Lepton Flavour Violation
(?and
Neutrino Cosmology ?)

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Flavour in the Standard Model (massless neutrinos)
- tree level for quarks and leptons
- loop FCNC in the quark sector and GIM suppression

some LFV processes
- rates for $\mu \to e\gamma$, $\mu \to e\bar{e}e$, $\mu - e$ conv.
- (meson decays, τs...)

EFT for LFV
- SM loops
- What can we learn ?

Neutrino Cosmology

1. what is LFV ? What do we know ?
2. review of flavour in the SM
   - leptons (flavour conservation if $m_\nu = 0$
   - quarks GIM-suppressed flavour-change in loops

lecture 2, outline : EFT for heavy New Physics in the leptonsector

1. parametrise LFV with contact interactions
2. calculate rates for :
   - $\mu \to e\gamma$
   - $\mu \to e\bar{e}e$
   - $\mu - e$ conv.
   - (what about mesons and taus ?)
3. what is EFT
4. a recipe for EFT in flavour physics
5. why do we need it and what can we learn ?

1. interaction rates in cosmology
2. fossils from the early Universe, and how ν affect them :
   - bounds on N_ν and m_ν
   - can neutrinos generate the observed matter excess ?

What is LFV ?

Lepton Flavour Violation $\equiv$ charged lepton flavour change at a point.

- sometimes, people write ChargedLFV = CLFV. But, curiosity :
  there are six quark flavours
  only *three* lepton flavours $\Leftrightarrow$ ν mass eigenstates not “flavours”
  (“flavour” states for νs are charged lepton mass eigenstates)

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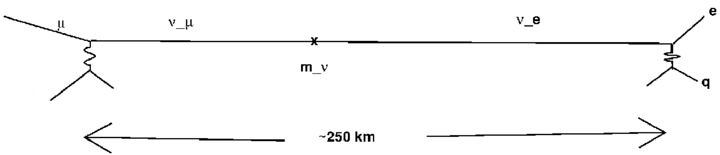
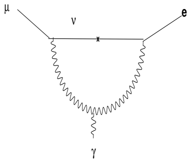
  there are six quark flavours
  only *three* lepton flavours $\Leftrightarrow$ ν mass eigenstates not "flavours"
  ("flavour" states for νs are charged lepton mass eigenstates)
- "at a point" is important : already see charged lepton flavour change in ν osc :



- examples : $\mu \to e\gamma$, $\tau \to 3\ell$, $B_s \to \bar{\mu}\mu$, $h \to \tau^{\pm}\mu^{\mp}$...



Why is LFV interesting ?

...because its New Physics that *must* exist (just not know rate)

1. in the Standard Model=SM (with massless neutrinos), lepton flavour is conserved. (we will see)
2. observed m_ν and mixing matrix $U \Rightarrow$ LF *not* conserved.
3. if m_ν is renormalisable Dirac mass, LFV amplitudes are GIM-suppressed like in quark sector (we will see)

$$\mathcal{A} \propto \frac{m_\nu^2}{m_W^2} \Rightarrow BR \lesssim 10^{-48}$$

4. if *see* LFV experimentally, then more New Physics in lepton sector than renormalisable Dirac masses :)

What do we know experimentally about Branching Ratios ?

some processes	current constraints on BR	future sensitivities
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$	6×10^{-14} (MEG)
$\mu \to e\bar{e}e$	$< 1.0 \times 10^{-12}$(SINDRUM)	10^{-16} (2018, Mu3e)
$\mu A \to eA$	$< 7 \times 10^{-13}$ Au, (SINDRUM)	10^{-16} (Mu2e,COMET)
		10^{-18} (PRISM/PRIME)
$\overline{K_L^0} \to \mu\bar{e}$	$< 4.7 \times 10^{-12}$ (BNL)	
$K^+ \to \pi^+\bar{\mu}e$	$< 1.3 \times 10^{-11}$ (E865)	10^{-12} (NA62)
$\tau \to \ell\gamma$	$< 3.3, 4.4 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to 3\ell$	$< 1.5 - 2.7 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II, LHCb ?)
$\tau \to e\phi$	$< 3.1 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$h \to \tau^\pm e^\mp$	$< 6.9 \times 10^{-3}$	
$Z \to e^\pm\mu^\mp$	$< 7.5 \times 10^{-7}$	

$\mu A \to eA \equiv \mu$ in 1s state of nucleus A converts to e

$BR = \Gamma_{LFV}/\Gamma_{total}$

Recall...the Standard Model

1. the SM is a successful *renormalisable* quantum field theory
   - *renormalisable* $\equiv$ predictive. Lagrangian contains $\sim$ 20 parameters(obtain from experiment), then the theory can predict every observable.
   - successful : explains everything but dark matter, the baryon asymmetry, inflation and m_ν.

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   - successful : explains everything but dark matter, the baryon asymmetry, inflation and m_ν.
2. is a beautiful theory : particle content and symmetries predict Lagrangian !
   - matter content : one generation

$$q_L \equiv \begin{pmatrix} u_L & u_L & u_L \\ d_L & d_L & d_L \end{pmatrix}; \quad \begin{matrix} u_R & = & (u_R \quad u_R \quad u_R) \\ d_R & = & (d_R \quad d_R \quad d_R) \end{matrix} \quad \ell_L \equiv \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}; e_R$$

   $\star$ Put three generations $\star$

   Also need a Higgs to give mass to SM particles :$H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix}$ $\langle H \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}$

   - gauge symmetries : $U(1)_Y \times SU(2) \times SU(3)$ with cplings $g' < g < g_s$ ($U(1)$ is hypercharge)

$$Y(\ell_L) = -\frac{1}{2}; Y(e_R) = -1; Y(q_L) = \frac{1}{6}; Y(u_R) = \frac{2}{3}; Y(d_R) = -\frac{1}{3}; Y(H) = -\frac{1}{2};$$

   - write all terms of dimension ≤ 4 consistent with syms.

$$\mathcal{L}_{(SM)} = +\mathcal{L}_{\rm kin\ f} + \mathcal{L}_{\rm kin\ gauge} + \mathcal{L}_{\rm kin\ H} + \mathcal{L}_{Yuk} + \mathcal{L}_{H\ self} \quad .$$

For leptons in any basis ($\alpha, \beta : 1..3$), kinetic terms and Yukawa :

$$\mathcal{L}_l \quad = \quad i\overline{\ell_L}_\beta^T \gamma^\mu \mathbf{D}_\mu \, \ell_{L\beta} + i\overline{e_R}_\alpha \gamma^\mu D_\mu \, e_{R\alpha} + \{ \, (\overline{\ell_L^\beta} \, [\mathbf{Y_e}]_{\beta\alpha} \widetilde{H}) e_R^\alpha + \text{h.c.} \}$$

where

$\mathbf{D}_\mu = \partial_\mu + i\frac{g}{2}\boldsymbol{\sigma}^a W^a_\mu + ig' Y(\ell_L)\boldsymbol{I} B_\mu$, $D_\mu = \partial_\mu + ig' Y(e_R) B_\mu$

B^μ hypercharge gauge boson, $Y(f) = T_3 + Q_{em}$ ($Y(f) \neq [Y_f]$!)

sign in front of Yukawa to obtain $(i\not{D} - m)\psi = 0$

$[\mathbf{Y_e}]$ arbitrary 3*3 complex "Yukawa" matrix

$\widetilde{H} = \begin{pmatrix} H^+ \\ -H_0^* \end{pmatrix}$ $\langle \widetilde{H} \rangle = \begin{pmatrix} 0 \\ -v \end{pmatrix}$

Lepton Interactions

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dimensions : action $S = \int d^4 x \mathcal{L}$ dimless (PI$\propto e^{iS}$) $\Rightarrow [\mathcal{L}] = m^4$

want [kinetic terms] $= m^4$ (massless particles propagate at all energies)

$[D_\alpha] = 1$, $[\ell] = [e] = 3/2$, $[H] = 1$, Yukawa,gauge cplings dimless.

Now diagonalise charged lepton mass matrix...

$$i\,\overline{\ell_L}^{\beta\,T}\gamma^\mu \mathbf{D}_\mu \ell_L^\beta + i\,\overline{e_R}^\alpha \gamma^\mu \mathbf{D}_\mu e_R^\alpha + \overline{\ell}_L^\beta [Y_e]_{\beta\alpha} \widetilde{H}\, e_{\alpha R} + h.c.)$$

$[Y_e]_{\beta\alpha}$ arbitrary 3×3 matrix
To obtain diagonal charged-lepton mass matrix, use different unitary transformations on left and right of Yukawa (makes sense : different fields on either side) :

$$V_L[\mathbf{Y_e}]V_R^\dagger = D_e = \mathrm{diag}(m_e, m_\mu, m_\tau)$$

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• order of my indices is LR
• obtain V_L, V_R by diagonalising hermitian matrices $[\mathbf{Y_e}][\mathbf{Y_e}]^\dagger = V_L^\dagger D_e^2 V_L$
$[\mathbf{Y_e}]^\dagger[\mathbf{Y_e}] = V_R^\dagger D_e^2 V_R$
• only basis choice in flavour space from Yukawa (no "interaction basis") $\Rightarrow$

$$i \sum_{\alpha\in\{e,\mu,\tau\}} \left\{ \overline{\ell_L}^{\alpha\,T}\gamma^\mu \mathbf{D}_\mu \ell_L^\alpha + i\,\overline{e_R}^\alpha \gamma^\mu \mathbf{D}_\mu e_R^\alpha + \overline{\ell}_L^\alpha y_\alpha \widetilde{H}\, e_{\alpha R} + h.c.) \right\}$$

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$$\left(\overline{\nu_{eL}}, \overline{e_L}\right) \not{D} \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \quad , \quad \left(\overline{\nu_{eL}}, \overline{e_L}\right) \widetilde{H} \tau_R$$

are gauge invariant $\Leftrightarrow$ why not the Lagrangien :

$$i\,\overline{\ell_L}^{\beta'} Z_{\beta\alpha} \gamma^\mu \mathbf{D}_\mu \ell_L^{\alpha'} + i\overline{e_R}^f \gamma^\mu D_\mu e_R^f + \overline{\ell}_L^{'\beta} [\tilde{Y}_e]_{\beta\delta} \widetilde{H} e_R^\delta + h.c.$$

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Because its equivalent ! To recover to canonical $\mathcal{L}$, diagonalise Z (hermitian pcq $\mathcal{L} \in \Re$)

$$\overline{\ell_L^{'\beta}} \mathbf{Z}_{\beta\alpha} \not{D}\, \ell_L^{'\alpha} = \overline{\ell_L^{'\beta}} [V_Z^\dagger \mathbf{D_Z} V_Z]_{\beta\alpha} \not{D}\, \ell_L^{'\alpha} = \overline{\ell_L^{\beta''}} \mathbf{D_Z}_{\beta\beta} \not{D}\, \ell_L^{\beta''} = \overline{\ell_L}^{\beta} \not{D}\, \ell_L^{\beta}$$

where absorb the eigenvalues of Z in field defns : $\ell_L^\beta = \sqrt{z^\beta} \ell_L^{\beta''}$.

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Then redefine Yukawas : $\mathbf{Y_e} = \mathbf{D_Z^{-1/2}} \mathbf{V_Z} \widetilde{\mathbf{Y}}_\mathbf{e}$, to get canonical kinetic terms :

$$\mathcal{L} \;\; = \;\; i \, \overline{\ell_L^\beta}^T \not{D} \; \ell_L^\beta + i \, \overline{e_R^a} \not{D} \; e_R^a + \{ (\overline{\ell_L^\beta} [\mathbf{Y_e}]_{\beta\alpha} \widetilde{H}) e_R^\alpha + \text{h.c.} \}$$

Consider the quarks

The quark part of the Lagrangien in arbitrary basis

$$\mathcal{L}_q = i\,\overline{q_L}^i \gamma^\mu \mathbf{D}_\mu q_L^i + i\,\overline{d_R}^j \gamma^\mu \mathbf{D}_\mu d_R^j + i\overline{u_R}^k \gamma^\mu \mathbf{D}_\mu u_R^k$$
$$+\overline{q_L}^i [\boldsymbol{Y_d}]_{ij} H d_R^j - \overline{q_L}^i [\boldsymbol{Y_u}]_{ik} \widetilde{H}\, u_R^k + h.c.$$

arbitrary 3*3 Yukawa matrices $\boldsymbol{Y_d}$, $\boldsymbol{Y_u}$.Rewrite 2nd line :

$$+(\overline{u_L}^i, \overline{d_L}^i)[\boldsymbol{Y_d}]_{ij} \begin{pmatrix} H^+ \\ -H_0^* \end{pmatrix} d_R^j - (\overline{u_L}^i, \overline{d_L}^i)[\boldsymbol{Y_u}]_{ik} \begin{pmatrix} H_0 \\ H^- \end{pmatrix} u_R^k + h.c.$$

Diagonalise by independent transformations on L and R :

$$V_{Ld}[\mathbf{Y_d}]V_{Rd}^\dagger = D_d \qquad V_{Lu}[\mathbf{Y_u}]V_{Ru}^\dagger = D_u$$

'ttention — basis choice : only basis choice in $\mathcal{L}$ for d_R, u_R, are eigenbases of $[\mathbf{Y_d}]^\dagger[\mathbf{Y_d}]$, $[\mathbf{Y_u}]^\dagger[\mathbf{Y_u}]$. But for doublets q_L, *two* mass bases :$[\mathbf{Y_d}][\mathbf{Y_d}]^\dagger$ and $[\mathbf{Y_u}][\mathbf{Y_u}]^\dagger$.

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Where does the CKM matrix appear?

Recall : gauge syms realised in $\mathcal{L}$, even if broken in "real world." Write (2-gen) $\mathcal{L}$ in terms of mass eigenstates, but gauge-invar doublets :

$$q_L^1 \equiv \begin{pmatrix} V_{dk} u_L^k \\ d_L \end{pmatrix} \text{ or } \begin{pmatrix} u_L \\ d_L^k V_{ku}^* \end{pmatrix} \quad , \; q_L^2 \equiv \begin{pmatrix} V_{sk} u_L^k \\ s_L \end{pmatrix}$$

so the doublet Kinetic Terms (KT), and Yukawas are

$$i(V_{dk}^* \overline{u_L}^k, \overline{d_L})\gamma^\mu \mathbf{D}_\mu \begin{pmatrix} V_{dk} u_L^k \\ d_L \end{pmatrix} + i(V_{sk}^* \overline{u_L^k}, \overline{s_L})\gamma^\mu \mathbf{D}_\mu \begin{pmatrix} V_{sk} u_L^k \\ s_L \end{pmatrix}$$
$$+y_d(V_{dk}^* \overline{u_L}^k, \overline{d_L}) \begin{pmatrix} \Phi_+ \\ -\Phi_0^* \end{pmatrix} d_R + y_s(V_{sk}^i \overline{u_L}^k, \overline{s_L}) \begin{pmatrix} \Phi_+ \\ -\Phi_0^* \end{pmatrix} s_R$$
$$-y_u(\overline{u_L}....) \begin{pmatrix} \Phi_0 \\ \Phi_- \end{pmatrix} u_R - y_c(\overline{c_L}, ...) \begin{pmatrix} \Phi_0 \\ \Phi_- \end{pmatrix} c_R + \text{h.c.}$$

So the mixing matrix appears in Kinetic Terms, + charged Higgs vertices. (But in unitary gauge, charged Higgs "eaten" by the $W^\pm$)

Where does CKM appear (ctd)

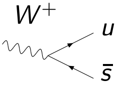
- The mixing matrix appears at W vertices, where u and d mass eigenstates meet :

$$\sum_{s\in\{d,s,b\}} \left\{ -\frac{g}{2}(V^*_{sj}\overline{u^j_L}, \overline{s_L})\gamma^\mu \begin{bmatrix} W^0_\mu - \frac{\tan\theta_W}{3}B_\mu & \sqrt{2}W^+_\mu \\ \sqrt{2}W^-_\mu & -W^0_\mu - \frac{\tan\theta_W}{3}B_\mu \end{bmatrix} \begin{pmatrix} V_{sk}u^k_L \\ s_L \end{pmatrix} \right\}$$

So CKM appears at $W^\pm$ vertices :

$$-i\frac{gV^*_{ij}}{\sqrt{2}}\overline{u^j_L}\gamma^\mu W^+_\mu d^i_L$$

$$\frac{ig}{\sqrt{2}}\gamma^\mu P_L V^*_{us}$$



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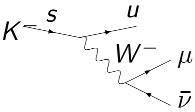

$$-i\frac{gV^*_{ij}}{\sqrt{2}}\overline{u^j_L}\gamma^\mu W^+_\mu d^i_L$$

Can measure CKM angles for instance in tree-level meson decays $K, D \to \pi\mu\nu$ (V_{us}, V_{cd}), $B \to D\ell\nu, \pi\ell\nu$ (V_{cb}, V_{ub})...

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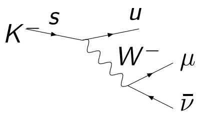

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• No need for dynamical W in low-energy tree diagrams
$\Rightarrow$ contact interaction for Kaon decay : ($m_K \sim .5$ GeV$\ll m_W \sim 80$ GeV)

$$[V]^*_{us} \frac{4 G_F}{\sqrt{2}} (\overline{u_L} \gamma^\mu P_L s)(\overline{\mu_L} \gamma^\mu P_L \nu_\mu)$$

CKM is almost diagonal

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \simeq \begin{bmatrix} 1 & s_{12} & s_{13}e^{-i\delta} \\ -s_{12} & 1 & s_{23} \\ s_{12}s_{23} - s_{13}e^{i\delta} & -s_{23} & 1 \end{bmatrix}$$

$$[|V_{ij}|] \approx \begin{bmatrix} 0.974 & 0.224 & -0.004 \\ -0.22 & 0.99 & 0.042 \\ 0.008 & -0.04 & 1.0 \end{bmatrix} \simeq \begin{bmatrix} 1 & \lambda & \lambda^3/2 \\ -\lambda & 1 & \lambda^2 \\ \lambda^3/2 & -\lambda^2 & 1 \end{bmatrix}$$

(neglecting CKM phase !), $\lambda \sim 0.2$.

Flavour Changing Neutral Currents : $K \to \mu\bar{\mu}$

- tree level Z, γ vertices are flavour diagonal :

$$\mathcal{L} \supset \sum_{d \in \{d,s,b\}} -i\frac{g}{2} V_{dj}^{*} \overline{u_L^j} \gamma^{\mu} Z_{\mu}^{+} V_{dk} u_L^k = \delta_{jk} \frac{g}{2} \overline{u_L^j} \gamma^{\mu} Z_{\mu}^{+} u_L^k$$

Flavour Changing Neutral Currents : $K \to \mu\bar{\mu}$

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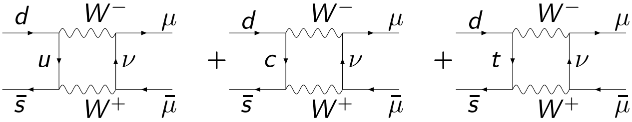
$$\mathcal{L} \supset \sum_{d\in\{d,s,b\}} -i\frac{g}{2} V^*_{dj} \overline{u^j_L} \gamma^\mu Z^+_\mu V_{dk} u^k_L = \delta_{jk} \frac{g}{2} \overline{u^j_L} \gamma^\mu Z^+_\mu u^k_L$$

- but...flavour change at W vertices...get FCNC from Ws in loops ?



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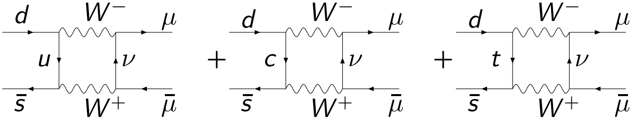

Neglect internal quark masses, and external momenta :

$$\begin{aligned}
\mathcal{M} &\propto \frac{g^4}{4}(V_{ud}V^*_{us} + V_{cd}V^*_{cs} + V_{td}V^*_{ts}) \times \\
&\quad \int \frac{d^4p}{(2\pi)^4} \overline{s}[\gamma^\mu P_L \frac{\not{p}}{p^2} \gamma^\nu P_L] d \frac{(-1)^2}{(p^2 - m_W^2)^2} \overline{\mu}[\gamma_\mu P_L \frac{\not{p}}{p^2} \gamma_\nu P_L]\mu \\
&\propto 0 \times \frac{g^2}{16\pi^2} G_F (\overline{s}\gamma^\alpha P_L d)(\overline{\mu}\gamma^\alpha P_L \mu) \times \text{integral}
\end{aligned}$$

finite$\leftrightarrow$ renorm theory. $2\sqrt{2}G_F = g^2/(2m_W^2)$.
effective four-fermion operator : dimension 6 in fields, coeff $\propto 1/\text{mass}^2$.

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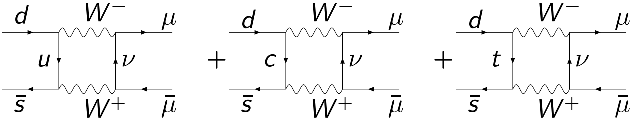

Keep $m_c \sim$ GeV :

$$\mathcal{M} \propto \frac{g^4}{4} V_{cd} V^*_{cs} \int \frac{d^4p}{(2\pi)^4} \overline{s}[\gamma^\mu P_L \frac{\not{p}}{p^2 - m_c^2} \gamma^\nu P_L] d \frac{(-1)}{(p^2 - m_W^2)^2} \overline{s}[\gamma_\mu P_L \frac{\not{p}}{p^2} \gamma_\nu P_L]\mu$$

$$\propto V_{cd} V^*_{cs} G_F \frac{g^2}{16\pi^2} \frac{m_c^2}{m_W^2} (\overline{s}\gamma^\alpha P_L d)(\overline{\mu}\gamma^\alpha P_L \mu) + \ldots$$

méchanisme de GIM (Glashow Iliapoulos, Maiani), qui reduit FCNC.

How to do that integral...

$$\mathcal{M} = \frac{g^4}{4} V_{cd} V_{cs}^* \int \frac{d^4p}{(2\pi)^4} \overline{s}[\gamma^\mu P_L \frac{\not{p}}{p^2 - m_c^2} \gamma^\nu P_L] d \frac{-1}{(p^2 - m_W^2)^2} \overline{s}[\gamma_\mu P_L \frac{\not{p}}{p^2} \gamma_\nu P_L]\mu$$

$$= \frac{g^4}{4} V_{cd} V_{cs}^* (\overline{s}\gamma^\mu \gamma^\alpha \gamma^\nu P_L d)(\overline{\mu}\gamma_\mu \gamma^\beta \gamma_\nu P_L \mu) \times \int \frac{d^4p}{(2\pi)^4} \frac{p_\beta p_\alpha}{p^2(p^2 - m_c^2)(p^2 - m_W^2)^2}$$

$\frac{p_\beta p_\alpha}{p^2} \to \frac{g_{\beta\alpha}}{4}$, Peskin+Schroeder A.39,6.40 : "Feynman" parametrisation

$$\frac{1}{(p^2 - m_c^2)(p^2 - m^2)^2} = \int_0^1 dx \frac{2x}{(p^2 - m_c^2 - x(m_W^2 - m - c^2))^3}$$

then P+S A.46

$$\int_0^1 dx \int \frac{d^4p}{(2\pi)^4} \frac{2x}{(p^2 - m_c^2 - x(m_W^2 - m_c^2))^3} = \frac{-i}{16\pi^2} \int_0^1 dx \frac{x}{(m_c^2 + x(m_W^2 - m_c^2))}$$

BurasHouches,chap3
hep-ph/9806471
InamiLim ...

$$= \frac{-i}{16\pi^2 m_W^2} \left[\frac{m_c^2}{m_W^2 - m_c^2} - 1 + \frac{m_c^2 m_W^2}{(m_W^2 - m_c^2)^2} \log \frac{m_c^2}{m_W^2} \right]$$

Finally Buras-Houches γ-identities 6.71-6.73 (also P+S, sec 3.4, eqn 18.37)

$$(\overline{s}\gamma^\mu \gamma^\alpha \gamma^\nu P_L d)(\overline{\mu}\gamma_\mu \gamma_a \gamma_\nu P_L \mu) = 16(\overline{s}\gamma^\alpha P_L d)(\overline{\mu}\gamma_a P_L \mu)$$

How to NOT do that integral...

...sometimes, people say "EFT is trivial", its just replacing

$$\frac{-g^2}{p^2 - m_W^2} \to \frac{g^2}{m_W^2}$$

So lets try that :

$$\begin{aligned}\mathcal{M} &= \frac{g^4}{4} V_{cd} V_{cs}^* \int \frac{d^4p}{(2\pi)^4} \bar{s}[\gamma^\mu P_L \frac{\not{p}}{p^2 - m_c^2} \gamma^\nu P_L] d \frac{-1}{(p^2 - m_W^2)^2} \bar{s}[\gamma_\mu P_L \frac{\not{p}}{p^2} \gamma_\nu P_L] \mu \\ &\to -\frac{m_c^2}{m_W^4} \frac{g^4}{4} V_{cd} V_{cs}^* (\bar{s}\gamma^\mu \gamma^\alpha \gamma^\nu P_L d)(\bar{\mu}\gamma_\mu \gamma^\beta \gamma_\nu P_L \mu) \times \int \frac{d^4p}{(2\pi)^4} \frac{p_\beta p_\alpha}{p^6}\end{aligned}$$

...which diverges.
We need to keep W momenta in this calculation, because we are trying to evaluate the coefficient of a four-fermion operator in the SM with dynamical W...

SM Flavour Summary

1. the mass basis for the $\{u_L, c_L, t_L\}$ quarks is rotated away from the mass basis for $\{d_L, s_L, b_L\}$ by V_{CKM}. V_{CKM} appears at vertices where $W^\pm$ interact with a $\{u_L, c_L, t_L\}$ and $\{d_L, s_L, b_L\}$.
2. no ν_L mass basis in SM, so $\{\nu_L\}$ in $\{e_L\}$ mass basis.
   $\Rightarrow$ lepton flavour *conserved* in SM.
3. There is no flavour-change at the Z vertex, (incoming+ outgoing fermions are of same charge). "no Flavour-Changing-Neutral-Currents" at tree level.
4. FCNC arise in loops in quark sector. However since V_{CKM} is unitary

$$\mathcal{A}_{ds} \propto V_{kd}^* V_{ks} \Big(1 + \mathcal{O}\frac{m_k^2}{m_W^2} + ... \to 0 + V_{kd}^* V_{ks} \frac{m_k^2}{m_W^2}$$

   FCNC amplitudes must include the internal fermion masses. Called GIM suppression (Glashow Iliopoulos Maiani) usually multiplicative : $\propto m_q^2/m_W^2$ ($\log(m_q/m_W)$ possible in quark sector. Not in leptons. I think.)