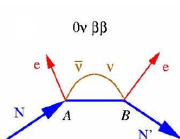


# Towards reliable nuclear matrix elements for neutrinoless $\beta\beta$ decay

Frédéric Nowacki



# Nuclear physics and neutrinoless $\beta\beta$ decay

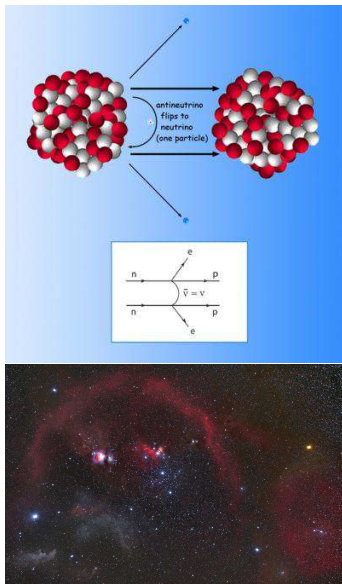
Neutrinos, dark matter studied in experiments using nuclei

Nuclear matrix elements depend on nuclear structure crucial to anticipate reach and fully exploit experiments

$$0\nu\beta\beta \text{ decay: } [T_{1/2}^{0\nu}]^{-1} \propto |M^{0\nu}|^2 \langle m_\nu \rangle^2$$

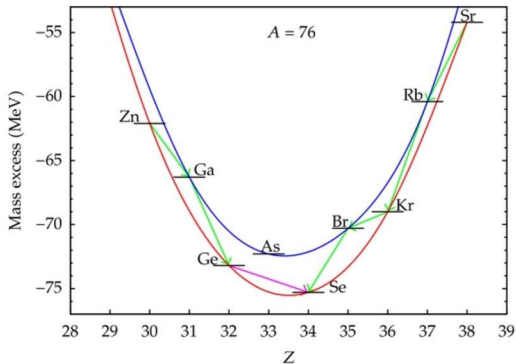
$$\text{Dark matter: } \frac{d\sigma_X \mathcal{N}}{dq^2} \propto \left| \sum_i c_i \zeta_i \mathcal{F}_i \right|^2$$

$M^{0\nu}$ : Nuclear matrix element  
 $\mathcal{F}_i$ : Nuclear structure factor



# Neutrinoless $\beta\beta$ decay

Lepton-number violation, Majorana nature of neutrinos  
 Second order process only observable in rare cases with  
 $\beta$ -decay energetically forbidden or hindered by  $\Delta J$

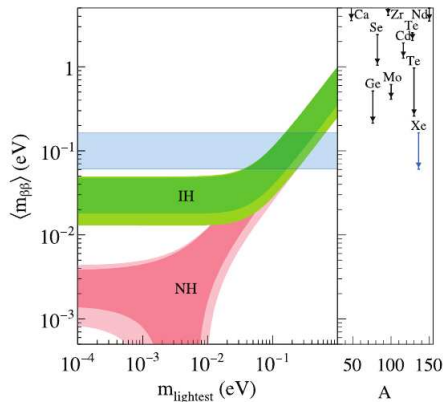
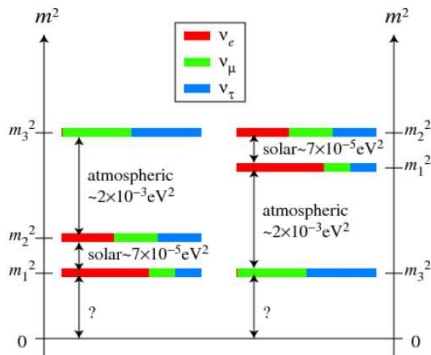


Present best limits  $T_{1/2}^{0\nu} \gtrsim 10^{25}$  y.:

$^{76}\text{Ge}$  (GERDA, Majorana),  $^{130}\text{Te}$  (CUORE),  $^{136}\text{Xe}$  (EXO, KamLAND-Zen)

# Next generation experiments: inverted hierarchy

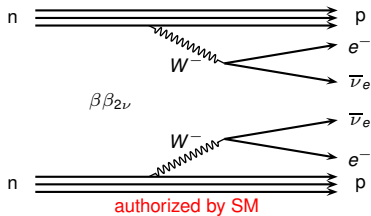
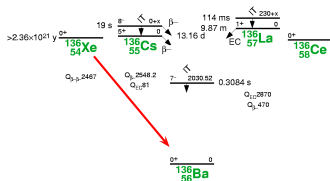
The decay lifetime is  $[T_{1/2}^{0\nu}(0^+ \rightarrow 0^+)]^{-1} = G_{0\nu} |M^{0\nu}|^2 \langle m_\nu^{\beta\beta} \rangle^2$   
 sensitive to absolute neutrino masses,  $\langle m_\nu^{\beta\beta} \rangle = \sum_i U_{ei}^2 m_i$



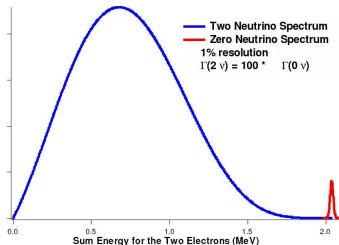
KamLAND-Zen, PRL117 082503 (2016)

Matrix elements needed to make sure next generation ton-scale experiments fully explore “inverted hierarchy”

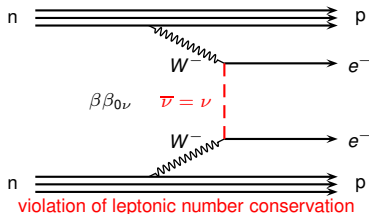
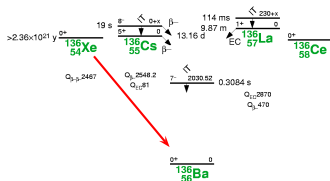
# $\beta\beta$ decay



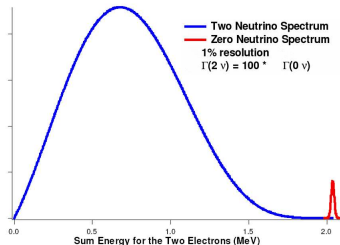
| Transition                                    | $Q_{\beta\beta}$ (keV) | Abundance<br>( $^{232}\text{Th} = 100$ ) |
|---|------------------------|--|
| $^{110}\text{Pd} \rightarrow ^{110}\text{Cd}$ | 2013                   | 12                                       |
| $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$   | 2040                   | 8  |
| $^{124}\text{Sn} \rightarrow ^{124}\text{Te}$ | 2288                   | 6  |
| $^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$ | 2479                   | 9  |
| $^{130}\text{Te} \rightarrow ^{130}\text{Xe}$ | 2533                   | 34                                       |
| $^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$ | 2802                   | 7  |
| $^{82}\text{Se} \rightarrow ^{82}\text{Kr}$   | 2995                   | 9  |
| $^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$ | 3034                   | 10                                       |
| $^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$   | 3350                   | 3  |
| $^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$ | 3667                   | 6  |
| $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$   | 4271                   | 0.2                                      |



# $\beta\beta$ decay



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# $(\beta\beta)_{0\nu}$ decay

Specificity of  $(\beta\beta)_{0\nu}$ :

NO EXPERIMENTAL DATA !!!

prediction for  $m_\nu$  very **difficult**  
**easier** for  $m_\nu(A)/m_\nu(A')$

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What is the best isotope to observe  $(\beta\beta)_{0\nu}$  decay ?

What is the influence of the structure of the nucleus on  $(\beta\beta)_{0\nu}$  matrix elements ?

# Calculating nuclear matrix elements

Nuclear matrix elements needed to study fundamental symmetries

$$\langle \text{Final} | \mathcal{L}_{\text{leptons-nucleus}} | \text{Initial} \rangle = \langle \text{Final} | dx j^\mu(x) J_\mu(x) | \text{Initial} \rangle$$

- Nuclear structure calculation of the initial and final states:

Shell model Retamosa, Caurier, FN...

Energy-density functional Rodriguez, Yao...

QRPA Vogel, Faessler, Simkovic, Suhonen...

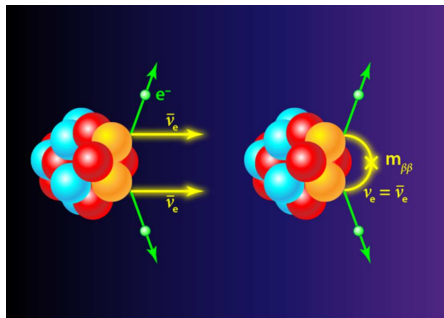
Interacting boson model Iachello, Barea...

Ab Initio many-body methods

Green's Function MC, Coupled-Cluster, IM-SRG

- Lepton-nucleus interaction:

Study hadronic current in nucleus:  
phenomenological approaches, effective theory of QCD



# Two neutrinos mode

The theoretical expression of the half-life of the  $2\nu$  mode can be written as:

$$[T_{1/2}^{2\nu}]^{-1} = G_{2\nu} |M_{GT}^{2\nu}|^2,$$

with

$$M_{GT}^{2\nu} = \sum_m \frac{\langle 0_f^+ || \vec{\sigma} t_- || 1_m^+ \rangle \langle 1_m^+ || \vec{\sigma} t_- || 0_i^+ \rangle}{E_m + E_0}$$

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- summation over intermediate states
- to quench or not to quench ? ( $\sigma\tau_{eff.}$ )
- **does a good  $2\nu$  ME guarantee a good  $0\nu$  ME ?**

$(\beta\beta)_{2\nu}$  structure function

$$M_{GT}^{2\nu} = \sum_m \frac{\langle 0_f^+ || \vec{\sigma}t_- || 1_m^+ \rangle \langle 1_m^+ || \vec{\sigma}t_- || 0_i^+ \rangle}{E_m + E_0}$$

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- at iteration N, N  $1^+$  states in the intermediate nucleus, with excitation energies  $E_m$

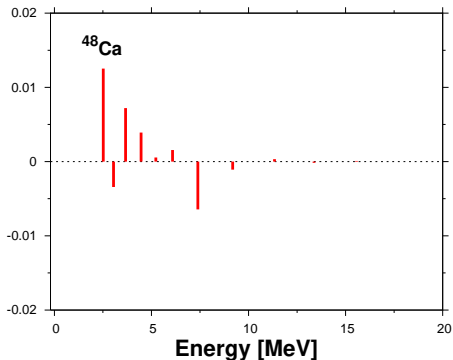
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  - at iteration N, N  $1^+$  states in the intermediate nucleus, with excitation energies  $E_m$
  - overlap with the other doorway, enter energy denominators and add up the N contributions

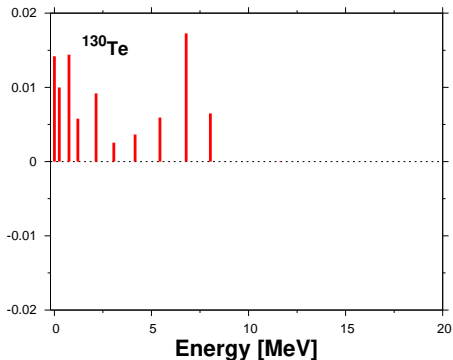
# $2\nu$ half-lives



$2\nu$  strength function in  $^{48}\text{Ca}$ ,  $^{130}\text{Te}$  and  $^{136}\text{Xe}$

| Parent nuclei              | $^{48}\text{Ca}$ | $^{76}\text{Ge}$ | $^{82}\text{Se}$ | $^{130}\text{Te}$ | $^{136}\text{Xe}$ |
|----------------------------|------------------|------------------|------------------|-------------------|-------------------|
| $T_{1/2}^{2\nu}(g.s.)$ th. | $3.7E19$         | $1.15E21$        | $3.4E19$         | $4E20$            | $6E20$            |
| $T_{1/2}^{2\nu}(g.s.)$ exp | $4.2E19$         | $1.4E21$         | $8.3E19$         | $2.7E21$          | $2.38E21$         |

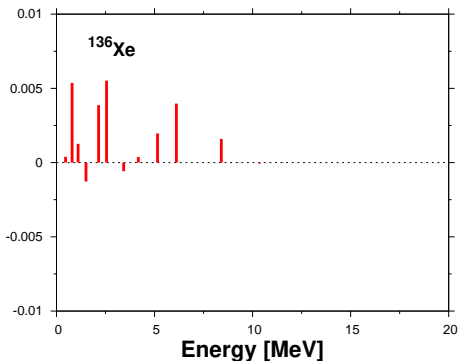
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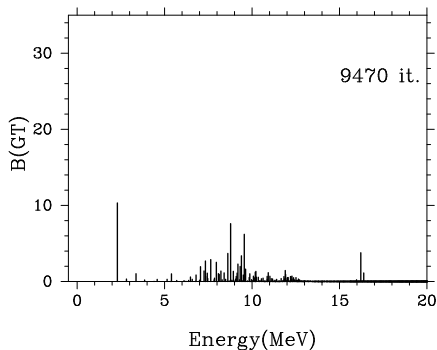
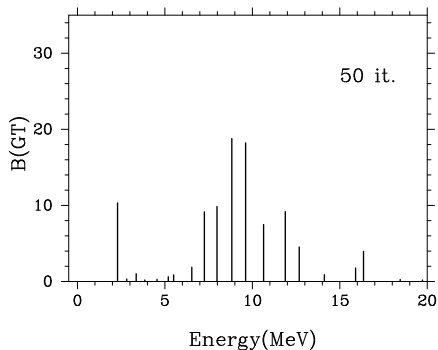
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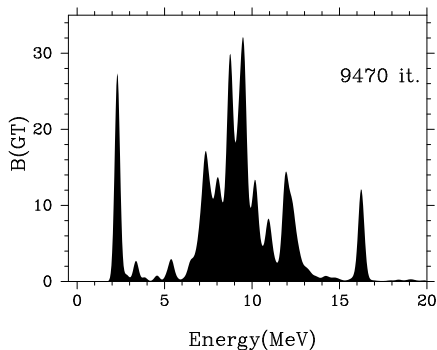
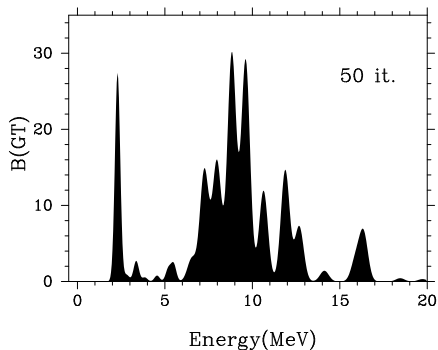
# Evolution of Strength Distribution

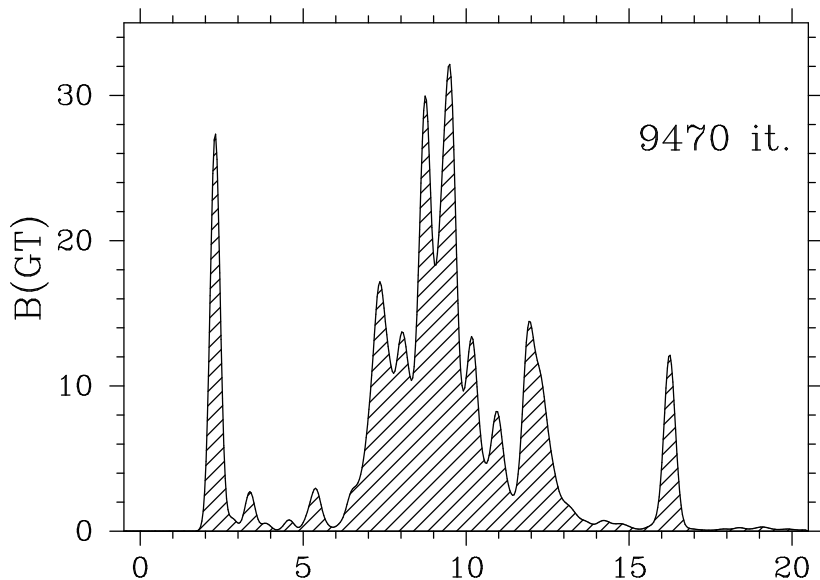
## GT Strength in $^{48}\text{Sc}$

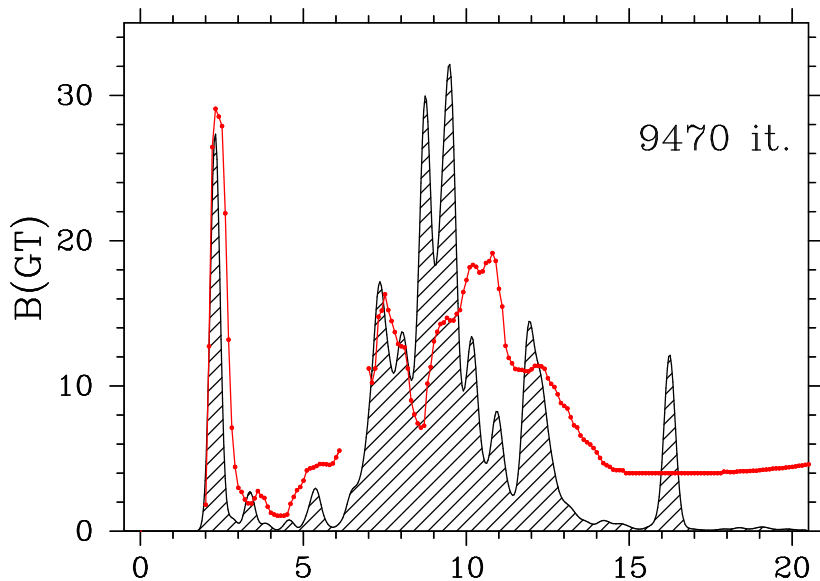


# Evolution of Strength Distribution

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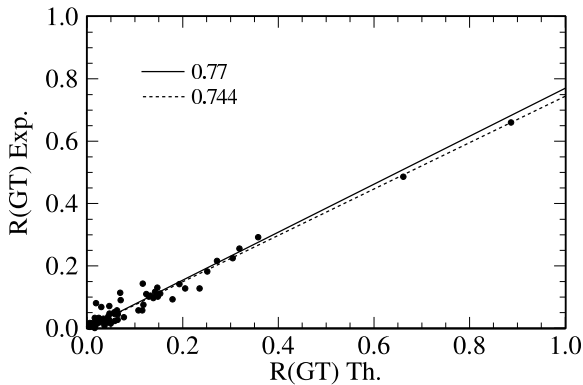


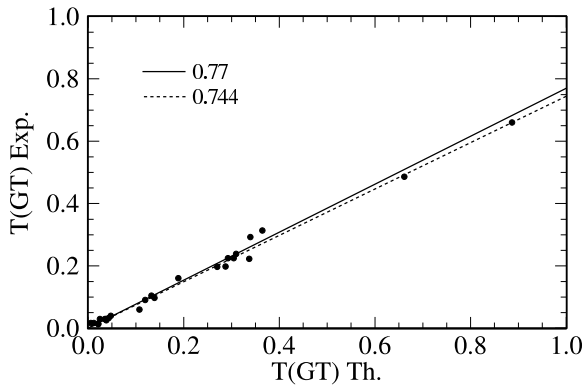
$^{48}\text{Ca}(p,n)^{48}\text{Sc}$  Strength Function

$^{48}\text{Ca}(p,n)^{48}\text{Sc}$  Strength Function

# Quenching of GT operator in the *pf*-shell

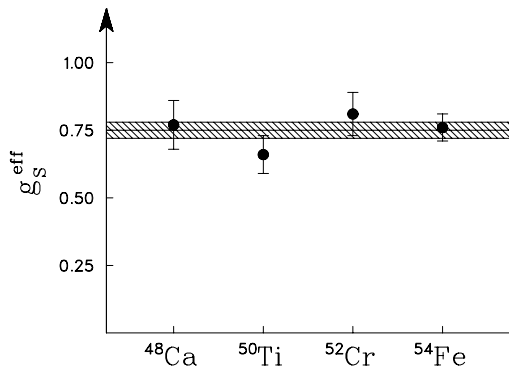
| Nucleus          | Uncorrelated | Correlated |            | Expt.         |
|------------------|--------------|------------|------------|---------------|
|                  |              | Unquenched | $Q = 0.74$ |               |
| $^{51}\text{V}$  | 5.15         | 2.42       | 1.33       | $1.2 \pm 0.1$ |
| $^{54}\text{Fe}$ | 10.19        | 5.98       | 3.27       | $3.3 \pm 0.5$ |
| $^{55}\text{Mn}$ | 7.96         | 3.64       | 1.99       | $1.7 \pm 0.2$ |
| $^{56}\text{Fe}$ | 9.44         | 4.38       | 2.40       | $2.8 \pm 0.3$ |
| $^{58}\text{Ni}$ | 11.9         | 7.24       | 3.97       | $3.8 \pm 0.4$ |
| $^{59}\text{Co}$ | 8.52         | 3.98       | 2.18       | $1.9 \pm 0.1$ |
| $^{62}\text{Ni}$ | 7.83         | 3.65       | 2.00       | $2.5 \pm 0.1$ |

Quenching of GT strength in the  $pf$ -shell

Quenching of GT strength in the  $pf$ -shell

Quenching of M1 operator in the  $pf$ -shell

KB3 interaction



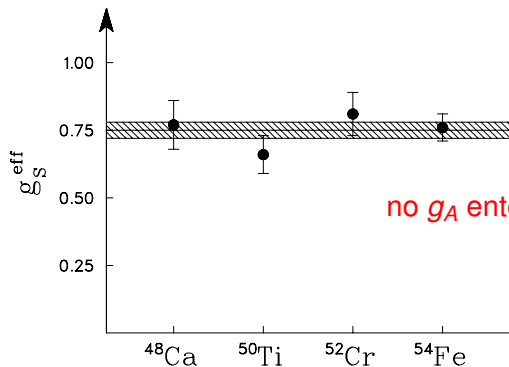
Neumann-Cosel et al.

Phys. Lett. **B433** 1 (1998)



Quenching of M1 operator in the  $pf$ -shell

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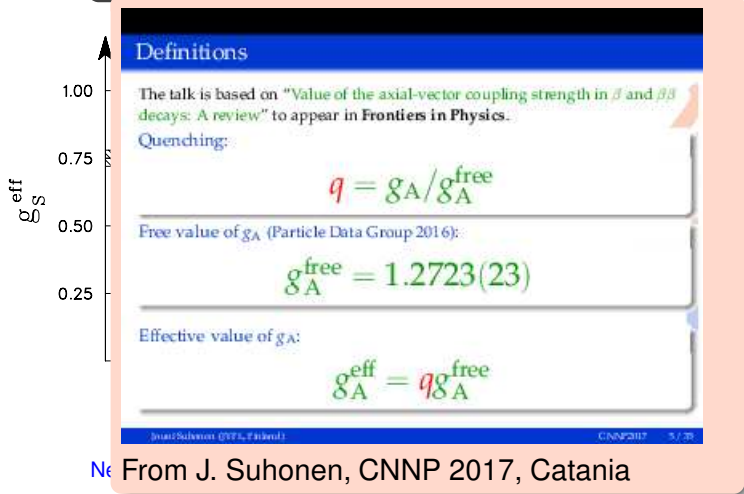
no  $g_A$  entering in the operator !

Neumann-Cosel et al.

Phys. Lett. **B433** 1 (1998)

# Quenching of M1 operator in the $pf$ -shell

## KB3 interaction



Phys. Lett. **D433** 1 (1990)

# Quenching of GT operator in the $pf$ -shell

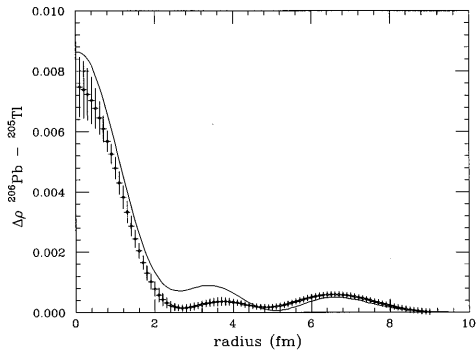
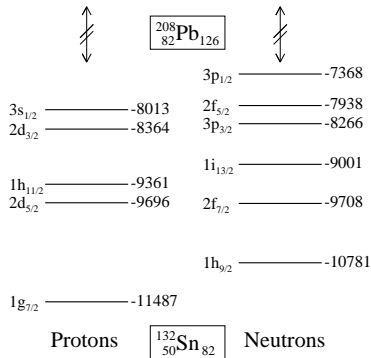


FIG. 3. Density difference between  $^{206}\text{Pb}$  and  $^{205}\text{Tl}$ . The experimental result of Cavendon *et al.* (1982) is given by the error bars; the prediction obtained using Hartree-Fock orbitals with adjusted occupation numbers is given by the curve. The systematic shift of  $0.0008 \text{ fm}^{-3}$  at  $r \leq 4 \text{ fm}$  is due to deficiencies of the calculation in predicting the core polarization effect.

V. R. Pandharipande, I. Sick and P. K. A. deWitt  
Huberts, *Rev. mod. Phys.* **69** (1997) 981



# Quenching of GT operator in the $pf$ -shell

If we write


$$|\hat{i}\rangle = \alpha|0\hbar\omega\rangle + \sum_{n \neq 0} \beta_n |n\hbar\omega\rangle,$$

$$|\hat{f}\rangle = \alpha'|0\hbar\omega\rangle + \sum_{n \neq 0} \beta'_n |n\hbar\omega\rangle$$

then

$$\langle \hat{f} \parallel \mathcal{T} \parallel \hat{i} \rangle^2 = \left( \alpha\alpha' T_0 + \sum_{n \neq 0} \beta_n \beta'_n T_n \right)^2,$$

- $n \neq 0$  contributions negligible
- $\alpha \approx \alpha'$

 projection of the physical wavefunction in the  $0\hbar\omega$  space is  $Q \approx \alpha^2$

 transition quenched by  $Q^2$

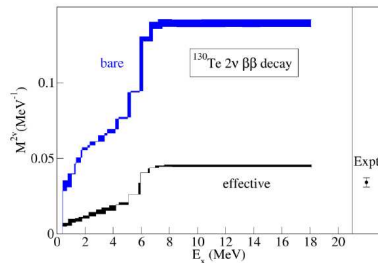
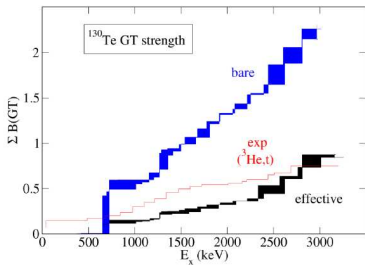
# Renormalisation of the GT operator by MBPT

PHYSICAL REVIEW C **95**, 064324 (2017)



## Calculation of Gamow-Teller and two-neutrino double- $\beta$ decay properties for $^{130}\text{Te}$ and $^{136}\text{Xe}$ with a realistic nucleon-nucleon potential

L. Coraggio,<sup>1,\*</sup> L. De Angelis,<sup>1</sup> T. Fukui,<sup>1</sup> A. Gargano,<sup>1</sup> and N. Itaco<sup>1,2</sup>



Renormalisation of the GT by Many-Body Perturbation Theory

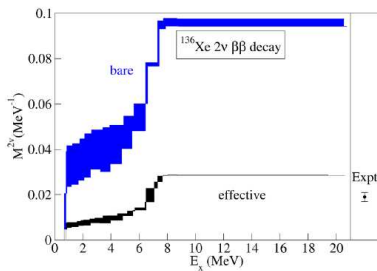
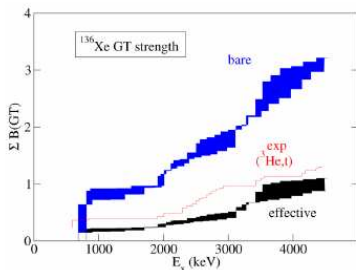
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PHYSICAL REVIEW C **95**, 064324 (2017)



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L. Coraggio,<sup>1,\*</sup> L. De Angelis,<sup>1</sup> T. Fukui,<sup>1</sup> A. Gargano,<sup>1</sup> and N. Itaco<sup>1,2</sup>



Renormalisation of the GT by Many-Body Perturbation Theory

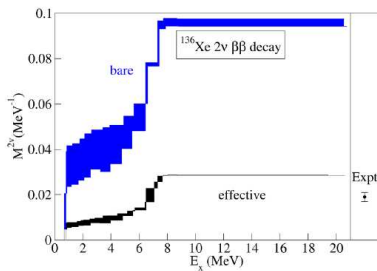
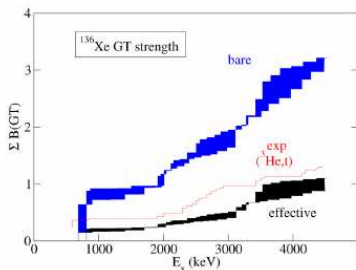
# Renormalisation of the GT operator by MBPT

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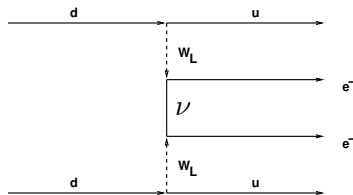
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Renormalisation of the GT by Many-Body Perturbation Theory  
 Further step, apply MBPT to Neutrinoless operator

# Neutrinoless mode:

Exchange of a light neutrino, only left-handed currents



The theoretical expression of the half-life of the  $0\nu$  mode can be written as:

$$[T_{1/2}^{0\nu}(0^+ \rightarrow 0^+)]^{-1} = G_{0\nu} |M^{0\nu}|^2 \langle m_\nu \rangle^2$$



# Neutrinoless mode:

CLOSURE APPROXIMATION then

$$\langle \Psi_f | \mathcal{O}^{(K)} | \Psi_i \rangle \quad \text{with} \quad \mathcal{O}^{(K)} = \sum_{ijkl} W_{ijkl}^{\lambda, K} \left[ (a_i^\dagger a_j^\dagger)^\lambda (\tilde{a}_k \tilde{a}_l)^\lambda \right]^K$$

## Neutrinoless mode:

CLOSURE APPROXIMATION then

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two-body operator

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two-body operator

We are left with a “standard” nuclear structure problem

$$M^{(0\nu)} = M_{GT}^{(0\nu)} - \left( \frac{g_V}{g_A} \right)^2 M_F^{(0\nu)} - M_T^{(0\nu)}$$

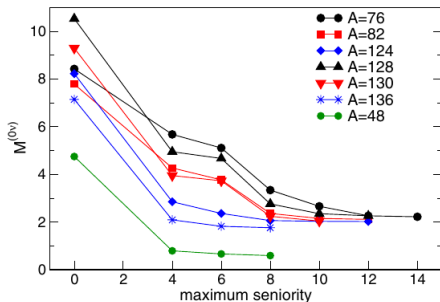
SM results for  $(\beta\beta)_{0\nu}$ 

| emitter           | $\langle m_\nu \rangle$<br>( $T_{\frac{1}{2}} = 10^{25}$ y.) | $M_{0\nu}^{tot}$ (UCOM) |
|-------------------|--|-------------------------|
| $^{48}\text{Ca}$  | 0.63   | 0.85                    |
| $^{76}\text{Ge}$  | 0.72   | 2.81                    |
| $^{82}\text{Se}$  | 0.37   | 2.64                    |
| $^{96}\text{Zr}$  |  |                         |
| $^{100}\text{Mo}$ |  |                         |
| $^{110}\text{Pd}$ |  |                         |
| $^{116}\text{Cd}$ | 0.46   | 1.60                    |
| $^{124}\text{Sn}$ | 0.37   | 2.62                    |
| $^{128}\text{Te}$ | 1.32   | 2.88                    |
| $^{130}\text{Te}$ | 0.28   | 2.65                    |
| $^{136}\text{Xe}$ | 0.38   | 2.19                    |
| $^{150}\text{Nd}$ | <b>heavy and deformed !</b>                                  |                         |

# Pairing correlations and $0\nu\beta\beta$ decay

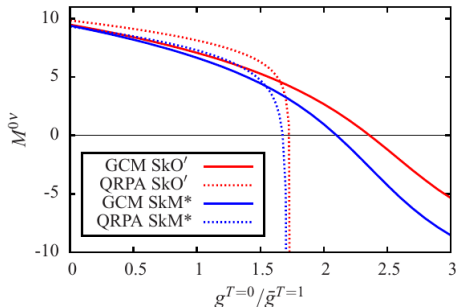
$0\nu\beta\beta$  decay favoured by proton-proton, neutron-neutron pairing,  
but it is disfavored by proton-neutron pairing

Ideal case: superfluid nuclei  
reduced with high-seniorities



E. Caurier et al., PRL100 052503 (2008)

Addition of isoscalar pairing  
reduces matrix element value



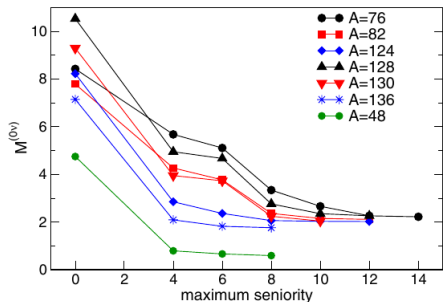
Hinohara, Engel, PRC 90 031301 (2014)

Related to approximate SU(4) symmetry of the  $\sum H(r)\sigma_i\sigma_j\tau_i\tau_j$  operator

# Pairing correlations and $0\nu\beta\beta$ decay

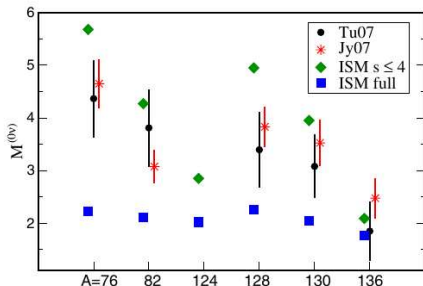
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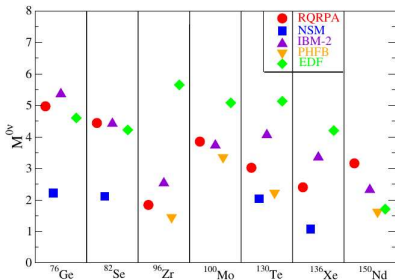


E. Caurier et al., PRL100 052503 (2008)

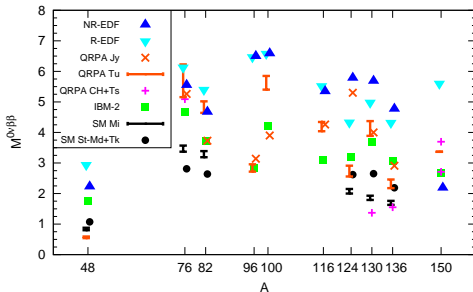
Related to approximate SU(4) symmetry of the  $\sum H(r)\sigma_i\sigma_j\tau_i\tau_j$  operator

# $0\nu\beta\beta$ matrix elements: last 5 years

## Comparison of nuclear matrix elements calculations: 2012 vs 2017



P. Vogel, J. Phys. G39 124002 (2012)



J. Engel, Rep. Prog. Phys.80 046301 (2017)

What have we learned in the last 5 years ?

# Shell model configuration space: two shells

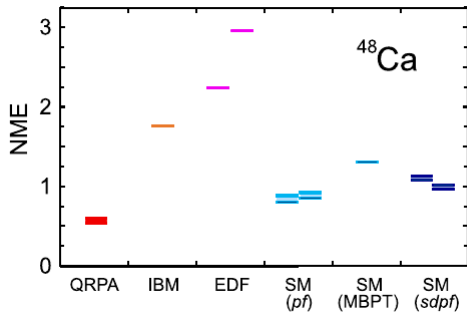
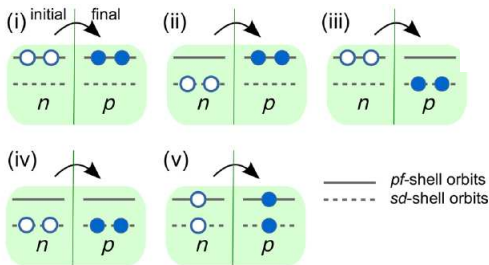
For  $^{48}\text{Ca}$  enlarge configuration space  
from *pf* to *sdpf*

4 to 7 orbitals, dimension  $10^5$  to  $10^9$

increases matrix elements

but only moderately 30%

Iwata et al. PRL116 112502 (2016)

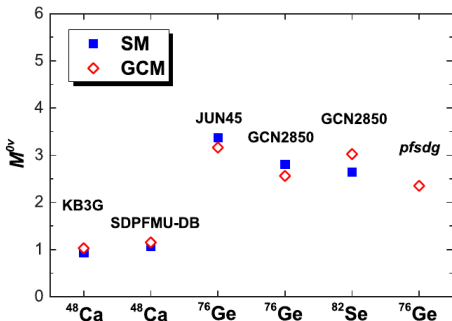


Contributions dominated by pairing  
2 particle - 2 hole excitations  
enhance the  $\beta\beta$  matrix element,  
Contributions dominated by  
1 particle - 1 hole excitations  
suppress the  $\beta\beta$  matrix element



# $^{76}\text{Ge}$ matrix element in two shells: approximate

Large configuration space calculations in 2 major oscillator shells include all relevant correlations: isovector/isoscalar pairing, deformation  
 Many-body approach: Generating Coordinate Method (GCM)



GCM approximates shell model calculation

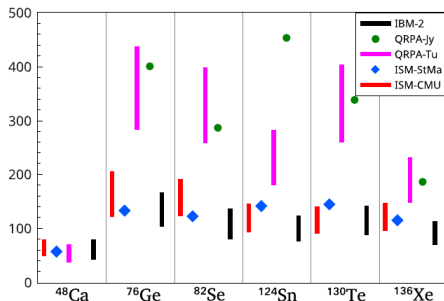
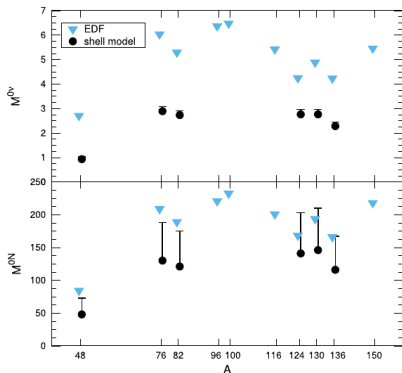
Degrees of freedom, or generating coordinates validated against exact shell model in restricted configuration space

Jiao et al., PRC96 054310 (2017)

$^{76}\text{Ge}$  nuclear matrix elements in 2 major shells  
 very similar to shell model nuclear matrix element in 1 major shell

# Heavy-neutrino exchange nuclear matrix elements

Contrary to light-neutrino exchange, for heavy-neutrino exchange decay shell model, IBM and EDF matrix elements agree reasonably!



A. Neacsu et al., PRC 93 024308 (2016)

Suggests differences in treating  
longer-range nuclear correlations  
dominant in light-neutrino exchange

J. Menendez, JPG 45 014003

(2018)

# Heavy-neutrino matrix element

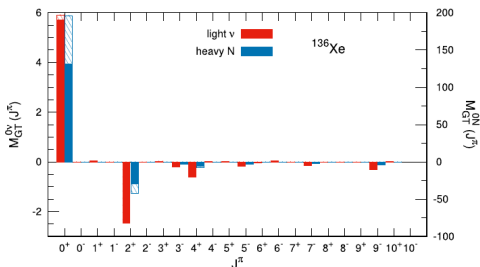
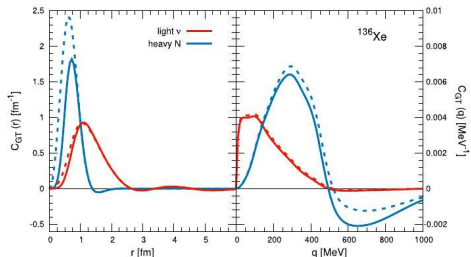
Compared to  
light-neutrino exchange

heavy neutrino exchange  
dominated by shorter inter-  
nucleon range,  
larger momentum transfers

heavy neutrino exchange  
contribution  
from  $J > 0$  pairs smaller:  
pairing most relevant

Long-range correlations  
(except pairing)  
not under control

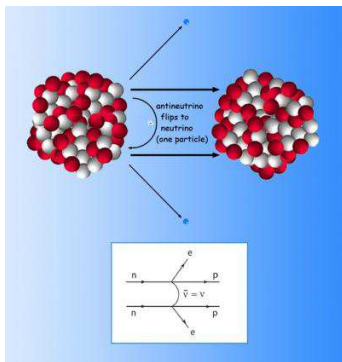
J. Menendez, JPG 45 014003 (2018)



# Summary

## Reliable nuclear matrix elements needed to plan and fully exploit impressive experiments looking for neutrinoless double-beta decay

- Matrix elements differences between present calculations, factor 2-3 besides additional “quenching” ?
- $^{48}\text{Ca}$  and  $^{76}\text{Ge}$  matrix elements in larger configuration space increase  $\approx 30\%$ , missing correlations introduced in IBM, EDF
- First Ab-initio calculations of  $\beta$  decays do not need additional “quenching”, Ab-initio  $^{48}\text{Ca}$  matrix elements in progress
- $2\nu\beta\beta$  decay,  $\mu$ -capture/ $\nu$ -nucleus scattering and double Gamow-Teller transitions can give insight on  $0\nu\beta\beta$  matrix elements



# $(\beta\beta)_{0\nu}$ matrix elements

$$\begin{aligned}
 M_{GT}^{(0\nu)} &= \langle 0_f^+ \| \sum_{n,m} h(\sigma_n \cdot \sigma_m) t_{n-} t_{m-} \| 0_i^+ \rangle, & \chi_F &= \langle 0_f^+ \| \sum_{n,m} h t_{n-} t_{m-} \| 0_i^+ \rangle \left( \frac{g_V}{g_A} \right)^2 / M_{GT}^{(0\nu)}, \\
 \chi'_{GT} &= \langle 0_f^+ \| \sum_{n,m} h'(\sigma_n \cdot \sigma_m) t_{n-} t_{m-} \| 0_i^+ \rangle / M_{GT}^{(0\nu)}, & \chi'_F &= \langle 0_f^+ \| \sum_{n,m} h' t_{n-} t_{m-} \| 0_i^+ \rangle \left( \frac{g_V}{g_A} \right)^2 / M_{GT}^{(0\nu)}, \\
 \chi_{GT}^\omega &= \langle 0_f^+ \| \sum_{n,m} h_\omega(\sigma_n \cdot \sigma_m) t_{n-} t_{m-} \| 0_i^+ \rangle / M_{GT}^{(0\nu)}, & \chi_F^\omega &= \langle 0_f^+ \| \sum_{n,m} h_\omega t_{n-} t_{m-} \| 0_i^+ \rangle \left( \frac{g_V}{g_A} \right)^2 / M_{GT}^{(0\nu)}, \\
 \chi_T &= \langle 0_f^+ \| \sum_{n,m} h' [(\sigma_n \cdot \hat{r}_{n,m})(\sigma_m \cdot \hat{r}_{n,m}) - \frac{1}{3} \sigma_n \cdot \sigma_m] t_{n-} t_{m-} \| 0_i^+ \rangle / M_{GT}^{(0\nu)}, \\
 \chi_P &= \langle 0_f^+ \| i \sum_{n,m} h' \left( \frac{r_{+n,m}}{2r_{n,m}} \right) [(\sigma_n - \sigma_m) \cdot (\hat{r}_{n,m} \times \hat{r}_{+n,m})] t_{n-} t_{m-} \| 0_i^+ \rangle \frac{g_V}{g_A} / M_{GT}^{(0\nu)}, \\
 \chi_R &= \frac{1}{6} (g_{-\frac{1}{2}}^s - g_{\frac{1}{2}}^s) \langle 0_f^+ \| \sum_{n,m} h_R(\sigma_n \cdot \sigma_m) t_{n-} t_{m-} \| 0_i^+ \rangle \frac{g_V}{g_A} / M_{GT}^{(0\nu)}.
 \end{aligned}$$

back

# $(\beta\beta)_{0\nu}$ matrix elements

$$h(r, \langle\mu\rangle) = \frac{R_0}{r} \phi(\langle\mu\rangle m_e r),$$

$$h'(r, \langle\mu\rangle) = h + \langle\mu\rangle m_e R_0 h_0(\langle\mu\rangle r),$$

$$h_\omega(r, \langle\mu\rangle) = h - \langle\mu\rangle m_e R_0 h_0(\langle\mu\rangle r),$$

$$h_R(r, \langle\mu\rangle) = -\frac{\langle\mu\rangle m_e}{M_j} \left( \frac{2}{\pi} \left( \frac{R_0}{r} \right)^2 - \langle\mu\rangle m_e R_0 h \right) + \frac{4\pi R_0^2}{M_p} \delta(r),$$

$$h_0(x) = -\frac{d\phi}{dx}(x),$$

$$\phi(x) = \frac{2}{\pi} [\sin(x) C_{int}(x) - \cos(x) S_{int}(x)],$$

$$\frac{d\phi}{dx} = \frac{2}{\pi} [\sin(x) C_{int}(x) + \cos(x) S_{int}(x)].$$

$S_{int}(x)$  and  $C_{int}(x)$  being the integral sinus and cosinus functions,

$$S_{int}(x) = -\int_x^\infty \frac{\sin(\zeta)}{\zeta} d\zeta, \quad C_{int}(x) = -\int_x^\infty \frac{\cos(\zeta)}{\zeta} d\zeta$$

back