New Dynamical Systems Tools to study atmospheric flows

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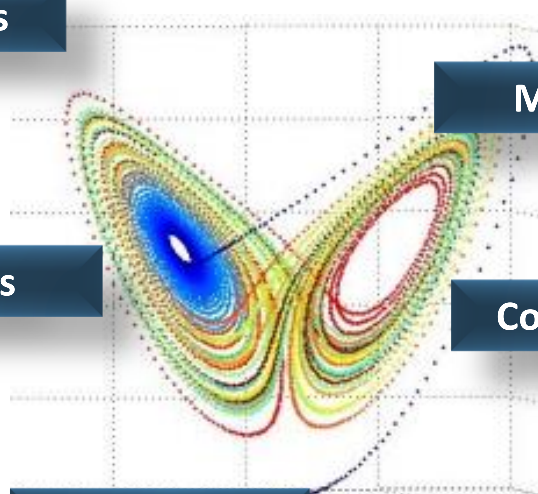
D Rodriguez, MC Alvarez-Castro, G Messori, Sandro Vaienti and P Yiou

Objectives

Method



Results

Conclusions

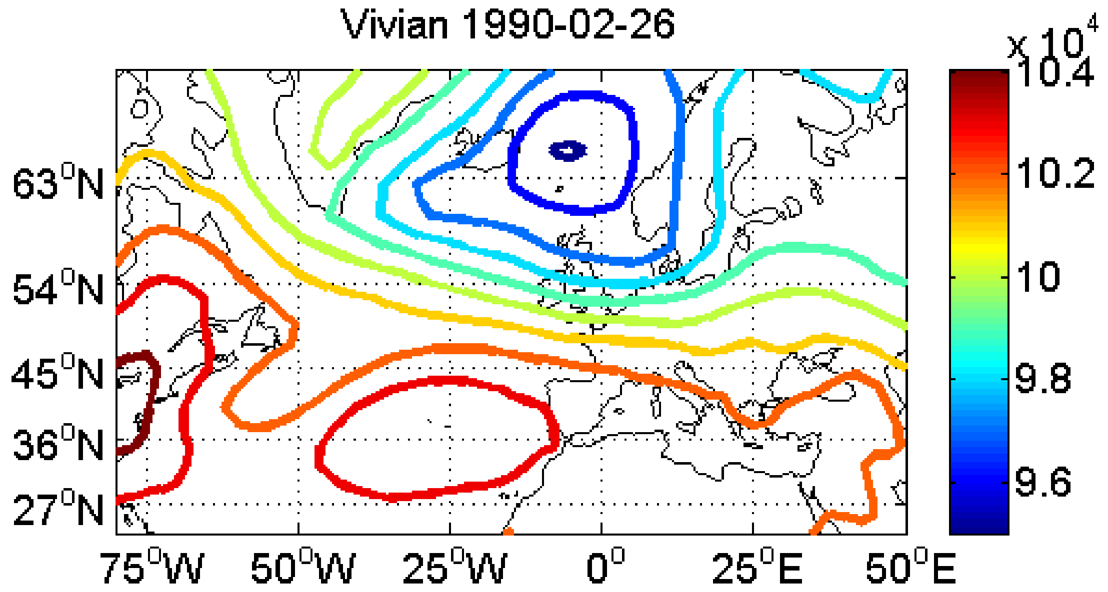
References

European Research Council
Established by the European Commission

OBJECTIVES

Characterize the predictability of Atmospheric Fields

- *How recurrent?*
- *How rare?*
- *How persistent?*
- *How predictable?*



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METHOD

Compute Dynamical Systems metrics to characterize atmospheric states, verifying that a long series of observations sample the underlying Attractor.

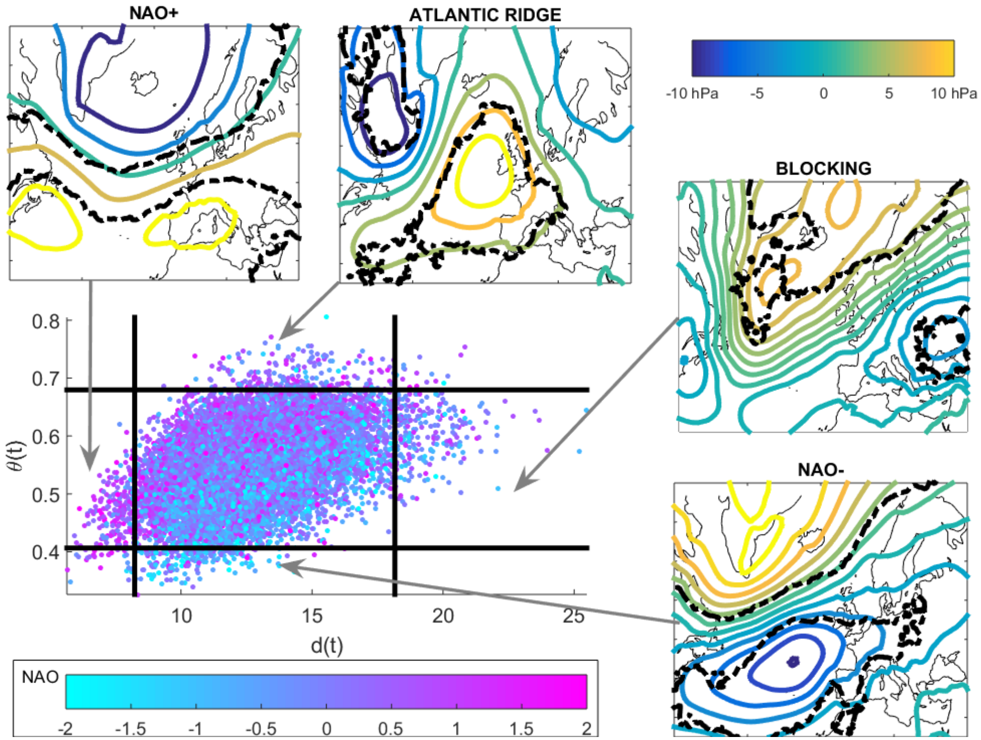
Local Dimensions *d*

It is proportional to the number of possible configurations (**number of degrees of freedom**) originating and resulting from the atmospheric field analyzed.

Persistence *ϴ*

Its inverse tells for **how long the atmospheric field will look like the one under examination**. For the present analysis ϴ is an inverse number of persistance days.

RESULTS – PERSISTENCE/DIMENSION DIAGRAM



The scatter plot displays the daily values of the instantaneous dimension *d* - the higher *d*, the more unpredictable is the atmospheric circulation - and the persistence *θ* - the lower θ the more stable is the atmospheric circulation - of the sea level pressure field (in hPa) extracted from the NCEP Database. The colorscale represents the North Atlantic Oscillation (NAO) index.

Seasonal Cycle

Resolution

Historical Storms

Predictability

CMIP5 models assessment

Winter 2013/2014

Real Time Analysis

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CONCLUSIONS

From an innovative application of recent results in dynamical system theory, we obtain that:

- The distribution of the local dimensions **capture the features of mid-latitude circulation dynamics**.

- Extremes of local dimensions **correspond to real-life extreme weather** (storms and blocking).

- One could use the dynamical indicators to prepare **better ensemble forecasts by adjusting the number of members and/or the accuracy of the forecast**

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REFERENCES

[1] Davide **Faranda**, Gabriele Messori and Pascal Yiou. Dynamical proxies of North Atlantic predictability and extremes**. Scientific Reports**, 7-41278, **2017**.

[2] Valerio Lucarini, Davide **Faranda**, Ana Cristina Gomes Monteiro Moreira de Freitas,Jorge Miguel Milhazes de Freitas, Mark Holland, Tobias Kuna, Matthew Nicol, Mike Todd, Sandro Vaienti. Book: **Extremes and Recurrence in Dynamical Systems.** ISBN 978-1-118-63219-2, 312 pages, **Wiley**, 2016.

[3] David Rodrigues, M Carmen Alvarez-Castro, Gabriele Messori, Pascal Yiou, Yoann Robin , Davide Faranda. Changes in the dynamical properties of the North Atlantic atmospheric circulation in the past 150 years. Submitted to *Journal of Climate*, 2017

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STORMS IN THE PRESS CORRESPOND TO MINIMA OF THE LOCAL DIMENSIONS


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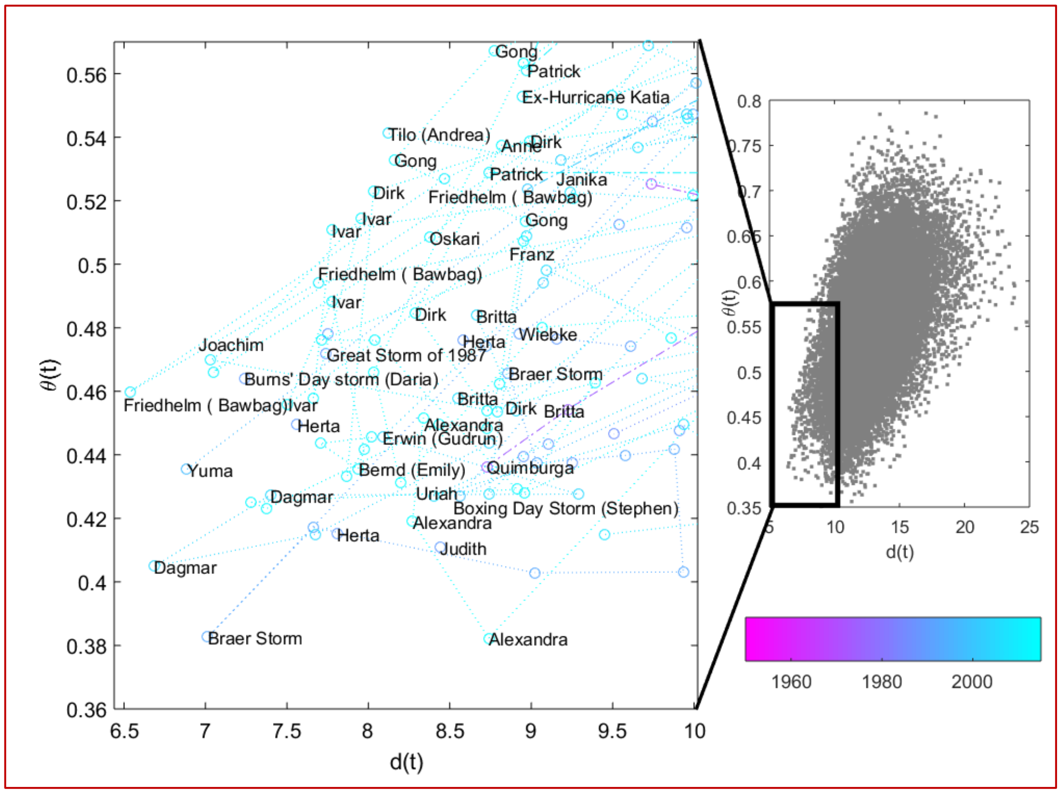
European windstorm

From Wikipedia, the free encyclopedia

This article is about storms crossing the Atlantic Ocean eastward and striking Europe. It is not to be confused with storms arising w Europe.

European windstorm is a name given to the strongest extratropical cyclones which occur across the continent of Europe.[2] They form as cyclonic windstorms associated with areas of low atmospheric pressure that track across the North Atlantic Ocean towards western Europe. They are most common in the autumn and winter months. On average, the month when most windstorms form is January.



Storms matching the <u>minima</u> of the instantaneous dimensions.

The instantaneous dimensions *d* (x-axis) and persistence *Θ* (y-axis) for the selected historical storms are plotted along with the storms' names and years of occurrence (colourscale). Repeated names indicate storms which persisted for several days. The inset shows the full distribution of *(d,Θ)* values. The black lines delimit the phase-space region in which the selected storms lie.

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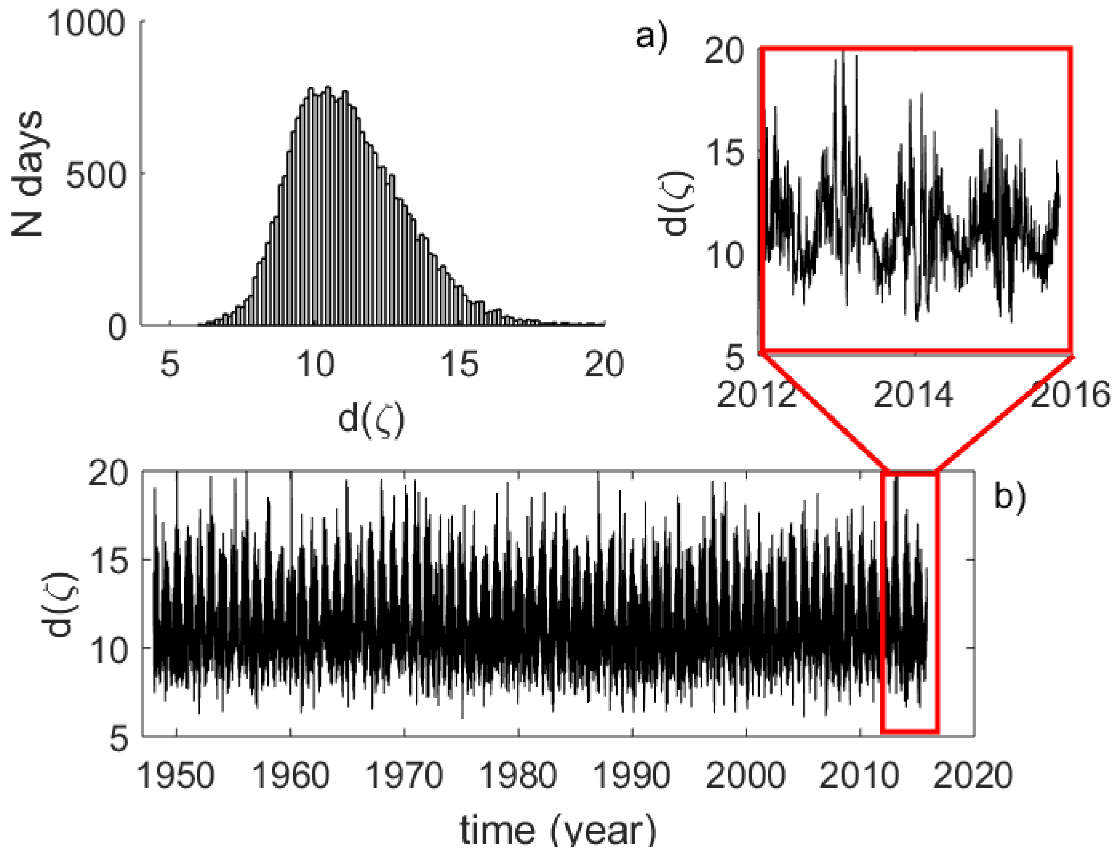
SEASONAL CYCLE



The instantaneous dimensions *d* (y-axis) versus the years of the database shows an interesting seasonal cycle. Extremes are found in wintertime, where sharp transitions occur between maxima and minima.

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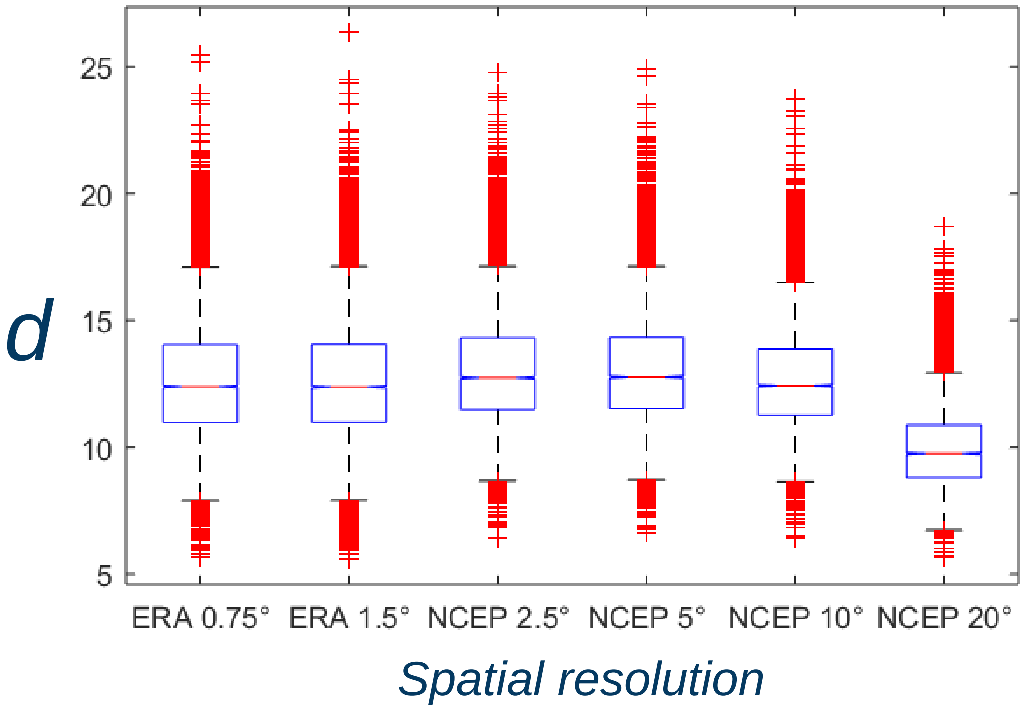
RESOLUTION



The instantaneous dimensions *d* (y-axis) boxplot versus the change in spatial resolution/dataset shows the stability of the method.

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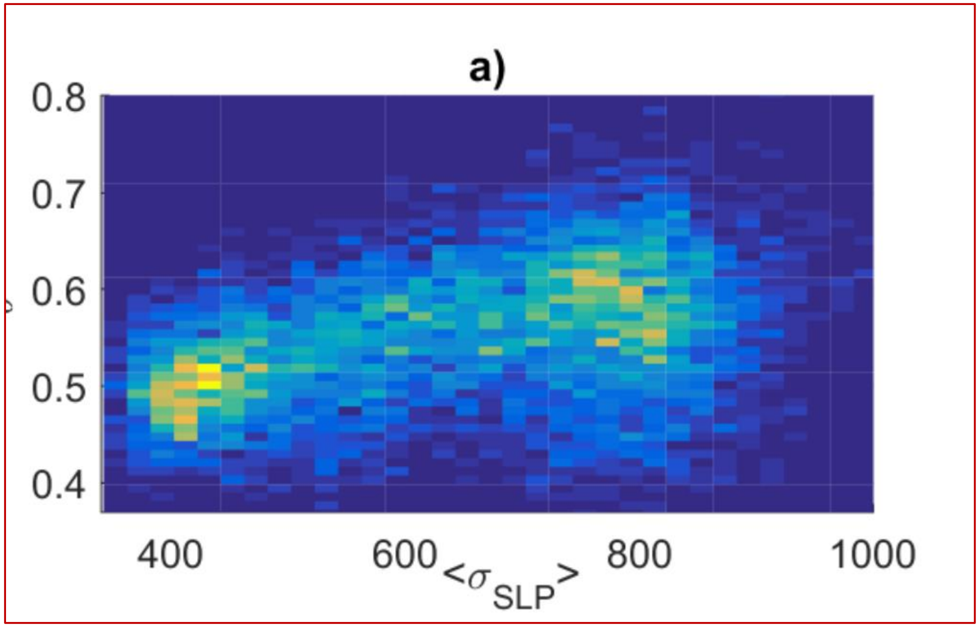
IMPLICATIONS ON WEATHER FORECASTS

θ



Error in the forecast at 384 h

Analysis of the relation between instantaneous properties and NOAA GER reforecast. Bivariate histograms of the ensemble spread $<\sigma_{SLP}>$ at a lead time of +384h as a function of the stability Θ of the initialisation field, for the period 2000-2015.

The colourscale indicates the number of days with the same pair of parameters.

There is a linear correlation between error in the forecat and persistence

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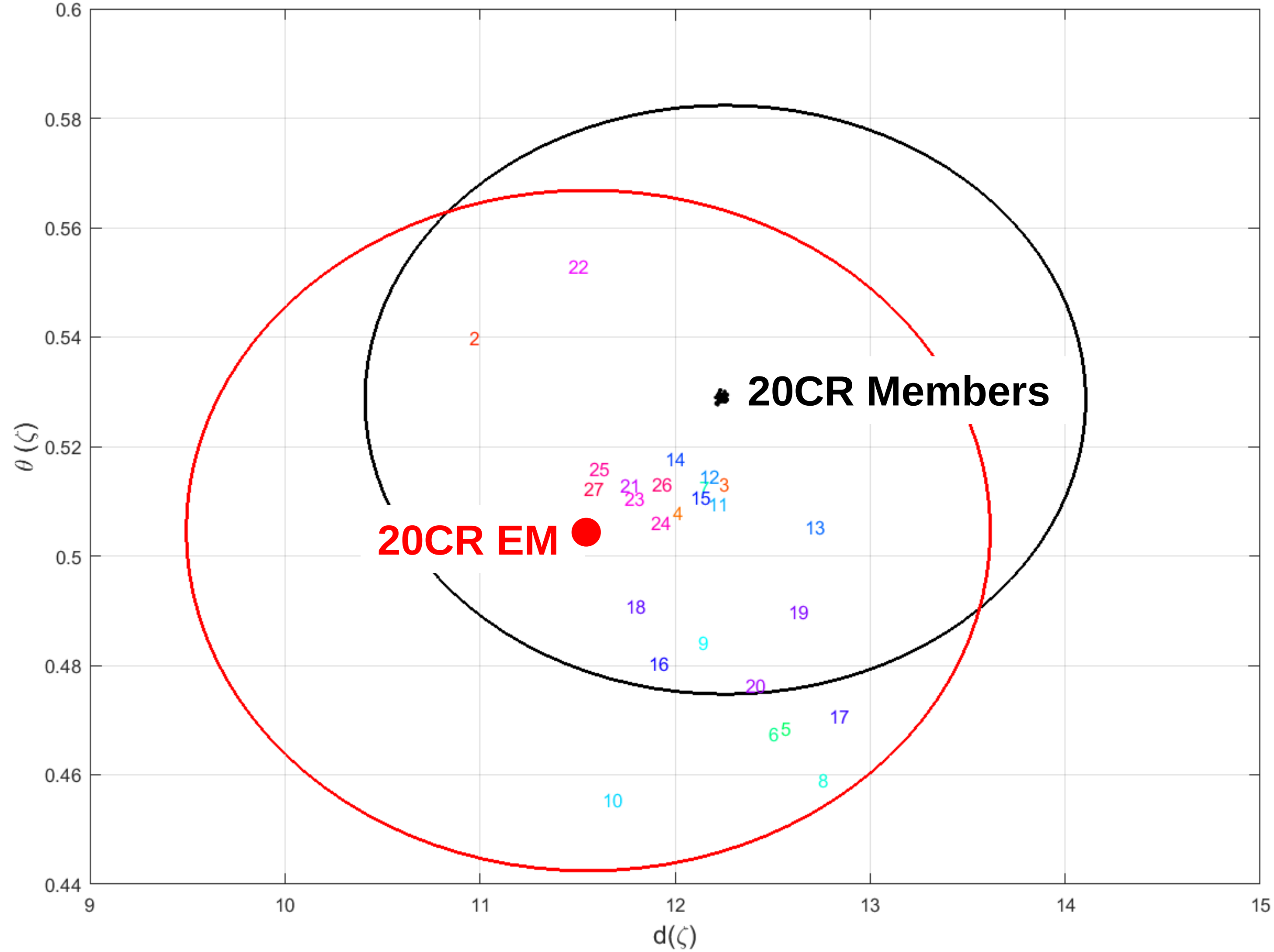
CMIP5 MODELS ASSESSMENT



02-CMCC-CESM
03-CanESM2
04-MIROC-ESM-CHEM
05-MIROC-ESM
06-BCC-CSM1-1
07-IPSL-CM5B
08-NorESM1-M
09-FGOALS-S2
10-MPI-ESM-P
11-MPI-ESM-LR
12-CSIRO-MK3-6-0
13-CMCC-CMS
14-MPI-ESM-MR
15-IPSL-CM5A
16-INM-CM4
17-ACCESS '1-0
18-MIROC5
19-CNRM-CM5
20-MRI-ESM1
21-BCC-CSM1-M
22-MRI-CGCM3
23-EC-EARTH
24-CESM1-FAST
25-CESM1-CAM5
CESM1-BGC
27-CCSM4

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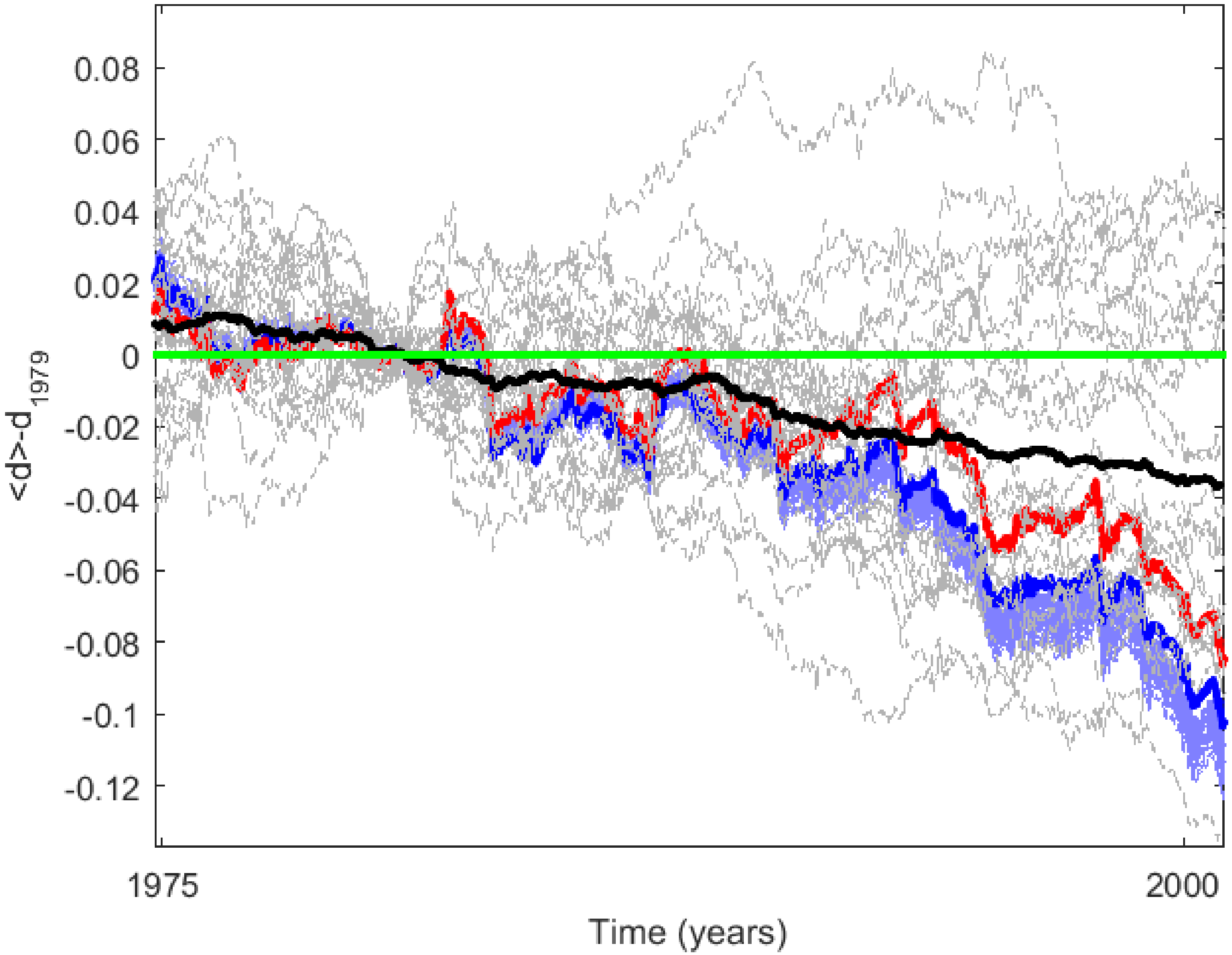
Median values of local dimensions *d* and the persistence *θ*. Semiaxes of each ellipse represent one standard deviation of *d* and θ.

CMIP5 MODELS ASSESSMENT – CHANGES FROM 1979



20CR EM
20CR Members
CMIP5 Models
Multimodel mean

30 Years moving average of local dimension showing a decreas in D

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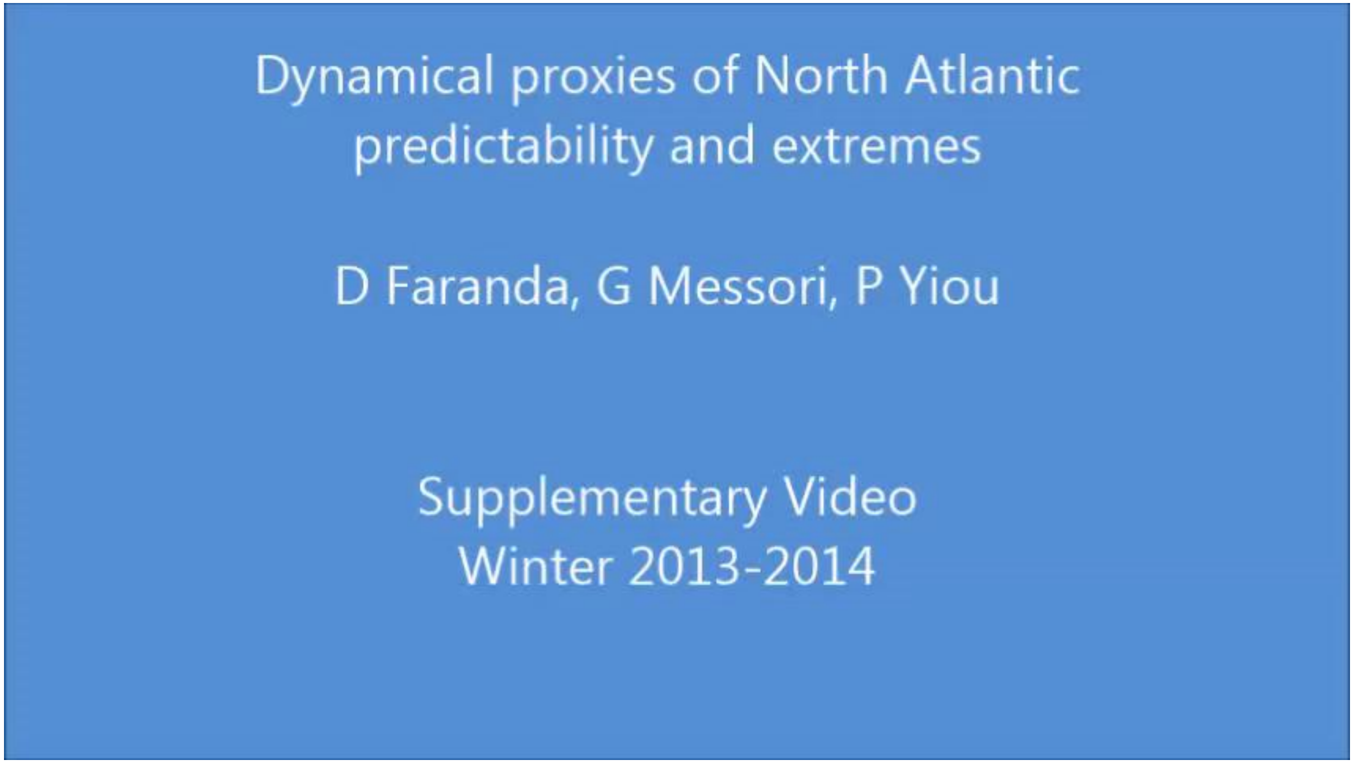
WINTER 2013/2014 ANIMATION



The video displays the daily values of the instantaneous dimension d - **the higher d, the more unpredictable is the atmospheric circulation -** and the persistence θ - **the lower θ the more stable is the atmospheric circulation** - of the sea level pressure field (in hPa) extracted from the [NCEP Database] for the winter 2013/2014.

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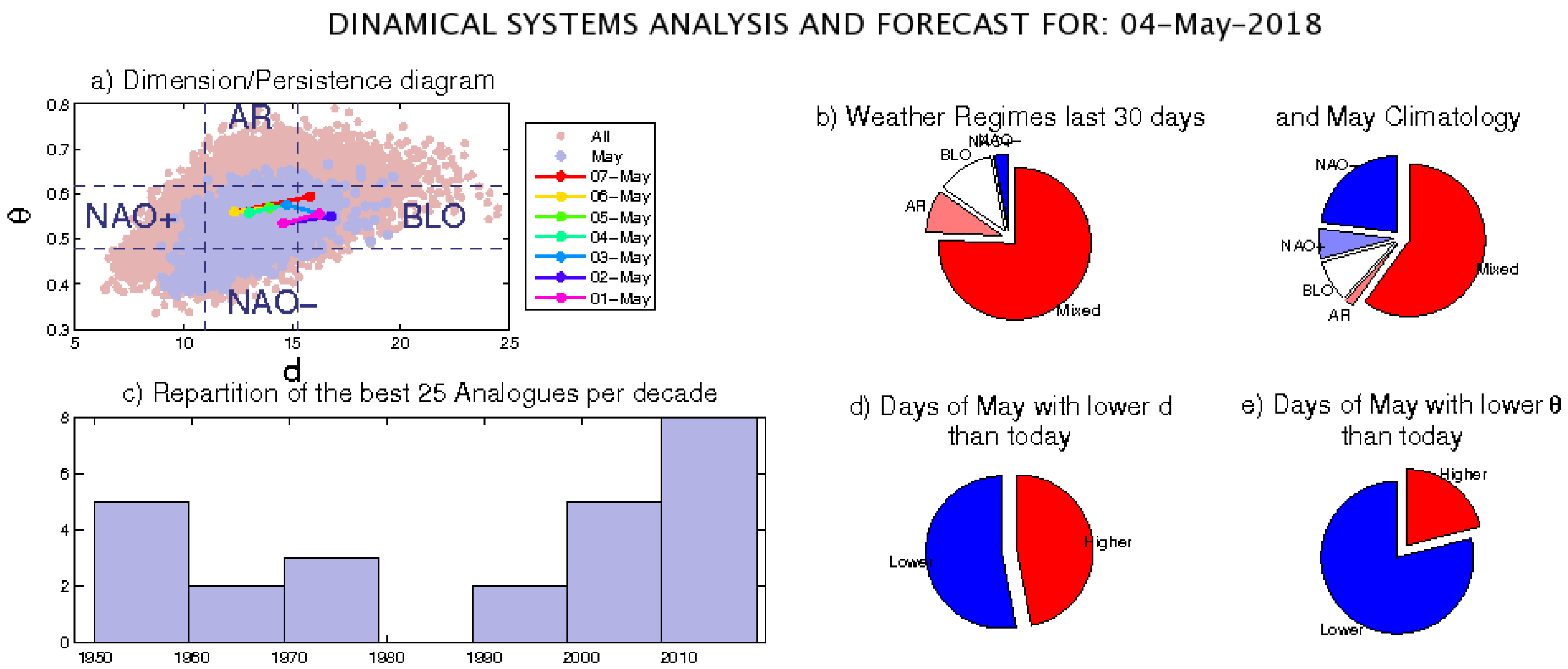
REAL TIME ANALYSIS FOR THE NORTH ATLANTIC



http://www.lsce.ipsl.fr/Pisp/davide.faranda/#dynAnalysis

The scatter plot displays the daily values of the instantaneous dimension d - **the higher d, the more unpredictable is the atmospheric circulation -** and the persistence θ - **the lower θ the more stable is the atmospheric circulation** - of the sea level pressure field (in hPa) extracted from the NCEP Database. The trajectory for the last 7 days is displayed in colors.

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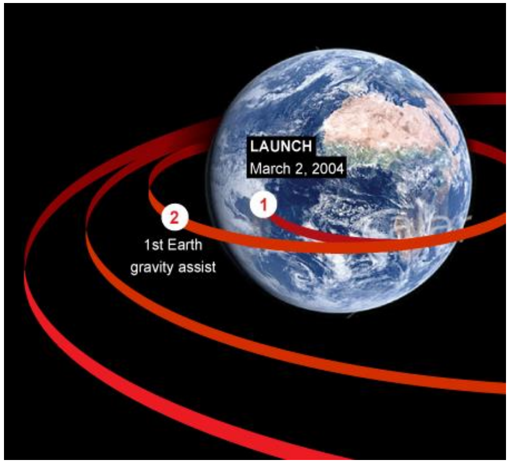
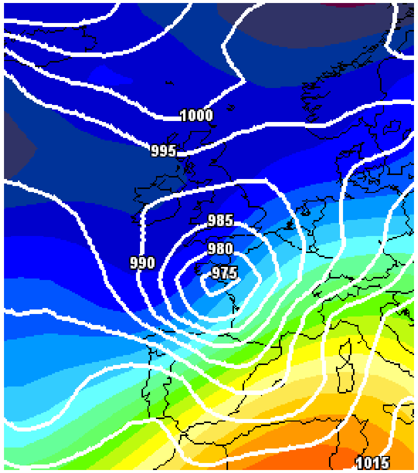
DYNAMICAL SYSTEMS THEORY

Use of differential equations
to predict the motion (trajectory) of a material point

Foucalt Pendulum



Rosetta Trajectory



Weather Forecast

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Lagrange - Mécanique analytique (1788),

ATTRACTOR

***Definition**:* Ensemble of numerical values toward which a system tends to evolve, for a wide variety of starting conditions of the system.

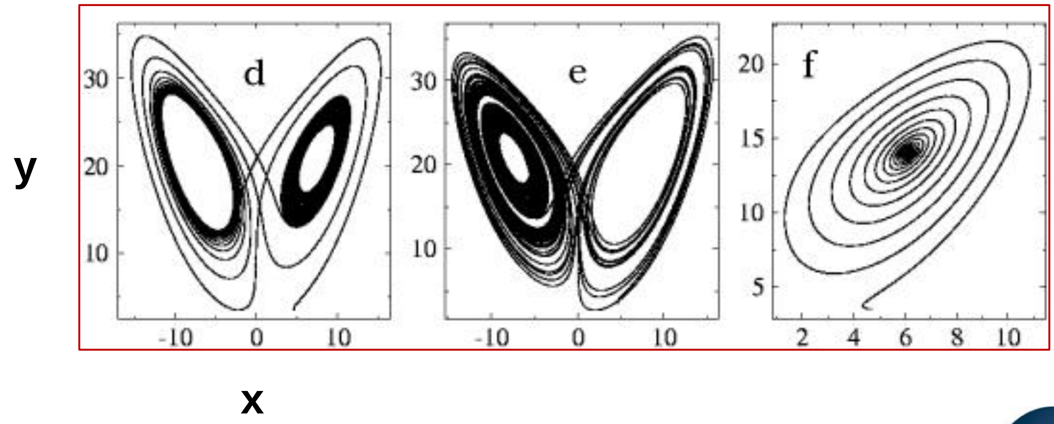
Example: Lorenz 63 attractor (Rayleigh Bénard convection)

$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = rx - y - xz$$

$$\frac{dz}{dt} = xy - bz$$

Attractors Change with σ , r ➔



σ , r Prandtl and Reyleigh Number, *b:* ratio of critical parameters
x: convection strength , *y*: difference of temperature, *z:* asymmetry

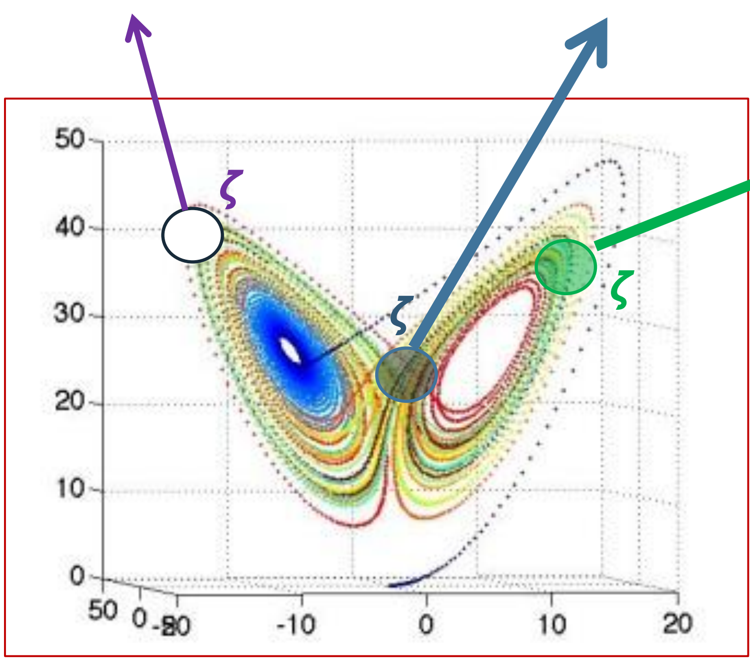
Lagrange - Mécanique analytique (1788),

LOCAL DIMENSIONS : GEOMETRY ⇐⇒ DYNAMICS

Line: d(ζ)=1

Patch: d(ζ)=2

Fractal:1 < d(ζ) < 2



The local dimension depends on the point of the attractor considered

How to compute Local Dimensions

Local Dimensions in Lorenz Attractor

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HOW TO COMPUTE LOCAL DIMENSIONS (1/3)

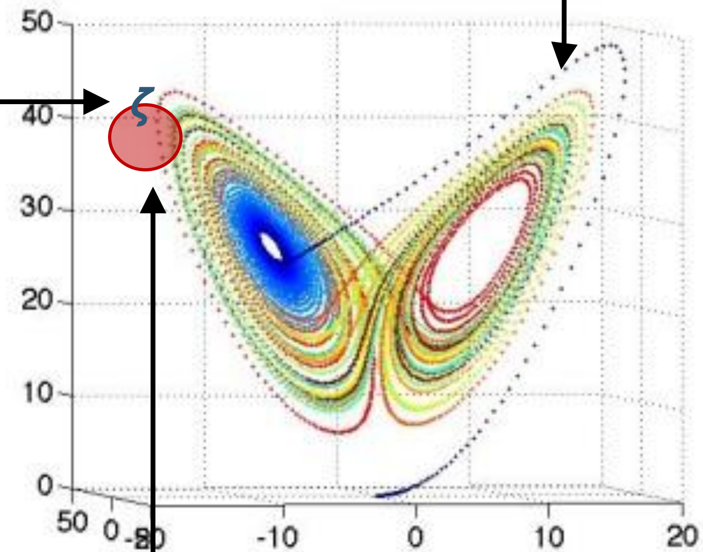
1) In a chaotic dynamical system, take a trajectory of the system: $f^m(x)$

2) Rare events are recurrences of a state ζ:

$$X_m(x) = g(\mathrm{dist}(f^m(x), \zeta))$$

3) Then, chose observables such that the maxima of *g* correspond to minima of the distances with respect to ζ:

$$g_1^m(\zeta) = -\log(\mathrm{dist}(f^m(x), \zeta))$$



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HOW TO COMPUTE LOCAL DIMENSIONS (2/3)

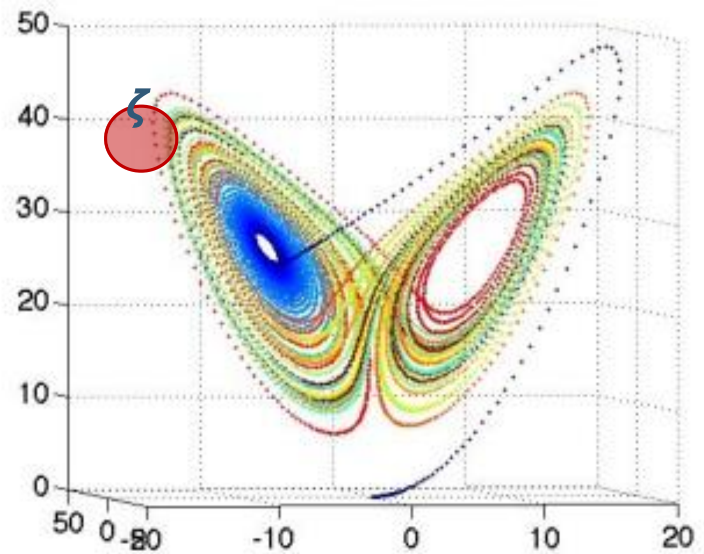
For any chaotic systems entering in a ball close to ζ, is equivalent to study threeshold exceedances of:

$$P(g(x(t)) > q, \zeta)$$

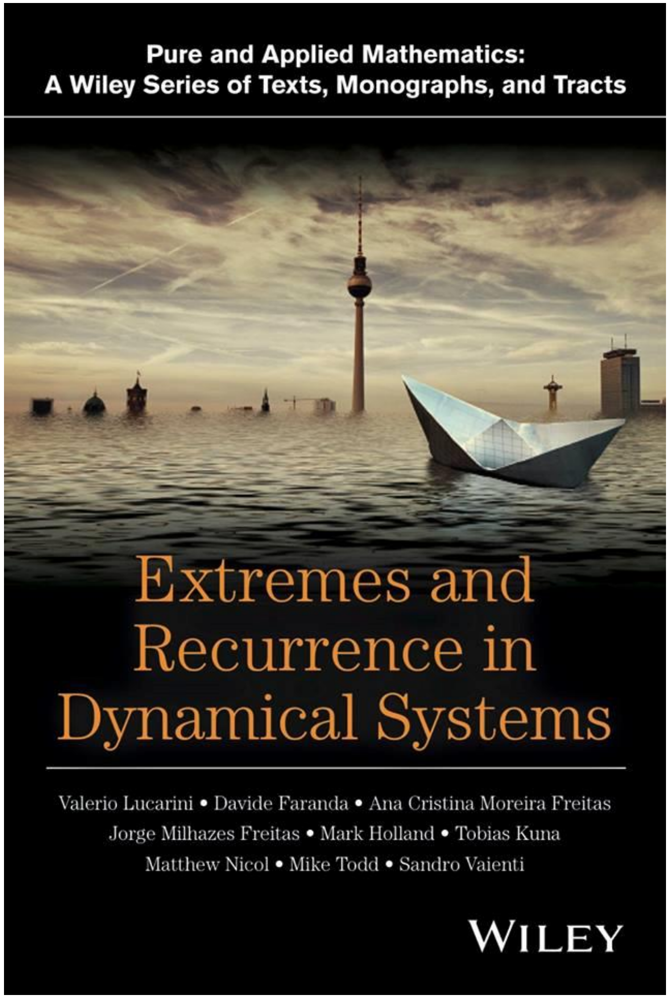
For the Freitas-Freitas-Todd theorem (2008) *P* converges asymptotically to the exponential member of the Generalized Pareto distribution (GEV) distribution:

$$P(y, \zeta) = \exp(-[y-a(\zeta)]/\sigma(\zeta))$$

And the local dimension is $d(\zeta)=1/\sigma(\zeta)$



HOW TO COMPUTE LOCAL DIMENSIONS (3/3)



"Extremes and recurrence in Dynamical systems" contains new tools for estimating the local dimensions d(ζ)

Book: Lucarini, Faranda et al. Wiley (2016)

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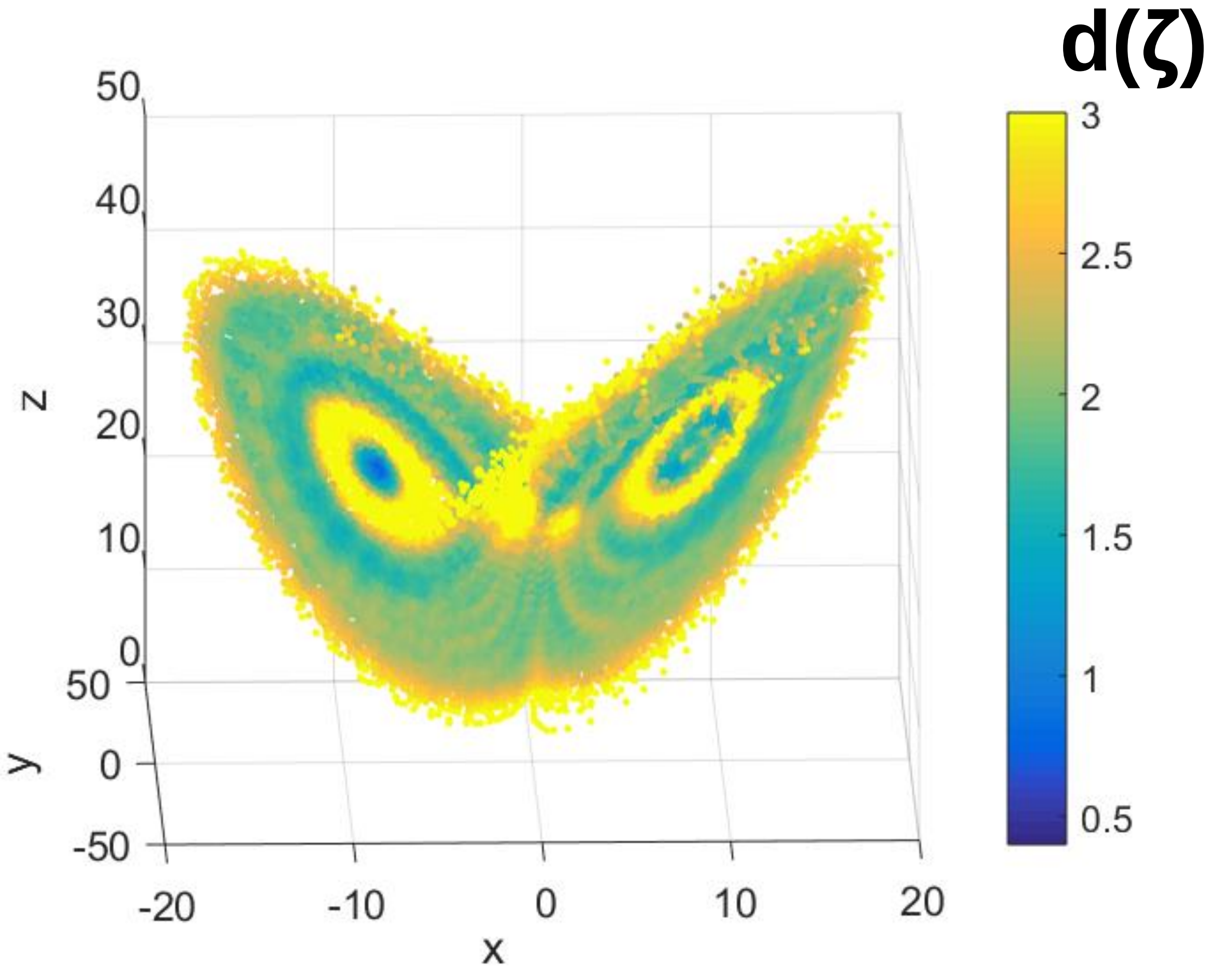
LOCAL DIMENSIONS IN LORENZ ATTRACTORS



The average $< D >=2.06$ is the same as classical estimates

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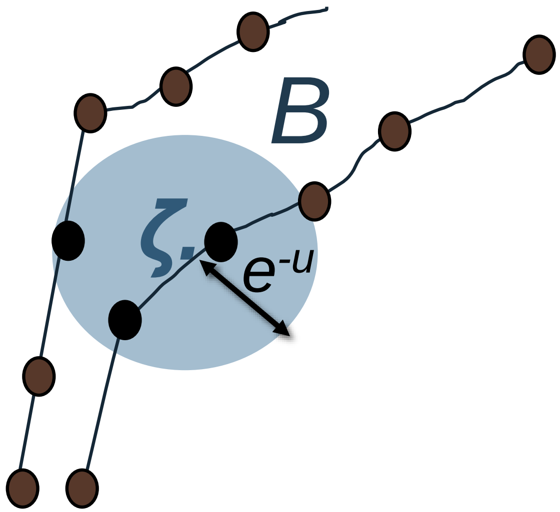
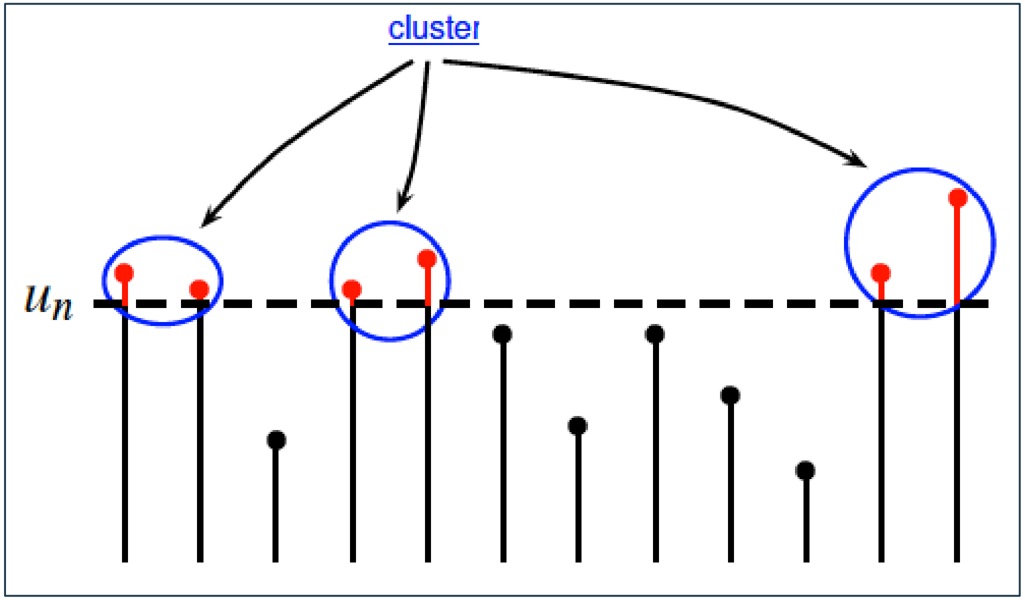
PERSISTENCE ⇔ STABILITY OF THE TRAJECTORY





If a threshold u is applied to a series of observations $x_1, x_2, \ldots, x_s$, the exceedances are those for which $x_i > u$. **The extremal index θ can then be thought of as the average inverse time spent above u.**

Local Stability in Lorenz Attractor

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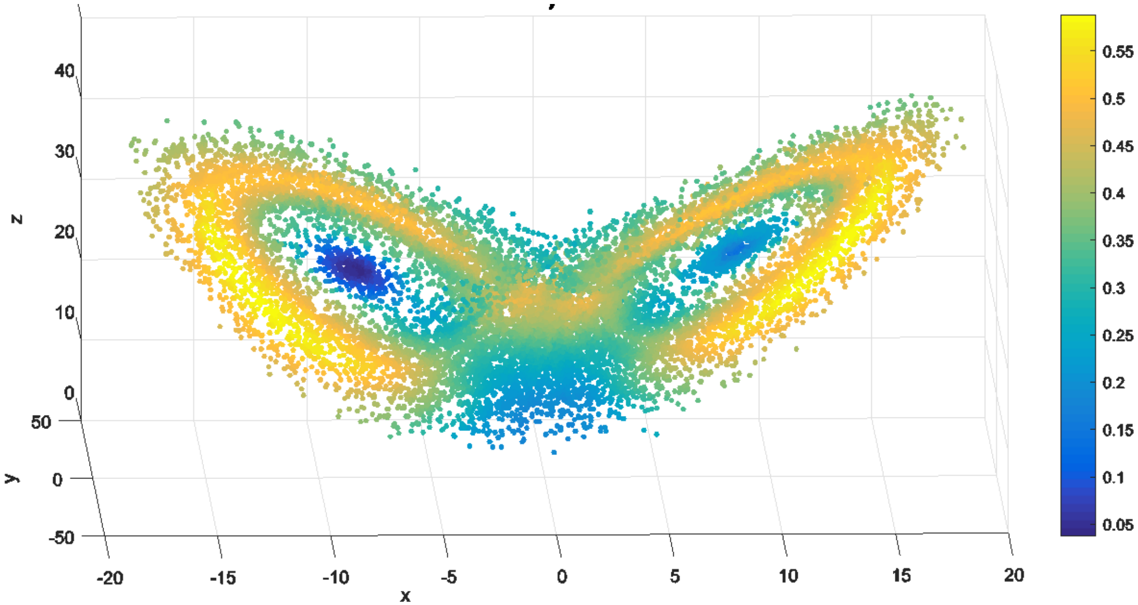
LOCAL STABILITY IN LORENZ ATTRACTOR

$\theta(\zeta)$



More Unstable

More Stable

θ is a good proxy for **unstable fixed points of the system**

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CORRELATION DIMENSIONS AND ENTROPY

Let us now consider the probability that two trajectories $x(t),y(t)$ of the same dynamical system get extremely close. By writing the observable

$$\psi(x(t),y(t)) = -log(dist(x(t),y(t))$$

$$P(\psi(x(t),y(t)) > q)$$

The results by Faranda and Vaienti (Chaos, 2018) show that again *P* converges asymptotically to the exponential member of the Generalized Pareto distribution (GEV) distribution:

$$P(y, \zeta) = \exp(-[y-a]/\sigma)$$

And the Correlation dimension is $D_2=1/\sigma$

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CORRELATION DIMENSIONS AND ENTROPY

The extremal index is instead related to the positive Lyapunov exponent Λ_μ in dimension 2

$$\theta \sim 1 - \int_M \frac{1}{|T'(x)|} d\mu(x) \leq 1 - e^{-\int_M \log|T'(x)| d\mu(x)} = 1 - e^{-\Lambda_\mu}.$$

And to the entropy *h* in dimension n:

$$\int d\mathcal{L}(x) |\det(DT(x)|_u)| = \sum_{j=1}^{d} \Lambda_j^+ = h_{\mathcal{L}},$$

$$\theta \sim 1 - e^{-h_{\mathcal{L}}}.$$

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CORRELATION DIMENSIONS AND ENTROPY

TABLE I. Estimates of correlation dimension D_2 and dynamical extremal index (DEI) θ obtained with $l = 100$ trajectories, consisting of $n = 10^6$ iterations or $n = 10^4$ iterations. The maxima of $\psi(x, y)$ are extracted in the block of $s = 10^3$ and $s = 10^2$ length, for a total of $m = 10^3$ or $m = 10^2$ blocks. The quantile for the estimate of the DEI is $\tilde{s} = 0.99$. For the Arnold Cat's map, the convergence to theoretical value is lower and the estimates are provided only for $\tilde{s} = 0.99999$ and $n = 10^7$.

Map	D_2 (classical)	D_2 ($n = 10^6$)	D_2 ($n = 10^4$)	θ (from Lyapunov)	θ ($n = 10^6$)	θ ($n = 10^4$)
Bernoulli's shifts	1	1.00 ± 0.02	1.01 ± 0.14	0.667	0.668 ± 0.004	0.69 ± 0.04
Gauss map	1	1.00 ± 0.03	0.96 ± 0.16	0.773	0.773 ± 0.005	0.78 ± 0.04
Cantor IFS	0.667	0.64 ± 0.01	0.59 ± 0.13	0.5	0.502 ± 0.005	0.50 ± 0.05
Baker map	1.41	1.46 ± 0.02	1.42 ± 0.25	0.47	0.49 ± 0.02	0.50 ± 0.04
Lozi map	1.38	1.39 ± 0.11	1.29 ± 0.25	0.37	0.37 ± 0.01	0.37 ± 0.05
Henon map	1.22	1.24 ± 0.03	1.13 ± 0.25	0.34	0.43 ± 0.01	0.43 ± 0.06
Solenoid $a = 1/3$	1.6309	1.64 ± 0.04	1.55 ± 0.17	0.5	0.51 ± 0.01	0.59 ± 0.03
Solenoid $a = 1/4$	1.5	1.52 ± 0.03	1.57 ± 0.20	0.5	0.51 ± 0.01	0.53 ± 0.03
Arnold Cat's map	1.987	2.00 ± 0.06	...	0.51	0.53 ± 0.06	...

CORRELATION DIMENSIONS AND ENTROPY

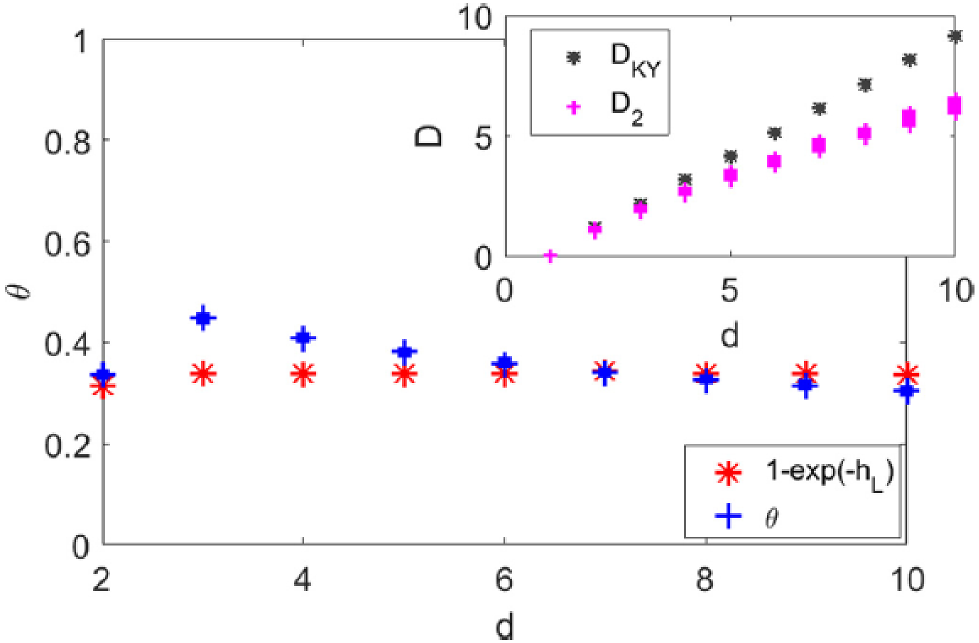

FIG. 1. Estimates of the dynamical extremal index θ and correlation dimension D_2 (inset) obtained for the Generalized Henon maps [Eq. (3.1)] in different dimensions d. The values represent the estimates obtained taking 30 couples of trajectories, iterated for $n = 10^6$ iterations. Each couple is displayed using a single marker, but the uncertainty is so small that the difference between couples is hardly recognizable. The quantile used for the estimation is $\tilde{s} = 0.98$. The results are compared to those obtained using the Kaplan-Yorke dimension D_{KY} and the entropy $h_{\mathcal{L}}$. This map has $d - 1$ positive Lyapunov exponents.

CORRELATION DIMENSIONS AND ENTROPY

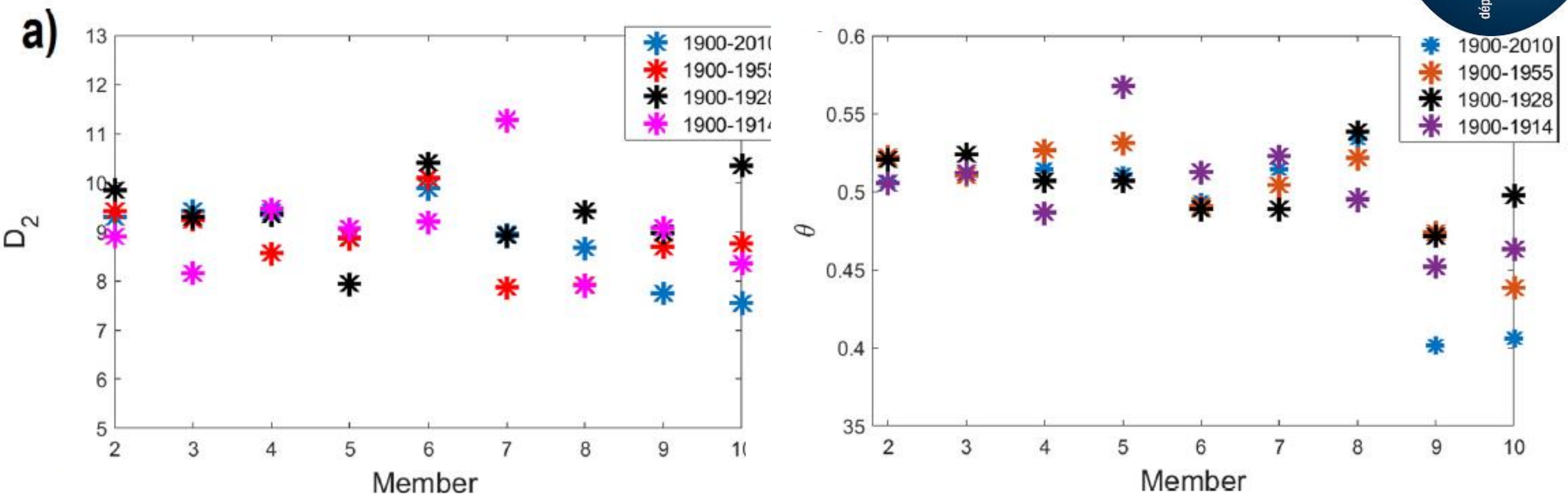

FIG. 2. Estimates of correlation dimension D_2 (a) and extremal index θ (b) obtained for daily sea-level pressure maps for four different periods in the ERA-20CM reanalysis. The values represent the estimates obtained taking as reference trajectory x the member M1 and as y, the remaining 9 ensemble members.

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