

Comprendre l'infiniment grand

V.Ruhlmann-Kleider
CEA/Saclay Irfu/DPhP

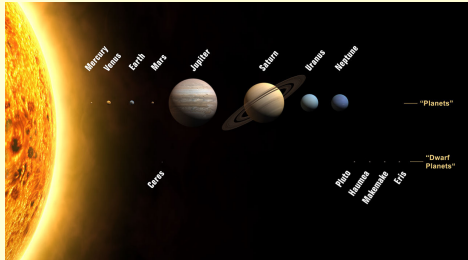
- 1) Du modèle du Big Bang au contenu de l'Univers
- 2) Mesures cosmologiques
- 3) Les grandes structures de l'Univers

The rise of the Big Bang model

Orders of magnitude. The three pillars of the Big Bang model

- Distances, cosmological scales
- Redshift
- Expansion of the Universe, the metric
- The rise of the Big Bang model

1. Distances

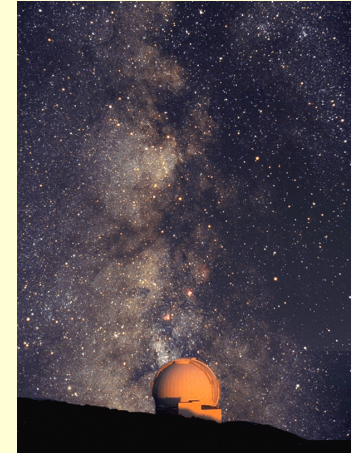


- **solar system:** Earth-Sun $\sim 150 \cdot 10^6$ km ~ 1 AU

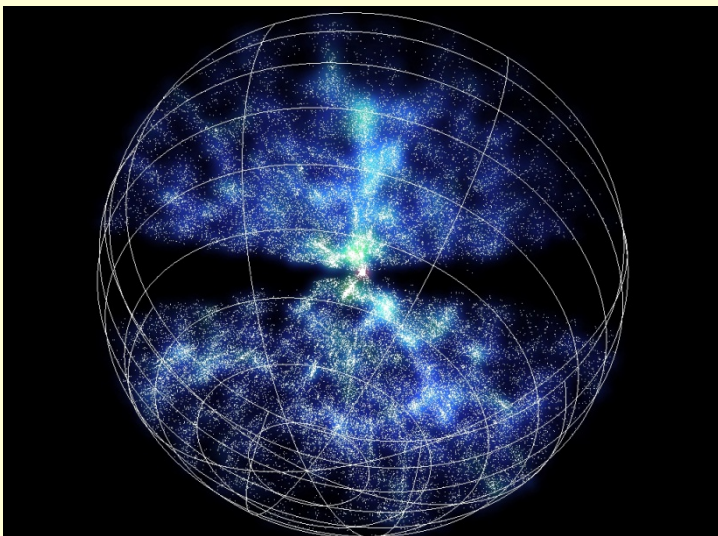
- **galaxies, eg the Milky Way:**

$\varnothing \sim 100,000$ lyr ~ 30 kpc Sun-Gal. center ~ 10 kpc

1lyr=63,240 AU 1pc=3.26 lyr 1Mpc=3.3 10^6 lyr



- **galaxy clusters:** largest and most massive gravitationally bound structures, 50 to 1,000's of galaxies, $\varnothing \sim 2$ to 10 Mpc



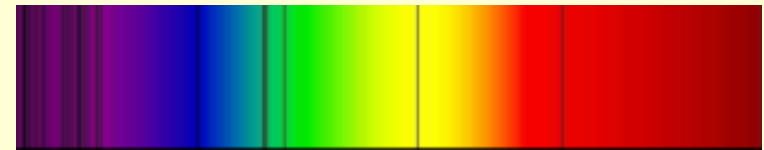
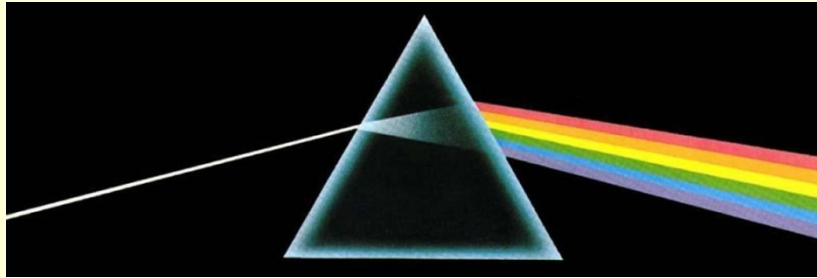
- **large scale structures:** galaxies \rightarrow clusters \rightarrow superclusters (15 - 100Mpc) making a network of voids (25 - 125Mpc) and filaments (90 - 300 Mpc)
 \Rightarrow beyond 100Mpc, homogeneous and isotropic Universe

6dF Galaxy Redshift Survey, (2009)

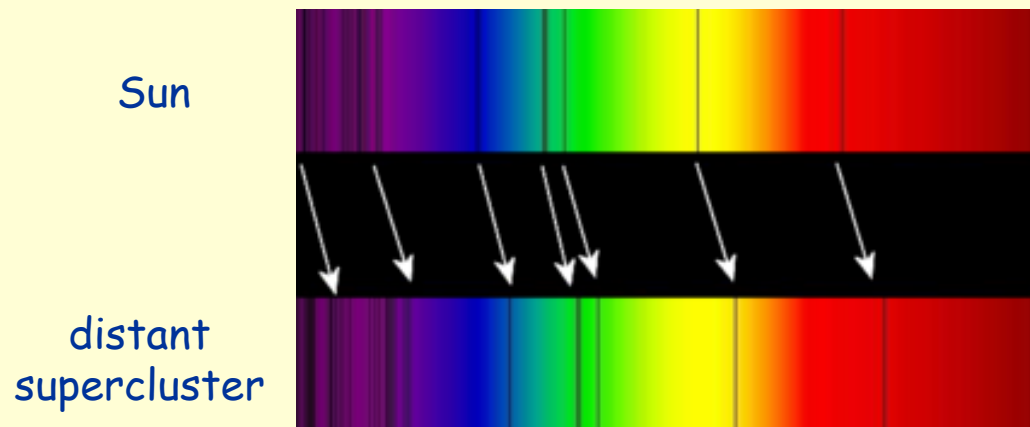
X

2. Redshift

- Emitted light spectrum \Rightarrow **spectral lines** \Rightarrow astro. object composition, environment ... and motion relative to Earth.



- Redshift : the object moves **away** from us $\Rightarrow \lambda_{\gamma}$ **increases**



$$z \equiv \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

or

$$1 + z = \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}}$$

See <http://astro.unl.edu/classaction/animations/cosmology/galacticredshift.html>

The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift):

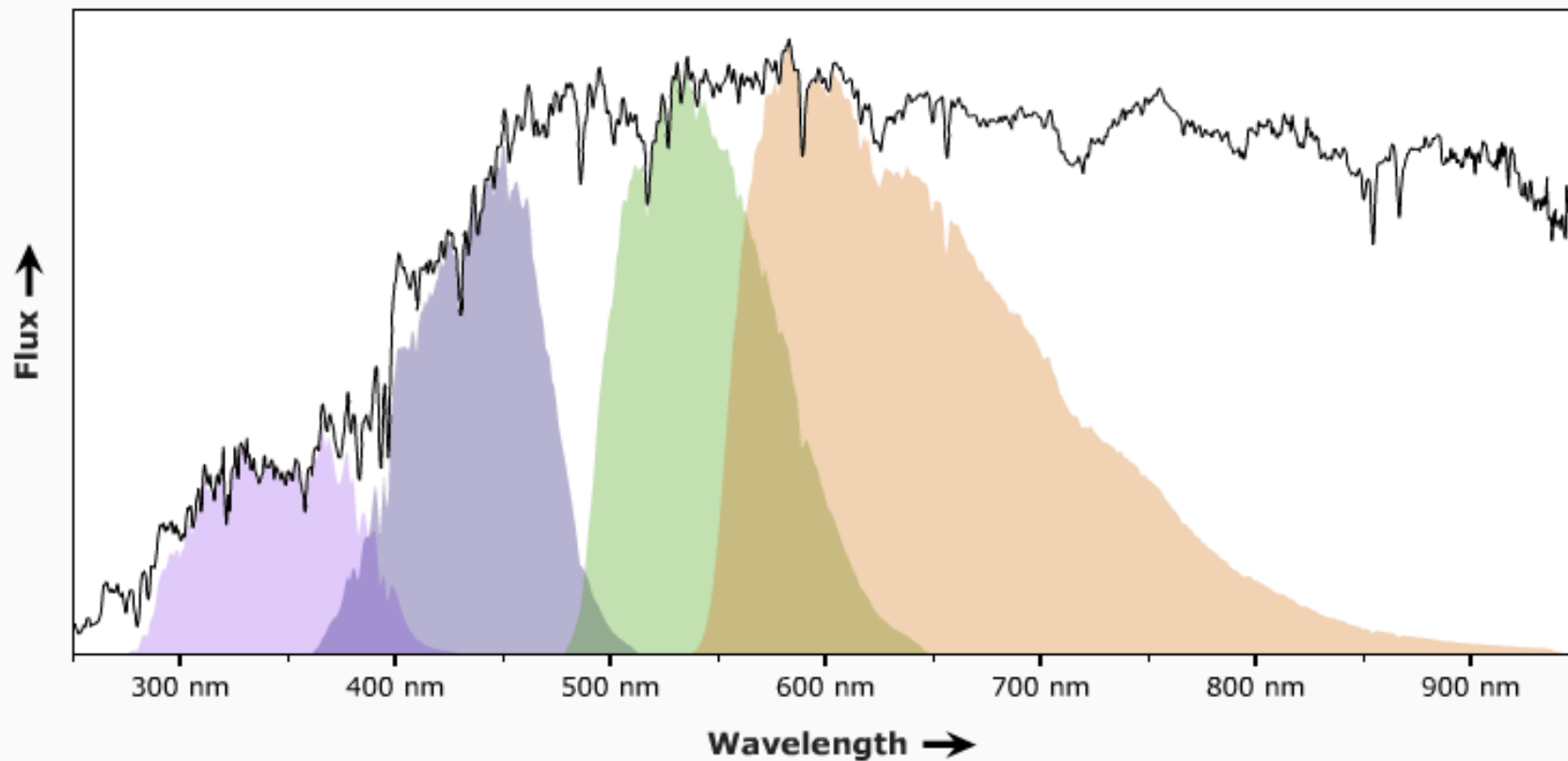
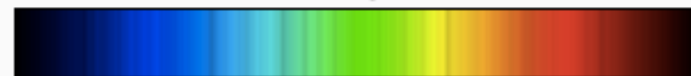


$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.



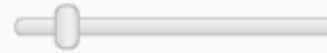
Visible Spectrum



The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift):

0.10

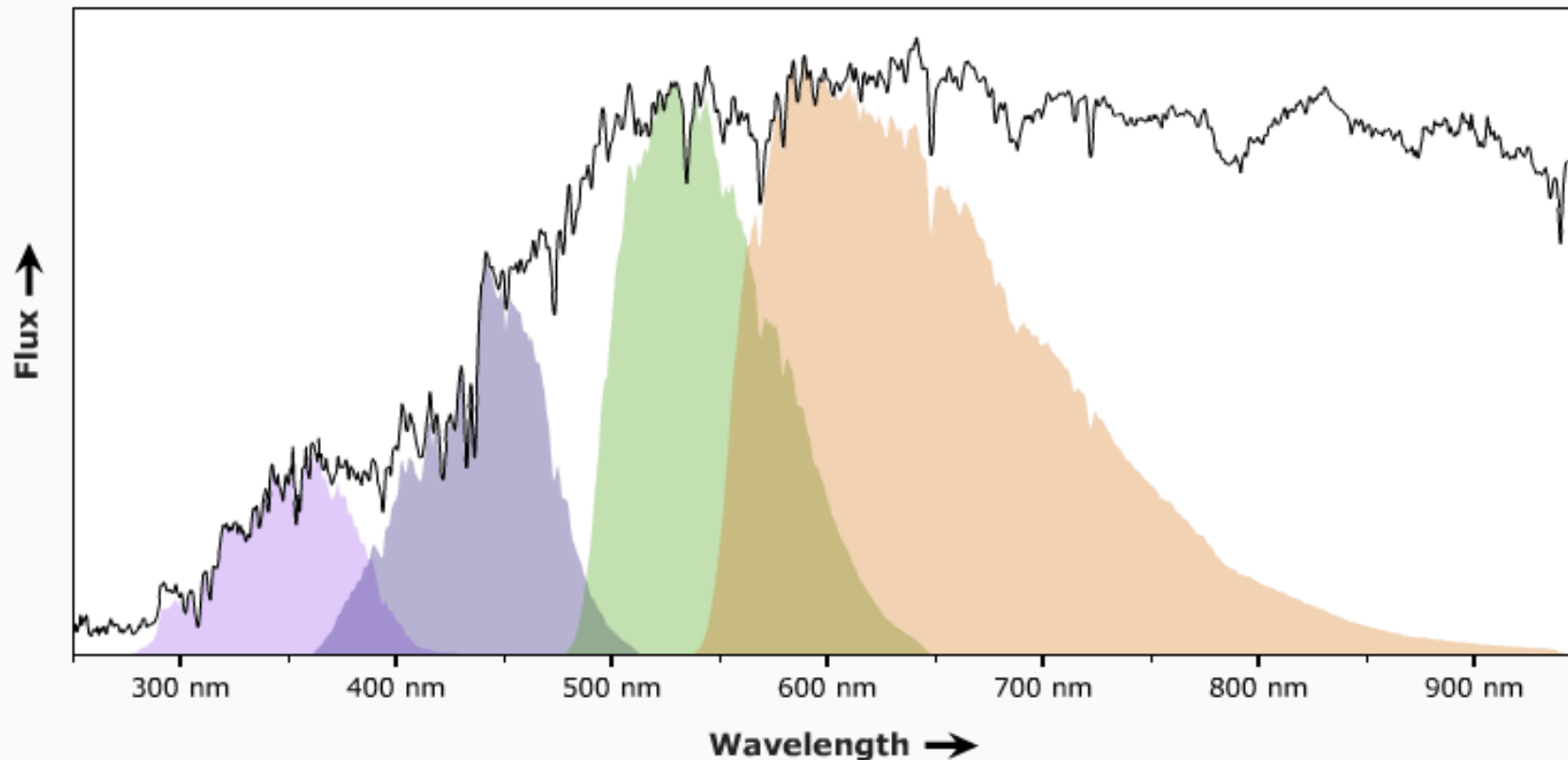
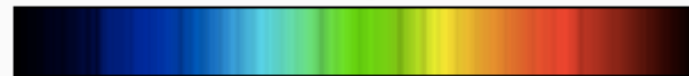


$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.



Visible Spectrum



The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift):

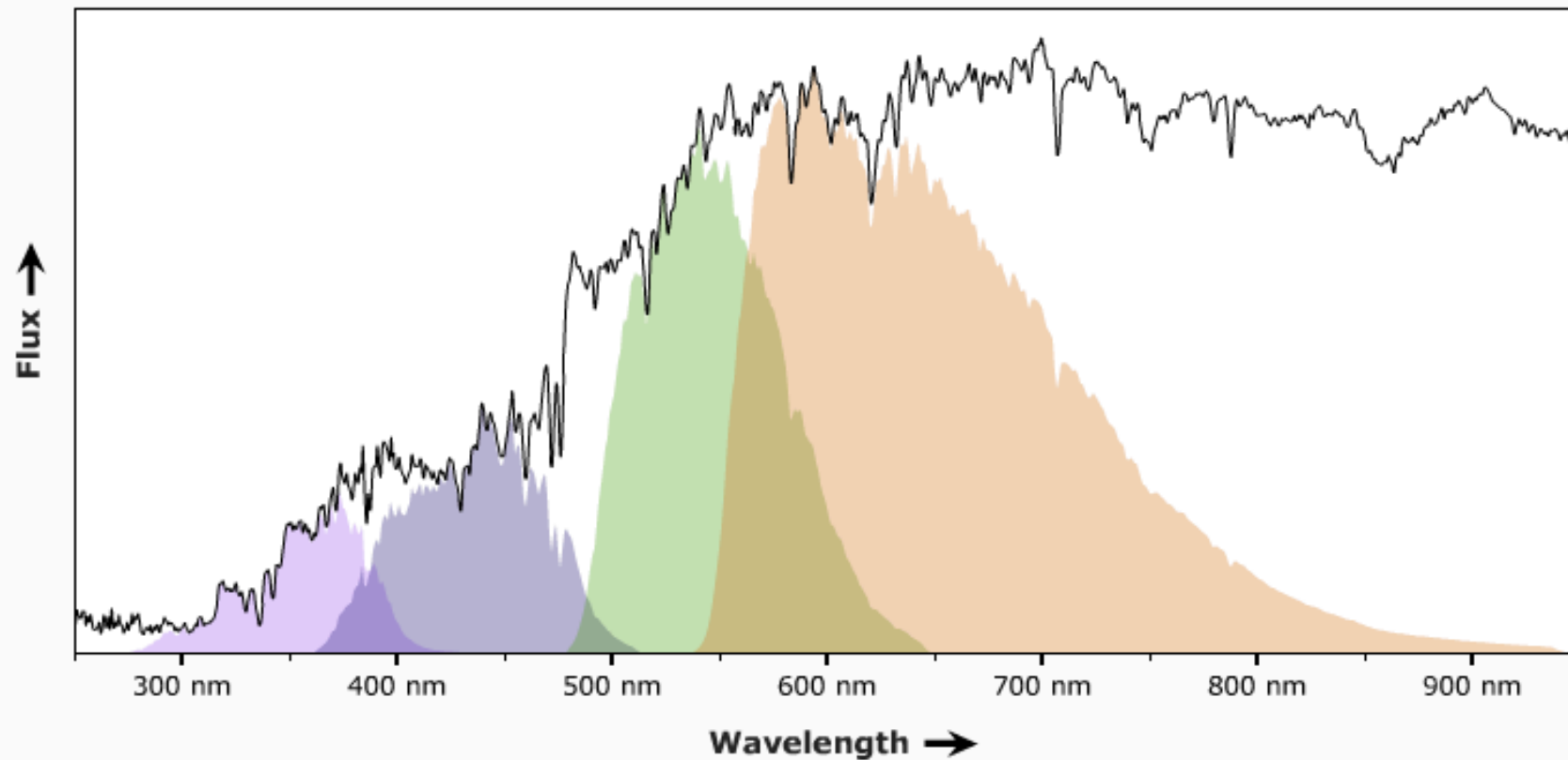


$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.



Visible Spectrum



The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift):

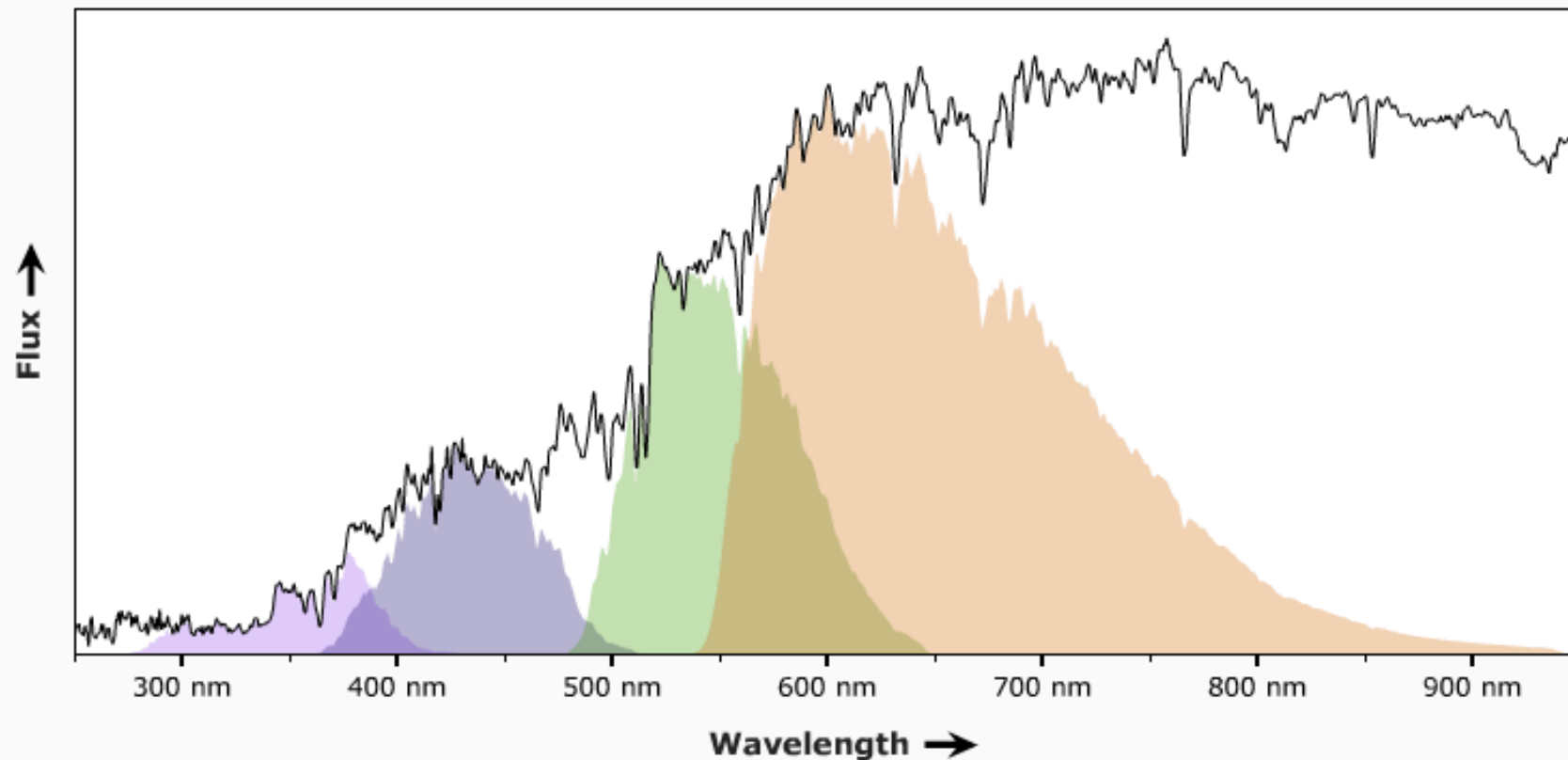


$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.



Visible Spectrum



The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift):



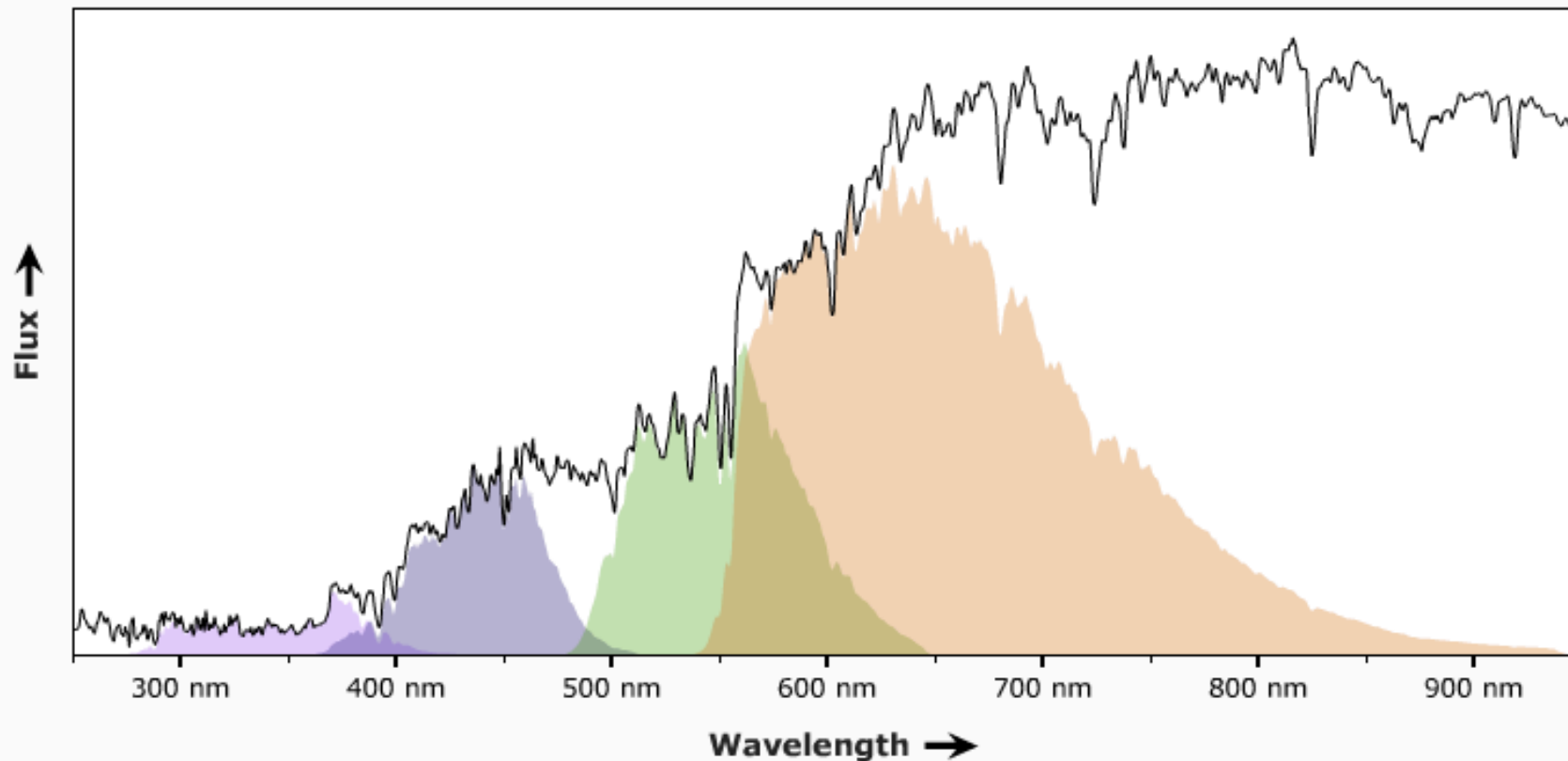
$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.

Visible Spectrum



hide



The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift):

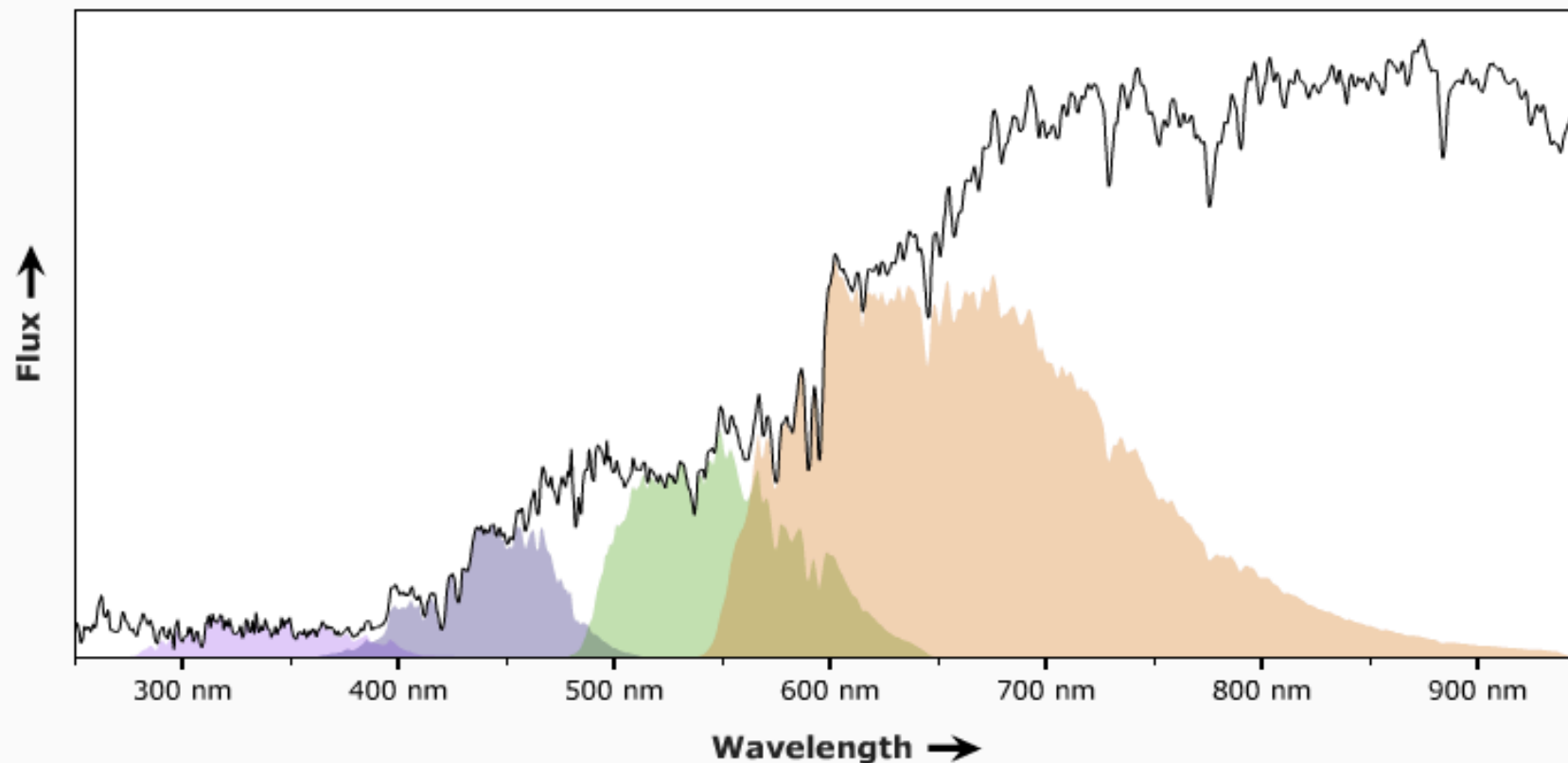


$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.



Visible Spectrum



The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift):



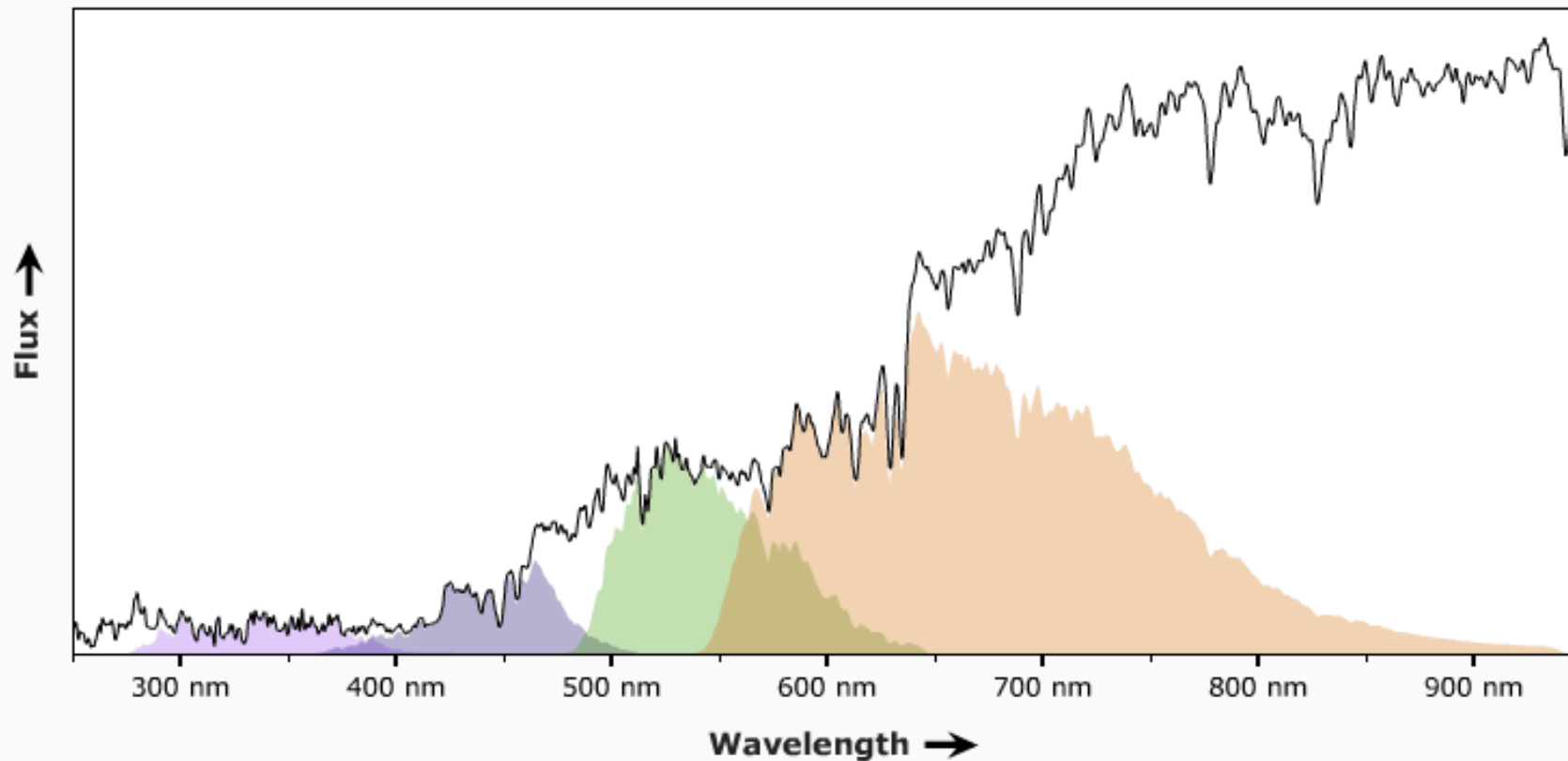
$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.

Visible Spectrum



hide



The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

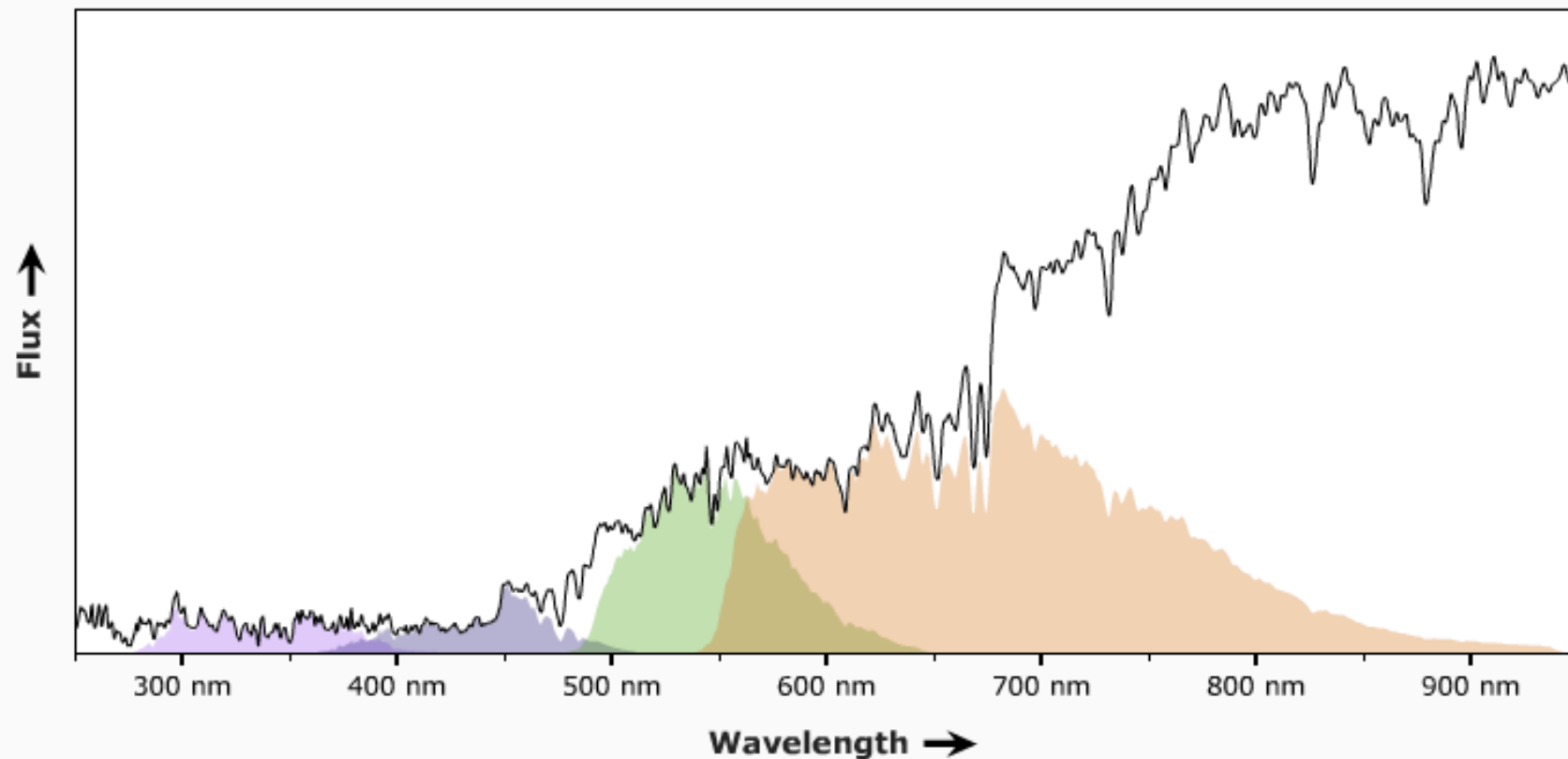
z (redshift):



$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

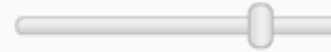
Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.

Visible Spectrum



The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

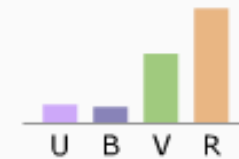
z (redshift):



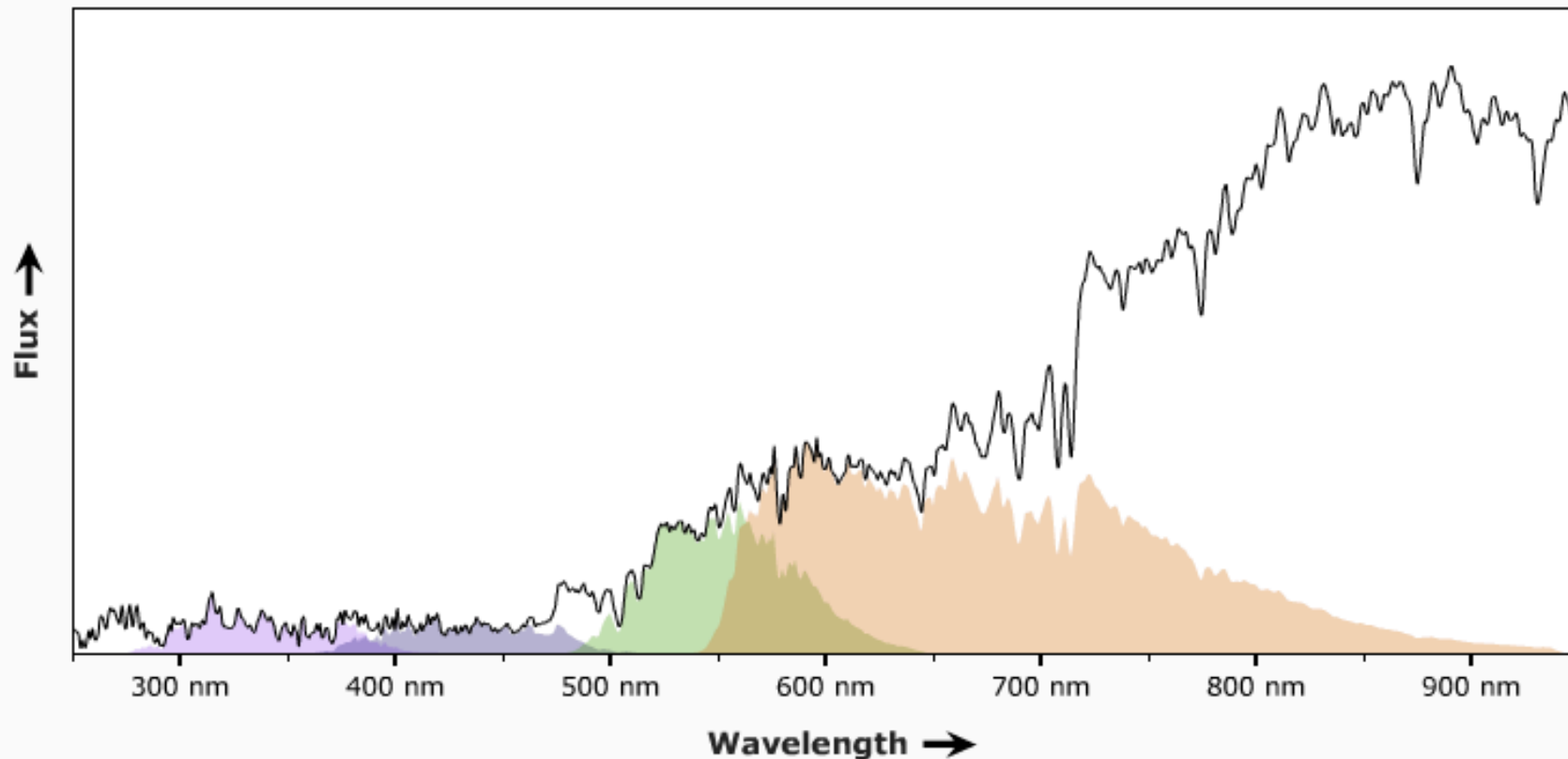
$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.

Visible Spectrum

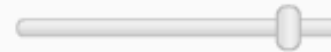


hide



The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift):



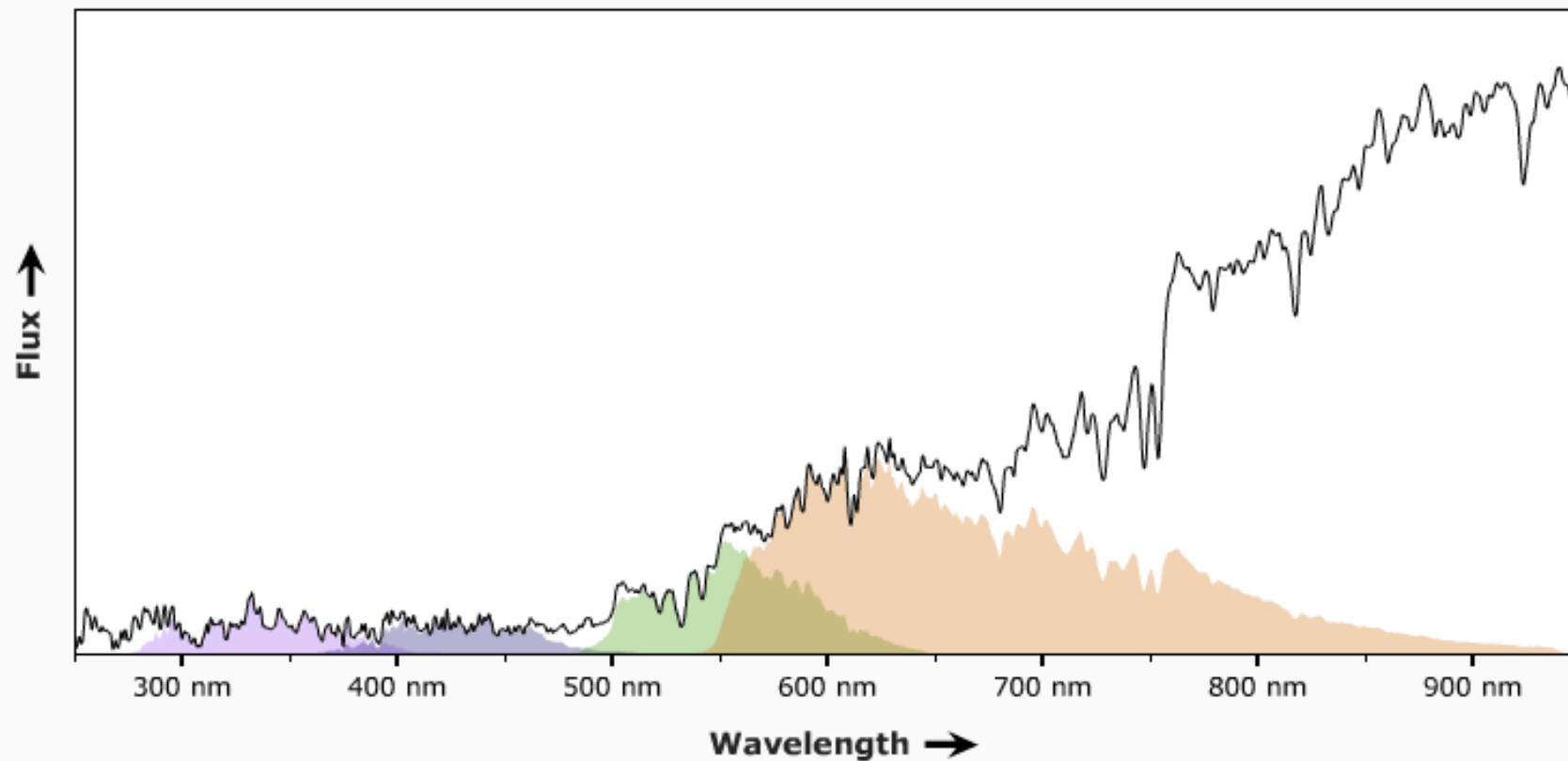
$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.

Visible Spectrum



hide



The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift):



$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

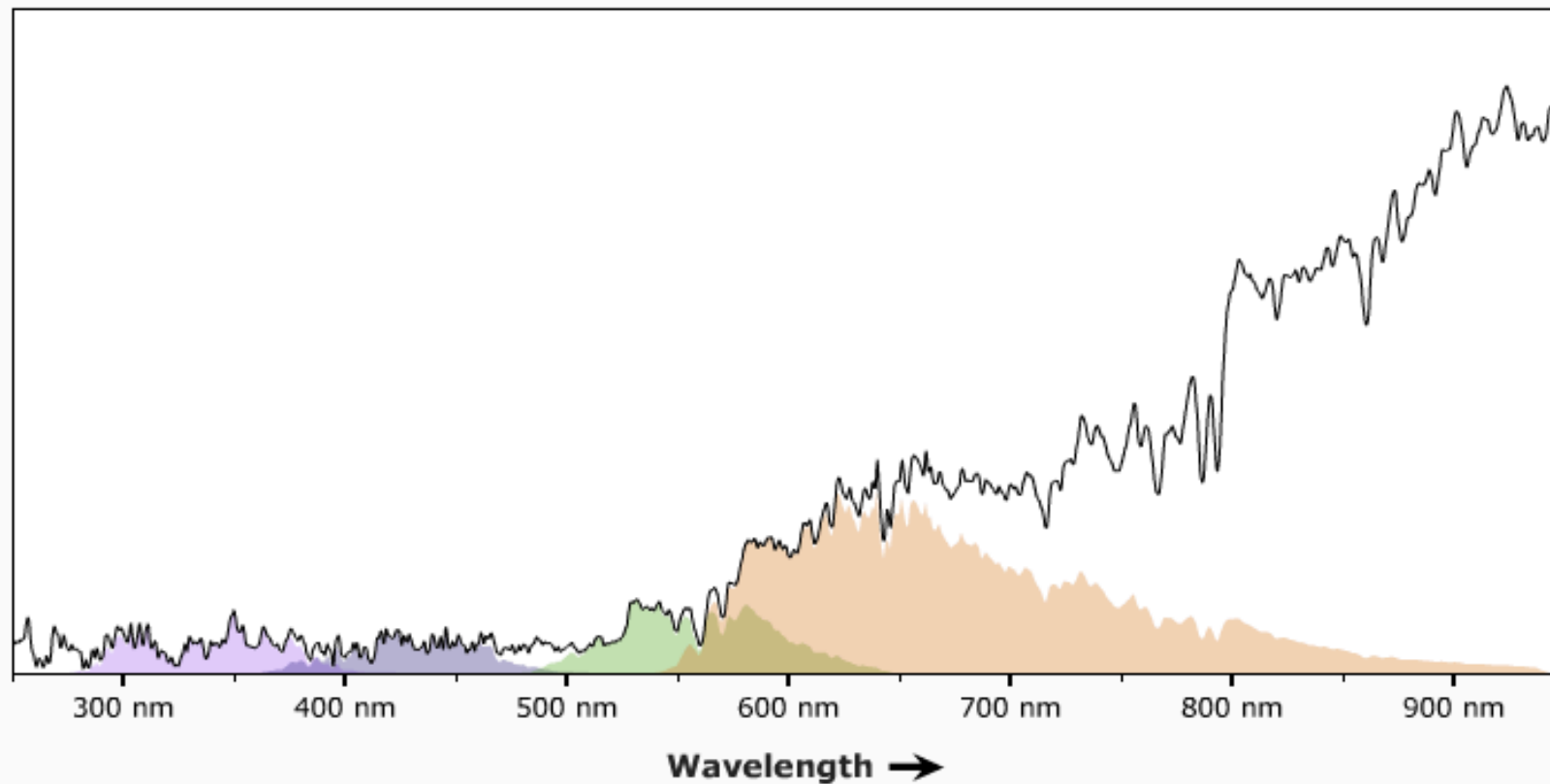
Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.

Visible Spectrum



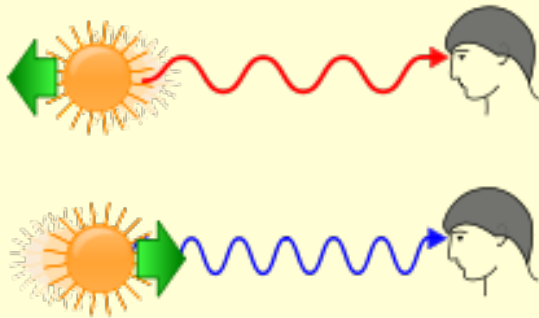
hide

Flux ↑



Different origins of redshifts/blueshifts

- Doppler effect : redshift/blueshift due to **relative motion**



$$1+z = \gamma \left(1 + \frac{v_{\parallel}}{c} \right) \quad z \approx \frac{v_{\parallel}}{c} \quad \text{for small } v_{\parallel}$$

(in Minkowski space i.e. flat spacetime)

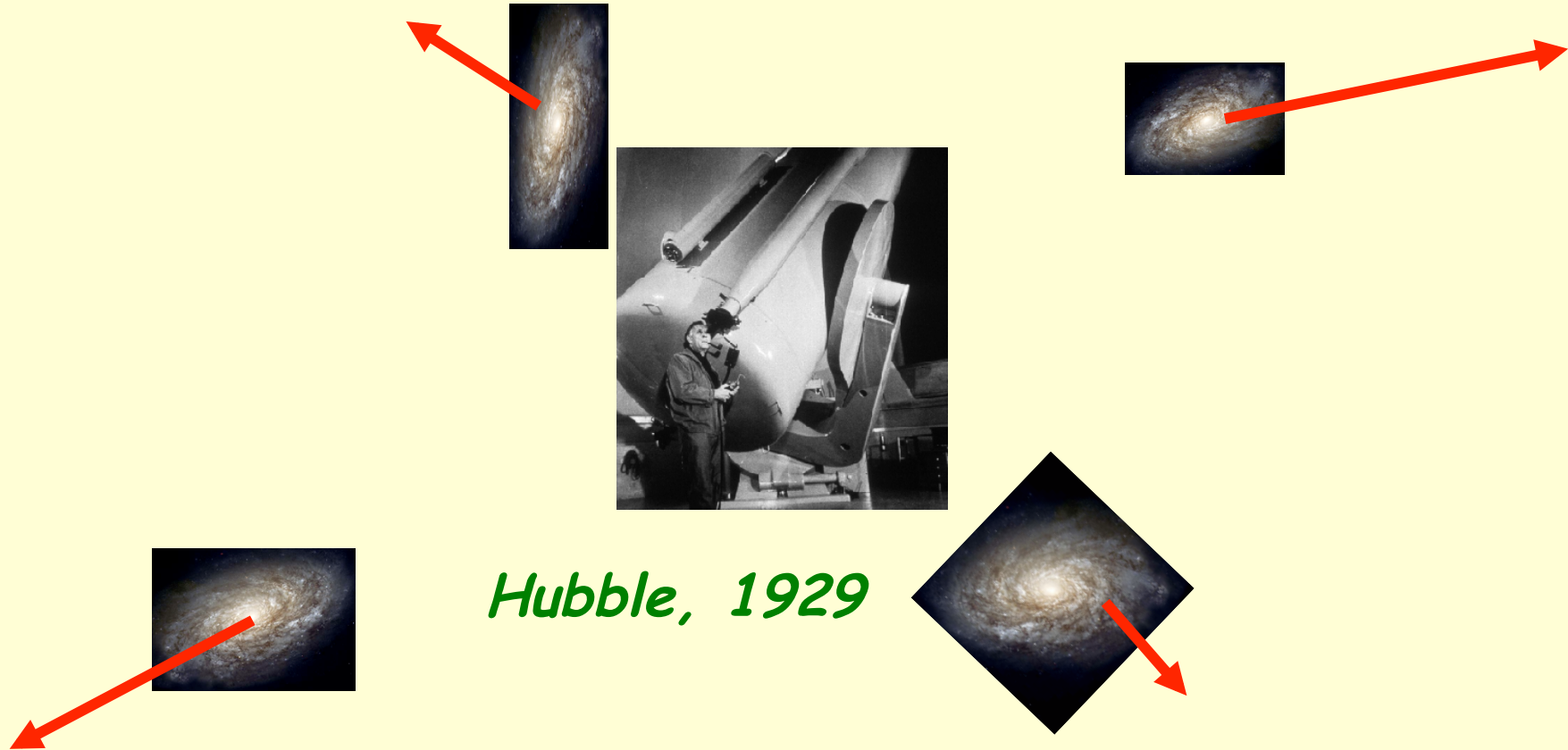
- **Cosmological redshift** : dominant for **distant sources** (above tens of Mpc or $z > 0.01$). Due to Universe expansion.

- **Gravitational red/blue shift** : radiation moving out of/into a **gravitational field**.

e.g. grav. redshift

$$1+z = \frac{1}{\sqrt{1-2GM/rc^2}}$$

The rise of modern cosmology



Hubble, 1929

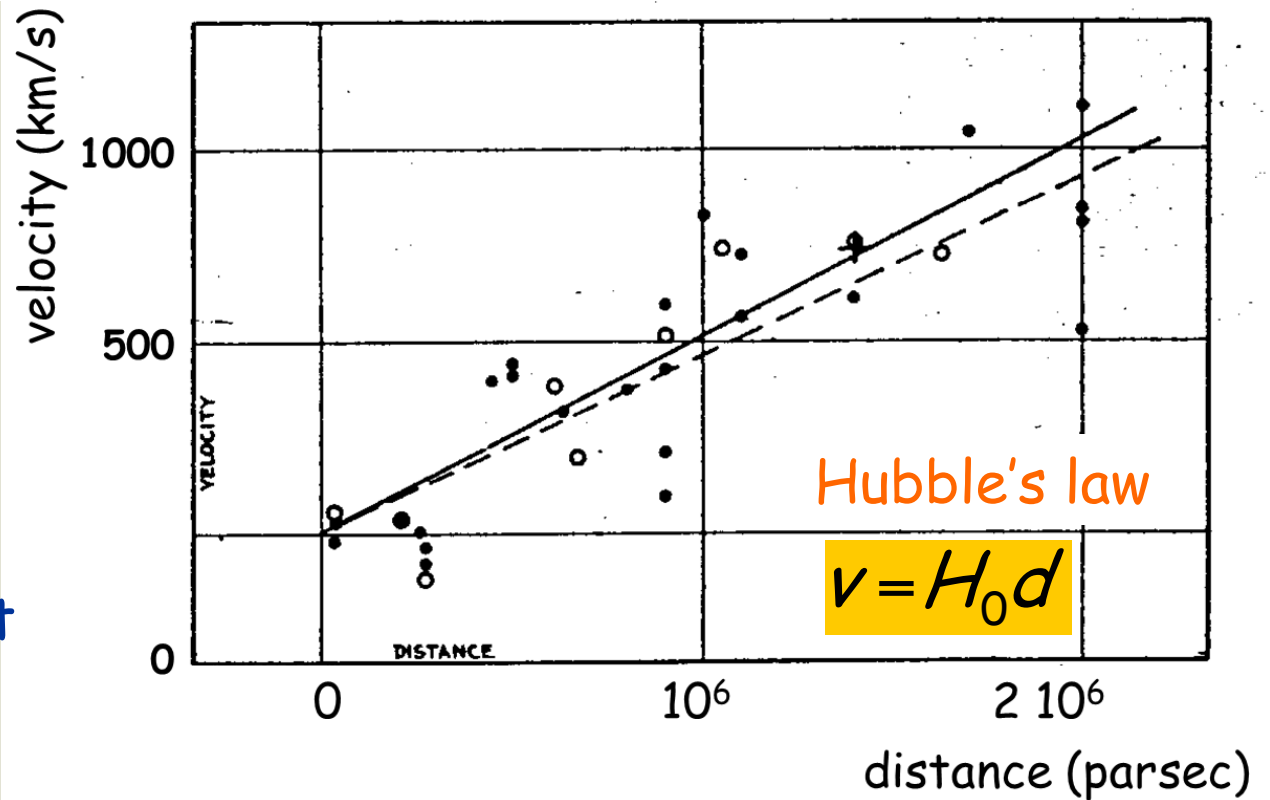
➡ distant galaxies are receding : the Universe is in **expansion**
(as predicted in General Relativity by A.Friedmann 1922 and G.Lemaître 1927)

Hubble data

Redshift converted into velocity

$$z \sim v/c$$

- H_0 : Hubble constant



- recent, precise measurements (2011): $H_0 = 73.8 \pm 2.4 \text{ km/s/Mpc}$
 (2014): $H_0 = 70.6 \pm 3.3 \text{ km/s/Mpc}$
 (2019): $H_0 = 74.0 \pm 1.4 \text{ km/s/Mpc}$

- critical density today

$$\rho_c^0 = 3H_0^2 / 8\pi G = 1.04 h^2 10^{10} \text{ eV} \cdot \text{m}^{-3} \quad h = H_0 / 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

↕
 same energy density as 1gal/Mpc³ ~ 5p/m³

Describing an *expanding* universe

- **General Relativity** : the simplest **relativistic** theory of **gravitatu** consistent with data. Gravity described as a **geometric** property of spacetime.
- **Metric**: allows to compute distances between two points

$$ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu$$

$g_{\mu\nu}$: metric tensor

ds^2 : line element, invariant

- Reminder: in special relativity (**no** gravity):

$$ds^2 \equiv \eta_{\mu\nu} dx^\mu dx^\nu = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

$$= c^2 dt^2 - (dr^2 + r^2 (d\theta^2 + \sin^2\theta d\varphi^2))$$

$\eta_{\mu\nu}$: Minkowski metric

- in a particle's rest frame:

$$ds = c d\tau \quad d\tau : \text{particle proper time}$$

- Friedmann-Lemaître-Robertson-Walker (FLRW) metric :
metric for a spatially homogeneous and isotropic expanding universe, with **scale/expansion factor** $a(t)$ and **curvature** k

$$ds^2 = c^2 dt^2 - a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right)$$

expansion factored out

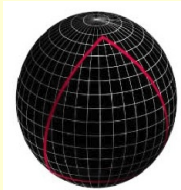
$H(t) \equiv \frac{\dot{a}(t)}{a(t)}$ expansion rate or Hubble parameter

$H_0 \equiv H(t_0)$ $t_0 = \text{today}$

r, θ, φ spherical **comoving** coordinates:
 r : dimensionless & **stationary** wrt expansion

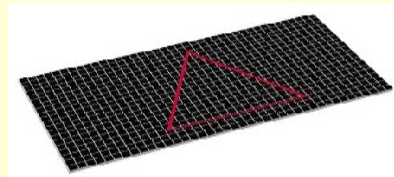
k , gaussian curvature:

closed universe



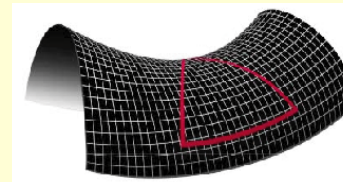
$k=+1$

flat universe



$k=0$

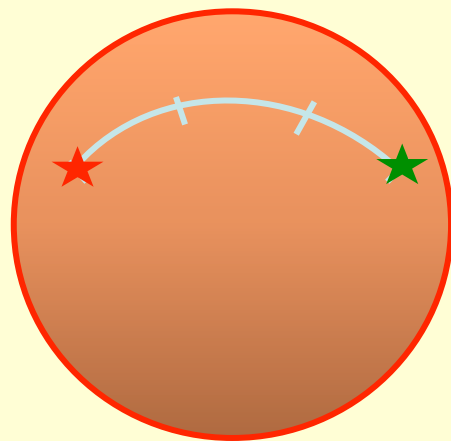
open universe



$k=-1$

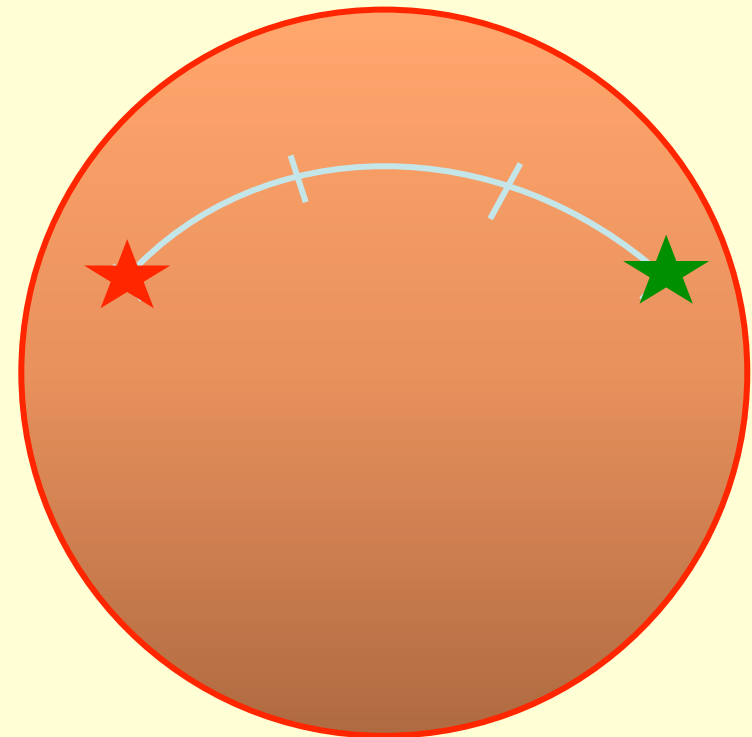
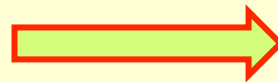
More on FLRW metric

- Introducing $d\chi \equiv \frac{dr}{\sqrt{1-kr^2}}$ to account for curvature :



$$\Delta\chi = 3 \quad D(t_1) = a(t_1)\Delta\chi$$

time



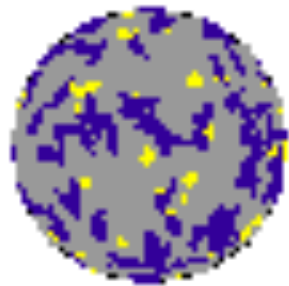
$$\Delta\chi = 3 \quad D(t_2) = a(t_2)\Delta\chi \geq D(t_1)$$

$a(t)$: scale factor

χ : comoving coordinate, stationary wrt expansion

$D(t)$: proper/physical distance $\Delta\chi$: comoving distance

The cosmological redshift



Universe in
expansion



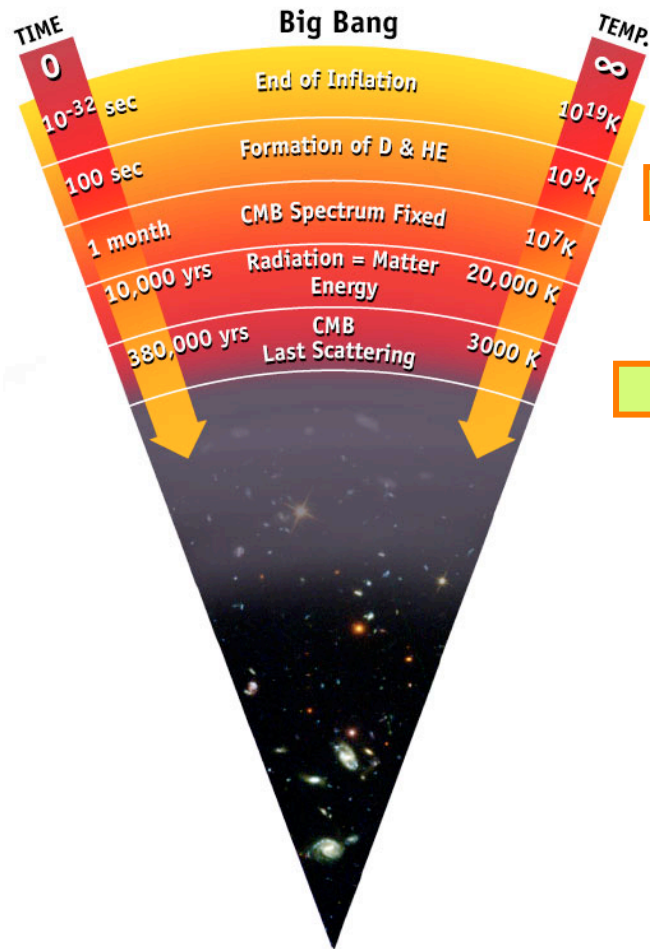
light from distant
sources is redshifted

$$\frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} \equiv 1 + z = \frac{a(t_{\text{observation}})}{a(t_{\text{emission}})}$$

3. The Big Bang model

Initial dense and hot phase followed by expansion and cooling

More confirmations of the Big Bang model



13,8 billions of years after Big Bang

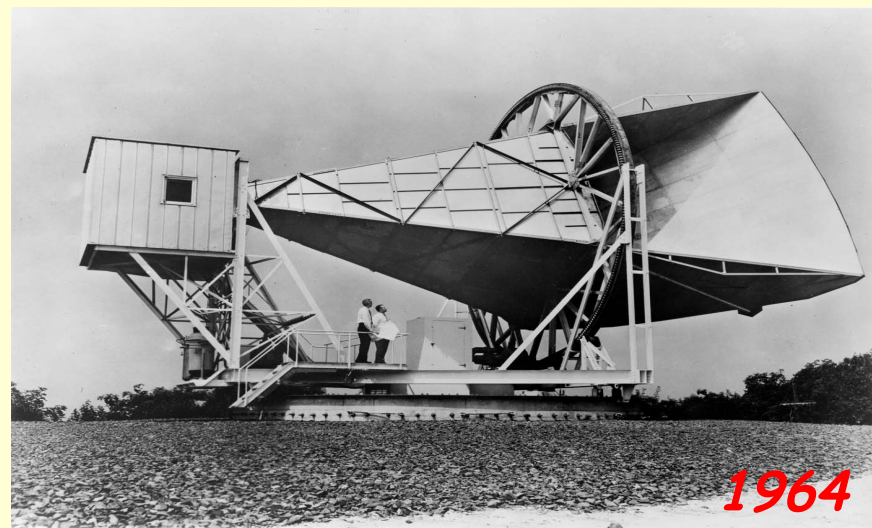
Primordial nucleosynthesis (He, D): $T \sim 1\text{MeV}$ predicted light element abundances = data

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \approx 10^{-10} \quad \text{1948: G. Gamow, R. Alpher}$$



Matter-radiation decoupling: $z \sim 1100$ $T \sim 3000\text{K}$
Cosmic **M**icrowave **B**ackground, relic radiation predicted and observed ($T \sim 2.725\text{K}$ 1992)

1948: G. Gamow, R. Alpher, R. Herman
1964: A. Penzias, R. Wilson



1964

The rise of the Big Bang model

General Relativity, Einstein's equations (1907-1915)

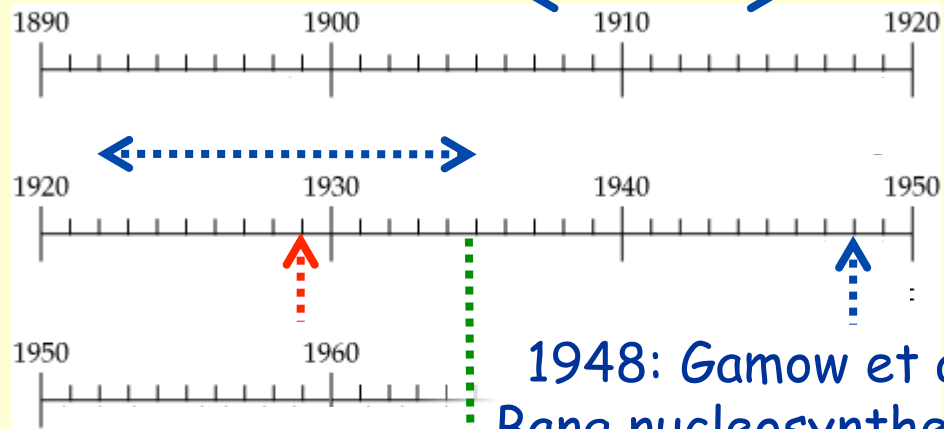
1922-1935 FLRW solutions
(expanding, homogeneous and isotropic Universe)

1929 Hubble's law

CMB detection

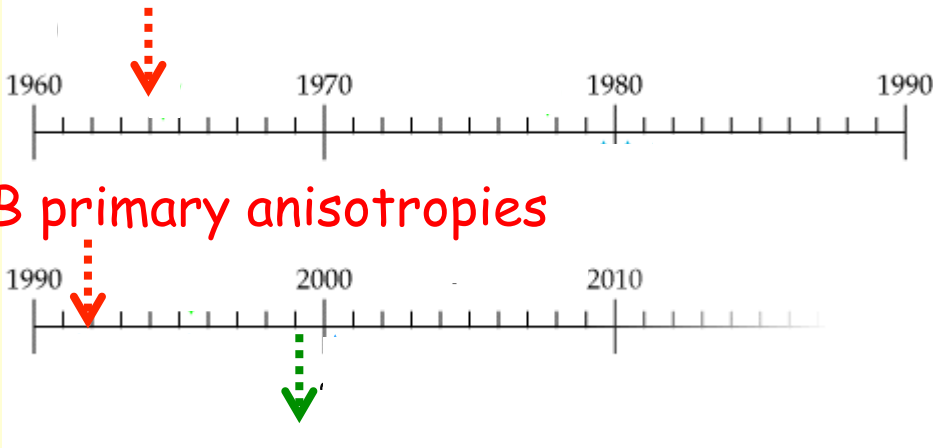
CMB primary anisotropies

Accelerated expansion: dark energy (or modified Einstein's gravity, e.g. cosmological constant)



1948: Gamow et al: Big Bang nucleosynthesis and CMB predictions

1934: 'missing mass' in orbital velocities of galaxies in clusters → dark matter postulated (F.Zwicky)



CONCLUSIONS (1)

- The Universe is spatially **homogeneous** and **isotropic** on cosmological scales (above 100Mpc)

- The Universe, initially in a hot, dense phase, is **expanding**

$$H_0 \equiv H(t_{\text{today}}) \approx 70 \text{ km/s/Mpc}$$

- **cosmological redshift**: due to the Universe expansion

$$\frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} \equiv 1 + z = \frac{a(t_{\text{observation}})}{a(t_{\text{emission}})}$$

- **General relativity** and **FLRW metric** are the basics of the Big Bang model
- Observational confirmations: **primordial abundances**, **CMB**

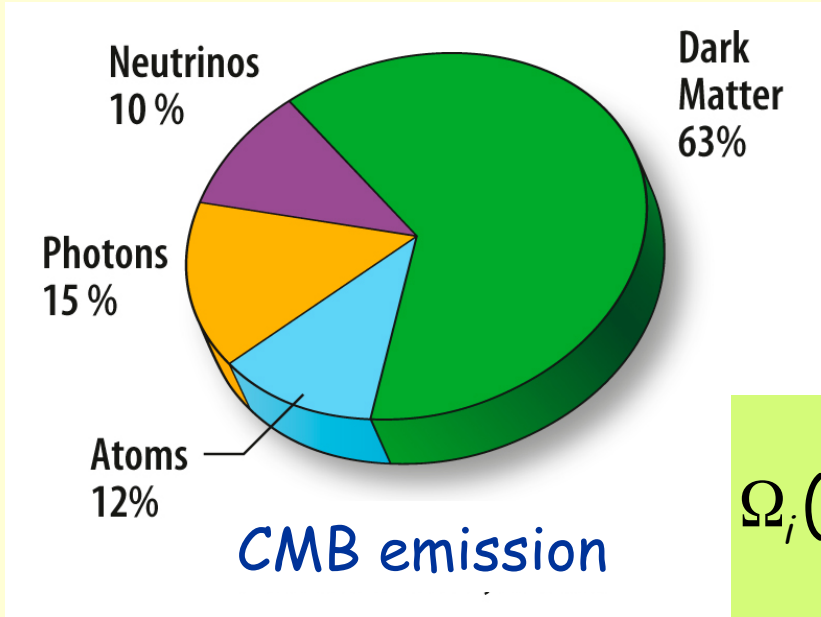
Content of the Universe

From General Relativity to the matter-energy content of the Universe

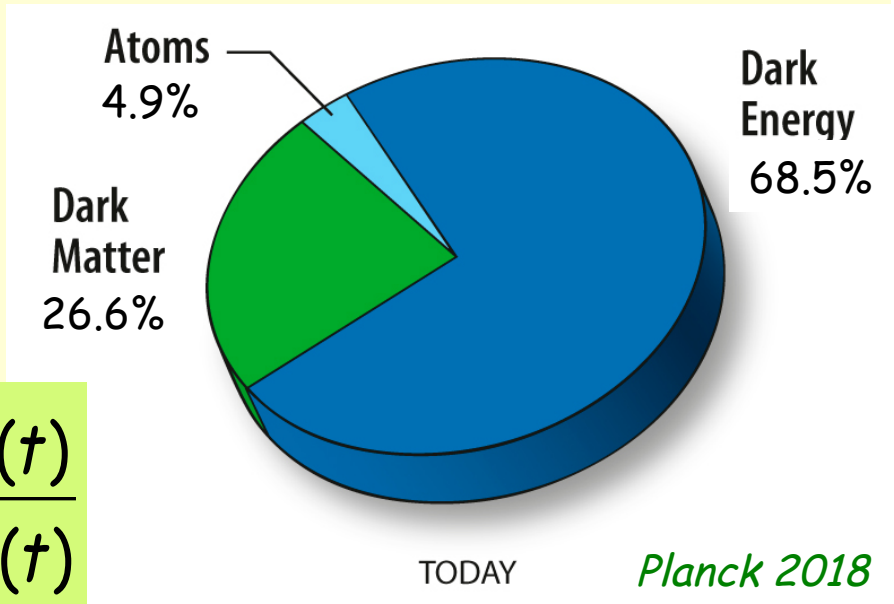
- The Ω_i parameters and the expansion history
- Thermal history of the Universe

1. Densities ρ_i and Ω_i parameters

Big Bang cosmology model+ many cosmological data
energy balance of the Universe



$$\Omega_i(t) \equiv \frac{\rho_i(t)}{\rho_c(t)}$$

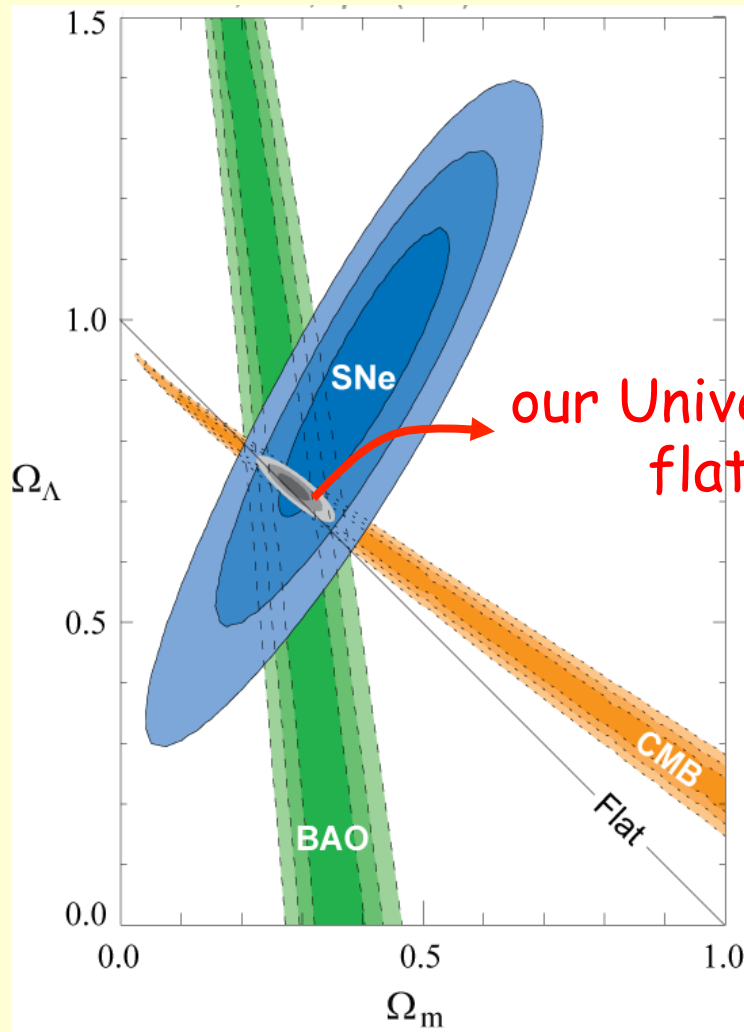


$$1 = \underbrace{\Omega_m(t) + \Omega_r(t) + \Omega_k(t)}_{\text{negligible today}} + \underbrace{\Omega_\Lambda(t)}_{\text{"dark energy" e.g. cosmological constant } \Lambda}$$

$\Omega_m(t)$ → NR matter
 $\Omega_r(t)$ → radiation
 $\Omega_k(t)$ → curvature
 $\Omega_\Lambda(t)$ → "dark energy" e.g. cosmological constant Λ

Present values of the Ω_i 's : the Ω_i^0 values

- CMB + BAO + SNe Ia data :



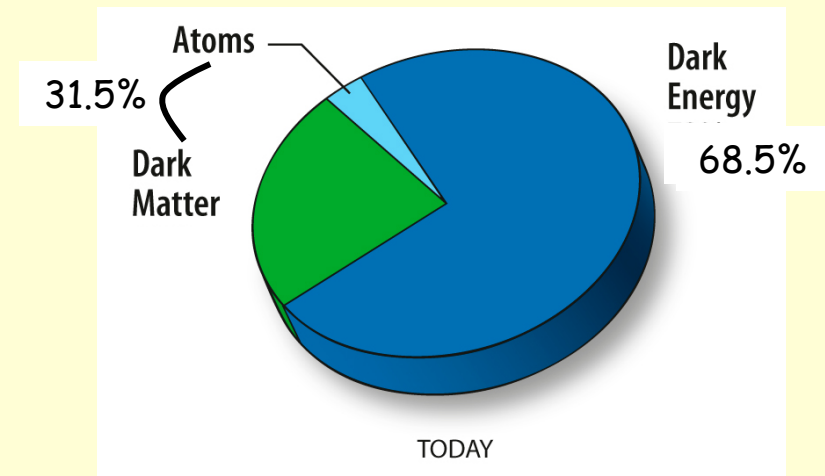
our Universe is flat !

$$\Omega_m^0 = 0.315 \pm 0.007$$

$$\Omega_\Lambda^0 = 0.685 \pm 0.007$$

2.2% precision

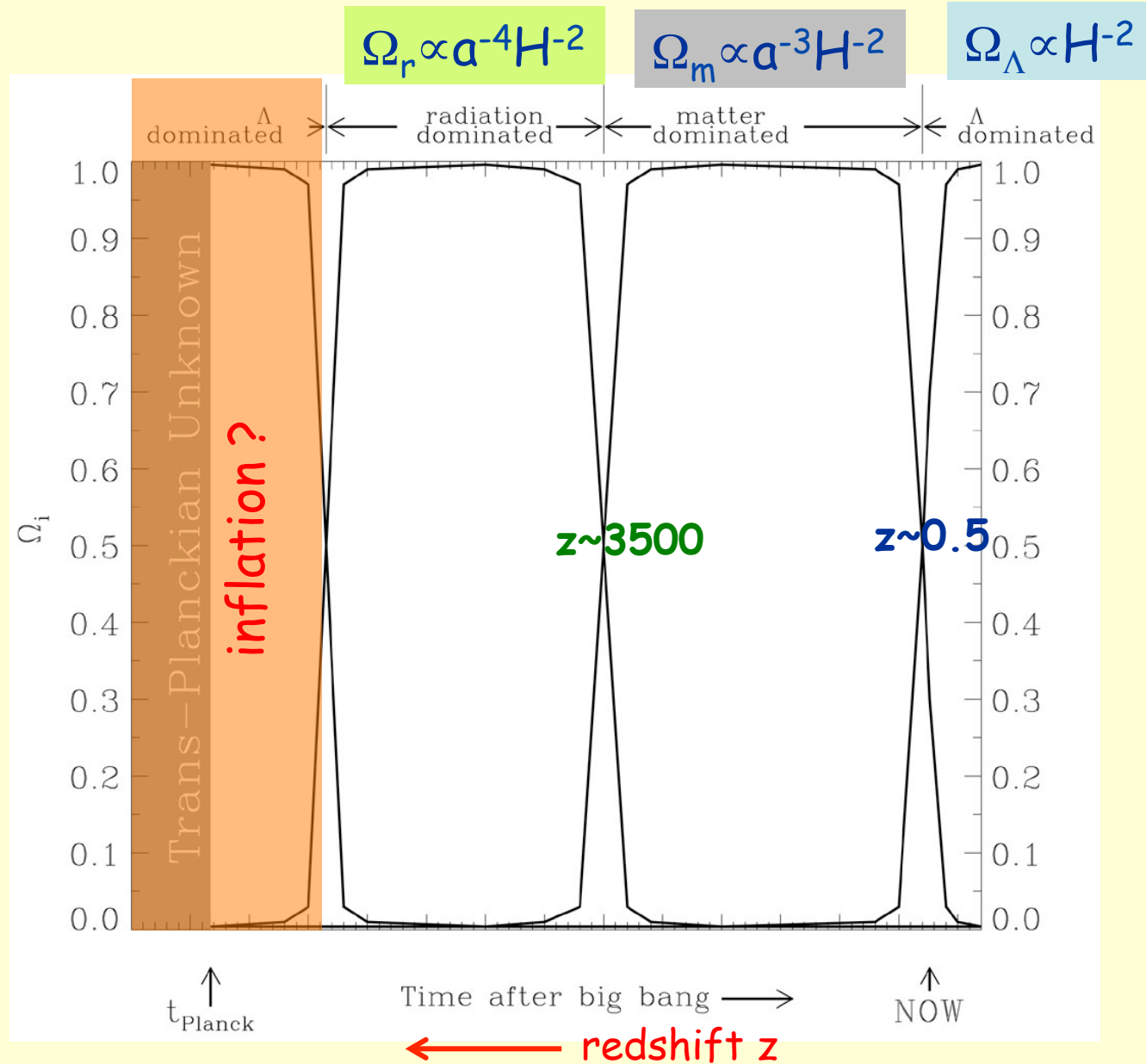
Planck collaboration. 2018, arXiv:1807.06209



atoms: gas (4%), stars (0.4%), ν (0.3%), heavy elements (0.03%)

M. Kowalski et al., 2008, ApJ, 686, 749

The rise and fall of the Ω_i 's



Expansion and temperature

- Photons emitted at t (e.g. CMB), received today:

$$1+z = \frac{\lambda_{observed}}{\lambda_{emitted}} = \frac{a_0}{a(t)} = \frac{E_\gamma(t)}{E_\gamma(t_0)} = \frac{T(t)}{T_0}$$

↗ at emission
↘ today

- Temperature was **hotter in the past**, expansion implies cooling down

- CMB now: $T_{CMB}^{now} \approx 2.725K \Rightarrow T_0 \approx 2.35 \cdot 10^{-4} eV \Rightarrow \Omega_\gamma^0 \approx 5 \cdot 10^{-5}$
 (COBE) $(k=8.617 \cdot 10^{-5} eV.K^{-1})$ ↙ negligible (today)

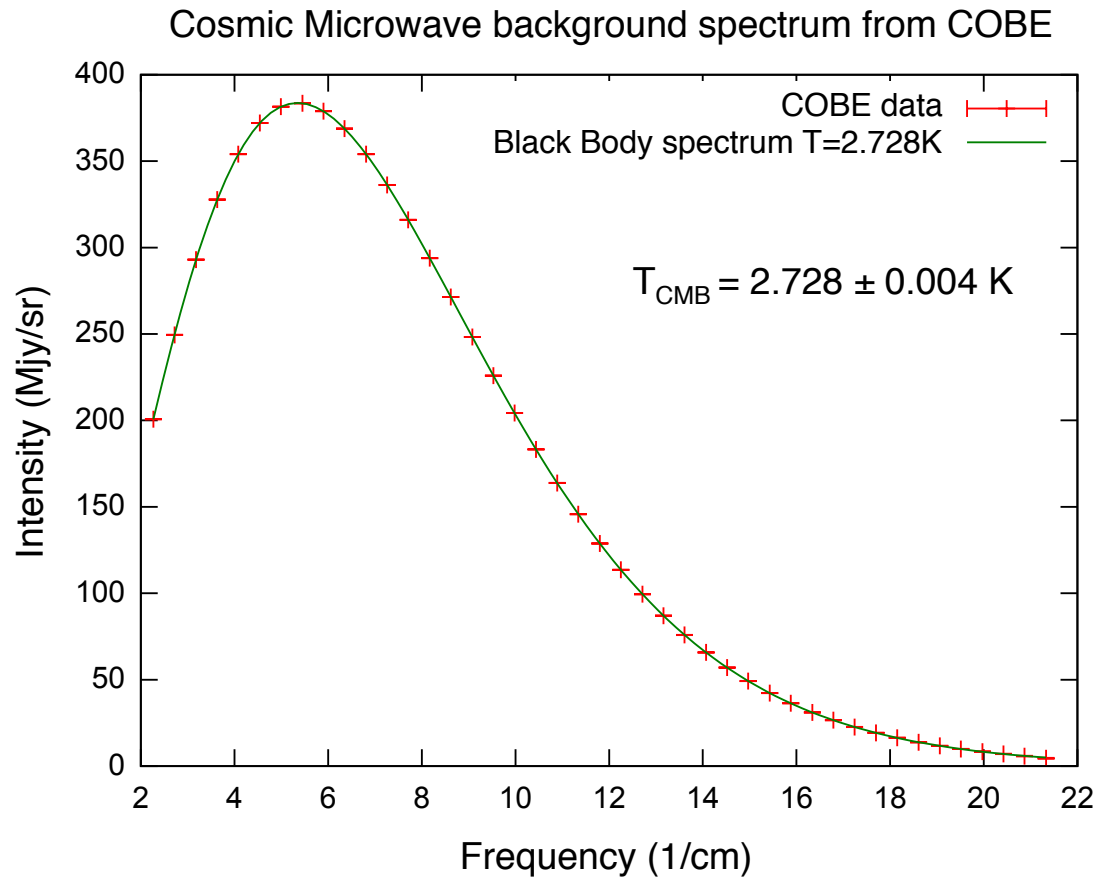
- CMB at emission (from anisotropy measurements):

$$z_{CMB}^{emission} \approx 1100 \Rightarrow T_{CMB}(t_{emission}) \approx 0.26 eV \approx 3,000K$$

- matter-radiation equality:

$$\rho_r(t_{eq}) = \rho_m(t_{eq}) \Rightarrow a_0 / a(t_{eq}) \Rightarrow T_{eq} \approx 1eV \quad z \approx 3500$$

The CMB spectrum as seen by COBE (1989-1993)



D.J. Fixsen et al. 1996, ApJ, 473, 576F

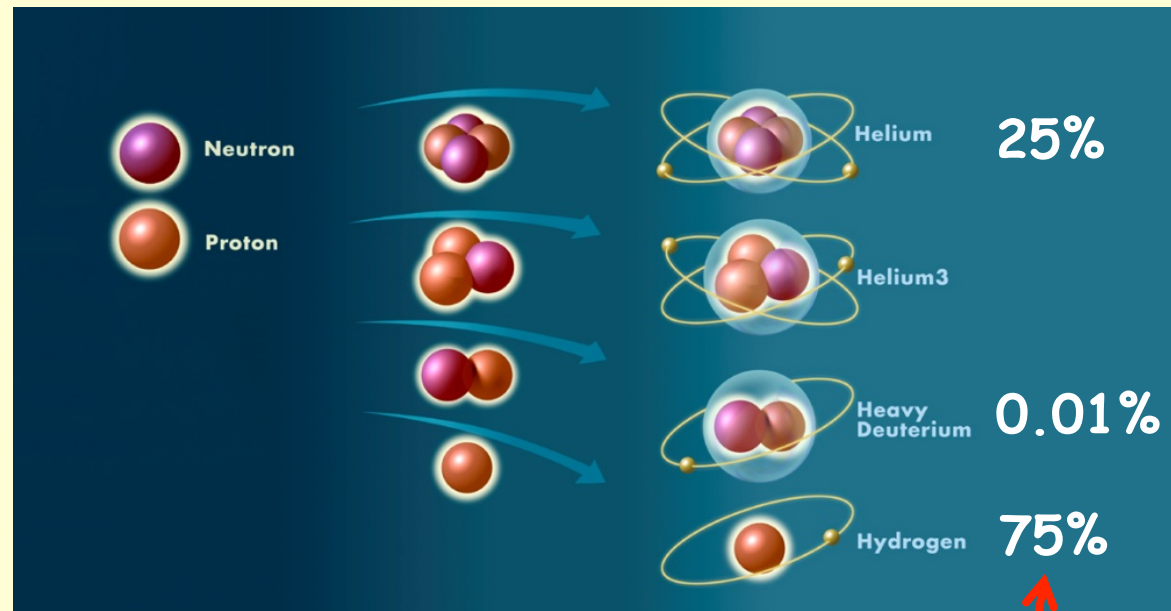
- Final : $T_{\text{CMB}} = 2.7255 \pm 0.0006 \text{ K}$

D.J. Fixsen, 2009, AJ, 707, 916

Primordial nucleosynthesis

- 1948: G.Gamow, R.Alpher "The origin of Chemical Elements"

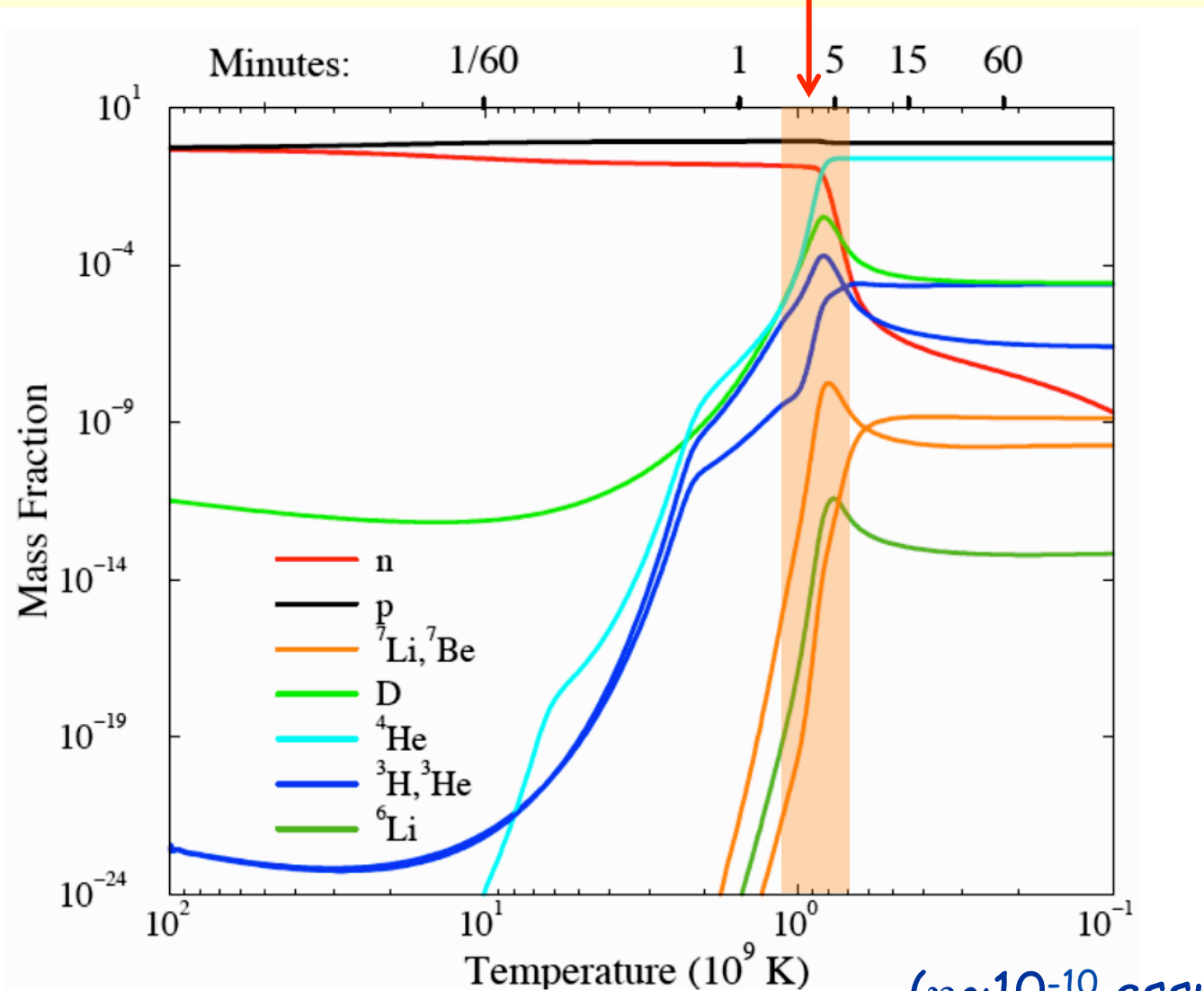
Expanding universe : deuterium and helium nuclei are formed by nuclear reactions inside the primordial plasma of p,n,e, γ when temperature and densities are adequate, leading to light element abundances as measured.



measured primordial abundances

The evolution of abundances

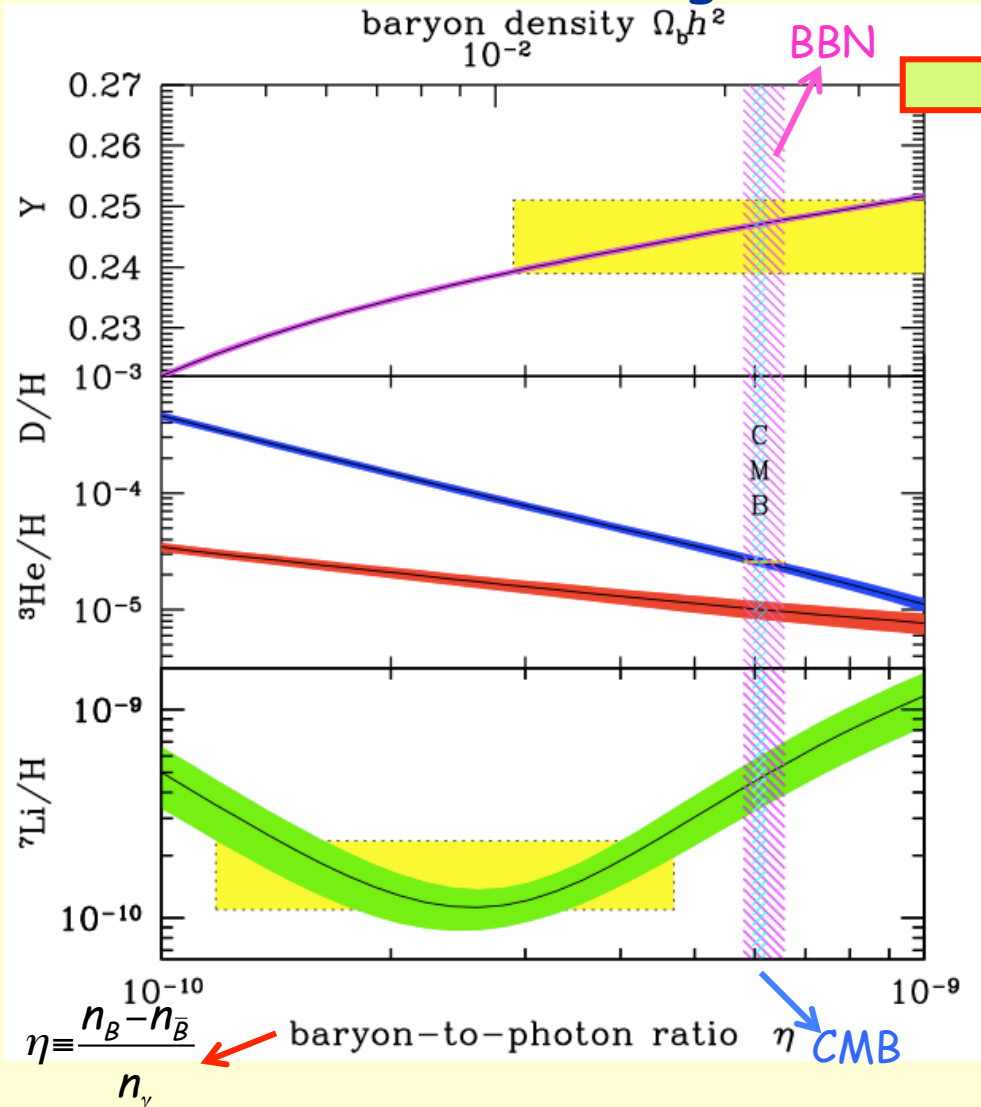
$t_{\text{BBN}} \sim 3\text{mn}$: abundances frozen



($\eta \sim 10^{-10}$ assumed)

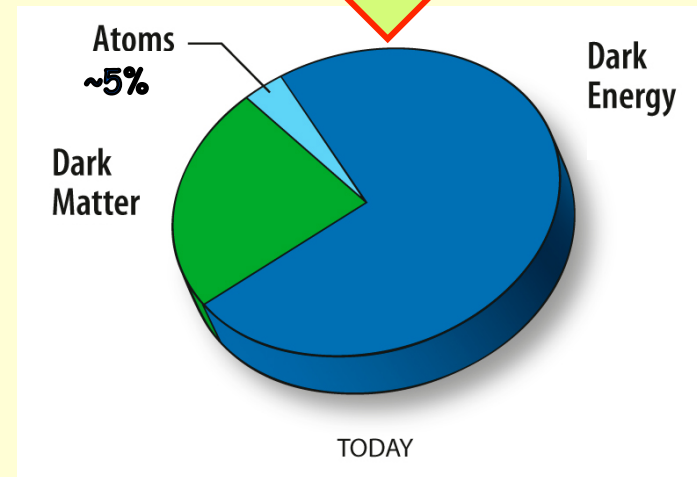
Present values of the Ω_i 's

- Measurements of light element abundances vs predictions:



$$0.046 \leq \Omega_b^0 \leq 0.053$$

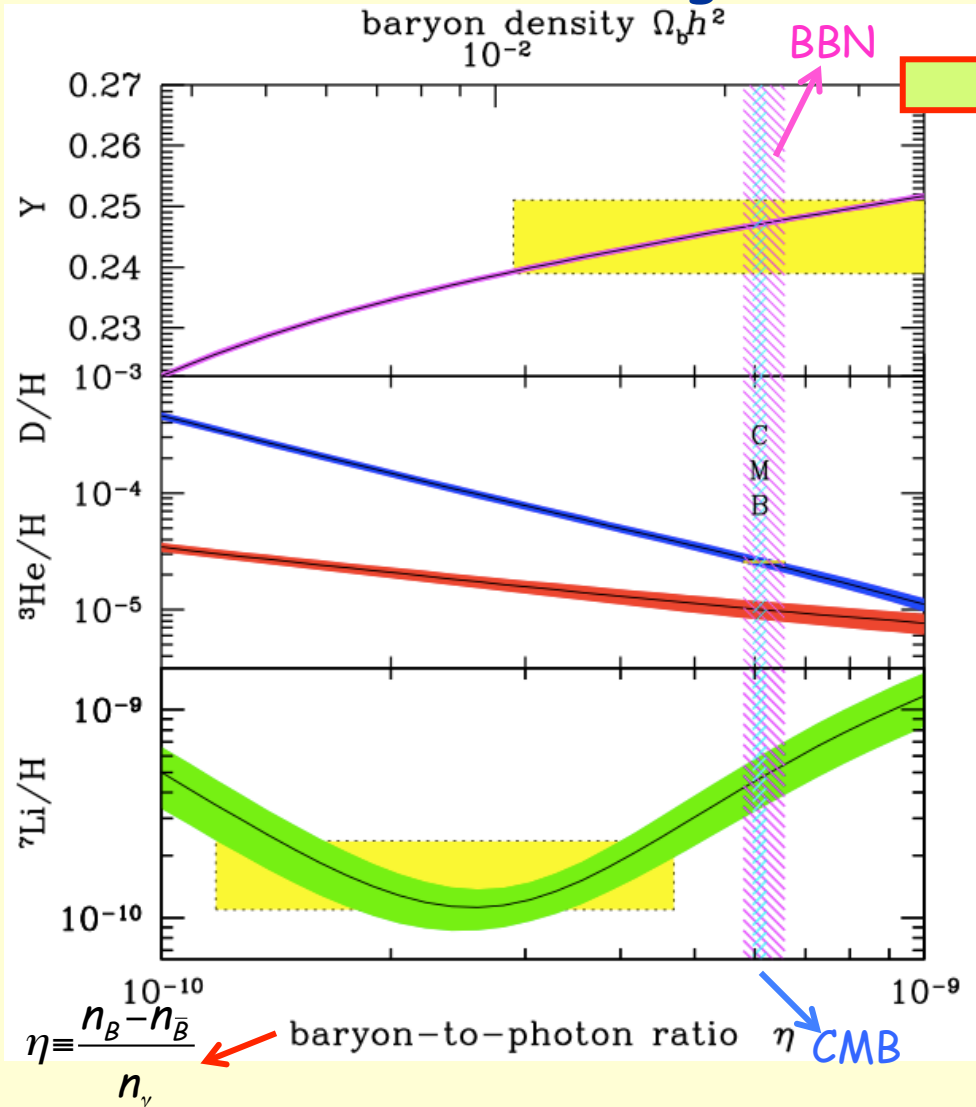
(confirmed independently by
CMB - Planck 2018)



Most matter in the
 Universe is **not** ordinary
 atoms : dark matter !

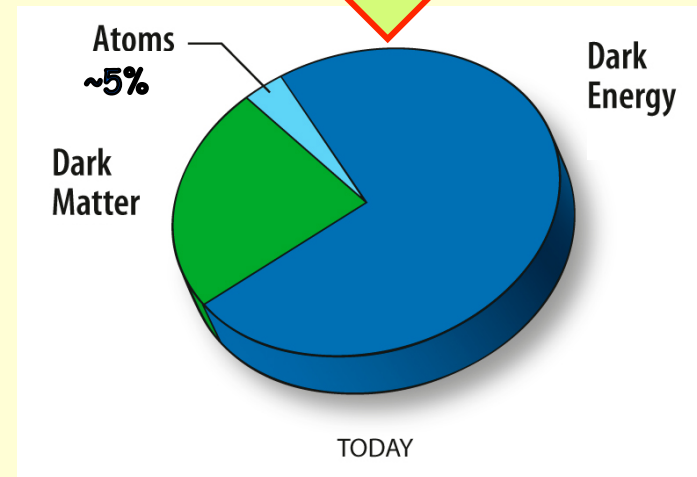
Present values of the Ω_i 's

- Measurements of light element abundances vs predictions:



$$0.046 \leq \Omega_b^0 \leq 0.053$$

(confirmed independently by
CMB - Planck 2018)



atoms: gas (4%), stars (0.4%),
 ν (0.3%), heavy elements (0.03%)

CONCLUSIONS (2)

- Einstein GR equations + Λ + FLRW metric :

$$1 = \Omega_m(t) + \Omega_r(t) + \Omega_k(t) + \Omega_\Lambda(t)$$

- COBE measurement: radiation today is negligible
- Cosmological measurements vs predictions:

- CMB+SNeIa+BAO:

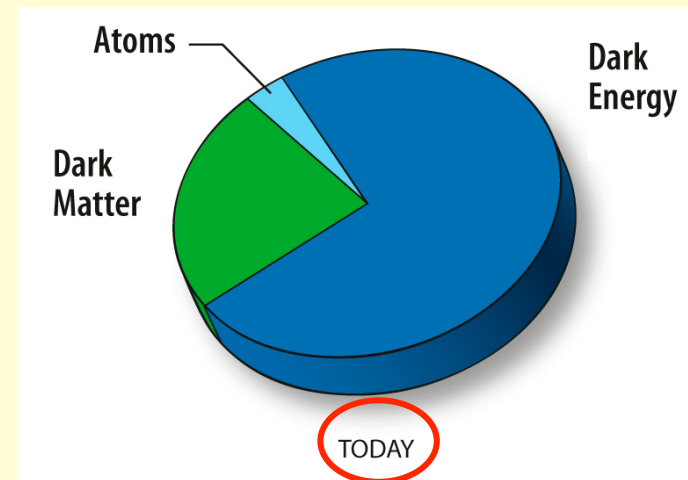
$$\Omega_m^0 = 0.315 \pm 0.007$$

$$\Omega_\Lambda^0 = 0.685 \pm 0.007$$

→ flat universe

- Primordial abundances:

$$0.043 \leq \Omega_b^0 \leq 0.049$$



Concordance model : Λ_{CDM}

radiation
domination

D, ${}^4\text{He}$ nucleosynthesis
 $T \sim 60\text{keV}$ $z \sim 10^8$

inflation ?



3mn

10^{-32}s

13.8Gyr

today

$T \sim 2.35 \cdot 10^{-4}\text{eV}$ $z=0$

Λ
domination

matter-radiation equality
 $T \sim 0.83\text{eV}$ $z \sim 3500$

10^5yr

380,000yr

9Gyr

expansion acceleration
 $T \sim 3.5 \cdot 10^{-4}\text{eV}$ $z \sim 0.5$

matter
domination

CMB

$T \sim 0.26\text{eV}$ $z=1100$