(Tackling Problems in Biomedical and Astrophysical Imaging)



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Outline











Compressed Sensing for Magnetic Resonance Imaging & Cosmology

DRF funded collaboration between CosmoStat and NeuroSpin to share knowledge and develop software tools for compressed sensing and image reconstruction.



http://cosmic.cosmostat.org/



N/eu/ro/Sp/j/n

NeuroSpin

Cutting-edge neuroimaging research centre based at CEA

- Magnetic Resonance Imaging (MRI)
- Cognitive Neuroscience
- Preclinical and Clinical Neuroscience
- Image Processing









Interdisciplinary lab at DAP combining cosmology and statistical methods.

- Cosmology
- Signal Processing
- Machine Learning







Biomedical Context

Just under a third of Europeans are affected by neurocognitive disorders and around 5% of children suffer from learning difficulties.

Characterising injuries in the brains of premature infants could potentially avoid long-term consequences for brain development. This, however, is extremely challenging given the long scan times required to acquire high resolution images.

COSMIC aims to significantly reduce MRI scan times while retaining full HR resolution in the reconstructed images. This will be achieved with combination of novel scanning schemes and cutting edge image reconstruction software.





[Lazarus, et al MRM 2019]





Astrophysical Context

Upcoming cosmological surveys (*e.g.* Euclid, LSST, SKA, *etc.*) will have to process petabyte scales of data in an efficient and automated way. Extracting the most from the data produced will require the best possible image analysis tools.

For radio data, the problem of reconstructing masked images is mathematically identical that of reconstructing subsampled MR images. This means that tools jointly developed by CosmoStat and NeuroSpin can also be applied to astrophysical data.



Outline

Inverse Problems





With an inverse problem one attempts to obtain information about a physical system from observed measurements.







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Linear Inverse Problem





Straight Line : Direct Problem

$$y = mx + b$$

 $x = \begin{bmatrix} 8 & 2 & 11 & 6 & 5 & 4 & 12 & 9 & 6 & 11 \end{bmatrix}$









Straight Line : Inverse Problem

$$y = mx + b$$

$$x = \begin{bmatrix} 8 & 2 & 11 & 6 & 5 & 4 & 12 & 9 & 6 & 11 \end{bmatrix}$$

$$y = \begin{bmatrix} 3 & 10 & 3 & 6 & 8 & 12 & 1 & 4 & 9 & 14 \end{bmatrix}$$





Straight Line : Inverse Problem

$$y = mx + b$$

$$x = \begin{bmatrix} 8 & 2 & 11 & 6 & 5 & 4 & 12 & 9 & 6 & 11 \end{bmatrix}$$

$$y = \begin{bmatrix} 3 & 10 & 3 & 6 & 8 & 12 & 1 & 4 & 9 & 14 \end{bmatrix}$$









Polynomial Line : Inverse Problem

$$y = a_0 + a_1 x + a_2 x^2 + \dots + a_k x^k$$





Well-Posed Problem

- 1. A solution exists
- 2. The solution is unique
- 3. The solution's behaviour changes continuously with the initial conditions

Ill-Posed Problem

- 1. No solution exists
- 2. The solution is not unique
- 3. The problem is ill-conditioned



Well-Conditioned Problem

$$\mathbf{y} \quad \mathbf{A} \quad \mathbf{x}$$

$$\begin{bmatrix} 4\\7 \end{bmatrix} = \begin{bmatrix} 1 & 2\\2 & 3 \end{bmatrix} \begin{bmatrix} x_1\\x_2 \end{bmatrix}$$

$$\begin{bmatrix} 4\\7 \end{bmatrix} = \begin{bmatrix} 1 & 2\\2.01 & 3 \end{bmatrix} \begin{bmatrix} x_1\\x_2 \end{bmatrix}$$

Ill-Conditioned Problem

$$\begin{bmatrix} 3\\1.47 \end{bmatrix} = \begin{bmatrix} 1 & 2\\0.48 & 0.99 \end{bmatrix} \begin{bmatrix} x_1\\x_2 \end{bmatrix}$$
$$\begin{bmatrix} 3\\1.47 \end{bmatrix} = \begin{bmatrix} 1 & 2\\0.49 & 0.99 \end{bmatrix} \begin{bmatrix} x_1\\x_2 \end{bmatrix}$$



Well-Conditioned Problem



Ill-Conditioned Problem

$$\begin{bmatrix} 3\\1.47 \end{bmatrix} = \begin{bmatrix} 1 & 2\\0.48 & 0.99 \end{bmatrix} \begin{bmatrix} x_1\\x_2 \end{bmatrix} \longrightarrow \begin{bmatrix} x_1\\x_2 \end{bmatrix} = \begin{bmatrix} 1\\1 \end{bmatrix}$$
$$\begin{bmatrix} 3\\1.47 \end{bmatrix} = \begin{bmatrix} 1 & 2\\0.49 & 0.99 \end{bmatrix} \begin{bmatrix} x_1\\x_2 \end{bmatrix}$$



Well-Conditioned Problem



Ill-Conditioned Problem

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$$\underset{\mathbf{x}}{\operatorname{argmin}} \quad \frac{1}{2} \|\mathbf{y} - A\mathbf{x}\|_{2}^{2} + \lambda R(\mathbf{x})$$

- 1. Find **x** such that **y**-**Ax** is small
- 2. We have some prior knowledge about the properties of **x** given by R(**x**)



$$F(\mathbf{x}) = \frac{1}{2} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_{2}^{2}$$
$$\nabla F(\mathbf{x}) = \mathbf{A}^{T} (\mathbf{y} - \mathbf{A}\mathbf{x})$$





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A sparse signal is one that is comprised mostly of zeros when expressed in the appropriate basis.





$$\mathbf{x} = \phi \alpha = \sum_{i=1}^{n} \phi_i \alpha_i$$

 ϕ is the dictionary that converts the signal to a sparse representation. (e.g. Fourier transform, wavelet transform, etc.)

Measuring Sparsity





Sparse Minimisation

$$\underset{\alpha}{\operatorname{argmin}} \quad \frac{1}{2} \|\mathbf{y} - A\phi\alpha\|_{2}^{2} + \lambda \|\alpha\|_{1}$$

Applications

- Denoising
- Deconvolution
- Component Separation
- Inpainting

- Blind Source Separation
- Minimisation algorithms
- Compressed Sensing





Compressed Sensing





Nyquist-Shannon Sampling Theorem

A bandlimited signal can perfectly be recovered if the sample frequency (*i.e.* number of sample taken per unit time/space) is at least twice the highest frequency contained in the signal.







What is Compressed Sensing?

Compressed (or compressive) sensing is a paradigm that allows one to sample certain singals at a rate lower than the Nyquist rate by exploiting the sparsity of the signal in a given domain.





Compressed Sensing







 $\frac{1}{2} \|y - H\phi\alpha\|_{2}^{2} + \lambda \|\alpha\|_{1}$ argmin α



Near perfect signal reconstruction!





 $\frac{1}{2} \|y - H\phi\alpha\|_{2}^{2} + \lambda \|\alpha\|_{1}$ argmin α



Near perfect signal reconstruction!











Python Sparse Data Analysis Package



https://github.com/cea-cosmic/pysap





Python Sparse Data Analysis Package



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Modular Optimisation



https://github.com/cea-cosmic/ModOpt





$$1: \tilde{\mathbf{X}}_{k+1} = \operatorname{prox}_{\tau G}(\mathbf{X}_{k} - \tau \nabla F(\mathbf{X}_{k}) - \tau \mathcal{L}^{*}(\mathbf{Y}_{k}))$$

$$2: \tilde{\mathbf{Y}}_{k+1} = \mathbf{Y}_{k} + \varsigma \mathcal{L}(2\tilde{\mathbf{X}}_{k+1} - \mathbf{X}_{k}) - \varsigma \operatorname{prox}_{K/\varsigma}\left(\frac{\mathbf{Y}_{k}}{\varsigma} + \mathcal{L}(2\tilde{\mathbf{X}}_{k+1} - \mathbf{X}_{k})\right)$$

$$3: (\mathbf{X}_{k+1}, \mathbf{Y}_{k+1}) := \xi(\tilde{\mathbf{X}}_{k+1}, \tilde{\mathbf{Y}}_{k+1}) + (1 - \xi)(\mathbf{X}_{k}, \mathbf{Y}_{k})$$

(Condat 2013, Vu 2013)

https://github.com/cea-cosmic/ModOpt





Gradient

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Proximity Operators

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Linear Operators

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Linear Operators

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Sparse2D





Sparse Dictionaries in **2D**

 PySAP provides Python bindings and wrappers for Sparse2D (C++) wavelet and curvet transforms.

Mass Mapping



Peel et al. (2017)



Deconvolution

Farrens et al. (2017)

Discrete Wavelet transforms $g[n] \rightarrow (2)$ $g[n] \rightarrow (2)$ Level 3 $g[n] \rightarrow (2)$ $h[n] \rightarrow (2)$ coefficients $x[n] \rightarrow h[n] \rightarrow (2)$ Level 1coefficients

Undecimated Wavelet transforms



https://github.com/CosmoStat/Sparse2D





Sparse Transforms in **2D**





Starlet Transform (Starck et al. 2015a)

https://github.com/CosmoStat/Sparse2D



Plug-Ins

- Space for PySAP tools to be combined for specific applications.
- Gives visibility to contributors.
- Currently includes example applications to astrophysical and MRI data.

k-Space

Observed Image

PySAP Denoising



https://github.com/cea-cosmic/pysap







An open source framework for medical image reconstruction



http://gadgetron.github.io/





The Future



The Future

BabyBrainScope

Fast High Resolution Brain Imaging in Preterm Infants

ANR Projet PRCI CES 45 – Mathematical and signal processing tools for life sciences



The Future





COSMIC constitutes an exciting collaboration between NeuroSpin and CosmoStat for developing new software and new ideas for image processing.

PySAP pre-release is available for beta testing.

Through the Gadgetron system PySAP could be made readily available on virtually any MRI scanner around the world.

The plug-in system opens up the software to various other image analysis application such as electron tomography.

https://github.com/cea-cosmic/pysap

