Precision Big-Bang Nucleosynthesis with improved He-4 predictions

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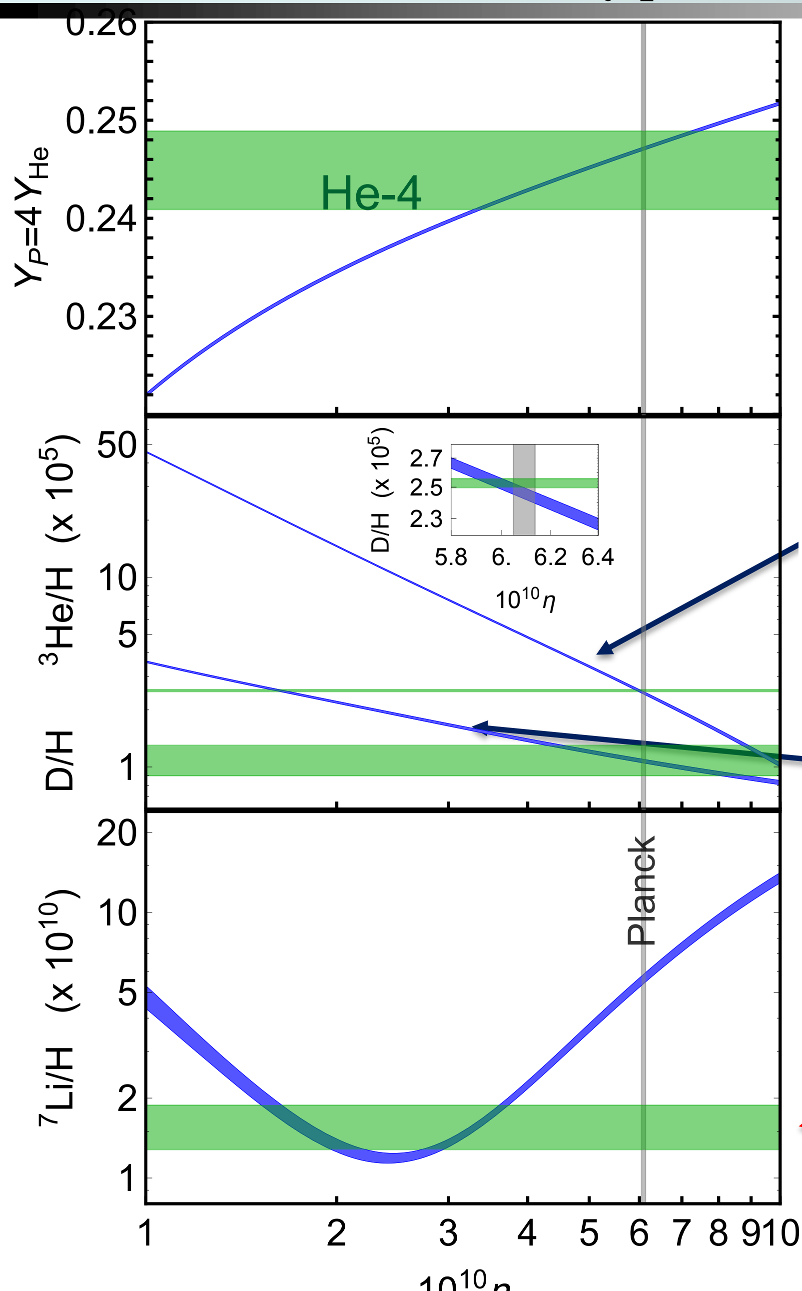
PRIMAT http://www2.iap.fr/users/pitrou/primat.htm (1801.08023, Physics Reports)

Why precision for BBN ?



Baryons abundance

Aver et al. 2015

$$Y_P = 0.2449 \pm 0.0040\,,$$

1.6 %

Deuterium

$$D/H = (2.527 \pm 0.030) \times 10^{-5}$$

1.2 %

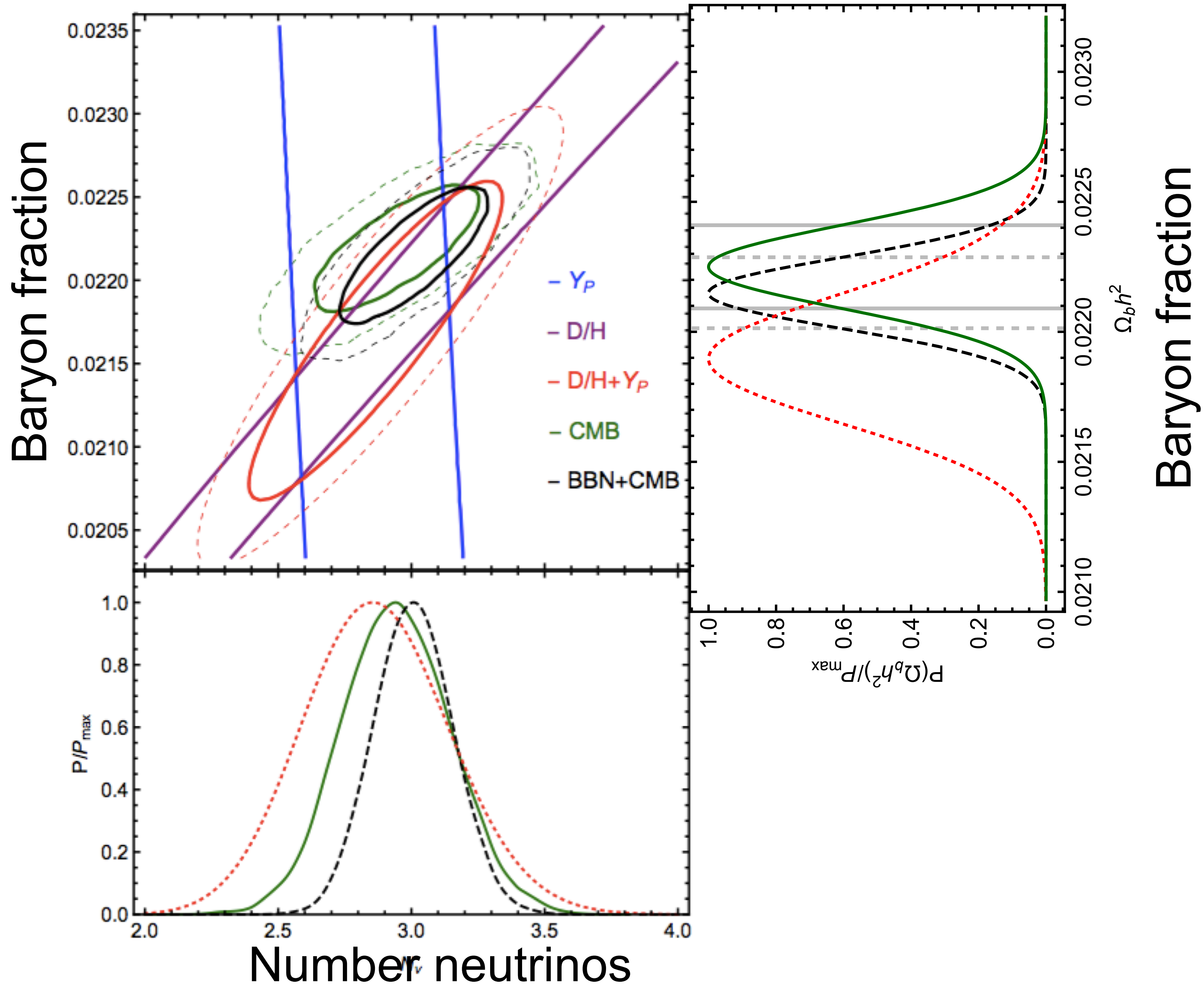
Cooke et al. 2017

He-3

Li-7

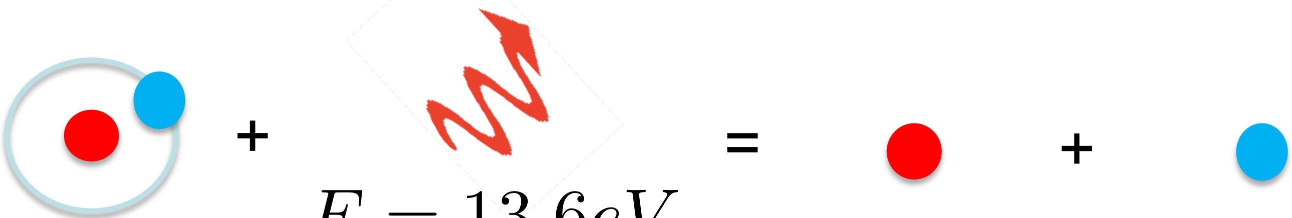
Lithium problem

(see Spite…)



Baryon fraction
0.0235
0.0230
0.0225
0.0220
0.0215
0.0210
0.0205
– Y_P
– D/H
– D/H+Y_P
– CMB
– BBN+CMB
P/P_{max}
1.0
0.8
0.6
0.4
0.2
0.0
2.0
2.5
3.0
3.5
4.0
Number neutrinos
N_ν
$\Omega_b h^2$
Baryon fraction
$P(\Omega_b h^2)/P_{max}$

Chemical reaction

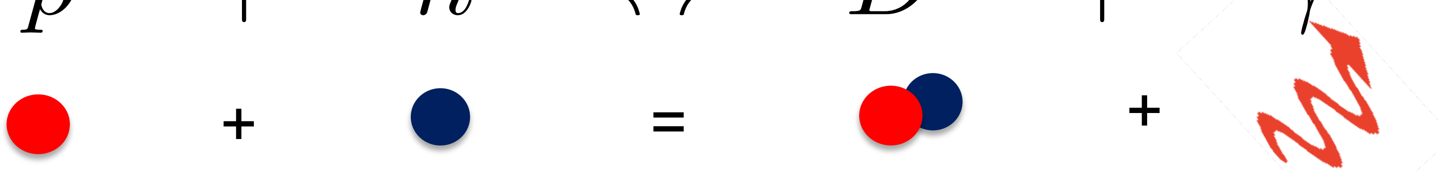


$E = 13.6eV$

$$H + \gamma \leftrightarrow p^+ + e^-$$

Nuclear reaction

$$p + n \leftrightarrow D + \gamma$$

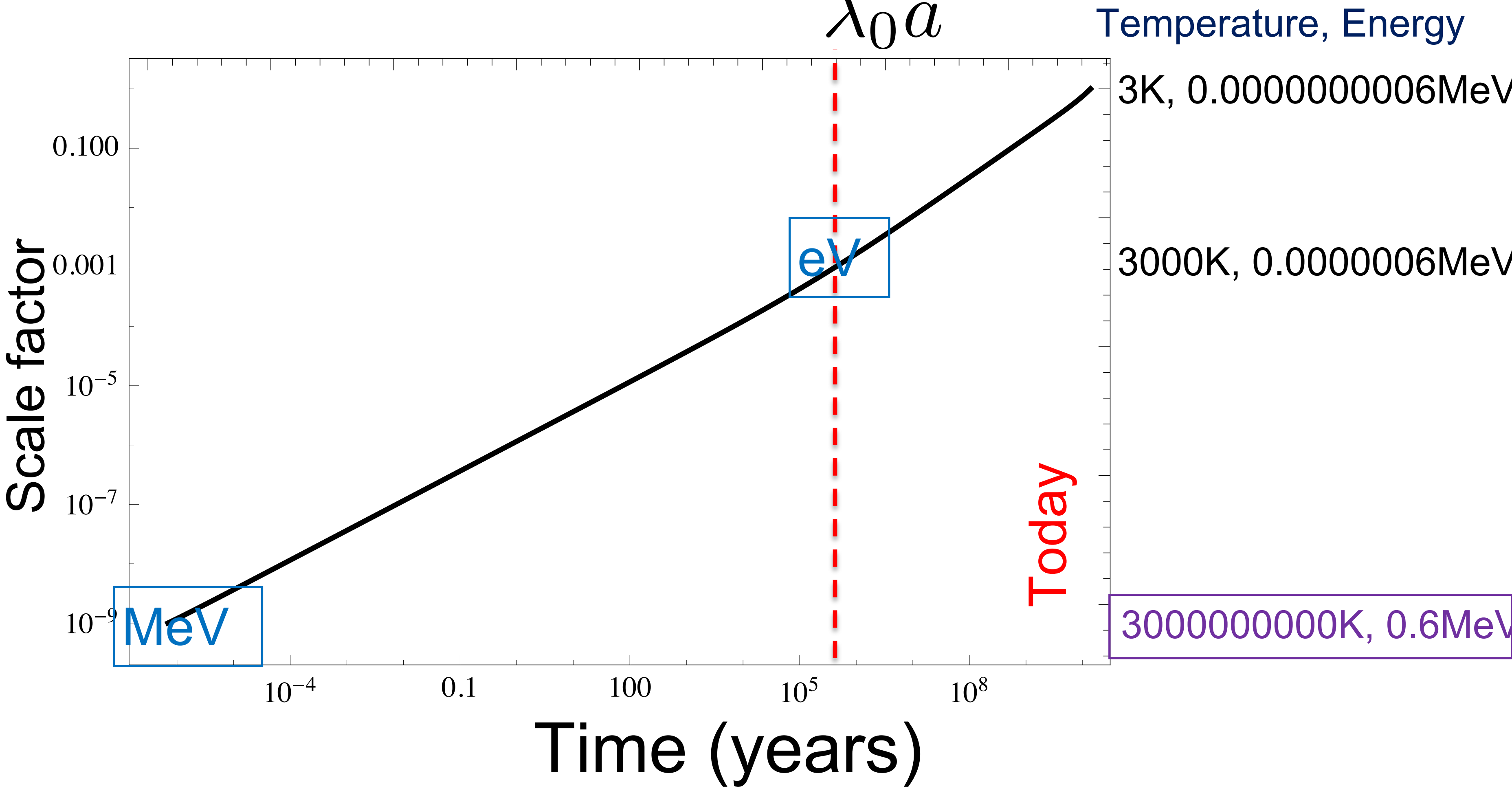


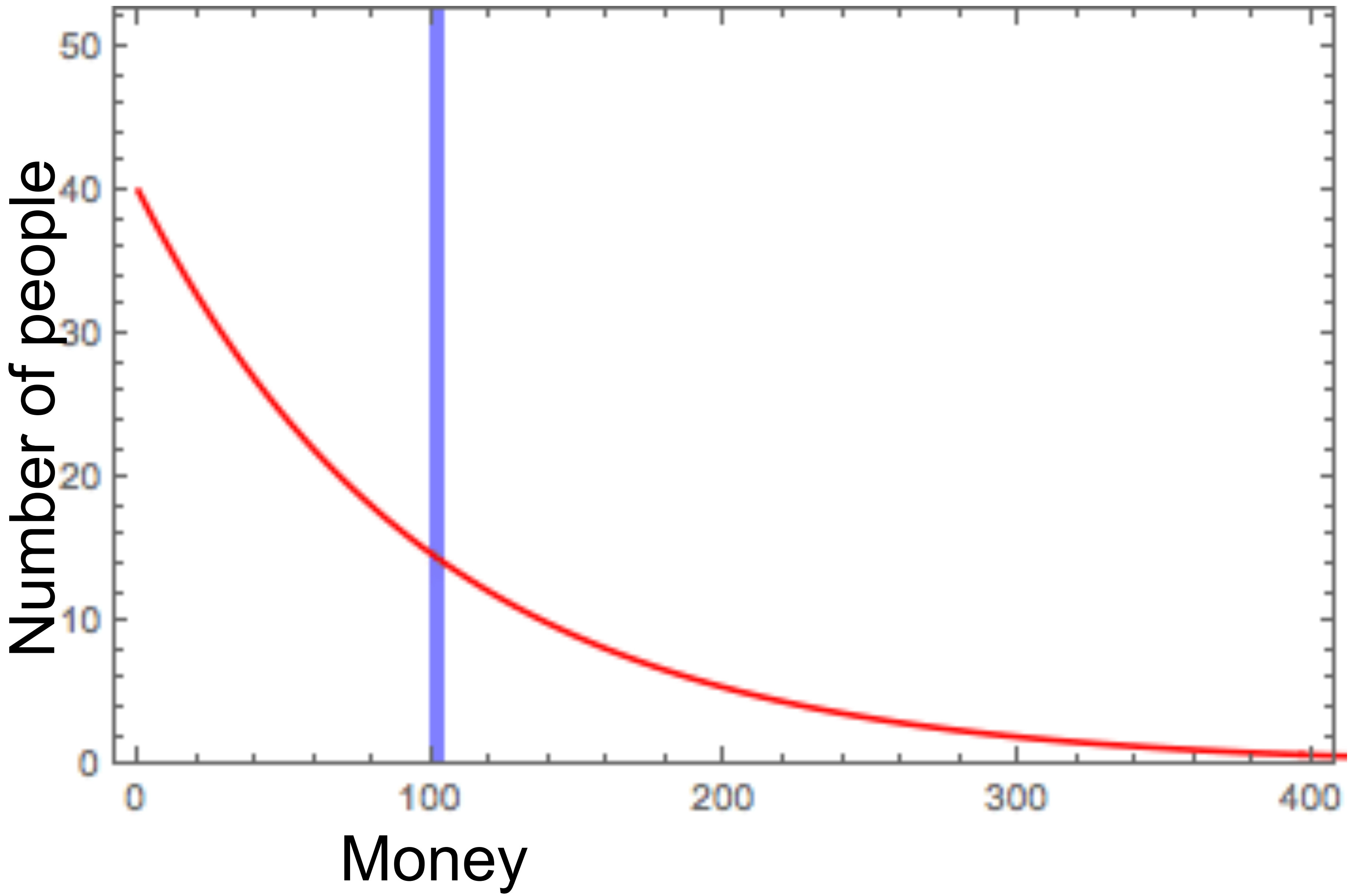
$2.2MeV = 2220000eV$

938.2 + 939.5 = 1875.5 + 2.2

Cosmic scaling

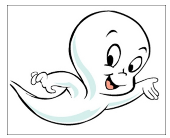
$$E = 2.70\, k_B T = \frac{hc}{\lambda_0 a}$$




Number of people
50
40
30
20
10
0
0
100
200
300
400
Money

Neutron abundances

Weak interactions

+  = +

$$n + \nu \leftrightarrow p^{+} + e^{-}$$

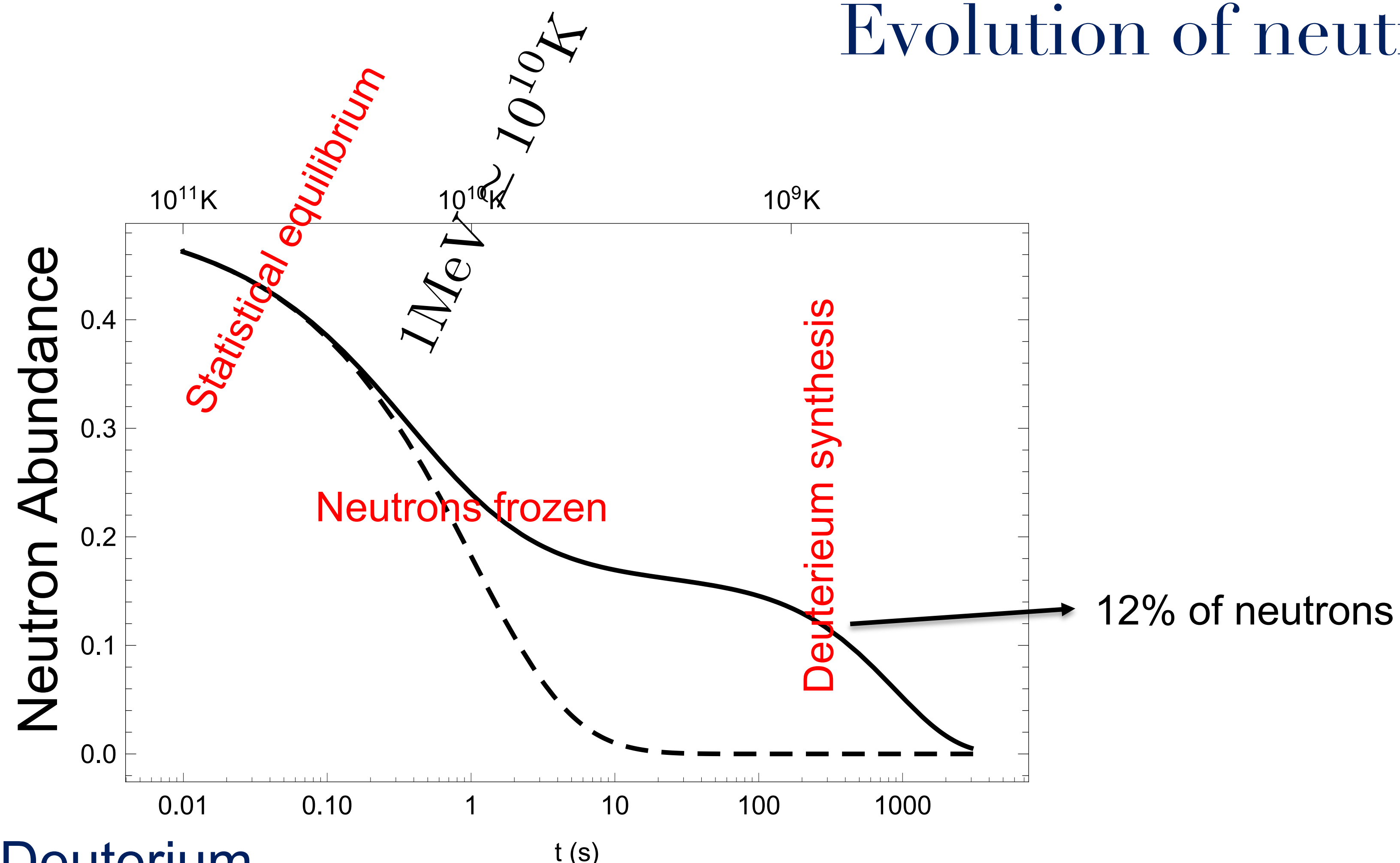
If enough interactions, then statistical equilibrium

$$n = \mathrm{e}^{-\frac{E}{k_B T}}$$

Protons $\quad n = \mathrm{e}^{-\frac{938.2}{k_B T}}$

Neutrons $\quad n = \mathrm{e}^{-\frac{939.5}{k_B T}} = \mathrm{e}^{-\frac{938.2}{k_B T}} \mathrm{e}^{-\frac{1.3}{k_B T}}$

Evolution of neutrons

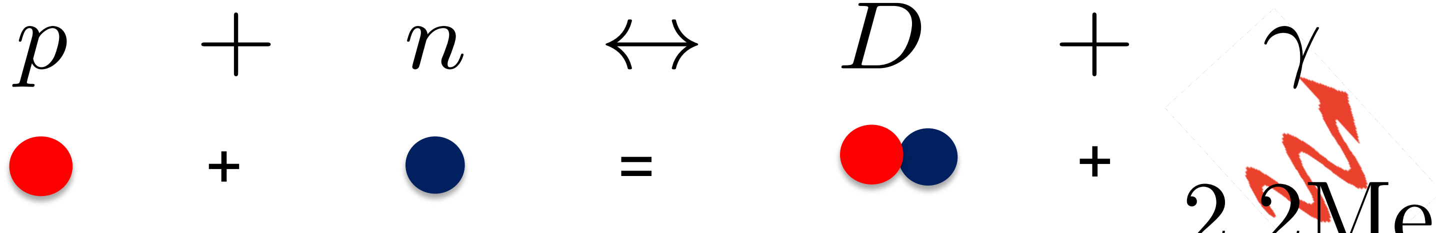


Deuterium

Baryons : $2.4\ 10^{-7}\,\mathrm{cm}^{-3}$

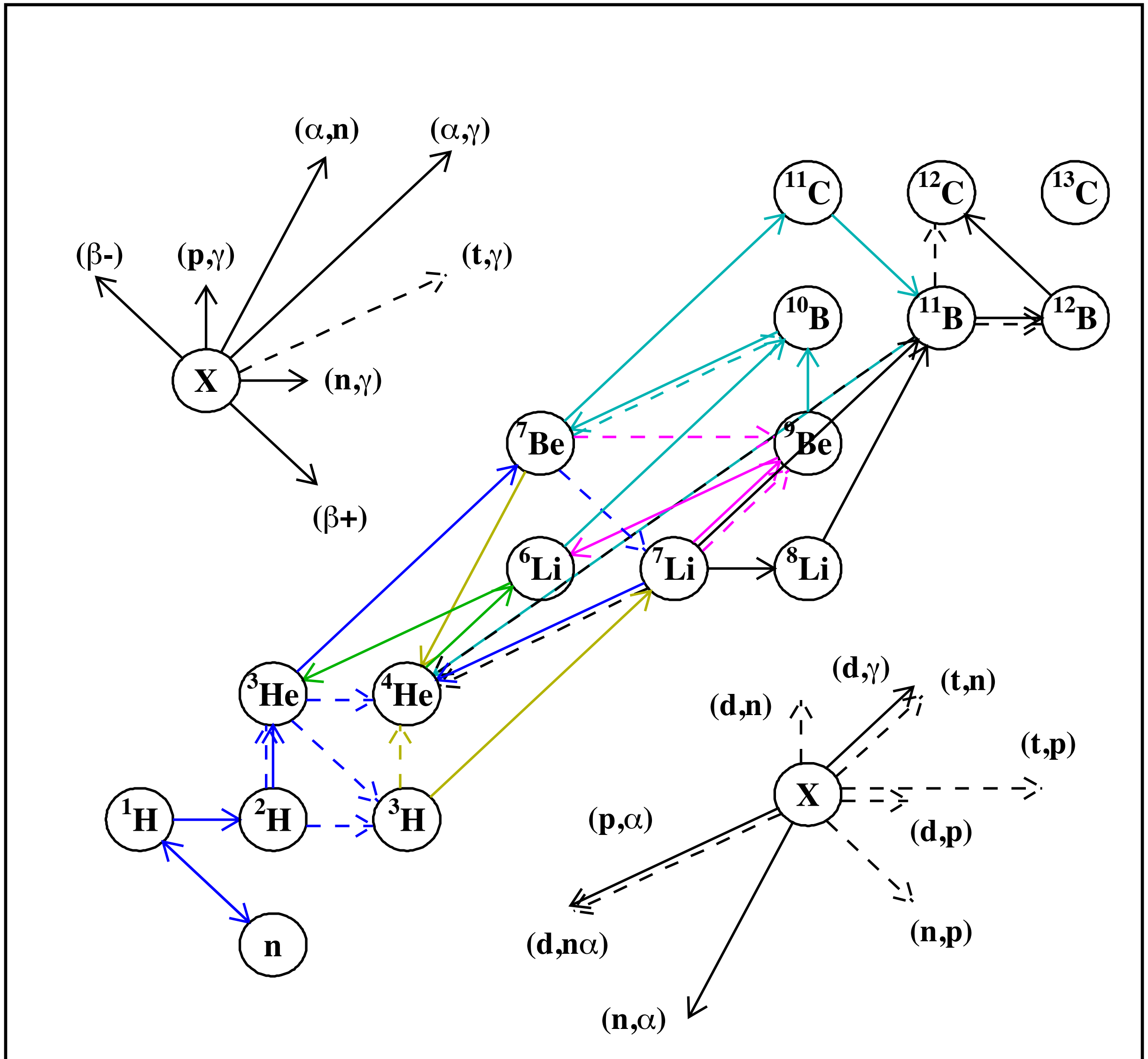
Photons : $411\,\mathrm{cm}^{-3}$

$$p \quad + \quad n \quad \leftrightarrow \quad D \quad + \quad \gamma$$



$2.2\mathrm{MeV}$

Other reactions



Nuclear Reactions

$$Y_i \equiv \frac{n_i}{n_{\rm tot}}$$

Creation Destruction

$$\dot{Y}_{i_1} = \sum_{i_2\dots i_p, j_1\dots j_q} N_{i_1} \left(\Gamma_{j_1\dots j_q \to i_1 \dots i_p} \frac{Y_{j_1}^{N_{j_1}} \dots Y_{j_q}^{N_{j_q}}}{N_{j_1}! \dots N_{j_q}!} - \Gamma_{i_1\dots i_p \to j_1 \dots j_q} \frac{Y_{i_1}^{N_{i_1}} \dots Y_{i_p}^{N_{i_p}}}{N_{i_1}! \dots N_{i_p}!} \right)$$

Tabulated nuclear rates (Alain Coc, Elisabeth Vangioni)
433 reactions + weak rates.

Reverse rates from detailed balance :

$$\frac{\gamma_{j_1\dots j_q \to i_1 \dots i_p}}{\gamma_{i_1\dots i_p \to j_1 \dots j_q}} = \frac{\Pi_{i=i_1\dots i_p} \frac{1}{N_i!} \left[g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} \right]^{N_i}}{\Pi_{j=j_1\dots j_q} \frac{1}{N_j!} \left[g_j \left(\frac{m_j T}{2\pi} \right)^{3/2} \right]^{N_j}} \exp \left(\frac{\sum_{j=1}^{q} m_j - \sum_{i=1}^{p} m_i}{T} \right)$$

Detailed balance

At statistical equilibrium, the rates and reverse rates must be such that

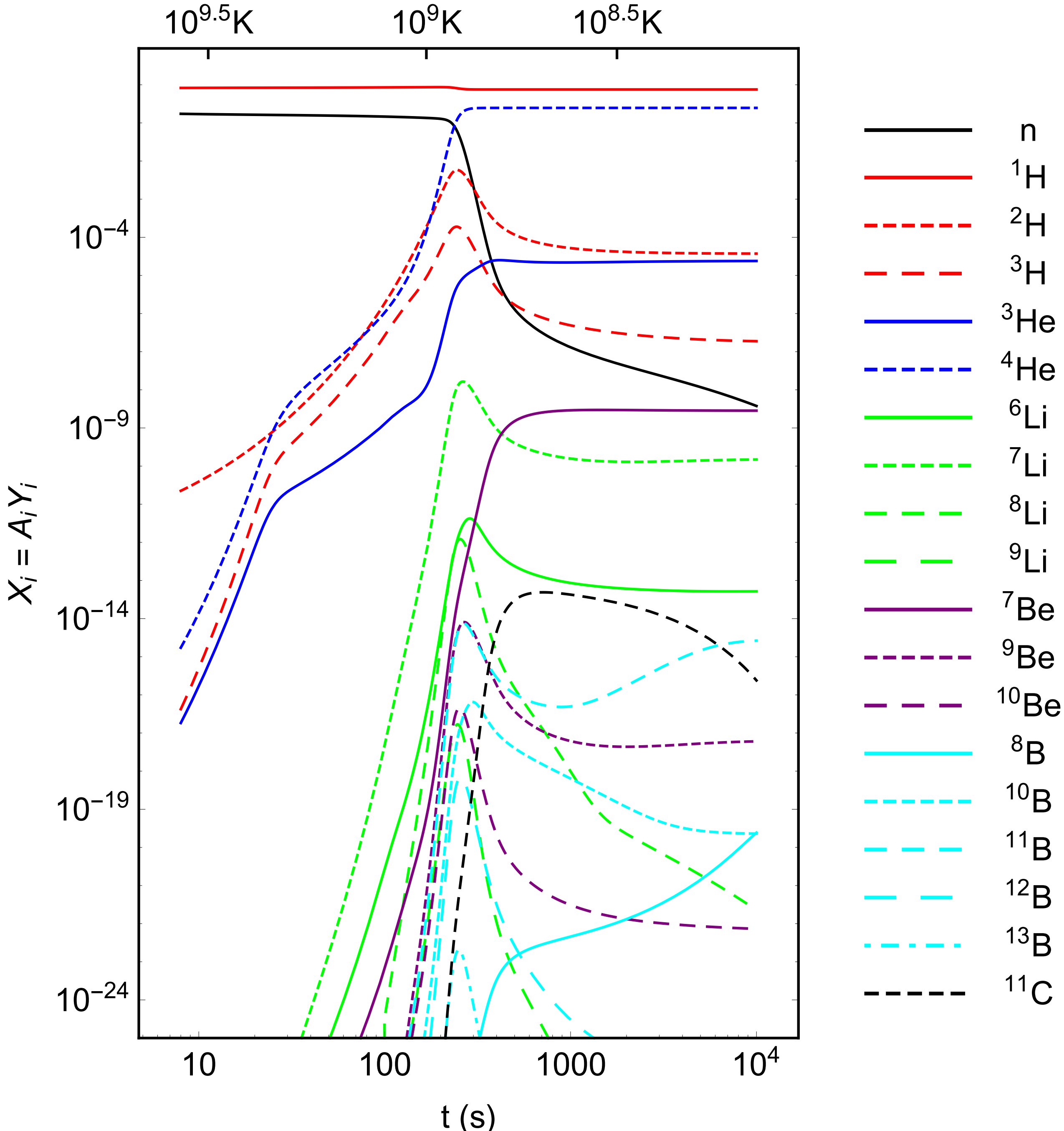
$$n_i^{\mathrm{NSE}} = \frac{g_i m_i^{3/2}}{2^{A_i}} \left(\frac{n_p}{m_p^{3/2}}\right)^{Z_i} \left(\frac{n_n}{m_n^{3/2}}\right)^{A_i - Z_i} \left(\frac{2\pi}{T}\right)^{\frac{3(A_i-1)}{2}} \mathrm{e}^{B_i/T}$$

PRIMAT (PRImordial MATter)

http://www2.iap.fr/users/pitrou/primat.htm

59 nuclides

Z \ N	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0		n												
1	H	^{2}H	^{3}H											
2		^{3}He	^{4}He	^{5}He	^{6}He									
3				^{6}Li	^{7}Li	^{8}Li	^{9}Li							
4				^{7}Be	^{8}Be	^{9}Be	^{10}Be	^{11}Be	^{12}Be					
5				^{8}B	^{9}B	^{10}B	^{11}B	^{12}B	^{13}B	^{14}B	^{15}B			
6				^{9}C	^{10}C	^{11}C	^{12}C	^{13}C	^{14}C	^{15}C	^{16}C			
7						^{12}N	^{13}N	^{14}N	^{15}N	^{16}N	^{17}N			
8						^{13}O	^{14}O	^{15}O	^{16}O	^{17}O	^{18}O	^{19}O	^{20}O	
9									^{17}F	^{18}F	^{19}F	^{20}F		
10									^{18}Ne	^{19}Ne	^{20}Ne	^{21}Ne	^{22}Ne	^{23}Ne
11										^{20}Na	^{21}Na	^{22}Na	^{23}Na	



10^9.5K
10^9K
10^8.5K
10^−4
10^−9
10^−14
10^−19
10^−24
X_i = A_i Y_i
10
100
1000
10^4
t (s)
n
1H
2H
3H
3He
4He
6Li
7Li
8Li
9Li
7Be
9Be
10Be
8B
10B
11B
12B
13B
11C

Precision BBN

Numerical Method

1) Solve for plasma (and cosmology) t, a, T

2) Compute weak rates *with all small corrections*

3) Solve nuclear network (uncertainty on nuclear rates)

This is valid because

1) Baryons are subdominant.

2) Energy release by weak reactions is negligible.

3) Energy release by nuclear reactions is also negligible

Plasma thermodynamics

$$n = g \int f(p) \frac{4\pi p^2 \mathrm{d}p}{(2\pi)^3}$$

$$\rho = g \int f(p) E \frac{4\pi p^2 \mathrm{d}p}{(2\pi)^3}$$

$$P = g \int f(p) \frac{p^2}{3E} \frac{4\pi p^2 \mathrm{d}p}{(2\pi)^3}$$

Conservation of Entropy $s = \frac{\rho + P}{T}$

$$sa^3 = \mathrm{Cte}$$

$$a(T) \leftrightarrow T(a)$$

Solve for cosmological evolution

$$\rho_{\text{plasma}} = \rho_{e^+} + \rho_{e^-} + \rho_\gamma$$

$$\rho_{\text{rad}} = \rho_{\text{neutrinos}} + \rho_{\text{plasma}}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\text{tot}}(T(a))$$

Allows to obtain a(t) and t(a)

a(t) : Friedmann equation (GR)

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

$$= \frac{8\pi G}{3}\left(\frac{\rho_{\rm matter}}{a^3} + \frac{\rho_{\rm rad}}{a^4}\right)$$

Radiation dominated universe $a \propto t^{1/2}$

$$\rho_{\rm rad} \propto T^4 \propto a^{-4}$$

$$\boxed{T^2 \propto 1/t}$$

QED Plasma effects

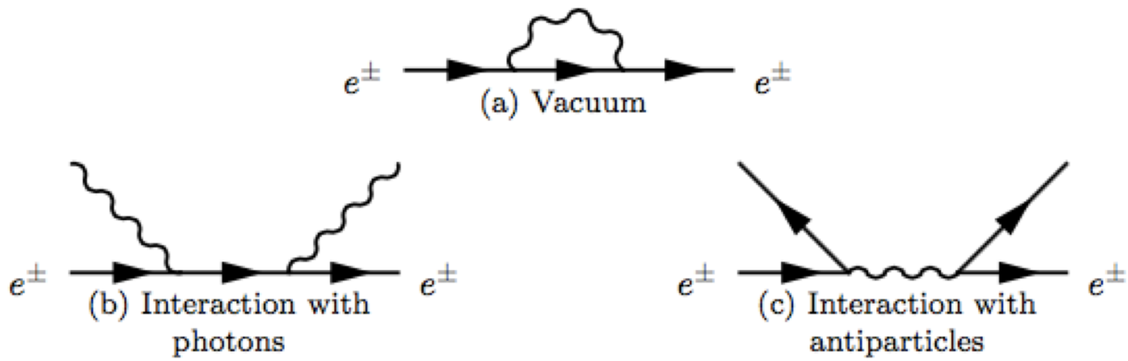


FIG. 5 *Top :* electron/positron self-energy. *Bottom :* electron/positron mass shift from interaction with plasma.

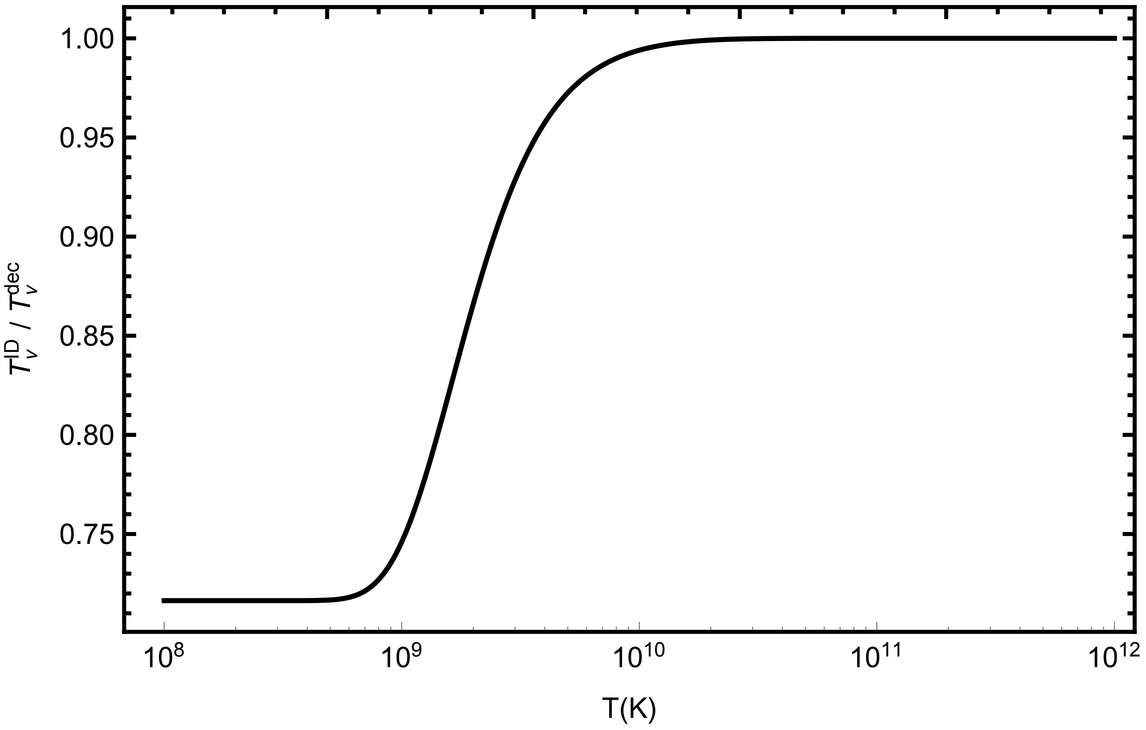
FIG. 6 *Left :* photon self-energy. *Right :* photon mass shift from interaction with electron/positron plasma.

1) Modified pressure
2) Modified energy density

Incomplete decoupling of neutrinos

Around $0.511\,\mathrm{MeV}$ $e^{+} + e^{-} \rightarrow 2\gamma$

$$\frac{T_\nu}{T_\gamma}$$



However there are some residual

$$e^{+} + e^{-} \rightarrow \nu + \bar{\nu}$$

General expression of weak rates

$$n + \nu \leftrightarrow p + e^-$$
$$n \leftrightarrow p + e^- + \bar{\nu}$$
$$n + e^+ \leftrightarrow p + \bar{\nu}$$

$$\dot{n}_n + 3Hn_n = -n_n\Gamma_{n\to p} + n_p\Gamma_{p\to n}$$
$$\dot{n}_p + 3Hn_p = -n_p\Gamma_{p\to n} + n_n\Gamma_{n\to p}$$

$$n_n\Gamma = \int \Pi_i[\mathrm{d}^3\mathbf{p}_i](2\pi)^4\delta^4\left(\underline{p}_n - \underline{p}_p + \alpha_\nu\underline{p}_\nu + \alpha_e\underline{p}_e\right)|M|^2 f_n(E_n)[1-f_p(E_p)]f_\nu(\alpha_\nu E_\nu)f_e(\alpha_e E_e)$$

$$[\mathrm{d}^3\mathbf{p}] \equiv \frac{\mathrm{d}^3\mathbf{p}}{2E(2\pi)^3} = \frac{4\pi p^2\mathrm{d}p}{2E(2\pi^3)}$$

$$g(-E) = 1 - g(E)$$

Fermi-Dirac Property

Interaction Hamiltonian

$$\mathcal{H}_I = \frac{G_F}{\sqrt{2}} J^{\mu}_{e\nu} J_{pn,\,\mu}$$

$$J^{\mu}_{e\nu} = \bar{\boldsymbol{\nu}} \gamma^{\mu}(1-\gamma^5)\mathbf{e}$$

$$J^{\mu}_{pn} = \cos\theta_{\mathrm{C}} \bar{\mathbf{p}} \left(\gamma^{\mu}(1 - g_A \gamma^5) + \mathrm{i}\frac{f_{\mathrm{wm}}}{m_N} 2\Sigma^{\mu\nu} q_{\nu} \right) \mathbf{n}$$

Axial current coupling

Weak-Magnetism

Matrix element

$$\frac{|M|^2}{2^7 G_F^2} = c_{LL}\mathcal{M}_{LL} + c_{RR}\mathcal{M}_{RR} + c_{LR}\mathcal{M}_{LR}$$

$$c_{LL} \equiv \frac{(1+g_A)^2}{4}$$
$$c_{RR} \equiv \frac{(1-g_A)^2}{4}$$
$$c_{LR} \equiv \frac{g_A^2 - 1}{4}.$$

$$\mathcal{M}_{LL} = (\underline{p}_n \cdot \underline{p}_\nu)(\underline{p}_p \cdot \underline{p}_e)$$
$$\mathcal{M}_{RR} = (\underline{p}_n \cdot \underline{p}_e)(\underline{p}_p \cdot \underline{p}_\nu)$$
$$\mathcal{M}_{LR} = m_p m_n (\underline{p}_\nu \cdot \underline{p}_e).$$

BORN approximation method

$$\Delta = m_n - m_p.$$

$$E_n - E_p = \Delta + \delta Q_1 + \delta Q_2 + \delta Q_3$$

$$\delta Q_1 \equiv -\frac{\mathbf{p}_n \cdot \mathbf{q}}{m_N}$$

$$\delta Q_2 \equiv -\frac{|\mathbf{q}|^2}{2m_N}$$

$$\delta Q_3 \equiv \frac{|\mathbf{p}_a|^2}{2}\left(\frac{1}{m_n} - \frac{1}{m_p}\right) \simeq -\frac{|\mathbf{p}_a|^2 \Delta}{2m_N^2}.$$

$$\mathbf{q} \equiv \mathbf{p}_p - \mathbf{p}_n = \alpha_\nu \mathbf{p}_\nu + \alpha_e \mathbf{p}_e$$

BORN approximation : Dirac expansion

$$n_n\Gamma = \int \frac{d^3\mathbf{p}_n d^3\mathbf{p}_e d^3\mathbf{p}_\nu}{2^4(2\pi)^8} \delta\left(E_n - E_p + \alpha_e E_e + \alpha_\nu E_\nu\right) \frac{|M|^2}{E_n E_p E_e E_\nu} f_n(E_n) f_\nu(\alpha_\nu E_\nu) f_e(\alpha_e E_e)$$

Born order

Finite Nucleon mass corrections

$$\delta\left(E_n - E_p + \alpha_e E_e + \alpha_\nu E_\nu\right) \simeq \delta(\Sigma) + \delta'(\Sigma)\left(\sum_{i=1}^{3}\delta Q_i\right) + \frac{1}{2}\delta''(\Sigma)(\delta Q_1)^2$$

$$\Sigma \equiv \Delta + \alpha_e E_e + \alpha_\nu E_\nu$$

Born order

Corrections

$$\frac{\mathcal{M}_{LL}}{\Pi_i E_i} \to 1 - \frac{\mathbf{p}_n}{m_N}\cdot\left(\frac{\mathbf{p}_e}{E_e} + \frac{\mathbf{p}_\nu}{E_\nu}\right) - \frac{\alpha_\nu|\mathbf{p}_\nu|^2}{m_N E_\nu}$$

$$\frac{\mathcal{M}_{RR}}{\Pi_i E_i} \to 1 - \frac{\mathbf{p}_n}{m_N}\cdot\left(\frac{\mathbf{p}_e}{E_e} + \frac{\mathbf{p}_\nu}{E_\nu}\right) - \frac{\alpha_e|\mathbf{p}_e|^2}{m_N E_e}$$

$$\frac{\mathcal{M}_{LR}}{\Pi_i E_i} \to \left(1 - \frac{|\mathbf{p}_n|^2}{m_N^2}\right)\left(1 - \frac{\mathbf{p}_e\cdot\mathbf{p}_\nu}{E_e E_\nu}\right).$$

BORN approximation

$$\overline{\Gamma}_{n\to p} = \overline{\Gamma}_{n\to p+e} + \overline{\Gamma}_{n+e\to p}$$

$$= K \int_0^\infty p^2 \mathrm{d}p [\chi_+(E) + \chi_+(-E)] \,,$$

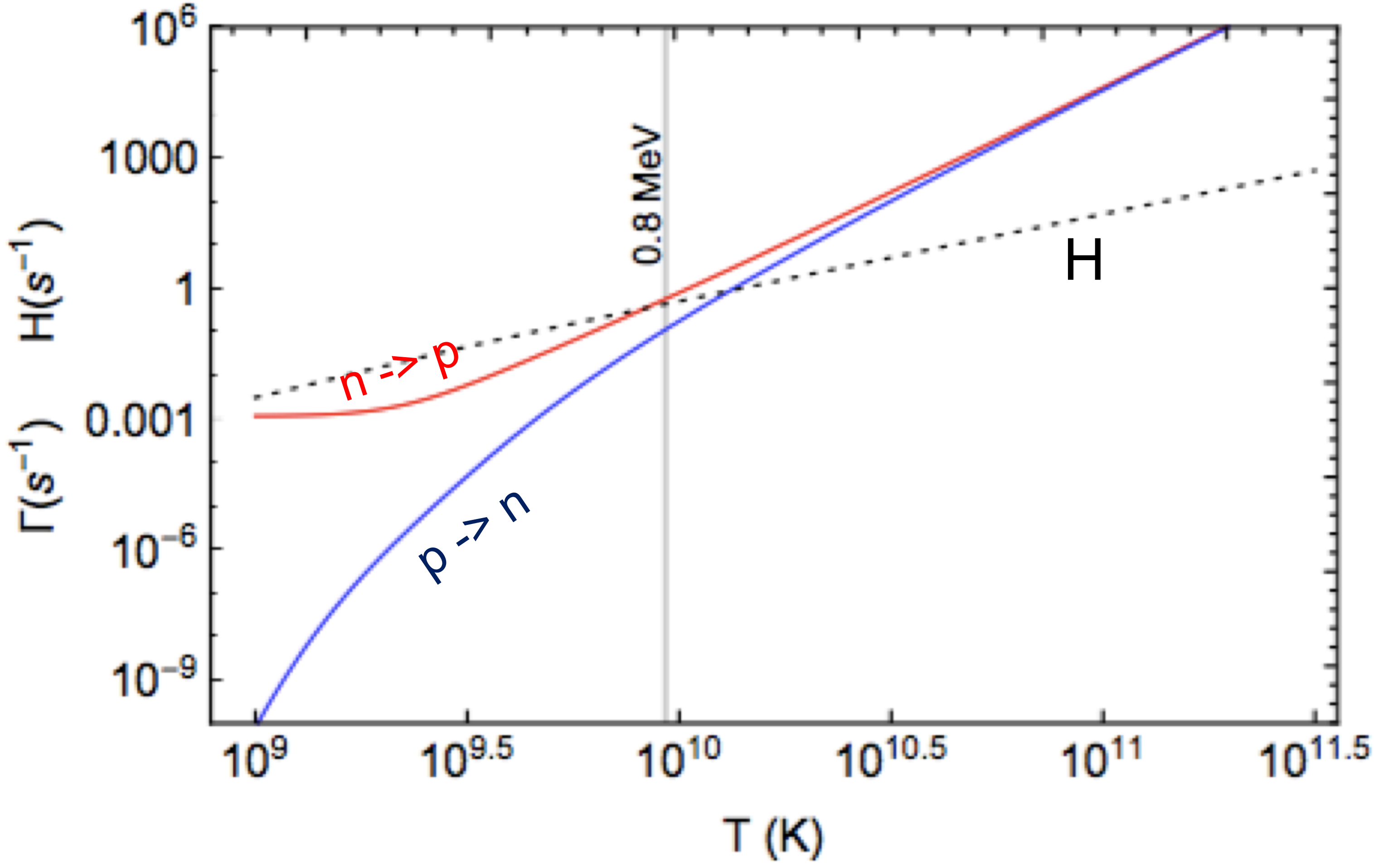
$$\chi_\pm(E) \equiv (E_\nu^\mp)^2 g_\nu(E_\nu^\mp) g(-E) \,,$$

$$E_\nu^\mp \equiv E \mp \Delta \,,$$

$$K \equiv \frac{4G_W^2(1+3g_A^2)}{(2\pi)^3} \,.$$

$$G_W = G_F V_{ud}$$

BORN approximation rates



Detailed balance

$$\begin{aligned} \dot{n}_n + 3Hn_n &= -n_n\Gamma_{n\to p} + n_p\Gamma_{p\to n} \\ \dot{n}_p + 3Hn_p &= -n_p\Gamma_{p\to n} + n_n\Gamma_{n\to p} \end{aligned} = 0$$

$$\frac{\Gamma_{p\to n}}{\Gamma_{n\to p}} = \mathrm{e}^{-(m_n - m_p)/T}$$

Detailed balance relation

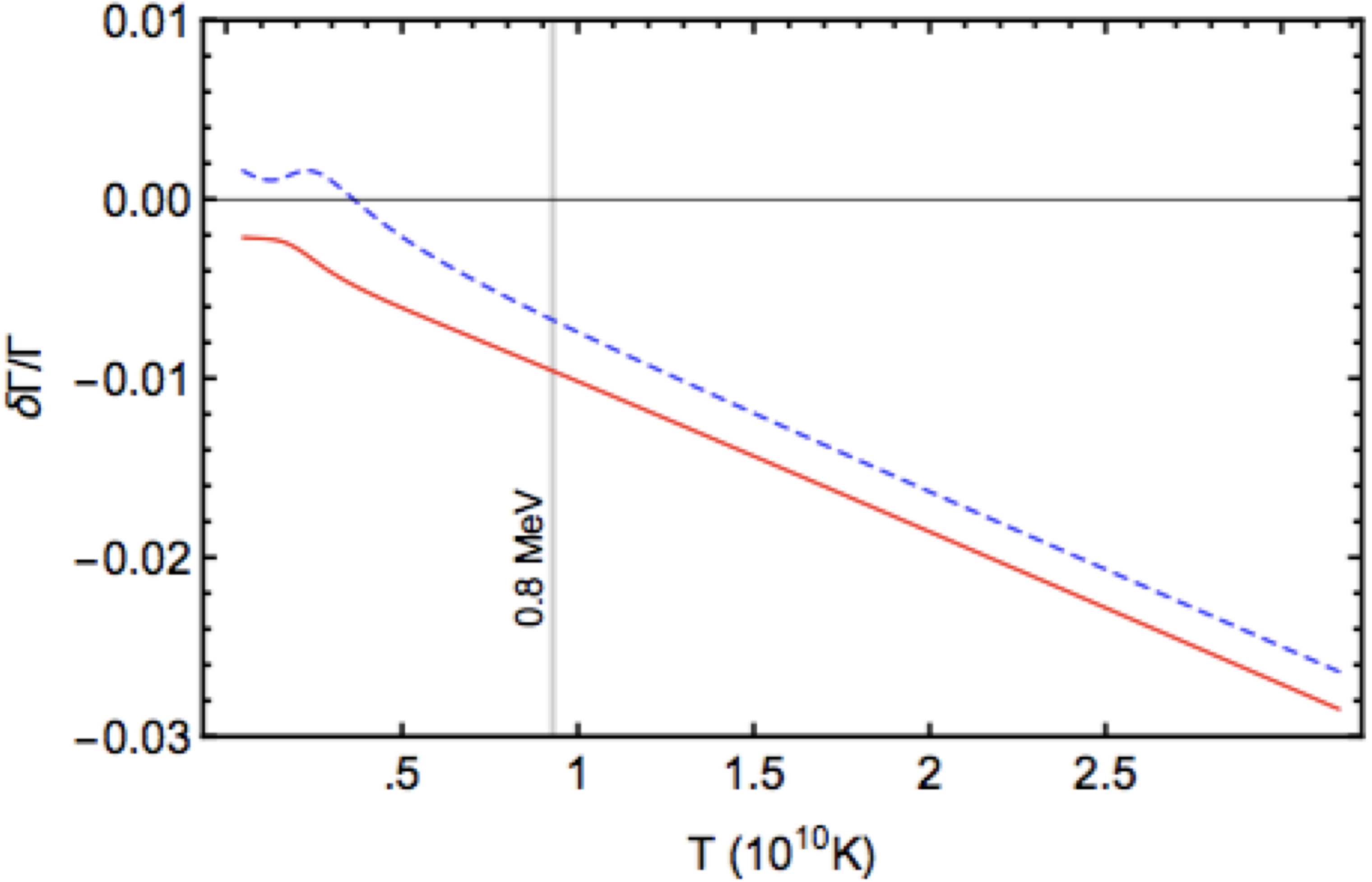
Finite nucleon mass corrections

$$\delta\Gamma^{\rm FM}_{n\to p} = K\int_0^\infty p^2 {\rm d}p\,[\chi^{\rm FM}_{+}(E,g_A)+\chi^{\rm FM}_{+}(-E,g_A)]$$

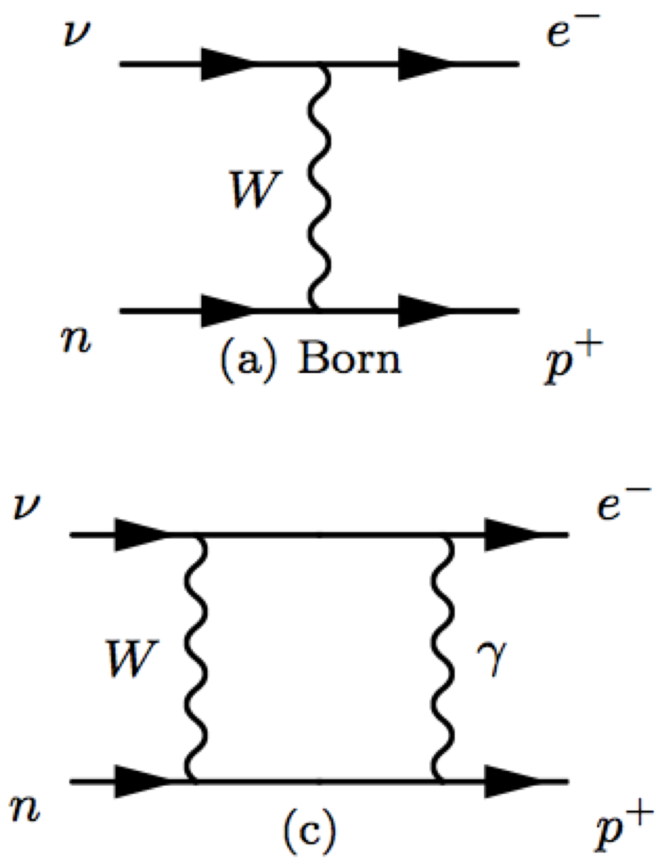
$$\delta\Gamma^{\rm FM}_{p\to n} = K\int_0^\infty p^2 {\rm d}p\,[\chi^{\rm FM}_{-}(E,-g_A)+\chi^{\rm FM}_{-}(-E,-g_A)]\,,$$

$$\begin{aligned}
\chi^{\rm FM}_{\pm}(E,g_A) =\; & \tilde{c}_{LL}\frac{p^2}{m_N E}g_\nu(E_\nu^\mp)g(-E) - \tilde{c}_{RR}\frac{E_\nu^\mp}{m_N}g_\nu^{(2,0)}(E_\nu^\mp)g(-E) \\
& +(\tilde{c}_{LL}+\tilde{c}_{RR})\,\frac{T}{m_N}\left(g_\nu^{(2,1)}(E_\nu^\mp)g(-E)\frac{p^2}{E} - g_\nu^{(3,1)}(E_\nu^\mp)g(-E)\right) \\
& +(\tilde{c}_{LL}+\tilde{c}_{RR}+\tilde{c}_{LR})\left[\frac{T}{2m_N}\left(g_\nu^{(4,2)}(E_\nu^\mp)g(-E)+g_\nu^{(2,2)}(E_\nu^\mp)g(-E)p^2\right)\right. \\
& \qquad\left. +\frac{1}{2m_N}\left(g_\nu^{(4,1)}(E_\nu^\mp)g(-E)+g_\nu^{(2,1)}(E_\nu^\mp)g(-E)p^2\right)\right] \\
& -(\tilde{c}_{LL}+\tilde{c}_{RR}+\tilde{c}_{LR})\,\frac{3T}{2}\left(1-\frac{m_n}{m_p}\right)g_\nu^{(2,1)}(E_\nu^\mp)g(-E) \\
& +\tilde{c}_{LR}\left[-\frac{3T}{m_N}g_\nu^{(2,0)}(E_\nu^\mp)g(-E)+\frac{p^2}{3m_N E}g_\nu^{(3,1)}(E_\nu^\mp)g(-E)+\frac{p^2 T}{3m_N E}g_\nu^{(3,2)}(E_\nu^\mp)g(-E)\right]
\end{aligned}$$

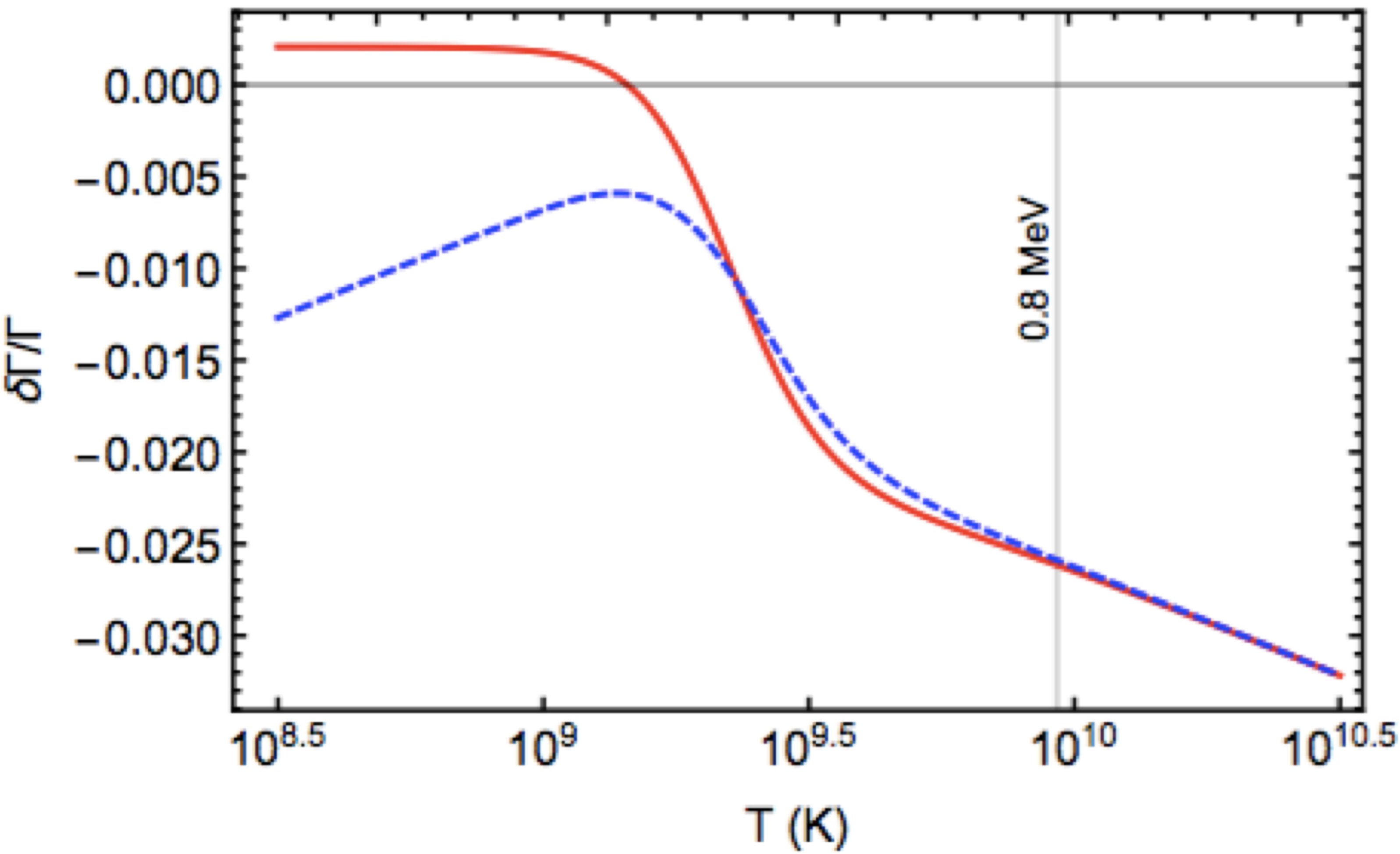
Finite nucleon mass corrections



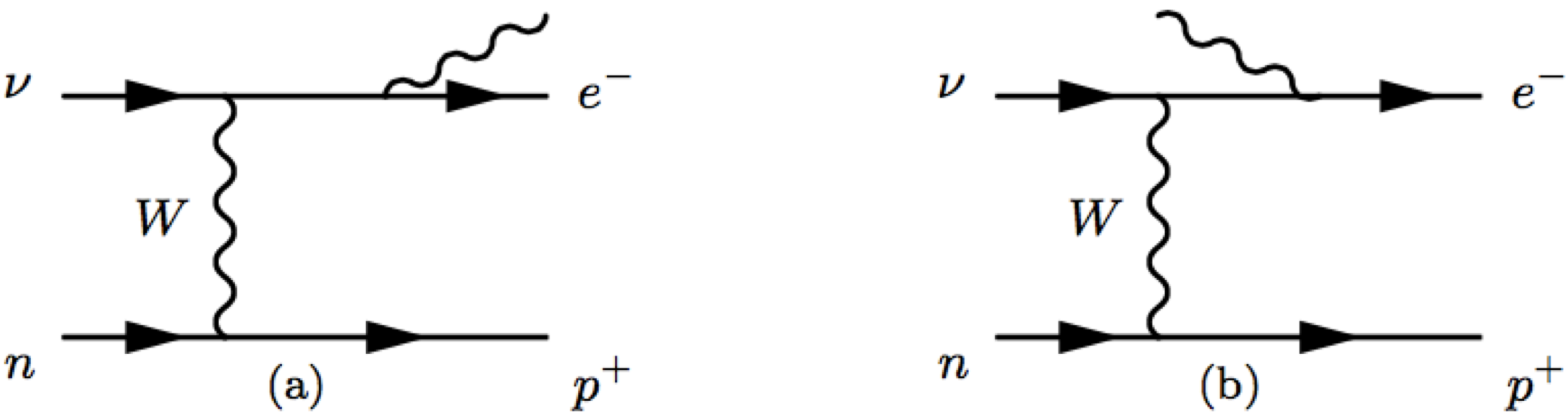
Radiative corrections



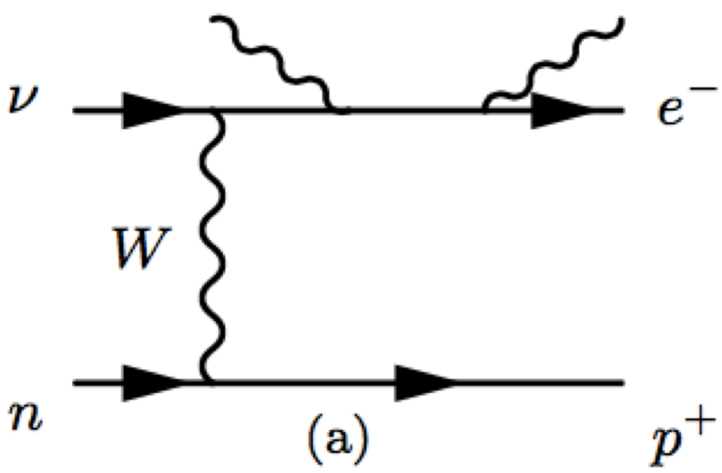
Radiative corrections

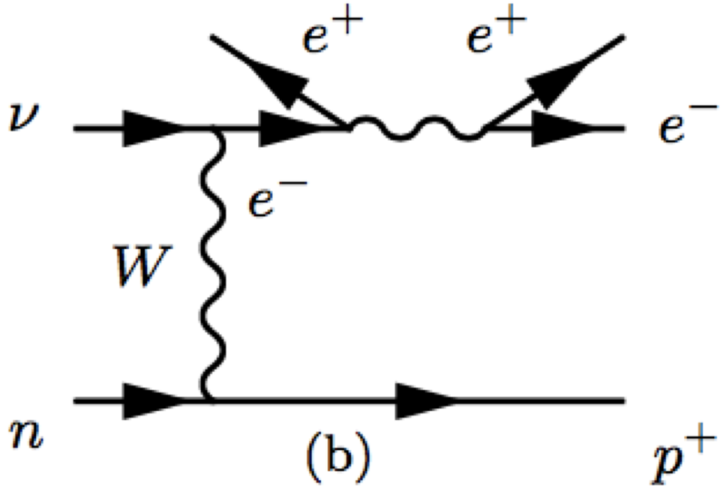


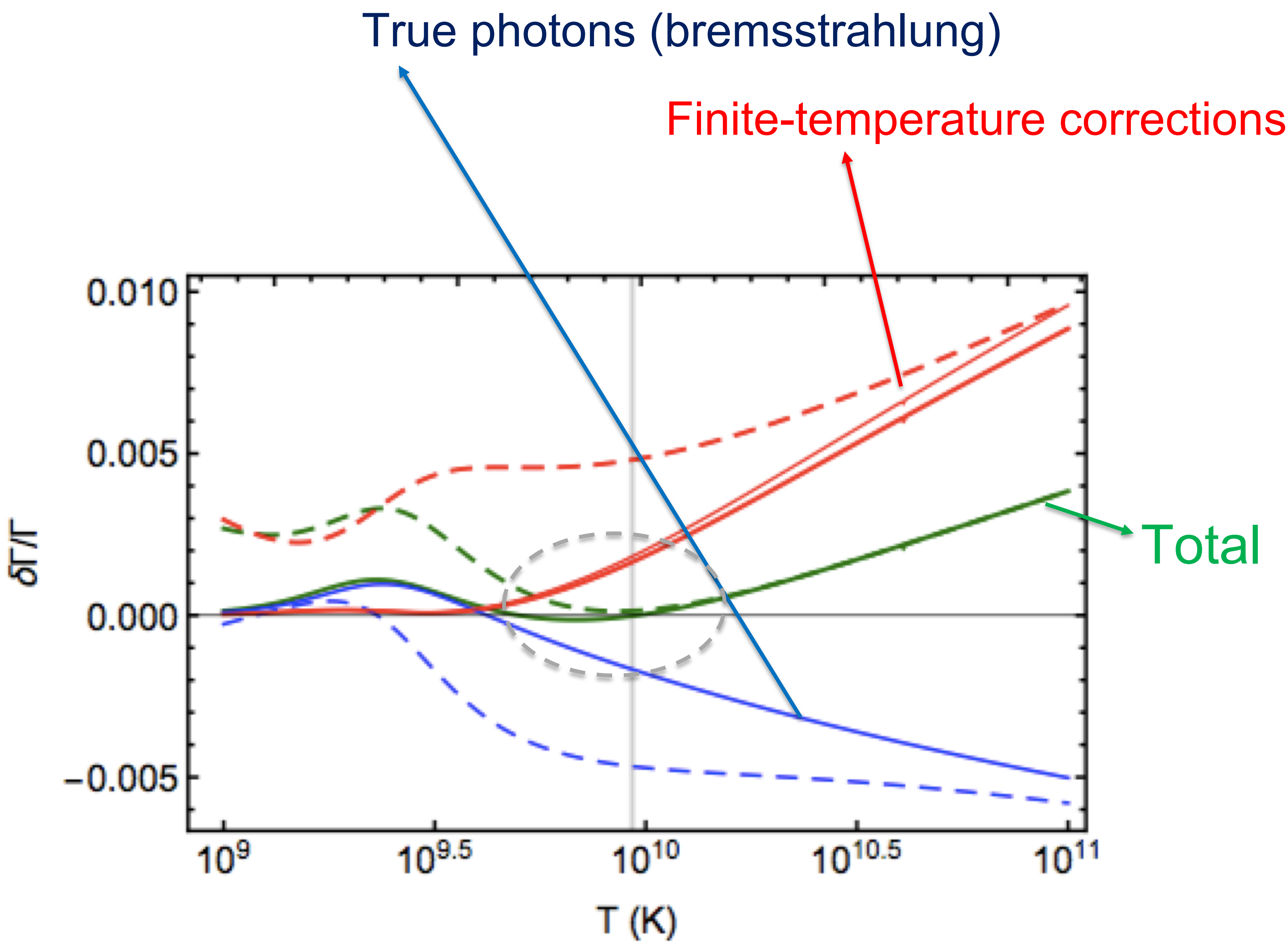
True photons



Finite Temperature corrections

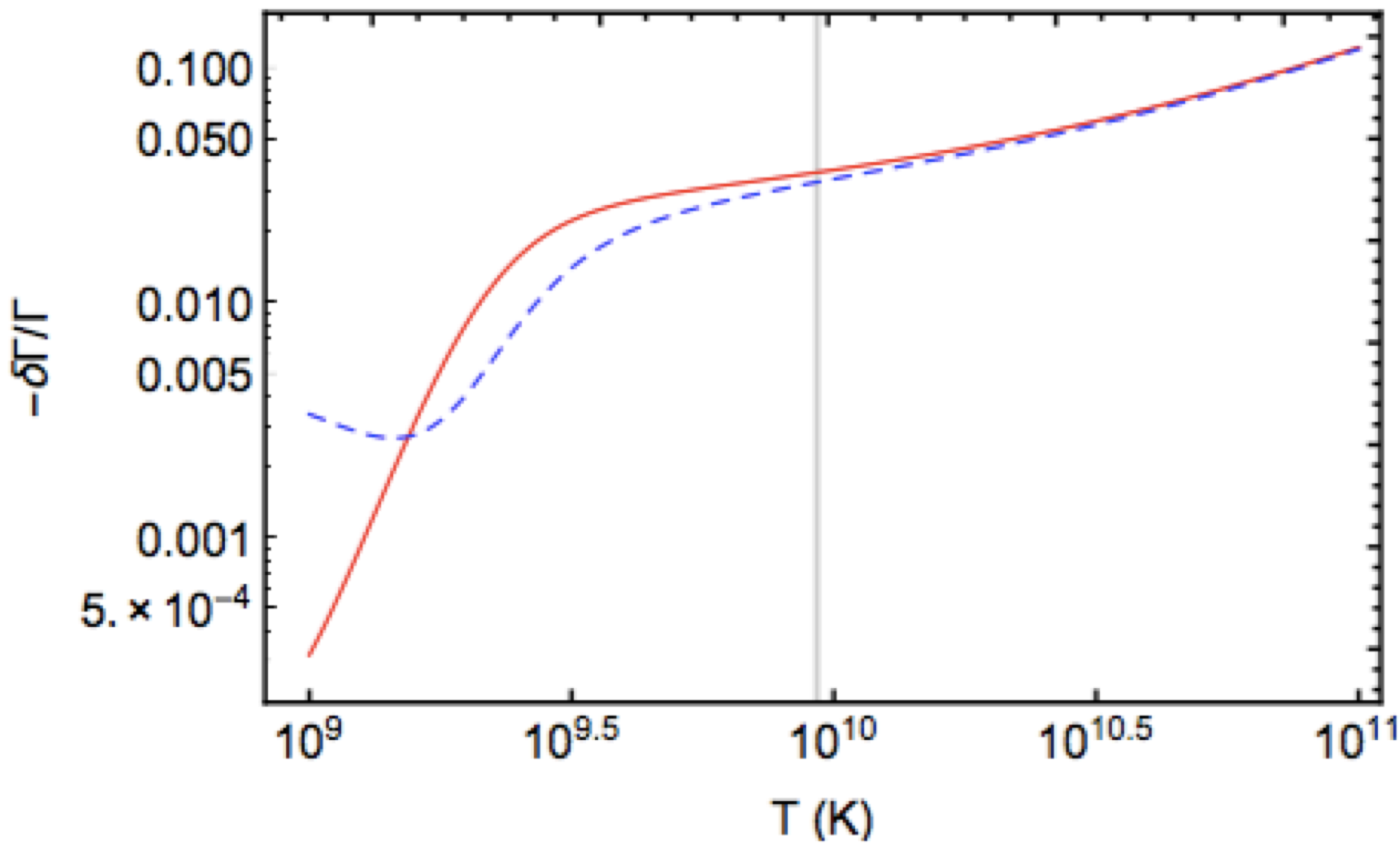






True photons (bremsstrahlung)
Finite-temperature corrections
Total
0.010
0.005
0.000
−0.005
δΓ/Γ
10^9
$10^{9.5}$
10^{10}
$10^{10.5}$
10^{11}
T (K)

Total corrections



Size of corrections

PRIMAT http://www2.iap.fr/users/pitrou/primat.htm

Corrections	Y_P	$\delta Y_P \times 10^4$	$\delta Y_P/Y_P(\%)$	$D/H \times 10^5$	$\Delta(D/H)$ (%)	$^3He/H \times 10^5$	$^7Li/H \times 10^{10}$
Born	0.24276	0	0	2.424	0	1.069	5.637
Born+ID	0.24289	1.2	0.05	2.433	0.37	1.070	5.615
Born+FM	0.24388	11.2	0.46	2.430	0.25	1.070	5.654
Born+FM+WM	0.24404	12.5	0.53	2.431	0.29	1.070	5.657
RCa [Eq. (B30), Non. Rel. Fermi]	0.24586	31.0	1.27	2.441	0.70	1.071	5.684
RCb [Eq. (B35), Non. Rel. Fermi]	0.24589	31.3	1.29	2.441	0.70	1.071	5.685
RC [Eq. (B35), Rel. Fermi]	0.24591	31.5	1.30	2.441	0.70	1.071	5.685
RC+QED-MS	0.24602	32.9	1.36	2.442	0.74	1.071	5.687
RC+QED-Pl	0.24591	31.5	1.30	2.444	0.82	1.072	5.677
RC+ID	0.24602	32.6	1.34	2.450	1.07	1.073	5.663
RC+ID+QED-Pl	0.24602	32.6	1.34	2.453	1.19	1.073	5.655
RC+FM+WM	0.24720	44.4	1.83	2.448	0.99	1.072	5.704
RC+FM+WM+QED-MS	0.24733	45.7	1.88	2.449	1.03	1.073	5.706
RC+FM+WM+QED-Pl	0.24719	44.3	1.82	2.451	1.11	1.073	5.696
RC+FM+WM+ID	0.24725	44.9	1.85	2.457	1.36	1.074	5.681
RC+FM+WM+ThRC (No BS)	0.24751	47.5	1.96	2.450	1.07	1.073	5.709
RC+FM+WM+ThRC+BS	0.24720	44.4	1.83	2.448	0.99	1.072	5.704
RC+FM+WM+ThRC+BS+ID+QED-Pl	0.24724	44.8	1.85	2.460	1.49	1.074	5.673

He-4 correction 1.85%

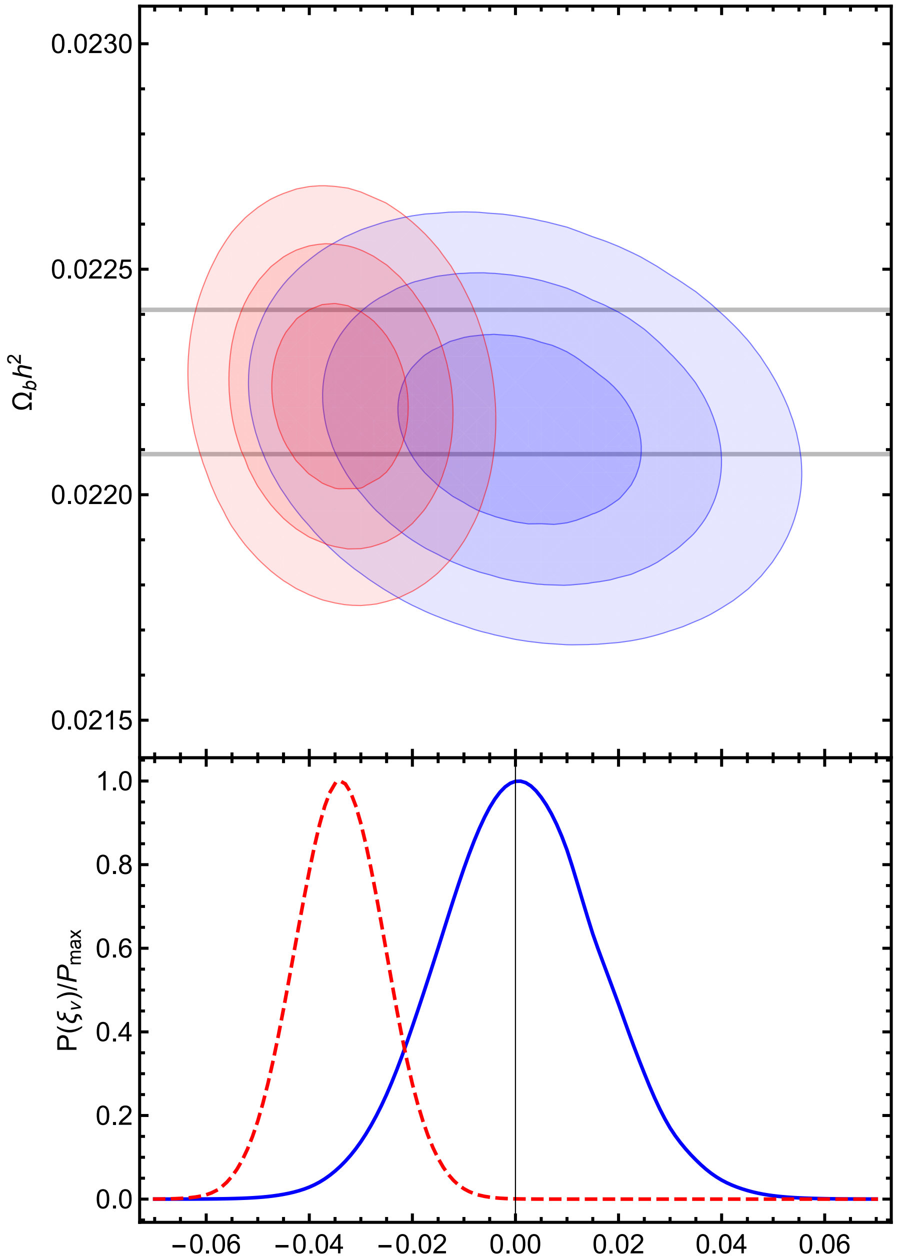
Deuterium correction 1.49%

Thank you

PRIMAT

http://www2.iap.fr/users/pitrou/primat.htm

Neutrino degeneracy



$$g_\nu(E) \equiv \begin{cases} g_\nu^+(E) \equiv \frac{1}{e^{(E/T_\nu - \xi_\nu)}+1} \\ g_\nu^-(E) \equiv \frac{1}{e^{(E/T_\nu + \xi_\nu)}+1} \end{cases}$$

Chemical potential