

Dark energy & scalar-tensor theories after GW170817

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Astroparticules
et Cosmologie

Scalar-tensor theories

- Traditional scalar-tensor theories : $\mathcal{L}(\nabla_\lambda \phi, \phi)$

$$S = \int d^4x \sqrt{-g} \left[F(\phi)^{(4)}R - Z(\phi)\partial_\mu\phi\partial^\mu\phi - U(\phi) \right] + S_m[\psi_m; g_{\mu\nu}]$$

- **Generalized theories** with second order derivatives

$$\mathcal{L}(\nabla_\mu \nabla_\nu \phi, \nabla_\lambda \phi, \phi)$$

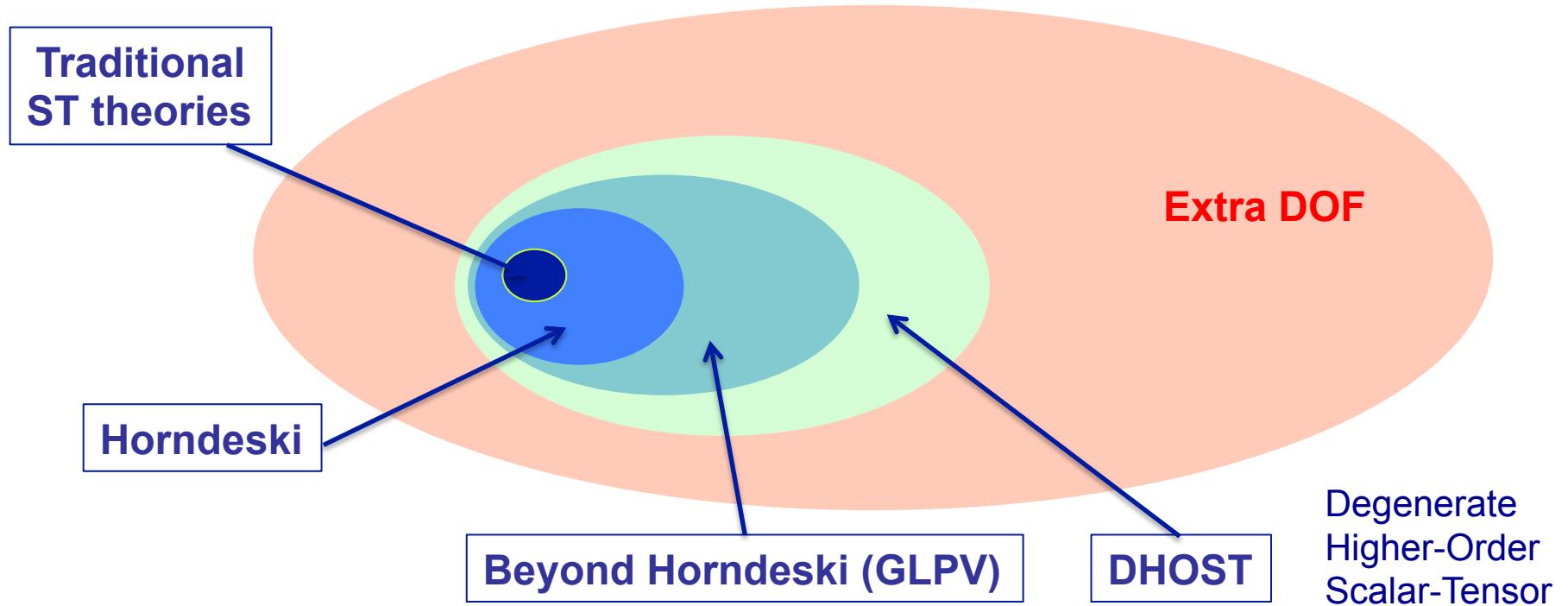
- In general, they contain an **extra degree of freedom**, expected to lead to **Ostrogradsky instabilities**

$$L(\ddot{q}, \dot{q}, q)$$

- But possible to avoid for special Lagrangians

Higher order scalar-tensor theories

- Traditional theories: $\mathcal{L}(\nabla_\lambda \phi, \phi)$
- Generalized theories: $\mathcal{L}(\nabla_\mu \nabla_\nu \phi, \nabla_\lambda \phi, \phi)$



DHOST theories

(Degenerate Higher-Order Scalar-Tensor)

- Action

DL & Noui '15

$$S[g, \phi] = \int d^4x \sqrt{-g} \left[P(X, \phi) + Q(X, \phi) \square \phi + f_2(X, \phi) {}^{(4)}R + \sum_{I=1}^5 a_I(X, \phi) L_I^{(2)} + f_3(X, \phi) G_{\mu\nu} \phi^{\mu\nu} + \sum_{I=1}^{10} b_I(X, \phi) L_I^{(3)} \right]$$

with the 5 **quadratic** Lagrangians

$$\begin{aligned} X &\equiv \nabla_\mu \phi \nabla^\mu \phi \\ \phi_{\mu\nu} &\equiv \nabla_\nu \nabla_\mu \phi \end{aligned}$$

$$L_1^{(2)} = \phi_{\mu\nu} \phi^{\mu\nu}, \quad L_2^{(2)} = (\square \phi)^2, \quad L_3^{(2)} = (\square \phi) \phi^\mu \phi_{\mu\nu} \phi^\nu$$

$$L_4^{(2)} = \phi^\mu \phi_{\mu\rho} \phi^{\rho\nu} \phi_\nu, \quad L_5^{(2)} = (\phi^\mu \phi_{\mu\nu} \phi^\nu)^2$$

and the 10 **cubic** ones

Ben Achour, Crisostomi, Koyama,
DL, Noui & Tasinato '16

$$L_1^{(3)} = (\square \phi)^3, \quad L_2^{(3)} = (\square \phi) \phi_{\mu\nu} \phi^{\mu\nu}, \quad \dots, \quad L_{10}^{(3)} = (\phi_\mu \phi^{\mu\nu} \phi_\nu)^3$$

DHOST theories

- The functions $f_2(X, \phi)$, $a_I(X, \phi)$, $f_3(X, \phi)$ and $b_I(X, \phi)$ must satisfy **degeneracy conditions**.



DHOST theories

(Degenerate Higher-Order Scalar-Tensor)

- Subsets of DHOST theories

- Beyond Horndeski (GLPV) Gleyzes, DL, Piazza & Vernizzi '14

$$f_2 = G_4, \quad a_1 = -a_2 = 2G_{4X} + XF_4, \quad a_3 = -a_4 = 2F_4$$

$$f_3 = G_5, \quad 3b_1 = -b_2 = \frac{3}{2}b_3 = G_{5X} + 3XF_5, \quad -2b_4 = b_5 = 2b_6 = -b_7 = 6F_5$$

- Horndeski theories: $F_4 = 0, \quad F_5 = 0$

DHOST in cosmology

- Cosmological background: $a(t)$, $\phi(t)$
- Quadratic action for **linear perturbations** (ADM form)

$$S_{\text{quad}} = \int d^3x dt a^3 \frac{M^2}{2} \left\{ \delta K_{ij} \delta K^{ij} - \left(1 + \frac{2}{3} \alpha_L \right) \delta K^2 + (1 + \alpha_T) \left(R \frac{\delta \sqrt{h}}{a^3} + \delta_2 R \right) \right. \\ \left. + H^2 \alpha_K \delta N^2 + 4H \alpha_B \delta K \delta N + (1 + \alpha_H) R \delta N + 4\beta_1 \delta K \delta \dot{N} + \beta_2 \delta \dot{N}^2 + \frac{\beta_3}{a^2} (\partial_i \delta N)^2 \right\}$$

9 time-dependent coefficients (only 6 independent)

DL, Mancarella,
Noui & Vernizzi '17

- **Degeneracy conditions** (2 categories)

$$\mathcal{C}_I : \alpha_L = 0, \beta_2 = -6\beta_1^2, \beta_3 = -2\beta_1 [2(1 + \alpha_H) + \beta_1(1 + \alpha_T)]$$

\mathcal{C}_{II} : gradient instability (either in the scalar or the tensor sector)

Tensor degrees of freedom

- Quadratic action for the **tensor modes**:

$$S_{\gamma}^{(2)} = \frac{1}{2} \int dt d^3x a^3 \left[\frac{M^2}{4} \dot{\gamma}_{ij}^2 - \frac{M^2}{4} (1 + \alpha_T) \frac{(\partial_k \gamma_{ij})^2}{a^2} \right]$$

Speed of gravitational waves: $c_T^2 = 1 + \alpha_T$

- From the Lagrangian...

$$f^{(4)}R \supset f(K_{ij}K^{ij} + {}^{(3)}R)$$

$$\nabla_i \nabla_j \phi \supset \dot{\phi} K_{ij} \implies \phi_{\mu\nu} \phi^{\mu\nu} \supset \dot{\phi}^2 K_{ij} K^{ij}$$

DHOST theories after GW170817

DHOST theories after GW170817

- Constraint on the speed of gravitational waves:

$$-3 \times 10^{-15} \leq \frac{c_T}{c} - 1 \leq 7 \times 10^{-16}$$

- Assuming $c_T = c$ implies

1. Quadratic terms: $a_1 = 0$

$$L_{\text{ADM}} = (f - Xa_1) K_{ij} K^{ij} + f^{(3)}R$$

2. No cubic term

- Remain quadratic DHOST theories of type I with $a_1 = 0$

DHOST theories with $c_g = c$

- Taking into account the degeneracy conditions,

$$a_1 = a_2 = 0 ,$$

$$a_4 = \frac{1}{8f_2} [48f_{2X}^2 - 8(f_2 - Xf_{2X})a_3 - X^2a_3^2] ,$$

$$a_5 = \frac{1}{2f_2} (4f_{2X} + Xa_3) a_3 \quad \textbf{2 free functions}$$

- Total Lagrangian

$$\begin{aligned} L_{c_g=1}^{\text{DHOST}} &= f_2(X, \phi) {}^{(4)}R + P(X, \phi) + Q(X, \phi) \square\phi \\ &\quad + a_3(X, \phi) \phi^\mu \phi^\nu \phi_{\mu\nu} \square\phi + a_4(X, \phi) \phi^\mu \phi_{\mu\nu} \phi_\lambda \phi^{\lambda\nu} \\ &\quad + a_5(X, \phi) (\phi_\mu \phi^{\mu\nu} \phi_\nu)^2 \end{aligned}$$

4 free functions of X and ϕ (as in Horndeski without $c_g = 1$!)

Horndeski and Beyond Horndeski with $c_g = c$

- Remaining Beyond Horndeski theories

$$a_1 = 2G_{4X} + XF_4 = 0 \implies F_4 = -\frac{2}{X}G_{4X}$$

$$S[g, \phi] = \int d^4x \sqrt{-g} \left\{ f(\phi, X) R - \frac{4}{X} f_X \left[(\square\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu - \phi^\mu\phi_{\mu\nu}\phi^{\nu\rho}\phi_\rho \right] \right\}$$

- Remaining Horndeski theories

$$G_2(X, \phi), \quad G_3(X, \phi), \quad G_4(\phi)$$

Other constraints

- GW decay into dark energy Creminelli, Lewandowski, Tambalo & Vernizzi ‘1809

$$L_{\gamma\delta\phi\delta\phi} = \frac{\alpha_H}{\alpha_K + 6\alpha_B^2} \frac{1}{\Lambda_3^3} \ddot{\gamma}_{ij}^{(c)} \partial_i \delta\phi^{(c)} \partial_j \delta\phi^{(c)} \quad \Lambda_3 \sim (M_P H_0^2)^{1/3}$$

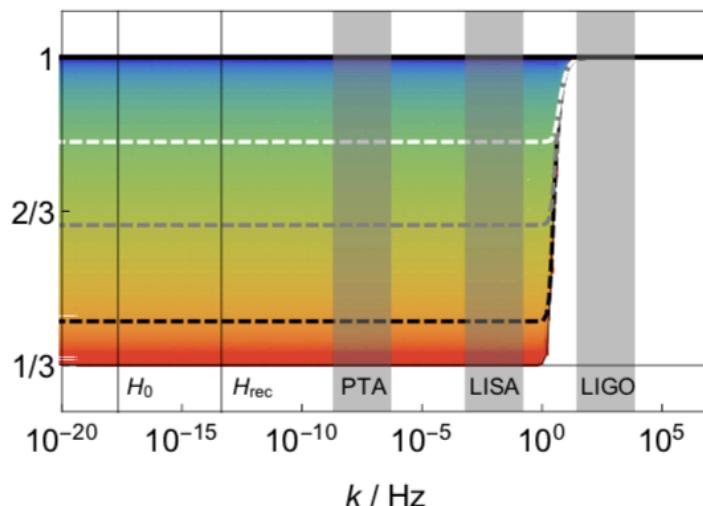
$$\Gamma_{h \rightarrow \delta\phi\delta\phi} \sim \frac{\alpha_H^2}{(\alpha_K + 6\alpha_B^2)^2} \frac{\omega_{\text{gw}}^7 (1 - c_s^2)^2}{c_s^7 \Lambda_3^6} \sim \alpha_H^2 \omega_{\text{gw}}$$

- Remaining DHOST theories: $a_3 = 0$

$$L_{c_g=1, \text{no decay}}^{\text{DHOST}} = P(X, \phi) + Q(X, \phi) \square \phi + f(X, \phi) {}^{(4)}R + 6 \frac{f_X^2}{f} \phi^\mu \phi_{\mu\nu} \phi^{\nu\rho} \phi_\rho$$

Conclusions

- **DHOST theories:** most general framework for scalar-tensor theories known at present
- **Drastic reduction of viable models after GW170817.**
- **Caveat:** LIGO/Virgo scales could be outside the regime of validity of the effective description as ST theory



De Rham & Melville '1807