

Neutrino physics and flavour mixing

Stéphane Lavignac (IPhT Saclay)

- neutrinos in the Standard Model
- massive neutrinos - Dirac versus Majorana
- flavour mixing - PMNS matrix
- neutrinos oscillations in vacuum and CP violation
- neutrino propagation in matter
- beta decay and neutrinoless double beta decay
- sterile neutrinos ?

Ecole internationale de Physique subatomique (EIPS)
IPN Lyon, 22-26 October 2018

Some references (textbooks and reviews)

- M. Fukugita, T. Yanagida, *Massive neutrinos in physics and astrophysics*, Springer, 2003
- A. Strumia, F. Vissani, *Neutrino masses and mixings and...*, arXiv:hep-ph/0606054 (last update: April 2010)
- M.C. Gonzalez-Garcia, M. Maltoni, *Phenomenology with massive neutrinos*, arXiv:hep-ph/07041800, Phys. Rept. 460 (2008) 1-129
- C. Giunti and C.W. Kim, *Fundamentals of neutrino physics and astrophysics*, Oxford University Press, 2007
- K. Zuber, *Neutrino Physics*, Taylor and Francis, 2011
- R. Zukanovich Funchal, B. Schmauch, G. Giesen, *The physics of neutrinos*, arXiv:1308.1029 [hep-ph]
- C. Giganti, S. Lavignac, M. Zito, *Neutrino oscillations: the rise of the PMNS paradigm*, arXiv:1710.00715 [hep-ex] (+ references therein)
- K. Nakamura and S.T. Petcov, *Neutrino masses, mixing and oscillations*, in the 2018 Review of Particle Physics (<http://pdg.lbl.gov>)

A popular book on neutrinos (in French)



(apologies for the self-promotion)

Neutrino physics and flavour mixing

Lecture 1

Stéphane Lavignac (IPhT Saclay)

- neutrinos in the Standard Model
- massive neutrinos - Dirac versus Majorana
- flavour mixing - PMNS matrix
- neutrinos oscillations in vacuum

Ecole internationale de Physique subatomique (EIPS)
IPN Lyon, 22-26 October 2018

Neutrinos in the electroweak Standard Model

Gauge group: $SU(2)_L \times U(1)_Y \longrightarrow U(1)_{\text{QED}}$
 (couplings) $(g) \quad (g') \quad (e = g \sin \theta_W)$

The spontaneous breaking of the electroweak gauge group leads to 2 massive gauge bosons (W^\pm, Z) and 1 massless gauge boson (the photon γ)

Fermions: come in three generations (family replication)

LH fermions \rightarrow SU(2) doublets ($T^3 = \pm 1/2$) \rightarrow couple to the W

RH fermions \rightarrow SU(2) singlets ($T^3 = 0$) \rightarrow do not couple to the W

$$\begin{array}{ccc}
 \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L & \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L & \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L & Y = -1 \\
 e_R & \mu_R & \tau_R & Y = -2
 \end{array}
 \qquad
 Q = T^3 + \frac{Y}{2}$$

Leptons from different generations are distinguished by their flavour (e, μ, τ), which labels the charged lepton mass eigenstates

Neutrinos are special fermions :

1) have only weak interactions \Rightarrow very small interaction rate (difficult to detecting them; cosmic messengers)

2) no electric charge \Rightarrow can be their own antiparticles (Majorana fermions)

$\bar{\nu} \neq \nu$ Dirac lepton number is conserved

$\bar{\nu} = \nu$ Majorana lepton number is violated

3) the SM as originally defined contains no RH neutrino, since only ν_L (or more precisely the left-helicity neutrino) has been observed [Goldhaber 1957]

\Rightarrow neutrinos are massless in the SM

A fermion mass term involves both chiralities:

$$\mathcal{L}_{\text{mass}} = -m \bar{\psi}\psi = -m (\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) \quad \psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

[way out: Majorana mass term, but cannot be generated in the SM]

4) neutrinos are actually massive, but their masses are much smaller (< 1 eV) than the ones of charged leptons and quarks; also their mixing angles (PMNS matrix) are large, while those of the quarks (CKM matrix) are small

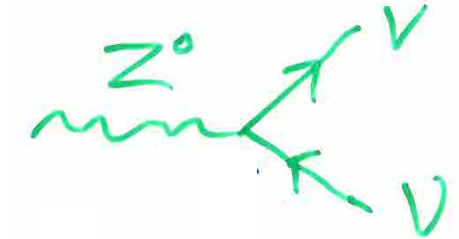
Neutrino interactions

Neutrinos only couple to the W and the Z bosons :

$$\begin{aligned}\mathcal{L}_{\text{CC}} &= \frac{g}{\sqrt{2}} W_{\mu}^{-} \sum_{\alpha=e,\mu,\tau} \bar{e}_{\alpha L} \gamma^{\mu} \nu_{\alpha L} \\ &= \frac{g}{\sqrt{2}} W_{\mu}^{-} (\bar{e}_L \gamma^{\mu} \nu_{eL} + \bar{\mu}_L \gamma^{\mu} \nu_{\mu L} + \bar{\tau}_L \gamma^{\mu} \nu_{\tau L})\end{aligned}$$



$$\begin{aligned}\mathcal{L}_{\text{NC}} &= \frac{g}{2 \cos \theta_W} Z_{\mu} \sum_{\alpha=e,\mu,\tau} \bar{\nu}_{\alpha L} \gamma^{\mu} \nu_{\alpha L} \\ &= \frac{g}{2 \cos \theta_W} Z_{\mu} (\bar{\nu}_{eL} \gamma^{\mu} \nu_{eL} + \bar{\nu}_{\mu L} \gamma^{\mu} \nu_{\mu L} + \bar{\nu}_{\tau L} \gamma^{\mu} \nu_{\tau L})\end{aligned}$$



θ_W = angle de Weinberg $\sin^2 \theta_W = \frac{g'^2}{g^2 + g'^2}$

Neutrinos only couple to the W and the Z bosons :

$$\mathcal{L}_{\text{CC}} = \frac{g}{\sqrt{2}} W_{\mu}^{-} (\bar{e}_L \gamma^{\mu} \nu_{eL} + \bar{\mu}_L \gamma^{\mu} \nu_{\mu L} + \bar{\tau}_L \gamma^{\mu} \nu_{\tau L})$$

$$\mathcal{L}_{\text{NC}} = \frac{g}{2 \cos \theta_W} Z_{\mu} (\bar{\nu}_{eL} \gamma^{\mu} \nu_{eL} + \bar{\nu}_{\mu L} \gamma^{\mu} \nu_{\mu L} + \bar{\nu}_{\tau L} \gamma^{\mu} \nu_{\tau L})$$

All SM interactions (including the charged lepton couplings to the photon and the Z, and their Yukawa couplings) preserve lepton number

$$L = \sum_{\alpha=e,\mu,\tau} (N_{e_{\alpha}^{-}} + N_{\nu_{\alpha}} - N_{e_{\alpha}^{+}} - N_{\bar{\nu}_{\alpha}}) = L_e + L_{\mu} + L_{\tau}$$

(accidental global symmetry of the SM; follows from gauge and Lorentz invariance + renormalizability; not a fundamental symmetry)

Thus e.g. $\pi^{+} \rightarrow \mu^{+} \nu_{\mu}$ is allowed, but $\pi^{+} \rightarrow \mu^{+} \bar{\nu}_{\mu}$ is forbidden

In the absence of neutrino masses, lepton flavour (i.e. the individual quantum numbers L_e, L_{μ}, L_{τ}) is also exactly conserved. Neutrino masses induce lepton flavour violating (LFV) transitions $\nu_{\alpha} \rightarrow \nu_{\beta \neq \alpha}$ (oscillations), but also LFV processes like $\mu^{+} \rightarrow e^{+} \gamma$ and $\mu^{+} \rightarrow e^{+} e^{+} e^{-}$, which however are extremely suppressed in the absence of new physics [see Sacha's lectures]

Massive neutrinos – Dirac versus Majorana

Dirac mass term

The simplest way to describe a massive neutrino is to add a ν_R to the SM and to write a Dirac mass term, as for the other fermions:

$$\mathcal{L}_{\text{mass}}^{\text{Dirac}} = -m_D (\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L) \equiv -m_D \bar{\nu}_D \nu_D \quad \nu_D \equiv \nu_L + \nu_R$$

The massive neutrino ν_D is a Dirac fermion (2 independent chiralities)

$$\begin{array}{c} \nu_R \qquad \nu_L \\ \longrightarrow \quad \text{X} \quad \longrightarrow \\ m_D \end{array} \quad \Delta L = 0 \quad \Delta T^3 = \frac{1}{2} \quad [\text{note: } \nu_R \text{ is an SM gauge singlet}]$$

not invariant under $SU(2)_L \times U(1)_Y$ but can be generated from a Yukawa coupling to the SM Higgs doublet (which has weak isospin 1/2)

$$\mathcal{L}_{\text{Yuk.}} = -y_D \bar{L} i\sigma^2 H^* \nu_R + \text{h.c.} \quad \longrightarrow \quad m_D = y_D \frac{v}{\sqrt{2}}$$

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \quad \langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad m_\nu \lesssim 1 \text{ eV} \quad \Rightarrow \quad y_D \lesssim 10^{-11}$$

caveat: possible to write a Majorana mass term for $\nu_R \Rightarrow$ end up with two Majorana neutrinos rather than one Dirac neutrino (see later)

Majorana mass term

Instead of introducing ν_R , form a RH spinor from ν_L

$$\nu_R^c \equiv C \bar{\nu}_L^T \quad \sim \text{CP conjugate of } \nu_L$$

C = charge conjugation matrix; defines the charge conjugate of a Dirac spinor

$$\psi(x) \rightarrow \psi^c(x) \equiv C \bar{\psi}^T(x) \quad \text{describes the corresponding antifermion}$$

\Rightarrow the existence of a LH neutrino (ν_L) implies the existence of a RH antineutrino ($\nu_R^c \sim \bar{\nu}_R$)

Can write a Majorana mass term :

$$\mathcal{L}_{\text{mass}}^{\text{Maj.}} = -\frac{1}{2} m_M (\bar{\nu}_L \nu_R^c + \bar{\nu}_R^c \nu_L) \equiv -\frac{1}{2} m_M \bar{\nu}_M \nu_M \quad \nu_M \equiv \nu_L + \nu_R^c$$

The massive neutrino $\nu_M = \nu_L + \nu_R^c$ satisfies the Majorana condition

$$\nu_M = \nu_M^c \rightarrow \text{Majorana fermion}$$

$$\begin{array}{c} \nu_R^c \quad \nu_L \\ \longrightarrow \quad \longrightarrow \\ \text{X} \\ m_M \end{array} \quad \Leftrightarrow \quad \begin{array}{c} \nu_L \quad \nu_L^c \\ \longleftarrow \quad \longleftarrow \\ \text{X} \\ m_M \end{array} \quad \Delta L = 2 \quad \Delta T^3 = 1$$

A Majorana mass term violates lepton number (signature of a Majorana neutrino) and cannot be generated from a coupling to the SM Higgs doublet
 \Rightarrow neutrino masses require an extension of the SM

Dirac versus Majorana neutrino

A Dirac neutrino is different from its antiparticle ($\nu \neq \nu^c$)

\Rightarrow describes 4 degrees of freedom: $\nu\uparrow, \nu\downarrow, \bar{\nu}\uparrow, \bar{\nu}\downarrow$

Described by a 4-component spinor $\nu_D = \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$ with independent LH and RH components

A Majorana neutrino satisfies the condition $\nu = \nu^c = C\bar{\nu}^T$

\Rightarrow describes only 2 degrees of freedom: $\nu\downarrow, \bar{\nu}\uparrow$

Can be described by a 4-component spinor $\nu_M = \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$, but the LH and RH components are not independent, as $\nu_M = \nu_M^c \Rightarrow \nu_R = C\bar{\nu}_L^T$

The Majorana condition is inconsistent with any conserved additive quantum number: if ψ possesses a conserved quantum number q ,

$$\psi \rightarrow e^{i\theta q} \psi \quad \Rightarrow \quad \psi^c \rightarrow e^{-i\theta q} \psi^c$$

Thus only neutrinos (not quarks, charged leptons) can be Majorana fermions

For the same reason, one cannot rephase a Majorana neutrino

How to distinguish Majorana from Dirac neutrinos?

Dirac and Majorana neutrinos have the same gauge interactions, since weak interactions only involve ν_L and its antiparticle $\nu_R^c \sim \bar{\nu}_R$ (ν_R , if it exists, is an SM gauge singlet and does not interact at all)

For the same reason, oscillations probabilities are the same for Dirac and Majorana neutrinos (production and detection are weak interaction processes)

The only practical difference between Dirac and Majorana neutrinos lies in their mass term, which violates lepton number by 2 units in the Majorana case

→ the Majorana nature of neutrinos can be established in $\Delta L = 2$ processes such as neutrinoless double beta decay

How to account for neutrino masses ?

Simplest possibility: add a RH neutrino to the SM

In addition to the Dirac mass term $-m_D \bar{\nu}_L N_R + \text{h.c.}$, must write a Majorana mass term for the RH neutrino, which is allowed by all (non-accidental) symmetries of the SM (or justify its absence):

$$-\frac{1}{2} M \bar{N}_L^c N_R + \text{h.c.} = -\frac{1}{2} M N_R^T C N_R + \text{h.c.} \quad \Delta L = 2 \quad \Delta T^3 = 0$$

[only lepton number, if imposed, can forbid this term]

Mass eigenstates : write the mass terms in a matrix form and diagonalize

$$\begin{aligned} \mathcal{L}_{\text{mass}} &= -\frac{1}{2} \begin{pmatrix} \bar{\nu}_L & \bar{N}_L^c \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \end{pmatrix} + \text{h.c.} \\ &= -\frac{1}{2} \begin{pmatrix} \bar{\nu}_{L1} & \bar{\nu}_{L2} \end{pmatrix} \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \begin{pmatrix} \nu_{R1}^c \\ \nu_{R2}^c \end{pmatrix} + \text{h.c.} \end{aligned}$$

$$\text{where } \begin{cases} \nu_{L1} = \cos \theta \nu_L - \sin \theta N_L^c \\ \nu_{L2} = \sin \theta \nu_L + \cos \theta N_L^c \end{cases}$$

Defining $\nu_{Mi} \equiv \nu_{Li} + \nu_{Ri}^c$ (such that $\nu_{Mi} = \nu_{Mi}^c$), one can see that the mass eigenstates are 2 Majorana neutrinos with masses m_1 and m_2 :

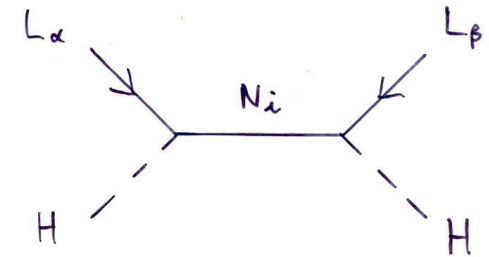
$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} \sum_{i=1,2} m_i \bar{\nu}_{Li} \nu_{Ri}^c + \text{h.c.} = -\frac{1}{2} \sum_{i=1,2} m_i \bar{\nu}_{Mi} \nu_{Mi}$$

“Seesaw” limit: $M \gg M_W \gtrsim m_D$

(N_R = gauge singlet \Rightarrow M unconstrained by electroweak symmetry breaking)

$$m_1 \simeq -m_D^2/M \ll M_W \quad m_2 \simeq M \gg M_W$$

$$\sin \theta \simeq \frac{m_D}{M} \ll 1 \quad \Rightarrow \quad \nu_{L1} \simeq \nu_L, \quad \nu_{R2}^c \simeq N_R$$



\rightarrow the light Majorana neutrino is essentially the SM neutrino

\rightarrow natural explanation of the smallness of neutrino masses

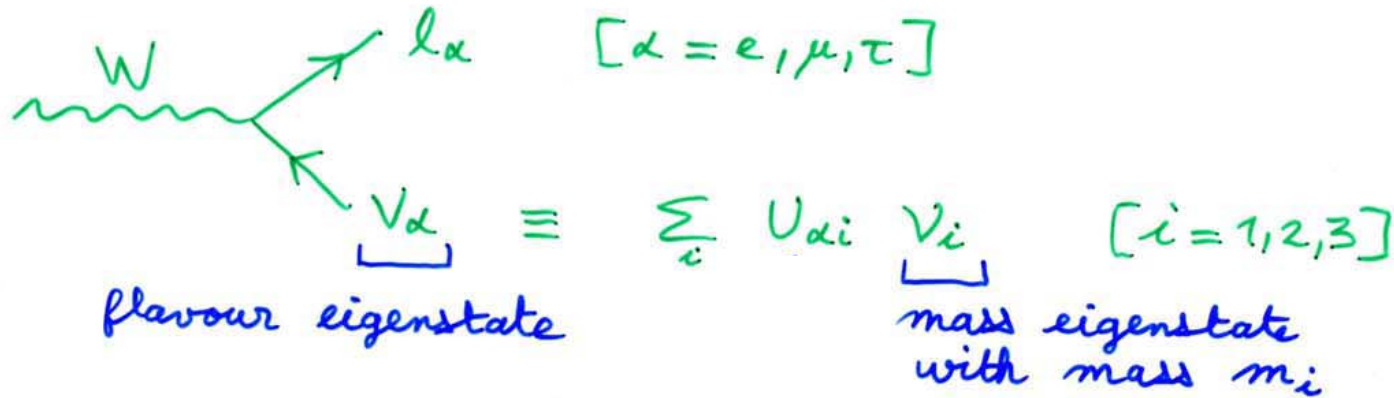
New physics interpretation: M = characteristic scale of the new physics responsible for lepton number violation – might be related to Grand Unification: the fermion content of $SO(10)$ includes a RH neutrino in addition to the SM fermions, and B-L is a generator of $SO(10)$

Alternative mechanisms of neutrino (Majorana) mass generation :

- other versions of the seesaw mechanism with heavy SU(2) triplets (scalar or fermionic)
- radiative models: neutrino masses generated at the one-loop (Zee) or two loop level (Babu-Zee)
- more exotic: supersymmetric models with R-parity violation (in which lepton number is violated), extra spatial dimensions...

Flavour mixing – PMNS matrix

When neutrinos are massive, possibility of flavour mixing : the neutrino to which a given charged lepton (e, μ or τ) couples via the W is not a mass eigenstate, but a coherent superpositions of mass eigenstates



As for quarks, the origin of flavour mixing is the mismatch between the basis of gauge (or flavour) eigenstates and of mass eigenstates. The relative rotation is the lepton mixing matrix, known as PMNS matrix (Pontecorvo-Maki-Nakagawa-Sakata)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L = U_{\text{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L$$

Physical parameters in UPMNS

U is a 3x3 unitary matrix \Rightarrow 3 mixing angles and 6 phases (not all physical)

$$\mathcal{L}_{\text{CC}} = \frac{g}{\sqrt{2}} W_{\mu}^{-} \sum_{\alpha} \bar{e}_{\alpha L} \gamma^{\mu} \nu_{\alpha L} = \frac{g}{\sqrt{2}} W_{\mu}^{-} \sum_{\alpha, i} \bar{e}_{\alpha L} \gamma^{\mu} U_{\alpha i} \nu_{i L}$$

(i) if neutrinos are Dirac fermions : analogous to quarks and CKM

can rephase the lepton fields $e_{\alpha L} \rightarrow e^{i\phi_{\alpha}} e_{\alpha L}$, $\nu_{i L} \rightarrow e^{i\phi_i} \nu_{i L}$ and absorb the phases in the PMNS matrix, so that CC interactions are unaffected

$$U_{\alpha i} \rightarrow e^{i(\phi_{\alpha} - \phi_i)} U_{\alpha i}$$

\Rightarrow removes $2 \times 3 - 1 = 5$ relative phases \Rightarrow a single physical phase δ_{PMNS}

(i) if neutrinos are Majorana fermions : cannot rephase the neutrino fields, since this would affect the Majorana condition

$$U_{\alpha i} \rightarrow e^{i\phi_{\alpha}} U_{\alpha i}$$

\Rightarrow removes only 3 phases \Rightarrow 3 physical phases : 1 "Dirac" phase δ_{PMNS} and 2 "Majorana" phases

Standard parametrization of the PMNS matrix

Analogous to CKM: written as the product of three rotations with angles θ_{23} , θ_{13} and θ_{12} , the second (complex) rotation depending on the phase δ

$$U \equiv U_{23}U_{13}U_{12}P \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P$$
$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P$$

P is the unit matrix in the Dirac case, and a diagonal matrix of phases containing 2 independent phases ϕ_i in the Majorana case

$$c_{ij} = \cos \theta_{ij}, \quad s_{ij} = \sin \theta_{ij}$$

$$\theta_{ij} \in [0, \pi/2], \quad \delta \in [0, 2\pi[, \quad \phi_i \in [0, \pi[$$

$\delta \neq 0, \pi \Rightarrow$ **CP violation in oscillations:** $P(\nu_\alpha \rightarrow \nu_\beta) \neq P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$

The Majorana phases play a role only in $\Delta L = 2$ processes like neutrinoless double beta decay

Neutrino oscillations in vacuum and CP violation

Oscillations are a quantum-mechanical process due to neutrino mass and mixing. An (ideal) oscillation experiment involves 3 steps:

1) production of a pure flavour state at $t = 0$ (e.g. a ν_μ from $\pi^+ \rightarrow \mu^+ \nu_\mu$)

This flavour state is a coherent superposition of mass eigenstates determined by the PMNS matrix, e.g. in the 2 flavour case

$$|\nu(t=0)\rangle = |\nu_\mu\rangle = -\sin\theta |\nu_1\rangle + \cos\theta |\nu_2\rangle$$

2) propagation

Each mass eigenstate, being an eigenstate of the Hamiltonian in vacuum, evolves with its own phase factor $e^{-iE_i t} \Rightarrow$ modifies the coherent superposition, which is no longer a pure flavour eigenstate:

$$|\nu(t)\rangle = -\sin\theta e^{-iE_1 t} |\nu_1\rangle + \cos\theta e^{-iE_2 t} |\nu_2\rangle$$

3) detection via a CC interaction which identifies a specific flavour

probability amplitude : $\langle \nu_e | \nu(t) \rangle = -\cos\theta \sin\theta e^{-iE_1 t} + \cos\theta \sin\theta e^{-iE_2 t}$

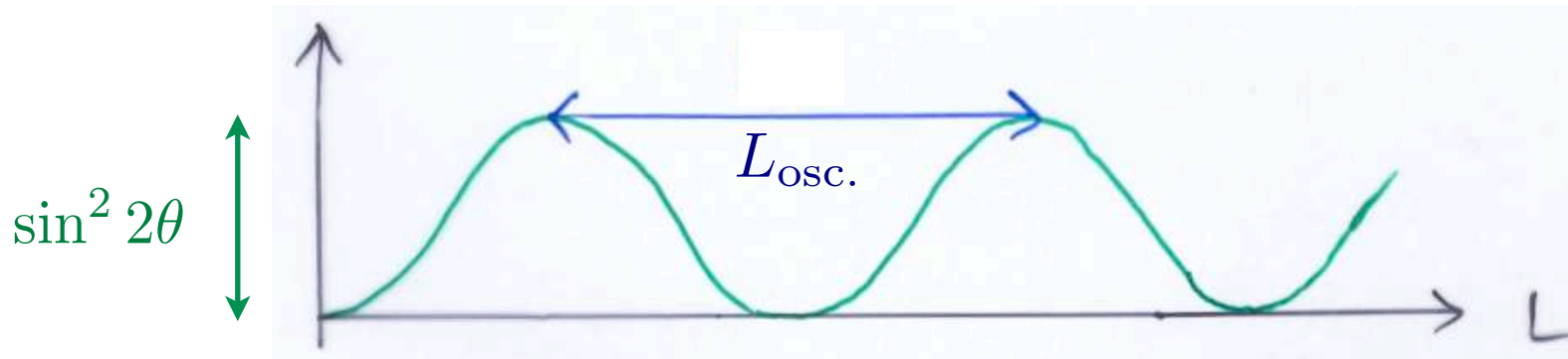
oscillation probability : $P(\nu_\mu \rightarrow \nu_e; t) = |\langle \nu_e | \nu(t) \rangle|^2 = \sin^2 2\theta \sin^2 \left(\frac{E_2 - E_1}{2} t \right)$

2-flavour oscillations in vacuum

Assuming ultra-relativistic neutrinos $L \simeq ct$, $m_i^2 \ll p^2$

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p} \quad \Rightarrow \quad \frac{E_2 - E_1}{2} \simeq \frac{m_2^2 - m_1^2}{4p}$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$



Amplitude of oscillations: $\sin^2 2\theta$

Oscillation length: $L_{\text{osc.}}(\text{km}) = 2.48 E(\text{GeV}) / \Delta m^2(\text{eV})$

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \quad \Delta m^2 \equiv m_2^2 - m_1^2$$

Question : where did I cheat in the derivation of the oscillation formula?

Question : where did I cheat in the derivation of the oscillation formula?

Answer: the propagating mass eigenstates ν_i were described as plane waves with well-defined (and identical) momenta p_i

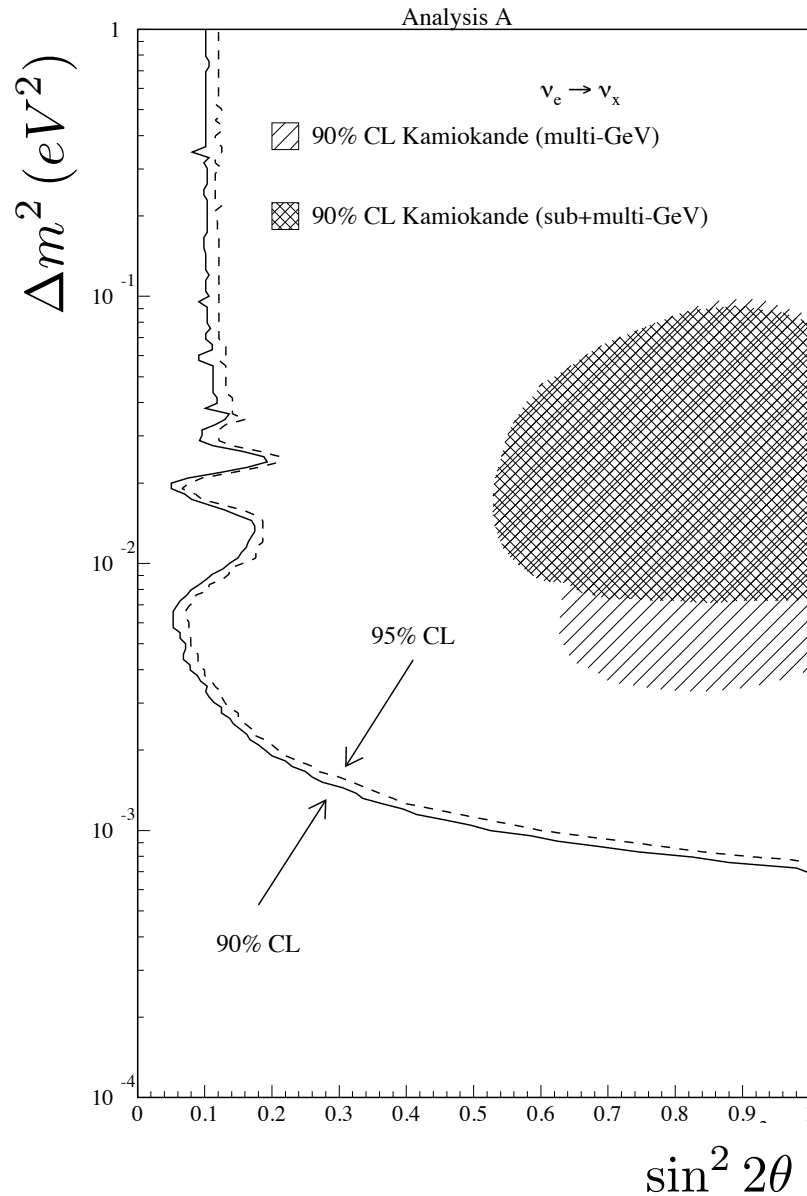
Should instead be described by wave packets with mean momenta p_i

Under appropriate coherence conditions at production and detection, and neglecting decoherence due to separation of the wave packets, the above oscillation formula is recovered (without the ad hoc assumption $p_i = p$)

[See e.g. Akhmedov and Smirnov, arXiv: 0905.1903 for details]

A typical exclusion curve (CHOOZ) :

$$P_{\text{th.}}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right) < P_{\text{exp.}}$$



CHOOZ Collaboration, hep-ex/9907037

N-flavour oscillations in vacuum

$$\nu_\alpha(x) = \sum_i U_{\alpha i} \nu_i(x) \quad (\text{fields}) \Rightarrow |\nu_\alpha\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle \quad (\text{states})$$

$$\text{and for antineutrinos} \quad |\bar{\nu}_\alpha\rangle = \sum_i U_{\alpha i} |\bar{\nu}_i\rangle$$

1) production: $|\nu(t=0)\rangle = |\nu_\alpha\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle$

2) propagation: $|\nu(t)\rangle = \sum_i U_{\alpha i}^* e^{-iE_i t} |\nu_i\rangle$

3) detection: $\langle \nu_\beta | \nu(t) \rangle = \sum_j U_{\beta j} \langle \nu_j | \nu(t) \rangle = \sum_i U_{\beta i} U_{\alpha i}^* e^{-iE_i t}$

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\langle \nu_\beta | \nu(t) \rangle|^2 = \left| \sum_i U_{\beta i} U_{\alpha i}^* e^{-iE_i t} \right|^2$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i < j} \text{Re} [U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}] \sin^2 \left(\frac{\Delta m_{ji}^2 L}{4E} \right) \\ + 2 \sum_{i < j} \text{Im} [U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}] \sin \left(\frac{\Delta m_{ji}^2 L}{2E} \right)$$

3-flavour oscillations

2 independent Δm^2 : Δm_{32}^2 (« atmospheric ») and Δm_{21}^2 (« solar »)

U contains 3 mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and one phase δ [+2 if Majorana]

$$U \equiv U_{23}U_{13}U_{12} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

[the « Majorana » phases are relevant only for processes that violate lepton number, such as neutrinoless double beta decay, and have no effect on oscillations]

→ oscillation probability = \sum oscillating terms with different « frequencies »

$\Delta m_{ji}^2 \equiv m_j^2 - m_i^2$ and amplitudes (which depend on the θ_{ij} and δ)

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i < j} \text{Re} [U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}] \sin^2 \left(\frac{\Delta m_{ji}^2 L}{4E} \right) \\ + 2 \sum_{i < j} \text{Im} [U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}] \sin \left(\frac{\Delta m_{ji}^2 L}{2E} \right)$$

For antineutrinos, $U \rightarrow U^*$ ($\delta \rightarrow -\delta$) and the last term changes sign

⇒ $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$ (if $\delta \neq 0, \pi$) → CP violation

In many experiments, oscillations are dominated by a single Δm^2 and can be described to a good approximation as 2-flavour oscillations:

- solar neutrinos (*), LBL reactors
($\nu_e/\bar{\nu}_e$ disappearance) $\Delta m_{21}^2, \theta_{12}$ $\Delta m_{21}^2 \simeq 7.5 \times 10^{-5} \text{ eV}^2$
- atmospheric, LBL accelerators
(ν_μ disappearance) $\Delta m_{31}^2, \theta_{23}$ $|\Delta m_{31}^2| \simeq 2.5 \times 10^{-3} \text{ eV}^2$
- SBL reactor experiments
($\bar{\nu}_e$ disappearance) $\Delta m_{31}^2, \theta_{13}$ $\sin^2 \theta_{13} \simeq 0.022$

(*) matter effects dominate for high-energy solar neutrinos

Notes: 1) θ_{13} is the only « small » leptonic angle $\theta_{13} < \theta_{12}, \theta_{23}$

2) $\Delta m_{21}^2 \ll |\Delta m_{31}^2| \simeq |\Delta m_{32}^2|$ [by convention, $\Delta m_{21}^2 > 0$]

sign of Δm_{31}^2 unknown \Rightarrow two types of spectra allowed

