The Proper Way to Extract Spectra (merci, M. Bongard)

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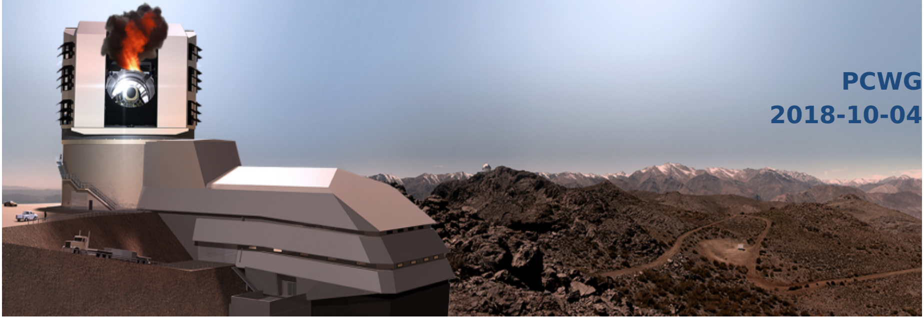

What are we trying to measure?

Wavelength-calibrated, flux-calibrated, 1-D spectra with the maximum possible S/N.Parameters which allow us to reconstruct the atmospheric absorption using `*tran` (and which have names such as "Pressure" and "PWV").

Longer exposure times decrease our *statistical* errors at the cost of *systematic* errors; If we pay attention to statistical efficiency we may be able to have our butter without paying for it.

Estimation problems such as this are usually best handled by forward modelling; we can incorporate priors if desired, and we asymptotically approach the MLE.

The model

In photometry, we measure $I(\mathbf{x})$:

$$I(\mathbf{x}) = B(\mathbf{x}) + t_{exp} D(\mathbf{x}) + \phi \otimes U(\mathbf{x}) + \epsilon$$

where $B(\mathbf{x})$ and $t_{exp} D(\mathbf{x})$ the bias and dark frames, $\phi(\lambda)$ the PSF, $U(\mathbf{x})$ the Universe. and ϵ the noise in the image.

We then proceed to measure the properties of U (for example positions and fluxes of objects) by writing down the likelihood (or posterior probability) for a model of U and maximimising (or sampling from) it.

The model

When we're doing spectroscopy, we measure $I(\mathbf{x})$:

$$I(\mathbf{x}) = B(\mathbf{x}) + t_{exp}D(\mathbf{x}) + \int \phi(\lambda) \otimes U(\mathbf{x},\lambda) \otimes d(\mathbf{x},\lambda)Q(\lambda)\,\frac{d\lambda}{\lambda} + \epsilon$$

where $B(\mathbf{x})$ and $t_{exp}D(\mathbf{x})$ the bias and dark frames, $\phi(\lambda)$ the PSF, $U(\mathbf{x})$ the Universe, and d the dispersion function $d(\mathbf{x},\lambda) = a\delta(\mathbf{x}) + d'(\mathbf{x},\lambda)$.

$$\begin{aligned} I(\mathbf{x}) = B(\mathbf{x}) + t_{exp}D(\mathbf{x}) + a\int \phi(\lambda) \otimes U(\mathbf{x},\lambda)Q(\lambda)\,\frac{d\lambda}{\lambda} + \\ \int \phi(\lambda) \otimes U(\mathbf{x},\lambda) \otimes d'(\mathbf{x},\lambda)Q(\lambda)\,\frac{d\lambda}{\lambda} + \epsilon \\ = (1-a)(B(\mathbf{x}) + t_{exp}D(\mathbf{x})) + aI_0(\mathbf{x}) + \\ \int \phi(\lambda) \otimes U(\mathbf{x},\lambda) \otimes d'(\mathbf{x},\lambda)Q(\lambda)\,\frac{d\lambda}{\lambda} + \epsilon \end{aligned}$$

The model

Assuming that we understand the bias and dark current and can drop them, we have

$$I(\mathbf{x}) = aI_0(\mathbf{x}) + \int \phi(\lambda) \otimes U(\mathbf{x}, \lambda) \otimes d'(\mathbf{x}, \lambda) Q(\lambda) \frac{d\lambda}{\lambda} + \boldsymbol{\epsilon}$$

Let's ignore the 0-order image and introduce the source spectrum, S and atmospheric absorption A

$$I(\mathbf{x}) = \int A(\lambda; \boldsymbol{\theta}) \phi(\lambda) \otimes S(\mathbf{x}, \lambda) \otimes d'(\mathbf{x}, \lambda) Q(\lambda) \frac{d\lambda}{\lambda} + \boldsymbol{\epsilon}$$

We can solve for $\boldsymbol{\theta}$ by ML.

What’s needed to model the data?

- Reliable CCD calibration products:
  - bias and overscan
  - dark
  - crosstalk
  - non-linearity
  - CTE
  - flatfield. This is a little tricky with slitless spectra
- A camera model:
  - position, $\lambda \rightarrow$ pixel
  - instrumental PSF
  - Variation of these with focus, temperature, rotation, . . .
- A CCD model:
  - gains/QE
- A Telescope model:
  - throughput
- The atmospheric PSF
  - as a function of λ ($\alpha \sim \lambda^{-1/5} \cdots \lambda^{-1/4}$?)

Is this possible?

Some of these are probably OK:

- The instrumental calibrations (including flats)
- The camera distortion model
- The camera PSF (after allowing for focus – which probably shows up in the distortion)

Some of these might be OK:

- telescope transmission
- gain/QE

Some of these are scary:

- telescope throughput
- $S(\mathbf{x}, \lambda)$
  - SED of primary source
  - Contamination

"When I was younger, boys of your age used to be nice and innocent."
"Now we are only nice. One must specialise in these days."

Equivalent Widths

"We can't know all that; I'll use equivalent widths instead" *I.e* Build a model for U and estimate it from the data; this is called a "spectrum".

Assume that Q is uniform over some wavelength interval. Measure the flux in some restricted wavelength ranges ("continuum[12]" and "line") Calculate a number from these fluxes.

You can describe this in the ML formalism as estimating the amplitude of three weight functions w_a, w_b, w_c:

$$w_i(\lambda) = \begin{cases} 1 & i_1 < \lambda < i_2 \\ 0 & \text{otherwise} \end{cases}$$

The advantage is that we have an error estimate, we can put a prior on $Q(\lambda)$, and that we can investigate other options (e.g. an equivalent width measured on T not U)

What should we do?

- Direct estimation of T
  - maybe with constrained parameters *e.g.* O_3
- Equivalent widths
- Ignoring certain spectral ranges makes the data equivalent to sets of narrow bands; using notch filters implements this in hardware.

Regressions against airmass enable us to test these approaches. . . and/or separate parameters such as the telescope transmission from the atmosphere.

The final answer will come from our ability to make FGCM irrelevant.