NS equation of state, phase transitions, tidal deformability and gravitational waves

Michał Bejger (APC/CNRS Paris & NCAC/PAN Warsaw)

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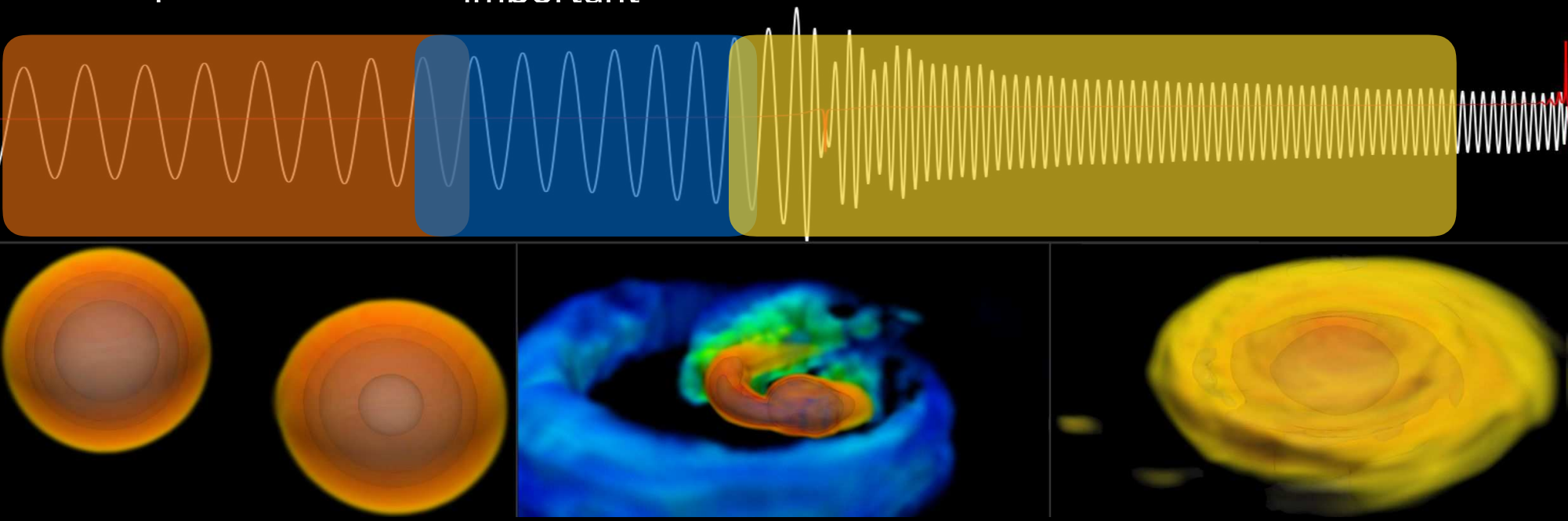
PHYSICAL EFFECTS IN BINARY NEUTRON STAR COALESCENCE WAVEFORMS

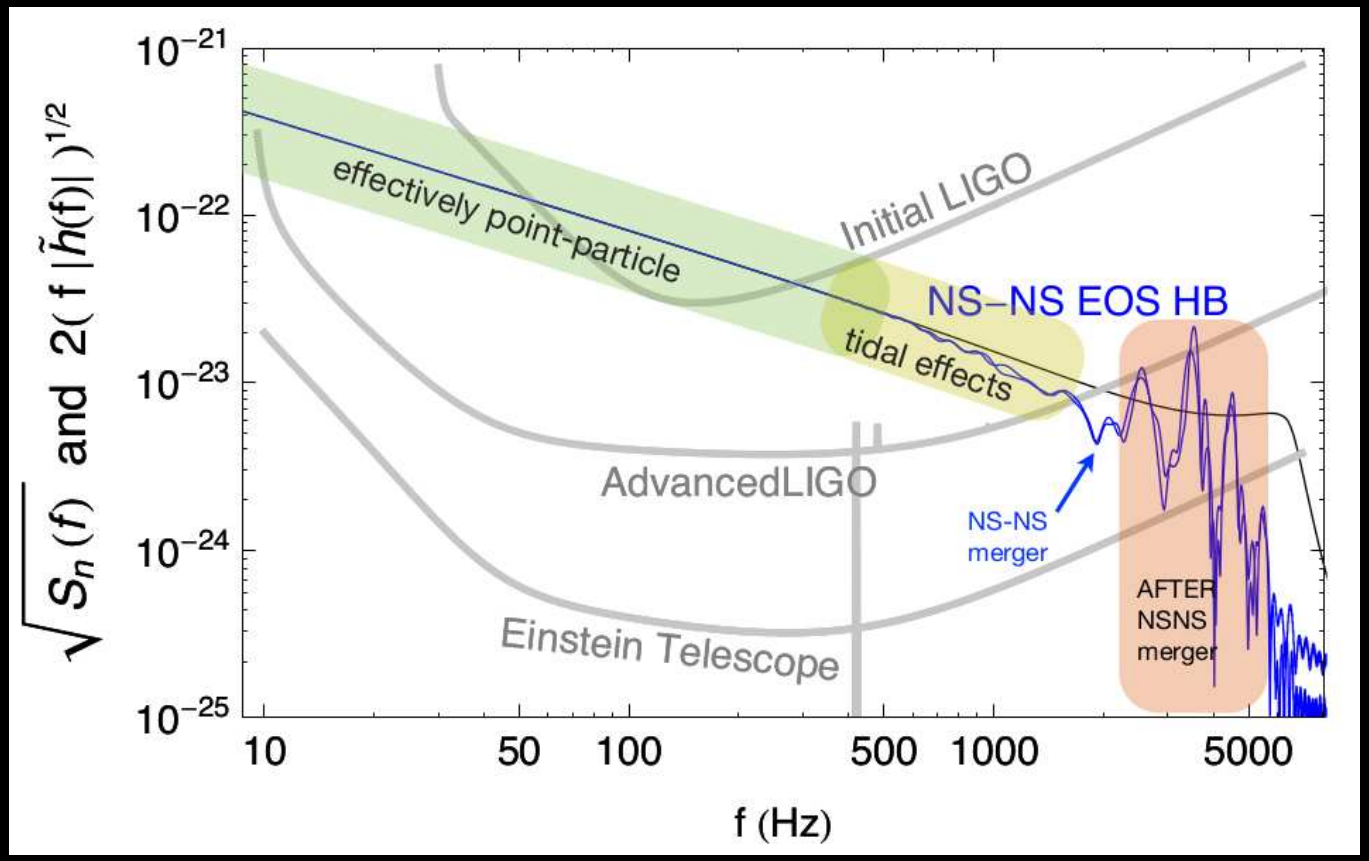
dominated by gravitational radiation back reaction - masses and spins

tidal effects appear at high PN order, dynamical tides might be important

complex physics of the merger remnant, multi-messenger source, signature of neutron star EoS



GW spectrum of ‘material’ binaries (e.g. BNSs)



Phase evolution differs from PP because of extended-body interactions:

$$\Psi(f) = \Psi_{PP}(f) + \Psi_{tidal}(f)$$

Ψ_{tidal} breaks the post-Newtonian v/c expansion degeneracy.

Binary inspiral vs the sensitivity curve

Frequency-domain signal model $\tilde{h}(f)$ (Fourier transform of $h(t)$) to compute the matched-filter SNR and compare with the sensitivity curves:

$$\tilde{h}(f) = Q(angles)\sqrt{\frac{5}{24}}\pi^{-2/3}\frac{\mathcal{M}^{5/6}}{r}f_{GW}^{-7/6}e^{-i\Psi(f)},$$

where the frequency domain phase Ψ is (in the point-particle approximation):

$$\Psi(f) \equiv \Psi_{PP}(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3M}{128\mu v^{5/2}}\sum_{k=0}^{N}\alpha_k v^{k/2},$$

with v denoting the orbital velocity:

$$v \propto (\pi\mathcal{M}f_{GW})^{1/3}$$

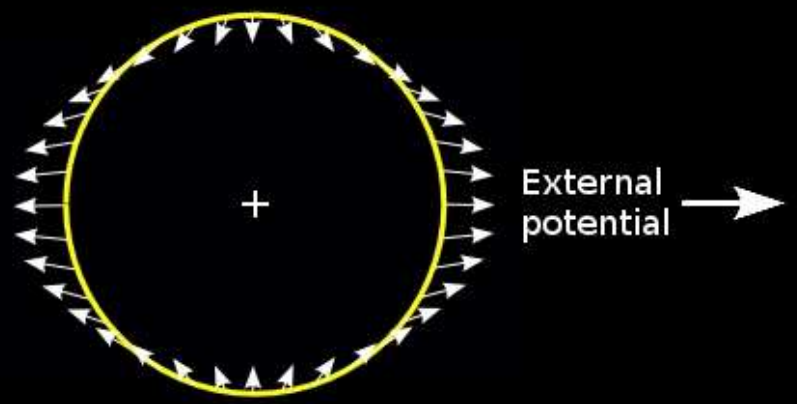
Signature of matter in binary NS waveforms

Tidal tensor $\mathcal{E}_{ij}$ of one of the components induces quadrupole moment Q_{ij} in the other:

$$Q_{ij} = -\lambda \mathcal{E}_{ij} \quad \rightarrow \quad \lambda = \frac{\text{size of quadrupole deformation}}{\text{strength of external tidal field}}$$

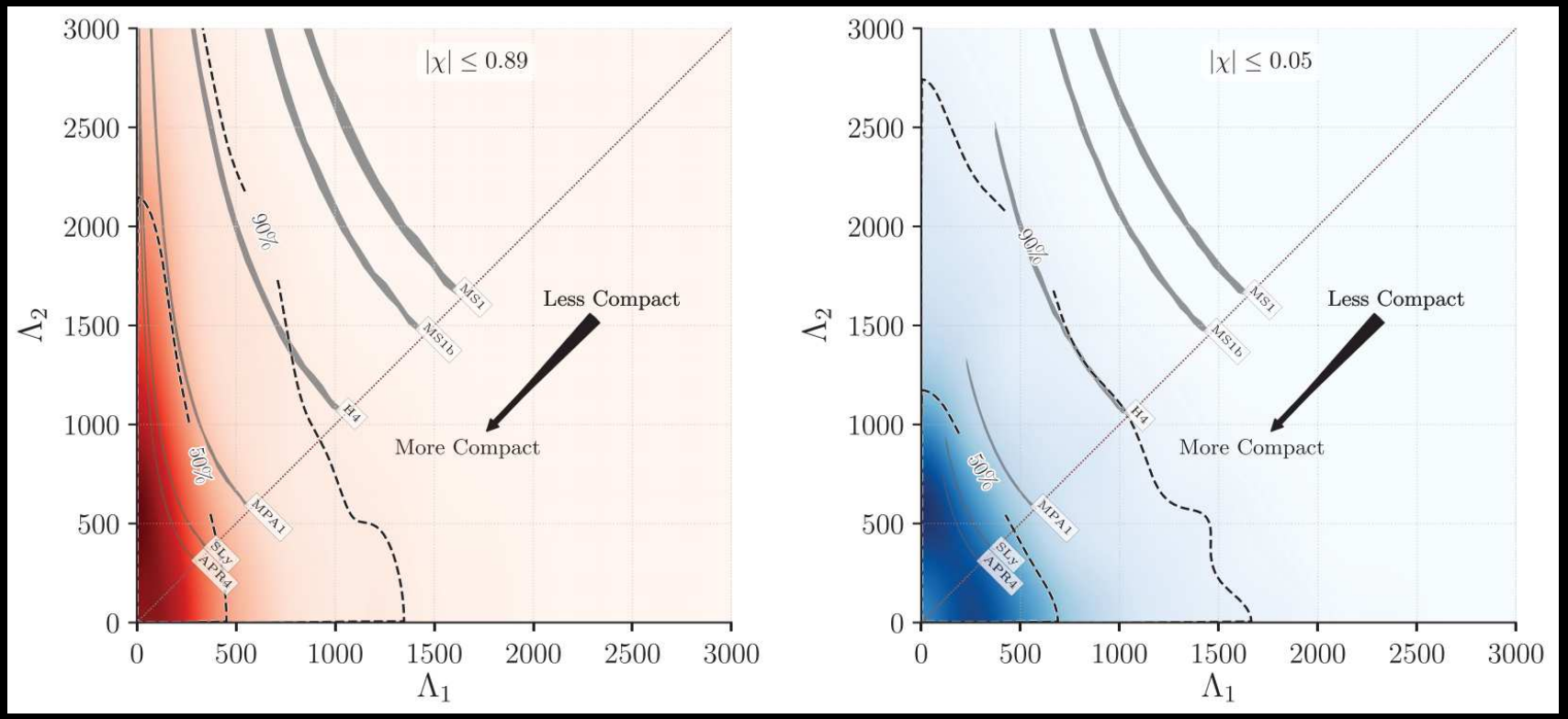
In lowest-order approximation: $\boxed{\lambda = \frac{2}{3} k_2 R^5}$

λ - tidal deformability,
$k_2 \in (0.05, 0.15)$ - the Love number (dependent on M and EOS).



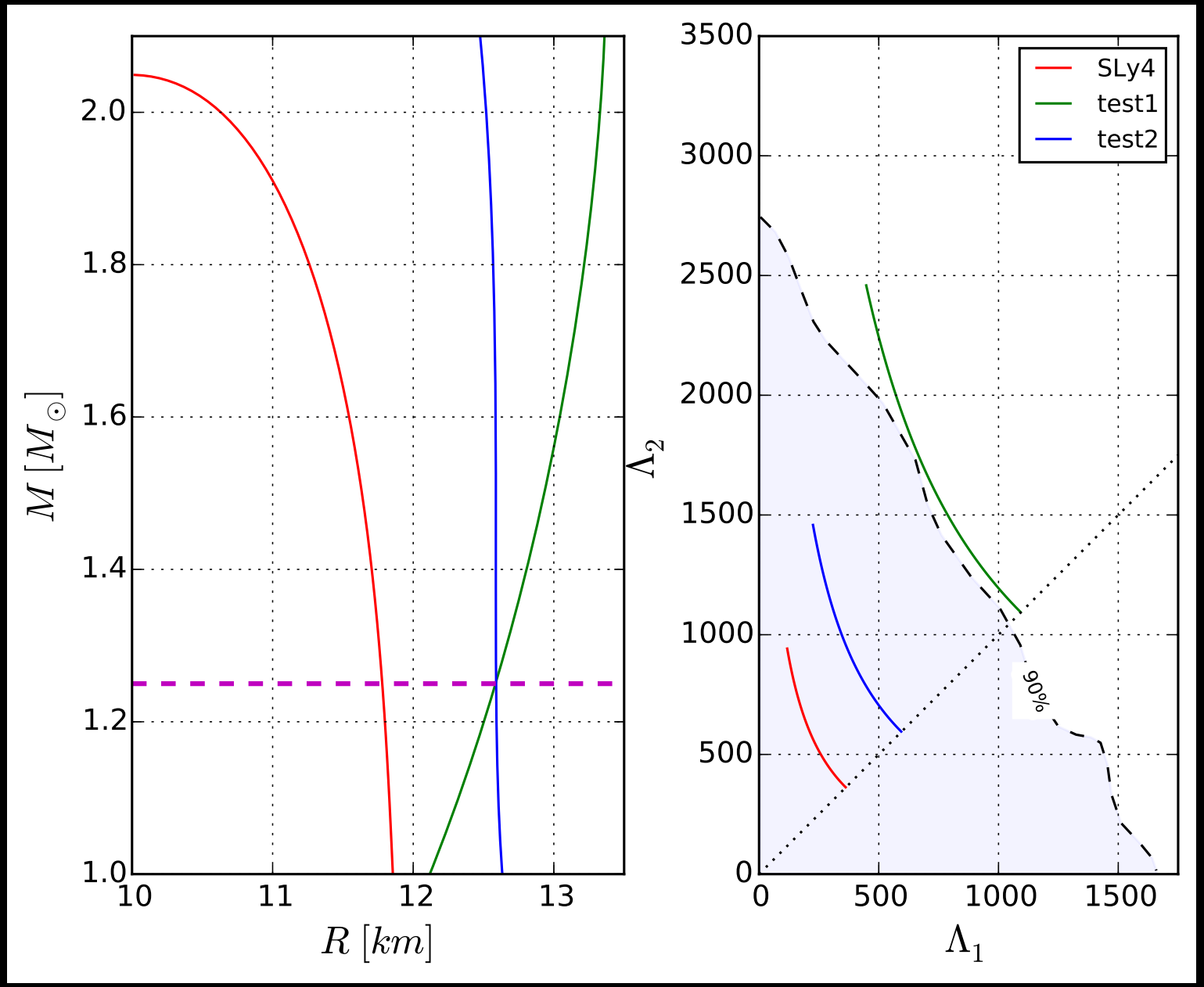
- ⋆ From the scaling this is a 5PN effect $(v/c)^{10}$
- ⋆ Convenient redefinition: $\boxed{\Lambda = G\lambda \left(\frac{GM}{c^2}\right)^{-5} \in (100, 1000)}$

GW170817: initial constraints on dense matter

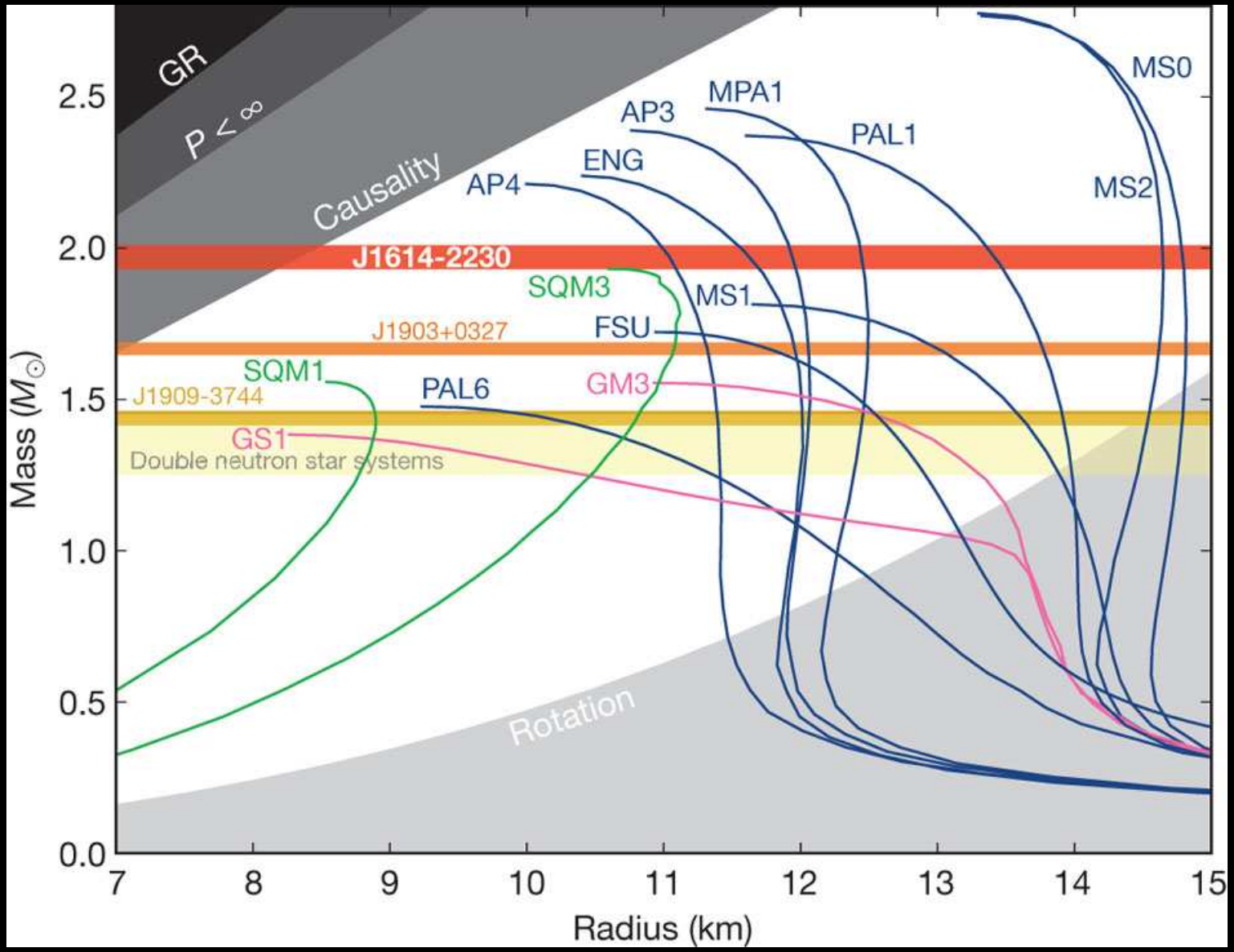


- ⋆ Chirp mass $\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}} = 1.188^{+0.004}_{-0.002}\, M_\odot$,
- ⋆ $m_1 \in (1.36, 1.60)$, $m_2 \in (1.17, 1.36)$,
- ⋆ $\tilde{\Lambda} = \frac{16}{13} \frac{(m_1+12m_2)m_1^4\Lambda_1+(m_2+12m_1)m_2^4\Lambda_2}{(m_1+m_2)^5}$,
- ⋆ High-spin prior $\tilde{\Lambda} \leq 800$, low-spin prior: $\tilde{\Lambda} \leq 900$

Λ is not (only) about *M* and *R*

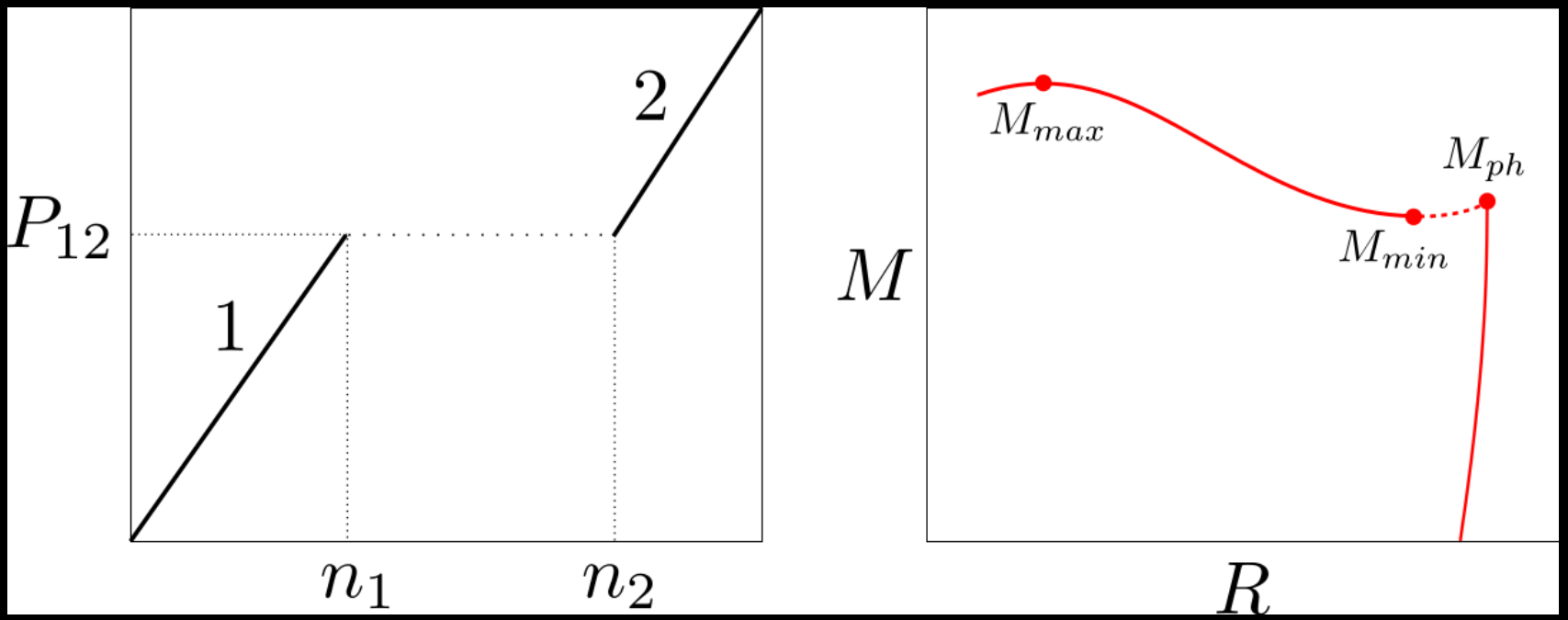


What about the phase transitions?



(Demorest et al., 2010)

What about the phase transitions?



Strong (destabilizing) phase transitions with a parametric EOS class (Sieniawska et al. 2018 [arXiv:1807.11581], in review):

- ⋆ SLy4 crust + polytrope (1) + simple quark bag model (2),
- ⋆ (1) $\leftrightarrow$ (2) connected with a density jump $\lambda = n_2/n_1$,
- ⋆ Speed of sound $\sqrt{\partial P/\partial \rho} < c$,
- ⋆ $M_{max} > 2\ M_\odot$.

relativistic polytrope (Tooper 1965) replaces the tabulated EOS. The definition of the pressure P and the energy-density ρc^2 are as follows:

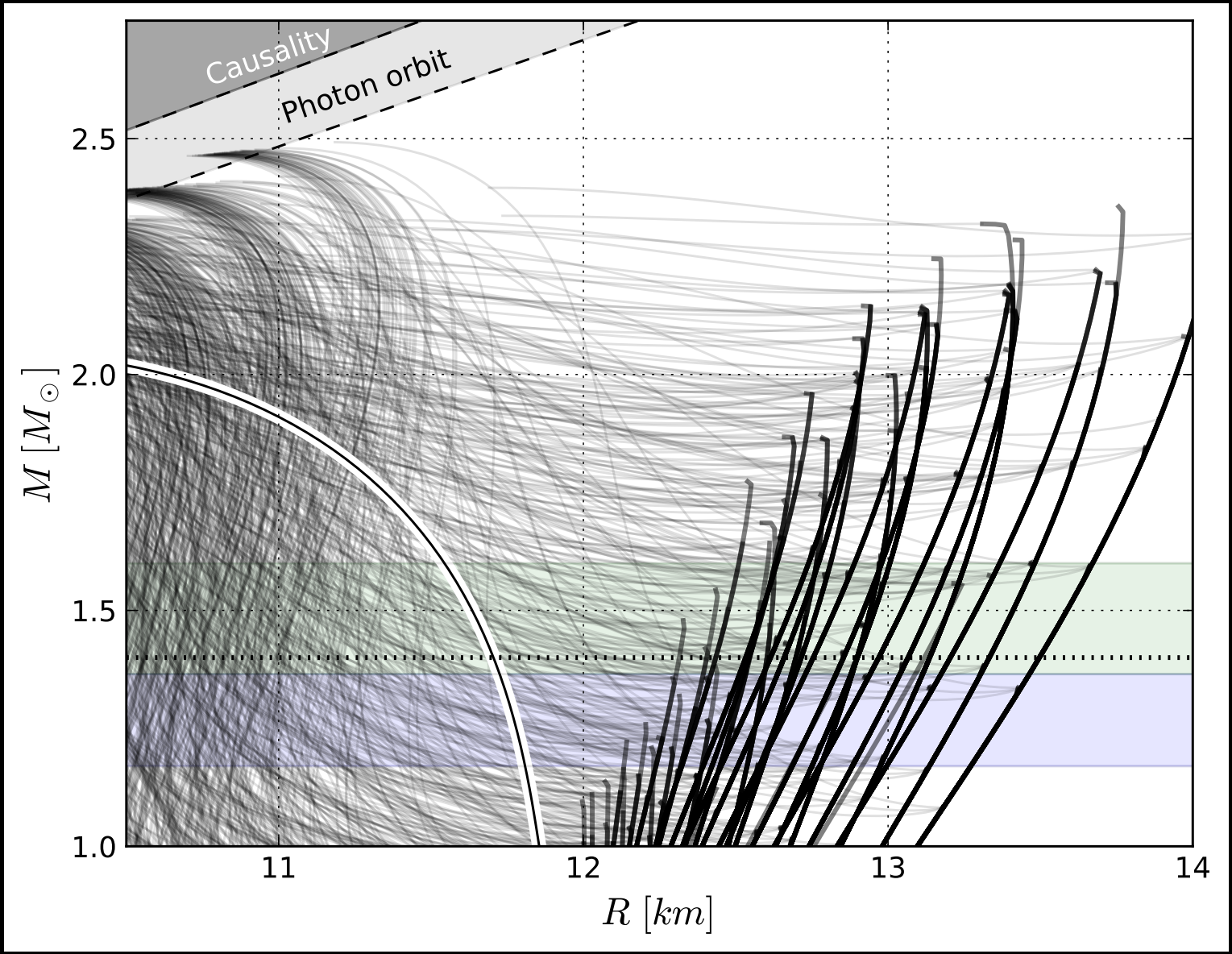
$$P = \kappa n^{\gamma}, \quad \rho c^2 = \frac{P}{\gamma - 1} + n m_b c^2, \tag{1}$$

where κ is the pressure coefficient, γ is the index of the polytrope (deciding about its stiffness) and m_b is the mass of the baryon in this phase. The index γ is a parameter of choice; consequently, by demanding the chemical and mechanical equilibrium at n_0, κ and m_b are fixed. First polytrope ends at some density $n_1 > n_0$, and is connected to a simple bag EOS, describing the quark matter. We use a linear pressure-density relation (Zdunik 2000),

$$P = \alpha(\rho - \rho_*), \tag{2}$$

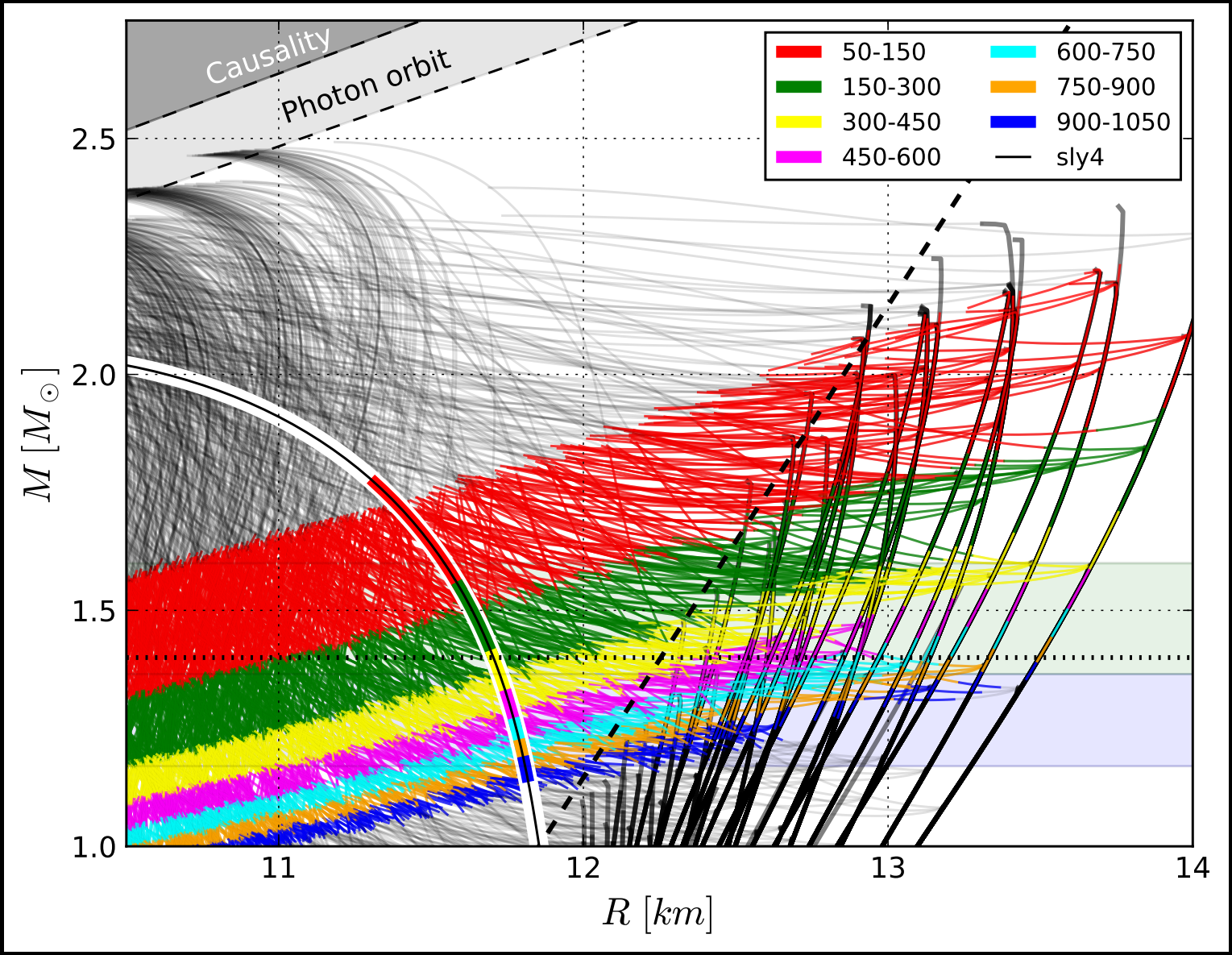
with α denoting the speed of sound in quark matter, and ρ_* the energy-density of such matter at zero pressure. In order to mimic the phase-transition, the connection features a density jump $\lambda = n_2/n_1$, that in a realistic EOS may be related to e.g., surface tension between two dense phases. The schematic

$M(R)$ diagram

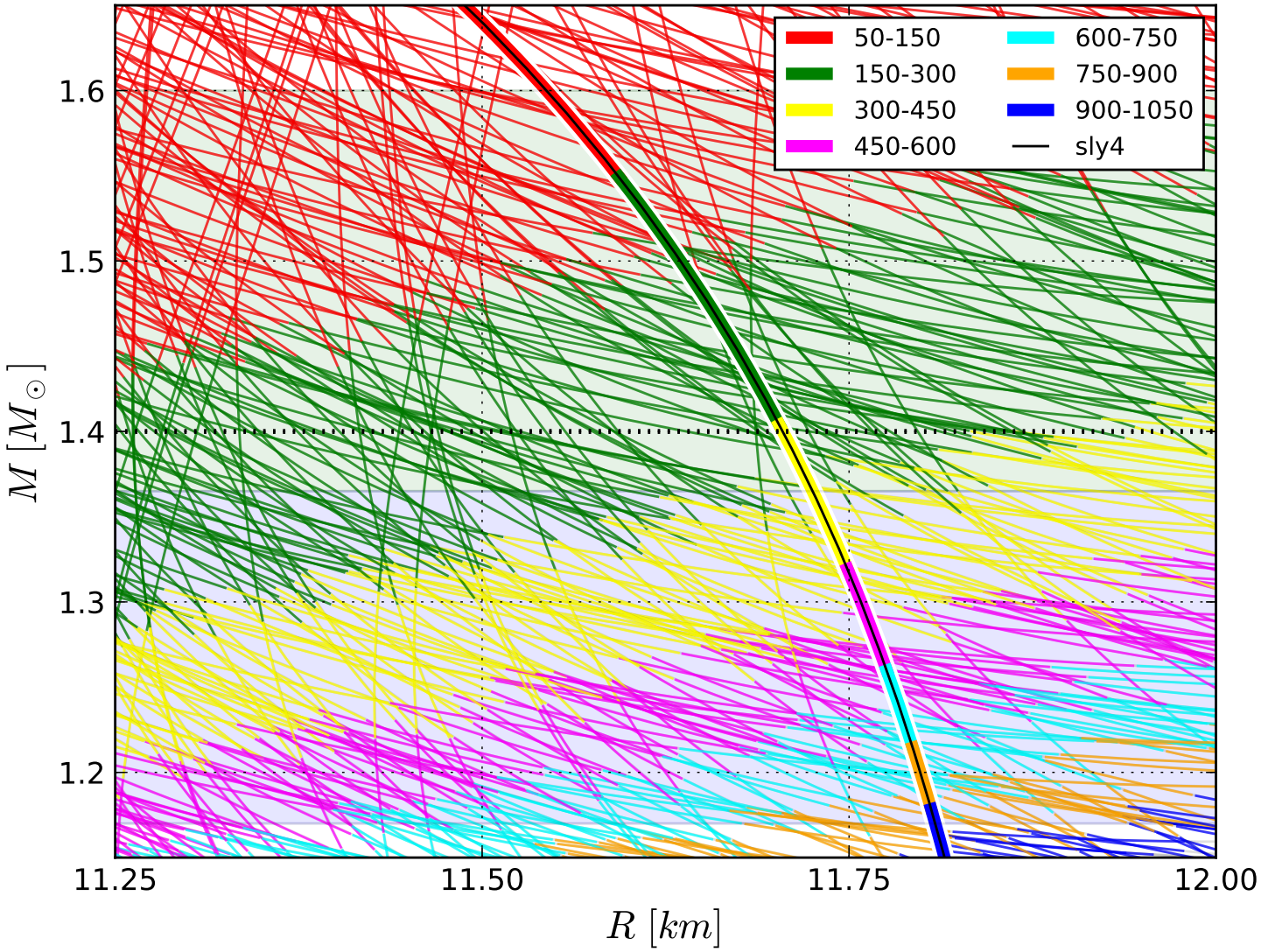


Speed of sound in the quark core $v_s = 1$

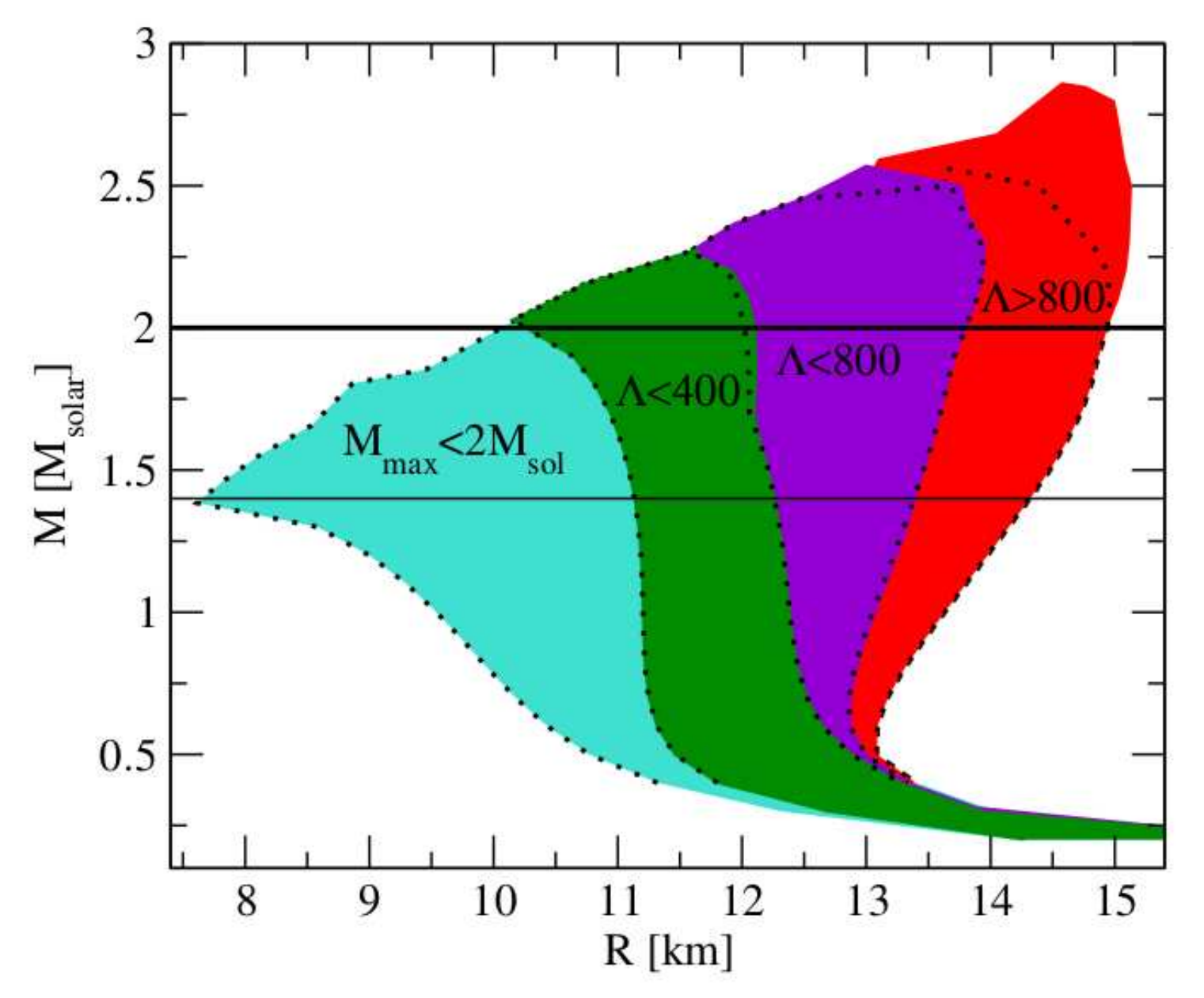
Λ on $M(R)$ diagram



Λ on $M(R)$ diagram



$\Lambda_{1.4}$ on $M(R)$ diagram (Annala et al., 2017)



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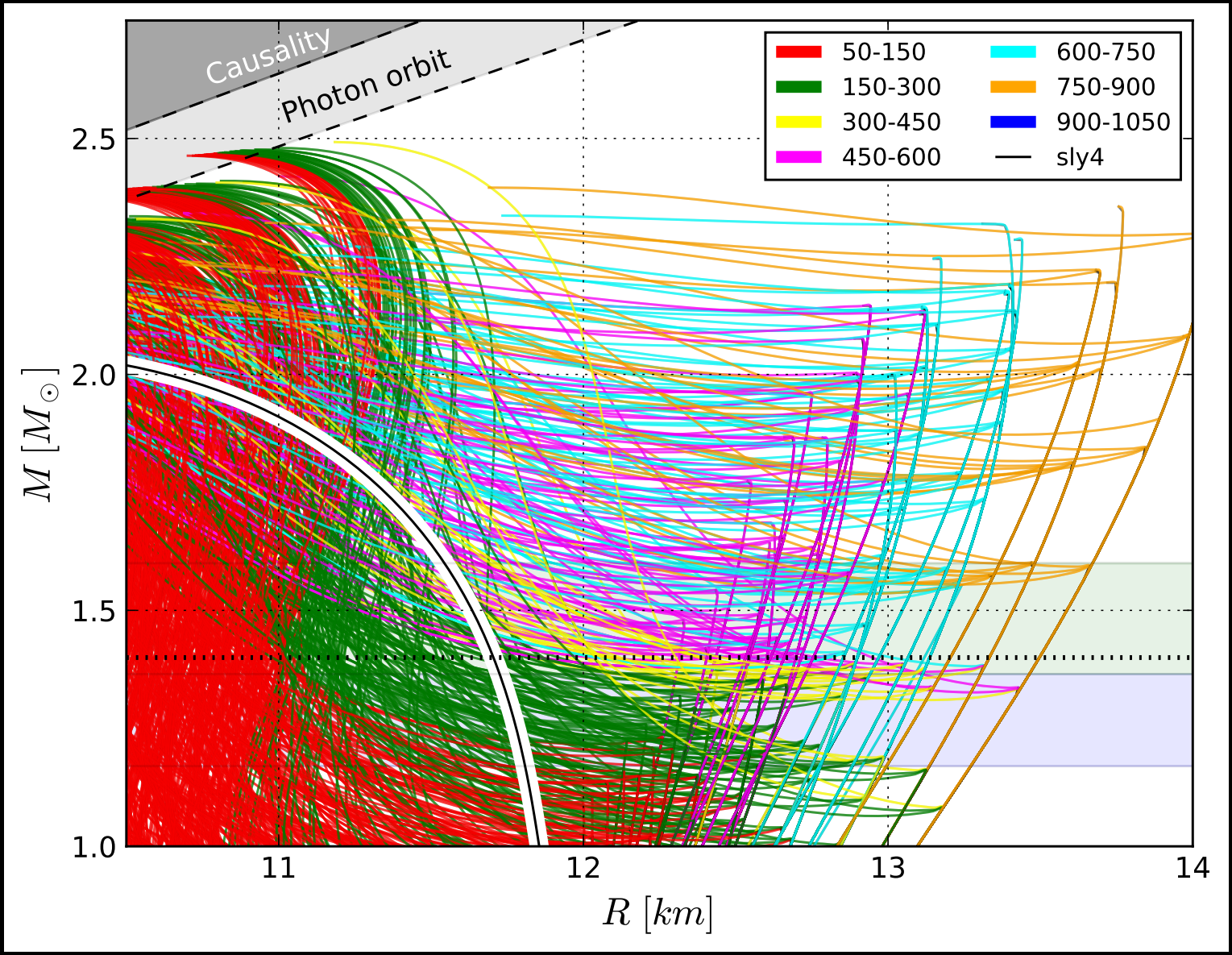
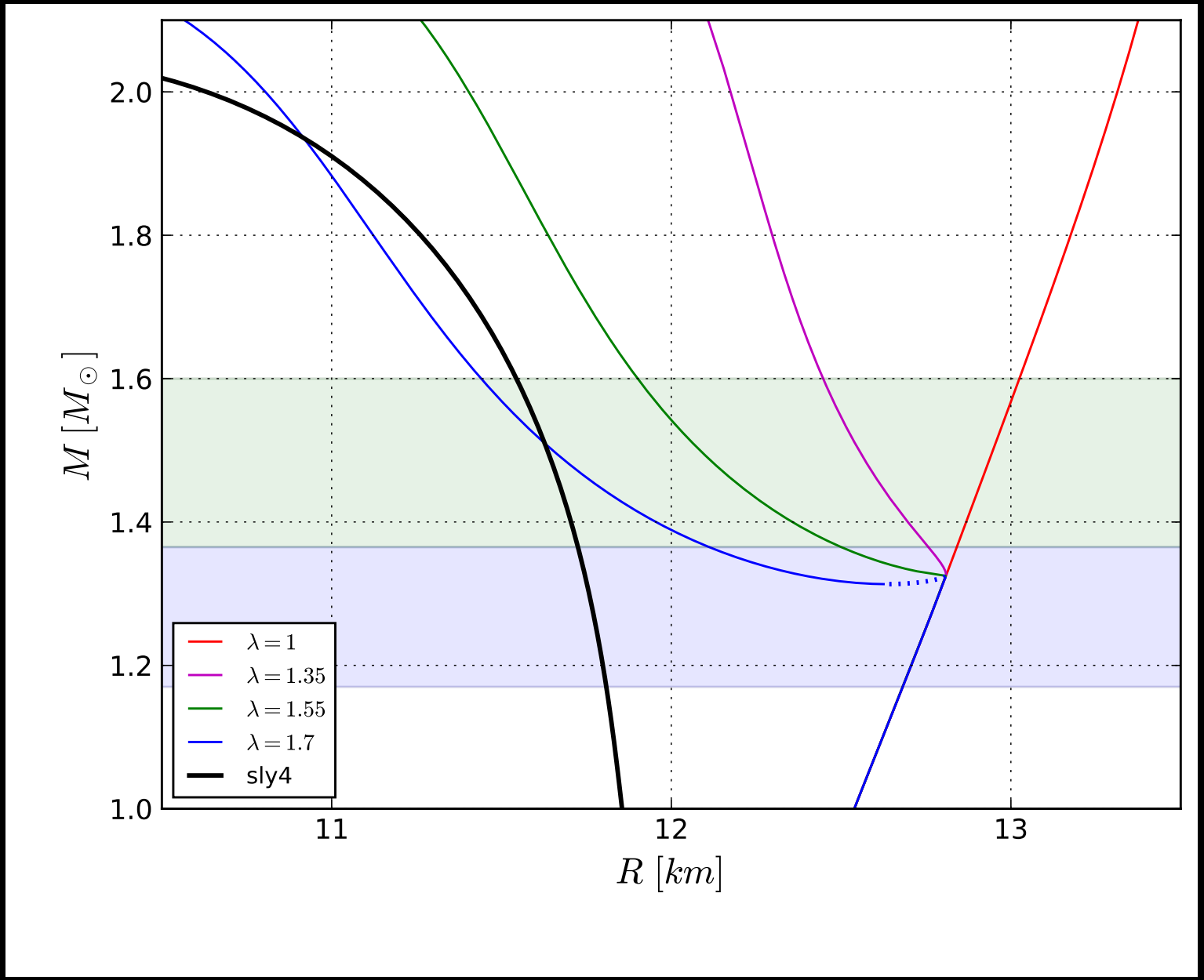


$M(R)$ vs $\Lambda_1 - \Lambda_2$



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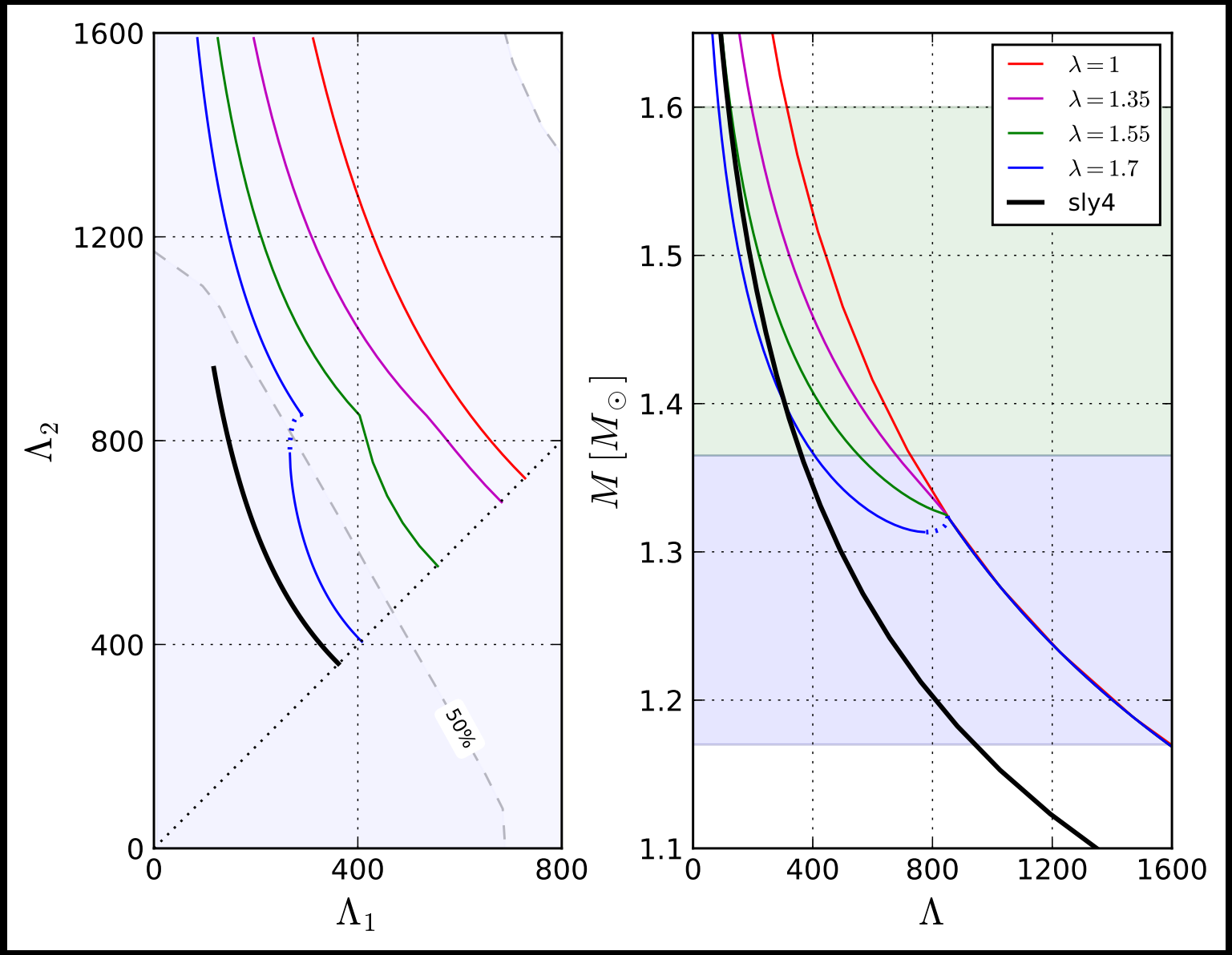
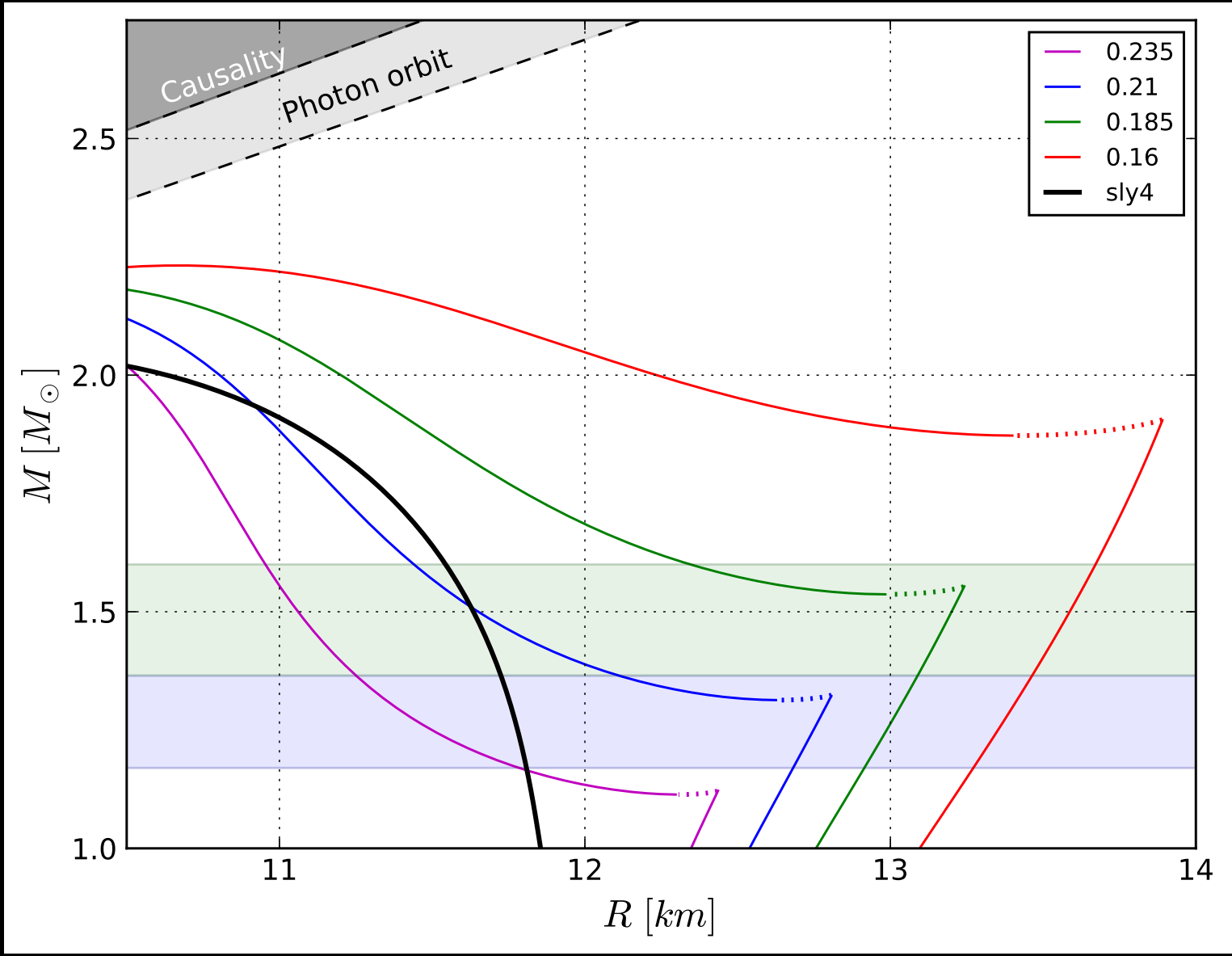
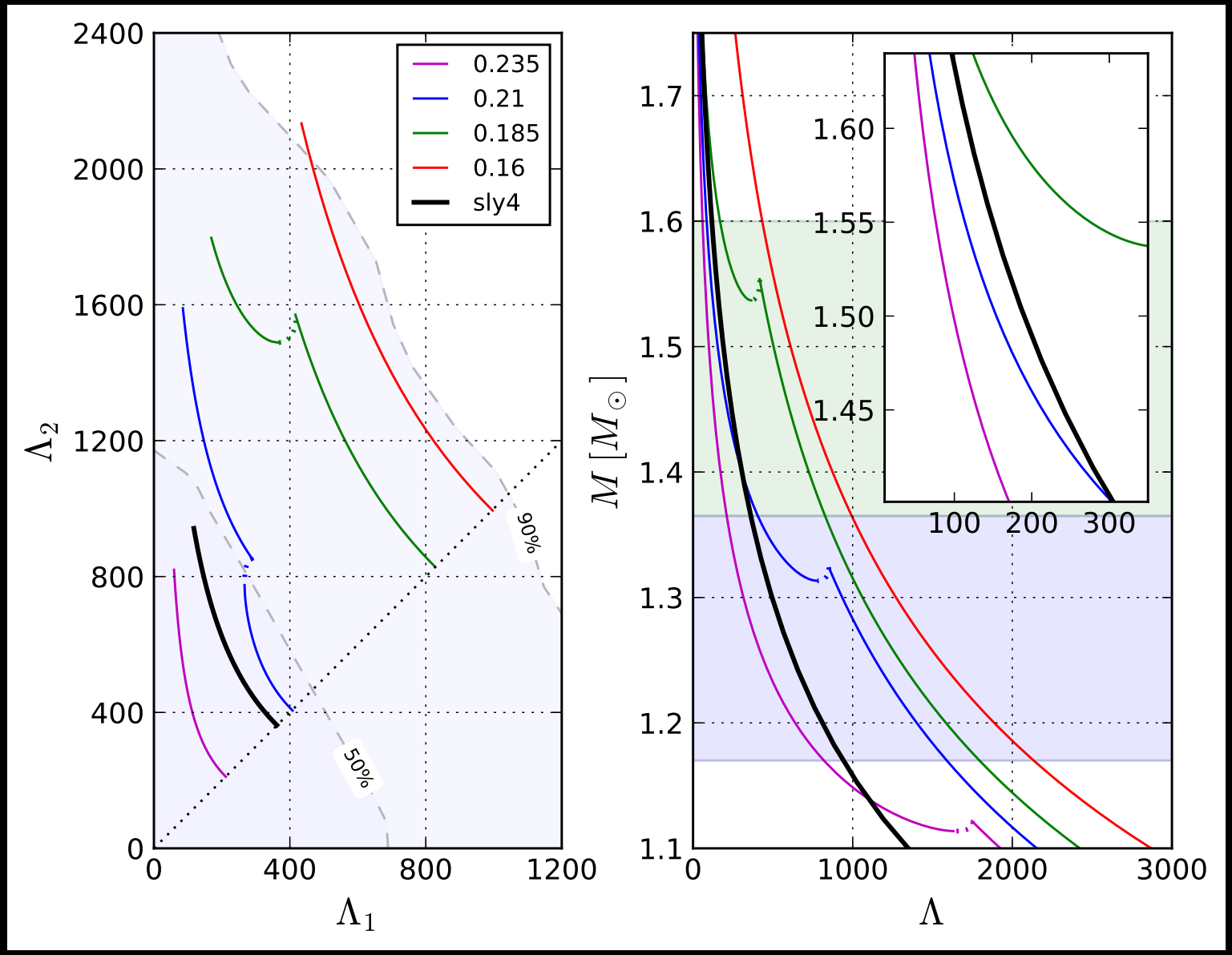


$M(R)$ vs $\Lambda_1 - \Lambda_2$: strong phase transitions

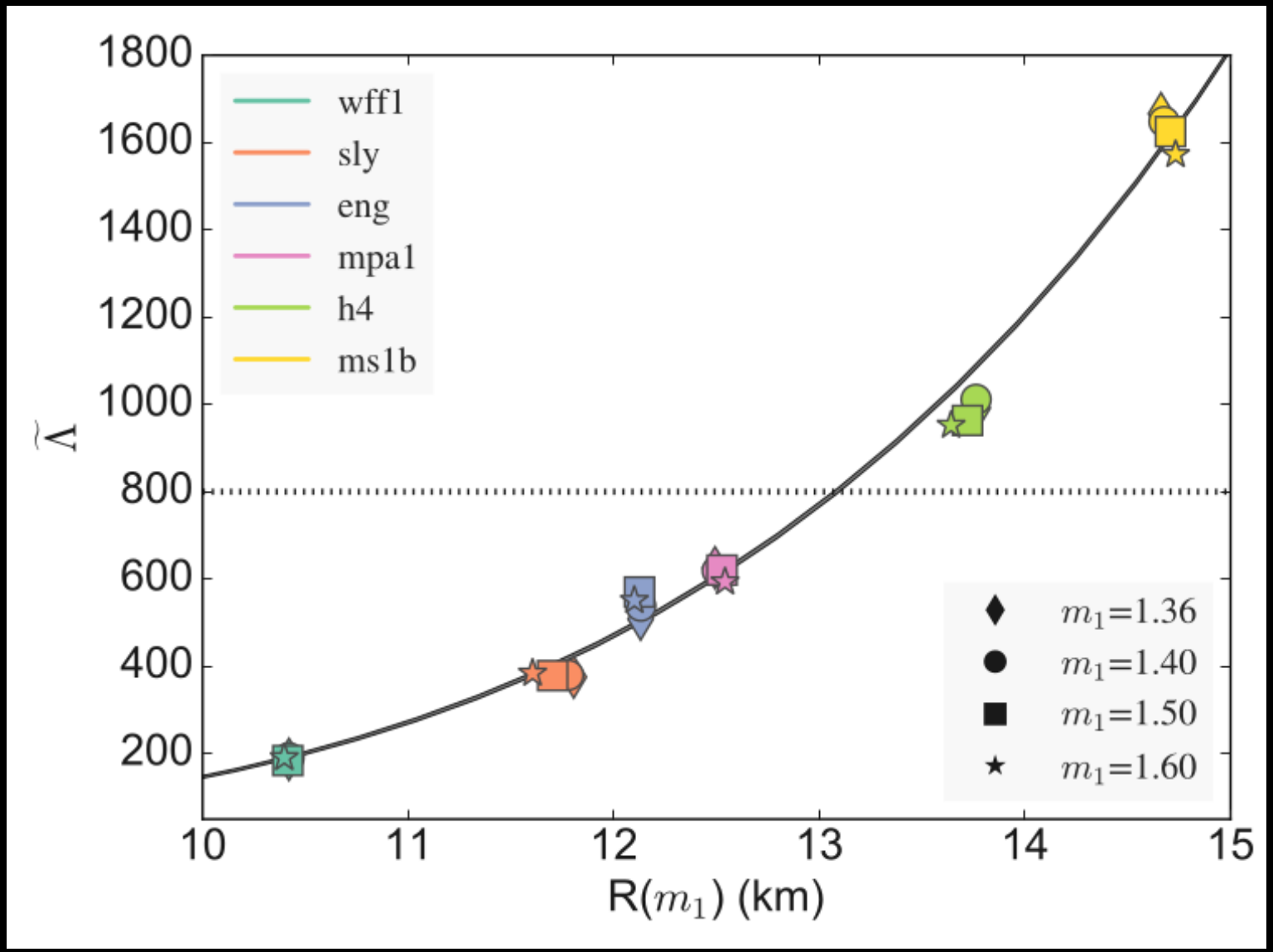


(EOS differ by only n_0, crust-polytrope connection point)

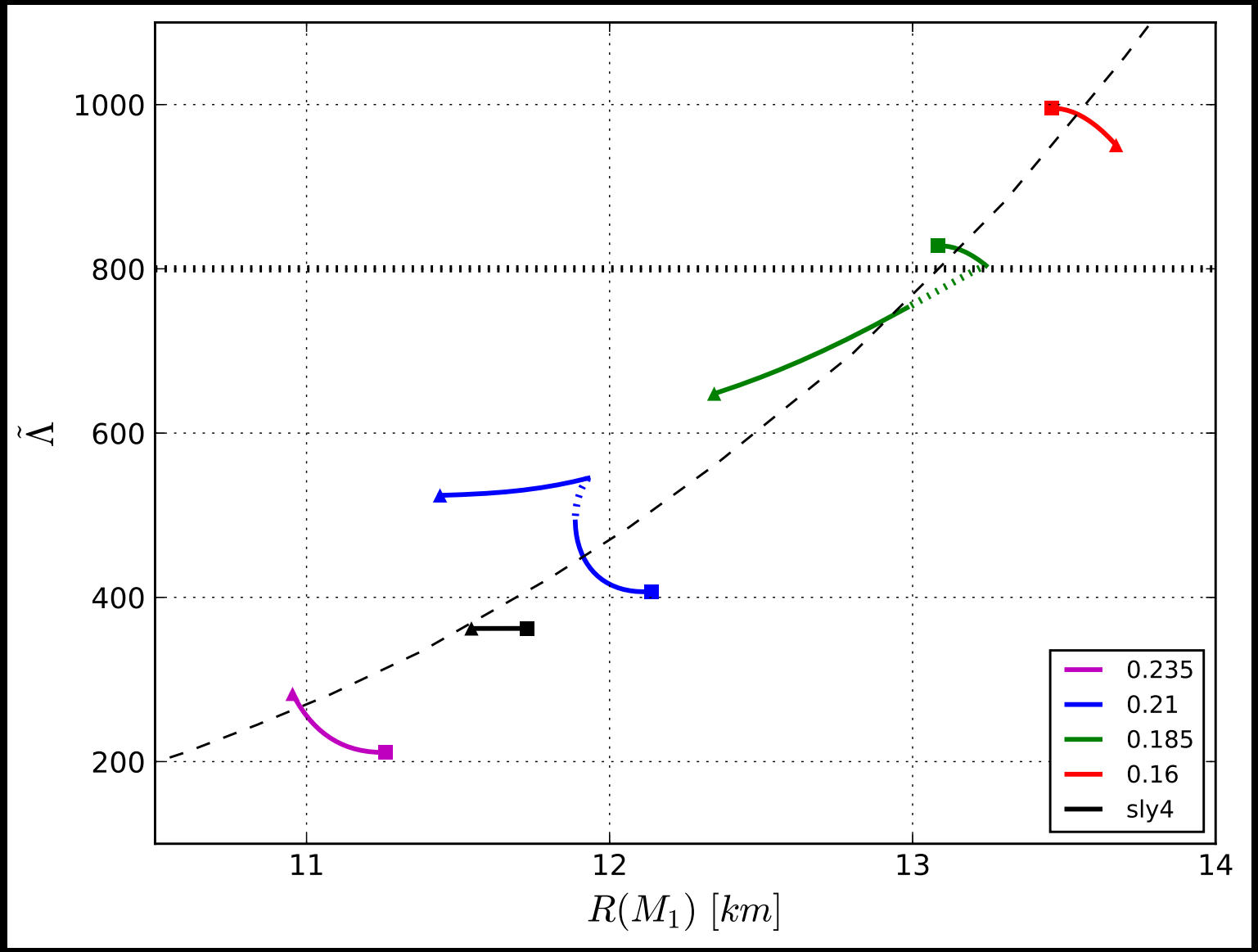
$M(R)$ vs $\Lambda_1 - \Lambda_2$: strong phase transitions



$\tilde{\Lambda}$ vs $R(m_1)$ (Raithel et al., 2018)



$\tilde{\Lambda}$ vs $R(m_1)$: strong phase transitions



($\triangle$: $m_1 = 1.6\ M_\odot$, $\square$: $m_1 = 1.36\ M_\odot$)

Conclusions

In `arXiv:1807.11581` we study generic features of tidal deformability with strong phase-transition EOS:

- ⋆ Softening due to phase transitions decreases Λ, $\tilde{\Lambda}$,
- ⋆ Non-trivial M-Λ relation,
- ⋆ Future observations of GWs can be used to constrain this degree of freedom (e.g. the LIGO-Virgo O3 predictions: a few BNS mergers),
- ⋆ NS radius from GW observations with $\sim$10% error (?)