# Gravitational waves implications on dark energy and modified gravity

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Colloque de l’IPhT - L’isle sur la Sorgue

# Motivations

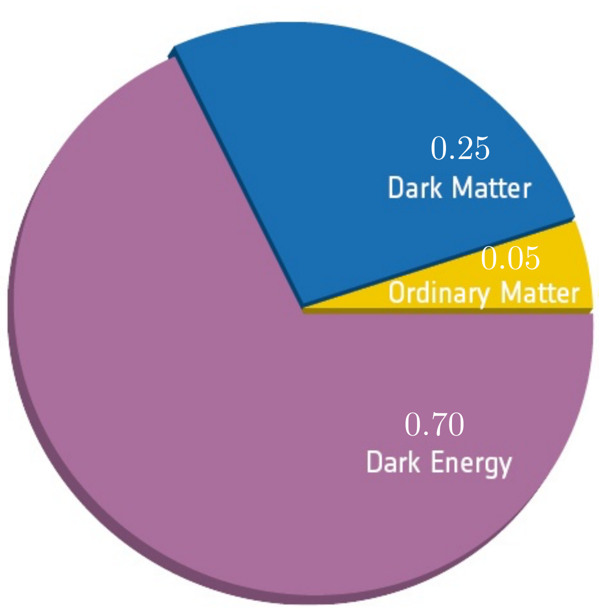
The expansion of the Universe is accelerating

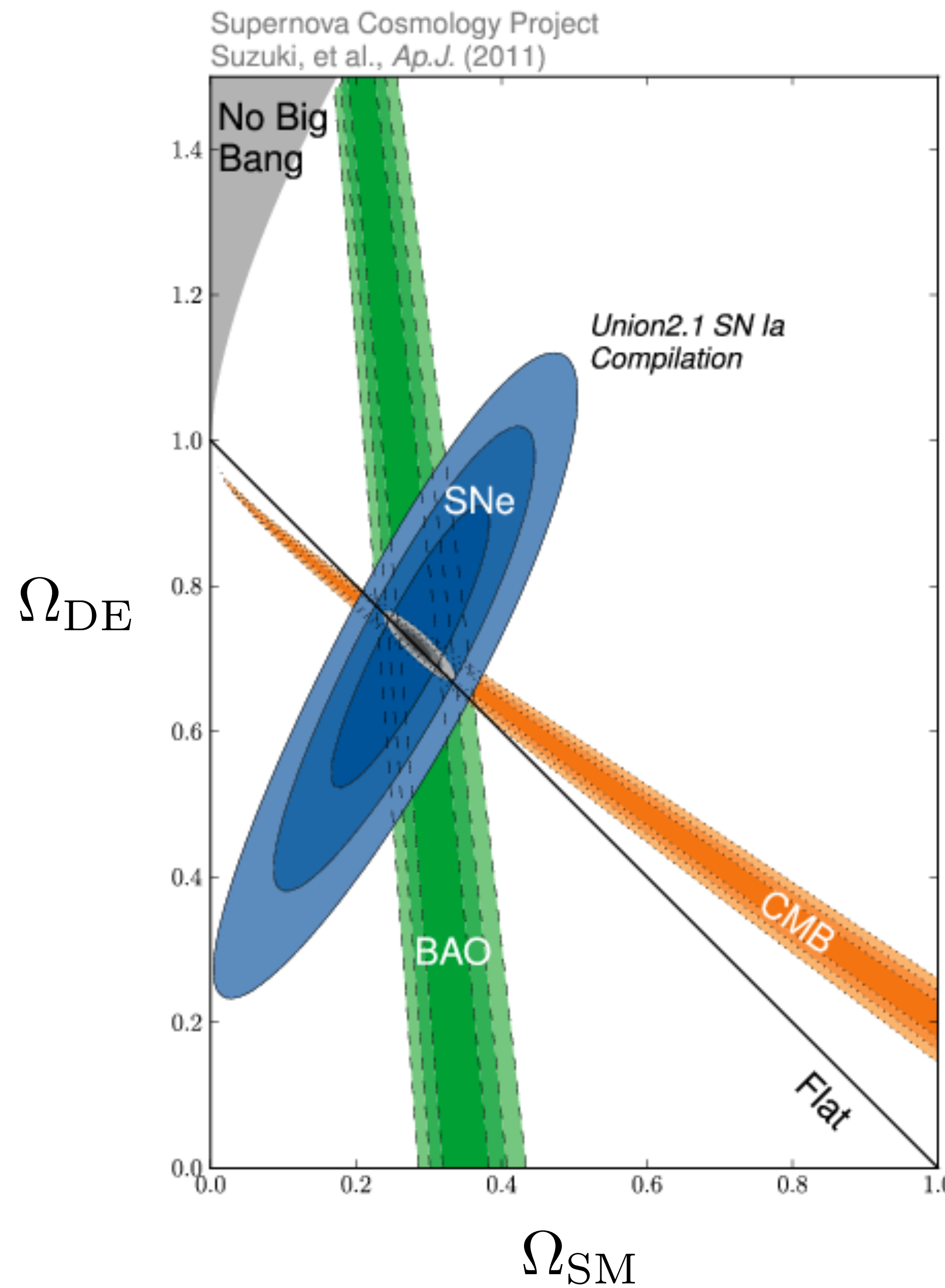
$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0$$

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}^{(\mathrm{matter})}$$

Acceleration implies some form of unknown matter with negative pressure: dark energy



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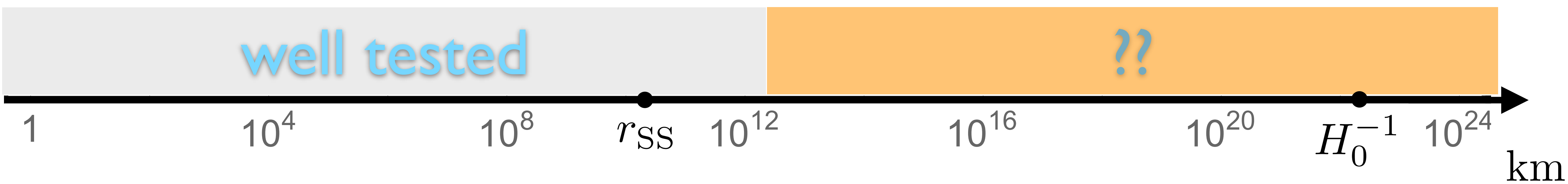
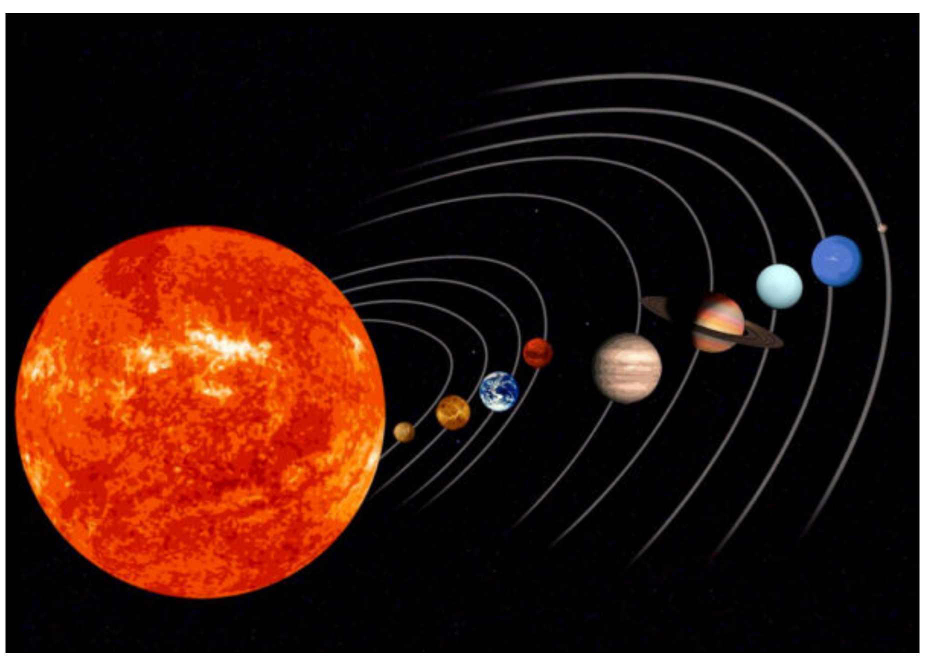
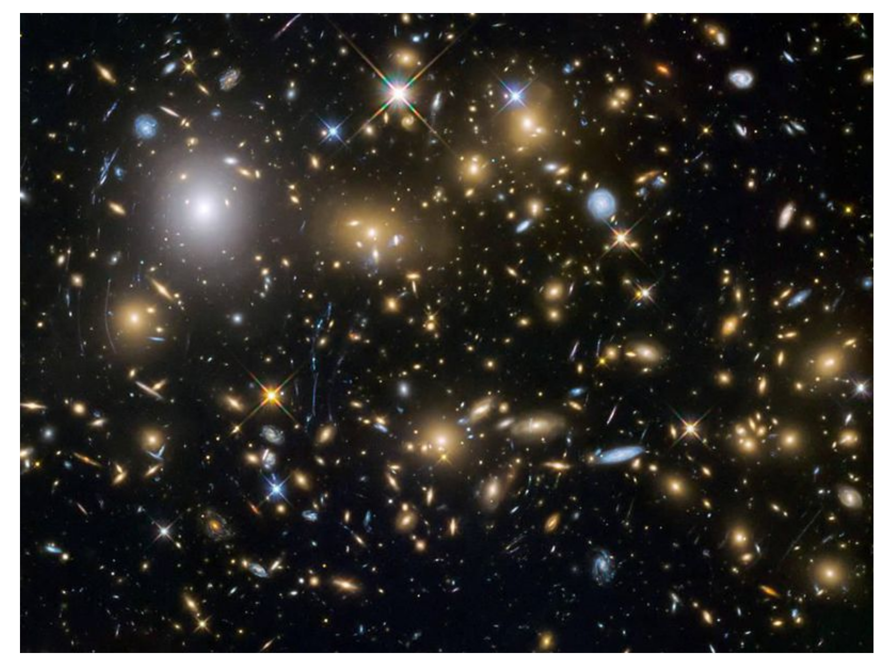
General relativity tested over special ranges of scales and masses. Cosmology is a window for testing it on very large distances. Distinguish among models and discover new physics. Cosmological precision tests of ΛCDM (precision tests of the Standard Model at the LHC)





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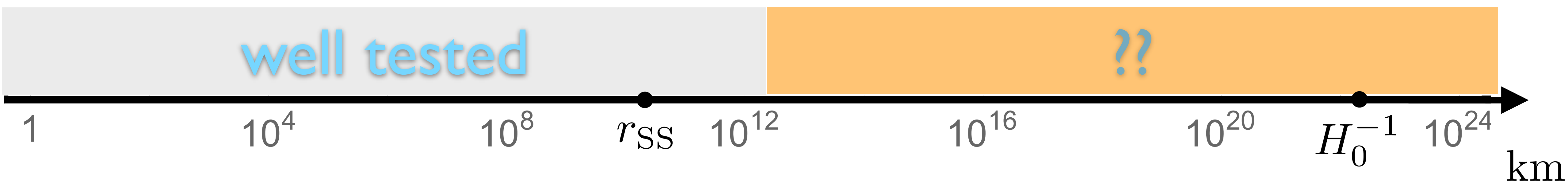
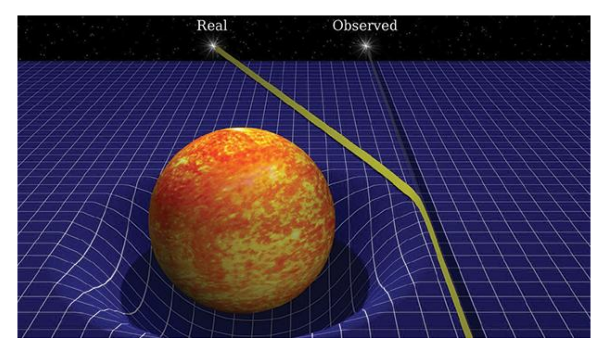



$$G_{\rm eff} = G_N(1+\mu)$$

fifth force

$$\Psi = (1+\Sigma)\Phi$$




anomalous light bending

Will '14 $|\mu| < 10^{-3} \div 10^{-6}$

$|\Sigma| < 10^{-5}$

Solar System scales

DES '18 $|\mu| < 8 \times 10^{-2}$

$|\Sigma| < 4 \times 10^{-1}$

cosmological scales

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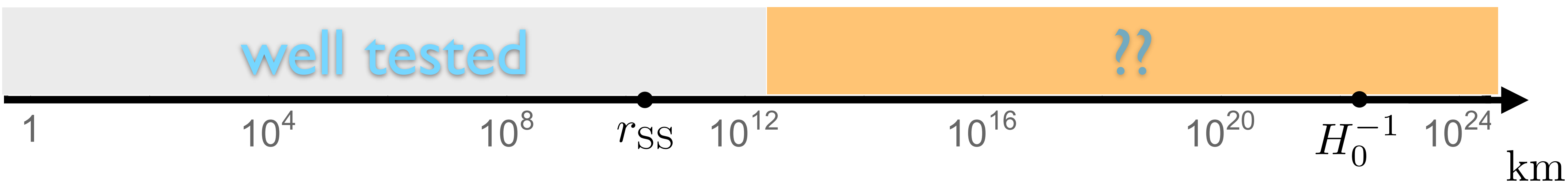


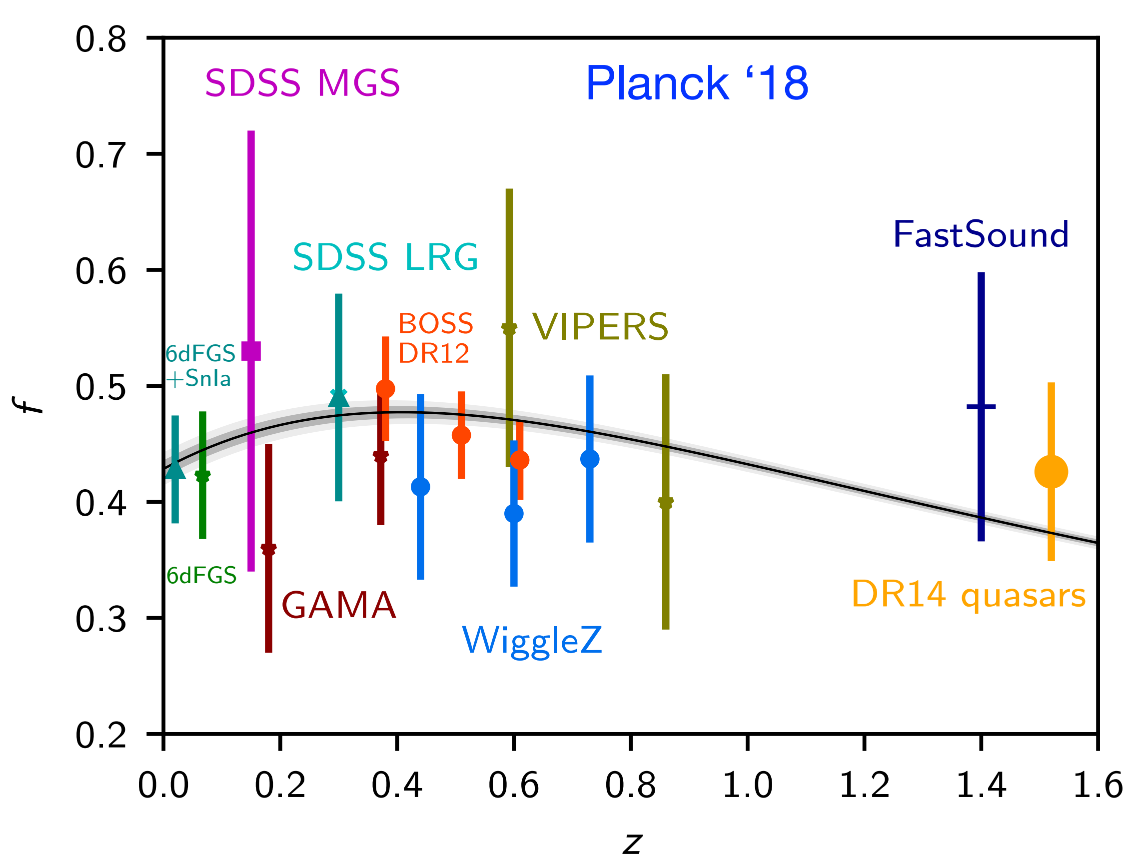


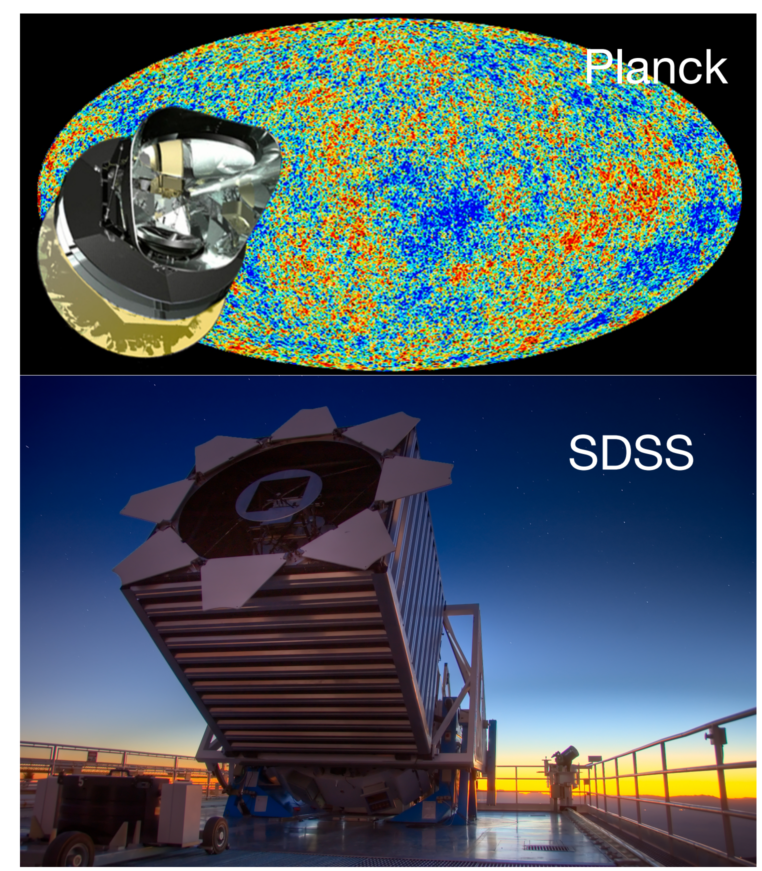

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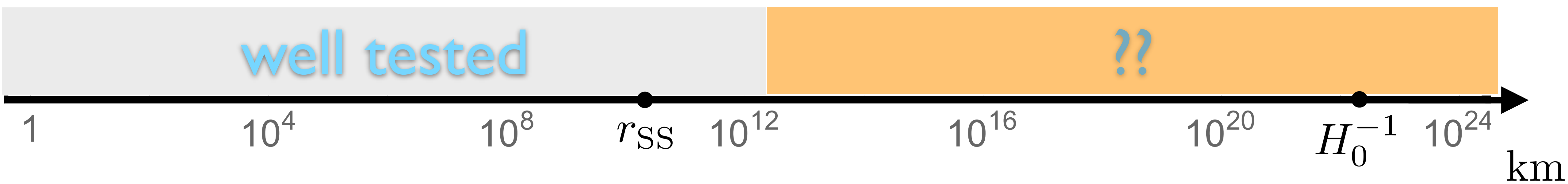
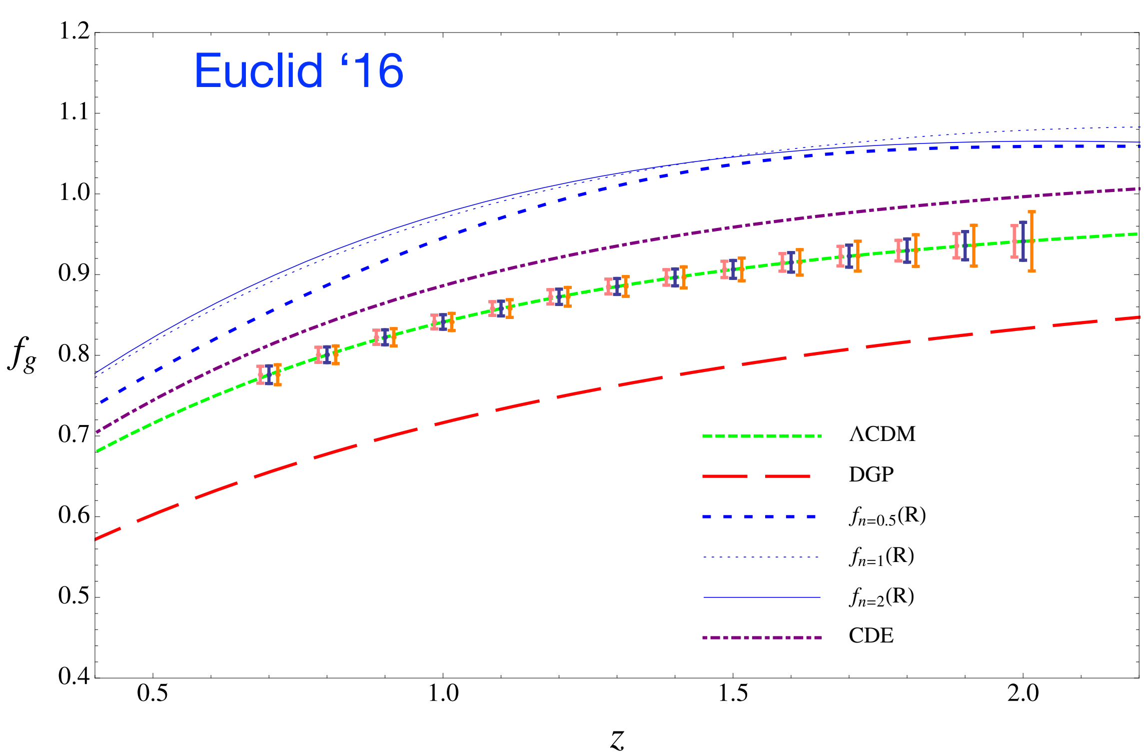
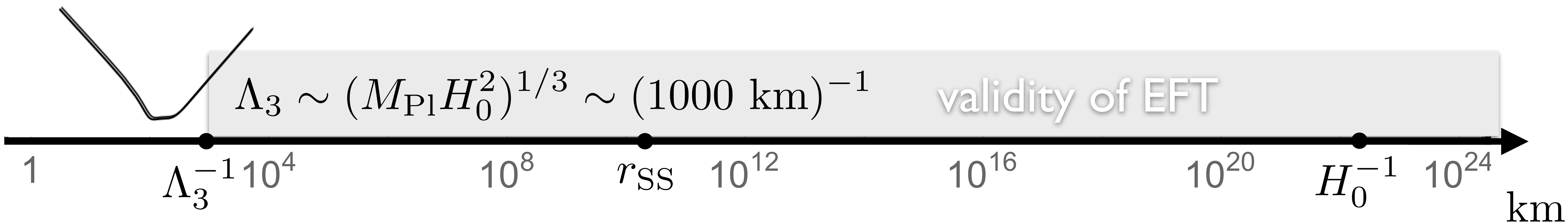
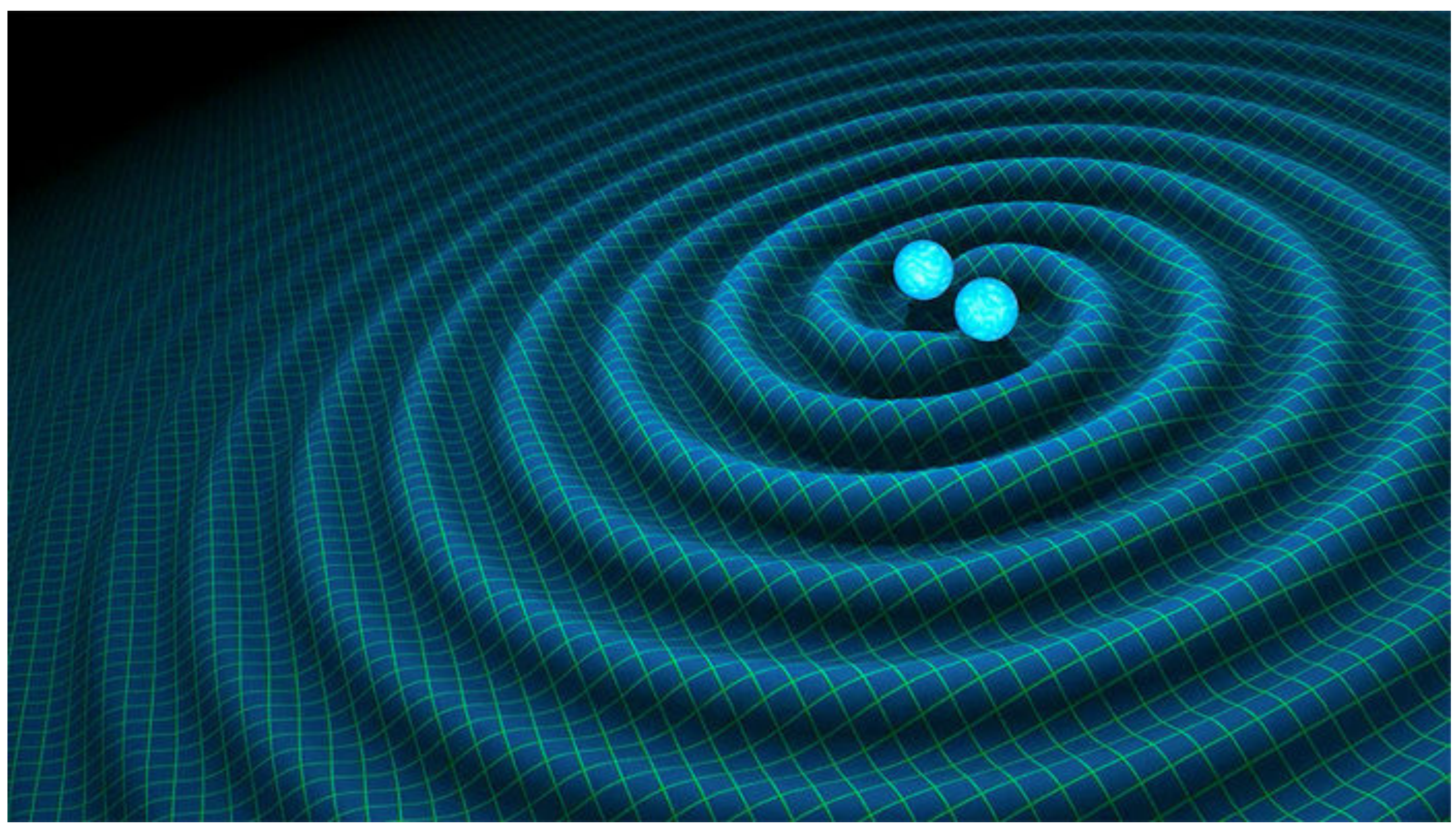



# Gravitational Waves propagation

Modified gravity spontaneously breaks Lorentz Invariance. Acts like a medium, where gravitons are absorbed and dispersed.



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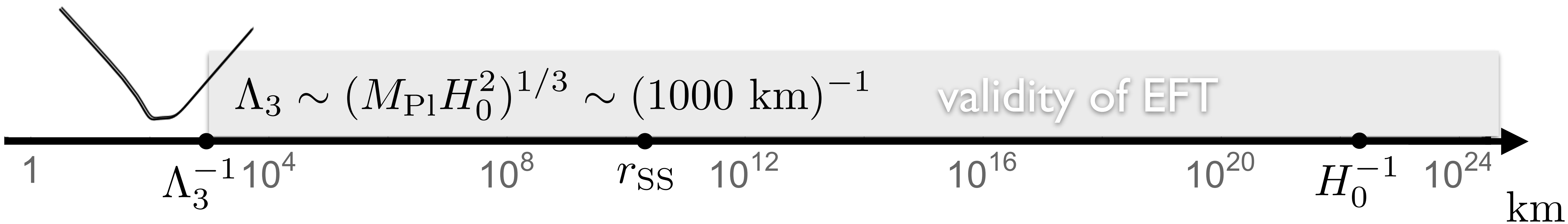


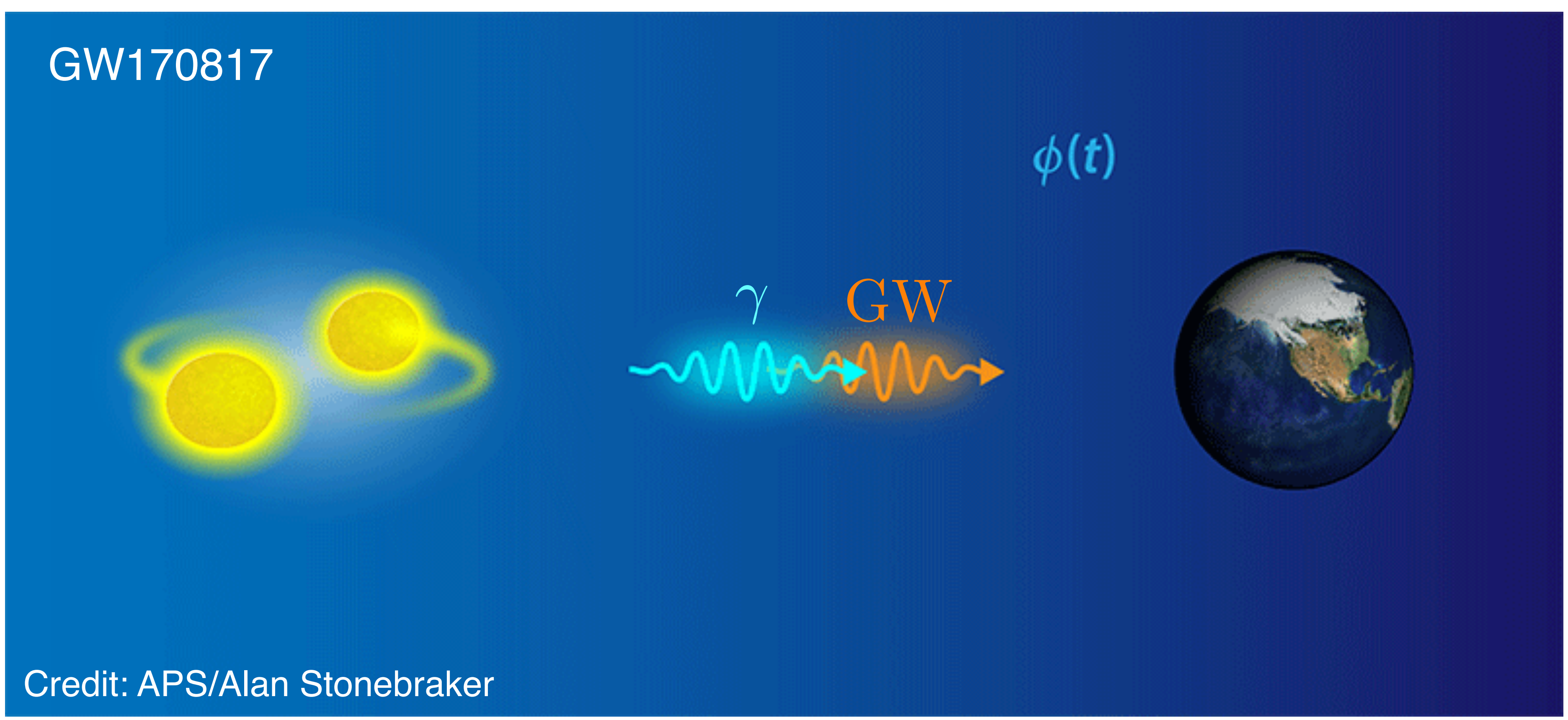

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$$ds^2 = -dt^2 + a^2(t)\left[\delta_{ij} + \gamma_{ij}\right] d\vec{x}^i d\vec{x}^j \ , \qquad \gamma_{ii} = 0 = \partial_i \gamma_{ij} \ , \qquad H = \dot{a}/a$$

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In general relativity:

$$\ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + k^2\gamma_{ij} = 0$$

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# Gravitational Waves propagation

Modified gravity spontaneously breaks Lorentz Invariance. Acts like a medium, where gravitons are absorbed and dispersed.

MG: Frequency independent effects:

$$\ddot{\gamma}_{ij} + (3 + \alpha_{\rm M}) H \dot{\gamma}_{ij} + c_T^2 k^2 \gamma_{ij} = 0$$

damping

speed of propagation

$$G_{\rm eff} = G_N (1 + \mu) \qquad \Psi = (1 + \Sigma)\Phi$$

$$\mu = \mu(\alpha_M, c_T^2, \ldots)\ , \qquad \Sigma = \Sigma(\alpha_M, c_T^2, \ldots)$$

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$$d_L^{\rm gw} \neq d_L^{\rm em}$$

Deffayet, Menou ’07;
Calabrese, Battaglia, Spergel, ’16;
Amendola et al. ’17, Belgacem et al. ’17,
etc…

LISA: $\sigma_{\alpha_M} \approx 0.03 - 0.1$

Amendola, Sawicki, Kunz, Saltas ‘18

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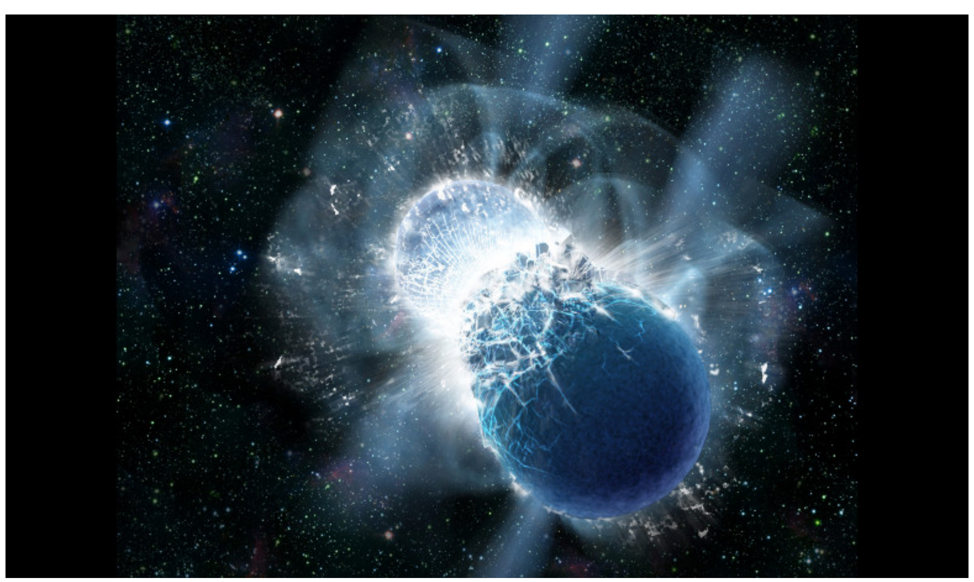
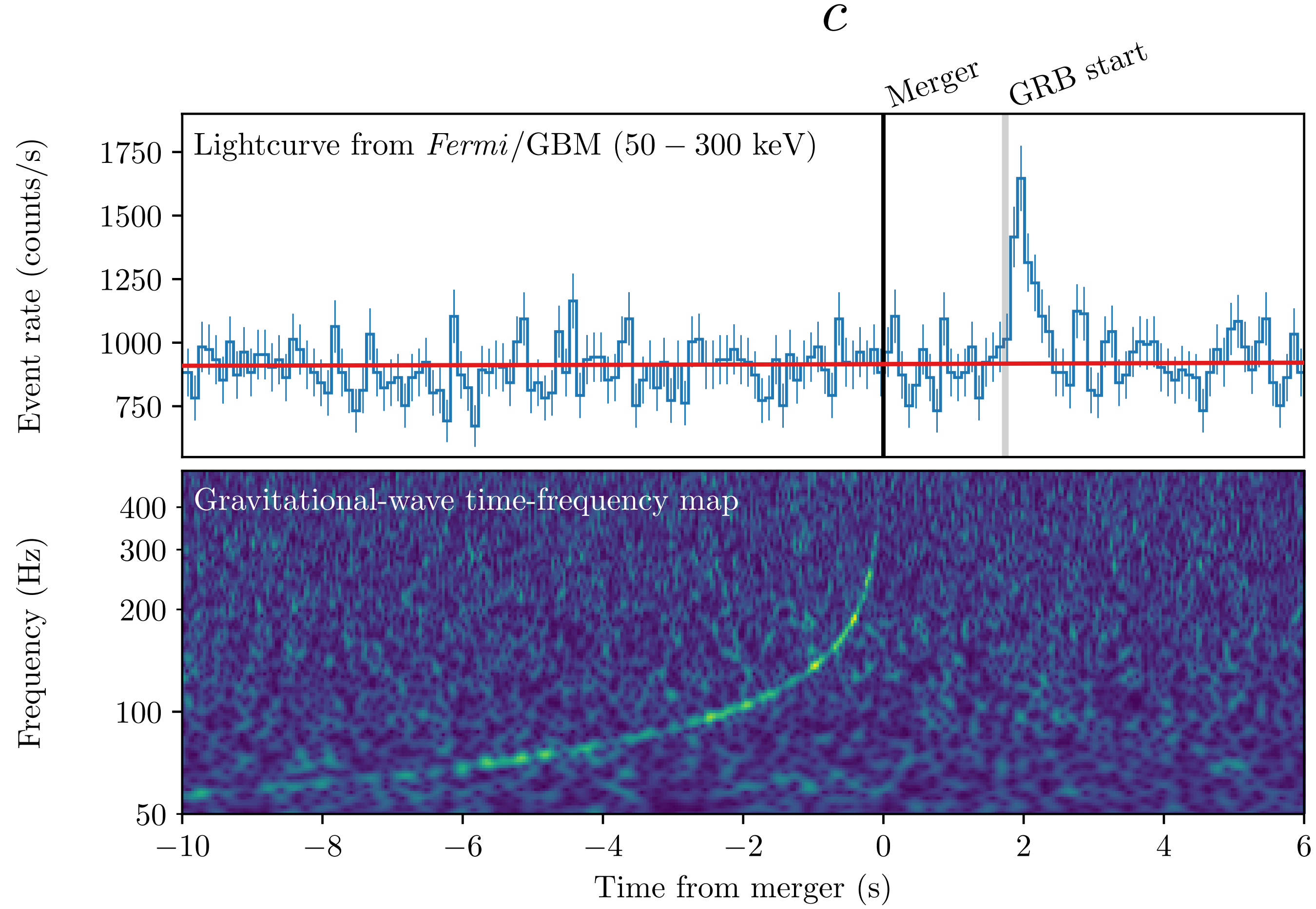
$$-3 \times 10^{-15} \leq \frac{c_g - c}{c} \leq 7 \times 10^{-16}$$

GW170817 = GRB170817A





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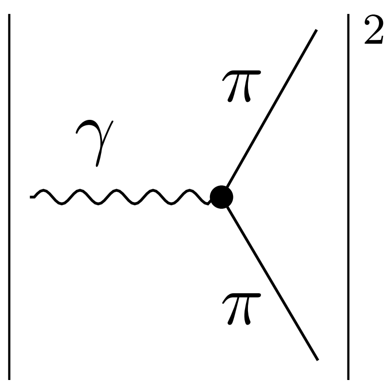
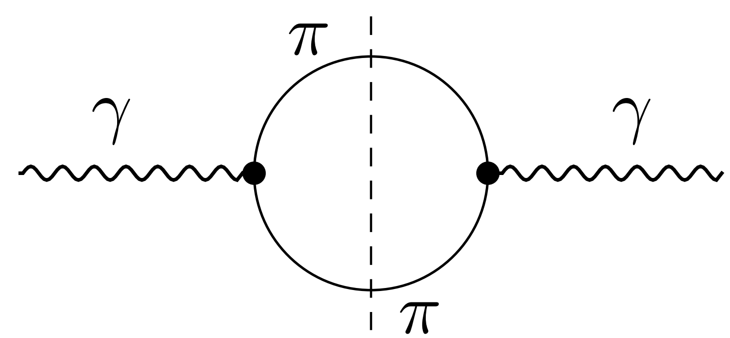
decay




dispersion




related by the optical theorem

$$\Gamma(k)\omega(k) = \mathrm{Im}\left[f(k)\right]$$

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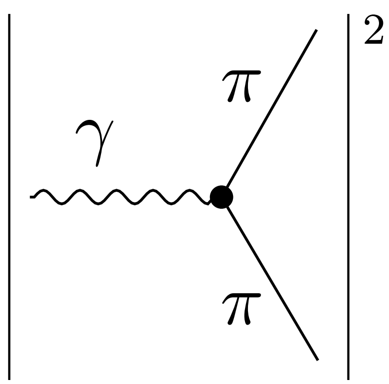


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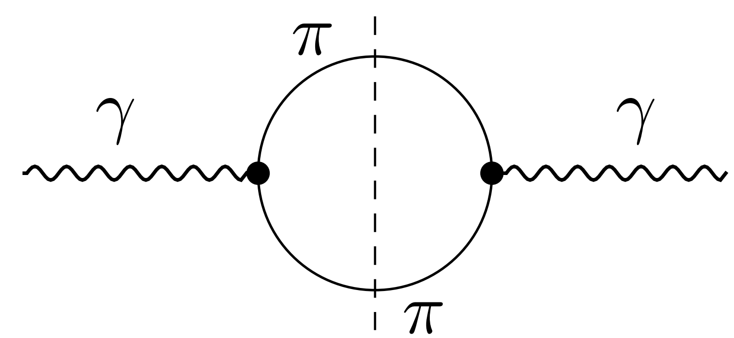


related by the optical theorem

$$\Gamma(k)\omega(k) = {\rm Im}\left[f(k)\right]$$

$$\frac{\Gamma(k)}{\omega} \lesssim \frac{1}{d_S\omega}$$

$$\frac{f(k)}{\omega^2} \lesssim \frac{1}{d_S\omega} \sim 10^{-18} \times \frac{2\pi \times 100\,{\rm Hz}}{\omega}\,\frac{40\,{\rm Mpc}}{d_S}$$

See e.g. Yunes, Yagi, Pretorius ’16; Abbott et al. ‘17

# Scalar-tensor theories

Simplest models of modified gravity are base on single scalar field (universal coupling)

Ex: $$\mathcal{L} = R + V(\phi) - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$ quintessence

$$G_{\mu\nu} = 8\pi G\left(T_{\mu\nu}^{(\mathrm{matter})} + T_{\mu\nu}^{(\phi)}\right)$$

w ≠ 1

# Scalar-tensor theories

Simplest models of modified gravity are base on single scalar field (universal coupling)

Ex: $$\mathcal{L} = R + G_2(\phi, X)\ , \quad X \equiv g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$ k-essence

$$G_{\mu\nu} = 8\pi G\left(T_{\mu\nu}^{(\mathrm{matter})} + T_{\mu\nu}^{(\phi)}\right)$$

$c_s^2 \neq 1$ : clustering

# Scalar-tensor theories

Simplest models of modified gravity are base on single scalar field (universal coupling)

Ex: $\mathcal{L} = f(\phi)R + G_2(\phi, X)\ , \quad X \equiv g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ scalar-tensor gravity

$$G_{\mu\nu}^{(\text{modified})} = 8\pi G \left( T_{\mu\nu}^{(\text{matter})} + T_{\mu\nu}^{(\phi)} \right)$$

self-acceleration

# Modified gravity

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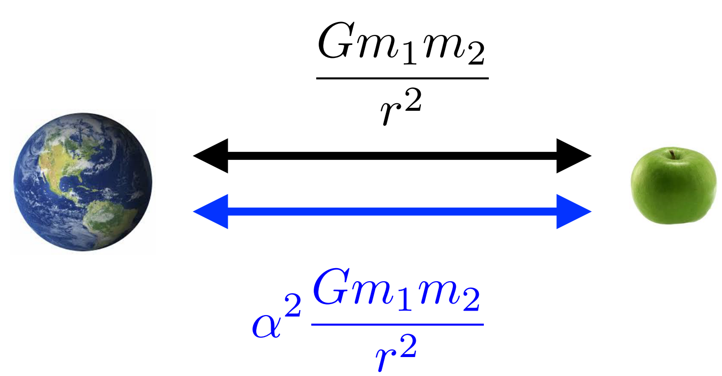

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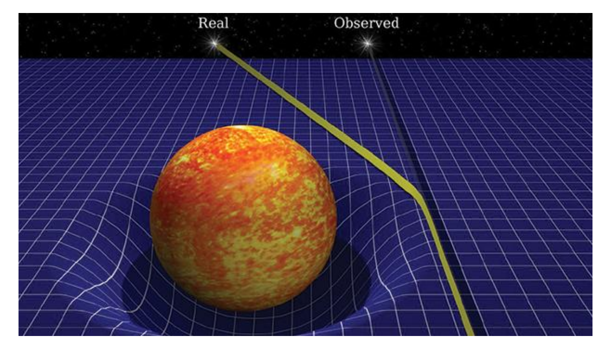
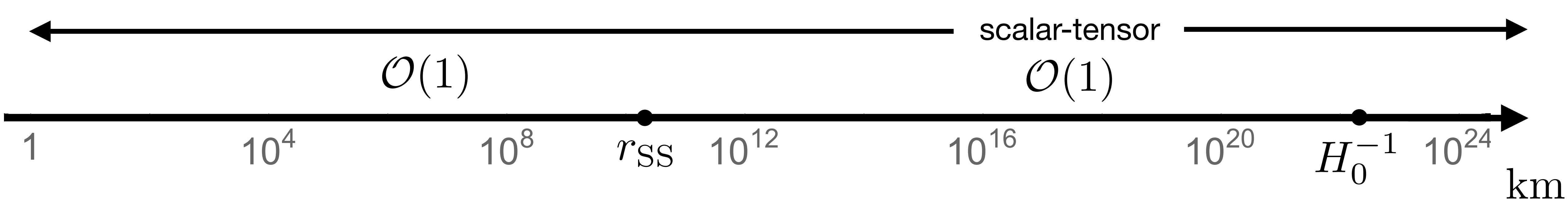


$\Psi \neq \Phi$

anomalous light bending



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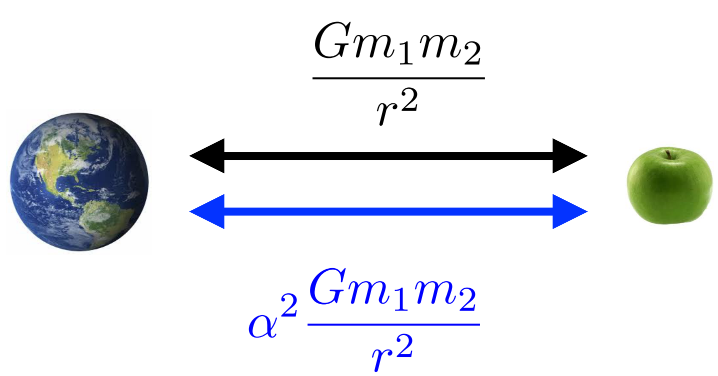
$$\frac{\Box\phi}{\Lambda_3^3} \gg 1$$ Vainshtein screening: large classical scalar field nonlinearities

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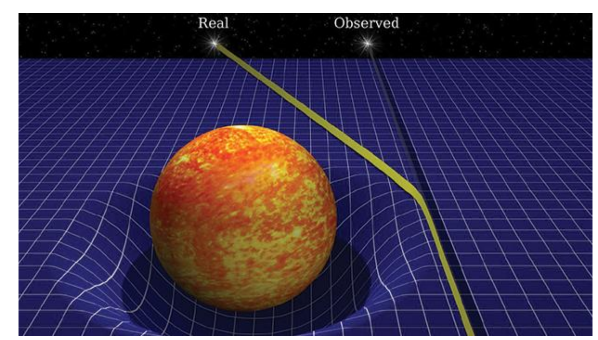


$\Psi \neq \Phi$

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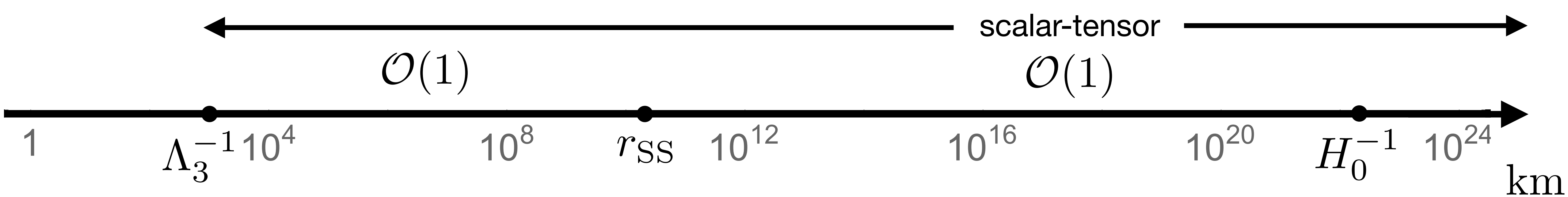
$$\Lambda_3 \sim (M_{\rm Pl}H_0^2)^{1/3} \sim (1000 \text{ km})^{-1}$$



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$$\frac{Gm_1m_2}{r^2}$$

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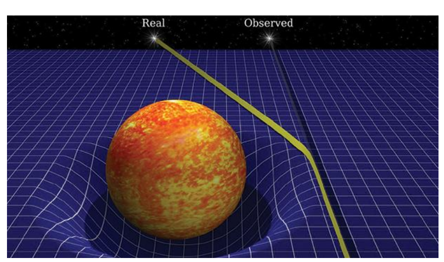
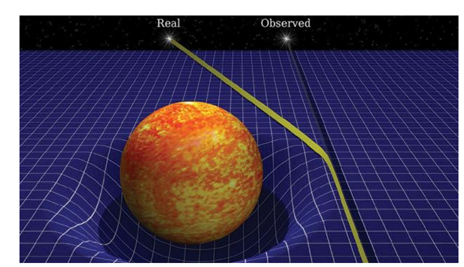
$$\alpha^2\frac{Gm_1m_2}{r^2}$$




$\Psi = \Phi$

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almost GR $\ll \mathcal{O}(10^{-2})$ | scalar-tensor $\mathcal{O}(1)$

1, $\Lambda_3^{-1}$, $10^4$, $10^8$, $r_{\rm SS}$, $10^{12}$, $10^{16}$, $10^{20}$, $H_0^{-1}$, $10^{24}$ km

$$\mathcal{L} = f(\phi)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi$$

Is this the end of the story?

# Generalized theories

Horndeski 73
Deffayet et al. 11

Most general Lorentz-invariant scalar-tensor theory with 2nd-order EOM.

$$\mathcal{L} = G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi \qquad \Box\phi \equiv \phi_{;\mu}^{;\mu} \quad X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu}$$

$$- 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - (\phi_{;\mu\nu})^2\right]$$

$$+ G_5(\phi, X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3\right]$$

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Degenerate theories: most general stable theory.

Langlois, Noui '15; Crisostomi, Koyama, Tasinato '16

Beyond Horndeski theories:

Gleyzes, Langlois, Piazza, FV '14

$$- F_4(\phi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'}$$
$$- F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'}\phi_{;\sigma\sigma'}$$

$$XG_{5,X}F_4 = 3F_5\left[G_4 - 2XG_{4,X} - (X/2)G_{5,\phi}\right]$$

# Setting $c_T$=1

$$\mathcal{L} = G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi \qquad \Box\phi \equiv \phi_{;\mu}^{;\mu} \quad X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu}$$

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Scalar field play with gravity through higher derivatives:

$$\nabla_\mu\nabla_\nu\phi \supset \Gamma^\rho_{\mu\nu}\partial_\rho\phi \qquad \Rightarrow \qquad \Gamma^0_{ij}\,\dot\phi \supset \dot\gamma_{ij}\,\dot\phi$$

$$\mathcal{L}_\gamma \sim (\dot\gamma_{ij})^2 - c_T^2(\partial_k\gamma_{ij})^2$$

$$c_T^2 - 1 \propto -2G_{4,X} - G_{5,\phi} - (H\dot\phi - \ddot\phi)G_{5,X} + XF_4 - 3HX\dot\phi F_5$$

Expected from LSS: $|c_T^2 - 1| \lesssim \text{few} \times 0.01$

# Setting $c_T$=1

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$$- F_4(\phi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'}$$

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$$- F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'}\phi_{;\sigma\sigma'}$$

Scalar field play with gravity through higher derivatives:

$$\nabla_\mu\nabla_\nu\phi \supset \Gamma^\rho_{\mu\nu}\partial_\rho\phi \qquad \Rightarrow \qquad \Gamma^0_{ij}\,\dot\phi \supset \dot\gamma_{ij}\,\dot\phi$$

$$\mathcal{L}_\gamma \sim (\dot\gamma_{ij})^2 - c_T^2(\partial_k\gamma_{ij})^2$$

$$c_T^2 - 1 \propto -2G_{4,X} - G_{5,\phi} - (H\dot\phi - \ddot\phi)G_{5,X} + XF_4 - 3HX\dot\phi F_5$$

Most general theory compatible with $c_T$=1: $G_5 = F_5 = 0\ ,\qquad XF_4 = 2G_{4,X}$

Creminelli, FV ’17; Sakstein, Jain ’17 ; Ezquiaga, Zumalacarregui ’17 ; Baker+ ’17

# What remains

$$\mathcal{L} = G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi \qquad \Box\phi \equiv \phi_{;\mu}^{;\mu} \quad X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu}$$

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$$XF_4 = 2G_{4,X}$$

$$\alpha_H \equiv -\frac{2XG_{4,X}}{X}$$

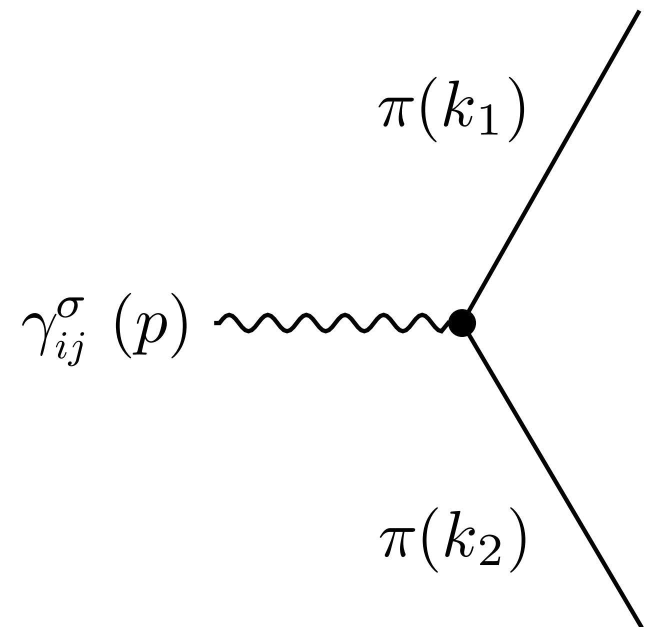
# The decay of GW

Creminelli, Lewandowski, Tambalo, FV '18

Beyond Horndeski with $c_T=1$ implies interactions between GW and scalar fluctuations $\pi$

$$\mathcal{L}_{\gamma\pi\pi} = \frac{\alpha_H}{\Lambda_3^3}\ddot{\gamma}^c_{ij}\partial_i\pi_c\partial_j\pi_c$$

$$\Lambda_3 \equiv (M_{\rm Pl}H_0^2)^{1/3} \qquad \pi \equiv \delta\phi/\dot{\phi}_0$$



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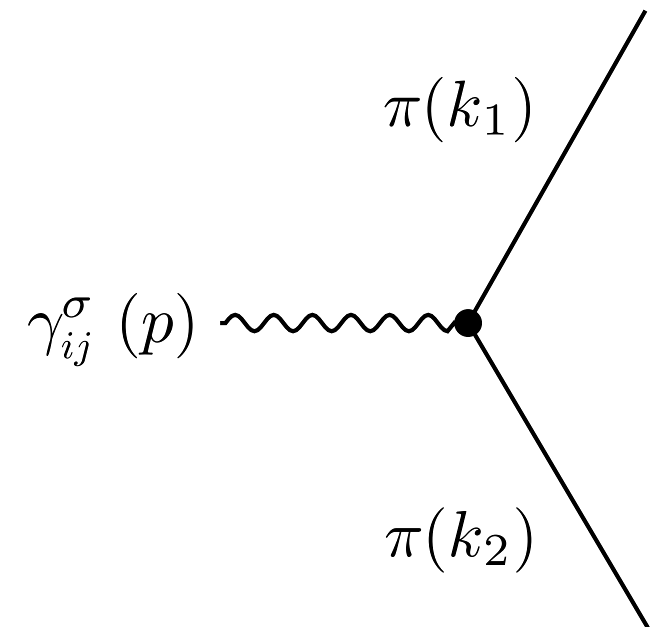



Perturbative decay of gravitons into $\pi$s ($c_s^2$ = sound speed of $\pi$ fluctuations):

- for $c_s > 1$ kinematically forbidden: $|\vec{p}| = c_s(|\vec{k}_1| + |\vec{k}_2|) > |\vec{k}_1 + \vec{k}_2| = |\vec{p}|$
- for $c_s = 1$ kinematics implies vanishing interaction: $\gamma_{ij}(\vec{p})\vec{k}_1^i \vec{k}_2^j = 0$
- for $c_s < 1$ allowed

$$\Gamma \sim \alpha_H^2 \frac{\omega_{\rm gw}^7 (1-c_s^2)^2}{c_s^7 \Lambda_3^6} \simeq \alpha_H^2\ \omega_{\rm gw} \qquad \Rightarrow \qquad \alpha_H = 0$$

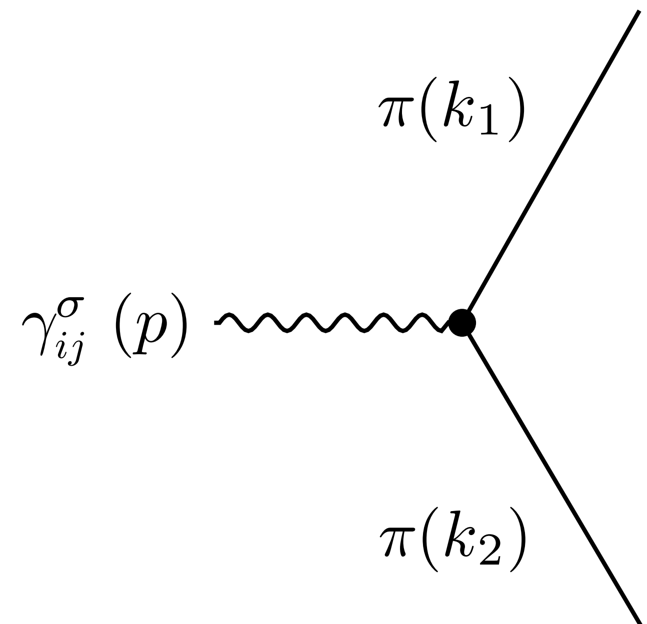
# Loop corrections

Creminelli, Lewandowski, Tambalo, FV '18

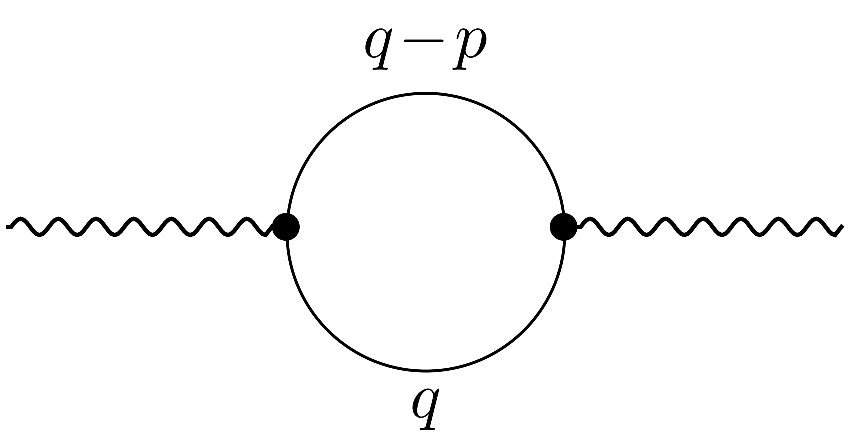
Spontaneous Lorentz-breaking implies modifications of the dispersion relation

$$\mathcal{L}_{\gamma\pi\pi} = \frac{\alpha_H}{\Lambda_3^3}\ddot{\gamma}^c_{ij}\partial_i\pi_c\partial_j\pi_c$$

$$\Lambda_3 \equiv (M_{\rm Pl}H_0^2)^{1/3} \qquad \pi \equiv \delta\phi/\dot{\phi}_0$$

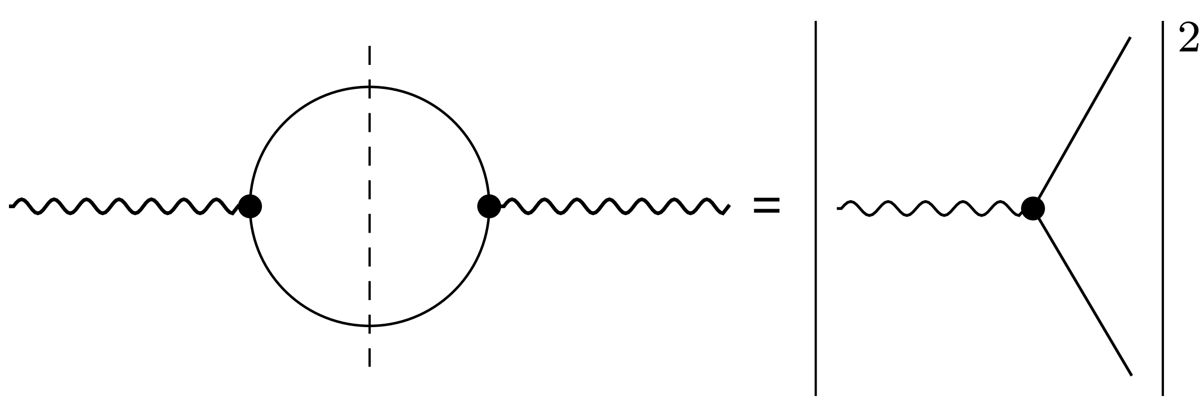



Graviton self-energy:




$$\omega^2 = k^2 - \alpha_H^2\frac{k^8(1-c_s^2)^2}{\Lambda_3^6 c_s^7}\log\left(-(1-c_s^2)\frac{k^2}{\mu^2}\right)$$

Optical theorem:



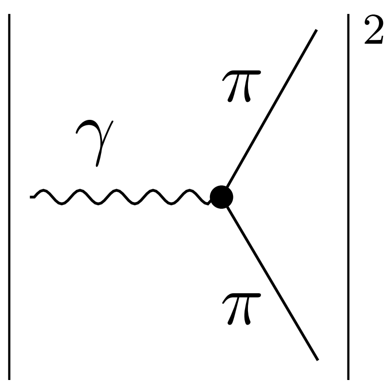

$$\mathrm{Im}\,\omega^2 = \Gamma\omega$$

# Gravitational Waves propagation

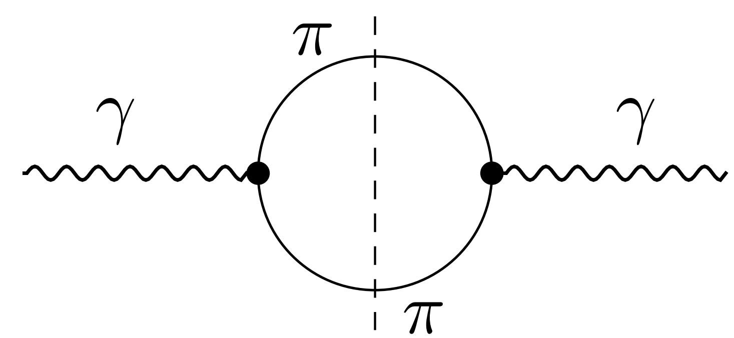
Modified gravity spontaneously breaks Lorentz Invariance. Acts like a medium, where gravitons are absorbed and dispersed.

$$\ddot{\gamma}_{ij} + [(3+\alpha_{\rm M})H + \Gamma(k)]\,\dot{\gamma}_{ij} + \left[c_T^2 k^2 + f(k)\right]\gamma_{ij} = 0$$

decay




dispersion




$$\mu = \mu(\alpha_M, c_T^2 = 1, \alpha_H = 0, \ldots)\ , \qquad \Sigma = \Sigma(\alpha_M, c_T^2 = 1, \alpha_H = 0, \ldots)$$

# What remains

$$\mathcal{L} = G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi \qquad \Box\phi \equiv \phi^{;\mu}_{;\mu} \quad X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu}$$

$$- 2G_{4,X}(\phi, X)\Big[(\Box\phi)^2 - (\phi_{;\mu\nu})^2\Big]$$

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$$- F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'}\phi_{;\sigma\sigma'}$$

$$XF_4 = 2G_{4,X} = 0$$

$$\alpha_H \equiv -\frac{2XG_{4,X}}{X} = 0$$

$$\mathcal{L} = f(\phi)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi$$

Is this the end of the story?

$$\mathcal{L} = f(\phi)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi$$

Is this the end of the story?

Yes.

# Summary

- For $c_s < 1$ decay is allowed and related to imaginary part of the calculable quantum correction of the dispersion relation
- For $c_s > 1$ no decay but real part of the calculable quantum correction of the dispersion relation
- For $c_s = 1$ no decay and no calculable quantum correction but other UV dependent quantum corrections suppressed by powers of $\partial/\Lambda_3$

$$\mathcal{L}_{c_T=1,\ \mathrm{no\ decay}} = G_4(\phi)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi$$

GW decaying rate and loop corrections suppressed

$$\mathcal{L}_{\gamma\pi\pi} \simeq \frac{1}{\Lambda_2^2}\dot{\gamma}^c_{ij}\partial_i\pi_c\partial_j\pi_c$$

$$\Lambda_2 \equiv (M_{\rm Pl}H_0)^{1/2} \gg \Lambda_3$$

The theory is radiatively stable

Luty, Porrati, Rattazzi ‘03

Pirstkhalava, Santoni, Trincherini, FV ‘15

# Conclusion

- GWs dramatically change the prospect for LSS: huge cut in available models
- Many theories are ruled out by cT=1, absence of GW decay and modifications in the graviton dispersion relation
- Constraints on other theories: massive (bi-)gravity (Ph. Brax)
- Future: Decay is perturbative. What happens with high occupation number?