

Theoretical introduction to (charged) lepton flavour violation

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- motivations for lepton flavour violation
- status of lepton flavour violation
- theoretical expectations/predictions

Motivations for lepton flavour violation (LFV)

We know that flavour is violated in the lepton sector, since neutrinos oscillate ($\nu_\mu \leftrightarrow \nu_e$ violates both L_e and L_μ)

$W \rightarrow l_\alpha \quad [\alpha = e, \mu, \tau]$
 $\nu_\alpha \equiv \sum_i U_{\alpha i} \nu_i \quad [i = 1, 2, 3]$
flavour eigenstate mass eigenstate with mass m_i

Since the PMNS matrix U appears in charged lepton current, would naively expect strong flavour violating effects in the charged lepton sector too (i.e. processes such as $\mu \rightarrow e \gamma$ and $\mu \rightarrow 3e$ should be observed).

This is not the case due to a GIM mechanism: LFV is strongly suppressed (and in practice unobservable) in the Standard Model

But we have good reasons to believe that there is new physics beyond the SM (dark matter, baryon asymmetry...) \Rightarrow generally new sources of LFV

Indeed, many well-motivated new physics scenarios predict large flavour violations in the charged lepton sector:

- supersymmetry
- extra dimensions
- little Higgs models
- ...

→ the absence of sizeable SM contributions makes LFV a unique probe of new physics

Further motivation: connection with neutrino physics

The smallness of neutrino masses suggests a specific mechanism of mass generation \Rightarrow new particles with flavour violating couplings to leptons

→ LFV could tell us something about the origin of neutrino masses

Status of lepton flavour violation

So far lepton flavour violation has been observed only in the neutrino sector (oscillations). Experimental upper bounds on LFV processes involving charged leptons:

Table 1.1: Present limits on rare μ decays.

mode	limit (90% C.L.)	year	Exp./Lab.
$\mu^+ \rightarrow e^+ \gamma$	1.2×10^{-11}	2002	MEGA / LAMPF
$\mu^+ \rightarrow e^+ e^+ e^-$	1.0×10^{-12}	1988	SINDRUM I / PSI
$\mu^+ e^- \leftrightarrow \mu^- e^+$	8.3×10^{-11}	1999	PSI
$\mu^- \text{ Ti} \rightarrow e^- \text{ Ti}$	6.1×10^{-13}	1998	SINDRUM II / PSI
$\mu^- \text{ Ti} \rightarrow e^+ \text{ Ca}^*$	3.6×10^{-11}	1998	SINDRUM II / PSI
$\mu^- \text{ Pb} \rightarrow e^- \text{ Pb}$	4.6×10^{-11}	1996	SINDRUM II / PSI
$\mu^- \text{ Au} \rightarrow e^- \text{ Au}$	7×10^{-13}	2006	SINDRUM II / PSI

Table 1.2: 90% C.L. upper limits on selected LFV tau decays by Babar and BELLE.

Channel	Babar		BELLE	
	\mathcal{L} (fb^{-1})	\mathcal{B}_{UL} (10^{-8})	\mathcal{L} (fb^{-1})	\mathcal{B}_{UL} (10^{-8})
$\tau^\pm \rightarrow e^\pm \gamma$	232	11	535	12
$\tau^\pm \rightarrow \mu^\pm \gamma$	232	6.8	535	4.5
$\tau^\pm \rightarrow \ell^\pm \ell^\mp \ell^\pm$	92	11 - 33	535	2 - 4
$\tau^\pm \rightarrow e^\pm \pi^0$	339	13	401	8.0
$\tau^\pm \rightarrow \mu^\pm \pi^0$	339	11	401	12
$\tau^\pm \rightarrow e^\pm \eta$	339	16	401	9.2
$\tau^\pm \rightarrow \mu^\pm \eta$	339	15	401	6.5
$\tau^\pm \rightarrow e^\pm \eta'$	339	24	401	16
$\tau^\pm \rightarrow \mu^\pm \eta'$	339	14	401	13

[W/G3 report]

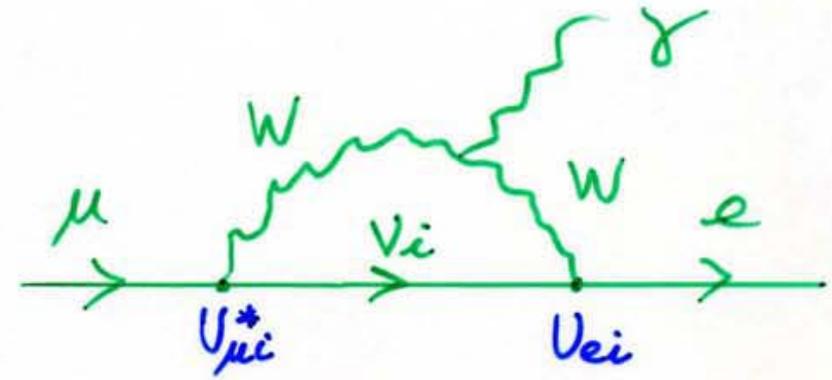
Also strong constraints on LFV rare decays of mesons:

$$\text{BR} (K_L^0 \rightarrow \mu e) < 4.7 \times 10^{-12}$$

$$\text{BR} (B_d^0 \rightarrow \mu e) < 1.7 \times 10^{-7} \quad [\text{Belle}]$$

$$\text{BR} (B_s^0 \rightarrow \mu e) < 6.1 \times 10^{-6} \quad [\text{CDF}]$$

This is consistent with the Standard Model, in which LFV processes involving charged leptons are suppressed by the tiny neutrino masses



e.g. $\mu \rightarrow e \gamma$:

$$\text{BR}(\mu \rightarrow e \gamma) = \frac{3\alpha}{32\pi} \left| \sum_i U_{\mu i}^* U_{ei} \frac{m_{\nu_i}^2}{M_W^2} \right|^2$$

Using known oscillations parameters ($U = \text{PMNS}$ lepton mixing matrix) and $|U_{e3}| < 0.2$, this gives $\text{BR}(\mu \rightarrow e \gamma) \lesssim 10^{-54}$: inaccessible to experiment!

This makes LFV a unique probe of new physics: the observation of e.g. $\mu \rightarrow e \gamma$ would be an unambiguous signal of new physics (no SM background)

→ very different from the hadronic sector

Conversely, the present upper bounds on LFV processes already put strong constraints on new physics (same as hadronic sector)

Prospects for LFV experiments

$\mu \rightarrow e \gamma$:

- the experiment MEG at PSI has started taking data in sept. 2008
- first results expected in summer 2009
- expects to reach a sensitivity of a few 10^{-13} (factor of 100 improvement) in 3 years of acquisition time

$\mu \rightarrow e$ conversion :

- the project mu2e is under study at FNAL - aims at $\mathcal{O}(10^{-16})$
- the project PRISM/PRIME at J-PARC aims at $\mathcal{O}(10^{-18})$

τ decays :

- LHC experiments limited to $\tau \rightarrow \mu\mu\mu$ – comparable to existing B fact.
- superB factories will probe the $10^{-9} - 10^{-10}$ level

Theoretical expectations/predictions

Many new physics scenarios predict “large” LFV rates: supersymmetry, extra dimensions, little Higgs models, ...

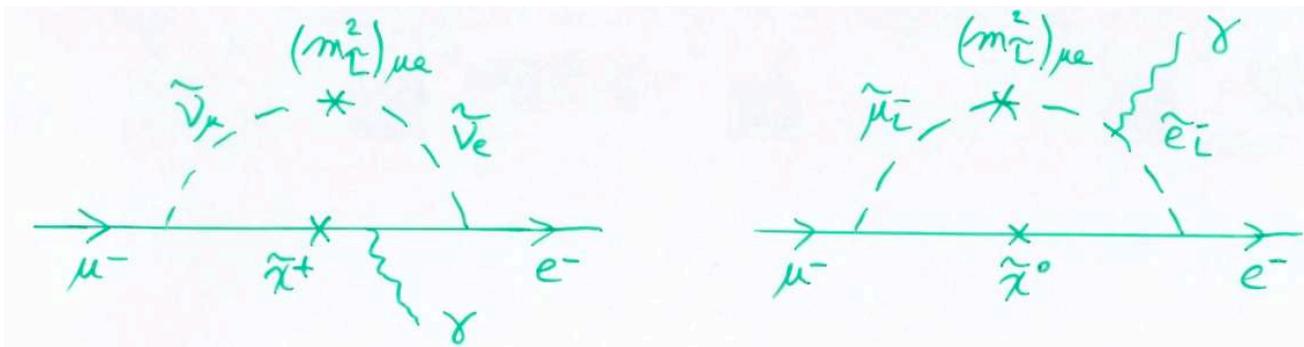
In (R-parity conserving) supersymmetric extensions of the Standard Model, LFV is induced by a misalignment between the lepton and slepton mass matrices, parametrized by the mass insertion parameters ($\alpha \neq \beta$):

$$\delta_{\alpha\beta}^{LL} \equiv \frac{(m_{\tilde{L}}^2)_{\alpha\beta}}{m_L^2}, \quad \delta_{\alpha\beta}^{RR} \equiv \frac{(m_{\tilde{e}}^2)_{\alpha\beta}}{m_R^2}, \quad \delta_{\alpha\beta}^{RL} \equiv \frac{A_{\alpha\beta}^e v_d}{m_R m_L}$$

(can be viewed as supersymmetric lepton mixing angles)

$$\Rightarrow \text{typical } \mu \rightarrow e \gamma \text{ rate: } B(\mu \rightarrow e \gamma) \sim 10^{-5} \frac{M_W^4}{M_{SUSY}^4} |\delta_{12}^{LL}|^2 \tan^2 \beta$$

where $\tan \beta \equiv \langle H_u^0 \rangle / \langle H_d^0 \rangle$



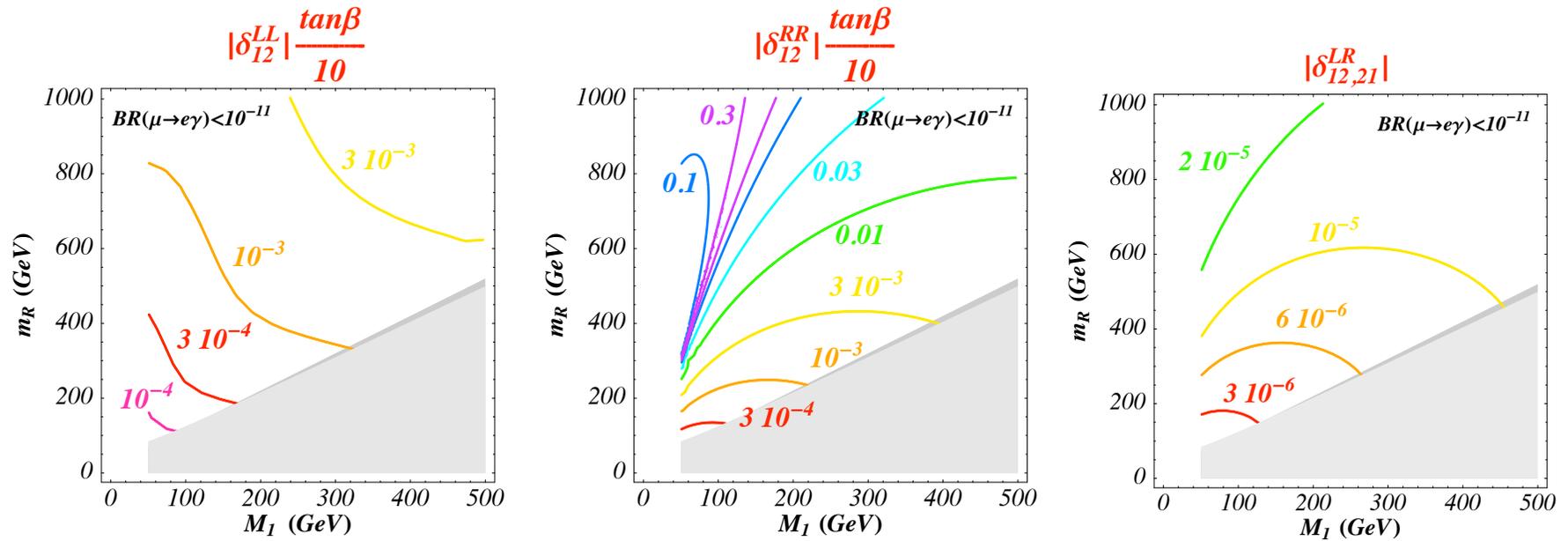


Fig. 5.3: Upper limits on δ_{12} 's in mSUGRA. Here M_1 and m_R are the bino and right-slepton masses, respectively.

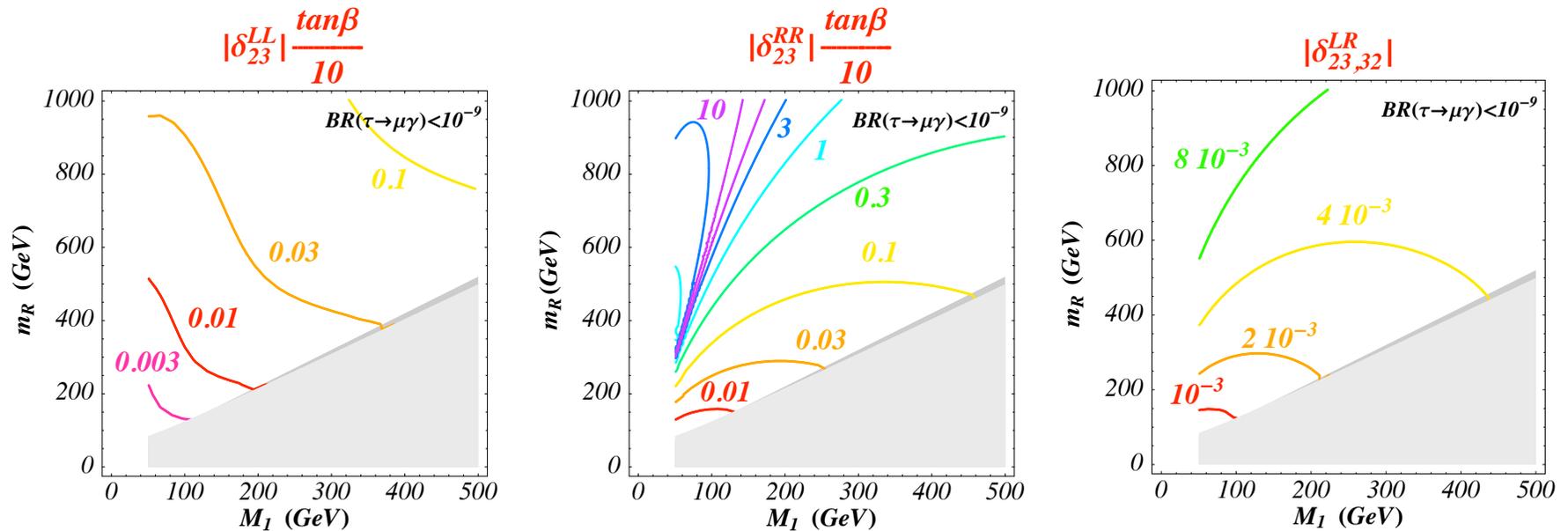


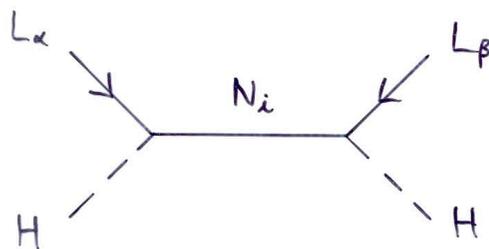
Fig. 5.4: Upper limits on δ_{23} 's in mSUGRA. Here M_1 and m_R are the bino and right-slepton masses, respectively.

Important difference with the quark sector: even if slepton mass matrices are flavour diagonal at some high scale, radiative corrections may induce large LFV [quark sector: controlled by CKM, pass most flavour constraints]

Such large corrections are due to heavy states with FV couplings to SM leptons, whose presence is suggested by $m_\nu \ll m_l$ [Borzumati, Masiero]

Well-known example: (type I) seesaw mechanism

$$\mathcal{L}_{seesaw} = -\frac{1}{2} M_i \bar{N}_i N_i - (\bar{N}_i Y_{i\alpha} L_\alpha H + \text{h.c.})$$



$$\Rightarrow (M_\nu)_{\alpha\beta} = -\sum_i \frac{Y_{i\alpha} Y_{i\beta}}{M_i} v^2 \quad (v = \langle H \rangle)$$

Assuming universal slepton masses at M_U , one obtains at low energy:

$$(m_{\tilde{L}}^2)_{\alpha\beta} \simeq -\frac{3m_0^2 + A_0^2}{8\pi^2} C_{\alpha\beta}, \quad (m_{\tilde{e}}^2)_{\alpha\beta} \simeq 0, \quad A_{\alpha\beta}^e \simeq -\frac{3}{8\pi^2} A_0 y_{e\alpha} C_{\alpha\beta}$$

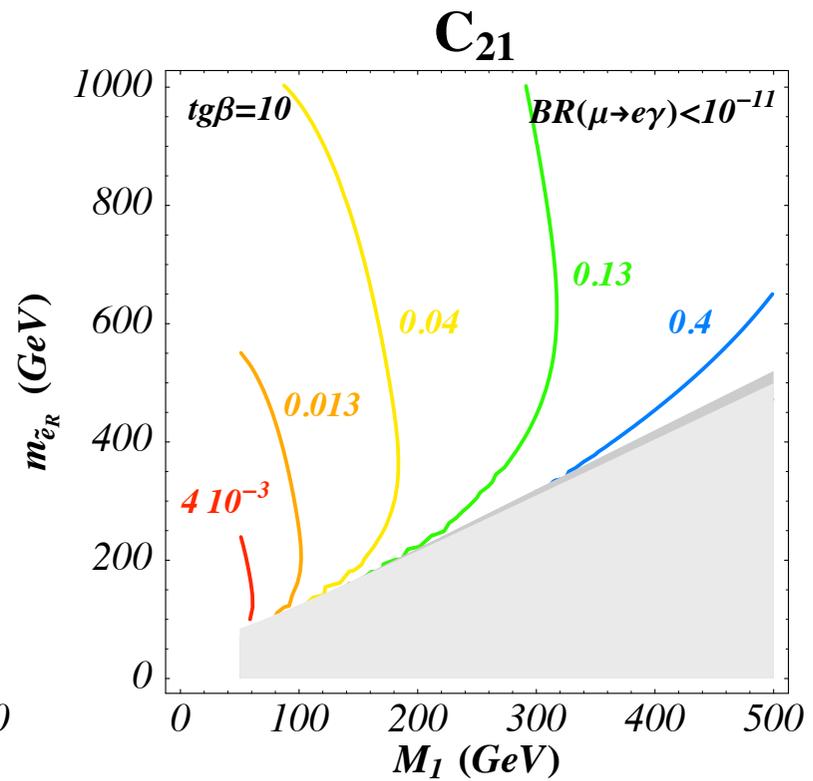
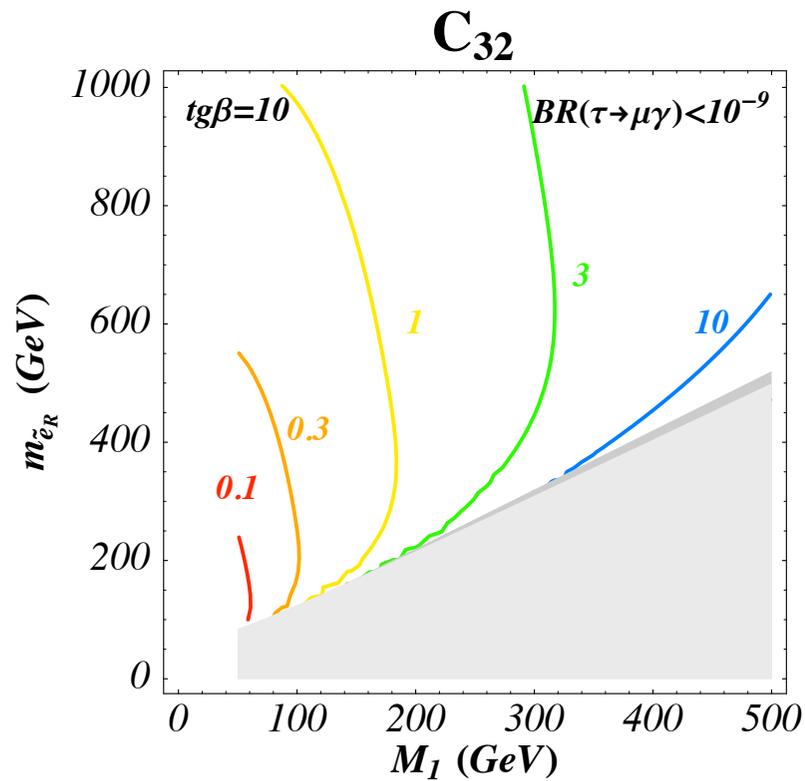
where $C_{\alpha\beta} \equiv \sum_k Y_{k\alpha}^* Y_{k\beta} \ln(M_U/M_k)$ encapsulates all the dependence on the seesaw parameters

$$\text{BR}(l_\alpha \rightarrow l_\beta \gamma) \propto |C_{\alpha\beta}|^2$$

$$\text{BR}(l_\alpha \rightarrow l_\beta \gamma) \propto |C_{\alpha\beta}|^2$$

$$C_{\alpha\beta} \equiv \sum_k Y_{k\alpha}^* Y_{k\beta} \ln(M_U/M_k)$$

[SL, Masina, Savoy]



Thus, in the supersymmetric seesaw mechanism, LFV processes probe the seesaw parameters

In general, however, cannot disentangle LFV induced by supersymmetry breaking from seesaw-induced LFV

Even in mSUGRA, there is no straightforward correlation between the measured neutrino parameters and the LFV rates, due to the degeneracy of seesaw parameters

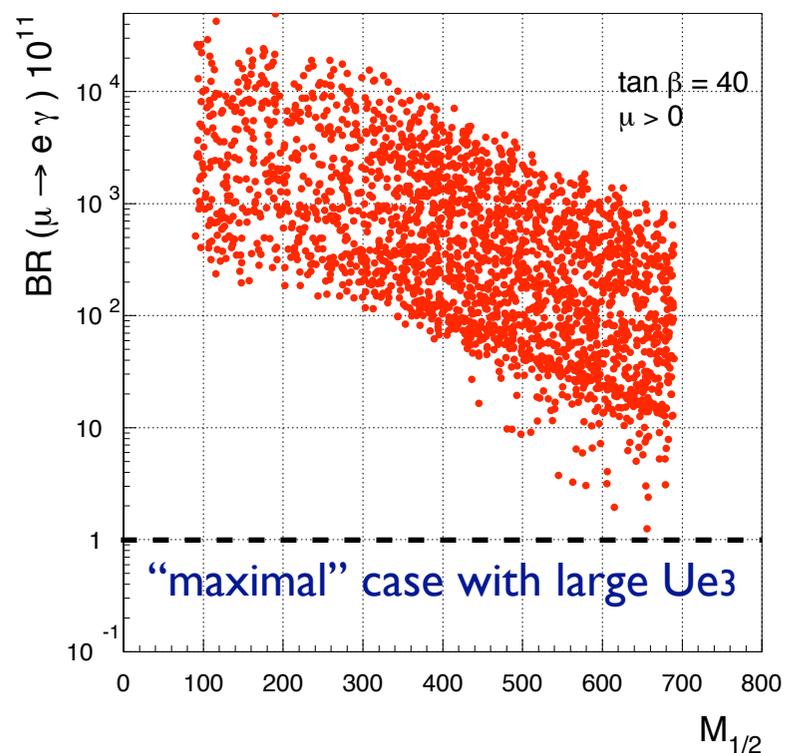
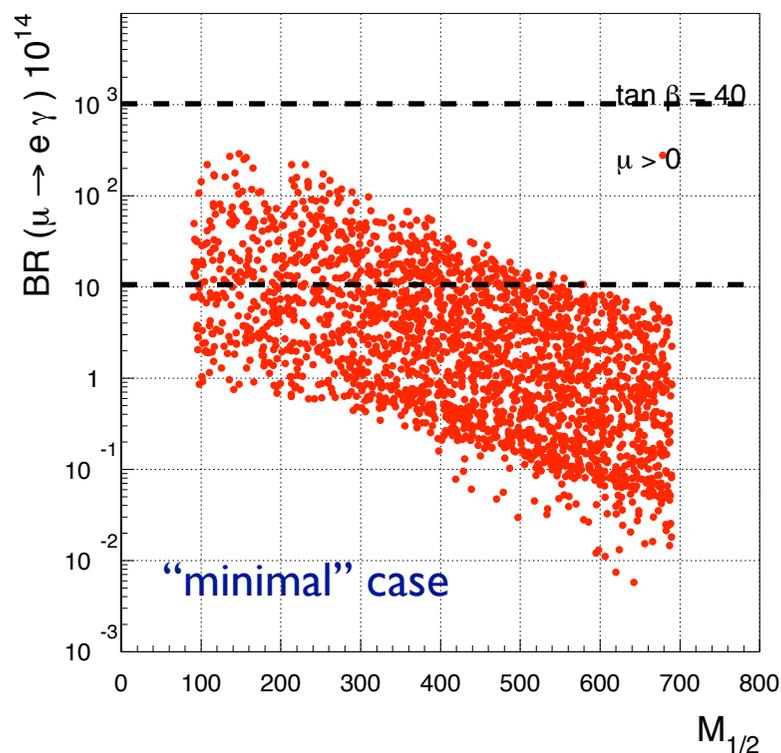
It is therefore fair to say that there is no definite prediction of the supersymmetric (type I) seesaw scenario for LFV processes, even in the mSUGRA case. This explains why different models give different predictions, although large rates are generic.

One can embed the supersymmetric seesaw in a Grand Unified Theory in order to reduce the arbitrariness in the seesaw parameters

Example [Masiero, Vempati, Vives]: SO(10)-motivated ansätze for the seesaw parameters

“minimal case”: CKM-like mixing in the Dirac couplings Y_{ij}

“maximal case”: PMNS-like mixing in the Dirac couplings Y_{ij} – $\mu \rightarrow e \gamma$ scales as U_{e3}^2 for $U_{e3}^2 \gtrsim 4 \times 10^{-5}$

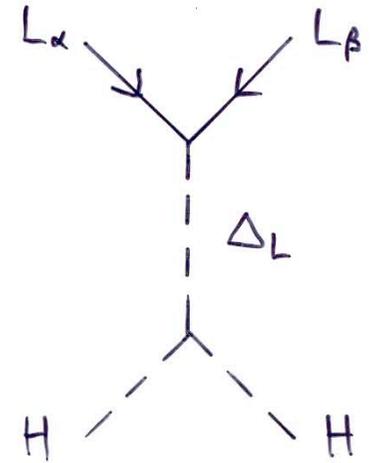


More predictive version of the seesaw mechanism:

Type II seesaw [heavy scalar SU(2)_L triplet exchange]

$$\frac{1}{\sqrt{2}} Y_T^{ij} L_i T L_j + \frac{1}{\sqrt{2}} \lambda H_u \bar{T} H_u + M_T T \bar{T}$$

$$\Rightarrow M_\nu^{ij} = \lambda Y_T^{ij} \frac{v_u^2}{M_T}$$



The radiative corrections to soft slepton masses are now controlled by

$$(Y_T^\dagger Y_T)_{\alpha\beta} \ln(M_U/M_T) \propto \sum_i m_{\nu_i}^2 U_{i\alpha} U_{i\beta}^*$$

\Rightarrow predictive (up to an overall scale) and leads to correlations between LFV observables (correlations controlled by the neutrino parameters)

[A. Rossi]

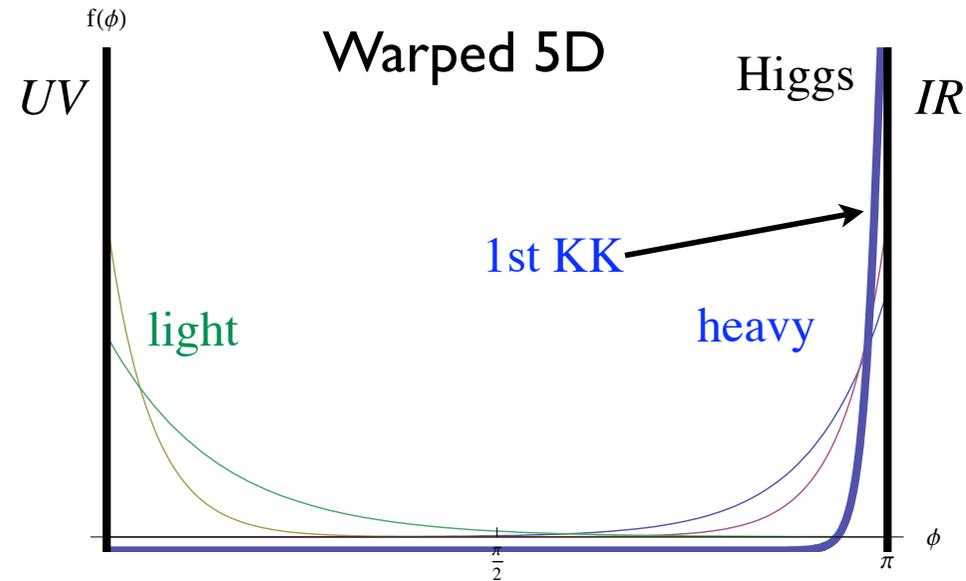
$$\frac{\text{BR}(\tau \rightarrow \mu \gamma)}{\text{BR}(\mu \rightarrow e \gamma)} \approx \left| \frac{(m_{\tilde{L}}^2)_{\tau\mu}}{(m_{\tilde{L}}^2)_{\mu e}} \right|^2 \frac{\text{BR}(\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu)}{\text{BR}(\mu \rightarrow e \nu_\mu \bar{\nu}_e)} \approx \begin{cases} 300 & [s_{13} = 0] \\ 2(3) & [s_{13} = 0.2] \end{cases}$$

$$\frac{\text{BR}(\tau \rightarrow e \gamma)}{\text{BR}(\mu \rightarrow e \gamma)} \approx \left| \frac{(m_{\tilde{L}}^2)_{\tau e}}{(m_{\tilde{L}}^2)_{\mu e}} \right|^2 \frac{\text{BR}(\tau \rightarrow e \nu_\tau \bar{\nu}_e)}{\text{BR}(\mu \rightarrow e \nu_\mu \bar{\nu}_e)} \approx \begin{cases} 0.2 & [s_{13} = 0] \\ 0.1(0.3) & [s_{13} = 0.2] \end{cases}$$

LFV in extra-dimensional scenarios

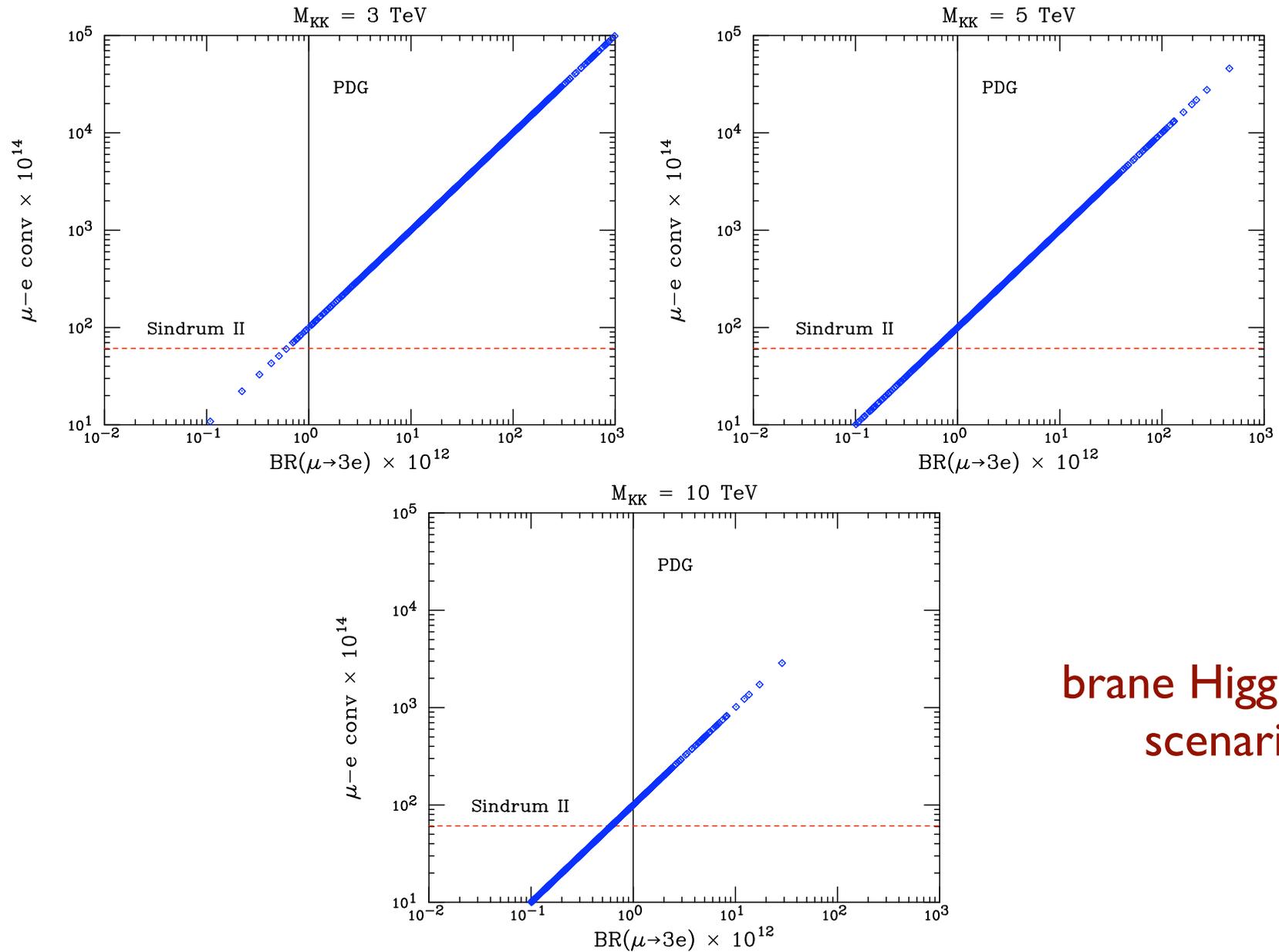
Source of flavour violation = couplings of light fermions to Kaluza-Klein excitations

Milder flavour violation in warped (Randall-Sundrum) models in which the fermion mass hierarchies are accounted for by different fermion localizations in extra dimensions (small overlap with KK wavefunction)



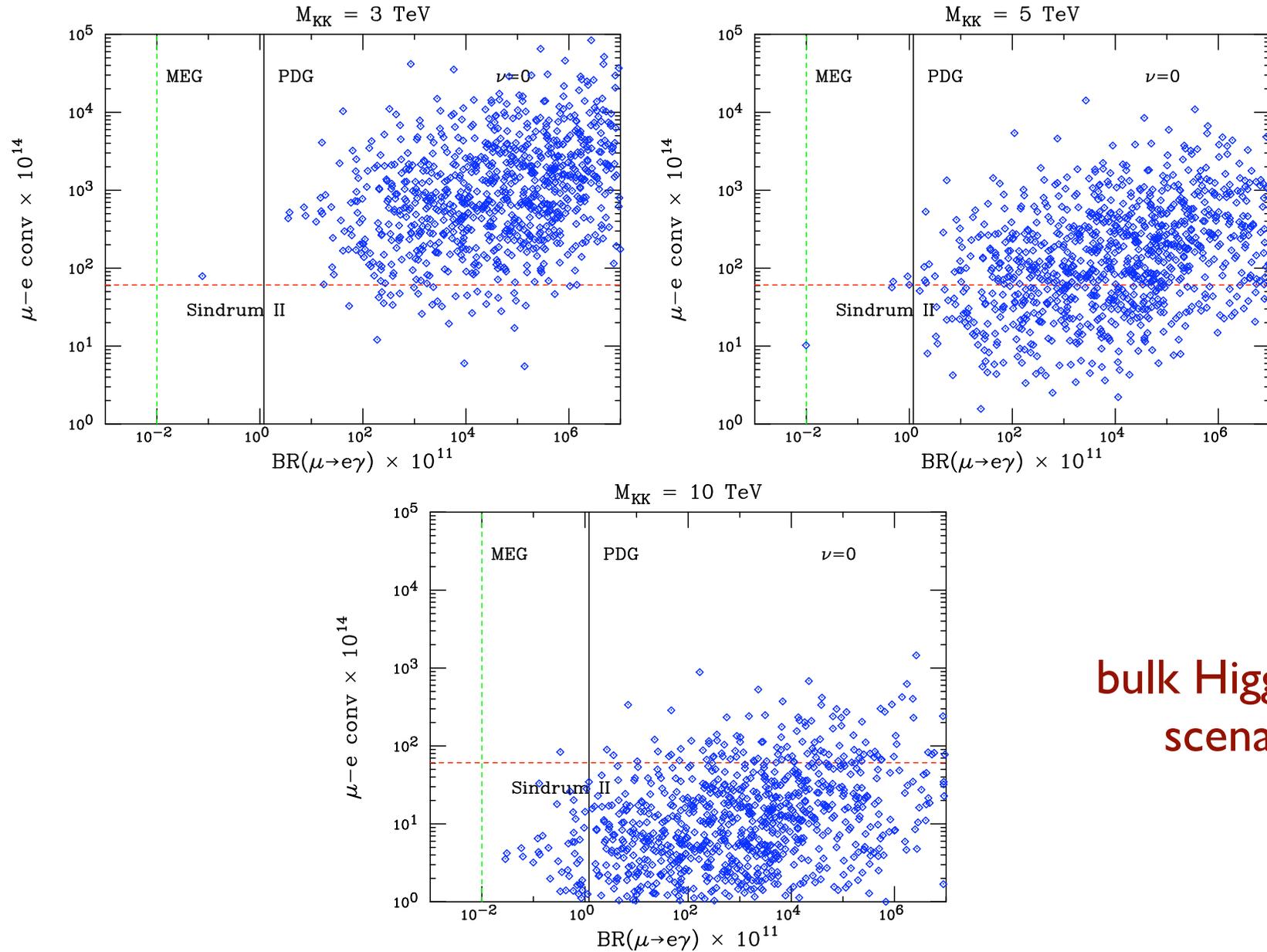
Agashe, Blechman, Petriello: RS model with Higgs propagating in the bulk ($l_i \rightarrow l_j \gamma$ UV sensitive if Higgs localized on the IR brane)

Present bounds on LFV processes compatible with $O(1 \text{ TeV})$ KK masses, with however some tension between loop-induced $l_i \rightarrow l_j \gamma$ and tree-level $\mu \rightarrow e$ conversion [can be improved with different lepton reps (2009)]



brane Higgs field
scenario

FIG. 4: Scan of the $\mu \rightarrow 3e$ and $\mu - e$ conversion predictions for $M_{KK} = 3, 5, 10$ TeV. The solid and dashed lines are the PDG and SINDRUM II limits, respectively.



**bulk Higgs field
scenario**

FIG. 6: Scan of the $\mu \rightarrow e\gamma$ and μ - e conversion predictions for $M_{KK} = 3, 5, 10$ TeV and $\nu = 0$. The solid line denotes the PDG bound on $BR(\mu \rightarrow e\gamma)$, while the dashed lines indicate the SINDRUM II limit on μ - e conversion and the projected MEG sensitivity to $BR(\mu \rightarrow e\gamma)$.

LFV in the littlest Higgs model with T-parity

Littlest Higgs model with T-parity (LHT) = model with a Higgs boson as a pseudo-Goldstone boson of a spontaneously broken global symmetry

The origin of LFV is the FV couplings of the mirror leptons to the SM leptons (via the heavy gauge bosons) = new flavour mixing matrices $V_{H\nu}$ and V_{Hl} , related by the PMNS matrix

Generally find large rate \Rightarrow constraints on the mirror lepton parameters

After imposing these constraints, find correlations between LFV processes that differ from the MSSM expectations

Ratios of LFV Branching Ratios

BBDRT, 0903.xxxx

	LHT	MSSM
$\frac{Br(\mu^- \rightarrow e^- e^+ e^-)}{Br(\mu^- \rightarrow e \gamma)}$	0.02... 1	$\sim 6 \cdot 10^{-3}$
$\frac{Br(\tau^- \rightarrow e^- e^+ e^-)}{Br(\tau^- \rightarrow e \gamma)}$	0.04... 0.4	$\sim 1 \cdot 10^{-2}$
$\frac{Br(\tau^- \rightarrow \mu^- \mu^+ \mu^-)}{Br(\tau^- \rightarrow \mu \gamma)}$	0.04... 0.4	$\sim 2 \cdot 10^{-3}$ ✱
$\frac{Br(\tau^- \rightarrow e^- \mu^+ \mu^-)}{Br(\tau^- \rightarrow e \gamma)}$	0.04... 0.3	$\sim 2 \cdot 10^{-3}$ ✱
$\frac{Br(\tau^- \rightarrow \mu^- e^+ e^-)}{Br(\tau^- \rightarrow \mu \gamma)}$	0.04... 0.3	$\sim 1 \cdot 10^{-2}$

✱ can be significantly enhanced by Higgs contributions

PARADISI, HEP-PH/0508054, HEP-PH/0601100

Conclusions

Experimental searches for flavour violating processes involving charged leptons are very interesting:

- 1) no SM background \Rightarrow unambiguous evidence for new physics
- 2) sensitive to the mechanism of neutrino mass generation [although its contribution to LFV processes might be negligible]

Different new physics scenarios give different predictions – If LFV is observed, will need to study correlations between several observables (including asymmetries in LFV charged lepton decays and LFV at colliders), and to take into account the complementary information from new particle searches, in order to be able to pin down its origin