Using Majorana Polarization and other local order parameters to predict and detect exotic phases

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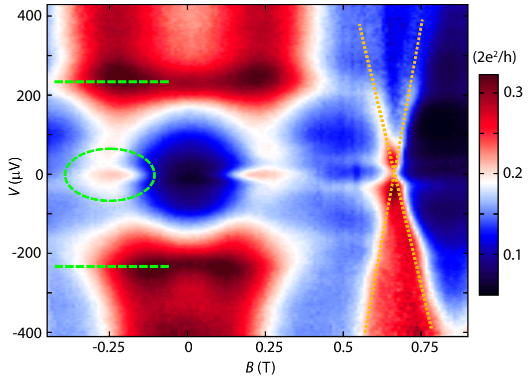
Introduction

- **Local electronic properties of 1D and 2D systems**
  - Graphene and Carbon Nanotubes
  - Andreev bound states
  - Impurity and Shiba states
  - Majorana
- **Importance**
  - fundamental: strong interactions, fractional charge, non-Abelian statistics, topological phases
  - applications: nanoelectronics, high-temperature superconductivity
- **Majorana states:**
  - condensed matter version of Majorana fermions (their own antiparticles) neutrinos ?
  - equal combination of electrons and holes
  - non-Abelian statistics, important pathway towards quantum computation
- **Majorana polarization**
  - new tool to characterize Majorana states in topological systems

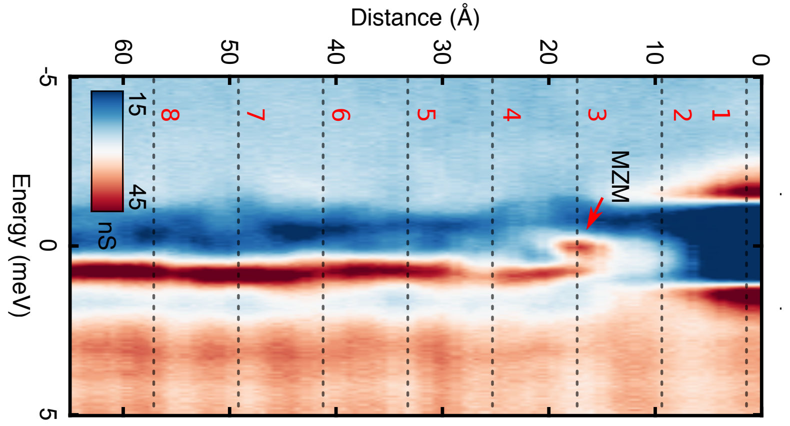
    *Sticlet, **Bena**, Simon PRL 2012, Sedlmayr, **Bena** PRB 2015*

Majorana states

- Do they exist? How to probe experimentally?
- Observed signatures (zero-bias peaks) are controversial
- Can come from non-Majorana states *(ABS Pillet, Quay, Morfin, **Bena**, Levy Yeyatti, Nat. Phys. 2010,* others *Lutchyn & al. Nat. Rev. Mat. 2018)*



Mourik, Kouwenhoven & al. Science 2012



Jeon, Yazdani & al. Science 2017

Majorana polarization (MP)

- fundamentally-different approach: ***local*** order parameter
- unambiguously establish whether a state is Majorana
- new experimental framework for exotic states

Majorana fermions

Ordinary fermions $\{c_i^+, c_j\} = \delta_{ij}$

Write in terms of Majorana fermions $c_j = (\gamma_{j1} + i\gamma_{j2})/2$

$\gamma_1 = (c^\dagger + c)/\sqrt{2}$

$\gamma_2 = i(c^\dagger - c)/\sqrt{2}$

$$\{\gamma_{i\alpha}^+, \gamma_{j\beta}\} = \delta_{ij}\delta_{\alpha\beta}$$

$$\gamma_{i\alpha}^+ = \gamma_{i\alpha}$$

Any fermionic Hamiltonian can be recast in terms of Majorana operators but very few can support solutions with **isolated** Majorana fermions

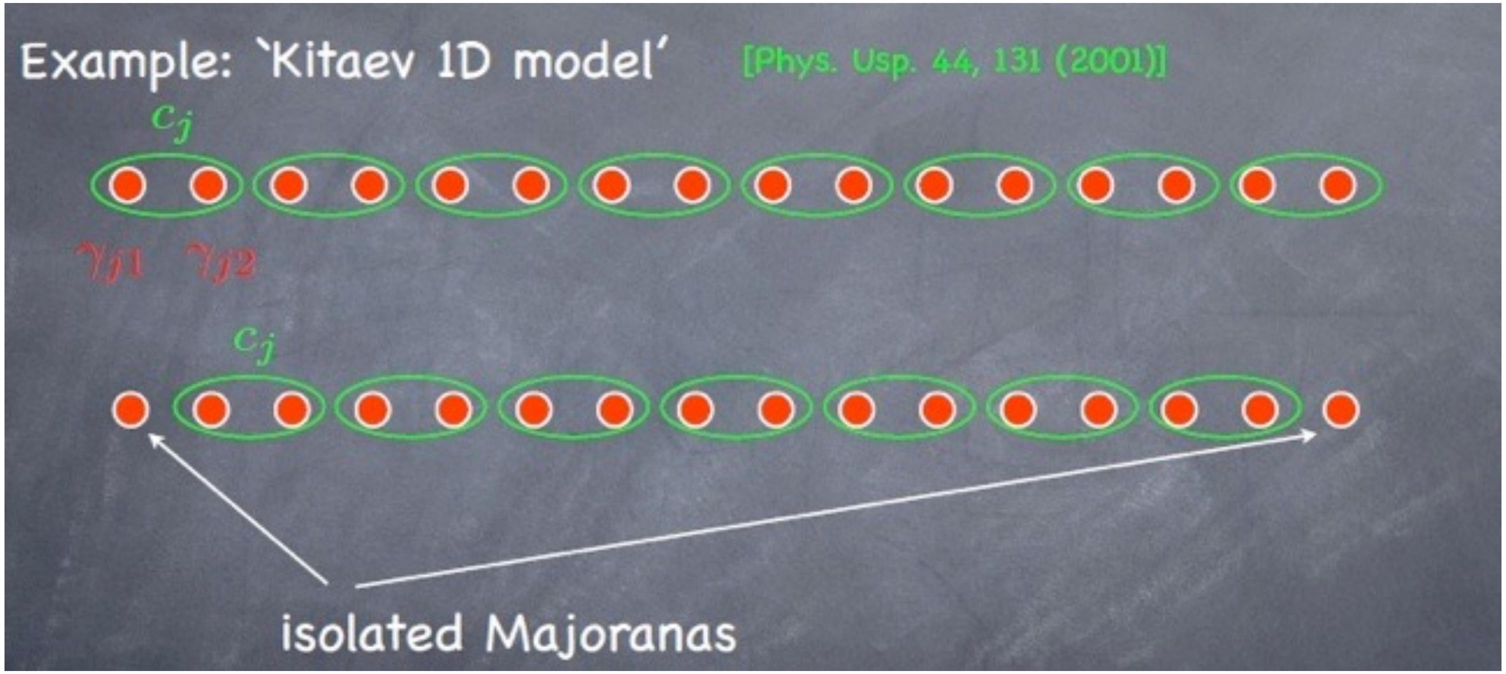
Majorana fermions

Hamiltonians with **isolated** localized Majorana fermions

$$H = -\mu \sum_j c_j^\dagger c_j + \sum_{j=0}^{N-1} \Big[-t \left(c_{j+1}^\dagger c_j + c_j^\dagger c_{j+1} \right) - |\Delta| \left(c_j c_{j+1} + c_{j+1}^\dagger c_j^\dagger \right) \Big].$$

$$\gamma_{j,1} = c_j + c_j^\dagger, \quad \gamma_{j,2} = i \left(c_j^\dagger + c_j \right)$$

$$H = -it \sum_{j=0}^{N-1} \gamma_{j,1} \gamma_{j+1,2}$$



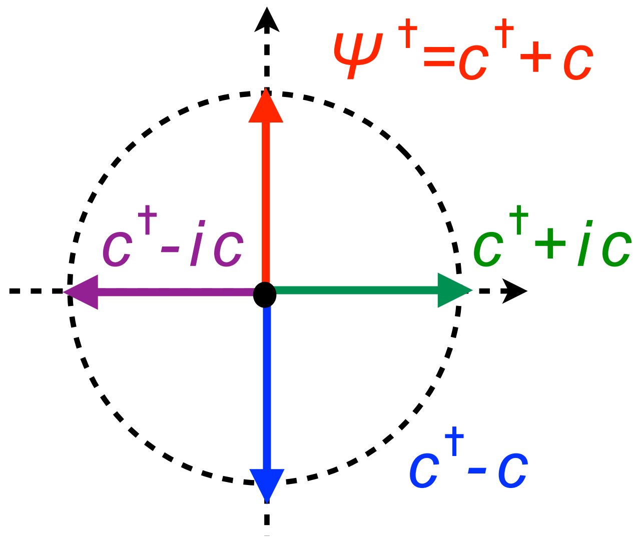
Majorana polarization

Majorana states = equal combinations of electrons and holes
need quantity to capture electron-hole overlap

General wavefunction:

$$\psi^{\dagger} = u\, c^{\dagger} + v\, c$$

$$P = 2uv$$



$P = 0$ if $u,v=0$ (*purely fermionic states*): c, $c^{\dagger}$

$|P|$ is maximal for *purely Majorana states* ($|u|=|v|$):

$|P|_{max} = 2|uv|_{max} = |u|^2 + |v|^2 =$ density

MP = vector in complex plane:
(pseudo-spin)

Finding right quantity subtle (naïve guess: $P=2uv^*$)
MP = expectation of particle-hole operator

*Sticlet, **Bena**, Simon PRL 2012*
*Sedlmayr, **Bena**, PRB 2015*

Majorana polarization

Spinful models $\Psi^{\dagger} = u_{\uparrow} c_{\uparrow}^{\dagger} + v_{\uparrow} c_{\uparrow} + u_{\downarrow} c_{\downarrow}^{\dagger} + v_{\downarrow} c_{\downarrow}$ $P = 2 u_{\uparrow} v_{\uparrow} + 2 u_{\downarrow} v_{\downarrow}$

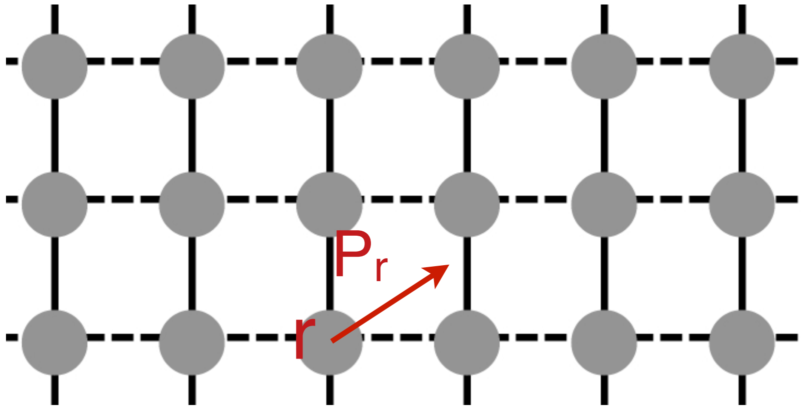
Same-spin combinations (opposite to BCS)

$\Psi_M^{\dagger} \propto c_{\uparrow}^{\dagger} + c_{\uparrow}$ or $c_{\downarrow}^{\dagger} + c_{\downarrow}$ Zero-energy BCS $\propto c_{\uparrow}^{\dagger} + c_{\downarrow}$

Spin structure → MP = 0 for any other zero-energy non-Majorana states (Andreev bound states, impurity, Shiba, etc.)

MP for a spatial distribution

- On each site define a MP vector P_r



$$\mathrm{P}(\boldsymbol{r}) \equiv \begin{pmatrix} P_x(\boldsymbol{r}) \\ P_y(\boldsymbol{r}) \end{pmatrix} \equiv \begin{pmatrix} -2\,\mathrm{Re}\left[u_{\boldsymbol{r}\uparrow}v_{\boldsymbol{r}\uparrow} + u_{\boldsymbol{r}\downarrow}v_{\boldsymbol{r}\downarrow}\right] \\ -2\,\mathrm{Im}\left[u_{\boldsymbol{r}\uparrow}v_{\boldsymbol{r}\uparrow} + u_{\boldsymbol{r}\downarrow}v_{\boldsymbol{r}\downarrow}\right] \end{pmatrix}$$

- Criterion to have a Majorana state

$$C = \left| \sum_{\boldsymbol{r}\in\mathcal{R}} \left[P_x(\boldsymbol{r}) + iP_y(\boldsymbol{r})\right] \right|^2 / \sum_{\boldsymbol{r}\in\mathcal{R}} \rho(r)$$

C=1

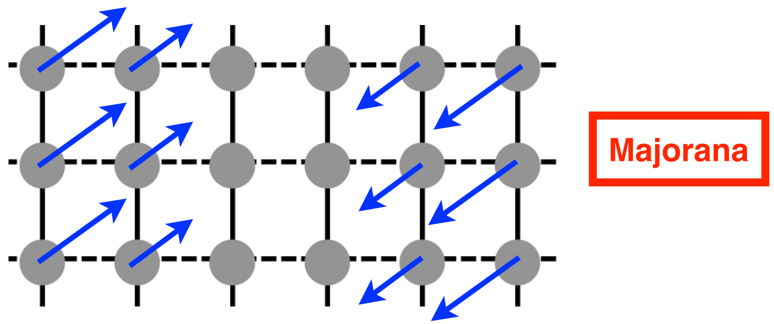
Sticlet, Bena, Simon PRL 2012
Sedlmayr, Bena, PRB 2015

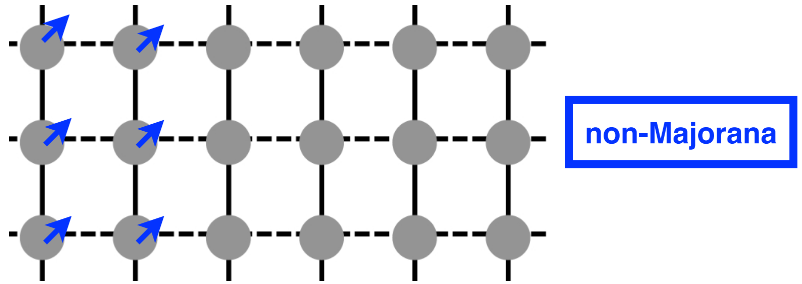
Majorana polarization

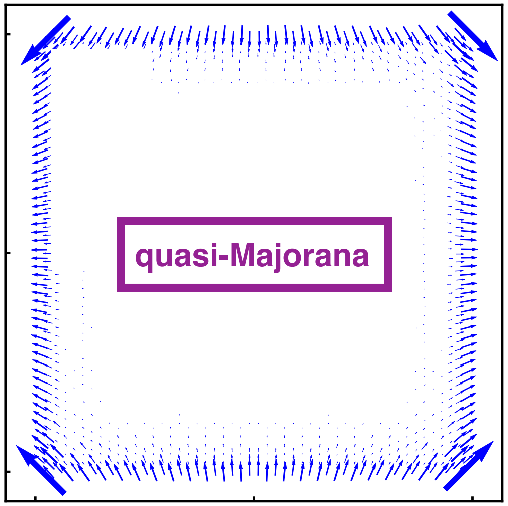
MP vector: Magnitude = electron-hole overlap
Direction - (e,h) phase

Criterion to test existence of topological phases

- Majorana states have
  - *|**MP**| = density* → locally Majorana
  - All MP vectors aligned
- Non-Majorana states
  - *|**MP**| < density*
- Discovery of ***Quasi-Majorana*** states
  - locally Majorana: *|**MP**|= density*
  - not globally: MP vectors not aligned
  - finite-size effects (experiments):
    Majorana → Quasi-Majorana







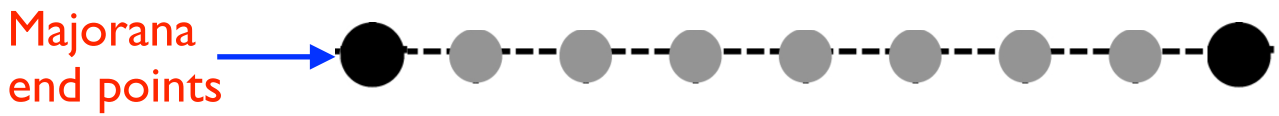
*Sedlmayr, Aguiar-Hualde, **Bena**, PRB 2016*

Applications: Quasi-1D systems, NS and SNS junctions, graphene

***Bena** & al: PRB 2012, 2013, SciPost 2017, EPJB 2017, arxiv:2018*

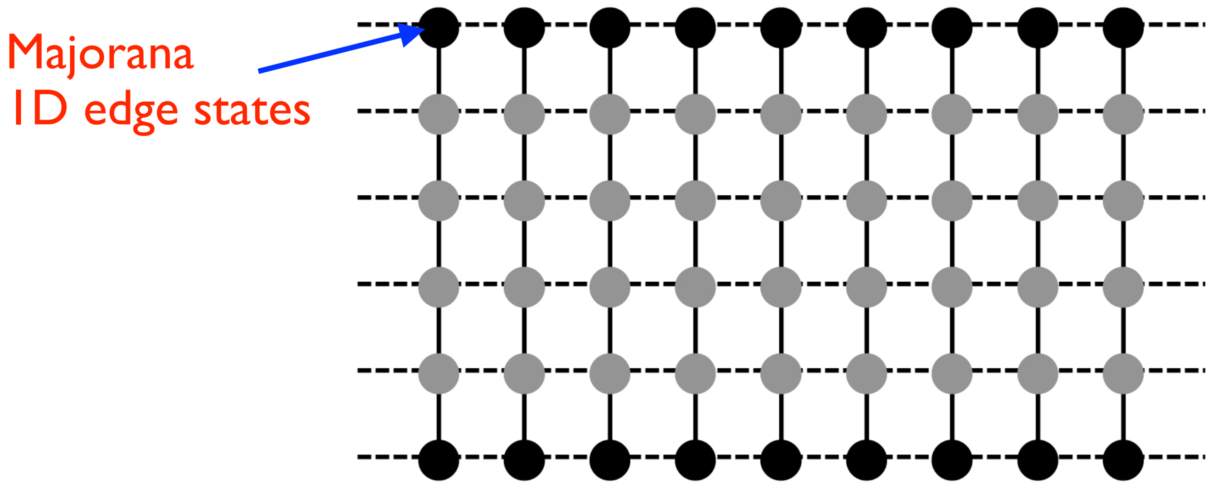
MP Applications

- 1D wires

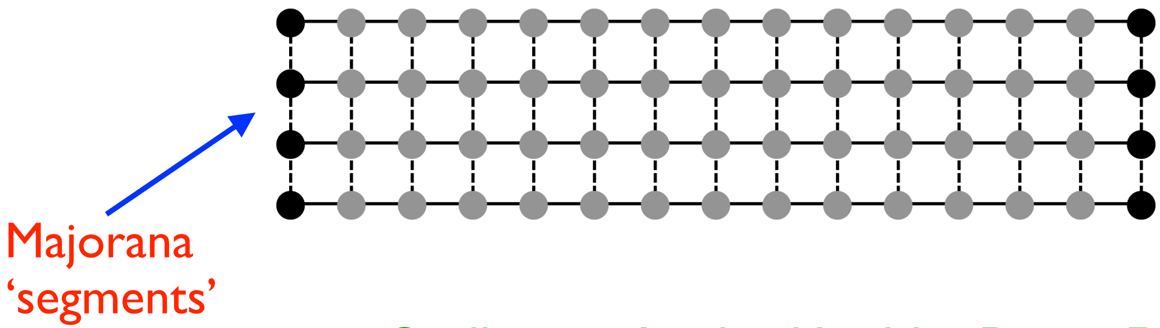


- NS junctions

- 2D ribbons



- Quasi-1D systems (finite-size strips)



Sedlmayr, Aguiar-Hualde, Bena, PRB 2016

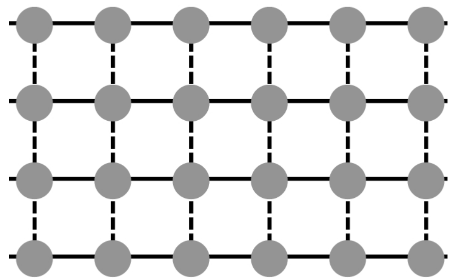
Model

Superconductivity

Zeeman

- perpendicular to the lattice plane
- in-plane

$$H = \sum_{\boldsymbol{r}} \left[\Psi^{\dagger}_{\boldsymbol{r}} \left(-\mu\tau_z + \Delta\tau_x + \boldsymbol{B}\cdot\boldsymbol{\sigma} \right) \Psi_{\boldsymbol{r}} + \right.$$
$$+ \Psi^{\dagger}_{\boldsymbol{r}} \left(-t_x - i\lambda_x\sigma_y \right) \tau_z \Psi_{\boldsymbol{r}+\boldsymbol{x}} + \mathrm{H.c.} +$$
$$\left. + \Psi^{\dagger}_{\boldsymbol{r}} \left(-t_y + i\lambda_y\sigma_x \right) \tau_z \Psi_{\boldsymbol{r}+\boldsymbol{y}} + \mathrm{H.c.} \right]$$



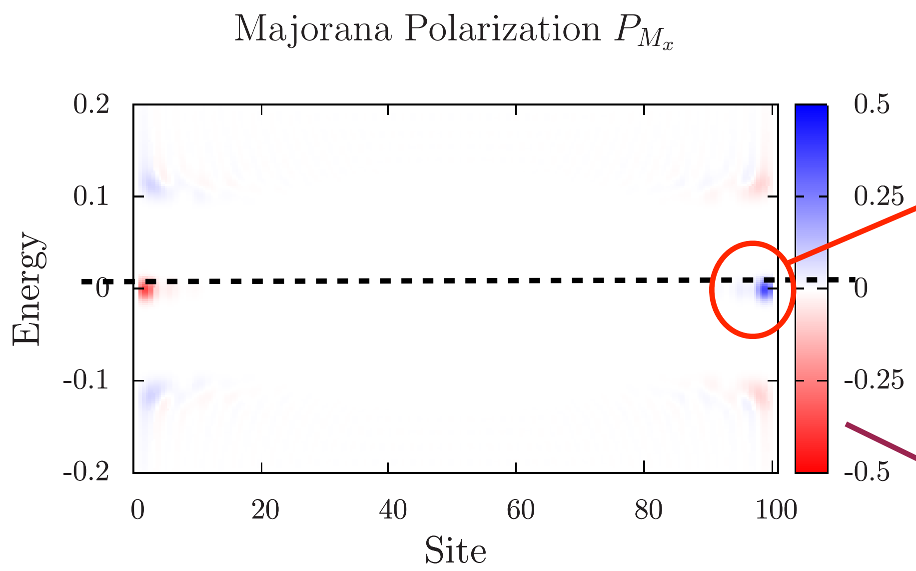
Rashba spin-orbit

Sedlmayr, Aguiar-Hualde, Bena, PRB 2016

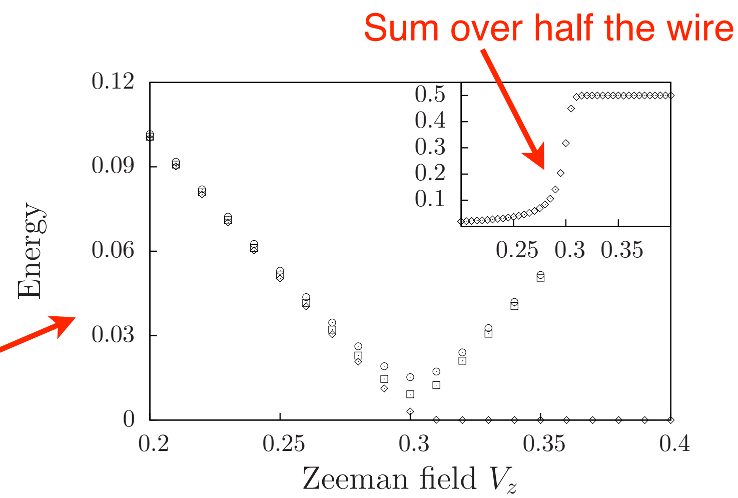
Application to quantum wires

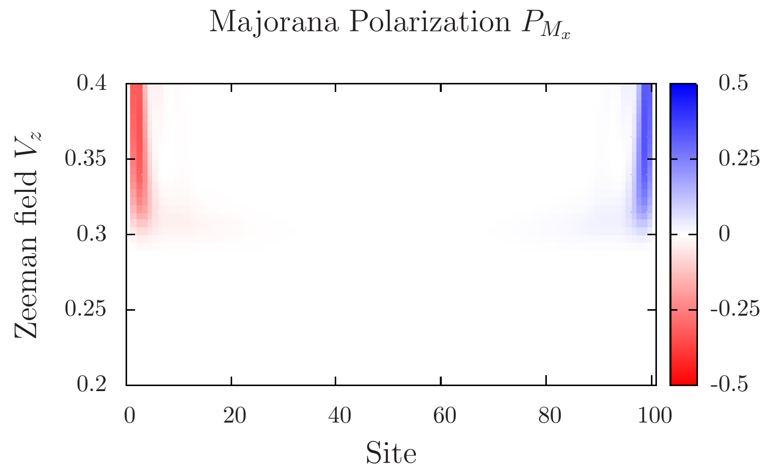
Condition for topological phase :

$$V_z^2 > \Delta^2 + \mu^2$$



$$\Delta = 0.3,\ V_z = 0.4,\ \alpha = 0.2,\ \beta = 0\ \ \mu = 0$$

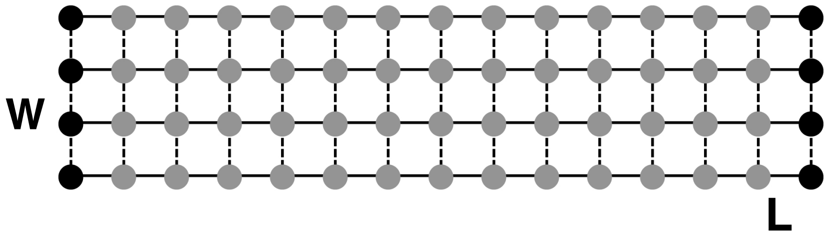




MP is a good order parameter for the topological transition

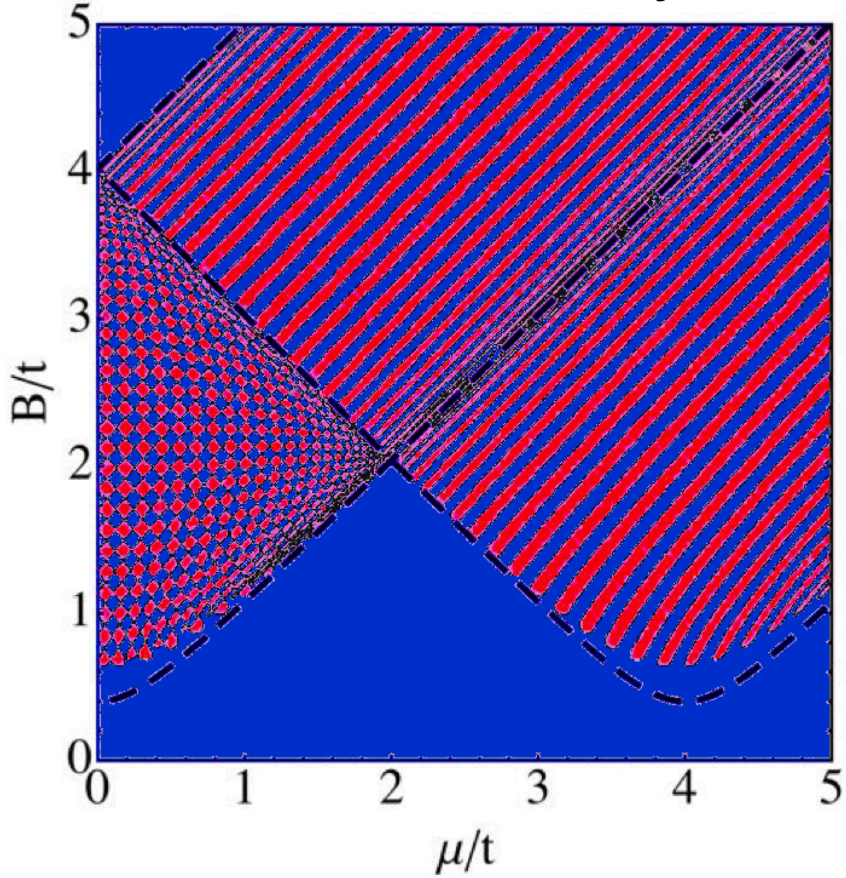
Sticlet, Bena, Simon PRL 2012

Quasi-1D: Zeeman field perpendicular to the plane



*Sedlmayr, Aguiar-Hualde, **Bena**, PRB 2016*

***L* >> *W* = 50 - Infinite system**

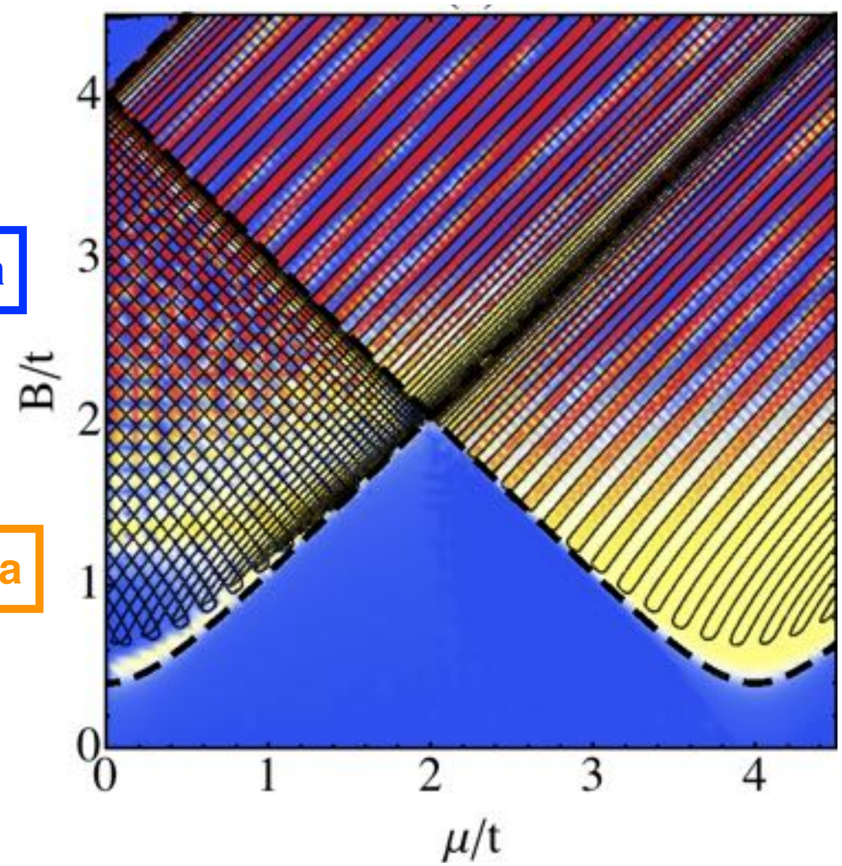


Majorana

non-Majorana

quasi-Majorana

***L* = 200, *W* = 50**

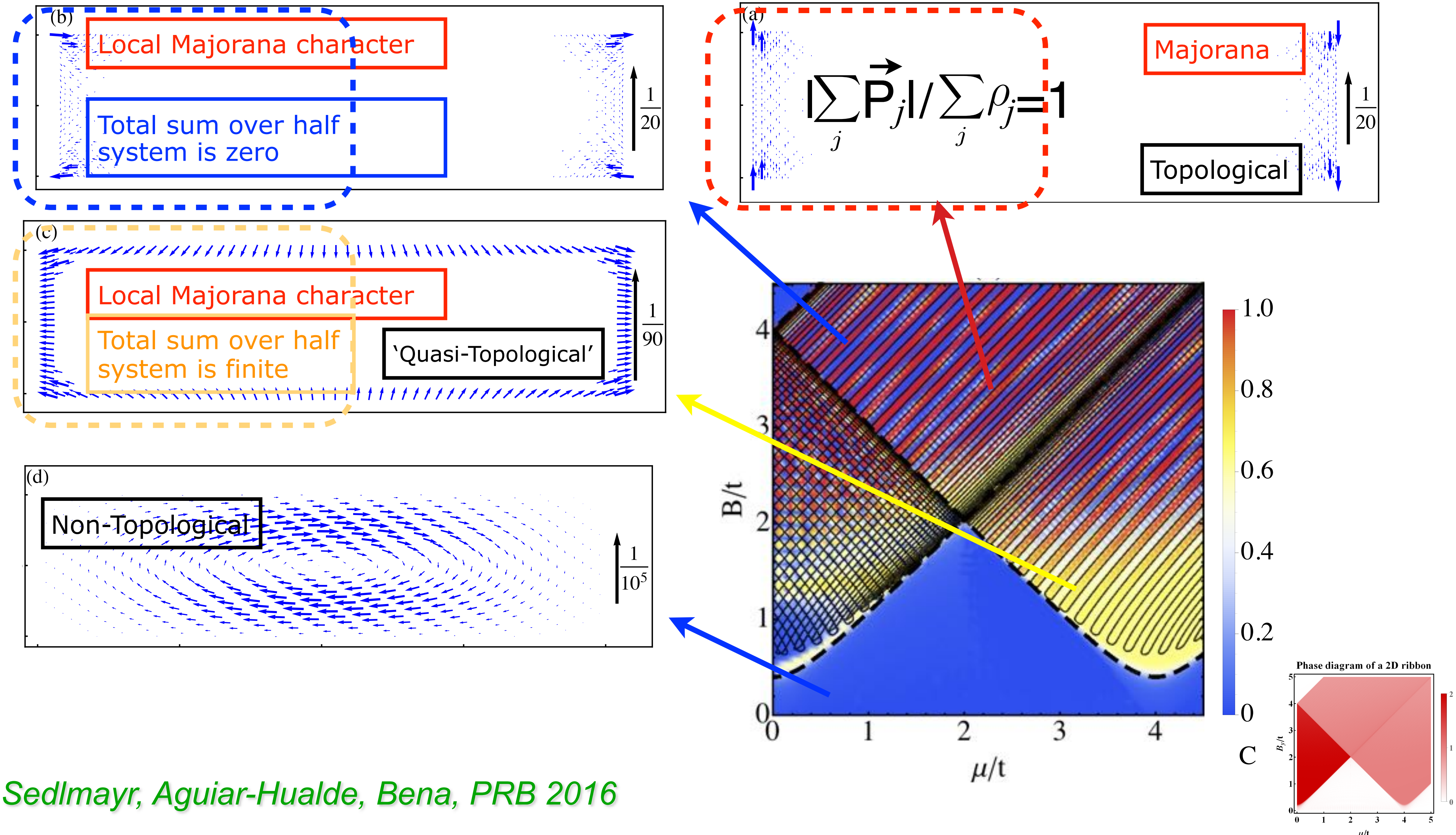


Proper order parameter - MP-based phase diagram:

- MP maximal in the topological phase
- zero in the trivial phase

Finite-size effects: formation of quasi-Majorana states

Quasi-1D: Zeeman field perpendicular to the plane



Sedlmayr, Aguiar-Hualde, Bena, PRB 2016

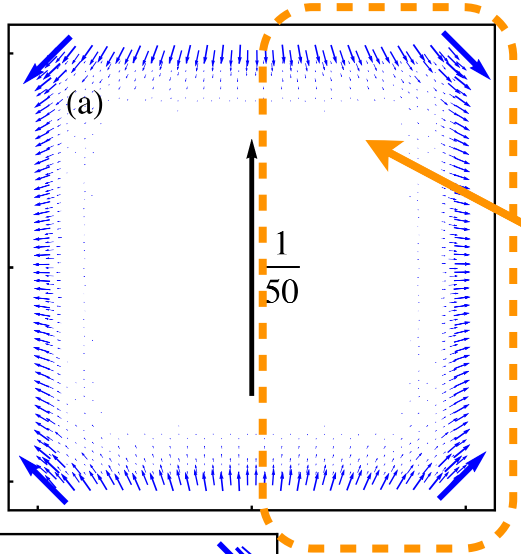
System with length comparable to width

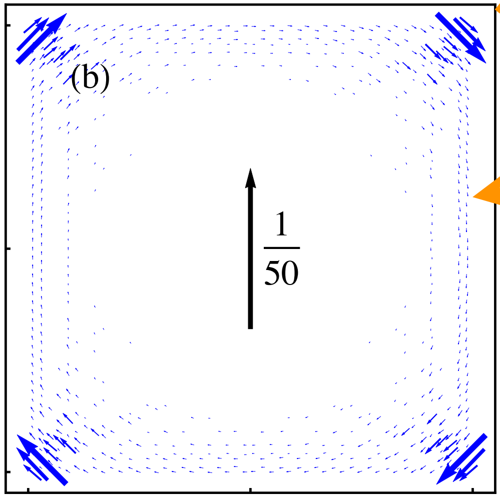
Sedlmayr, Aguiar-Hualde, Bena, PRB 2016

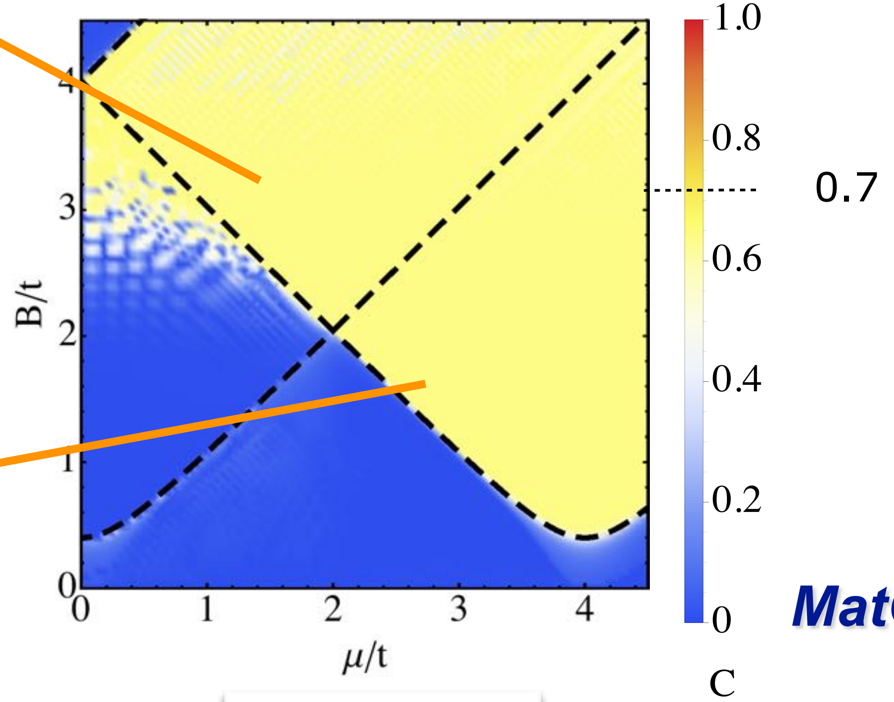
Local Majorana character

Total sum over half system is finite

2D ‘Quasi-Topological’

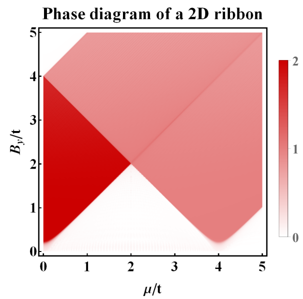






MatQ

100 x 100



MP recovers the shape of the topological phase diagram for the 2D case

Low-energy quasi/chiral Majorana states - no Majoranas for L~W!

Future directions

I: How to measure Majorana polarization

II: Applications of Majorana polarization

III: Exchange statistics of quasi-Majorana states

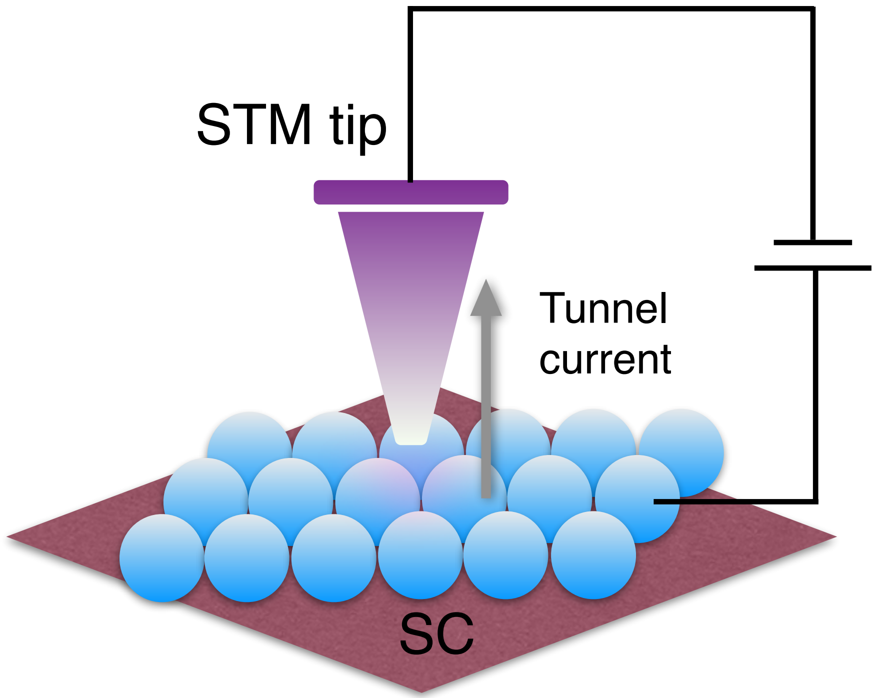
IV: New topological local order parameters

I. How to measure MP

$$\psi^\dagger = u_\uparrow c_\uparrow^\dagger + v_\uparrow c_\uparrow + u_\downarrow c_\downarrow^\dagger + v_\downarrow c_\downarrow$$

Measuring the MP - need $P = 2\, u_\uparrow v_\uparrow + 2\, u_\downarrow v_\downarrow$

- impossible *directly:*
  Majoranas cannot tunnel in and out
- indirect methods



- Tunnel current to STM tip:
  local density of states = $|u_\uparrow|^2 + |u_\downarrow|^2$
- Spin-polarized STM:
  $|u_\uparrow|^2 - |u_\downarrow|^2$, $\mathrm{Re}[u_\uparrow u_\downarrow]$, $\mathrm{Im}[u_\uparrow u_\downarrow]$

Need also $v_\uparrow$, $v_\downarrow$!!!

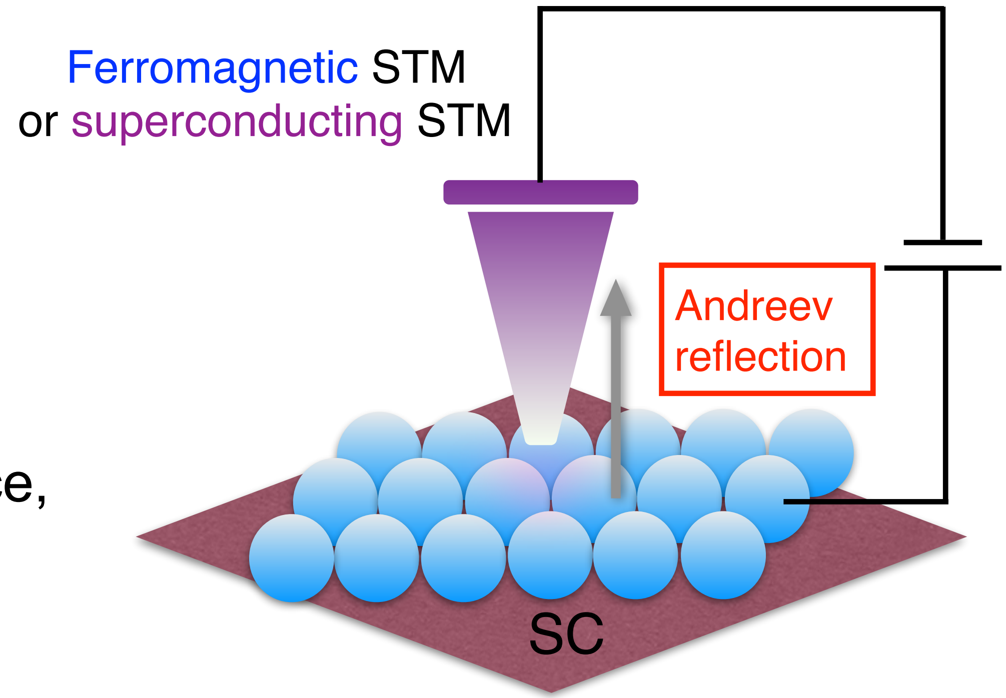
I. How to measure MP

Measuring the MP - need $\boldsymbol{P = 2\, u_{\uparrow} v_{\uparrow} + 2\, u_{\downarrow} v_{\downarrow}}$

Andreev reflection → $\boldsymbol{u}$, $\boldsymbol{v}$

- normal / ferromagnetic tip
- superconducting tip, eventually p-wave
- calculate conductance and **noise**
- vary Andreev / normal tunnelling
- modify superconducting phase difference,

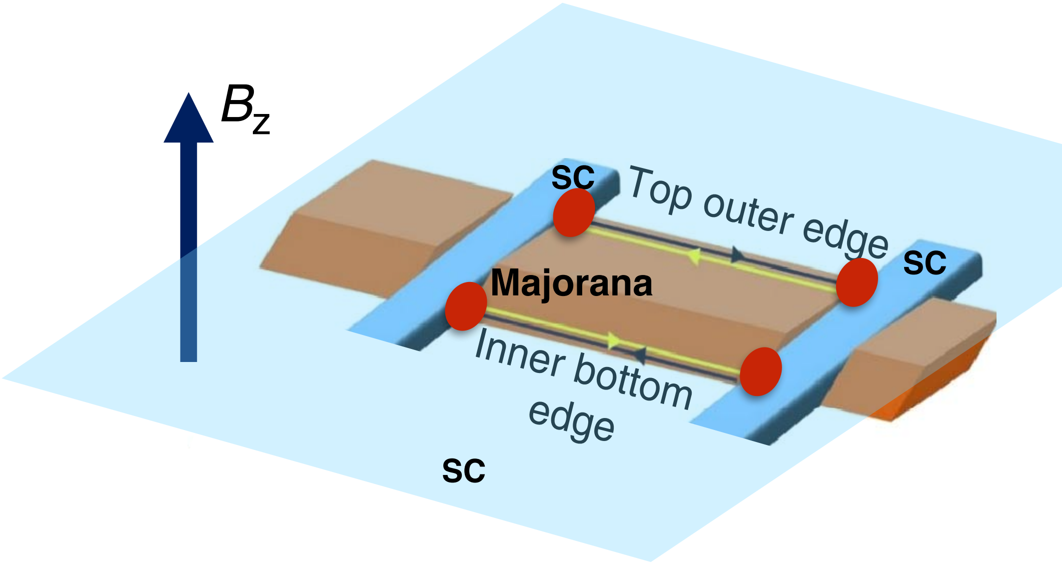
ferromagnetic angle



Discussions and collaboration with experimentalists:
H. Beidenkopf, N. Avraham (Weizmann), F. Massee (LPS Orsay), L. Simon (Mulhouse), T. Cren, D. Roditchev (Paris)

II. Applications of the MP

- Topological candidates:
  - bismuth
  - doped graphene
  - transition metal dichalcogenides

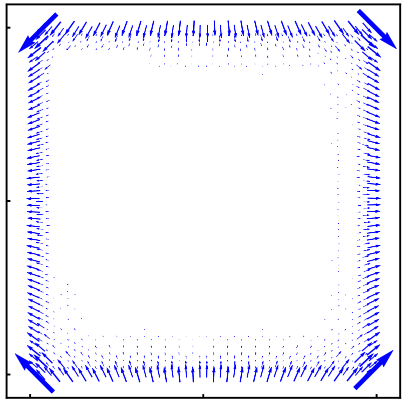


- Interacting systems:
  - derive MP within a Green's function formalism
  - test the stability of Majoranas

Tight-binding model, MatQ code

Experiments *H. Bouchiat (Orsay)*

III. Exchange statistics of quasi-Majorana states



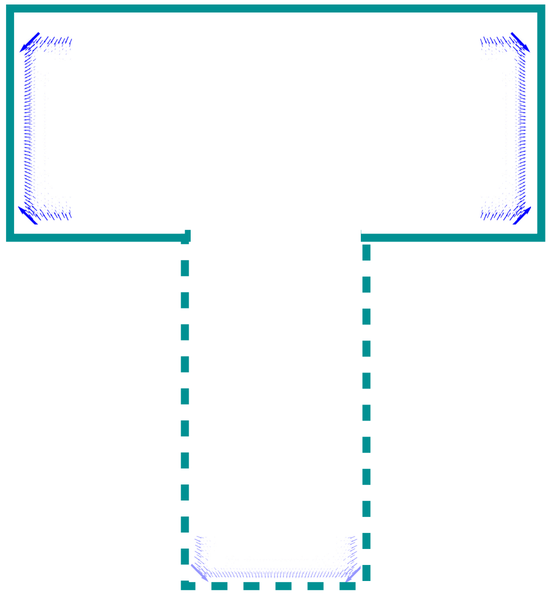
Bena et al, PRB 2016

Quasi-Majorana states:

- finite-size effects - very important for experiments
- locally but not globally Majorana, quasi-null energy

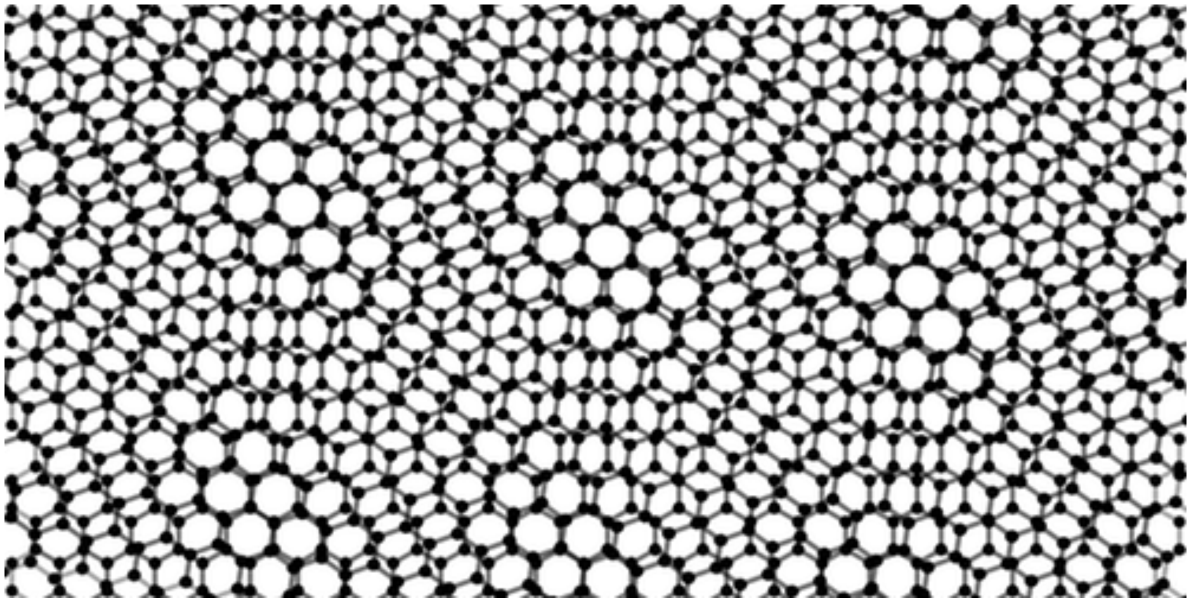
Abelian or non-Abelian exchange statistics ?

- quantum computing ?
- calculate wave function resulting from braiding
- **T-junction:**
  - change chemical potential adiabatically
  - move quasi-Majorana states around each other



IV. New local order parameters

- Aim: construct local order parameters ➞ new local probes of exotic bound states
  - analogous to local density of states
  - zero for trivial states, non-zero for topological/exotic states: phase transitions
- Twisted graphene bilayers: superconducting for specific twist angle ➞ flat band *Cao & al. Nature 2018*



IV. New local order parameters

Twisted graphene bilayers: what kind of superconductivity ?

- ***d* + *i d*** and other singlet / triplet topological superconductivity proposed
  Xu & al PRL2018, Fidrysiak & al PRB2018, Roy & al arXiv:2018
- SC phases support topological edges states
- local order parameters ➞ induced SC pair symmetry
  - for edge states - MP: $\boldsymbol{P}_{MP} = 2\, u_{\uparrow} v_{\uparrow} + 2\, u_{\downarrow} v_{\downarrow}$
  - for Andreev bound states - ABS density:
    $\boldsymbol{P}_{ABS} = 2\, u_{\uparrow} v_{\downarrow} + 2\, u_{\downarrow} v_{\uparrow}$?
    (opposite-spin electron-hole overlap)
  - calculate spatial distribution of MP and ABS density
  - construct method to measure