Neutrino physics and flavour mixing
Lecture 3

Stéphane Lavignac (IPhT Saclay)

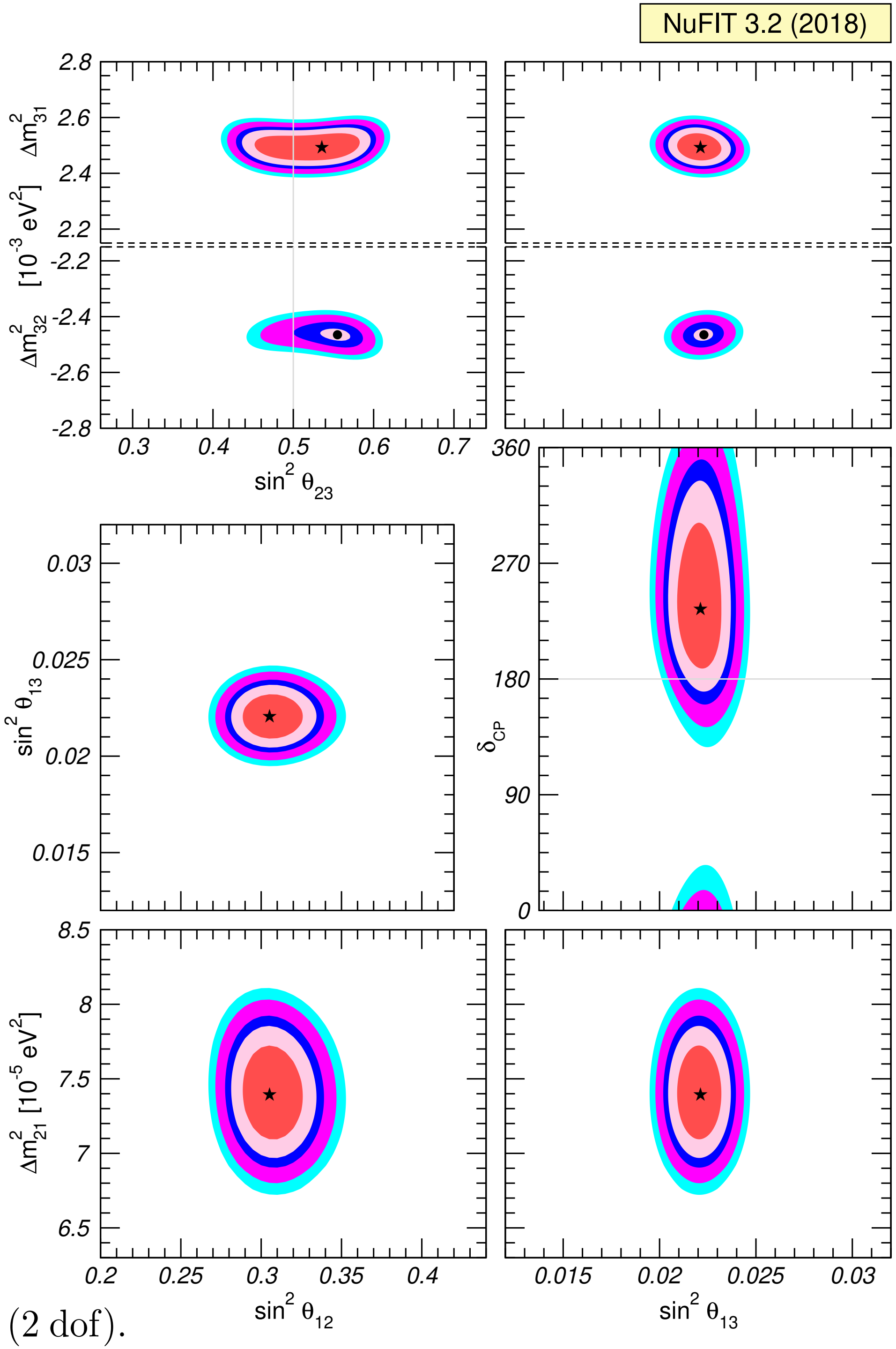
- summary of experimental results
- the absolute neutrino mass scale
- the neutrino nature : neutrinoless double beta decay
- sterile neutrinos

Ecole internationale de Physique subatomique (EIPS)
IPN Lyon, 22-26 October 2018

Summary of experimental results

[I. Esteban et al., NuFit 3.2 (2018), www.nu-fit.org]

All experimental data (leaving aside a few anomalies) is very well described in the 3-flavour framework, and the determination of oscillation parameters is becoming more and more precise



The different contours correspond to 1σ, 90%, 2σ, 99%, 3σ CL (2 dof).

A recent 3-flavour fit (January 2018)

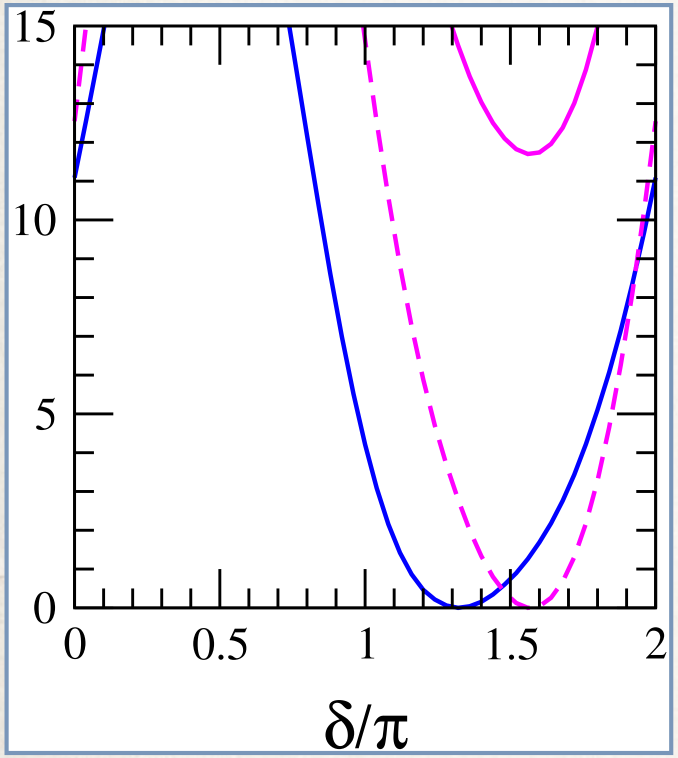
[I. Esteban et al., NuFit 3.2 (2018), www.nu-fit.org]

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 4.14$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.307^{+0.013}_{-0.012}$	$0.272 \to 0.346$	$0.307^{+0.013}_{-0.012}$	$0.272 \to 0.346$
$\theta_{12}/^\circ$	$33.62^{+0.78}_{-0.76}$	$31.42 \to 36.05$	$33.62^{+0.78}_{-0.76}$	$31.43 \to 36.06$
$\sin^2\theta_{23}$	$0.538^{+0.033}_{-0.069}$	$0.418 \to 0.613$	$0.554^{+0.023}_{-0.033}$	$0.435 \to 0.616$
$\theta_{23}/^\circ$	$47.2^{+1.9}_{-3.9}$	$40.3 \to 51.5$	$48.1^{+1.4}_{-1.9}$	$41.3 \to 51.7$
$\sin^2\theta_{13}$	$0.02206^{+0.00075}_{-0.00075}$	$0.01981 \to 0.02436$	$0.02227^{+0.00074}_{-0.00074}$	$0.02006 \to 0.02452$
$\theta_{13}/^\circ$	$8.54^{+0.15}_{-0.15}$	$8.09 \to 8.98$	$8.58^{+0.14}_{-0.14}$	$8.14 \to 9.01$
$\delta_{\mathrm{CP}}/^\circ$	234^{+43}_{-31}	$144 \to 374$	278^{+26}_{-29}	$192 \to 354$
$\dfrac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.40^{+0.21}_{-0.20}$	$6.80 \to 8.02$	$7.40^{+0.21}_{-0.20}$	$6.80 \to 8.02$
$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.494^{+0.033}_{-0.031}$	$+2.399 \to +2.593$	$-2.465^{+0.032}_{-0.031}$	$-2.562 \to -2.369$

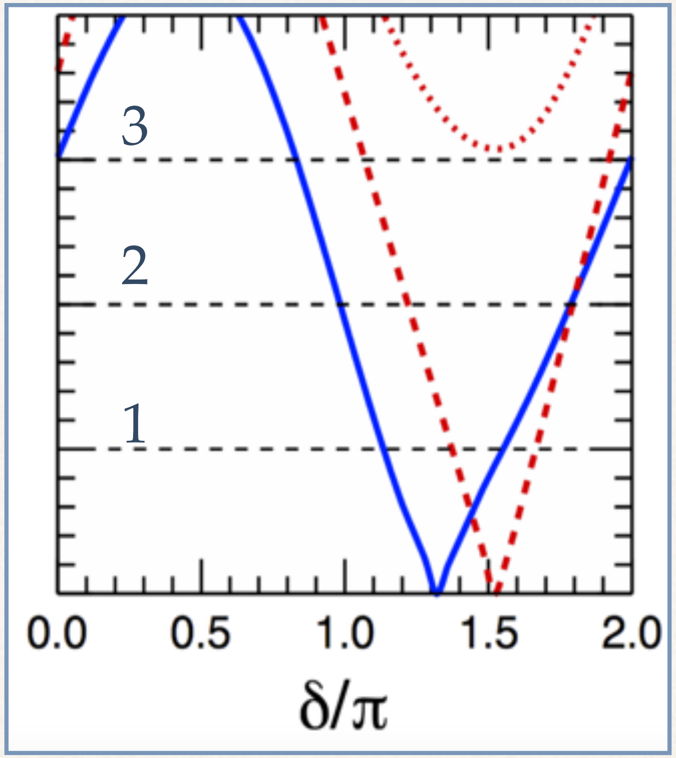
3σ uncertainty around 15% for θ_{12} and Δm^2_{21}, less than 10% for θ_{13} and $\Delta m^2_{3\ell}$

Sensitivity to the mass ordering

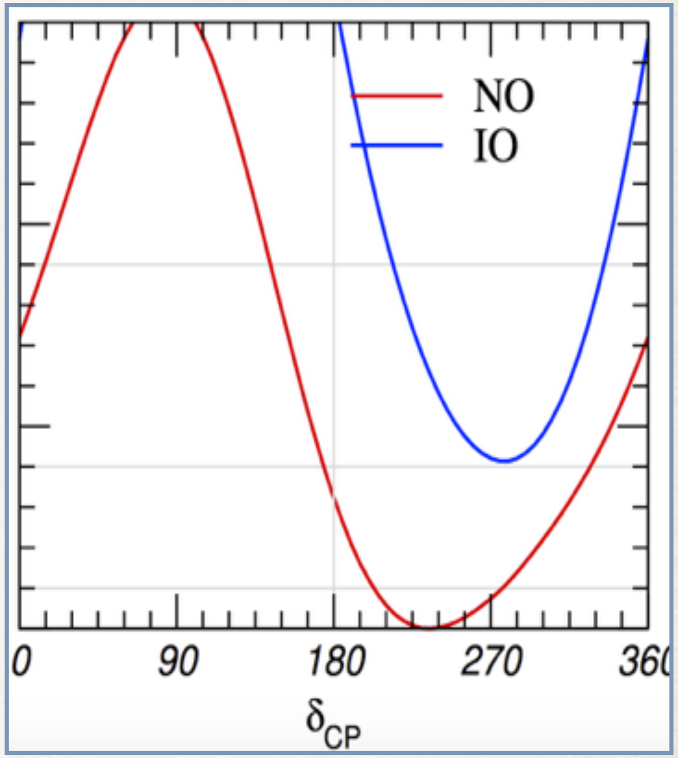
$\Delta\chi^2$, Valencia [1708.01186]



Nσ, Bari [1804.09678]



$\Delta\chi^2$, NuFit v3.2



IO disfavoured at 3.4σ

IO disfavoured at 3.1σ

IO disfavoured at 2σ*

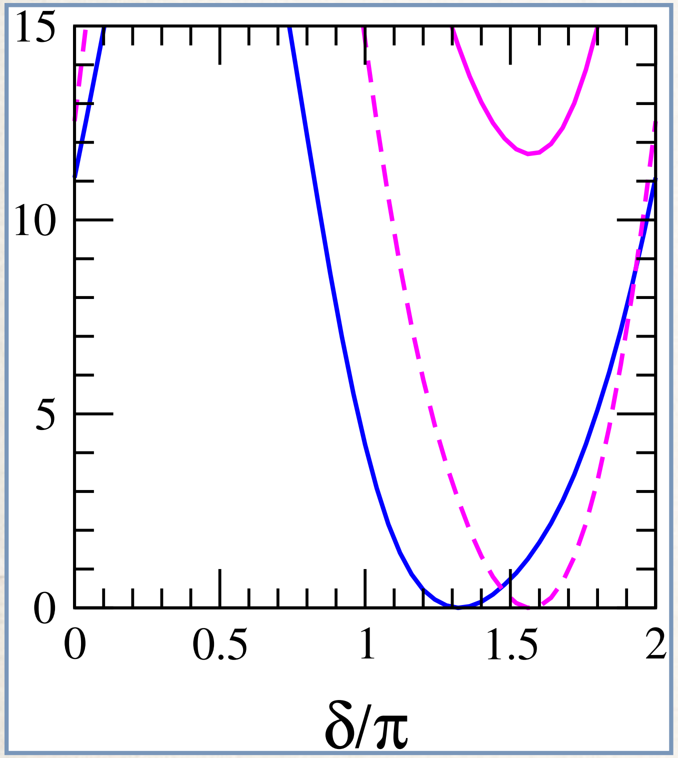
Preference for NO at ~3σ when all data are considered

*SK-atm not included

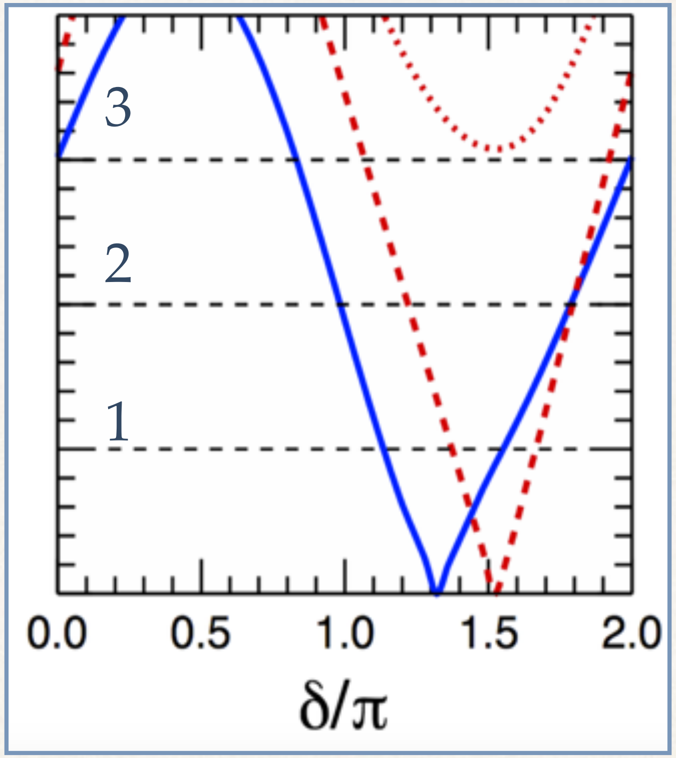
M. Tortola, Neutrino 2018

Measurement of the CP phase

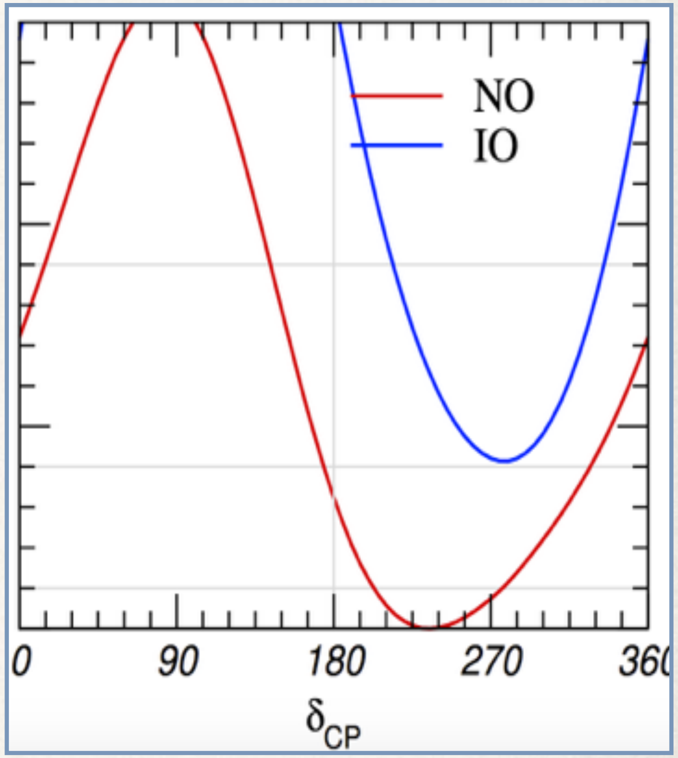
$\Delta\chi^2$, Valencia [1708.01186]



Nσ, Bari [1804.09678]



$\Delta\chi^2$, NuFit v3.2



SK-atm not included

- preference for $\pi < \delta < 2\pi$, with CP conservation allowed at 2σ (3.8σ) for NO (IO)
- preferred value depends on mass ordering: $\delta_{NO}=1.32\,\pi$ $\delta_{IO}=1.56\,\pi$

The absolute neutrino mass scale

Oscillation experiments measure only mass squared differences
→ information on the neutrino mass scale from beta decay or cosmology

Cosmology

Upper bound on sum of neutrino masses from CMB and large structure data
[eV-scale SM neutrinos would be hot dark matter and affect structure formation, leading to fewer small structures than observed ⇒ must be a subdominant DM component]

$$\sum m_\nu < 0.12\ \text{eV} \quad (95\,\%,\ \textit{Planck}\ \text{TT,TE,EE+lowE+lensing+BAO}).$$

[Planck 2018]

Kinematic measurements (beta decay)

The non-vanishing neutrino mass leads to a distortion of the Ee spectrum close to the endpoint

Present bound (Troitsk [Mainz]) : $m_{\nu_e} < 2.05\,\text{eV}\ [2.3\,\text{eV}] \quad (95\%\ \text{C.L.})$

Tritium beta decay

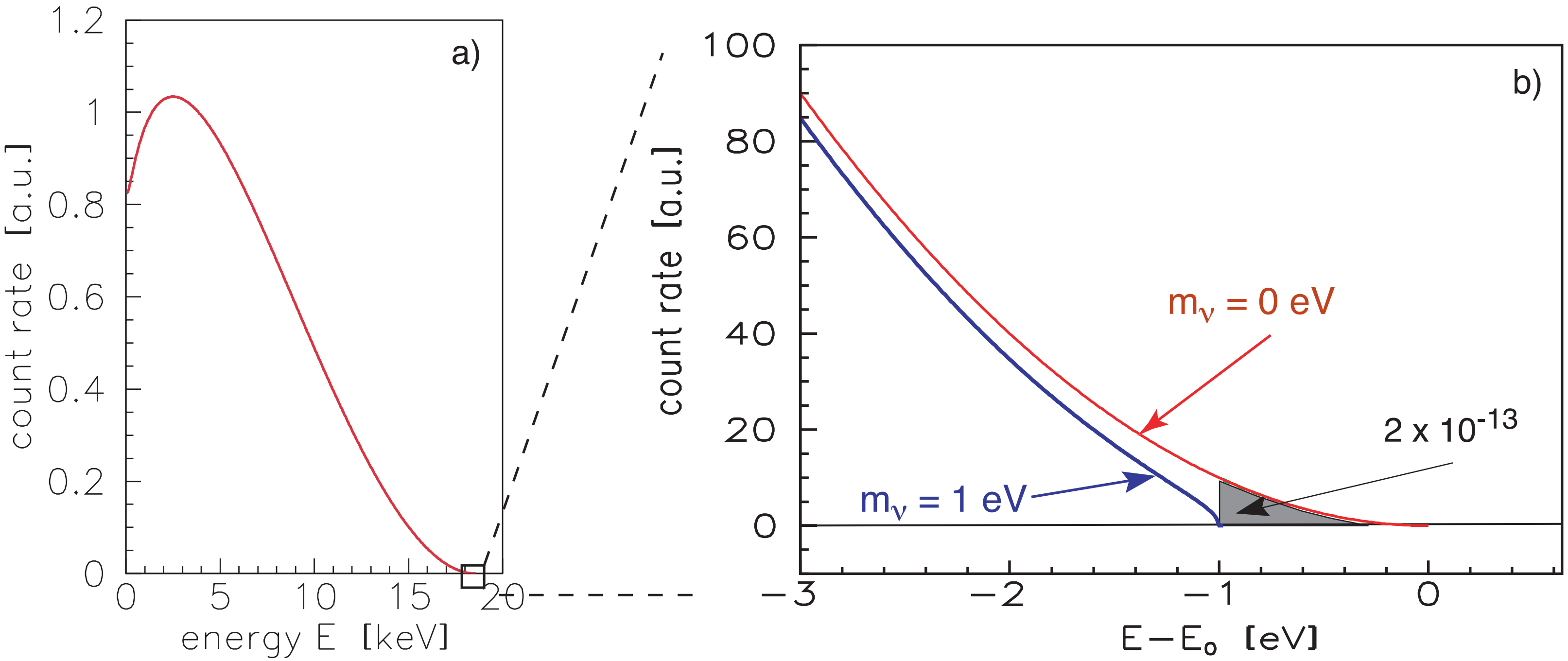
$${}^3H \rightarrow {}^3H_e + e^- + \bar{\nu}_e \qquad\qquad E_0 = m_{{}^3H} - m_{{}^3H_e}$$

The electron energy spectrum is given by:

$$\frac{dN}{dE_e} = R(E_e)\sqrt{(E_0 - E_e)^2 - m_\nu^2} \qquad\qquad E_e = E_0 - E_\nu$$

Effect of the non-vanishing neutrino mass: $E_e^{max} = E_0 \rightarrow E_0 - m_\nu$

$\Rightarrow$ distorsion of the Ee spectrum close to the endpoint



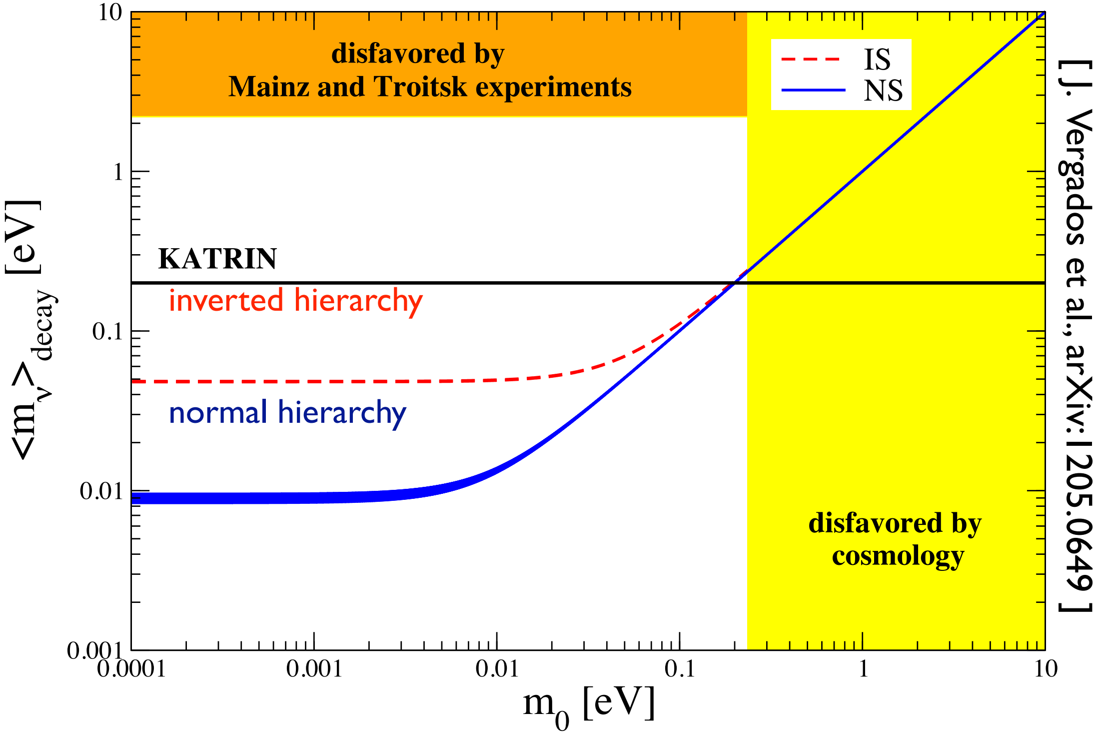
[Katrin Collaboration, hep-ex/0109033]

Present bound (Troitsk [Mainz]) : $m_{\nu_e} < 2.05\,\mathrm{eV}\,[2.3\,\mathrm{eV}] \quad (95\%\,\mathrm{C.L.})$

KATRIN will reach a sensitivity of about 0.2 eV (90% CL) in 5 years
(5σ discovery potential 0.35 eV)

In pratice, there is no electron neutrino mass, but 3 (or more) strongly mixed mass eigenstates. However the energy resolution does not allow to resolve them, and what is measured is the effective mass

$$m_\beta^2 \equiv \sum_i m_i^2 |U_{ei}|^2$$



(lightest neutrino mass)

[J. Vergados et al., arXiv:1205.0649]

KATRIN will test only
the degenerate case

data taking started in May 2018

Future experiments like Project 8 aim at the 50 meV level

In pratice, there is no electron neutrino mass, but 3 (or more) strongly mixed mass eigenstates, and

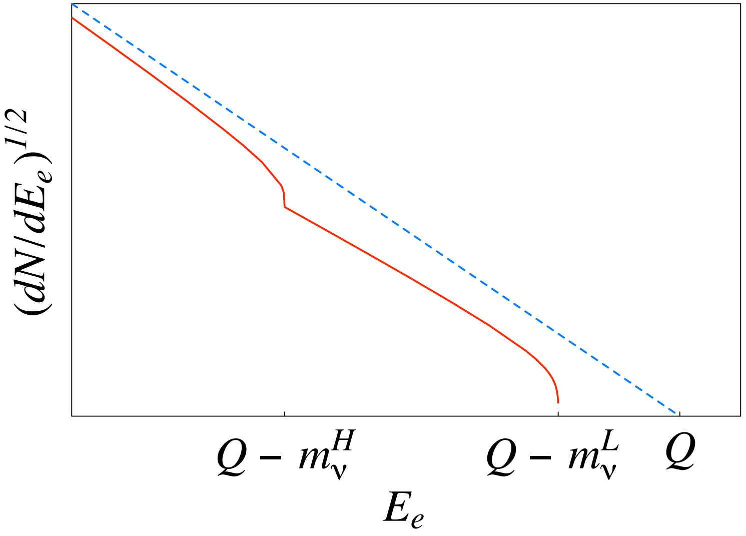
$$\frac{dN}{dE_e} = R(E_e) \sum_i |U_{ei}|^2 \sqrt{(E_0 - E_e)^2 - m_i^2}\, \Theta(E_0 - E_e - m_i)$$

If all m_i's are smaller than the energy resolution, this can be rewritten as:

$$\frac{dN}{dE_e} = R(E_e) \sqrt{(E_0 - E_e)^2 - m_\beta^2} \qquad m_\beta^2 \equiv \sum_i m_i^2 |U_{ei}|^2$$

If there is an eV-scale sterile neutrino (comparable to the energy resolution of KATRIN), its mass may be resolved (but difficult measurement):

$$\frac{1}{R(E_e)} \frac{dN}{dE_e} = (1 - |U_{e4}|^2) \sqrt{(E_0 - E_e)^2 - m_\beta^2}$$
$$+ |U_{e4}|^2 \sqrt{(E_0 - E_e)^2 - m_4^2}\, \Theta(E_0 - E_e - m_4)$$

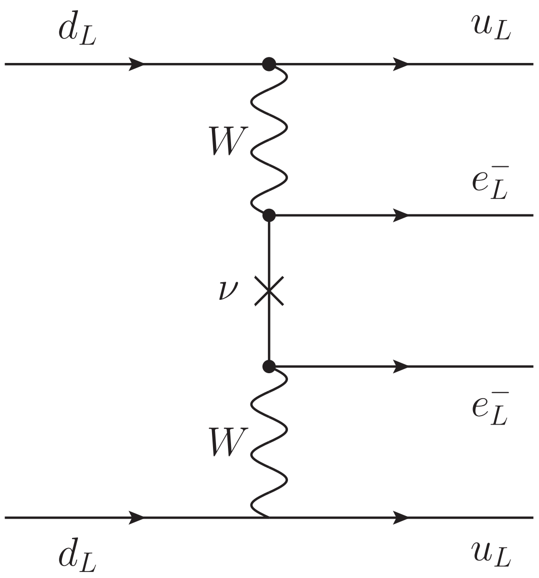


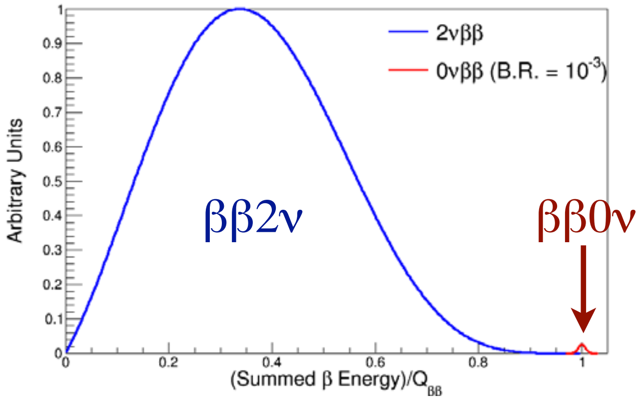
The neutrino nature: neutrinoless double beta decay

$$(A,Z) \;\rightarrow\; (A,Z+2)+e^{-}+e^{-}$$

violates lepton number by 2 units

⇒ possible only for Majorana neutrinos





$$Q_{\beta\beta} \equiv M_i - M_f - 2m_e = T_{e_1} + T_{e_2}$$

Half-life: $$\left[T^{0\nu}_{1/2}\right]^{-1} = \Gamma_{0\nu} = G_{0\nu}(Q_{\beta\beta},Z)\;|M_{0\nu}|^2\;|m_{\beta\beta}|^2$$

integrated phase-space factor

nuclear matrix element (NME)
(large theoretical uncertainty)

Sensitive to the effective mass parameter:

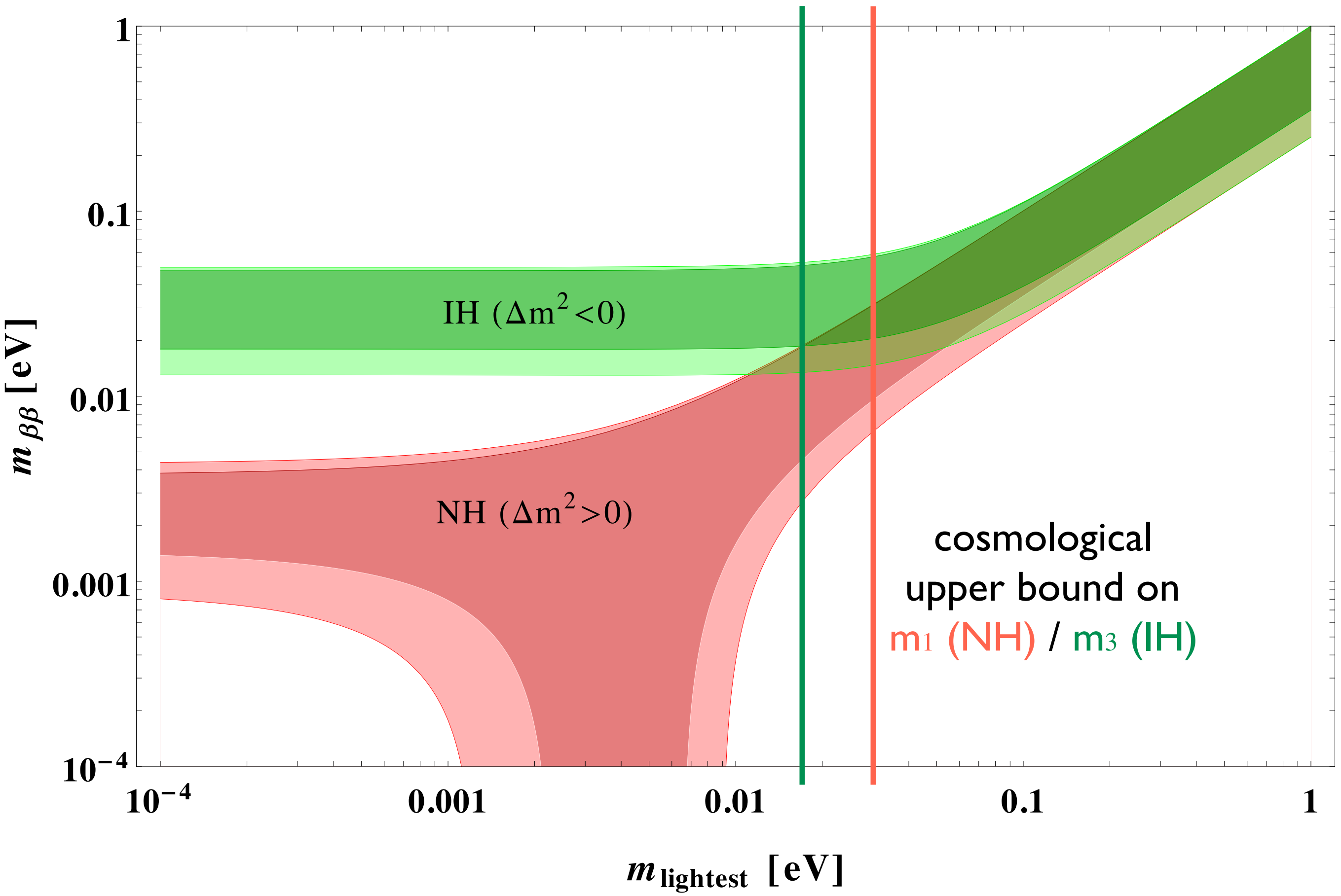
$$m_{\beta\beta} \equiv \sum_i m_i U^2_{ei} = m_1 c^2_{13} c^2_{12} e^{2i\alpha_1} + m_2 c^2_{13} s^2_{12} e^{2i\alpha_2} + m_3 s^2_{13}$$

possible cancellations in the sum (Majorana phases α_1, α_2 in U)

[Dell'Oro et al., arXiv:1404.2616]

IH

NH

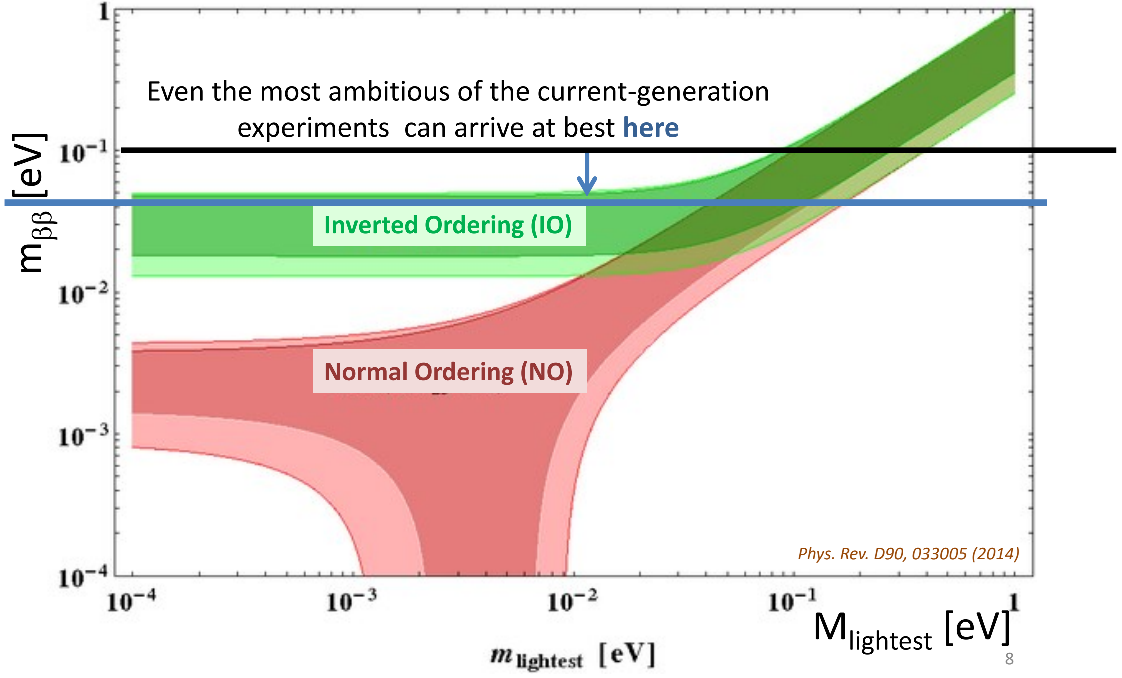


dark shaded areas
= best fit values of
oscillation parameters
(only α_1, α_2 vary)

light shaded areas
= 3σ regions due
to uncertainties on
oscillation parameters
(+ dependence on α_i)

- need to reach 10 meV to exclude IH (lower bound on $m_{\beta\beta}$)
- need to reach few meV to test NH (if no mass degeneracy)
- if unlucky (m_1 ~ 1-10 meV), may not observe $\beta\beta0\nu$ even if neutrinos are Majorana (cancellation in $m_{\beta\beta}$ due to α_1, α_2)

A. Giuliani, Neutrino 2018



Even the most ambitious of the current-generation experiments can arrive at best **here**

currently here, around 100 meV
(experimental upper bounds
depend on NME calculations
⇒ 2 - 4 uncertainty factor)

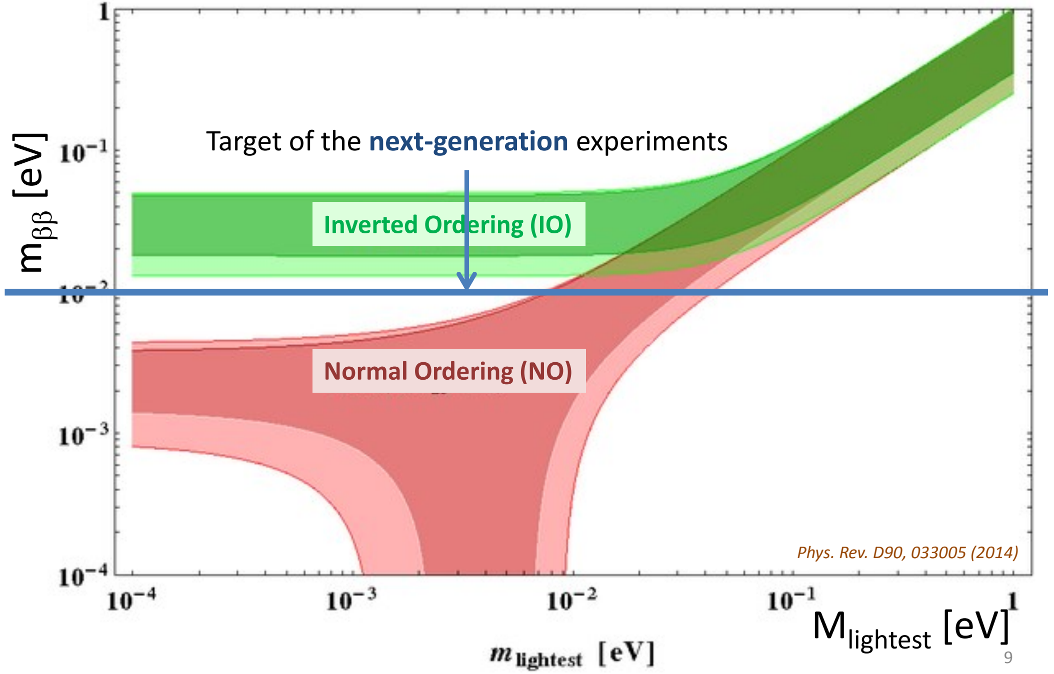
Current best limit (90% C.L.) :
KamLAND-Zen (2016)
^{136}Xe-loaded liquid scintillator

$$T_{1/2}^{0\nu} > 1.07 \times 10^{26}\,\text{yr}$$

$$m_{\beta\beta} < (61-165)\ \text{meV}$$

(uncertainty from NMEs)

A. Giuliani, Neutrino 2018



Target of the **next-generation** experiments

Sterile neutrinos

Only 3 light neutrinos ($m_\nu < M_Z/2$) couple to the Z boson :

$$N_\nu \equiv \Gamma_Z^{\text{invisible}} / \Gamma(Z \to \nu\bar{\nu})_{\text{SM}} = 2.9840 \pm 0.0082 \qquad \text{[LEP]}$$

Still additional light neutrino species without electroweak interactions may exist. These "sterile neutrinos" would interact only through their mixing with the "active neutrinos" ν_e, ν_μ, ν_τ and affect their oscillations.

(eV-scale) sterile neutrinos have been invoked to explain experimental anomalies that cannot be accounted for within 3-flavour oscillations

Sterile neutrinos are present in models where the SM neutrino masses arise from their coupling to RH neutrinos with a Majorana mass. In the seesaw limit, the sterile neutrinos are very heavy and mix very weakly with the SM neutrinos. But in general, their masses may lie anywhere between the eV and the Grand Unification scale. Generic prediction : the lighter the sterile neutrinos, the stronger their mixing with active neutrinos

$$m_\nu \sim \frac{m_D^2}{M}\,, \quad m_s \sim M\,, \quad \sin\theta \sim \frac{m_D}{M} \quad \Rightarrow \quad \sin\theta \sim \sqrt{\frac{m_\nu}{m_s}}$$

Active-sterile neutrino mixing

Standard case (3 flavours)

$$\nu_\alpha = \sum_{i=1}^{3} U_{\alpha i}\,\nu_i$$

3x3 lepton mixing matrix (PMNS)

3 flavour eigenstates $(\alpha = e, \mu, \tau)$

3 mass eigenstates with masses mi $(i = 1, 2, 3)$

Add a sterile neutrino :

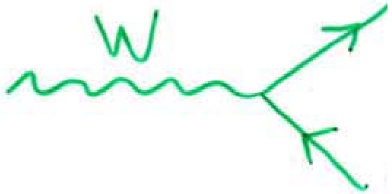
$$\nu_\alpha = \sum_{i=1}^{4} U_{\alpha i}\,\nu_i \qquad (\alpha = e, \mu, \tau, s)$$

ν_s flavour eigenstate

ν_4 mass eigenstate (m_4)

lepton mixing matrix U = 4x4 unitary matrix

Only ν_e, ν_μ, ν_τ couple to electroweak gauge boson, but all four mass eigenstates are produced in a weak process like beta decay



e^-

$\nu_e = \sum_{i=1}^{4} U_{ei}\,\nu_i$

(if kinematically accessible, as assumed in the following)

New oscillation parameters : $\Delta m^2_{43}\,,\ \ \Delta m^2_{42}\,,\ \ \Delta m^2_{41}$

$\theta_{14}\,,\ \ \theta_{24}\,,\ \ \theta_{34} \qquad (\text{or } U_{e4},\, U_{\mu 4},\, U_{\tau 4})$

<u>Consider short baseline oscillations with</u> $\Delta m^2_{41} \gg \Delta m^2_{31}$

$$\frac{\Delta m^2_{41} L}{4E} \lesssim 1 \quad \Longrightarrow \quad \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \gg \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right),\ \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

→ approximate $\Delta m^2_{31} = \Delta m^2_{21} = 0\,, \quad \Delta m^2_{43} = \Delta m^2_{42} = \Delta m^2_{41} \equiv \Delta m^2_{\rm SBL}$

$$\begin{aligned} P_{\nu_\alpha \to \nu_\alpha} &\simeq 1 - 4\left(|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2 + |U_{\alpha 3}|^2\right)|U_{\alpha 4}|^2 \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \\ &\equiv 1 - \sin^2 2\theta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \end{aligned}$$

where $\sin^2 2\theta_{\alpha\alpha} \equiv 4\,(1 - |U_{\alpha 4}|^2)|U_{\alpha 4}|^2$

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta} &\simeq -4\,\mathrm{Re}\left[\left(U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} + U_{\alpha 3} U^*_{\beta 3}\right) U^*_{\alpha 4} U_{\beta 4}\right] \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \\ &\equiv \sin^2 2\theta_{\alpha\beta}\, \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \end{aligned}$$

where $\sin^2 2\theta_{\alpha\beta} \equiv 4\,|U_{\alpha 4} U_{\beta 4}|^2$

$$|U_{e4}| = \sin\theta_{14}\,, \quad |U_{\mu 4}| = \cos\theta_{14}\sin\theta_{24}\,, \quad |U_{\tau 4}| = \cos\theta_{14}\cos\theta_{24}\sin\theta_{34}$$

Experimental status of oscillation anomalies

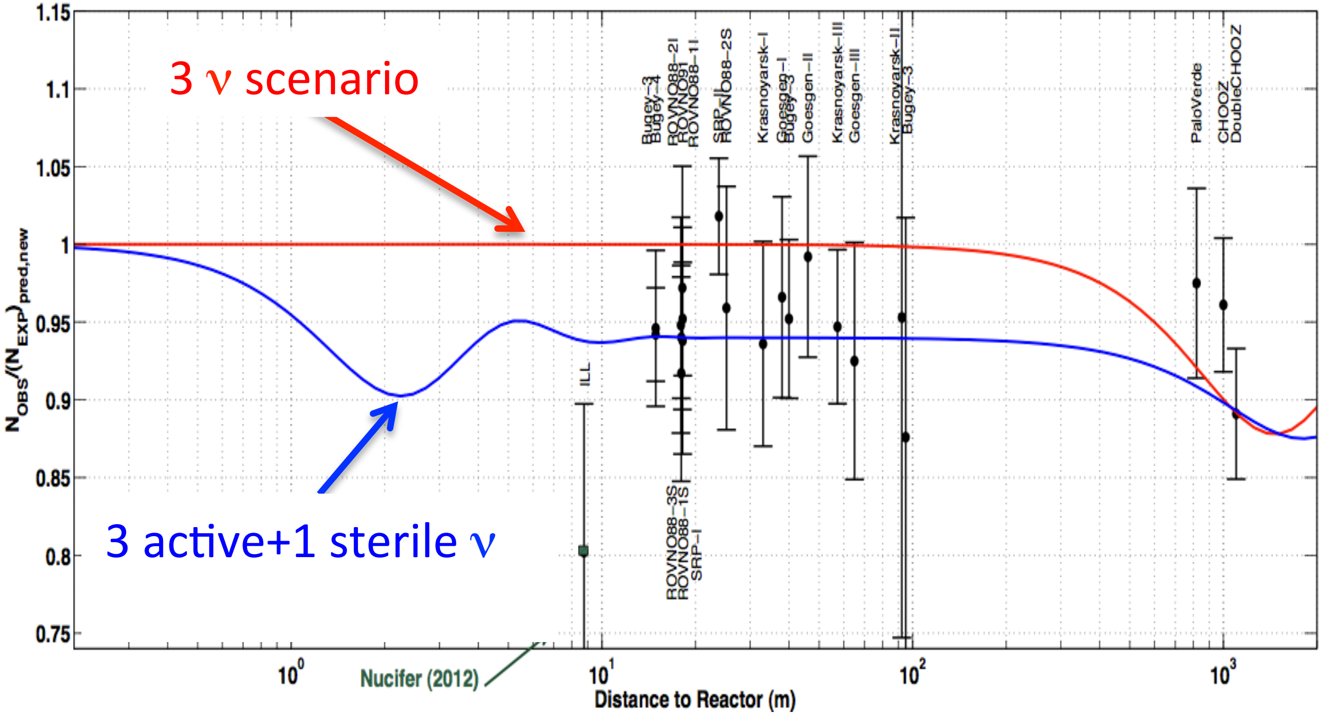
Short-baseline $\nu_e(\bar{\nu}_e)$ disappearance experiments

The reactor antineutrino anomaly (2011)

New computation of the reactor $\bar{\nu}_e$ spectra [Th. Mueller et al., 2011 - P. Huber, 2011]
$\Rightarrow$ increase of the flux by about 3.5%
$\Rightarrow$ deficit of antineutrinos in SBL reactor experiments

Mean observed to predicted rate 0.943 ± 0.023 [G. Mention et al., arXiv:1101.2755]

can be explained by
$\bar{\nu}_e \to \bar{\nu}_s$ oscillations
with $\Delta m^2_{41} \sim 1\,\mathrm{eV}^2$
and $\sin^2 2\theta_{ee} \sim 0.1$

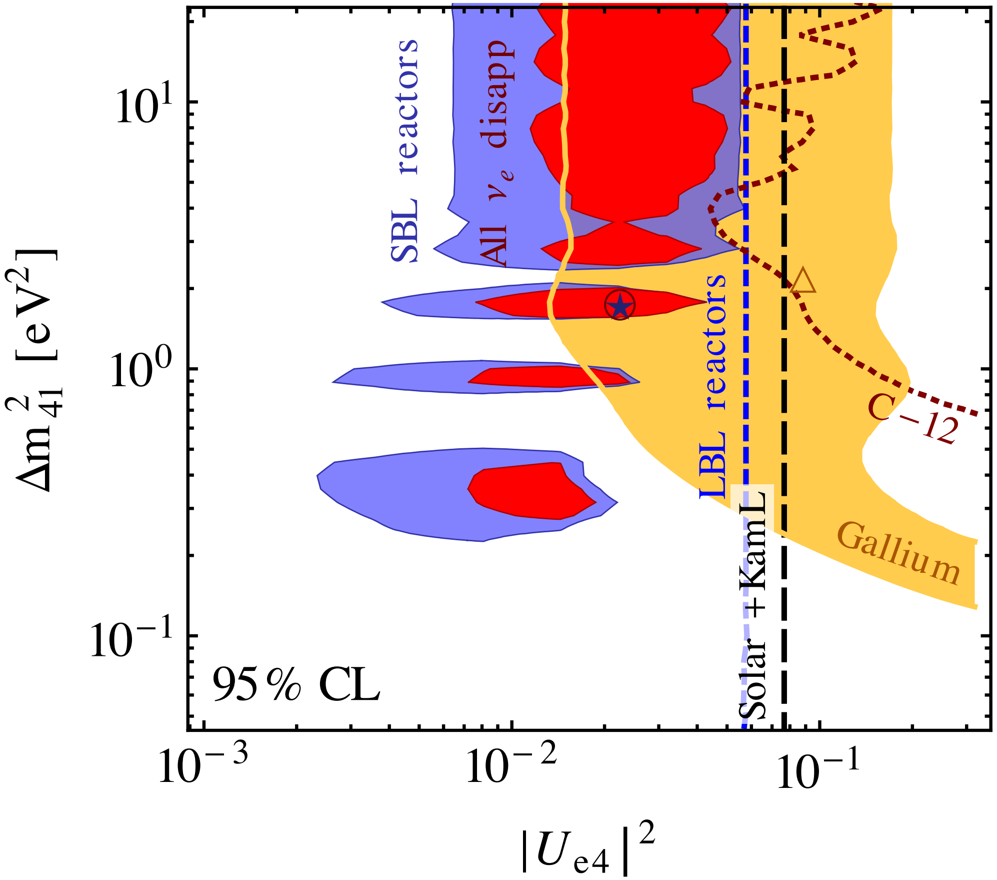


[D. Lhuillier, talk at IPA 2016]

The Gallium anomaly

Calibration of the Gallex and SAGE experiments with radioactive sources
⇒ observed 3σ deficit of ν_e with respect to predictions (R = 0.84 ± 0.05)

The reactor and gallium anomalies suggest oscillations into a sterile neutrino with $\Delta m^2_{41} \gtrsim 1\,\mathrm{eV}^2$ and $\sin^2 2\theta_{ee} \sim 0.1$ $[\,\sin^2 2\theta_{ee} \equiv 4(1-|U_{e4}|^2)|U_{e4}|^2\,]$



[Kopp et al., arXiv:1303.3011]

(Other) unexplained features of the reactor antineutrino flux

- excess ("bump") around 5 MeV in the reactor antineutrino spectra (observed by RENO, Day Bay, Double Chooz)
- dependence of the antineutrino flux on the fuel composition (proportion of $^{235}\mathrm{U}$, $^{238}\mathrm{U}$, $^{239}\mathrm{Pu}$, $^{241}\mathrm{Pu}$) does not agree with predictions (3.1 σ) : deficit wrt calculations for $^{235}\mathrm{U}$, not for $^{239}\mathrm{Pu}$ (observed by Day Bay)

These features (which cannot be explained by oscillations into sterile neutrinos) cast some doubt on the flux calculations

→ important to test the reactor anomaly independently of flux predictions (both normalization and shape)

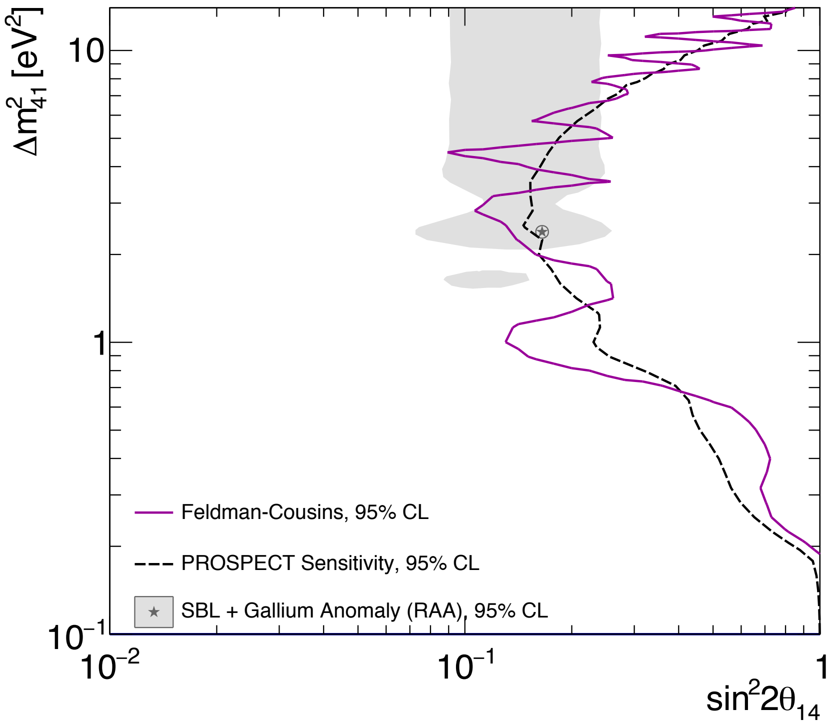
Ongoing short-baseline reactor neutrino experiments :

NEOS (Korea, L = 23.7 m, comparison with Daya Bay spectrum - finished)
DANSS (Russia, movable detector, L = 10.7, 11.7 and 12.7 m)
Neutrino-4 (Russia, movable detector, L = 6-12 m)
STEREO (France, segmented detector with 6 target cells at distances 9-11 m)
PROSPECT (US, segmented detector, L = 7-9 m)
SoLid (Belgium, long segmented detector, L = 6-9 m)

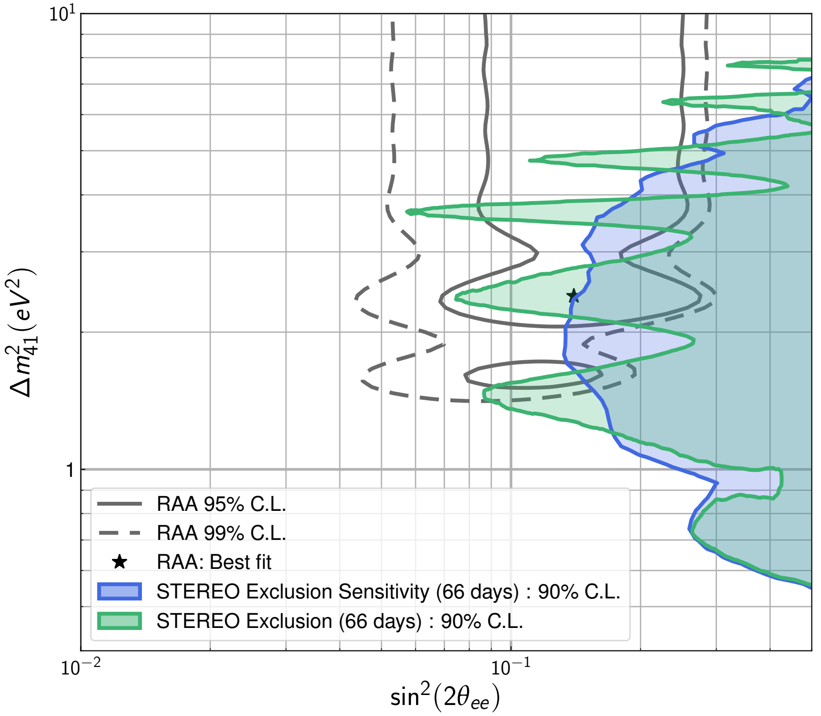
Searches for short-baseline $\bar{\nu}_e$ disappearance : first results

- NEOS and DANSS exclude a significant portion of the reactor anomaly parameter space around $1\,\mathrm{eV}^2$ (namely, $\sin^2 2\theta_{ee} \gtrsim 0.1$). More optimistic claims but systematic errors underestimated (NEOS) / not included (DANSS)
- STEREO, PROSPECT exclude the reactor anomaly best fit point at 2.2 σ

[PROSPECT Collaboration, arXiv:1806.02784]



[STEREO Collaboration, arXiv:1806.02096]



These experiments (+ SoLid) should fully test the $\Delta m^2_{41} \sim 1\,\mathrm{eV}^2$ region of the reactor anomaly parameter space in the near future

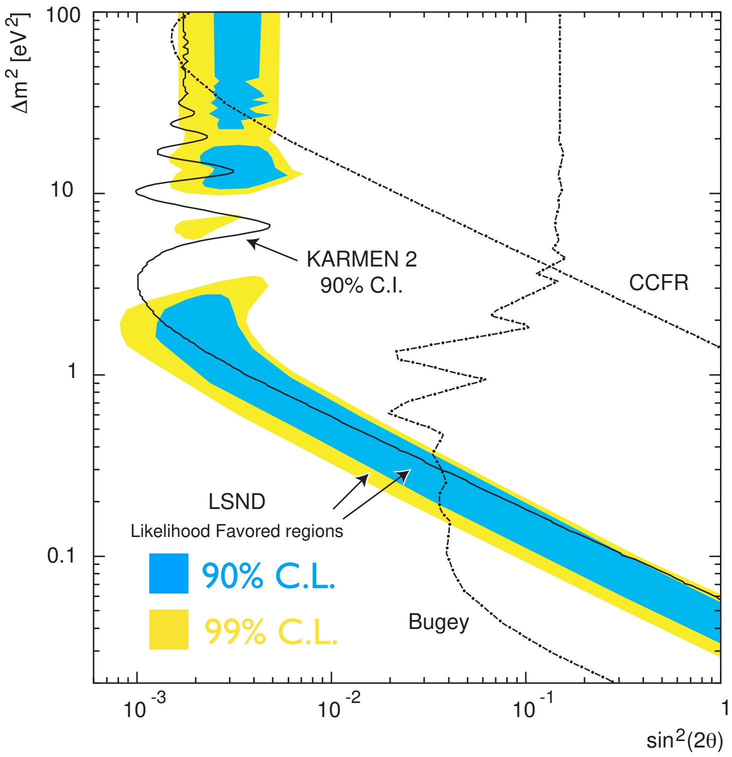
Short-baseline appearance experiments [$\nu_e(\bar{\nu}_e)$ appearance in a $\nu_\mu(\bar{\nu}_\mu)$ beam]

LSND (1993-1998) [$\bar{\nu}_\mu$ beam, $L \approx 35\,\mathrm{m}$]

Excess of $\bar{\nu}_e$ events over background at 3.8 σ interpreted by LSND as $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations

Not observed by KARMEN

[LSND allowed region in the $(\sin^2 2\theta, \Delta m^2)$ plane (2-neutrino fit) - hep-ex/0203021]



$\sin^2 2\theta_{\mu e} \equiv 4|U_{e4}U_{\mu 4}|^2$

MiniBooNE (2002-2017) [ν_μ and $\bar{\nu}_\mu$, $L = 541\,\mathrm{m}$]

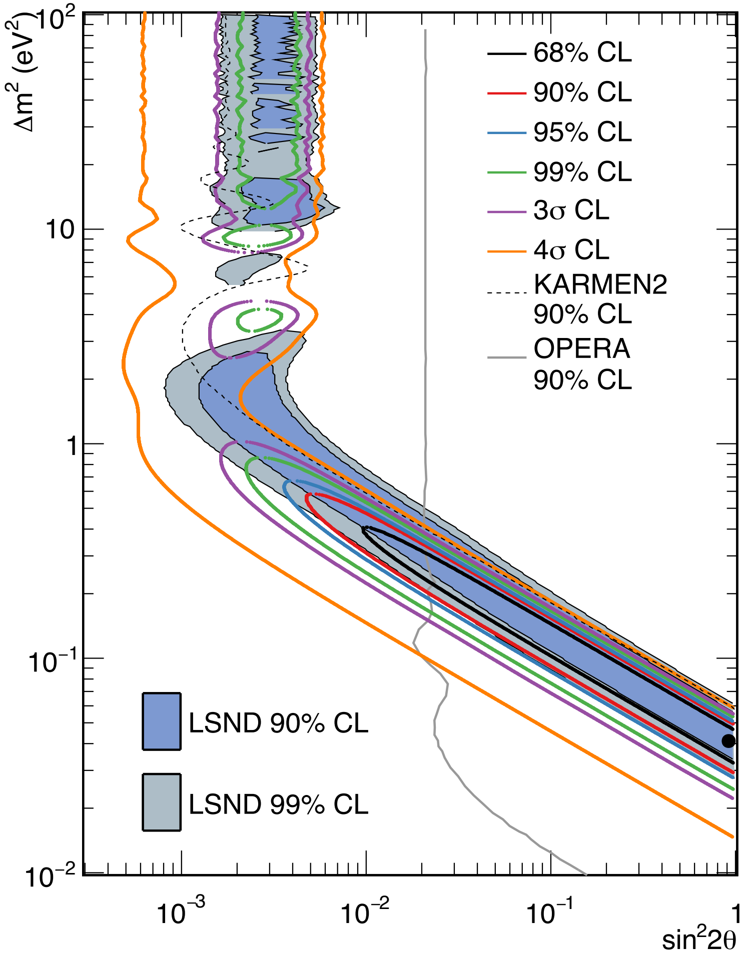
Designed to test the LSND anomaly with a different L but a similar L/E

2002-2012 : inconclusive/contradictory results

Full 2002-2017 data : excess of $\nu_e(\bar{\nu}_e)$ CC events both in the ν and $\bar{\nu}$ modes (4.8 σ in total)

→ MiniBooNE collaboration : *either* $\nu_\mu \to \nu_e$ / $\bar{\nu}_\mu \to \bar{\nu}_e$ *oscillations or new anomalous background processes*

2002-2017 MiniBooNE results



[MiniBooNE Collaboration, arXiv:1805.12028]

FIG. 5: MiniBooNE allowed regions for a combined neutrino mode (12.84×10^{20} POT) and antineutrino mode (11.27×10^{20} POT) data sets for events with $200 < E_\nu^{QE} < 1250$ MeV within a two-neutrino oscillation model. The shaded areas show the 90% and 99% C.L. LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ allowed regions. The black circle shows the MiniBooNE best fit point. Also shown are 90% C.L. limits from the KARMEN [34] and OPERA [35] experiments.

MiniBooNE + LSND excesses : 6.1 σ significance

Oscillation interpretation requires a 4th massive neutrino in the eV range

$$\Delta m^2_{41} \gtrsim 0.1\,\mathrm{eV}^2, \quad \sin^2 2\theta_{\mu e} \approx (10^{-3} - 10^{-2})$$

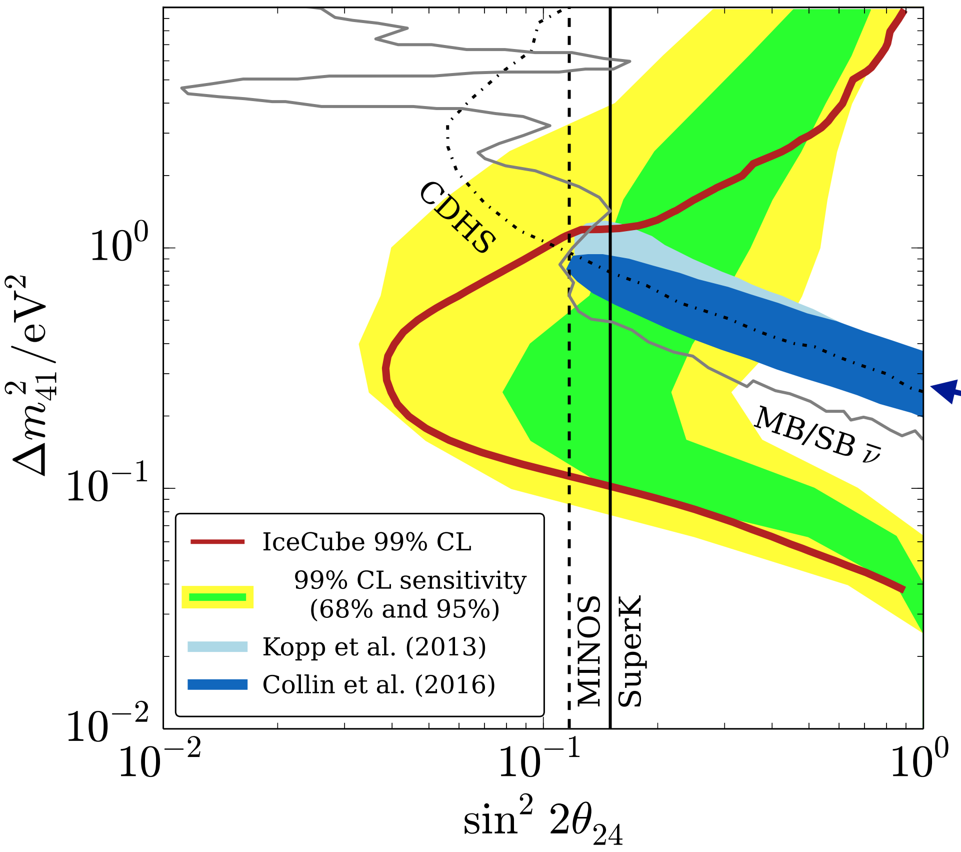
However, this interpretation is essentially excluded by $\nu_\mu(\bar{\nu}_\mu)$ disappearance data :

- MINOS/MINOS+ (long-baseline oscillation experiment)
- IceCube (neutrino telescope located under the Antarctic ice)

$\nu_\mu(\bar{\nu}_\mu)$ disappearance experiments

IceCube (2016) : a sterile neutrino in the eV range would affect the survival probability of atmospheric ν_μ and $\bar{\nu}_\mu$ passing through the Earth (MSW resonance). Not observed by IceCube $\Rightarrow$ limits on Δm^2_{41} and $\sin^2 2\theta_{\mu\mu}$

[IceCube Collaboration, arXiv:1605.01990]



excludes most of the LSND and MiniBooNE allowed region

99% C.L. allowed region by $\nu_e(\bar{\nu}_e)$ appearance experiments from a 2013 [2016] fit (2016-2017 MiniBooNE data not included)

assumes $|U_{e4}|^2 = 0.023\ [0.027]$ (best fit value from Kopp et al. [Collin et al.])

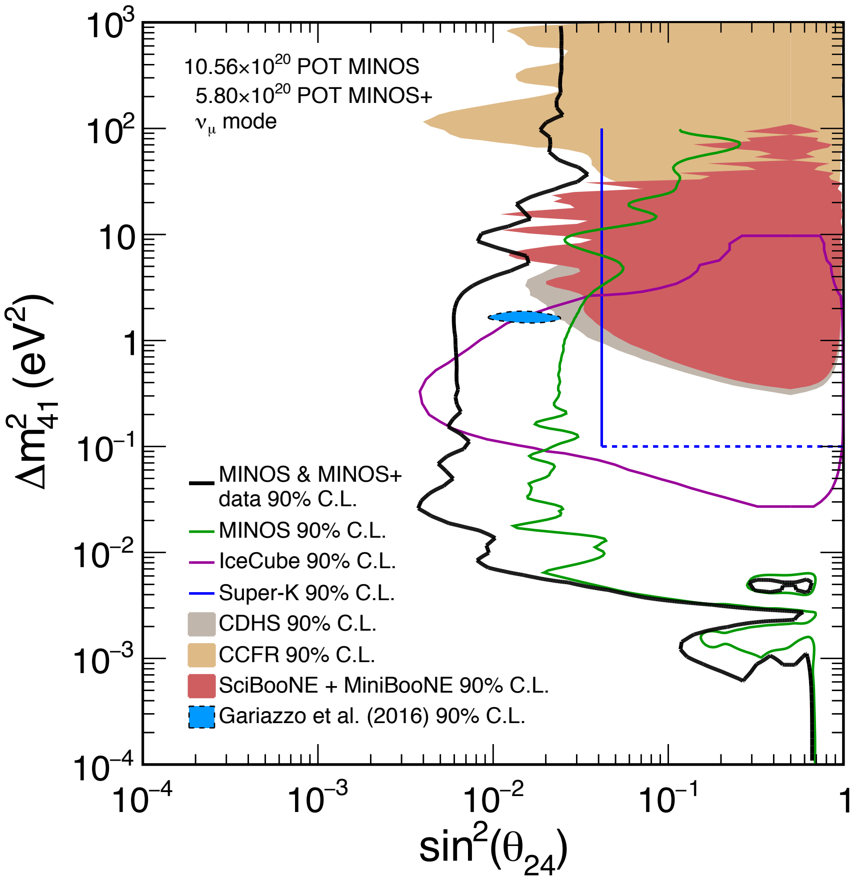
the IceCube contour (red) assumes
$U_{e4} = U_{\tau 4} = 0 \quad (\theta_{14} = \theta_{34} = 0)$

$\sin^2 2\theta_{\mu\mu} \equiv 4(1 - |U_{\mu 4}|^2)|U_{\mu 4}|^2 = \sin^2 2\theta_{24}$
in the limit $|U_{e4}| = \sin\theta_{14} = 0$

MINOS/MINOS+ (2017)

Long-baseline oscillation experiment (ν_μ beam, L_{near} = 1.04 km, L_{far}= 735 km)

Neutral current data are sensitive to the total flux of active neutrinos, hence to $\nu_\mu \to \nu_s$ oscillations $\Rightarrow$ limits on Δm^2_{41} and $\sin^2 2\theta_{24}$



[MINOS Collaboration, arXiv:1710.06488]

black curve : 90% C.L. excluded region from a fit to charged current and neutral current data from MINOS and MINOS+

blue region : pre-IceCube fit to all oscillation anomalies (except low-energy MiniBooNE data)

Further restricts the parameter space with respect to IceCube and worsens the conflict with the oscillation interpretation of LSND and MiniBooNE data

the MINOS/MINOS+ contour assumes

$|U_{e4}| = \sin\theta_{14} = 0$ (weak dependence on θ_{14})

Origin of the conflict between appearance (LSND + MiniBooNE) and disappearance experiments (reactors, accelerators, IceCube…)

Reactors :

$$P_{\bar{\nu}_e \to \bar{\nu}_e} \simeq 1 - \sin^2 2\theta_{ee} \sin^2 \left(\frac{\Delta m^2_{41} L}{4E} \right)$$

require relatively small $\sin^2 2\theta_{ee} \equiv 4\,(1 - |U_{e4}|^2)|U_{e4}|^2 \simeq 4\,|U_{e4}|^2$

($|U_{e4}|^2 \approx 1$ excluded by SNO)

MINOS, IceCube… : $\nu_\mu (\bar{\nu}_\mu)$ disappearance not observed

require relatively small $\sin^2 2\theta_{\mu\mu} \equiv 4\,(1 - |U_{\mu 4}|^2)|U_{\mu 4}|^2 \simeq 4\,|U_{\mu 4}|^2$

($|U_{\mu 4}|^2 \approx 1$ excluded by SK and LBL experiments)

Appearance experiments (LSND + MiniBooNE) :

$$P_{\bar{\nu}_\mu \to \bar{\nu}_e} \simeq \sin^2 2\theta_{\mu e} \sin^2 \left(\frac{\Delta m^2_{41} L}{4E} \right)$$

require relatively large $\sin^2 2\theta_{\mu e} \equiv 4\,|U_{e4} U_{\mu 4}|^2 \simeq \frac{1}{4} \sin^2 2\theta_{ee} \sin^2 2\theta_{\mu\mu}$

Quantifying the tension between appearance and disappearance data

(M. Dentler et al., arXiV:1803.10661)

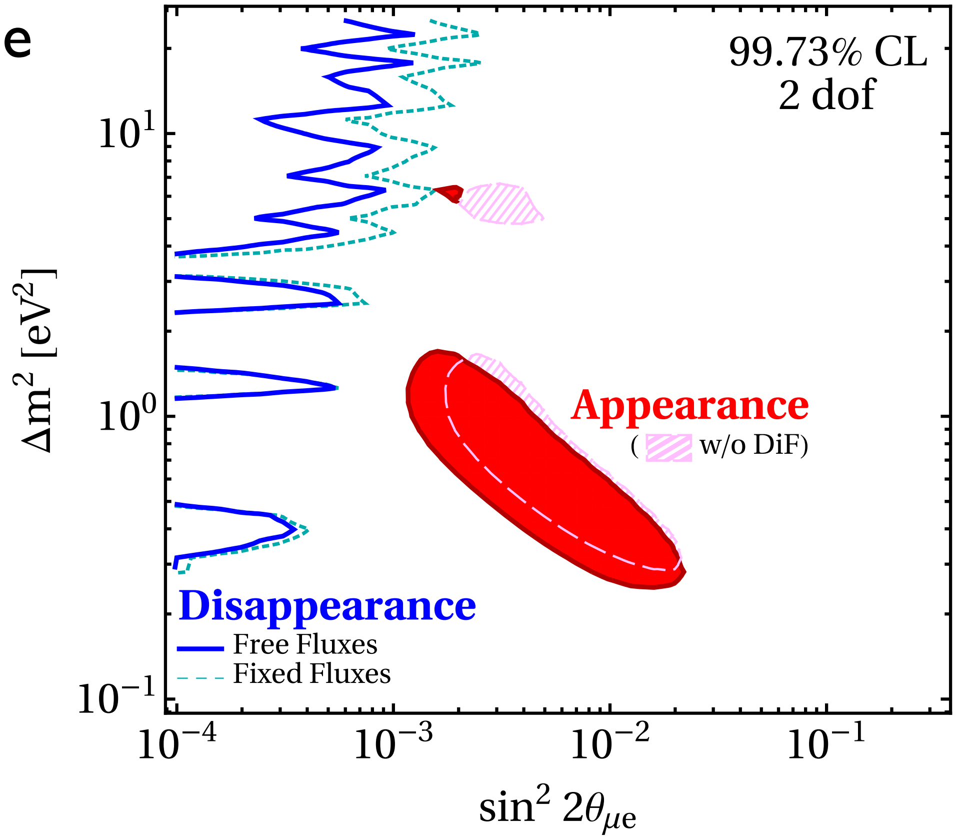
$\left(\sin^2 2\theta_{\mu e} \equiv 4|U_{e4}U_{\mu 4}|^2,\ \Delta m^2_{41}\right)$ plane

red region is allowed at 3σ by appearance data

[pink hatched: without LSND DiF]

blue curve defines 3σ excluded region by disappearance data

[dashed = fixed reactor fluxes]



(M. Dentler et al., arXiV:1803.10661)

→ sterile neutrino interpretation of LSND and MiniBooNE data excluded at the 4.7 σ level

Cosmological constraints on sterile neutrinos

Cosmological measurements constrain the number of stable, relativistic degrees of freedom (other than photons) in the early Universe :

$$N_{\rm eff} = 2.99^{+0.34}_{-0.33} \quad (95\,\%, \ \text{TT,TE,EE+lowE+lensing +BAO}).$$ [Planck 2018]

A given species contributes to Neff proportionally to its contribution to the relativistic energy density (normalization : $N_{\rm eff} = 1$ for a neutrino)

The Standard Model value, due to neutrinos, is $N_{\rm eff} = 3.046$

[not exactly 3, since neutrino decoupling is not fully completed when e+ and e- annihilate]

In the presence of a sterile neutrino, the cosmological constraint becomes :

$$\left.\begin{aligned} N_{\rm eff} &< 3.29, \\ m^{\rm eff}_{\nu,\,\rm sterile} &< 0.65\ \text{eV}, \end{aligned}\right\} \quad 95\,\%,\ \textit{Planck}\ \text{TT,TE,EE+lowE +lensing+BAO},$$ [Planck 2018]

A sterile neutrino with the mixing angles suggested by oscillation anomalies would be fully thermalized and contribute as $\Delta N_{\rm eff} = 1$

→ strongly disfavored by standard cosmology [at 6σ according to Planck]

Ways out : non-standard cosmological model, sterile neutrino interactions that would prevent thermalization…