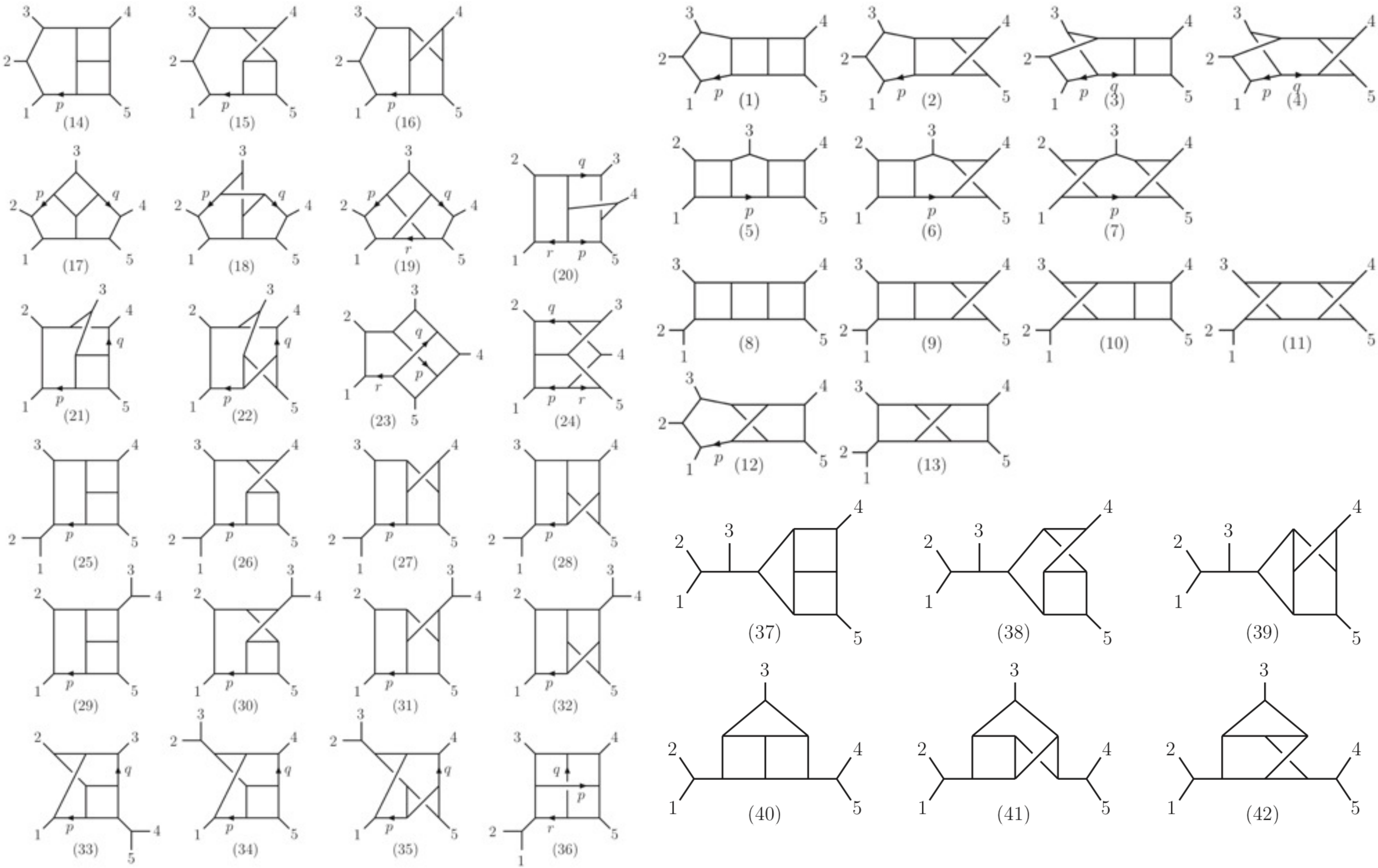
(B)

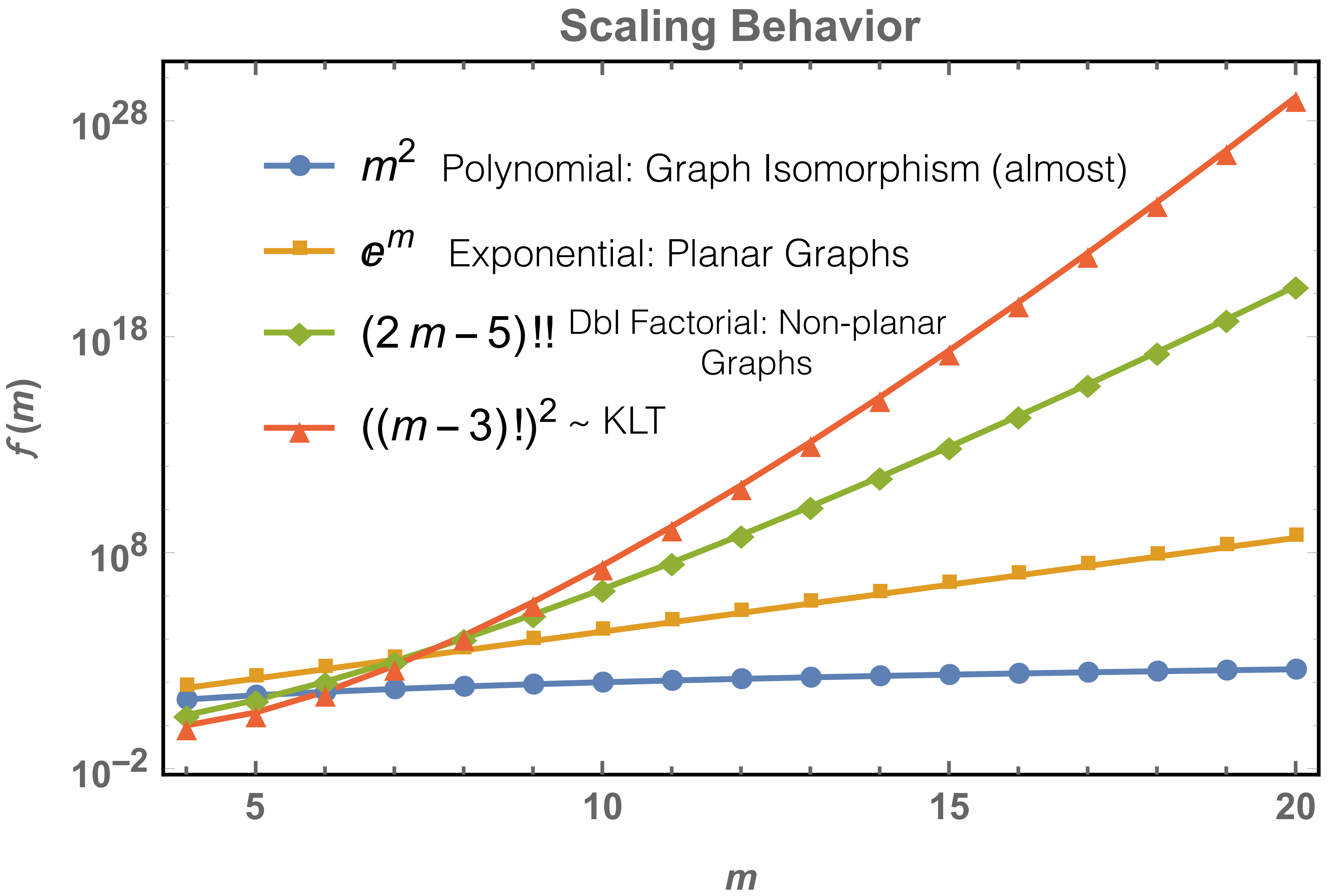
Five point 2-loop (no triangles, no bubbles)



Five point 3-loop (no bubbles, no triangles)

JJMC, Johansson (to appear)





Scaling Behavior
10^28
10^18
10^8
10^-2
f(m)
m^2 Polynomial: Graph Isomorphism (almost)
e^m Exponential: Planar Graphs
(2 m − 5)!! Dbl Factorial: Non-planar Graphs
((m − 3)!)^2 ~ KLT
5
10
15
20
m

the game of **Scattering Amplitudes**

IN
free states
(no interactions)

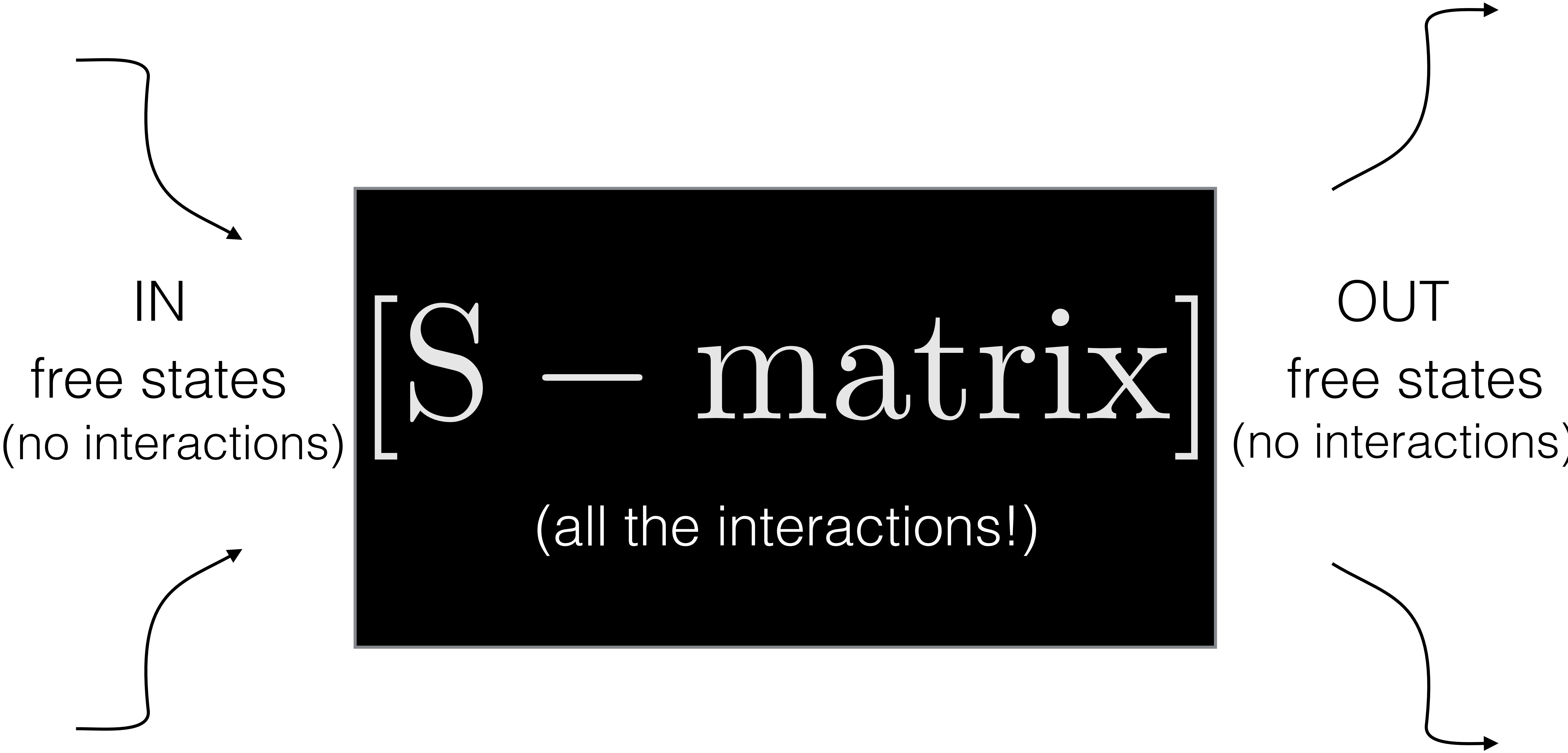
$$[\mathrm{S-matrix}]$$

(all the interactions!)

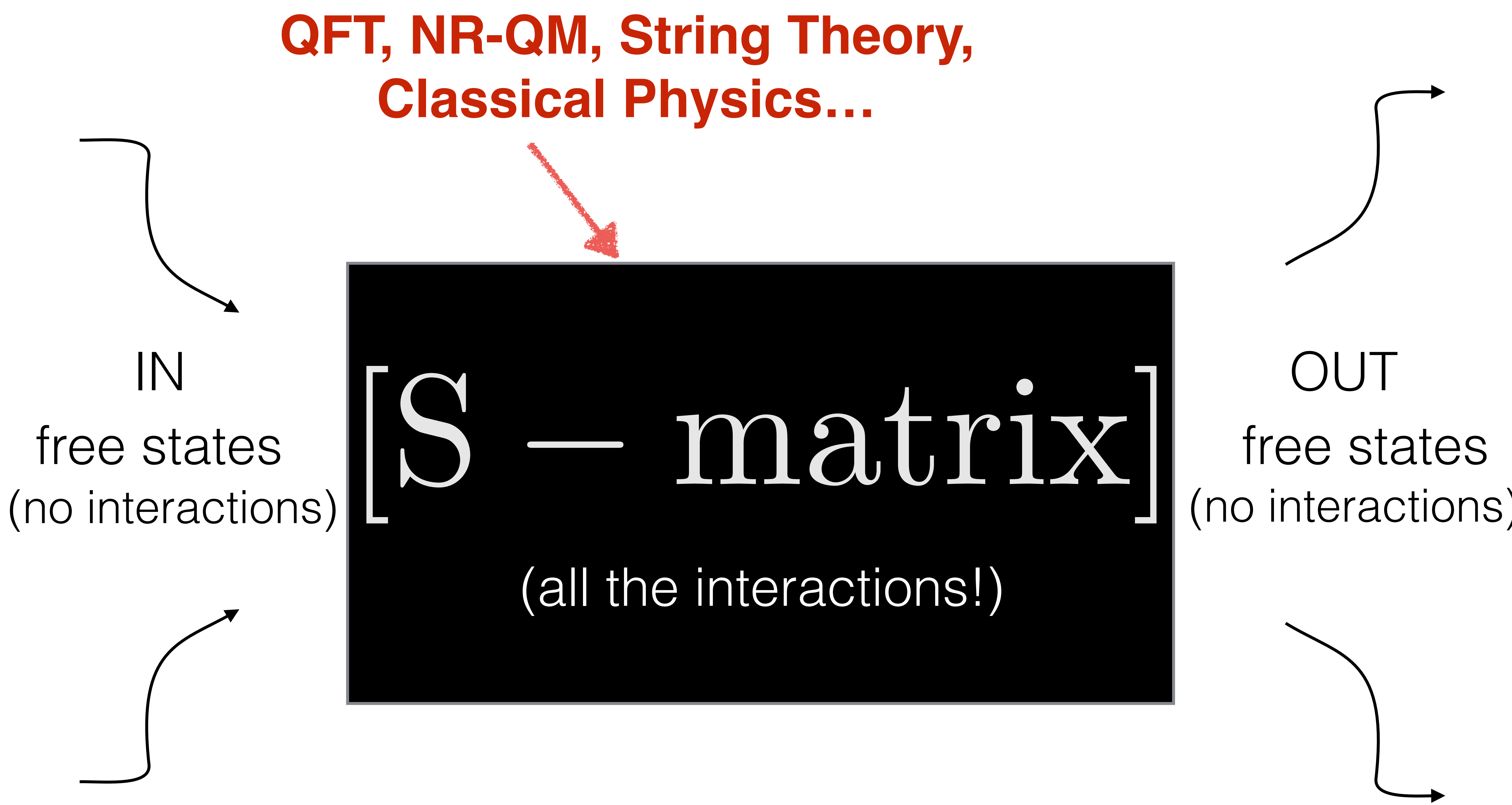
OUT
free states
(no interactions)

Key Property: GAUGE INVARIANT

the game of Scattering Amplitudes

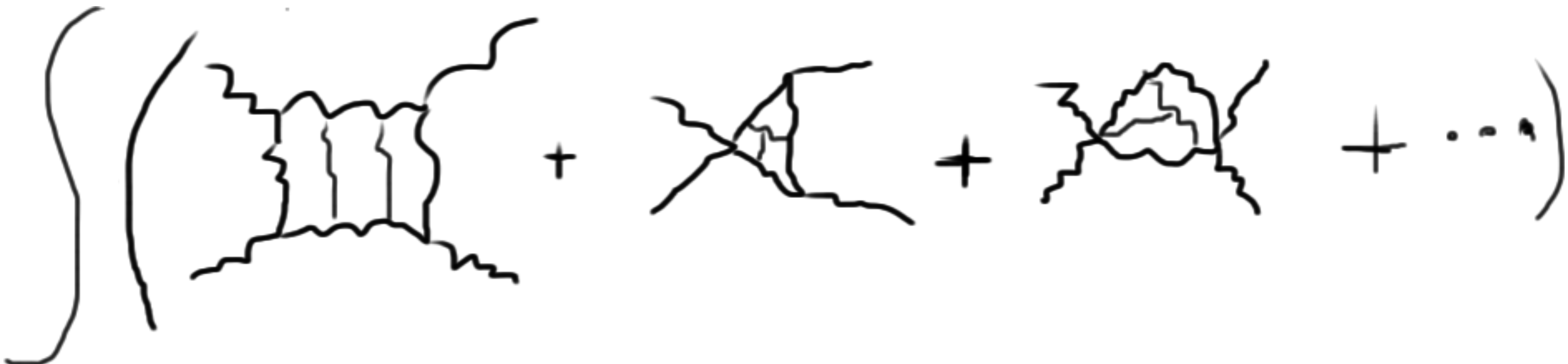


the game of Scattering Amplitudes



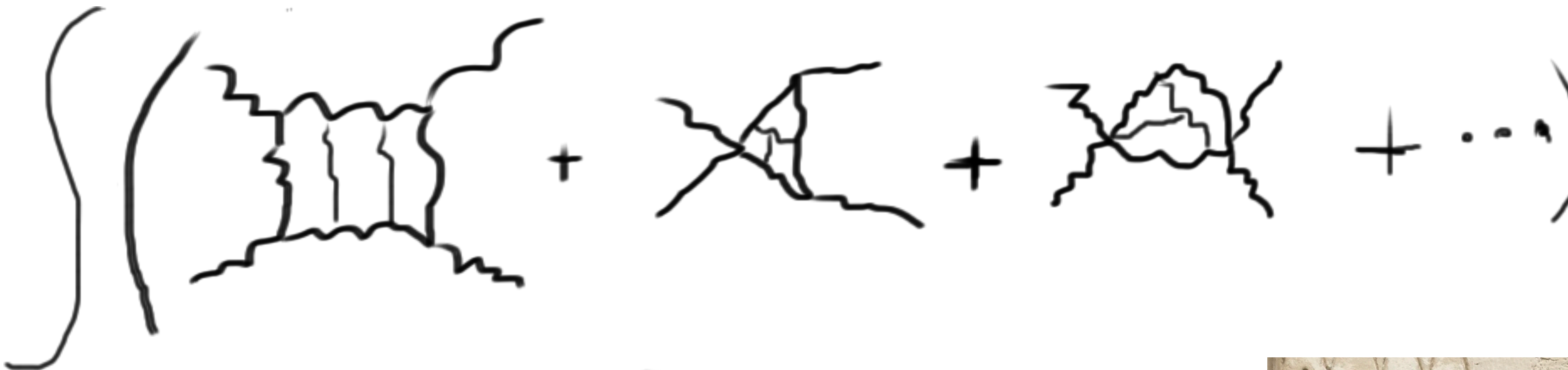
Same predictions, but definitely different stories

NECESSARY



$= \int$ expression

NECESSARY



$$= \int \sum_{g} \frac{n \circ g}{d \circ g}$$

SUFFICIENT

$\mathcal{U}_c \circ \Big($  $+ \cdots \Big)$

$$= \mathcal{U}_c \circ \sum_{g} \frac{n \circ g}{d \circ g}$$

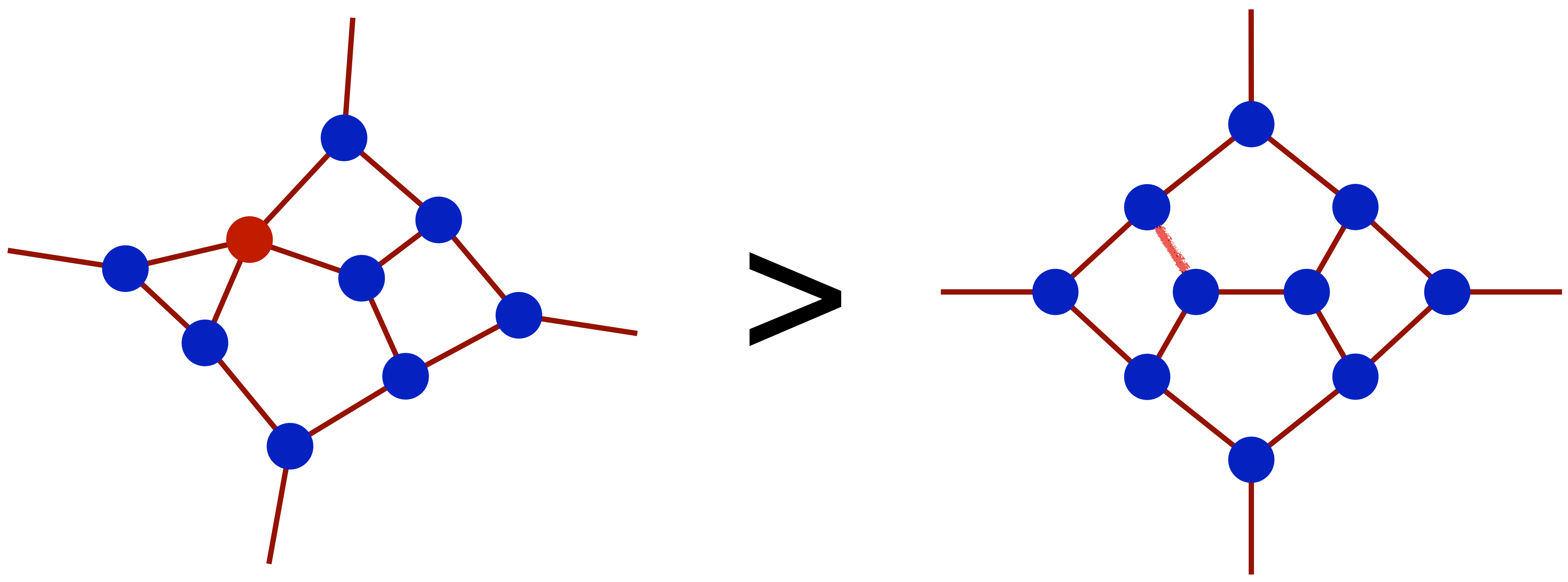
$\forall\, \mathcal{U}_c \in$ unitarity cuts

Bern, Dixon, Dunbar, and Kosower ('94,'95)

Bern, Dixon, and Kosower ('96)

Britto, Cachazo, and Feng ('04)

SPANNING CUTS



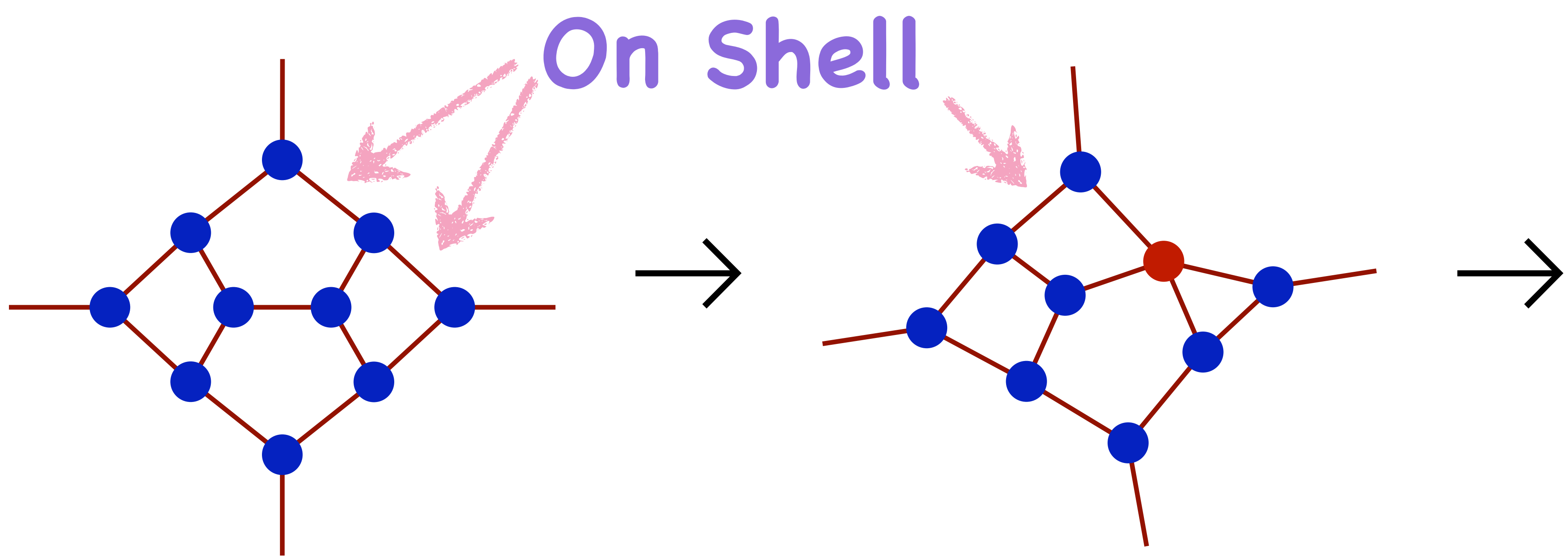
leads to notion of a **Minimal Spanning Set**

EASY VERIFICATION

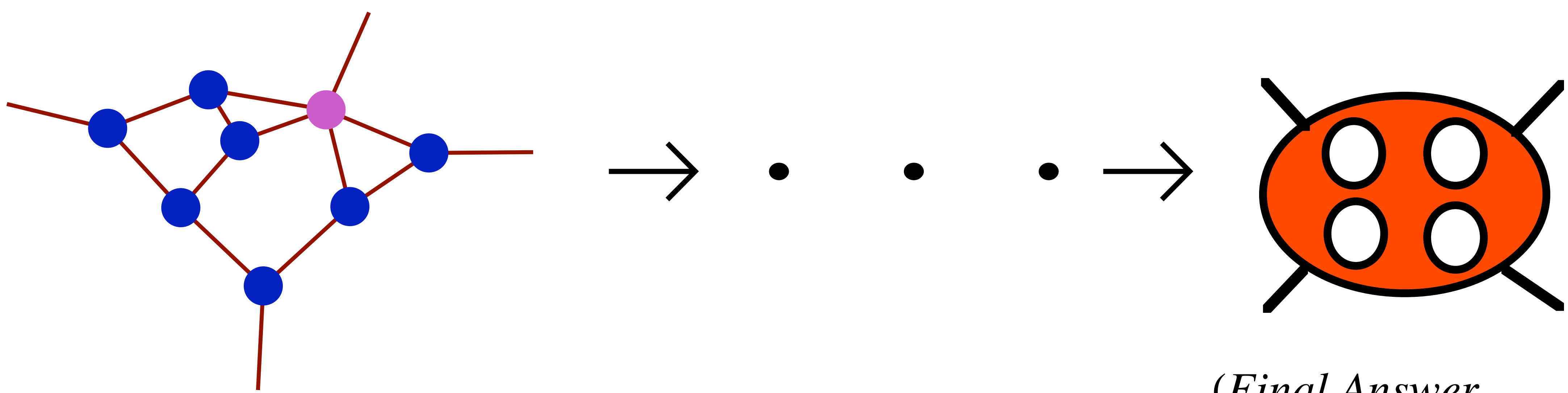
EASY VERIFICATION ⟶ NATURAL CONSTRUCTION

METHOD OF MAXIMAL CUTS

Bern, JJMC, Kosower, Johansson

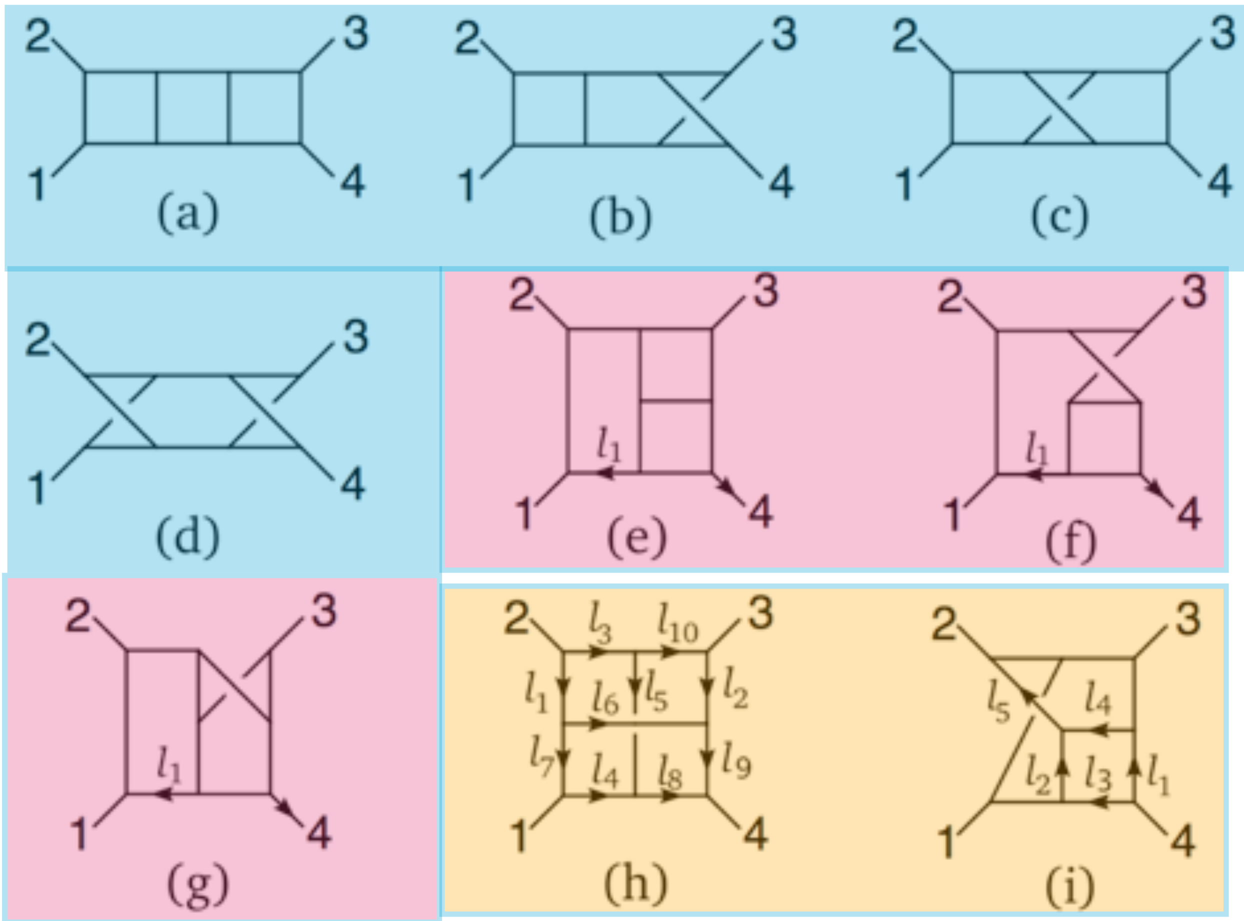


$$(\forall \text{ exposed propagators } p^2 = 0)$$



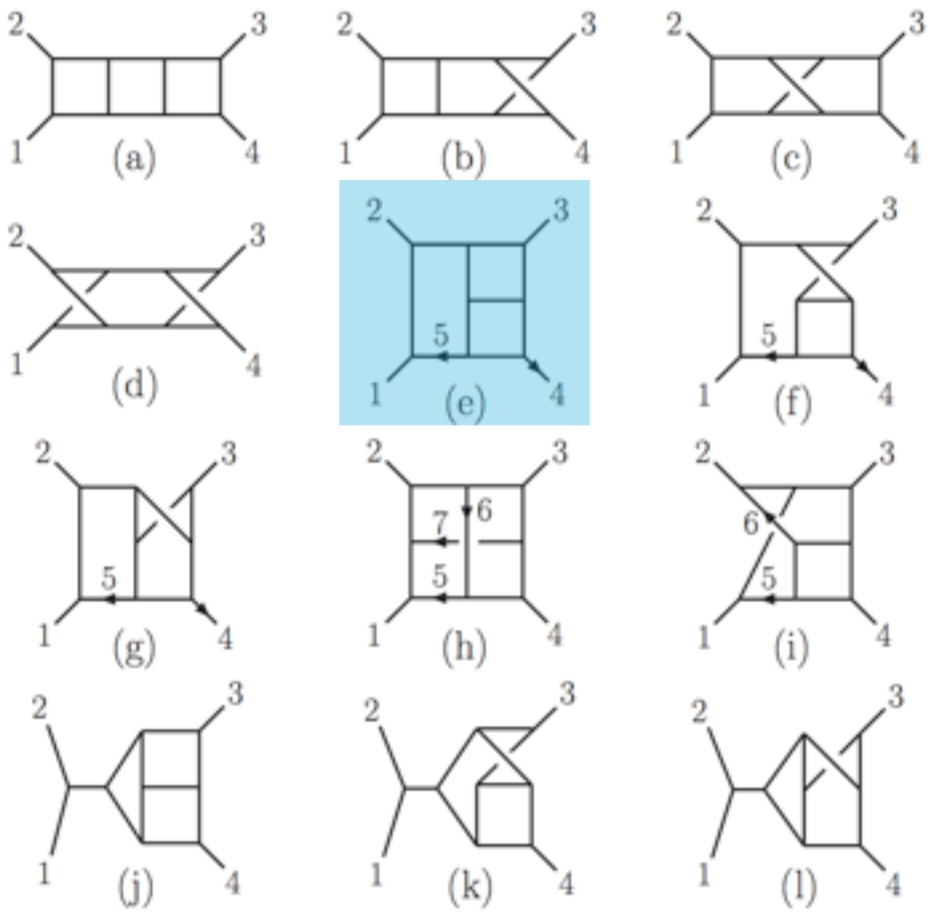
(*Final Answer,*
no cut conditions!)

Bern, JJMC, Dixon, Kosower, Johansson, Roiban '07



Original solution of three-loop four-point N=4 sYM and N=8 sugra

Integral	$\mathcal{N}=4$ Yang-Mills	$\mathcal{N}=8$ Supergravity
(a)–(d)	s^2	$[s^2]^2$
(e)–(g)	$s(l_1+k_4)^2$	$[s\,(l_1+k_4)^2]^2$
(h)	$s(l_1+l_2)^2+t(l_3+l_4)^2 - sl_5^2 - tl_6^2 - st$	$(s(l_1+l_2)^2+t(l_3+l_4)^2 - st)^2 - s^2(2((l_1+l_2)^2-t)+l_5^2)l_5^2 -t^2(2((l_3+l_4)^2-s)+l_6^2)l_6^2 - s^2(2l_7^2l_2^2+2l_1^2l_9^2+l_2^2l_9^2+l_1^2l_7^2) - t^2(2l_3^2l_8^2+2l_{10}^2l_4^2+l_8^2l_4^2+l_3^2l_{10}^2)+2stl_5^2l_6^2$
(i)	$s(l_1+l_2)^2-t(l_3+l_4)^2 -\frac{1}{3}(s-t)l_5^2$	$(s(l_1+l_2)^2-t(l_3+l_4)^2)^2 - (s^2(l_1+l_2)^2+t^2(l_3+l_4)^2+\frac{1}{3}stu)l_5^2$



Bern, JJMC, Johansson (2010)

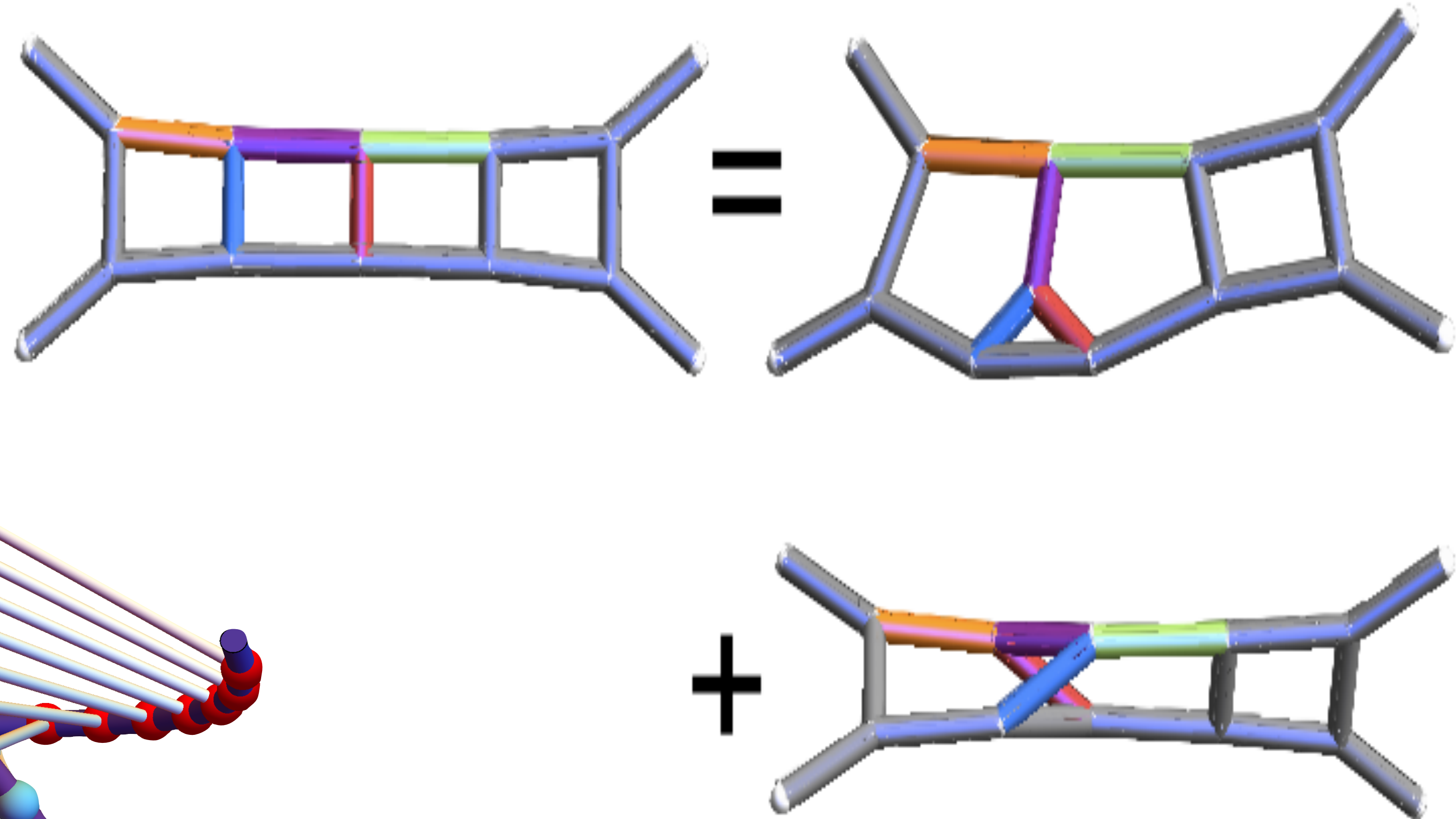
Cubic Double-Copy Solution

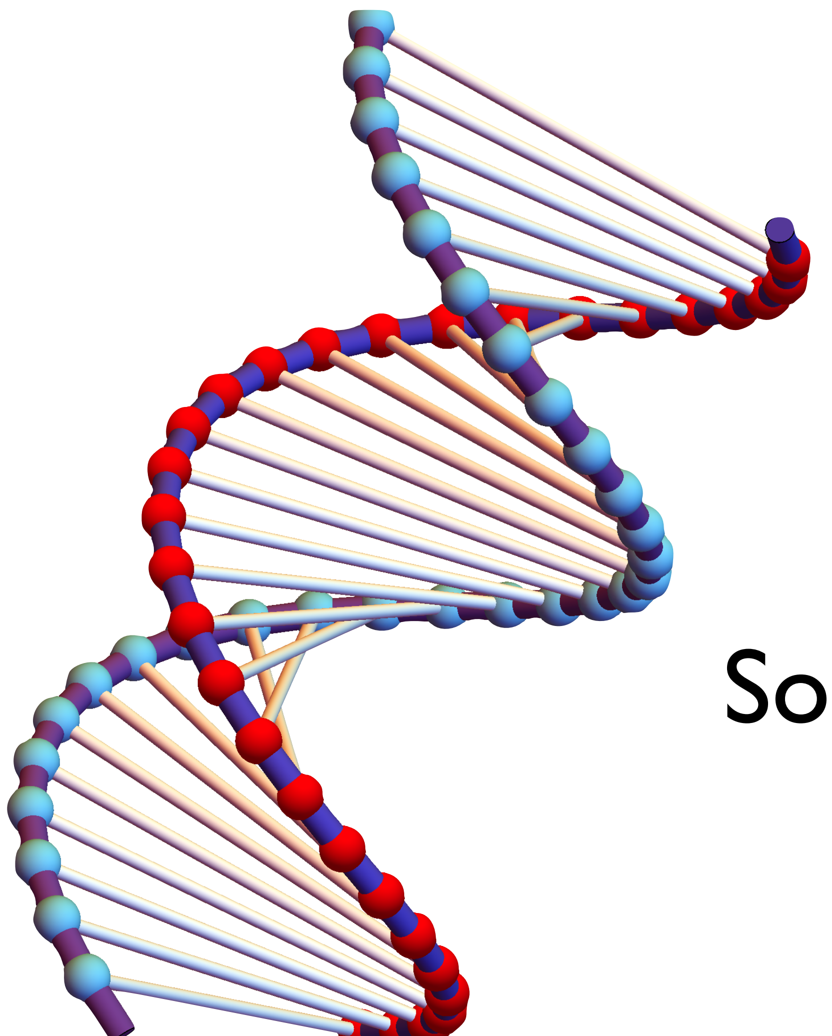
$s = (k_1 + k_2)^2 \quad t = (k_1 + k_4)^2 \quad u = (k_1 + k_3)^2 \quad \tau_{i,j} = 2k_i \cdot l_j$

Integral $I^{(x)}$	$\mathcal{N} = 4$ Super-Yang-Mills ($\sqrt{\mathcal{N} = 8 \text{ supergravity}}$) numerator
(a)–(d)	s^2
(e)–(g)	$\big(s\,(-\tau_{35} + \tau_{45} + t) - t\,(\tau_{25} + \tau_{45}) + u\,(\tau_{25} + \tau_{35}) - s^2\big)/3$
(h)	$\big(s\,(2\tau_{15} - \tau_{16} + 2\tau_{26} - \tau_{27} + 2\tau_{35} + \tau_{36} + \tau_{37} - u)$ $+t\,(\tau_{16} + \tau_{26} - \tau_{37} + 2\tau_{36} - 2\tau_{15} - 2\tau_{27} - 2\tau_{35} - 3\tau_{17}) + s^2\big)/3$
(i)	$\big(s\,(-\tau_{25} - \tau_{26} - \tau_{35} + \tau_{36} + \tau_{45} + 2t)$ $+t\,(\tau_{26} + \tau_{35} + 2\tau_{36} + 2\tau_{45} + 3\tau_{46}) + u\,\tau_{25} + s^2\big)/3$
(j)-(l)	$s(t-u)/3$

Bern, JJMC, Johansson ('08,'10)

Color and Kinematics dance together.



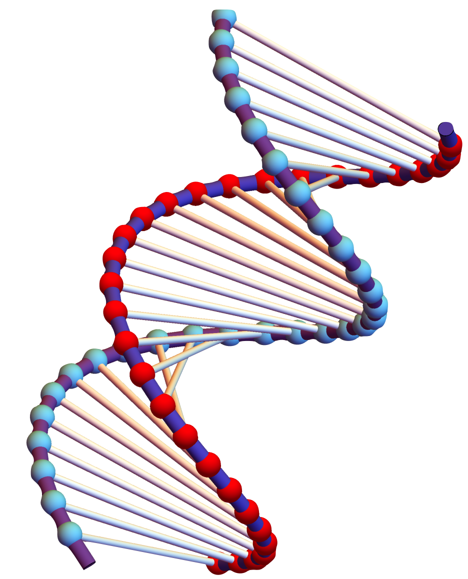


Solving Yang-Mills theories means solving Gravity theories.

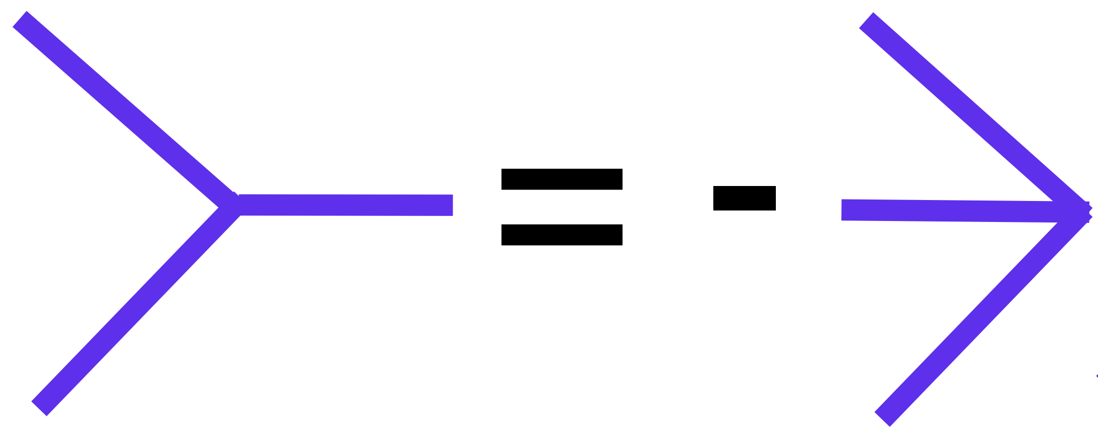
Bern, JJMC, Johansson ('08,'10)

Generic D-dimensional YM theories have a fascinating structure at tree-level

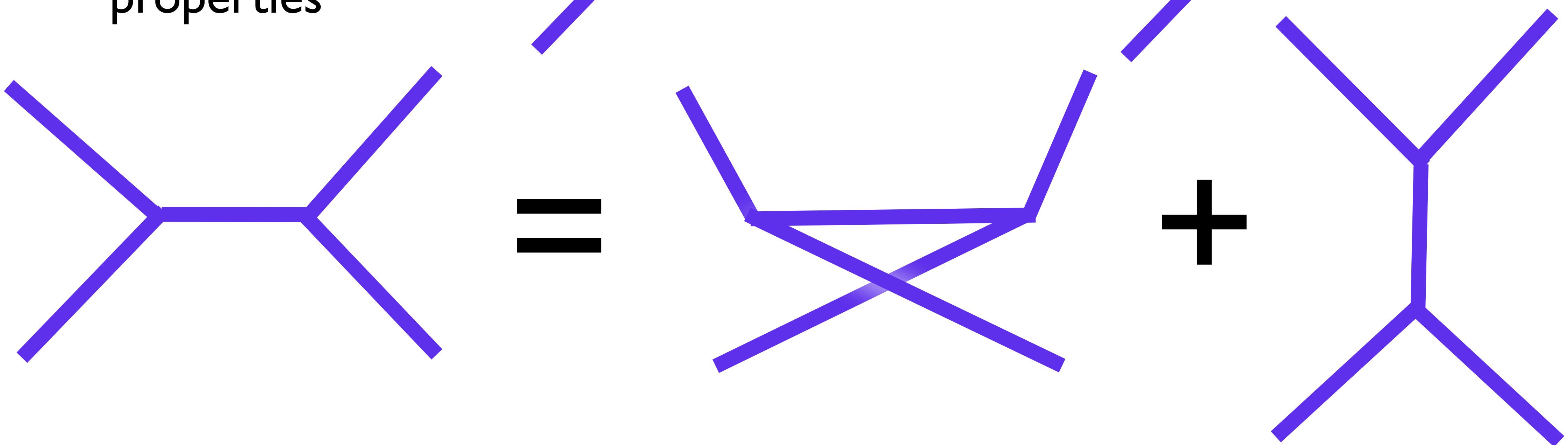
$$\mathcal{A}_m^{\text{tree}} = \sum_{\mathcal{G}\in\text{cubic}} \frac{c(\mathcal{G})\,n(\mathcal{G})}{D(\mathcal{G})}$$



Color factors and numerator factors satisfy similar lie algebra properties



Vertex Antisymmetry



Jacobi

Color-Kinematic Duality!

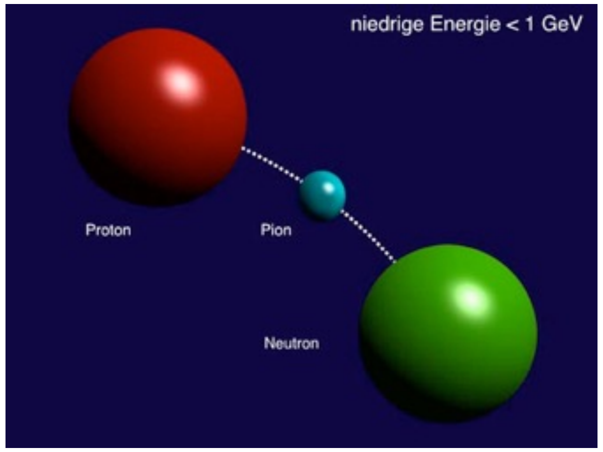
Tree level example that doesn't hurt the eyes…

Non-Linear sigma model…

$$\mathcal{L}_{\mathrm{NLSM}} = \frac{1}{2}\mathrm{Tr}\left\{ \partial_\mu \varphi \, \frac{1}{1-\varphi^2} \, \partial^\mu \varphi \, \frac{1}{1-\varphi^2} \right\}$$

(Cayley Parameterization)

Leading O(p^2) contribution to Chiral Lagrangian



Ref.TH.3689-CERN

CHIRAL PERTURBATION THEORY TO ONE LOOP *)

J. Gasser
Institut für Theoretische Physik der
Universität Bern, 3012 Bern

and

H. Leutwyler **)
CERN -- Geneva

Ref.TH.3798-CERN

CHIRAL PERTURBATION THEORY:
EXPANSIONS IN THE MASS OF THE STRANGE QUARK*)

J. Gasser

Institut für Theoretische Physik der Universität Bern
3012 Bern

and

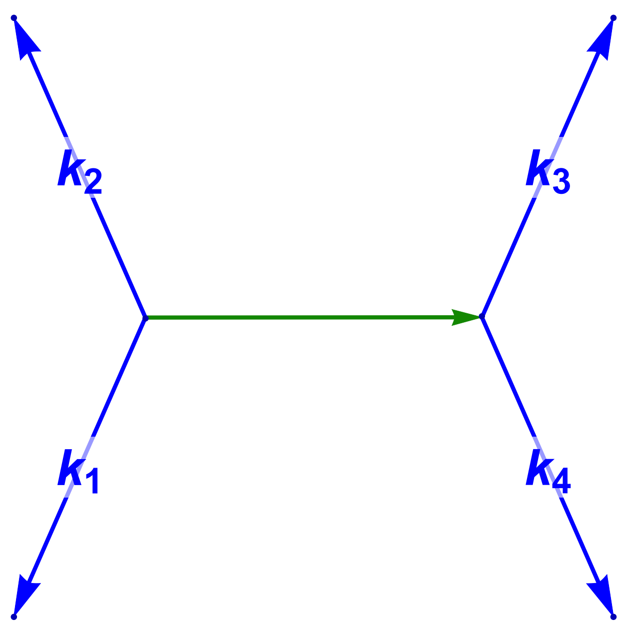
H. Leutwyler+)

CERN - Geneva

Non-Linear sigma model…

$$\mathcal{L}_{\rm NLSM} = \frac{1}{2}{\rm Tr}\left\{\partial_\mu\varphi\,\frac{1}{1-\varphi^2}\,\partial^\mu\varphi\,\frac{1}{1-\varphi^2}\right\}$$

(Cayley Parameterization)

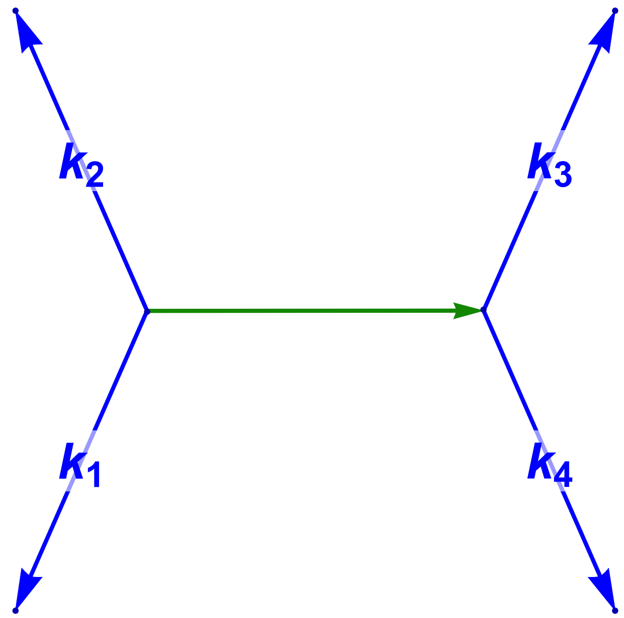


For SU(2) $f^{abc} \propto \epsilon^{abc}$

$$c_s = f^{a_1\,a_2\,b} f^{\,b\,a_3\,a_4} \propto \delta^{a_1,a_3}\,\delta^{a_2,a_4} - \delta^{a_1,a_4}\,\delta^{a_2,a_3}$$

$$n_s \propto (k^2_{(1,2)} + k^2_{(3,4)}) \times (k_{[1,2]} \cdot k_{[3,4]})$$

$$k_{(ab)} = k_a + k_b \qquad k_{[ab]} = k_a - k_b$$

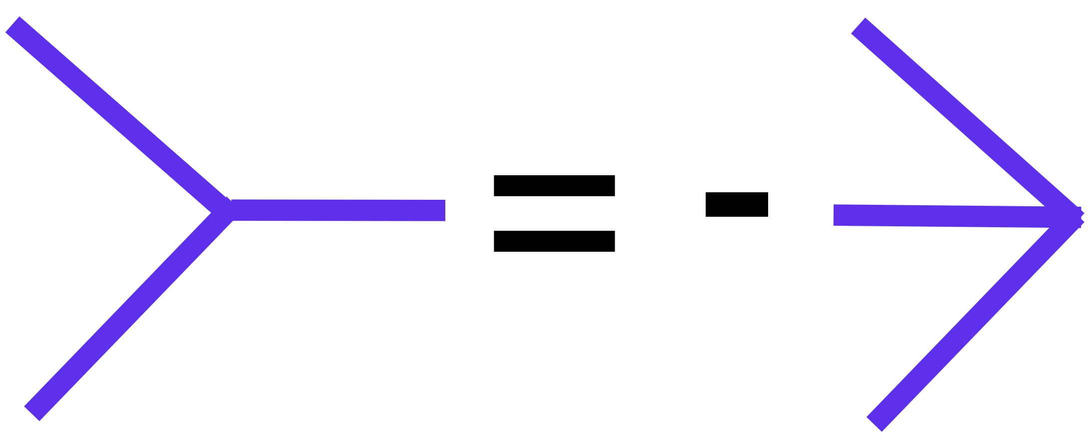


For SU(2) $f^{abc} \propto \epsilon^{abc}$

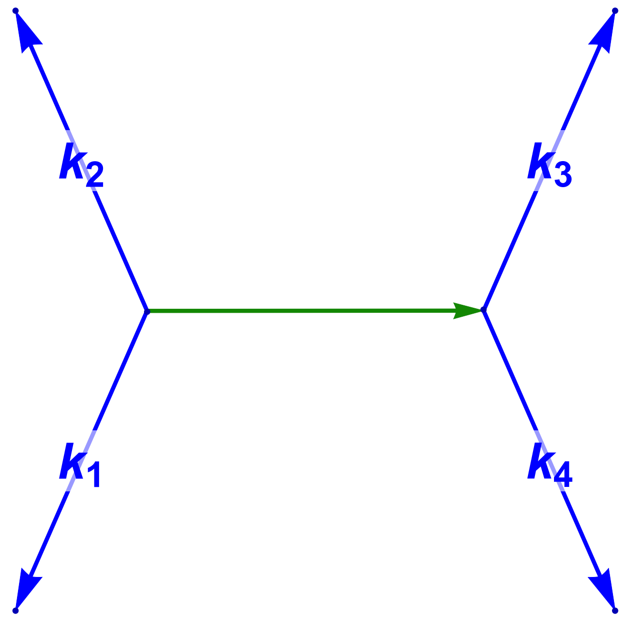
$$c_s = f^{a_1 a_2 b} f^{b a_3 a_4} \propto \delta^{a_1, a_3} \delta^{a_2, a_4} - \delta^{a_1, a_4} \delta^{a_2, a_3}$$

$$n_s = (k_{(1,2)}^2 + k_{(3,4)}^2) \times (k_{[1,2]} \cdot k_{[3,4]})$$

$$k_{(ab)} = k_a + k_b \qquad k_{[ab]} = k_a - k_b$$



Vertex Antisymmetry

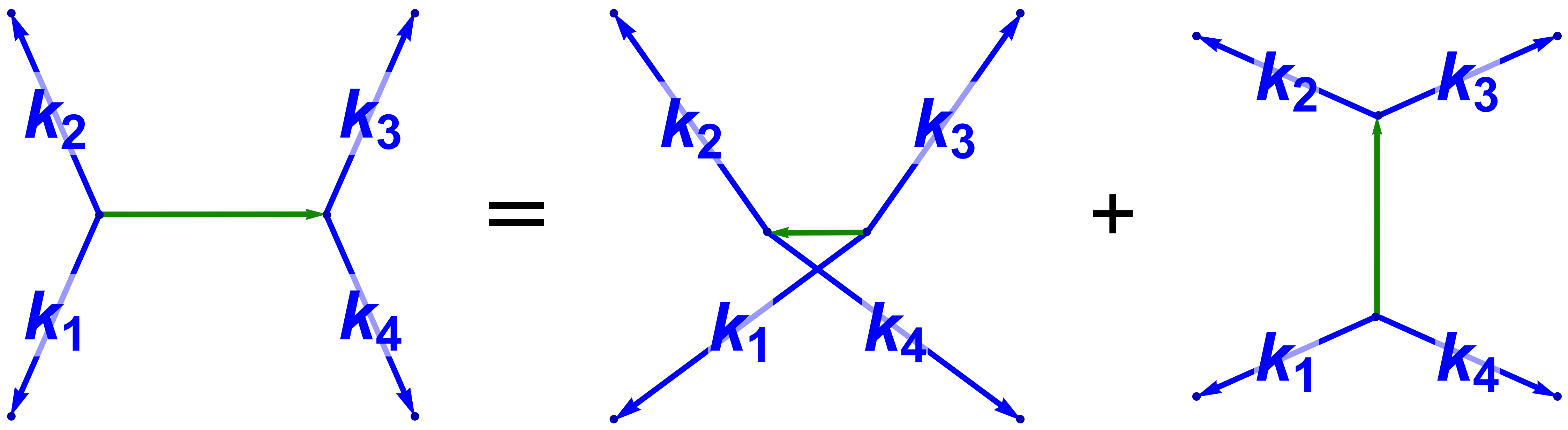


For SU(2) $f^{abc} \propto \in^{abc}$

$$c_s = f^{a_1 a_2 b} f^{b a_3 a_4} \propto \delta^{a_1,a_3}\delta^{a_2,a_4} - \delta^{a_1,a_4}\delta^{a_2,a_3}$$

$$n_s = (k^2_{(1,2)} + k^2_{(3,4)}) \times (k_{[1,2]} \cdot k_{[3,4]})$$

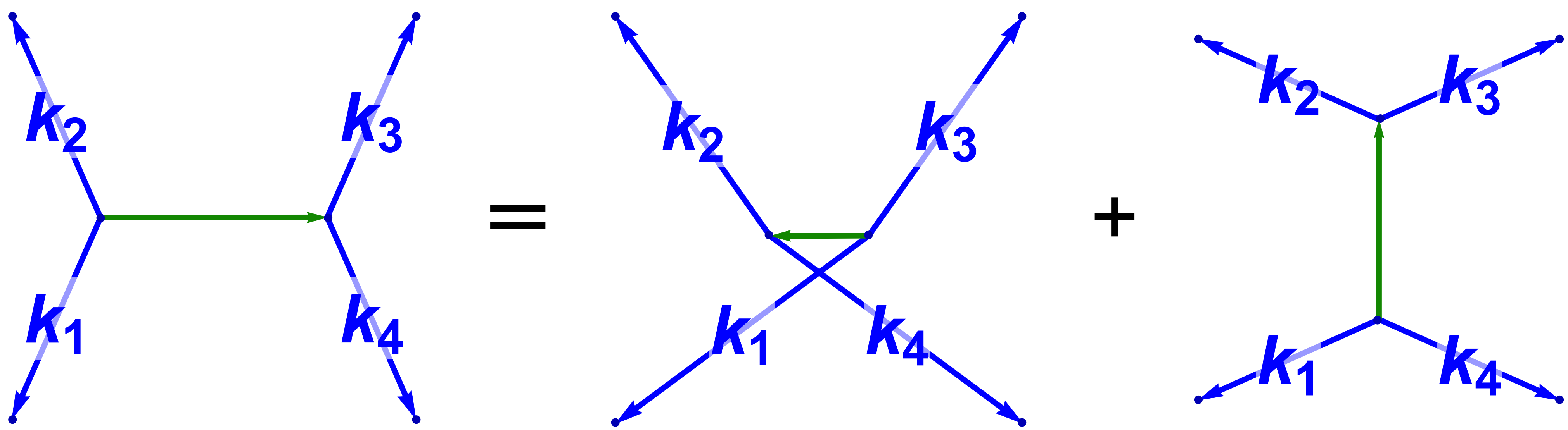
$$k_{(ab)} = k_a + k_b \qquad k_{[ab]} = k_a - k_b$$



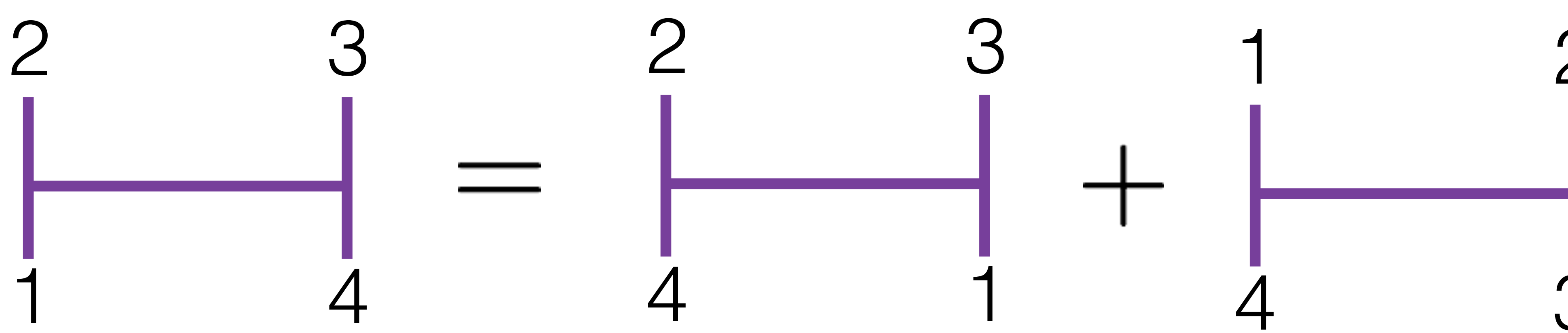
Jacobi

$$n_s \propto (k^2_{(1,2)} + k^2_{(3,4)}) \times (k_{[1,2]} \cdot k_{[3,4]})$$
$$\propto s \times (u - t - t + u)$$
$$\propto s \times (u - t)$$

$$k_{(ab)} = k_a + k_b$$
$$k_{[ab]} = k_a - k_b$$



Jacobi



n_s n_u n_t

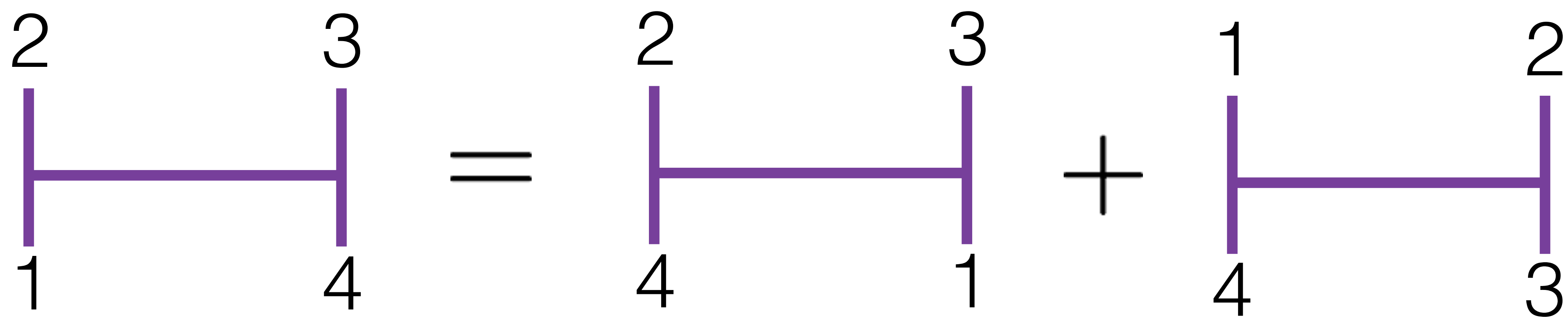
$$n_s \propto (k^2_{(1,2)} + k^2_{(3,4)}) \times (k_{[1,2]} \cdot k_{[3,4]})$$

$$\propto s \times (u - t - t + u)$$

$$\propto s \times (u - t)$$

$$k_{(ab)} = k_a + k_b$$

$$k_{[ab]} = k_a - k_b$$



Jacobi

$$0 \stackrel{?}{=} \boldsymbol{n_s - n_u - n_t}$$

$$\propto n_s - n_s\big|_{s\leftrightarrow u} - n_s\big|_{s\leftrightarrow t}$$

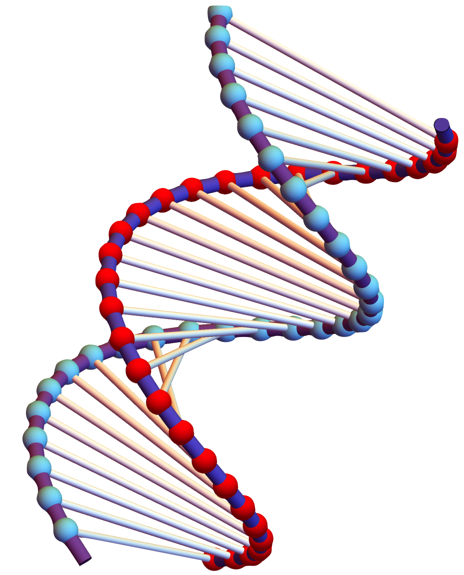
$$\propto s(u-t) - u(s-t) - t(u-s)$$

$$\propto su - st - us + ut - tu + ts = 0$$

Bern, JJMC, Johansson ('08,'10)

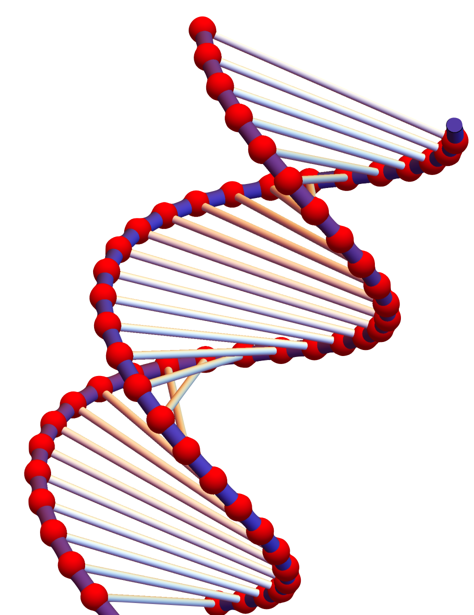
Generic D-dimensional YM theories have a fascinating structure at tree-level

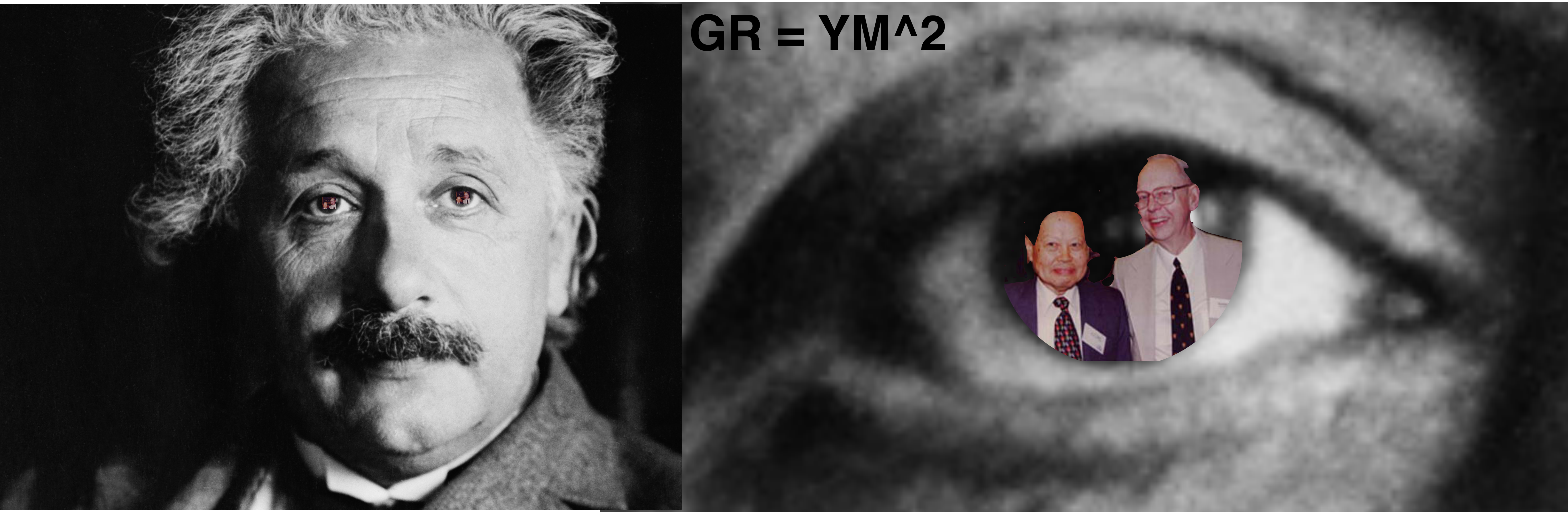
$$\mathcal{A}_m^{\text{tree}} = \sum_{\mathcal{G}\in\text{cubic}} \frac{c(\mathcal{G})n(\mathcal{G})}{D(\mathcal{G})}$$



YM: Color-Kinematic Duality, makes manifest gravitational double copy structure:

$$-iM_n^{\text{tree}} = \sum_{\mathcal{G}\in\text{cubic}} \frac{n(\mathcal{G})\tilde{n}(\mathcal{G})}{D(\mathcal{G})}$$

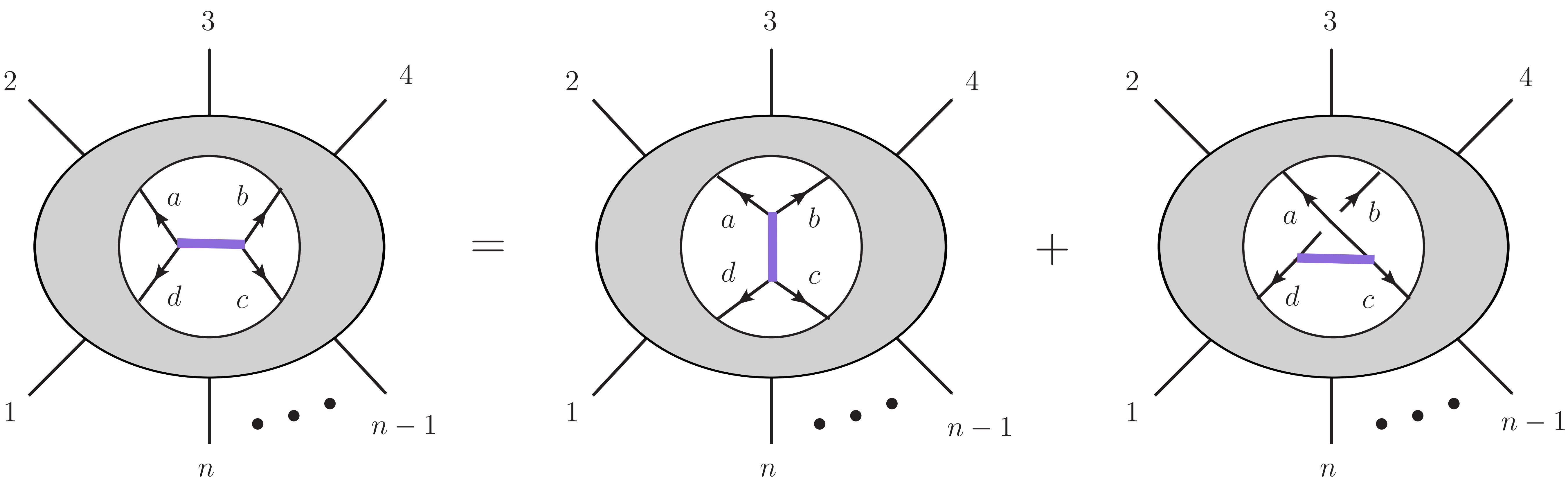



GR = YM^2

Valid multi-loop generalization?

$$\frac{(-i)^L}{g^{n-2+2L}}\mathcal{A}^{\text{loop}} = \sum_{\mathcal{G}\in\text{cubic}} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S(\mathcal{G})} \frac{n(\mathcal{G})c(\mathcal{G})}{D(\mathcal{G})}$$

CONJECTURE: for all graphs, can impose CK on every edge:

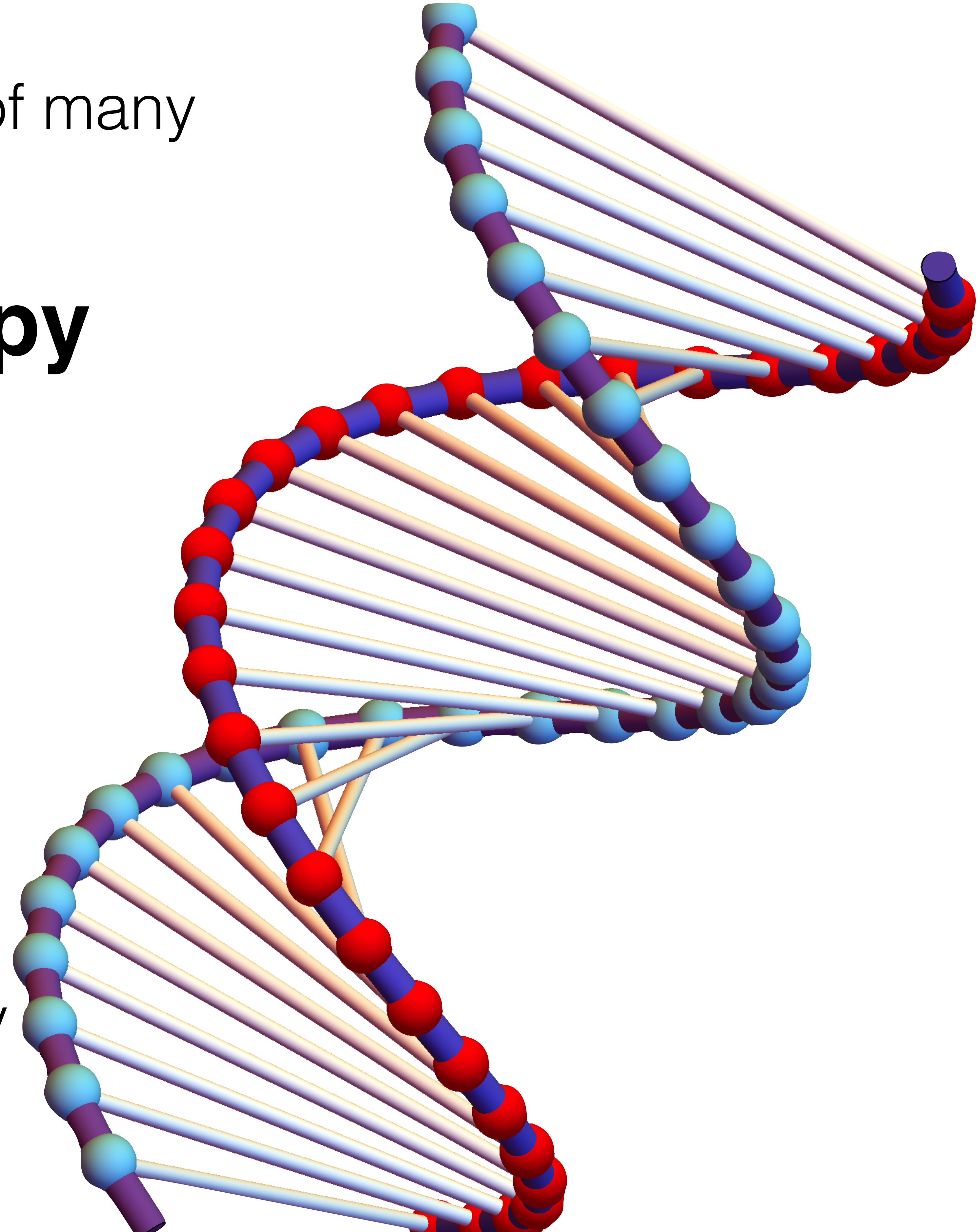


Consequence of unitarity: double copy structure holds.

$$\frac{(-i)^{L+1}}{(\kappa/2)^{n-2+2L}}\mathcal{M}^{\text{loop}} = \sum_{\mathcal{G}\in\text{cubic}} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S(\mathcal{G})} \frac{n(\mathcal{G})\tilde{n}(\mathcal{G})}{D(\mathcal{G})}$$

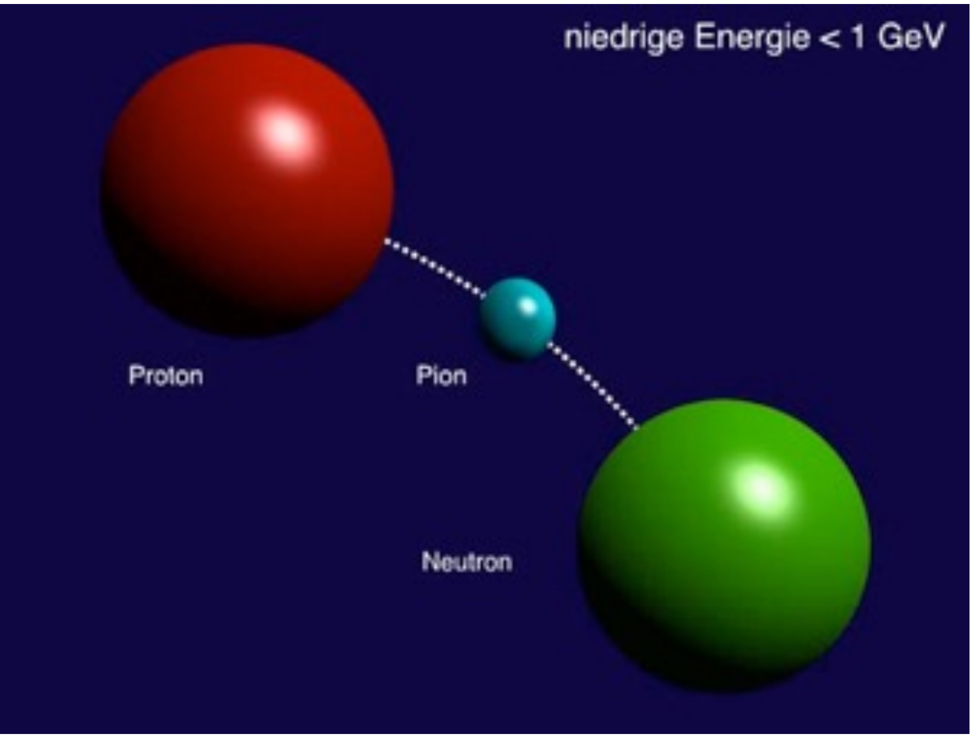
The scattering amplitudes of many relativistic theories admit a:

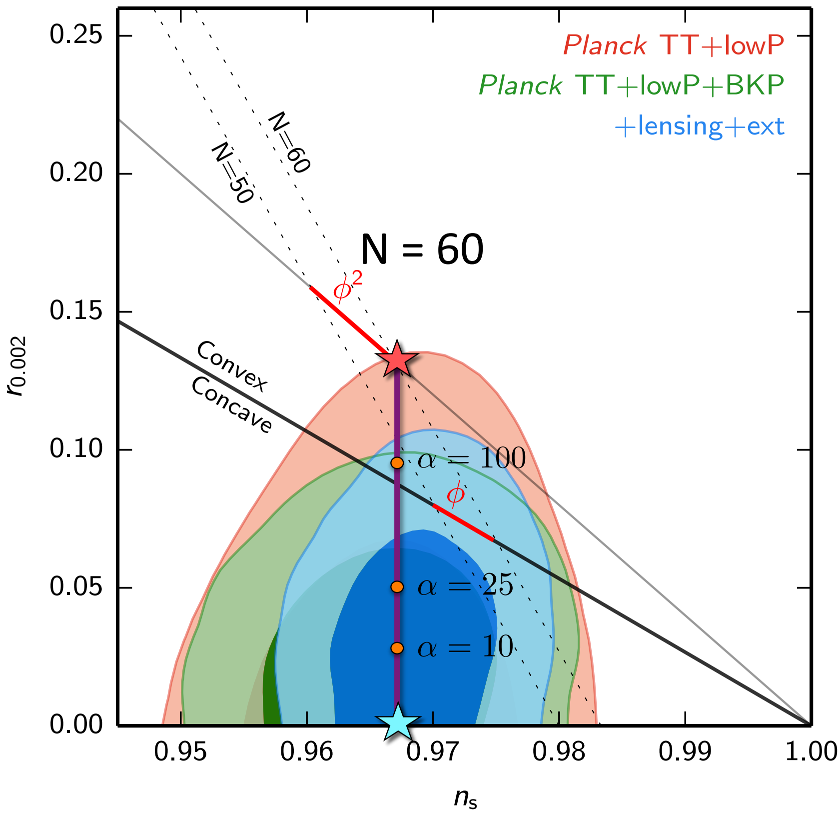
Double-copy
Numerator
Algebra



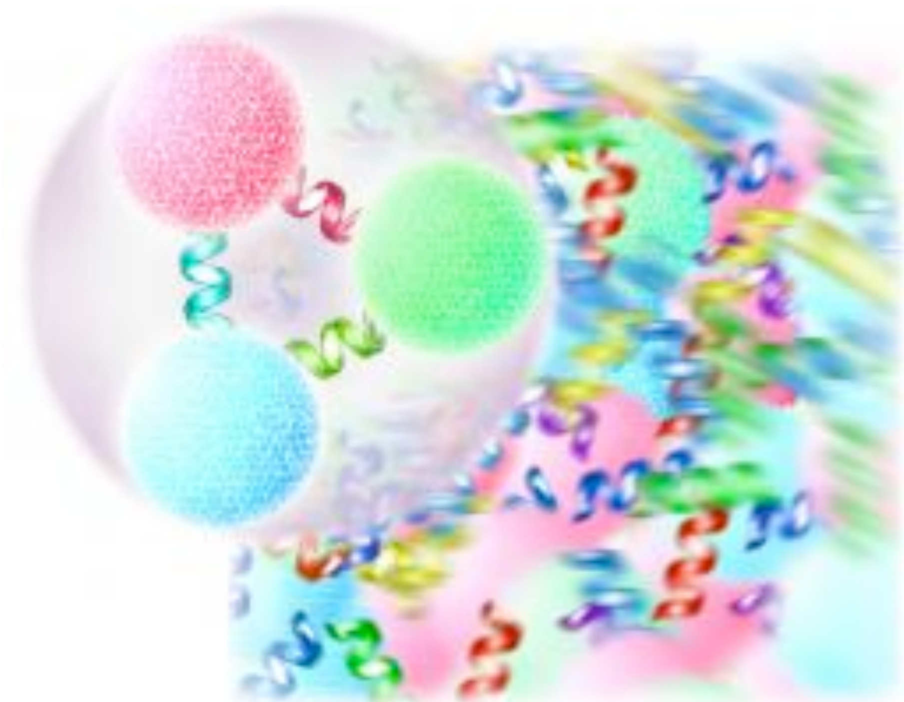
This points to previously hidden structure in many theories.

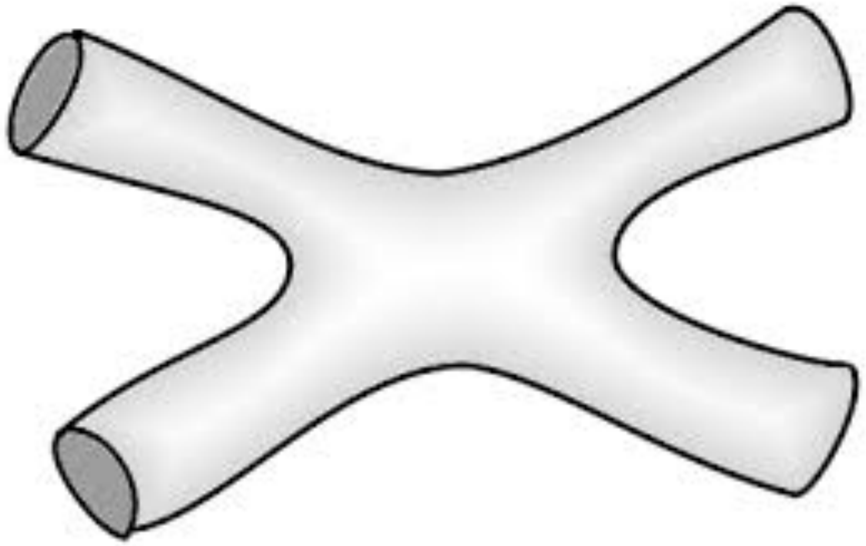
Structure yet to be generally understood at the level of the action.





Many theories are double copy!





Key Point: **MANY Theories are Double Copies**

Bi-Adjoint Scalar:	color	$\otimes$	color
Bern, de Freitas, Wong ('99); Bern, Dennen, Huang; Du, Feng, Fu; Bjerrum-Bohr, Damgaard, Monteiro, O'Connell			
(S)YM (…(S)QCD…):	color	$\otimes$	spin-1
BCJ ('08) Bjerrum-Bohr, Damgaard, Vanhove; Steiberger; Feng et al; Mafra, Schlotterer, ('08-'11); Johansson, Ochirov			
(S)Gr (…(S)Einstein-YM…):	spin-1	$\otimes$	spin-1
KLT('86); BCJ ('08); Chiodaroli, Gunaydin, Johansson, Roiban; Johansson, Ochirov; Johansson, Kälin, Mogull			
(S) Conformal Gravity:	$(DF)^2$	$\otimes$	spin-1
Johansson, Mogull, Teng '17,'18; Azevedo, Engelund '17			
NLSM / Chiral Lagrangian:	"color"	$\otimes$	even-spin-0
Chen, Du '13 Cachazo, He, Yuan '14 Cheung, Shen '16			
(S)Born-Infeld:	spin-1	$\otimes$	even-spin-0
Cachazo, He, Yuan '14			
Special Galileon:	even-spin-0	$\otimes$	even-spin-0
Cachazo, He, Yuan '14 Cheung, Shen '16			
Open String:	α'	$\otimes$	spin-1
Broedel, Schlotterer, Stieberger			
Closed String:	spin-1	$\otimes$	α' corrected spin-1
Broedel, Schlotterer, Stieberger;			
Z-theory:	α'	$\otimes$	"color"
Broedel, Schlotterer, Stieberger; JJMC, Mafra, Schlotterer			

Key Point: **MANY Theories are Double Copies**

Ingredients:

α'

$(DF)^2$

color

spin 0,1/2,1

For all these theories:

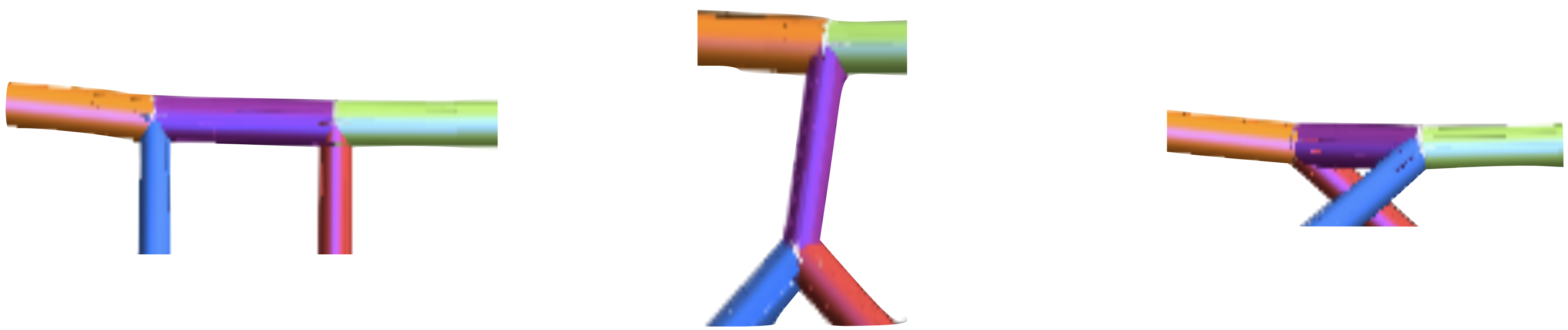
Bi-Adjoint Scalar	(S) YM (…(S) QCD…)	Conf. (S) Gr+…
NLSM	(S) Born-Infeld	(S) Gr (…(S) Einstein-YM…)
		Special Galileon
Z-theory	**Open String**	**Closed String**

a geometric guide to color-kinematics

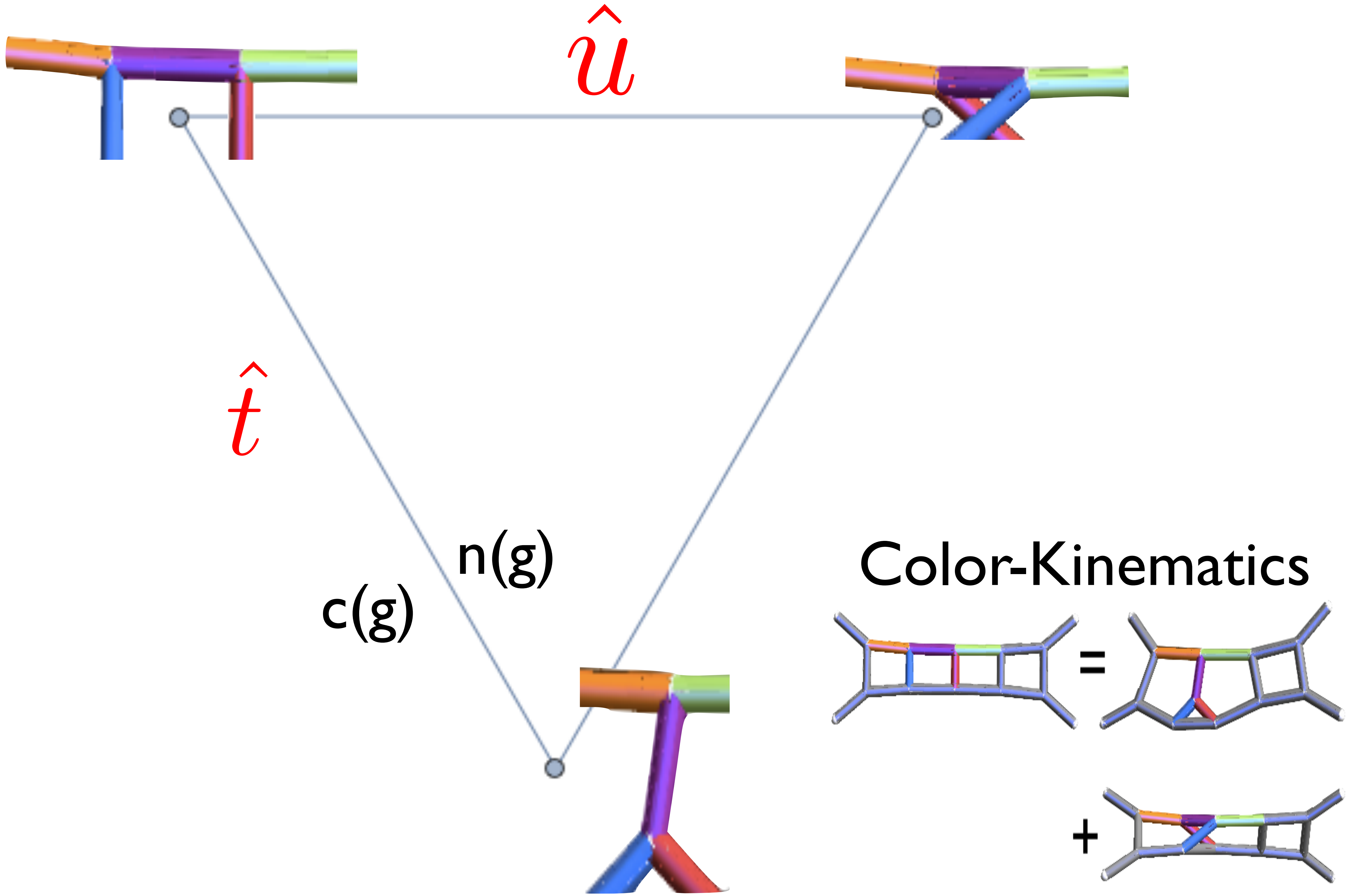
Physics = Geometry

(the best polytopes are graphs of graphs!)

Cubic graphs contributing to 4-pt Tree



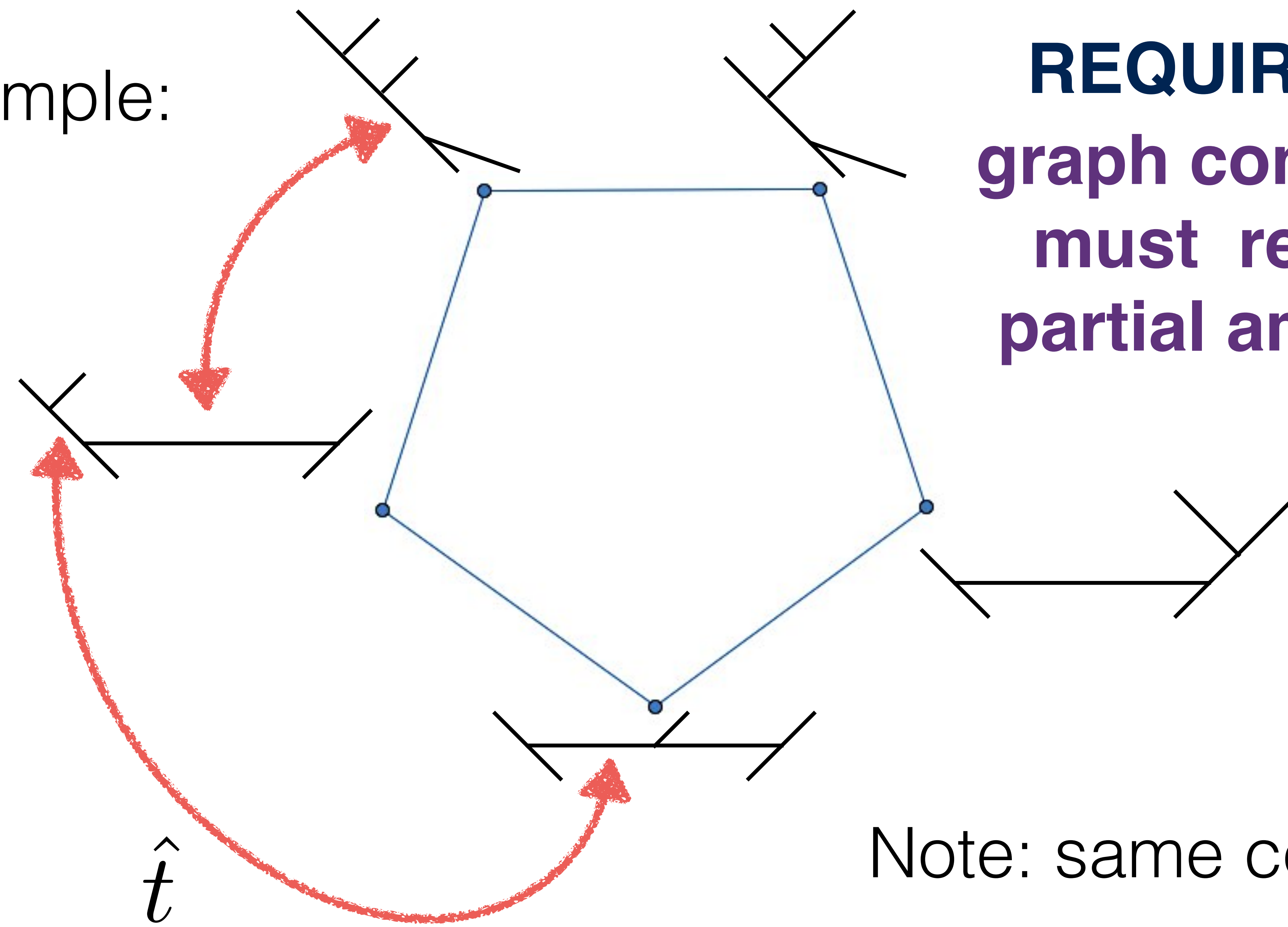
Convenient language: graphs of graphs



Theory specific input: Partial amplitudes

Graphs contributing to a tree-level **color-stripped YM partial amplitude**, generate the 1-skeleton of **Stasheff polytopes** joined only by $\hat{t}$

5pt example:



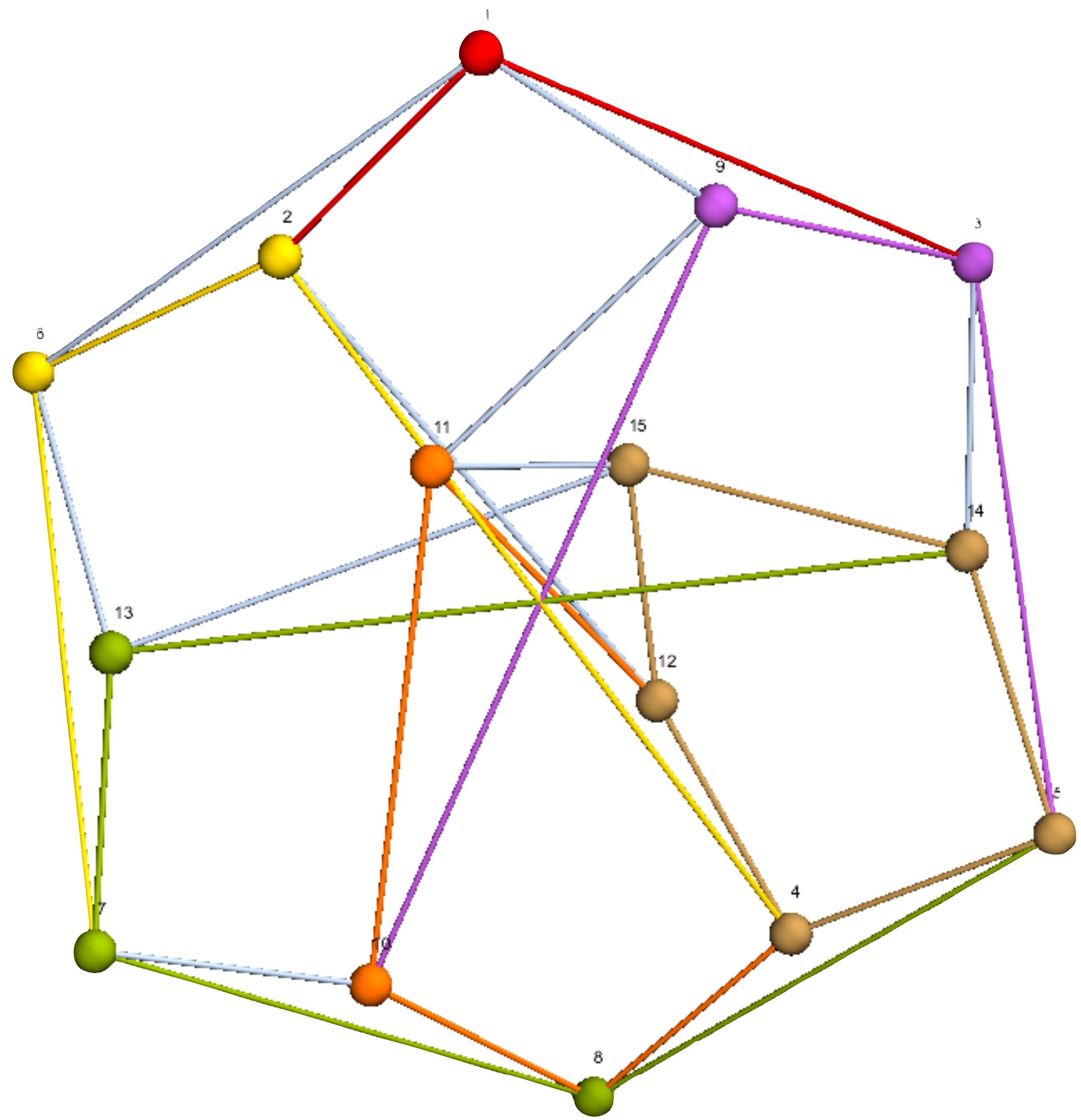
REQUIREMENT

graph contributions must reproduce partial amplitudes.

Note: same color-order!

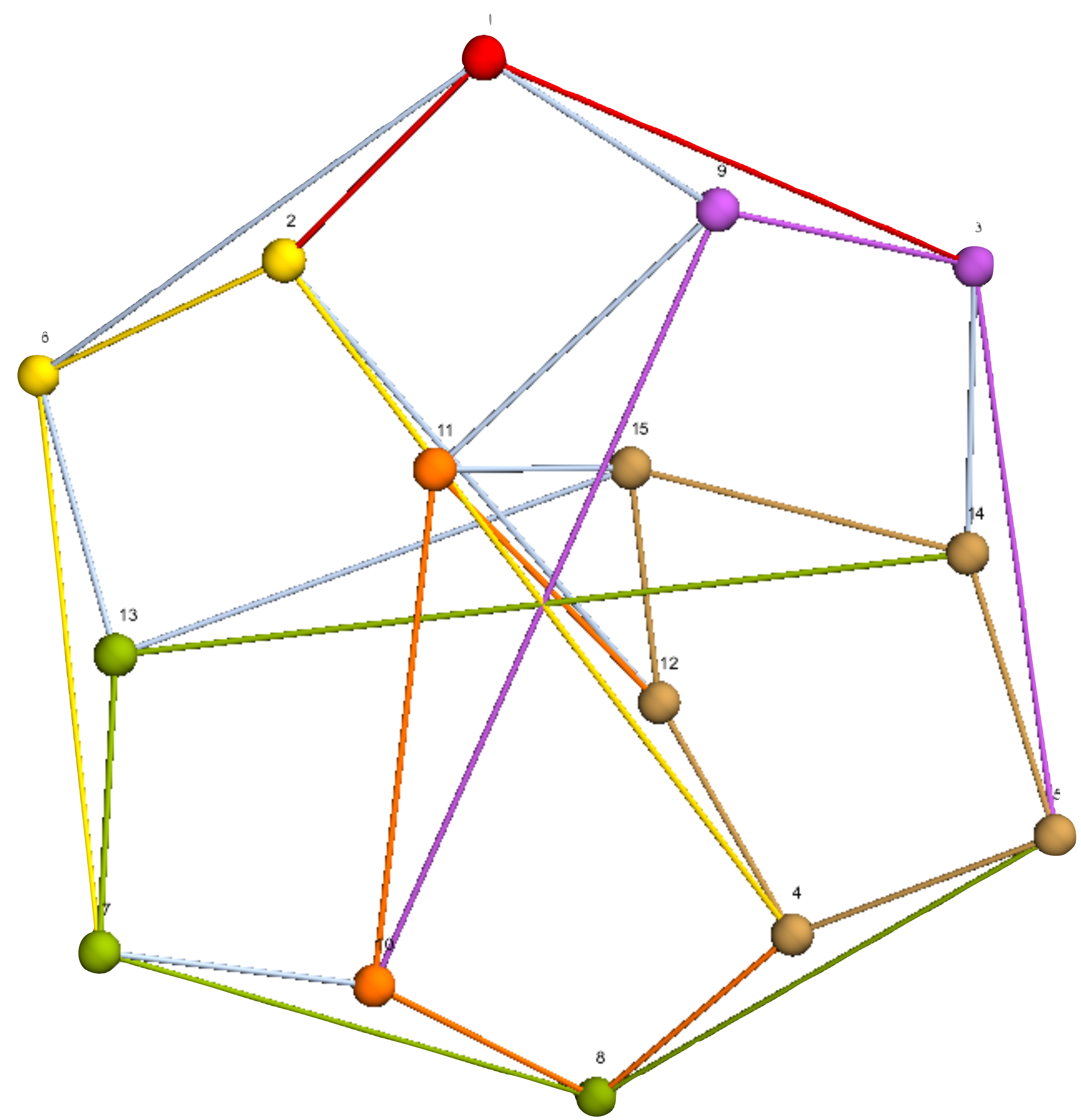
(these polytopes are also called **associahedra**)

You might think you need (m-2)! of these color-stripped amplitudes to capture everything because this is what is required to touch every vertex at least once:



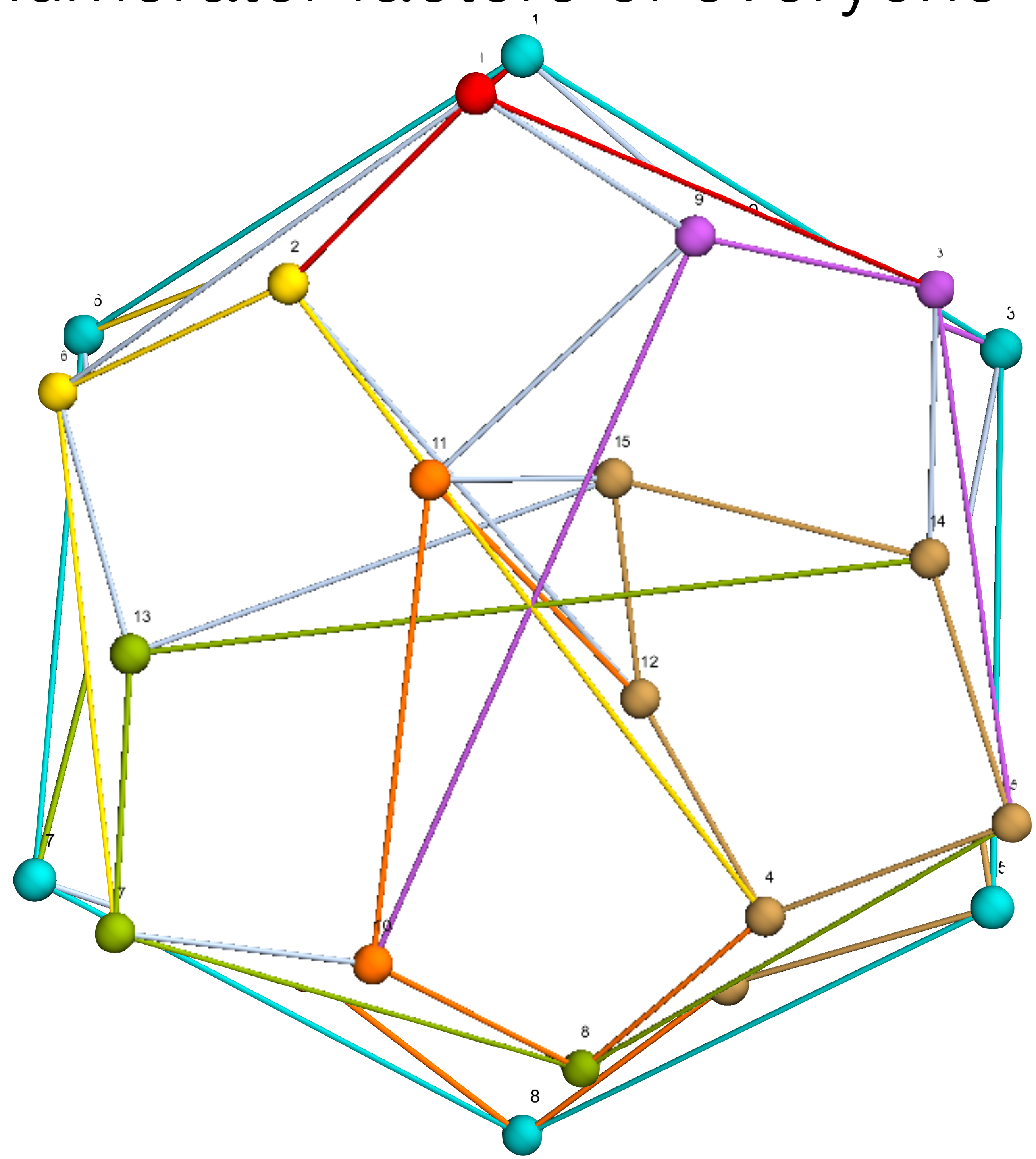
This ensures everyone talks well with each other.

You might think you need (m-2)! of these color-stripped amplitudes to capture everything because this is what is required to touch every vertex at least once:

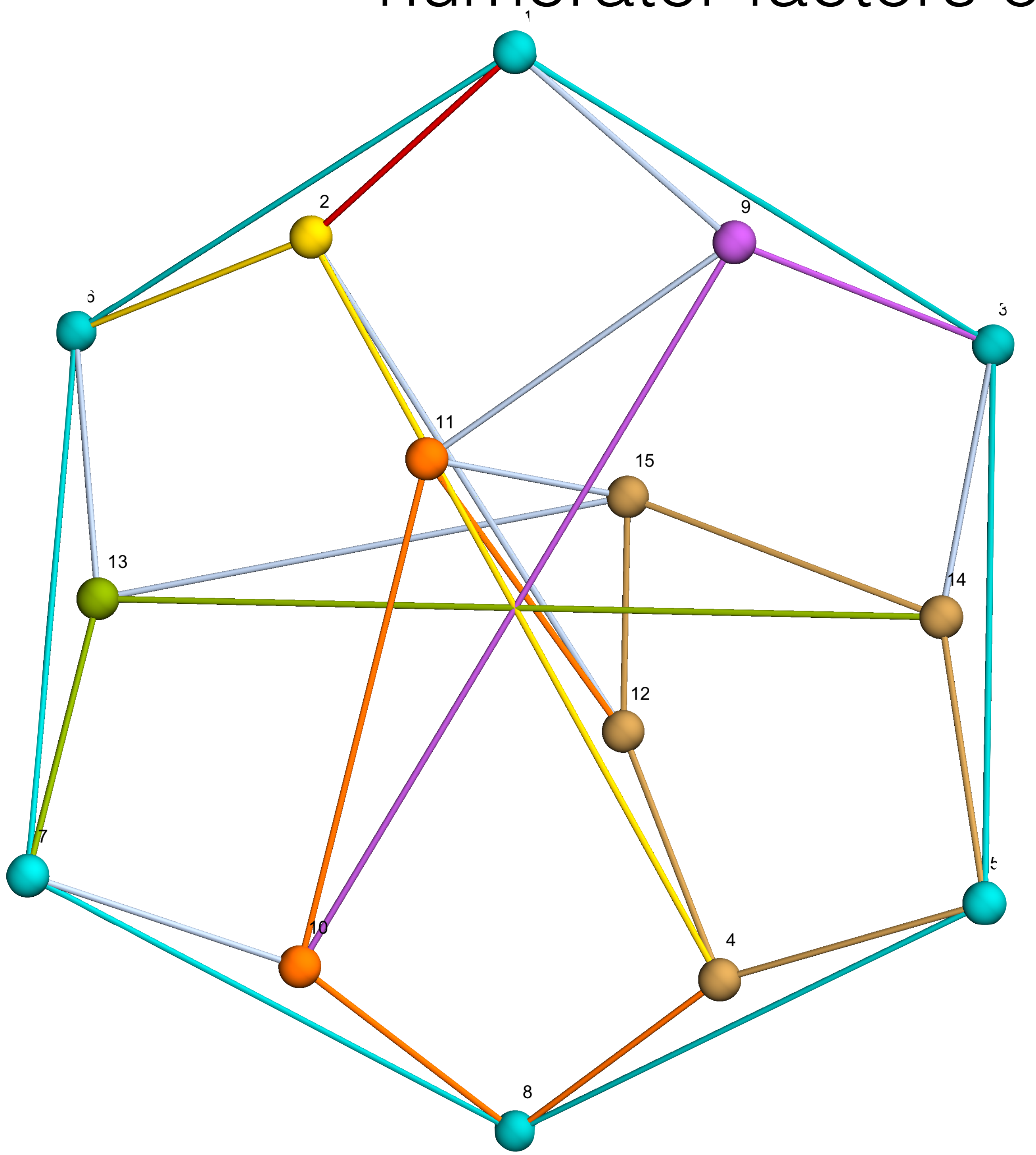


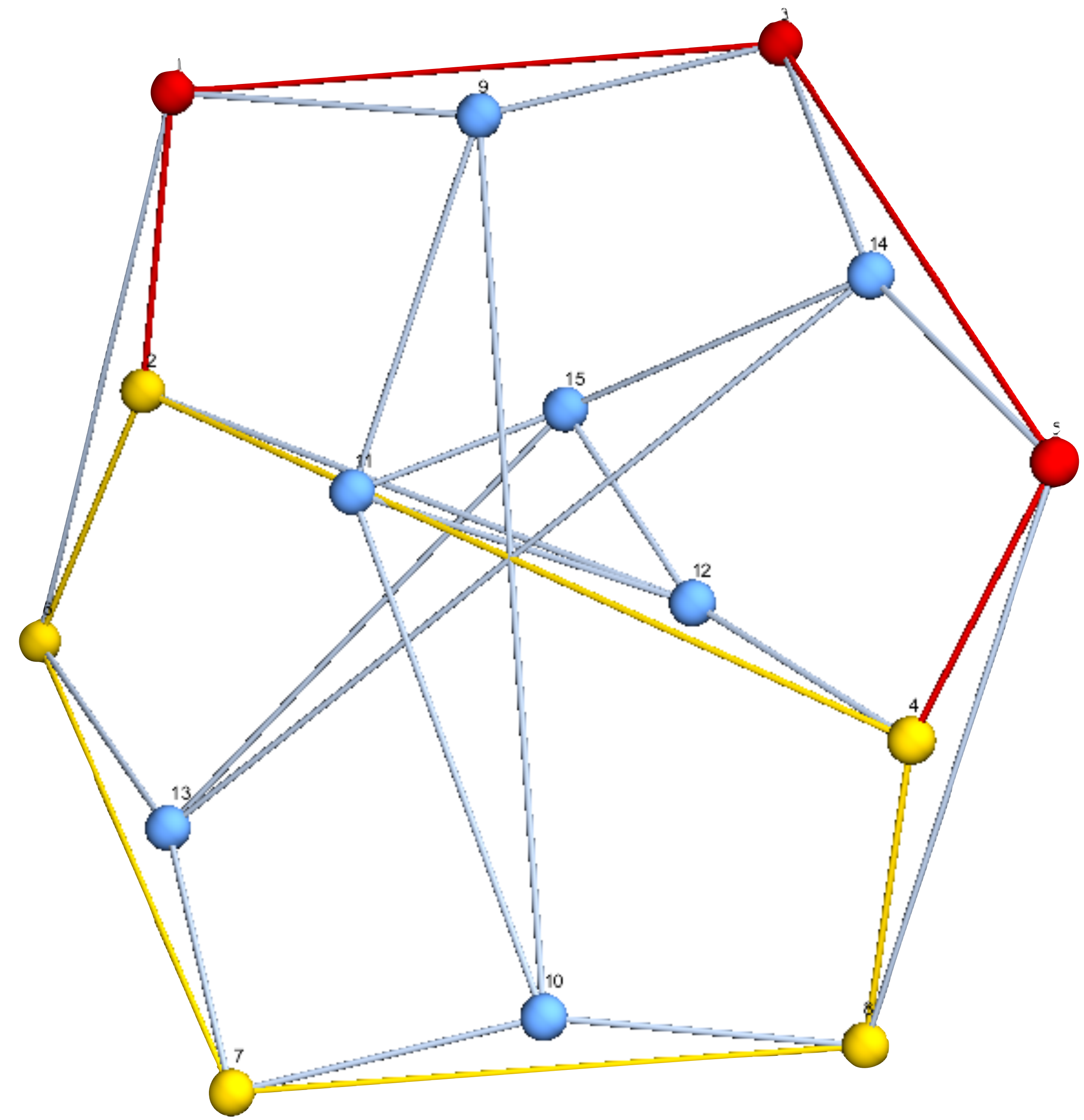
In fact, such a choice is the KK-basis, proven sufficient by Del Duca, Dixon, and Maltoni

But notice, because of color-kinematics, only (m-2)! nodes are needed to specify both the color factors and numerator factors of everyone



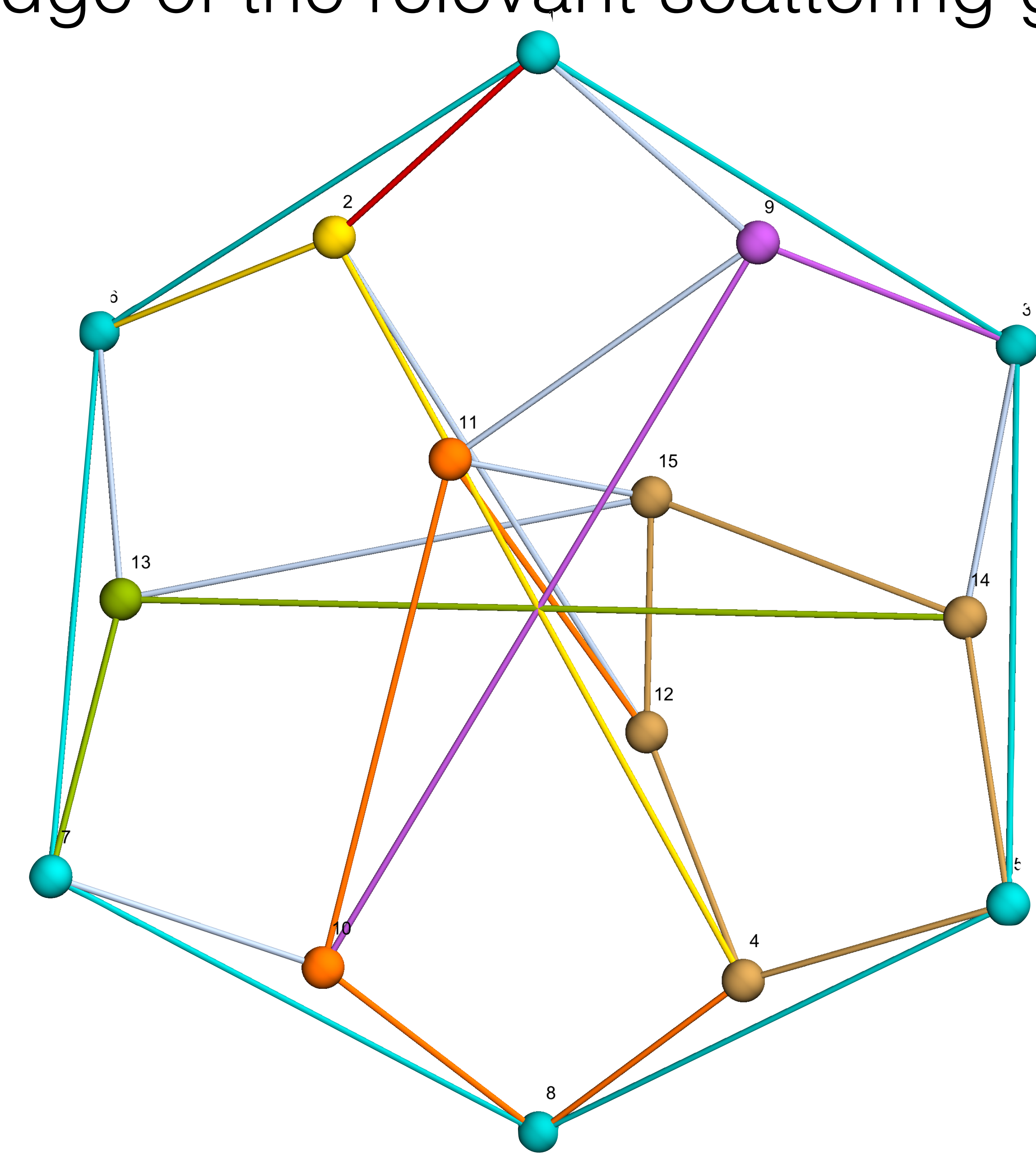
But notice, because of color-kinematics, only (m-2)! nodes are needed to specify both the color factors and numerator factors of everyone

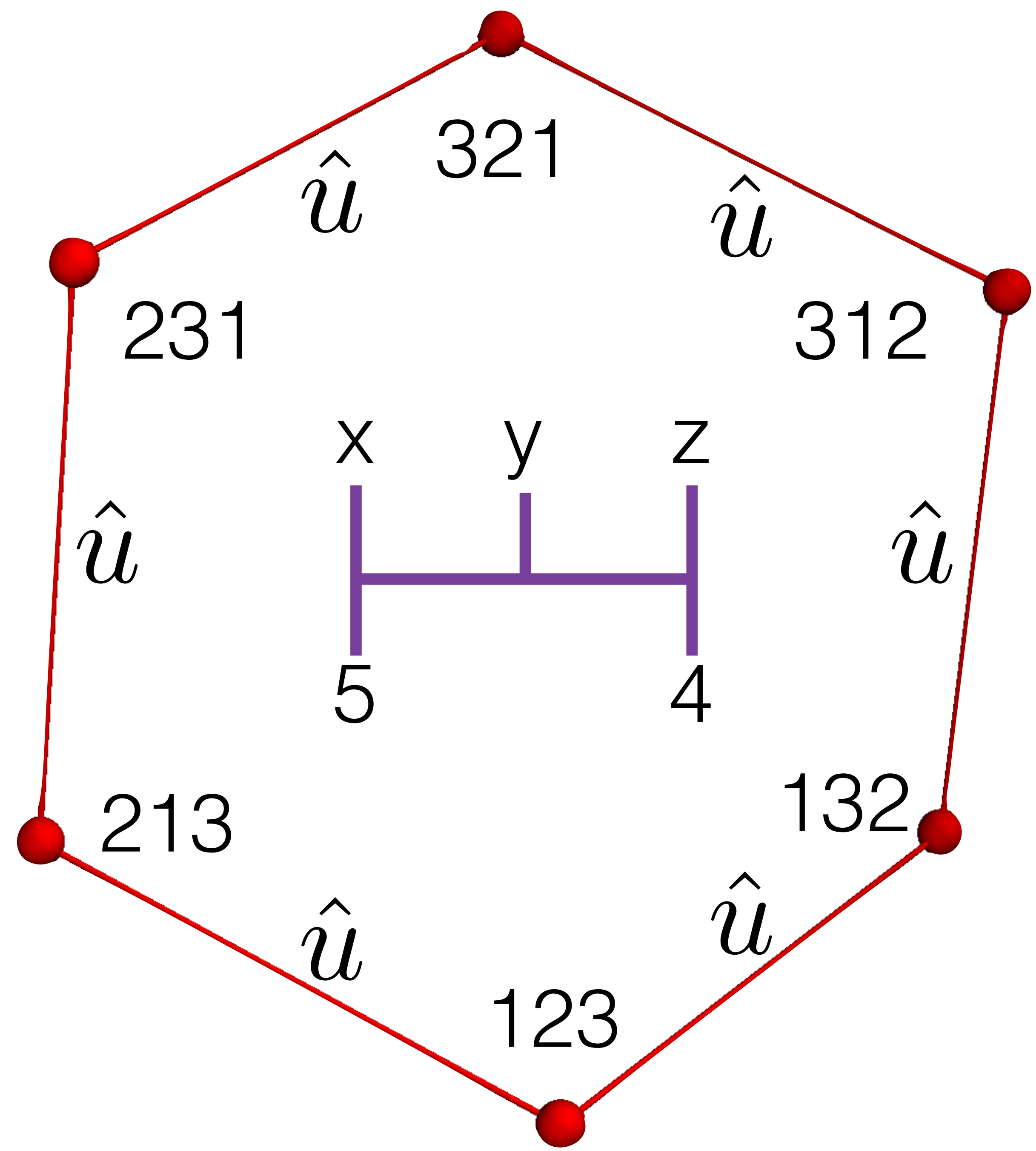




This reduces the set of necessary ordered partial amplitudes (associahedra) to (m-3)! : “BCJ” relations

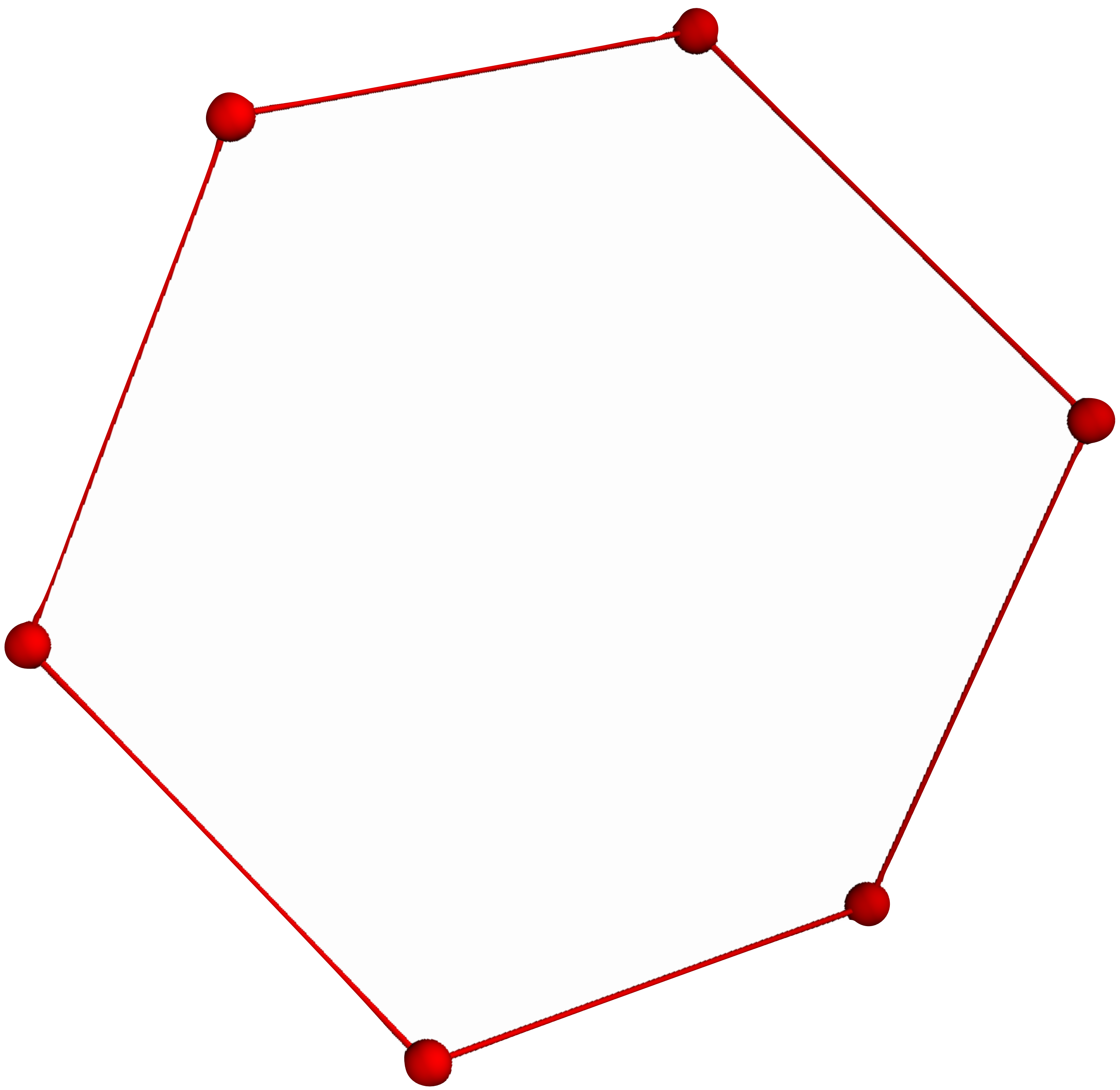
At every multiplicity the masters can be chosen to form the 1-skeleton of a polytope related by $\hat{u}$ on every internal edge of the relevant scattering graphs

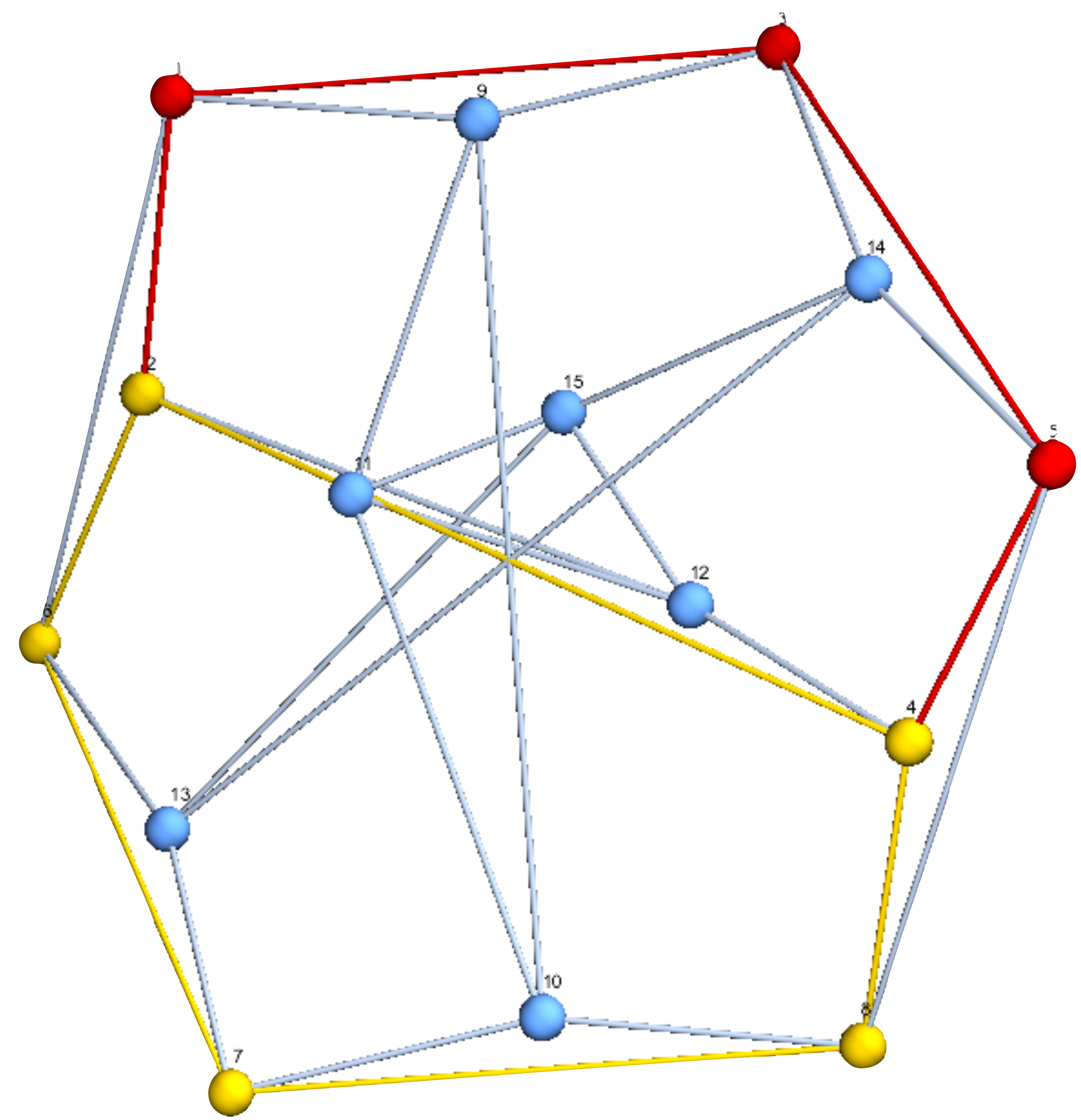




(these polytopes are called **permutahedra**)

Can linearly solve for the (m-2)! numerators of the masters in terms of the (m-3)! “BCJ” independent color-ordered amplitudes. In fact you get (m-3)! numerators in terms of the ordered partial amplitudes and (m-3)(m-3)! free functions.



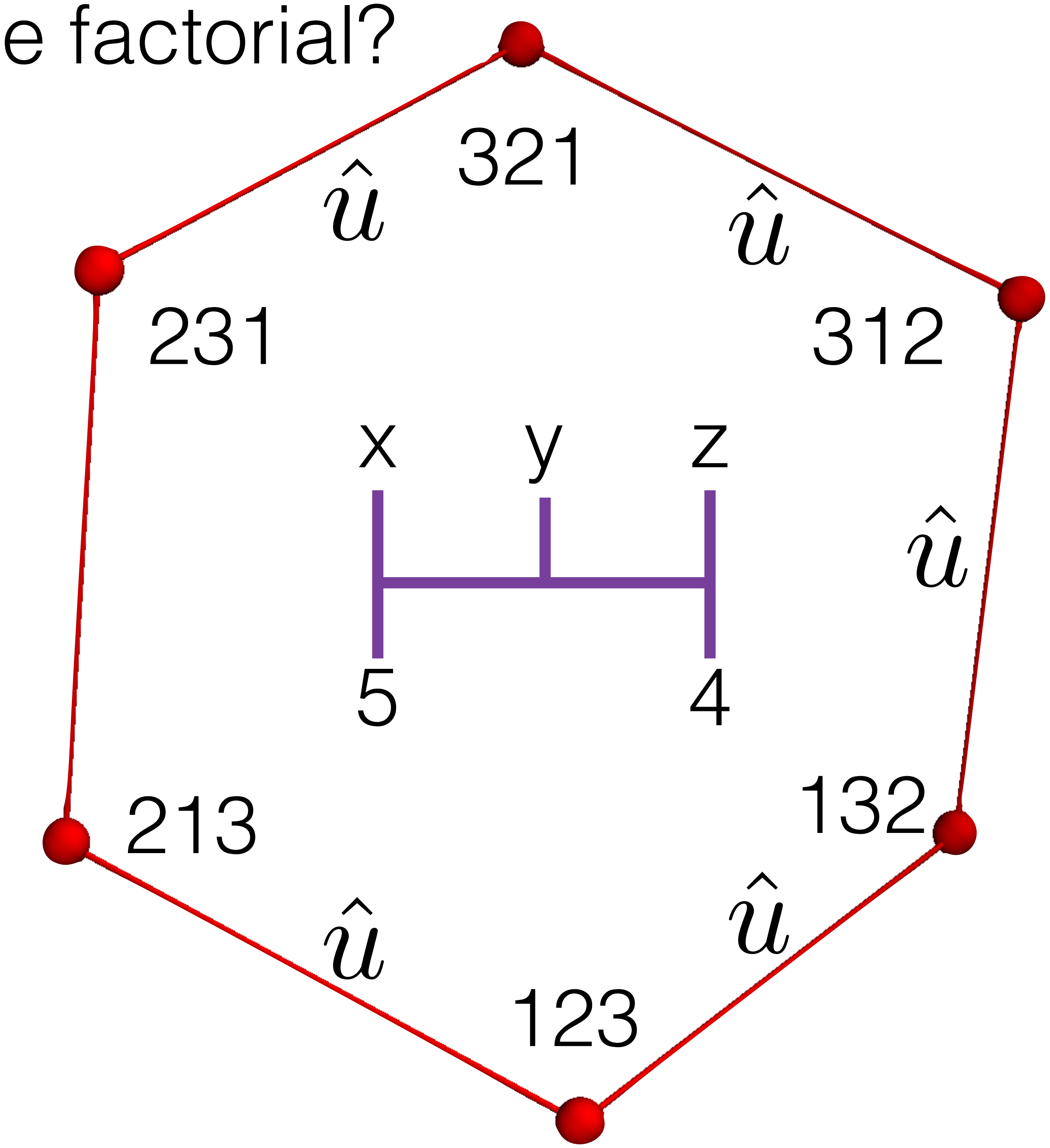


(generalized gauge freedom)

Can linearly solve for the **(m-2)!** numerators of the masters in terms of the (m-3)! “BCJ” independent color-ordered amplitudes. In fact you get **(m-3)!** numerators in terms of the ordered partial amplitudes and **(m-3)(m-3)!** free functions.

But what about beating down the factorial?

Color-kinematics and automorphic invariance reduce everything to 1 function at tree-level at all multiplicity:
the half-ladder dressing



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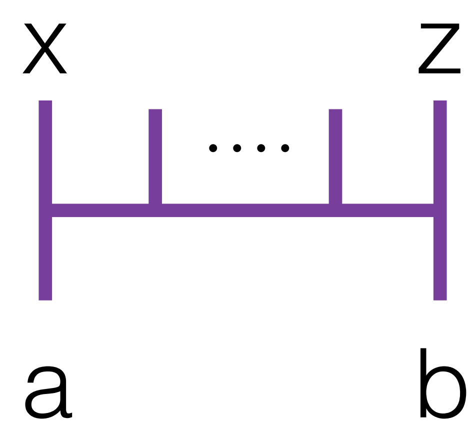
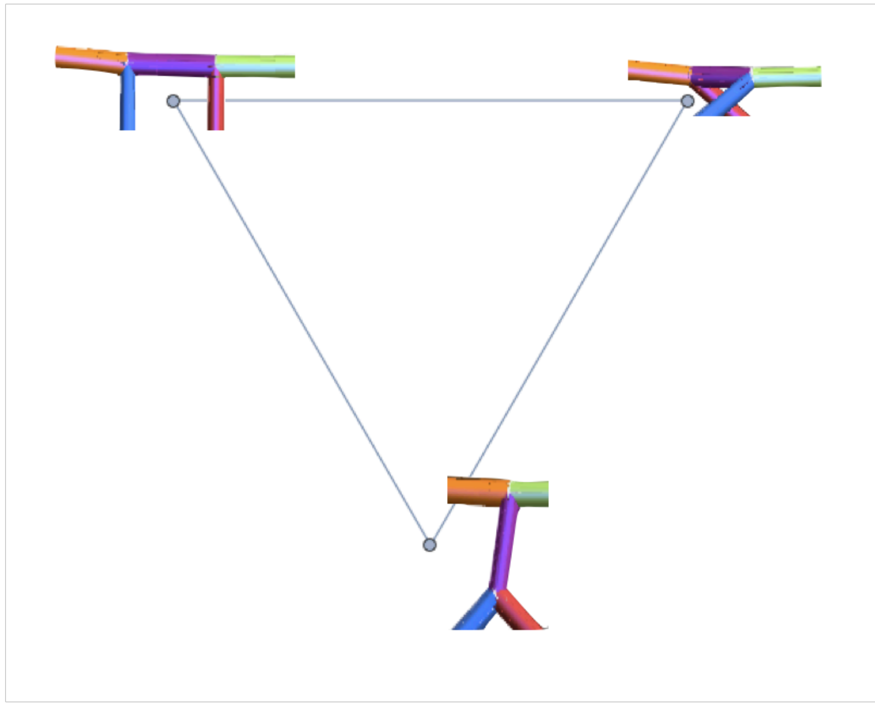




Recall our automorphic invariant Jacobi satisfying dressing for NLSM: $n_s \propto s \times (u - t)$

This is not the only dressing. Can instead solve:

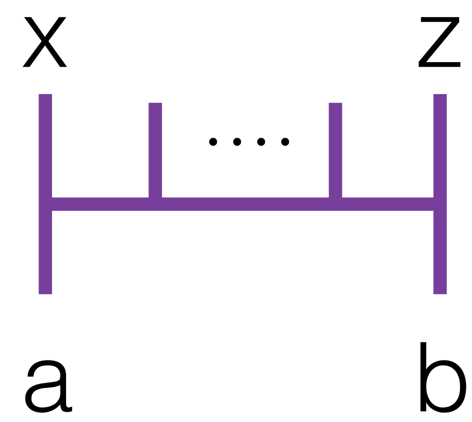
$$A(s,t) = \frac{n_s}{s} + \frac{n_t}{t} \qquad A(s,u) = -\frac{(n_u \equiv n_s - n_t)}{u} - \frac{n_s}{s}$$

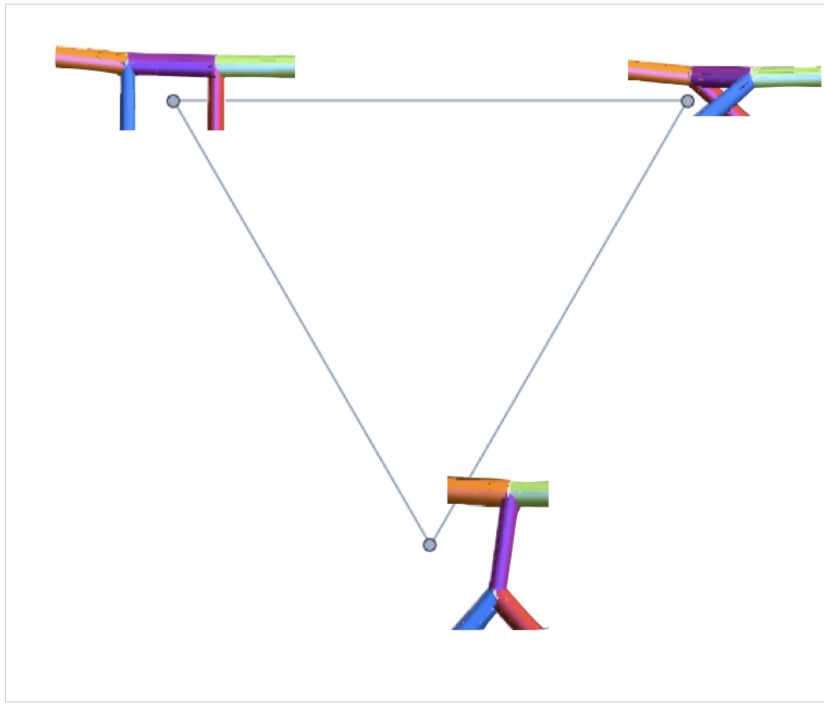
We find that:

$$n_s = s\,A(s,t) - \frac{s}{t} n_t \qquad A(s,u) = A(s,t)\frac{t}{u}$$

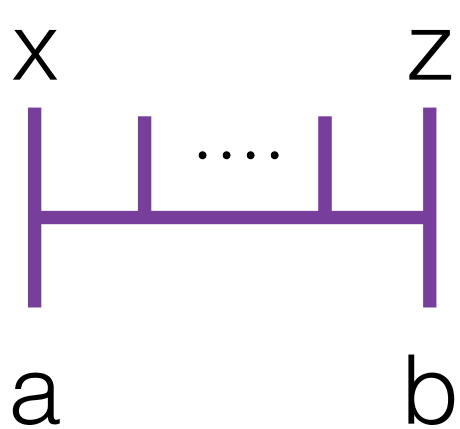
Color-kinematics and automorphic invariance reduce everything to 1 function at tree-level at all multiplicity:

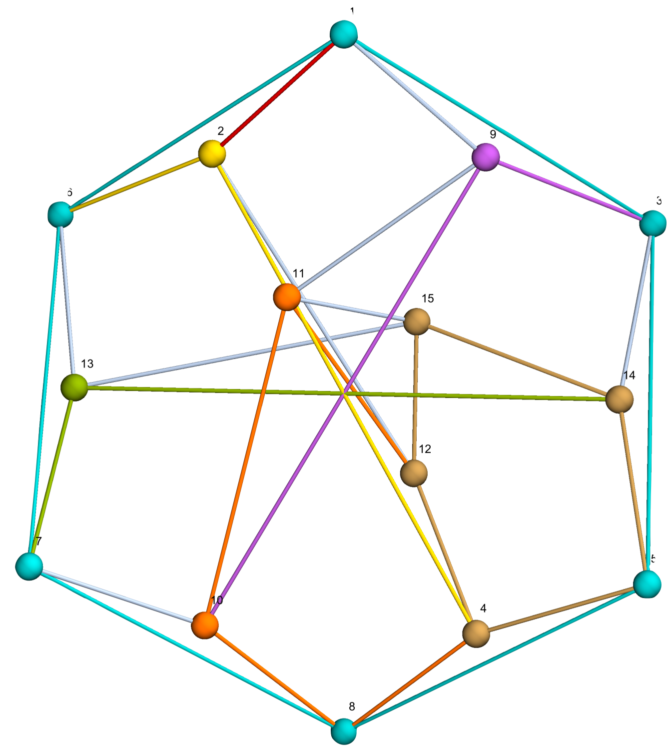
the half-ladder dressing





$$n_s \propto {\color{blue}s} \times ({\color{red}u} - {\color{red}t}) \qquad \text{vs} \qquad n_s = s\,A(s,t) - \frac{s}{t}{\color{blue}n_t}$$



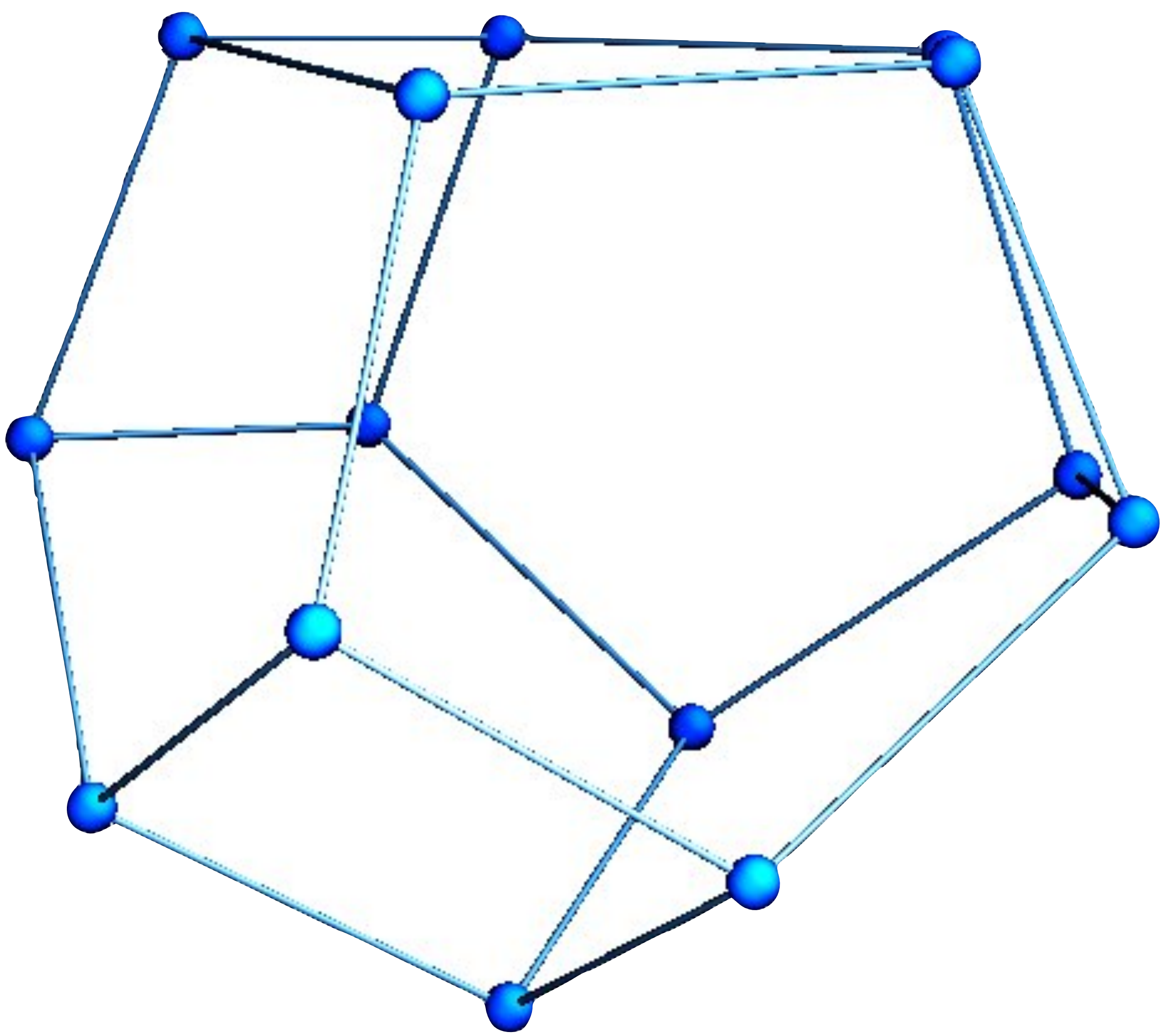


1 function at each **m**

(m-2)! functions at each **m**

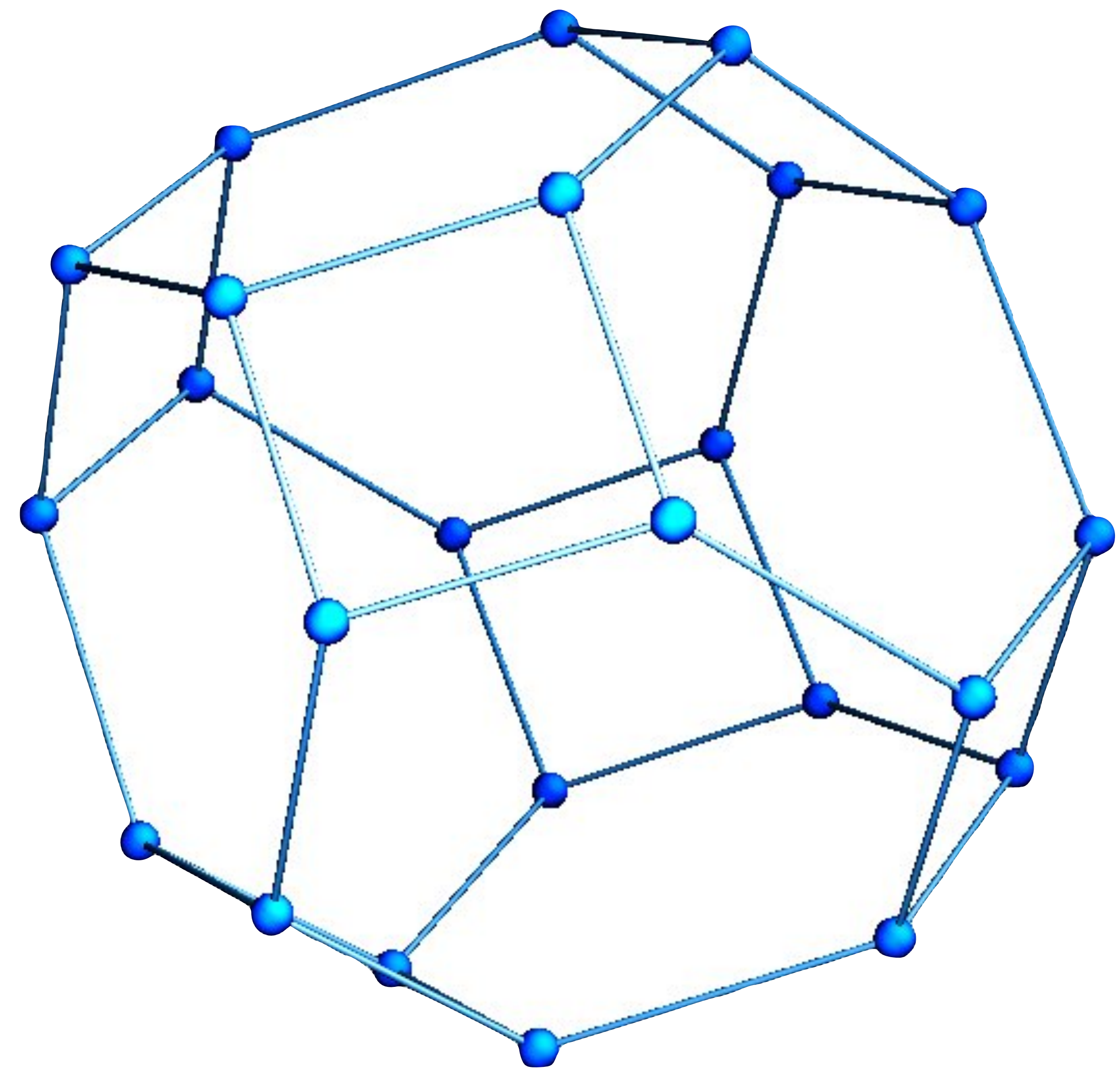
Building blocks at 6-points:

color-ordered amplitude



associahedron

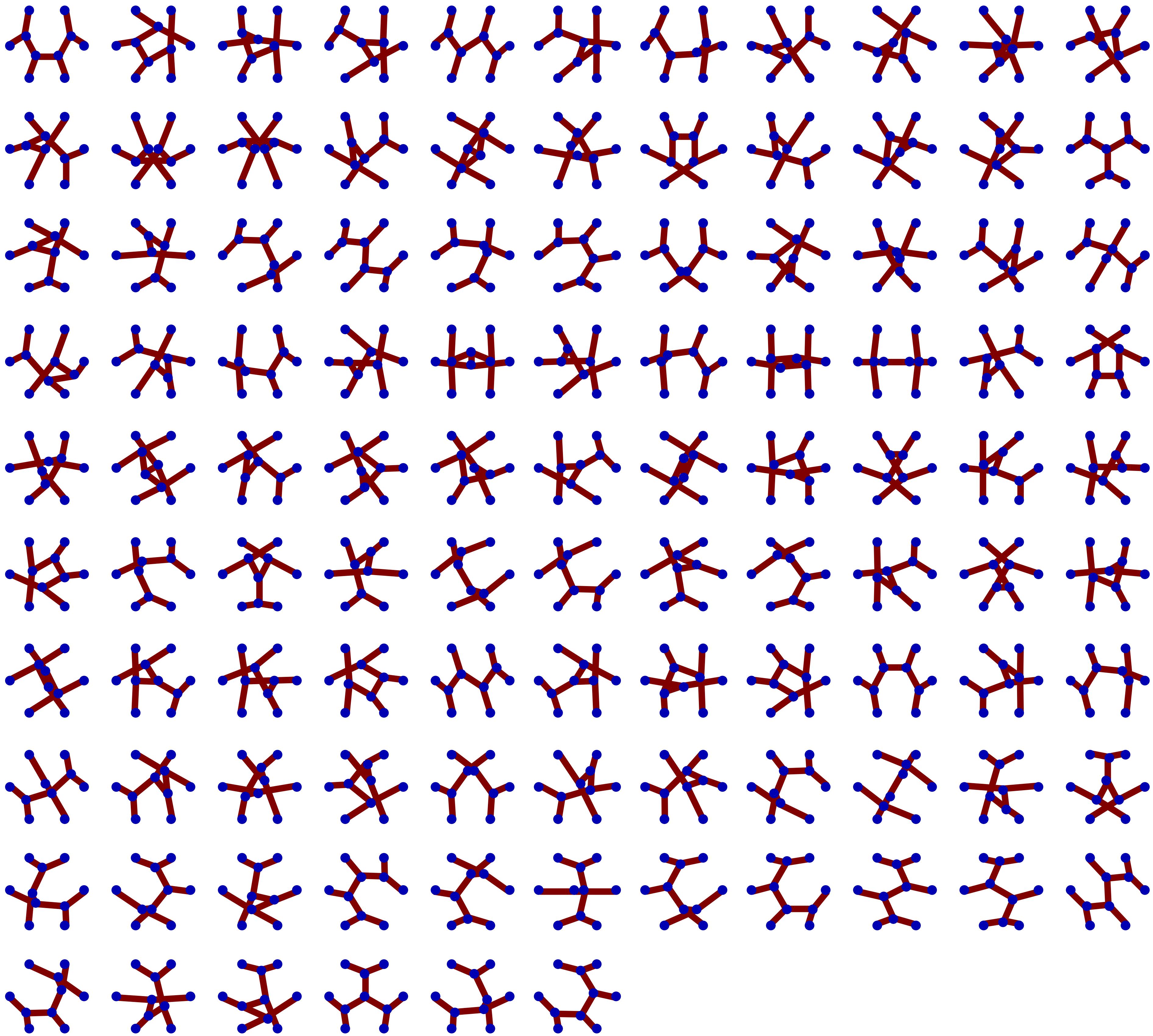
set of masters

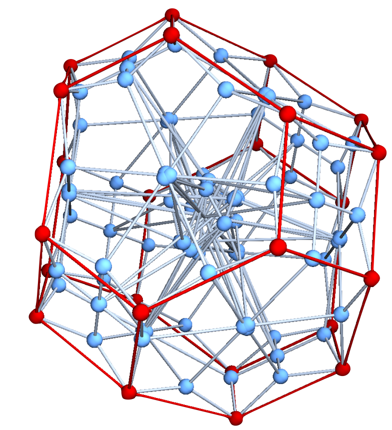


permutahedron

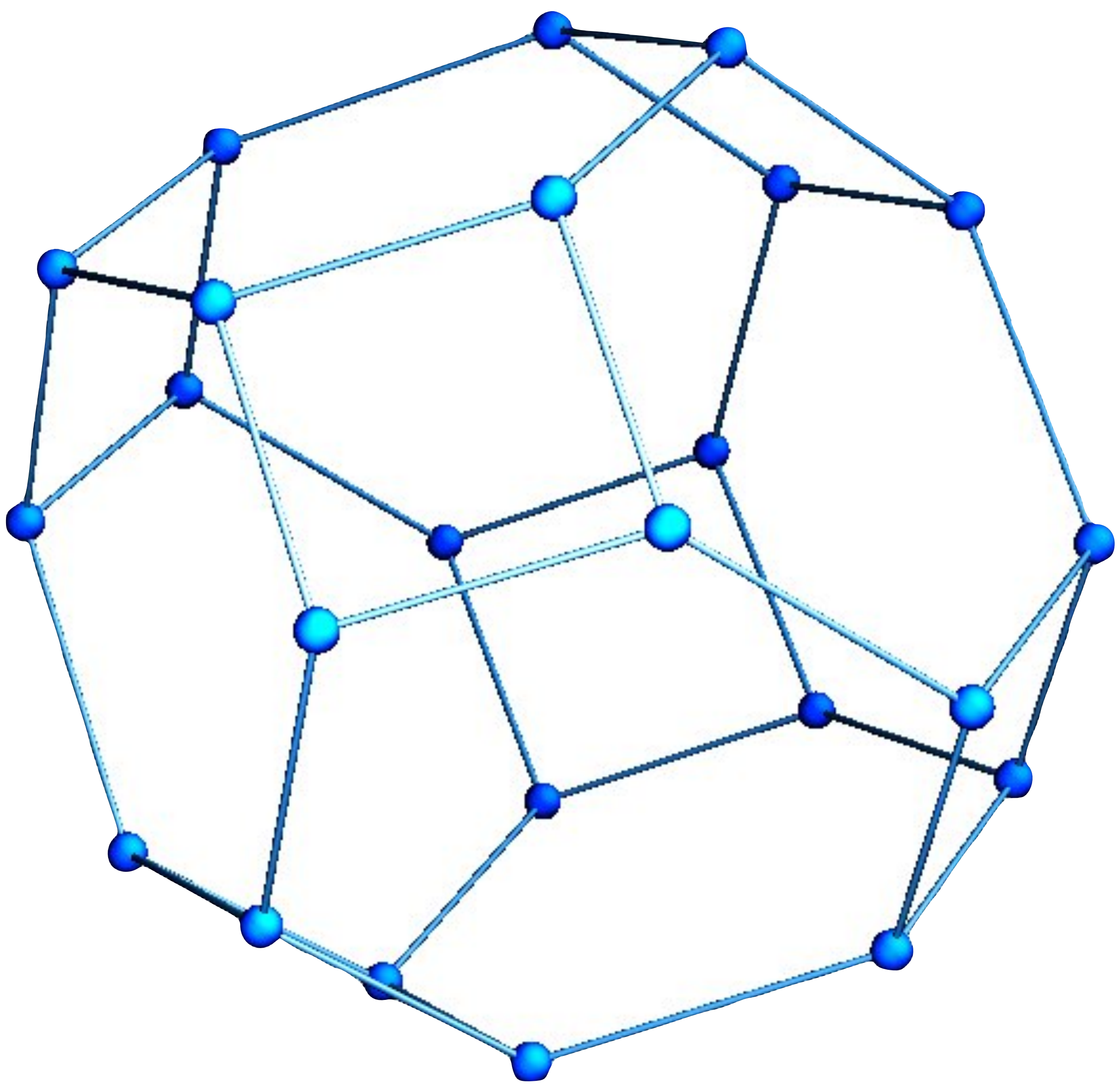
JJMC

105
cubic graphs at 6 pt

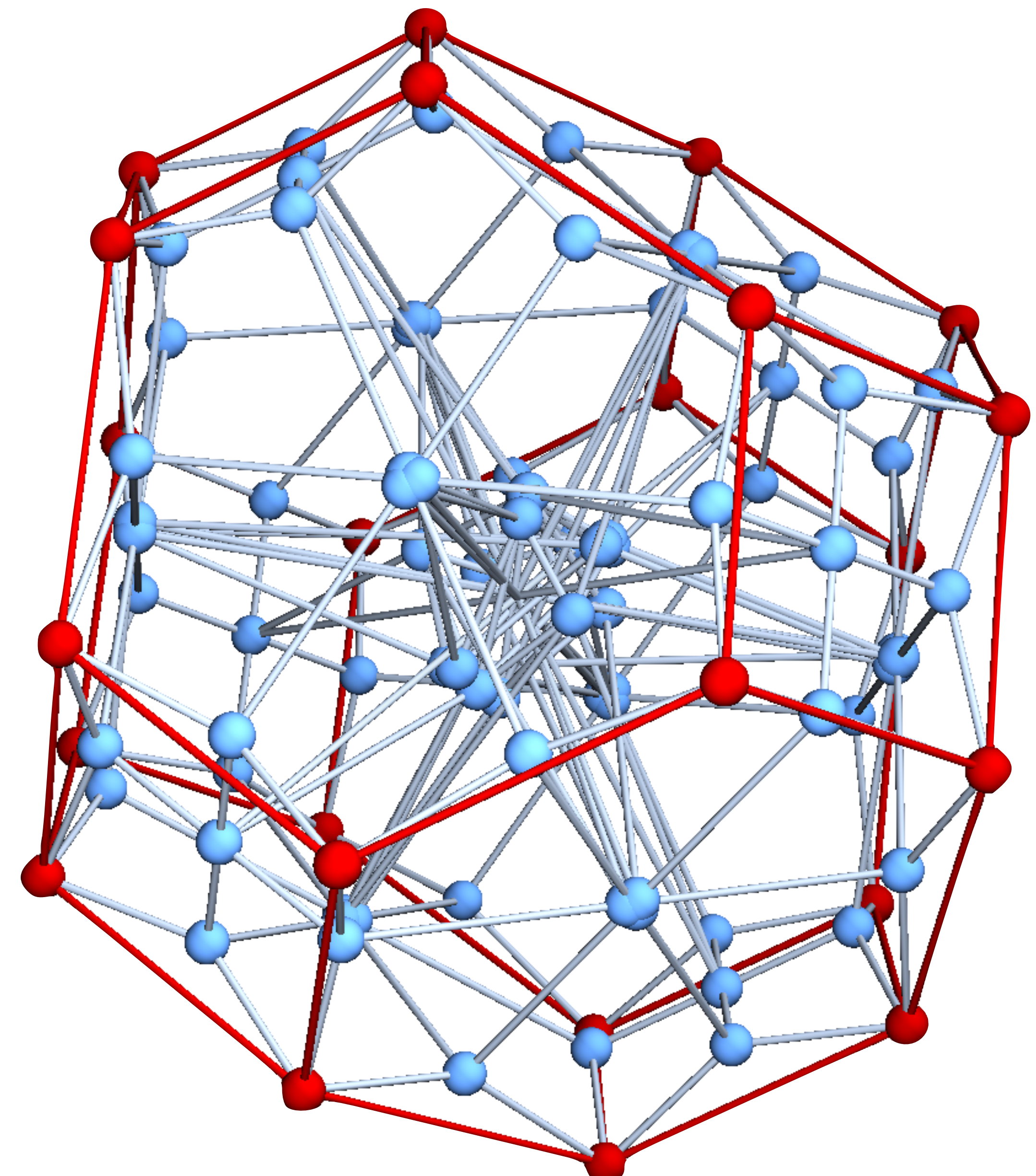


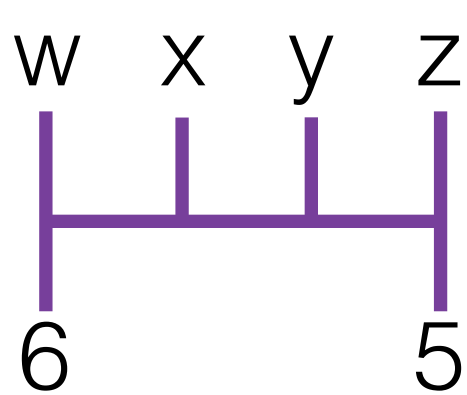


set of masters

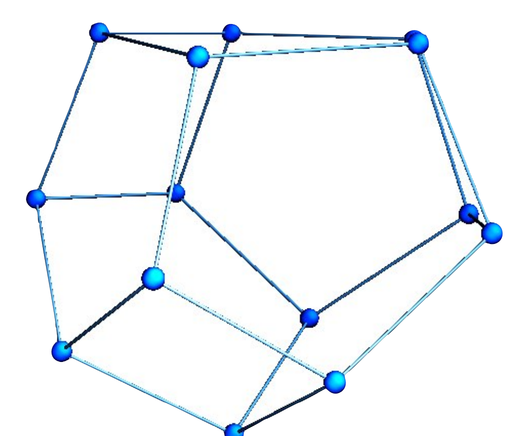


full amplitude



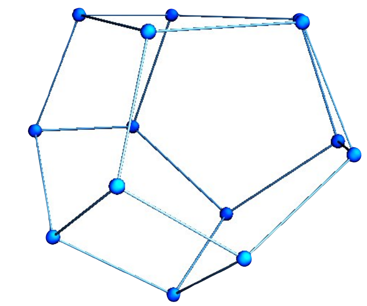


masters fixed by 6

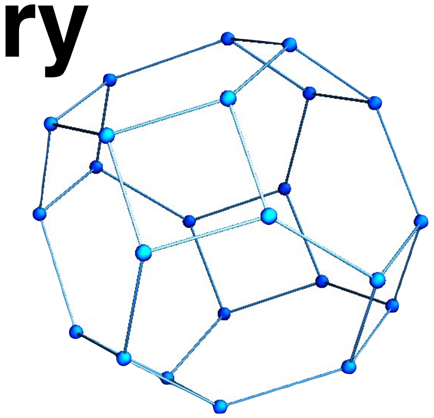


TREE-LEVEL SUMMARY

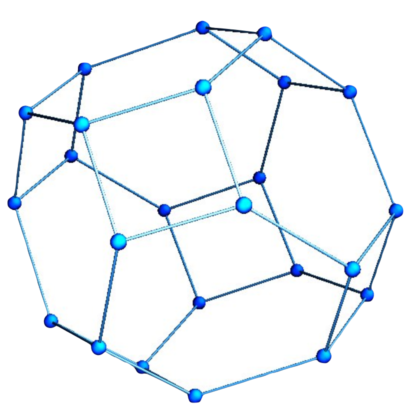
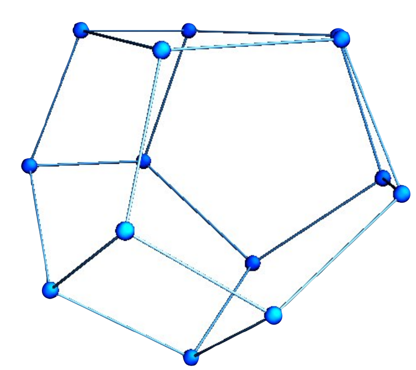
1. **Gauge invariant building blocks that speak to the theory:** color-ordered amplitudes, *associahedra*



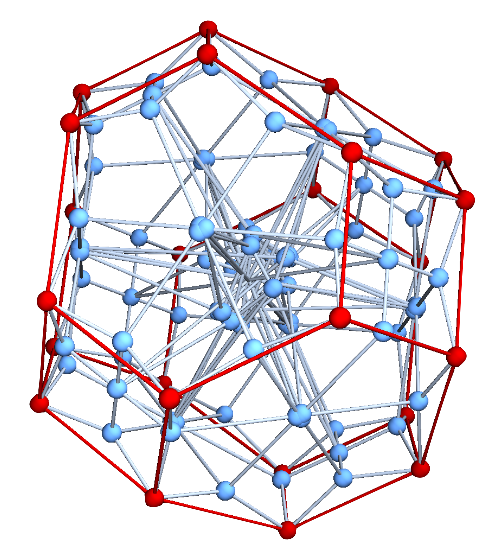
2. **CK means only need to specify the boundary data**: the master graphs, given by the relevant *permutahedron*



3. **Can solve for the *full amplitude efficiently* in terms of the (n-3)! independent *associohedra***

 = f()

(linear)

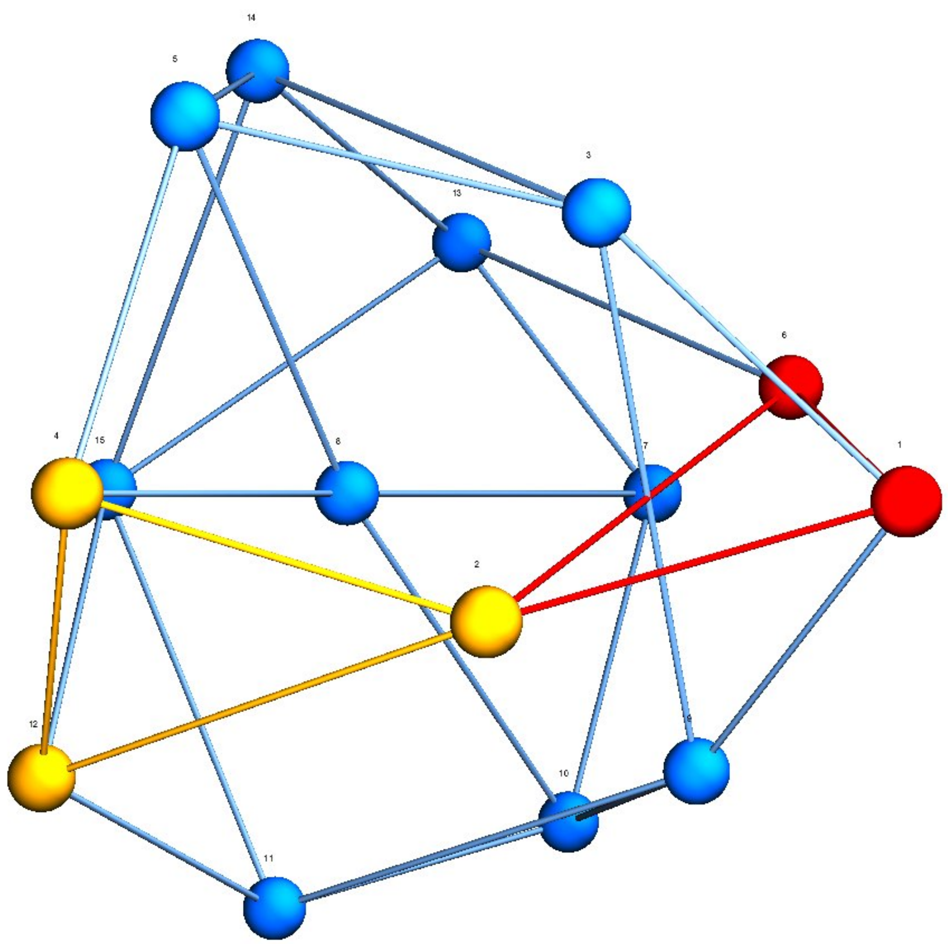


physics <—> geometry

Can do this on loop-level cuts. Can generalize to the off-shell integrand either by introducing ansatze or with a massive over-redundancy of graphs (the pre-Integrand).

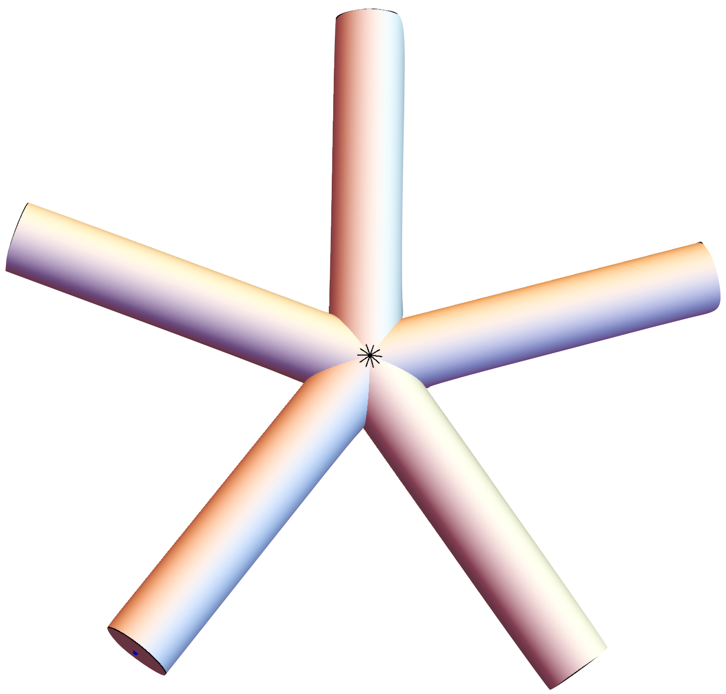
JJMC

Natural question, given a generic (non color-dual) representation for a **gauge amplitude**, and all you want is the related **gravity amplitudes**.

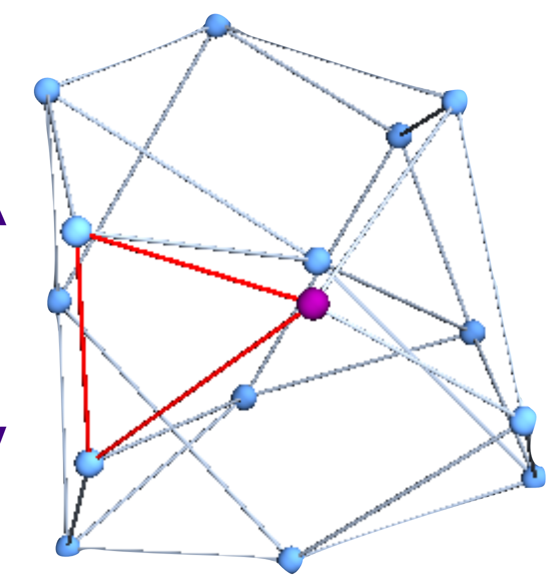


Is there a simple path forward?

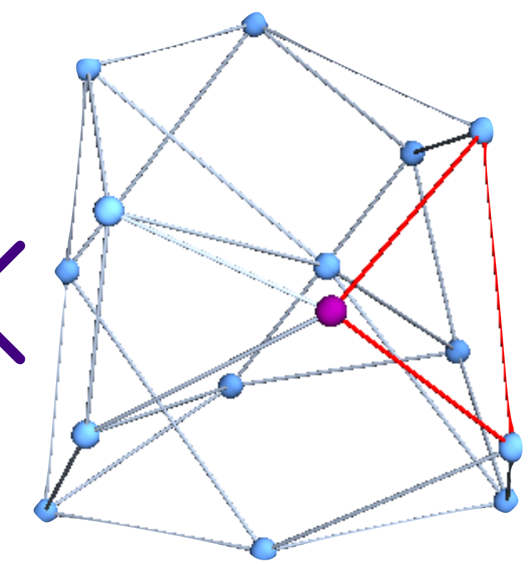
The idea is natural: take all non-vanishing kinematic-Jacobi combinations (the triangles), double-copy them with each other, use this information to ***define*** *off-shell* contact graphs in the double-copy theory.



$$=\sum$$



$$\times$$



$$= -\frac{1}{6}\sum_i \frac{J_{i,1}J'_{i,2}+J_{i,2}J'_{i,1}}{d^{(1)}_{i,1}\,d^{(1)}_{i,2}}$$

with Bern, Chen, Johansson, Roiban (2017)

Playful construction

the game of Scattering Amplitudes

QFT, NR-QM, String Theory, Classical Physics...

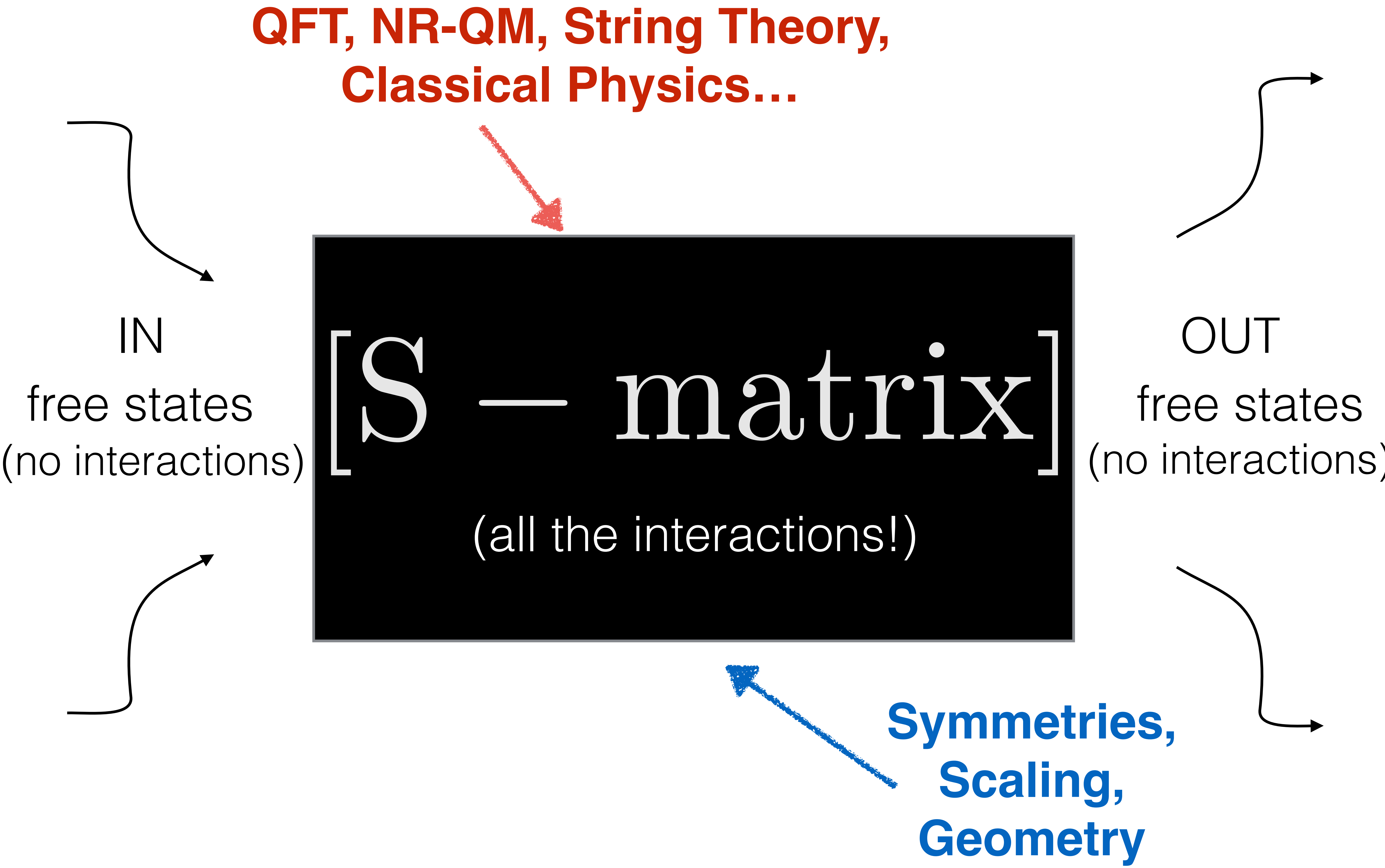
IN
free states
(no interactions)

$$[\mathrm{S-matrix}]$$

(all the interactions!)

OUT
free states
(no interactions)

the game of Scattering Amplitudes



Playful Construction Using Double-Copy as a Principle

$$U = V \otimes W$$

1) Take theories that manifest Double-Copy, strip one “factor” replace with something else that obeys the same algebra.

2) Start with generic ansatze, constrain engineering weight, impose algebra.

Example of playful construction

Open String:

Broedel, Schlotterer, Stieberger

$\alpha' \otimes$ spin-1

Chan-Paton Stripped open string

$$\mathrm{OS}(P(1,\ldots,n)) = Z_P \otimes A$$

Doubly-ordered Z-functions: obey monodromy relations on P

But obey field theory (n-3)! relations on its field theory KLT with Yang-Mills A.

$$Z_P(q_1, q_2, \ldots, q_n) \equiv \alpha'^{n-3} \int_{-\infty \le z_{P(1)} \le z_{P(2)} \le \ldots \le z_{P(n)} \le \infty} \frac{\mathrm{dz}_1 \ \mathrm{dz}_2 \ \cdots \ \mathrm{dz}_n}{\mathrm{vol}(SL(2,\mathbb{R}))} \frac{\prod_{i<j}^{n} |z_{ij}|^{\alpha' s_{ij}}}{z_{q_1 q_2} z_{q_2 q_3} \ldots z_{q_{n-1} q_n} z_{q_n q_1}} .$$

Take seriously Z-functions as encoding predictions for some (effective) field theory.

JJMC, Mafra, Schlotterer

Replace sYM in OS with a color-stripped bi-adjoint Scalar

$$\mathrm{OS}(P(1,\ldots,n)) = Z_P \otimes A_{\mathrm{YM}}$$
$$\mathbf{Z}(P(1,\ldots,n)) = Z_P \otimes C$$

Dressing with Chan-Paton factors renders something that can has the possibility of being interpreted as doubly-colored field-theory scattering amplitudes: we call it Z theory.

Color-Ordered tree-level Z-amplitude:

$$\mathcal{Z}\left(\tau(1,2,\ldots,n)\right) \equiv \sum_{P\in S_{n-1}} \mathrm{Tr}(t^1 t^{P(2)} \cdots t^{P(n)}) Z_{1,P}(\tau(1,2,\ldots,n))$$

Color-Ordered tree-level Z-amplitude

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Now look at: $\mathcal{Z} \otimes C$

"Low energy limit" -> bi-adjoint scalar: $\sum_g \frac{\tilde{c}(g)c(g)}{D(g)}$

Higher order in α': $\sum_g \frac{z(g)c(g)}{D(g)}$

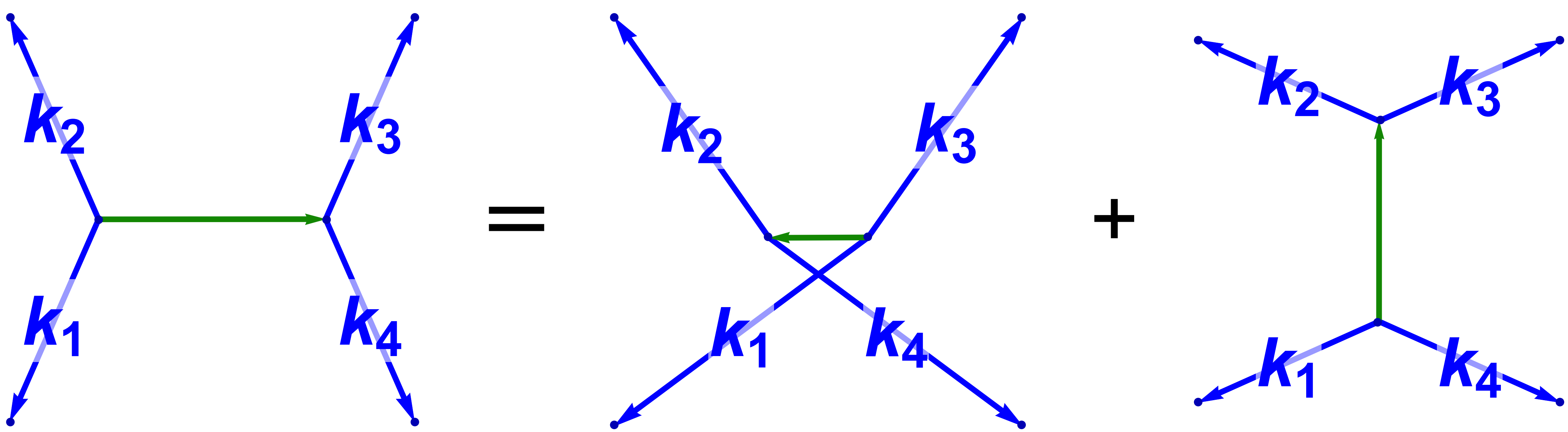
both CP-weights and kinematics conspire in z(g) to obey algebraic identities.

JJMC, Mafra, Schlotterer

"Low energy limit" -> bi-adjoint scalar: $$\sum_g \frac{\tilde{c}(g)c(g)}{D(g)}$$

Higher order in α': $$\sum_g \frac{z(g)c(g)}{D(g)} \qquad Z \otimes C$$

Get building block of open string predictions **only** when **z** numerators depend on kinematics AND Chan-Paton factors. Their **algebra** depends on both playing well together



Jacobi

JJMC, Mafra, Schlotterer

Color-Ordered tree-level Z-amplitude

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Play with CP factors. Abelian CP generators means no-longer a bi-colored scalar.

$$\mathcal{Z}_\times \otimes C = \sum_g \frac{z_\times(g)c(g)}{D(g)}$$

Color-Ordered tree-level Z-amplitude

$$\mathcal{Z}\left(\tau(1,2,\ldots,n)\right) \equiv \sum_{P\in S_{n-1}} \mathrm{Tr}(t^1 t^{P(2)} \cdots t^{P(n)}) Z_{1,P}(\tau(1,2,\ldots,n))$$

Play with CP factors. Abelian CP generators means no-longer a bi-colored scalar.

$$\mathcal{Z}_\times \otimes C = \sum_g \frac{z_\times(g) c(g)}{D(g)}$$

Low energy limit: $\lim_{\alpha'\to 0} \mathcal{Z}_\times \otimes C \to \mathrm{NLSM}$

JJMC, Mafra, Schlotterer

$$\mathcal{L}_{\mathrm{NLSM}} = \frac{1}{2}\mathrm{Tr}\left\{ \partial_\mu \varphi \, \frac{1}{1-\varphi^2} \, \partial^\mu \varphi \, \frac{1}{1-\varphi^2} \right\}$$

Color-Ordered tree-level Z-amplitude

$$\mathcal{Z}\,(\tau(1,2,\ldots,n)) \equiv \sum_{P\in S_{n-1}} \mathrm{Tr}(t^1 t^{P(2)}\cdots t^{P(n)}) Z_{1,P}(\tau(1,2,\ldots,n))$$

Abelian Z: $\lim_{\alpha'\to 0} \mathcal{Z}_\times \otimes C \to \mathrm{NLSM}$

JJMC, Mafra, Schlotterer

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(Cayley Parameterization)

Completely different story for the same prediction. Chen, Du '13 showed obeyed (n-3)! relns. Cheung,Shen '16 found an action that directly gives the color-dual kinematic story.

$$\mathcal{L}_{\mathrm{NLSM}} = Z^{a\mu}\Box X^a_\mu + \frac{1}{2}Y^a\Box Y^a - f^{abc}\left(Z^{a\mu}Z^{b\nu}X^c_{\mu\nu} + Z^{a\mu}(Y^b\overset{\leftrightarrow}{\partial}_\mu Y^c)\right)$$

Color-Ordered tree-level Z-amplitude

$$\mathcal{Z}\left(\tau(1,2,\ldots,n)\right) \equiv \sum_{P\in S_{n-1}} \mathrm{Tr}(t^1 t^{P(2)} \cdots t^{P(n)}) Z_{1,P}(\tau(1,2,\ldots,n))$$

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JJMC, Mafra, Schlotterer

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Somehow abelianization is encoding a story related to SSB

Color-Ordered tree-level Z-amplitude

$$\mathcal{Z}\left(\tau(1,2,\ldots,n)\right) \equiv \sum_{P\in S_{n-1}} \mathrm{Tr}(t^1 t^{P(2)} \cdots t^{P(n)}) Z_{1,P}(\tau(1,2,\ldots,n))$$

Abelian Z: $$\lim_{\alpha'\to 0} \mathcal{Z}_\times \otimes C \to \text{NLSM}$$

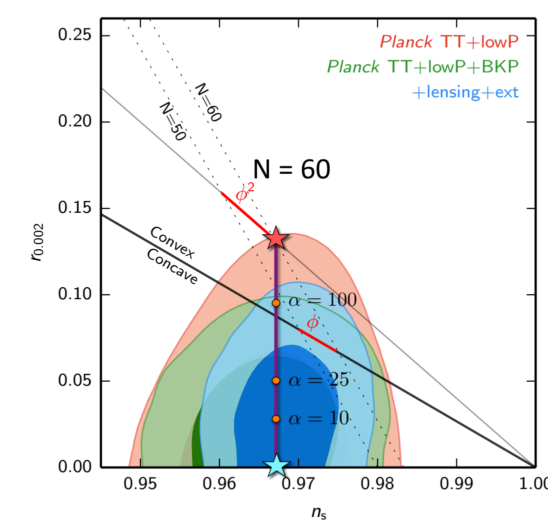
JJMC, Mafra, Schlotterer

Let's look at another copy, back to the superstring:

Abelian Open Superstring: $$\left[\left(\lim_{\alpha'\to 0} \mathcal{Z}_\times\right) \otimes A_{\text{YM}}\right] \to \left[\text{NLSM} \otimes A_{\text{YM}}\right]$$

Recall **He, Liu, Wu '16; Cachazo, Cha, Mizera '16** found:

$$\left[\text{NLSM} \otimes A_{\text{YM}}\right] = \text{SDBIVA}$$



For maximal sYM, 16 linearly realized, 16 nonlinearly realized,

Bergshoeff, Coomans, Kallosh, Shahbazi Van Proeyen '13

$$U = V \otimes W$$

Order by order in higher derivatives can play all these constructive games and more using ansatze with the correct ingredients.

These are stories whose actions may be complicated but whose *predictions* may be maximally compact.

Open question as to what theories can be understood as nontrivial double copies and what their dual-stories are.

The amplitudes can still be interesting even if “crazy” from some perspectives.

Clearly lots of fun games yet to be played — very much an open field.

Classical Solutions

Do classical solutions double-copy?

(See also work of Saotome & Akhourycombinations of Anastasiou, Borsten, Duff, Hughes, Nagy)

Monteiro, O'Connell, and White began a program amassing evidence that the answer could be **yes**, at least for a certain class of solutions.

Monteiro, O'Connell, White '14

Luna, Monteiro, O'Connell, White '15

Luna, Monteiro, Nicholson, O'Connell, White '16

for general perturbative solutions:

Goldberger, Ridgeway '16

Luna, Monteiro, Nicholson, Ochirov, O'Connell, Westerberg, White '16

Goldberger, Prabhu, Thompson '17

scattering on sandwich plane-waves:

Adamo, Casali, Mason, Nekovar '17

Monteiro, O'Connell, and White

3-pt Scattering Amplitude

$$\frac{\mathbf{c(g)n(g)}}{\mathbf{d(g)}} \xrightarrow{\text{Double Copy}} \frac{\mathbf{n(g)n(g)}}{\mathbf{d(g)}}$$

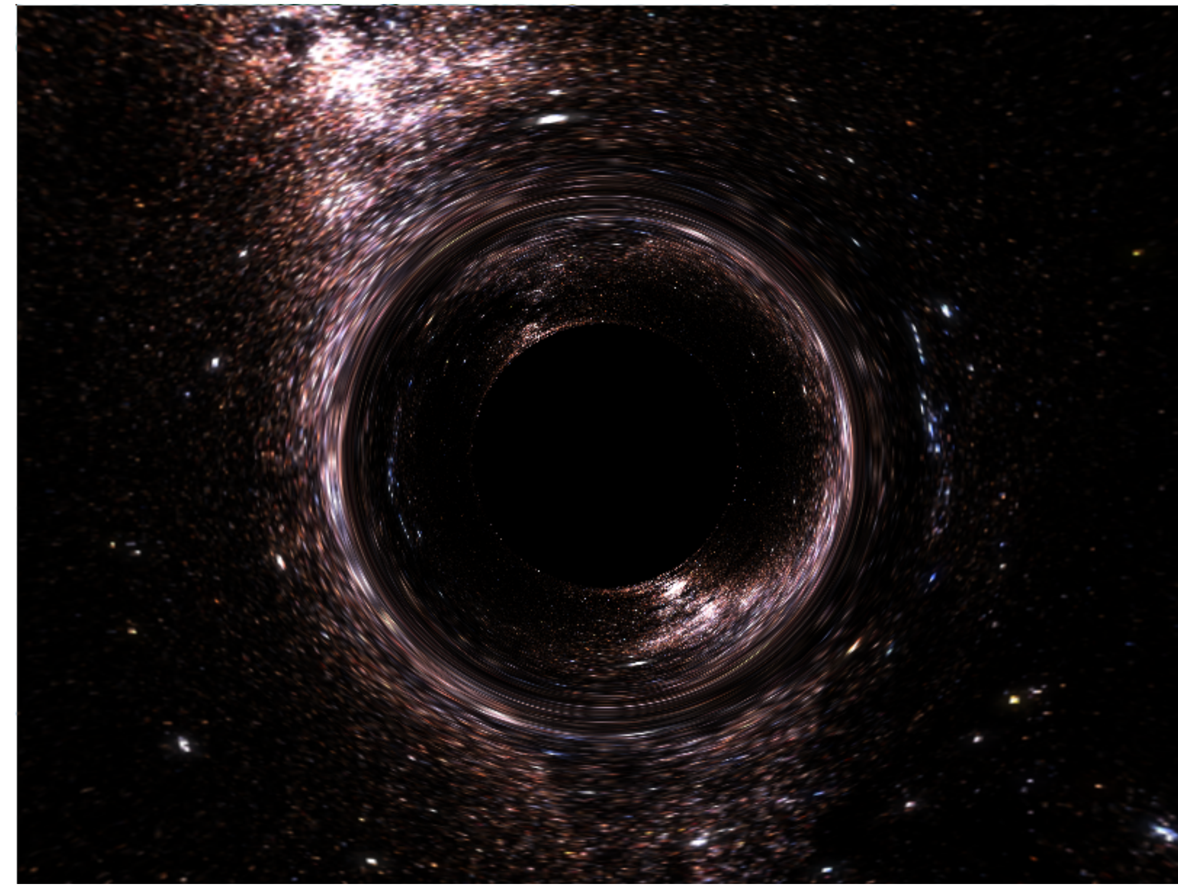
Classical Solutions (in a special class called Kerr-Schild)

$$\mathbf{A^a_m u = c^a k_\nu \phi} \xrightarrow{\text{Double Copy}} \mathbf{g}_{\mu\nu} - \eta_{\mu\nu} = \mathbf{k}_\mu \mathbf{k}_\nu \phi$$

Schwarzschild

$$\mathbf{g}_{\mu\nu} - \eta_{\mu\nu} = \frac{\mathbf{2GM}}{\mathbf{r}}\mathbf{k}_\mu \mathbf{k}_\nu$$

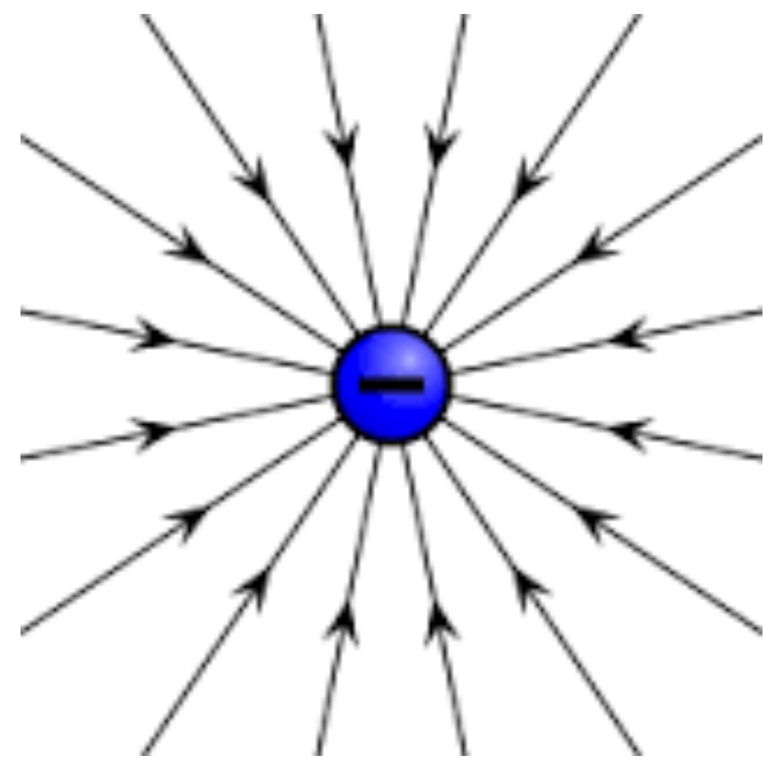
$$\mathbf{k}_\mu = \{\mathbf{1}, \hat{\mathbf{r}}\}$$



The double copy of

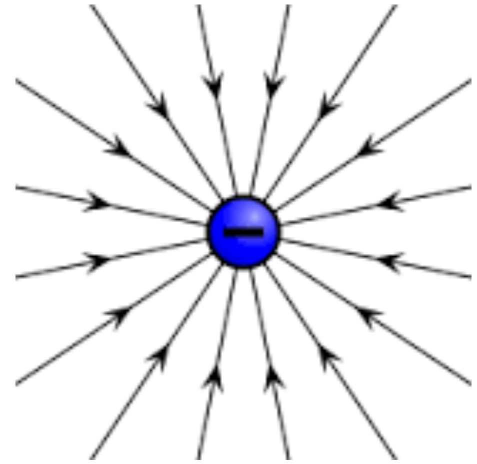
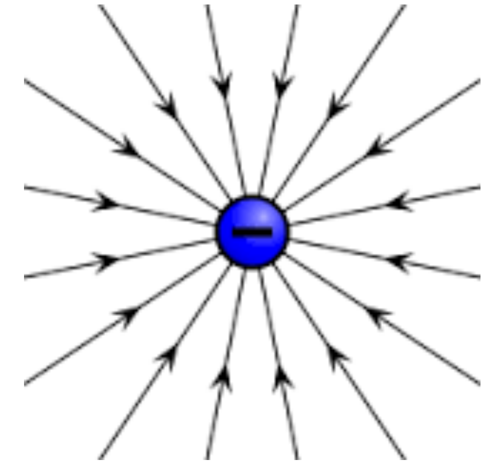
$$\mathbf{A}_\mu = \frac{\mathbf{2GM}}{\mathbf{r}}\mathbf{k}_\mu$$

abelianized point charge



Monteiro, O'Connell, and White

Monteiro, O’Connell, and White

 =  $\otimes$ 

Natural question:

What process double copies to Hawking radiation?

Suggestive answer:

Schwinger pair production

JJMC, Torroba (to appear)

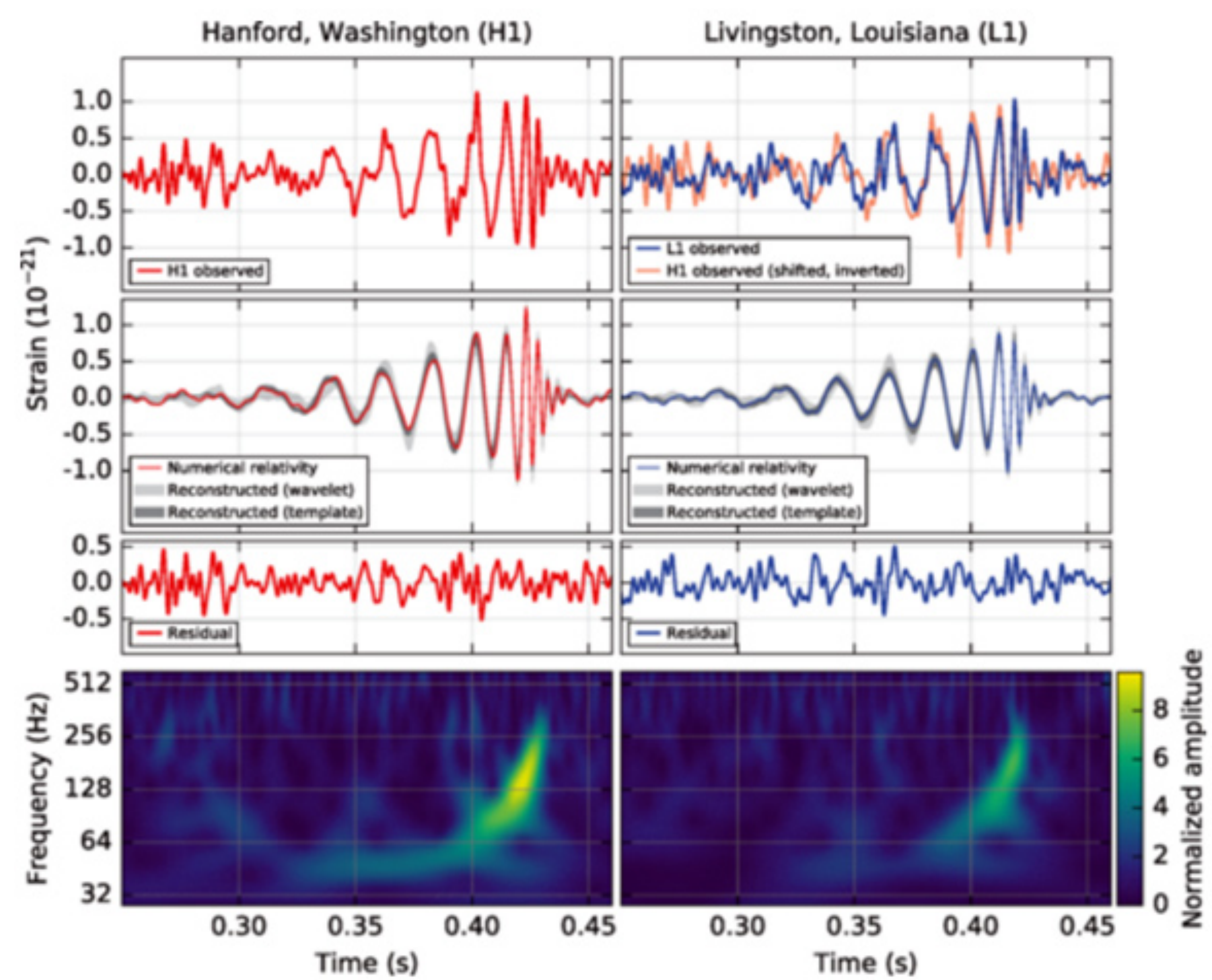
Classical gravity is a Double Copy?

Remind you of some of the double-copy positives:

+ Constrained solutions => can exploit for technical simplicity in prediction

+ Unifying web of relationships between theories

Open question: how far can this go?



Two Fantastic Postdocs at Saclay

Dr. Michele Levi

EFT of Binary Inspiral (Spin effects)

Dr. Laurentiu Rodina

Formal Scattering Amplitudes

Learn more about our group at fancyphysics.org