Systèmes binaires : l'approche de la théorie effective

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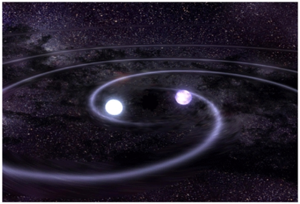
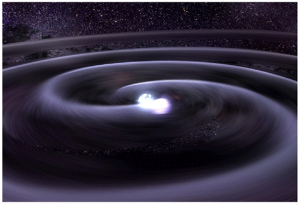
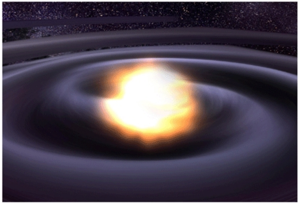
24 octobre 2018

Outline

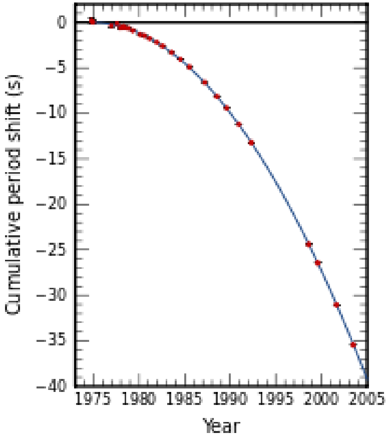
1. Binary system overview
2. NRGR in a scalar-tensor theory
   - Building up the action
   - Conservative dynamics
   - Dissipative dynamics
3. Outlook and conclusions

The object under study

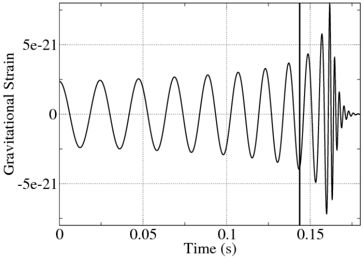
Which observables ?

Prehistoric times



$\dot{E}$

Today



$\phi(t)$

Some orders of magnitude

- VIRGO/LIGO band : 10Hz - 1kHz

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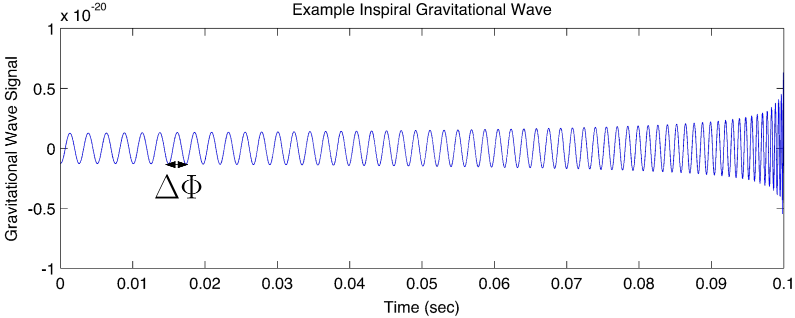
- $t \sim 5$ min.
- $\Phi = 2\int_{t_i}^{t_f} \omega(t)dt \sim 4 \times 10^4$ rad

The impressive precision of GW detectors

Corrections 3 PN $\Delta\Phi \sim v^4 \Phi \sim 1$!



Modified gravity effects : Scalar-tensor theory

- Additional power loss

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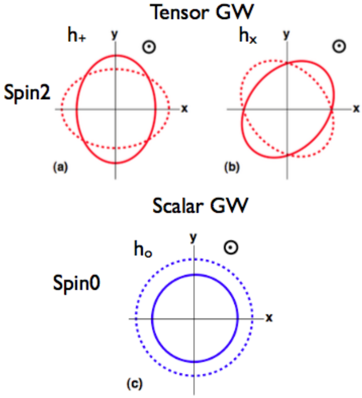
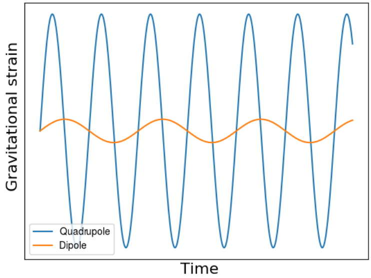
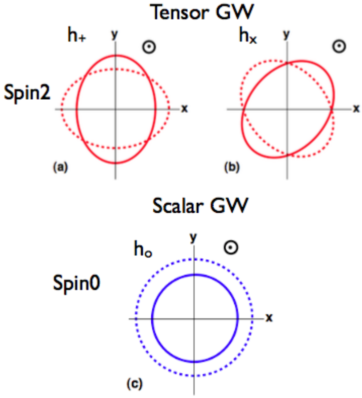


Modified gravity effects : Scalar-tensor theory

- Additional power loss
- Supplementary polarisation
- Dipolar radiation !





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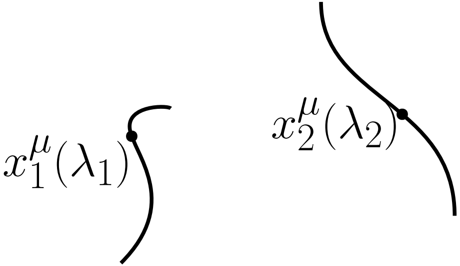
Invariances of the system



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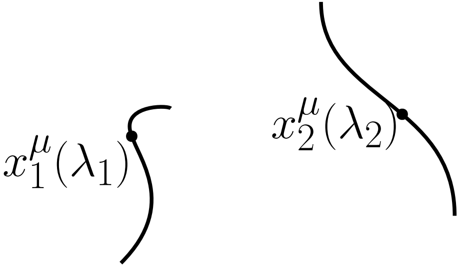

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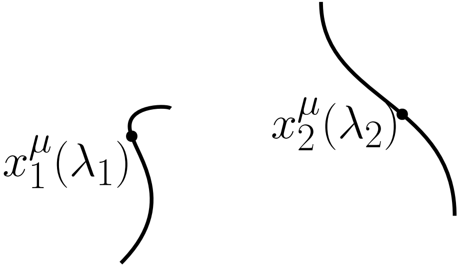

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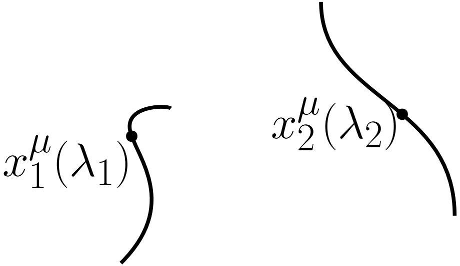

Fluctuating field

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

- $x^\mu \to x'^\mu(x) \Rightarrow$ Use R and $g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$
- $\lambda_a \to \lambda'_a(\lambda_a) \Rightarrow$ Use $d\tau = \sqrt{g_{\mu\nu}dx^\mu dx^\nu} = dt\sqrt{1 - v^2 - h_{\mu\nu}v^\mu v^\nu}$

Point particles action

$$S = S_{grav} + S_{pp,1} + S_{pp,2}$$

$$S_{grav} = \frac{m_P^2}{2} \int d^4x \sqrt{-g} R$$

$$S_{pp,a} = - m_a \int d\tau_a$$

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$$S_{grav} = \frac{m_P^2}{2}\int d^4x\sqrt{-g}R - \boxed{\frac{1}{2}\int d^4x\sqrt{-g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}$$

$$S_{pp,a} = -m_a\int d\tau_a + \boxed{a\frac{m_a}{m_P}\int d\tau_a\phi + b\frac{m_a}{m_P^2}\int d\tau_a\phi^2} + \dots$$

'Quantum' gravity

Integrate out fluctuating fields :

$$e^{iS_{\rm ef}[\mathbf{x}_1(t),\mathbf{x}_2(t)]} = \int \mathcal{D}[h_{\mu\nu}]\mathcal{D}[\phi]e^{iS[\mathbf{x}_1(t),\mathbf{x}_2(t),h_{\mu\nu},\phi]}$$

$S_{\rm ef}$ contains the dynamics of the point-particles only

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S_{ef} contains the dynamics of the point-particles only

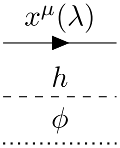
$$\Re(S_{\text{ef}}) = \int dt L[\mathbf{x}_a, \mathbf{v}_a]$$ Conservative dynamics

and

$$\Im(S_{ef}) = \frac{T}{2}\int dE d\Omega \frac{d^2\Gamma}{dE d\Omega}$$ Dissipative dynamics

In practice

- Feynman expansion



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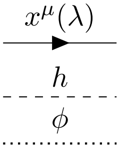
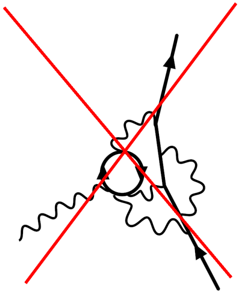
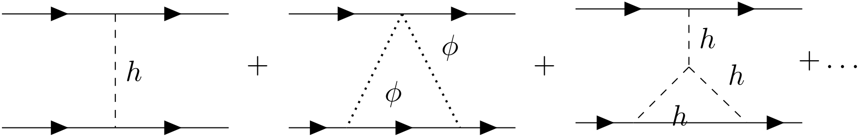




- No loops ! $\frac{\hbar}{L} \sim 10^{-76}$ where $L = mrv$

In pratice

$iS_{\text{ef}} =$



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v^0 (or Newtonian) Lagrangian

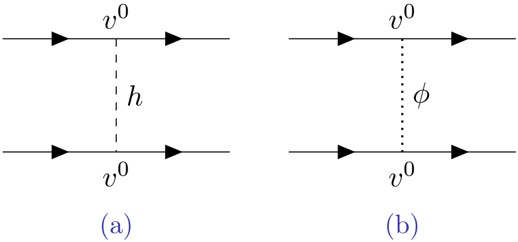

Figure: Feynman diagrams contributing to the Newtonian potential

v^0 (or Newtonian) Lagrangian

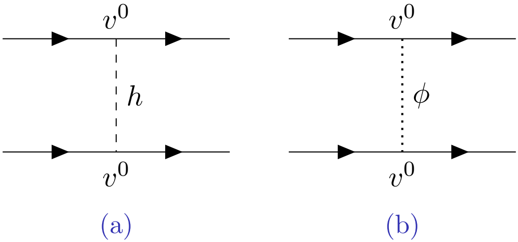

Figure: Feynman diagrams contributing to the Newtonian potential

$$L_{v^0} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{G_N m_1 m_2}{|\mathbf{x_1}(t) - \mathbf{x_2}(t)|}(1 + 2a^2)$$

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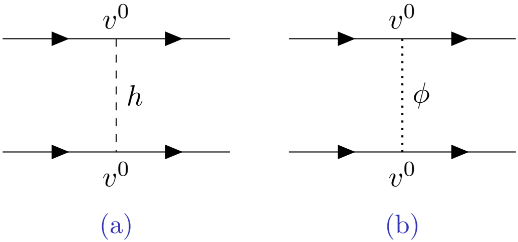

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$$\tilde{G} = G_N(1 + 2a^2)$$

v^2 (or EIH) Lagrangian

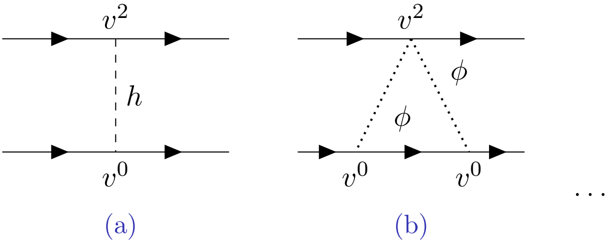

Figure: Some Feynman diagrams contributing to the v^2 Lagrangian

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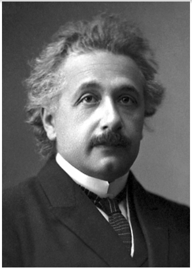

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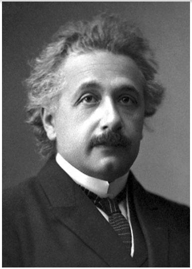

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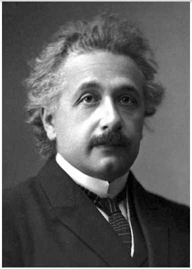

$$L_{EIH} = \frac{1}{8}\sum_a m_a v_a^4$$

$$+ \frac{\tilde{G} m_1 m_2}{2|\mathbf{x}_{12}|}\left[(v_1^2 + v_2^2) - 3\mathbf{v}_1\cdot\mathbf{v}_2 - \frac{(\mathbf{v}_1\cdot\mathbf{x}_{12})(\mathbf{v}_2\cdot\mathbf{x}_{12})}{|\mathbf{x}_{12}|^2} + 2\gamma(\mathbf{v}_1 - \mathbf{v}_2)^2\right]$$

$$- \frac{\tilde{G}^2 m_1 m_2 (m_1 + m_2)}{2|\mathbf{x}_{12}|^2}(2\beta - 1)$$

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Multipole expansion

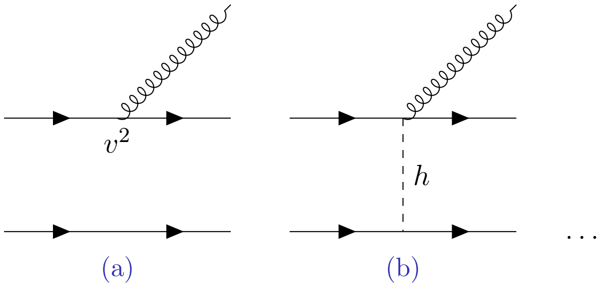

Figure: Some Feynman diagrams for the emission of one radiation scalar.

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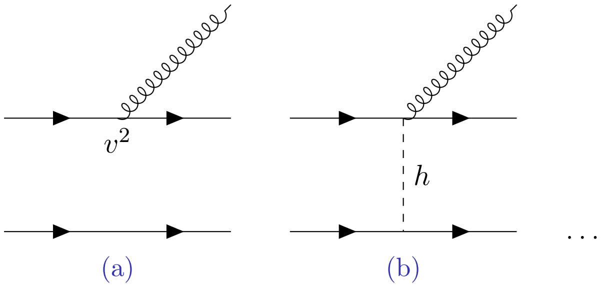

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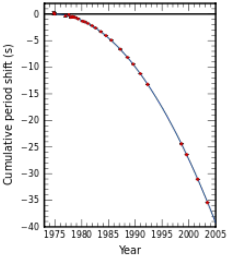
Net result :

$$S_{\text{int}} = \frac{1}{2}\int dt I_h^{ij} R_{0i0j} + \frac{1}{m_P}\int dt \left(I_\phi \bar{\phi} + I_\phi^i \partial_i \bar{\phi} + \frac{1}{2} I_\phi^{ij} \partial_i \partial_j \bar{\phi} \right) + \dots$$

Radiated power

Quadrupole formula :

$$P_h = \frac{G_N}{5} \left\langle \dddot{I}_h^{ij\,2} \right\rangle + \dots$$



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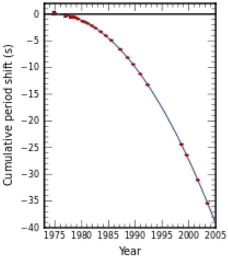

Monopole, dipole and quadrupole scalar radiation :

$$P_\phi = 2G_N\left(\left\langle \dot{I}_\phi^{\,2}\right\rangle + \frac{1}{3}\left\langle \ddot{I}_\phi^{i\,2}\right\rangle + \frac{1}{30}\left\langle \dddot{I}_\phi^{ij\,2}\right\rangle + \dots\right)$$

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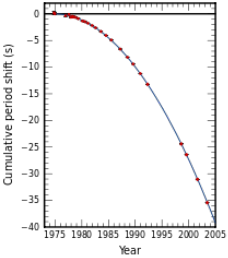

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$I_\phi = \text{Const} + \text{v}^2\text{correction}$, $I_\phi^i \propto a_1 - a_2$

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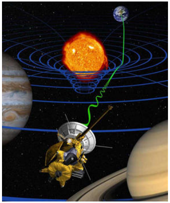
Conformal tranformation

$a\frac{m_a}{m_P}\int d\tau_a\phi$ and $b\frac{m_a}{m_P^2}\int d\tau_a\phi^2$ can be seen as

Conformal coupling

$$\tilde{g}_{\mu\nu} = A(\phi)g_{\mu\nu}$$



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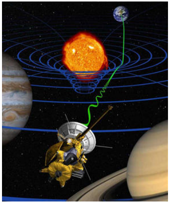
Conformal coupling

$\tilde{g}_{\mu\nu} = A(\phi)g_{\mu\nu}$

$$\Rightarrow -m\int d\tilde{\tau} = -m\int d\tau\sqrt{A}$$
$$= -m\int d\tau\left(1 - a\frac{\phi}{m_P} - b\frac{\phi^2}{m_P^2}\right)$$



Disformal operator...

Other operator allowed :

$$m \int d\tau \partial_\mu \phi \frac{dx^\mu}{d\tau}$$

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is a total derivative !

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is a total derivative !

but

$$m \int d\tau \left(\frac{d\phi}{d\tau} \right)^2$$

is allowed...

...from disformal transformation

Disformal coupling

$$\tilde{g}_{\mu\nu} = A(\phi)g_{\mu\nu} + B(\phi)\partial_\mu\phi\partial_\nu\phi$$

...from disformal transformation

Disformal coupling

$\tilde{g}_{\mu\nu} = A(\phi)g_{\mu\nu} + B(\phi)\partial_\mu\phi\partial_\nu\phi$

$$\Rightarrow d\tilde{\tau}^2 = d\tau^2 \left(A + B \left(\partial_\mu \phi \frac{dx^\mu}{d\tau} \right)^2 \right)$$

We plan to study the effects of such an operator, as we done for the conformal coupling

Conclusion

- We generalized NRGR to a scalar-tensor theory
- Effective theory point of view : operators allowed by symmetries
- We plan to examine the effects of disformal couplings

Power-counting rules

Small expansion parameter

$\frac{Gm}{r} \sim v^2$

Vertex

Expression

Weight

Power-counting rules

Small expansion parameter

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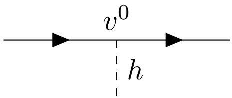

Vertex	
Expression	$\frac{m}{2}\int dt h_{00}$
Weight	v^0

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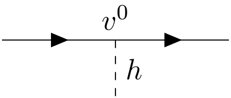
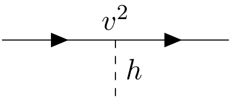
Vertex		
Expression	$\frac{m}{2}\int dt h_{00}$	$\frac{m}{2}\int dt h_{ij} v^i v^j$
Weight	v^0	v^2

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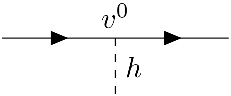
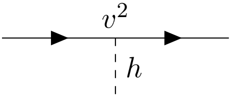
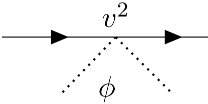
Vertex			
Expression	$\frac{m}{2}\int dt h_{00}$	$\frac{m}{2}\int dt h_{ij} v^i v^j$	$bm \int dt \phi^2$
Weight	v^0	v^2	v^2