The effect of backreaction in cosmology — some challenges

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Colloque National DARK ENERGY - 2018/10/25

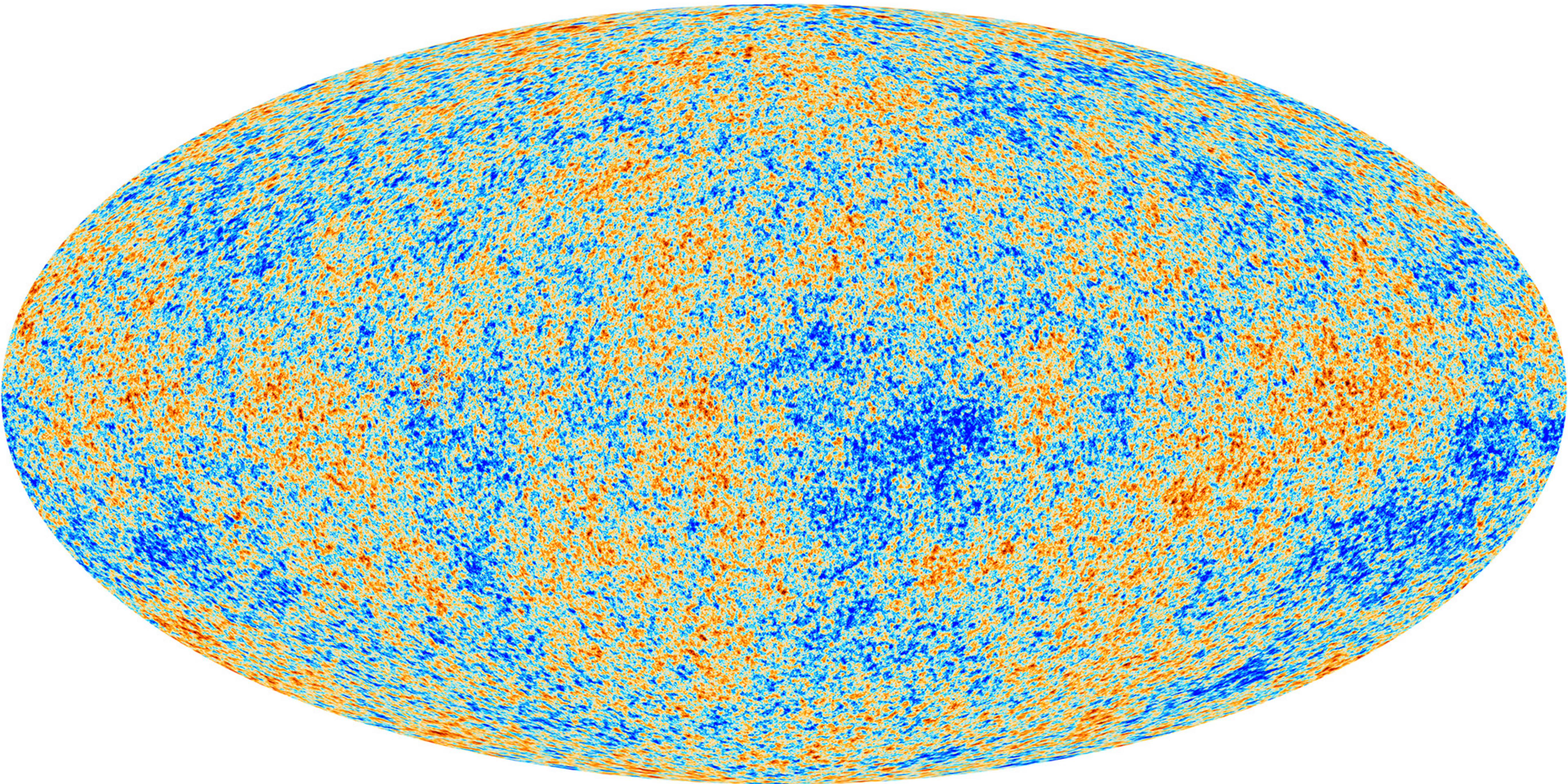
Outline

- Does lensing by structure <u>bias</u> the distance-redshift relation?
- Why there is no Newtonian backreaction
- Some challenges for relativistic backreaction

Context: cosmological parameters from the CMB

It is usually assumed that we are looking here at a spherical surface at $z{\sim}1100$ with $D = D_0(z{=}1100)$

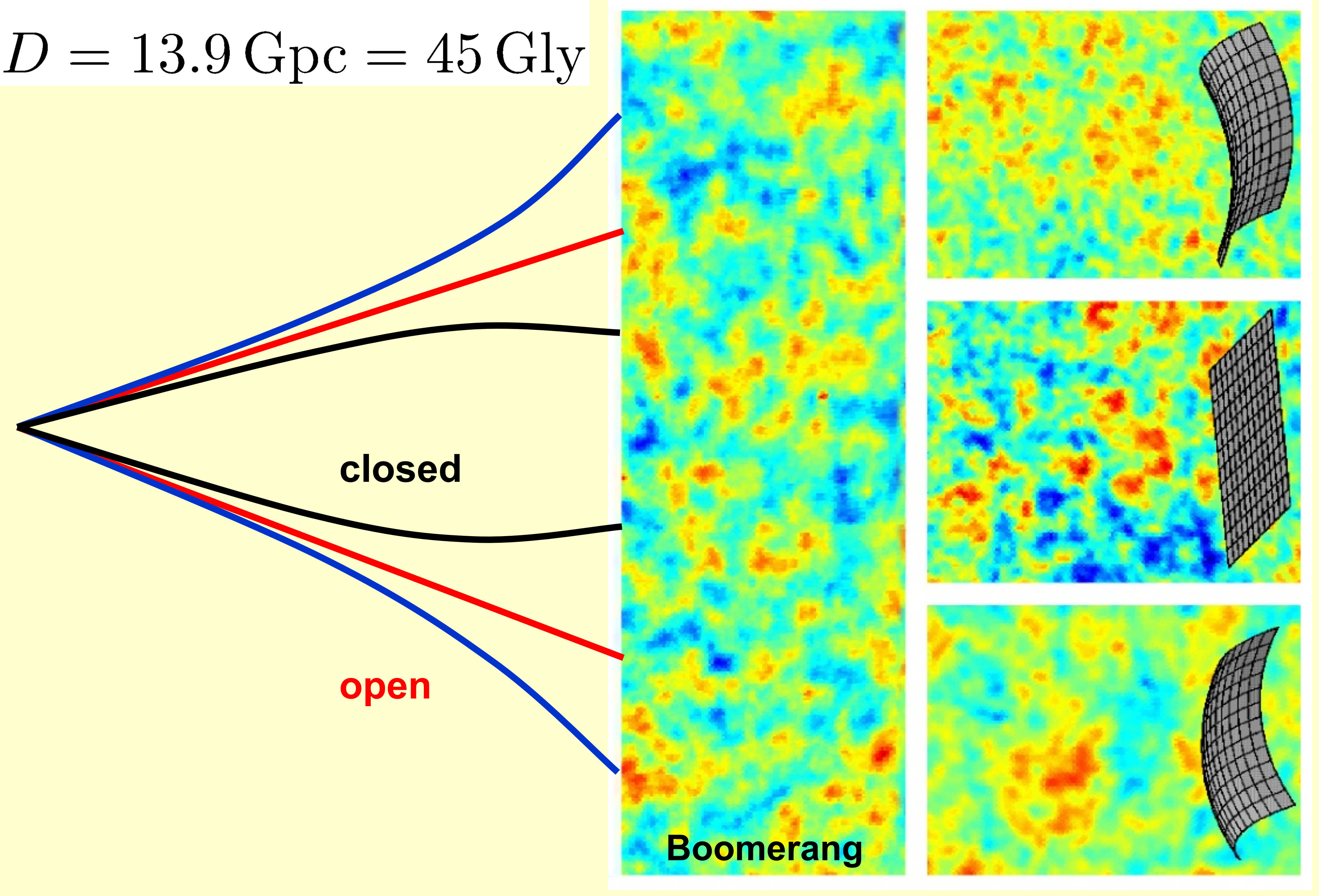
But are we?



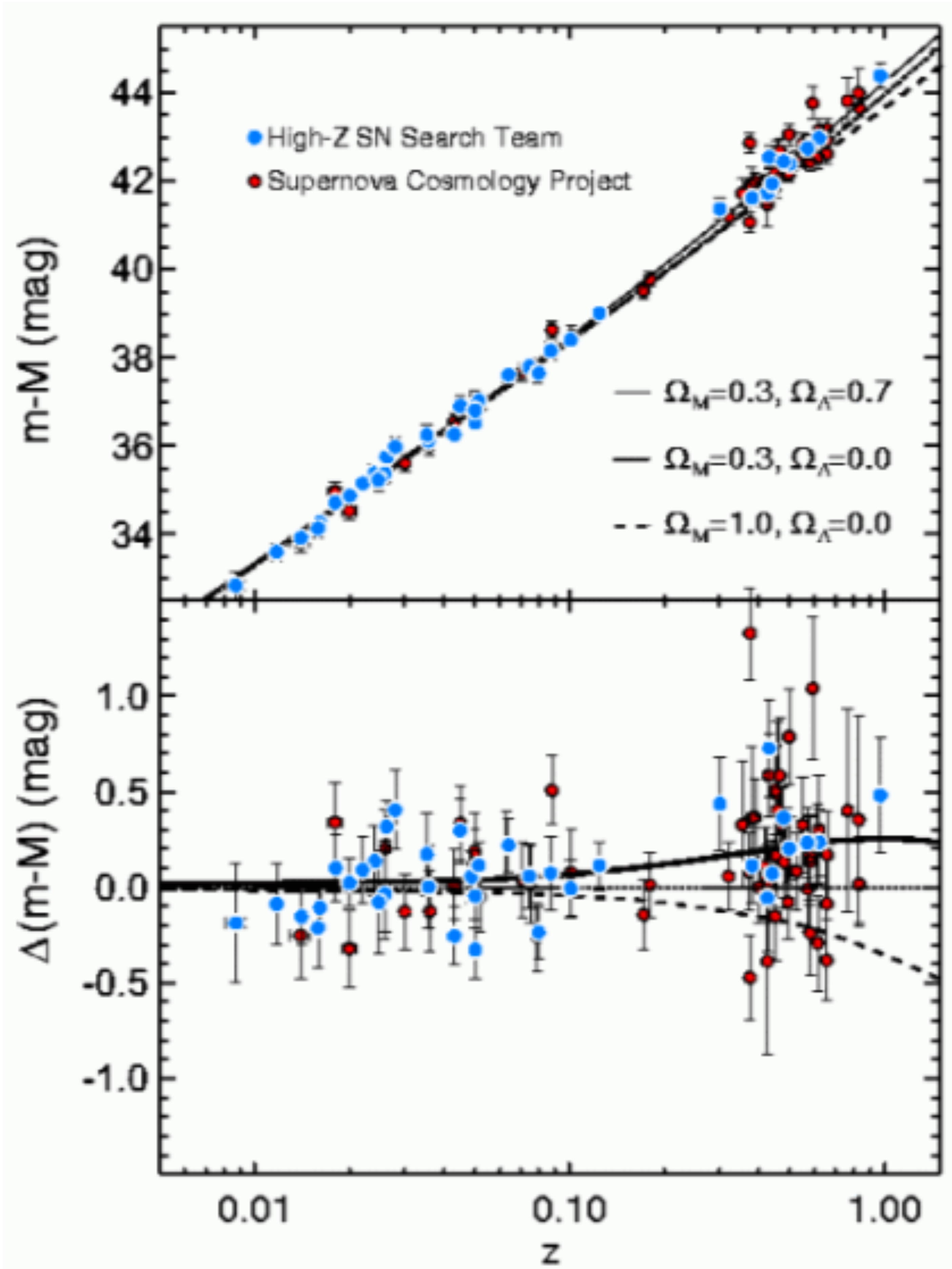
How far away is the CMB?

$$D = \int \frac{c}{H(z)}\, dz$$

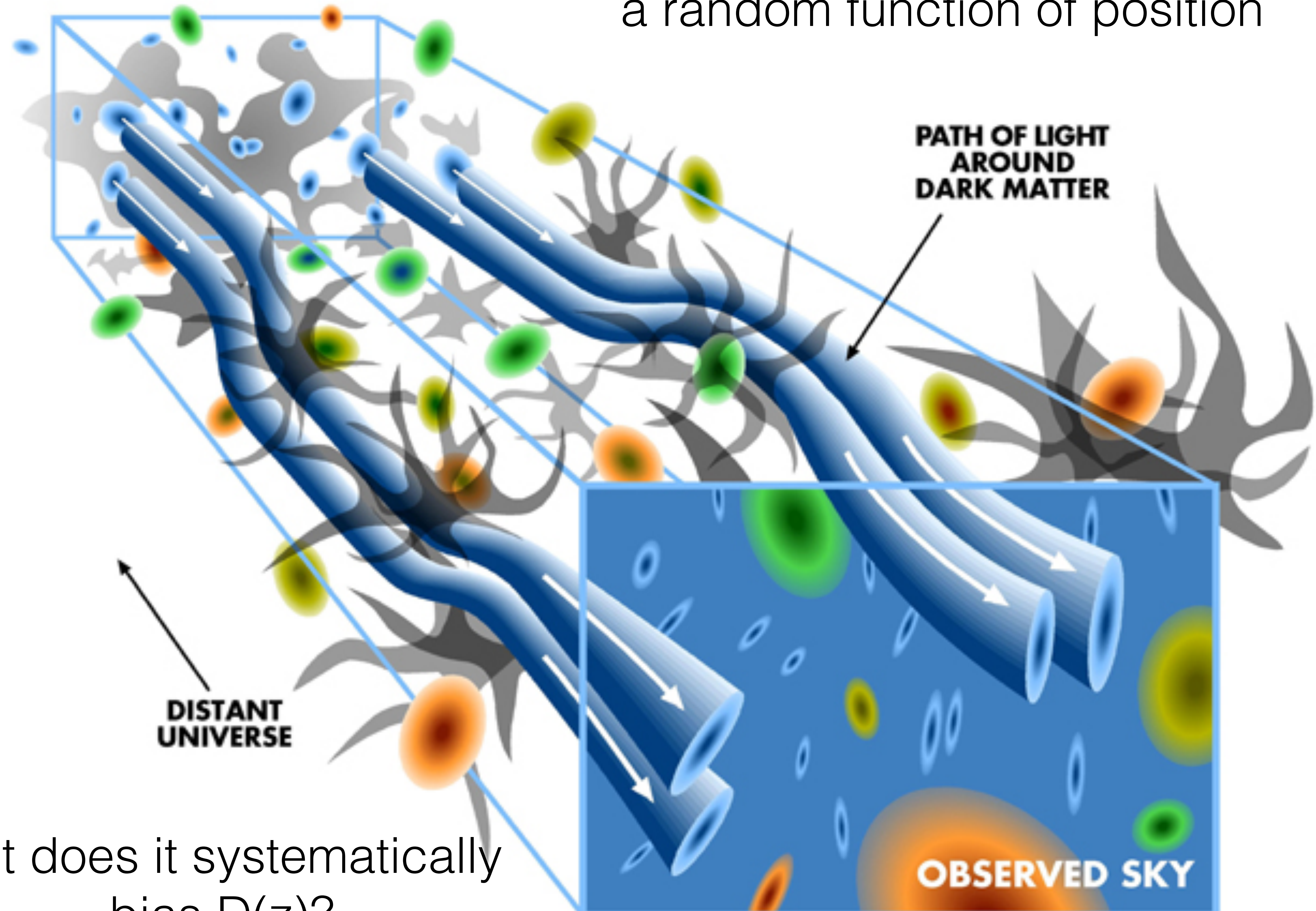
$$z = 1080 \Rightarrow D = 13.9\,\mathrm{Gpc} = 45\,\mathrm{Gly}$$



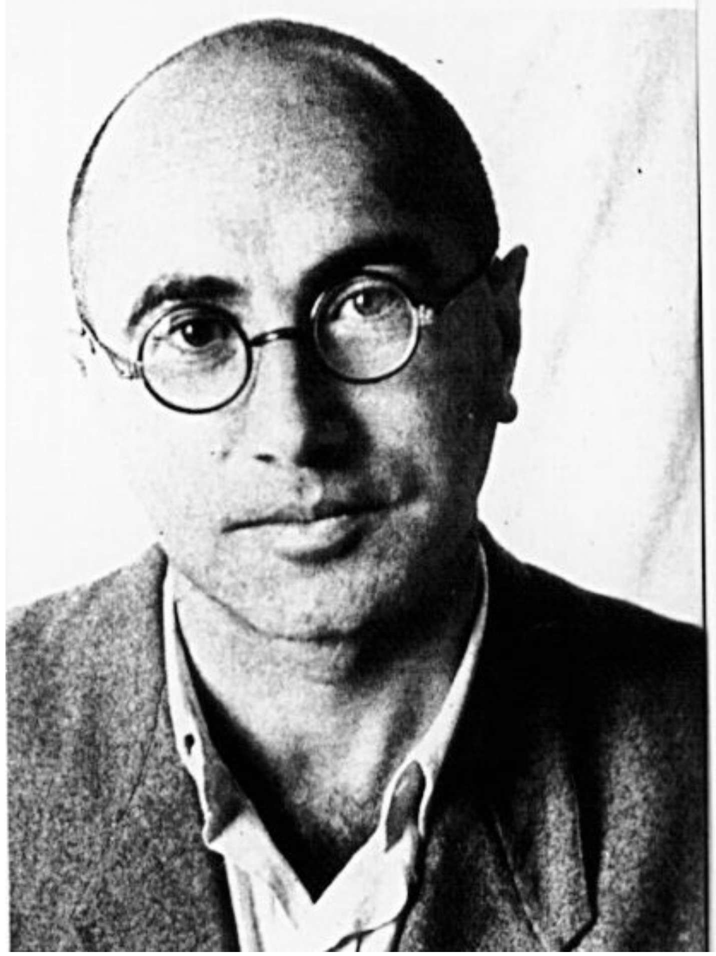
Hubble diagram from SN1a - assumes no flux *bias* from lensing



Lensing makes distance D(z) a random function of position



But does it systematically <u>bias</u> D(z)?



OBSERVATIONS IN A UNIVERSE HOMOGENEOUS IN THE MEAN

Ya. B. Zel'dovich

Translated from Astronomicheskii Zhurnal, Vol. 41, No. 1,
pp. 19-24, January-February, 1964
Original article submitted June 12, 1963

A local nonuniformity of density due to the concentration of matter of the universe into separate galaxies produces a significant change in the angular dimensions and luminosity of distant objects as compared to the formulas for the Friedman model.

The propagation of light in a homogeneous and isotropic model of the expanding universe (first studied by A. A. Friedman) has been investigated in a number of papers [1, 2, 3].

In these papers expressions were obtained for the observed angular diameter Θ and the observed brightness of an object with a known absolute diameter and absolute brightness as a function of the distance or, strictly speaking, the red shift of the object $\Delta = (\omega_0 - \omega)/\omega_0$.

In particular, there is a remarkable feature in the function Θ(Δ), namely, the presence of a minimum when Δ is approximately equal to 1/2. Formula (10) and Fig. 6 in the appendix show the variation of the function $f(\Delta) = rH/c\Theta$ which is inversely proportional to Θ for a given density of matter. Here r is the radius of the object, H is Hubble's

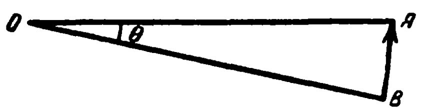


Fig. 1.

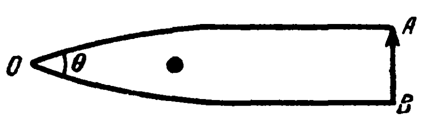
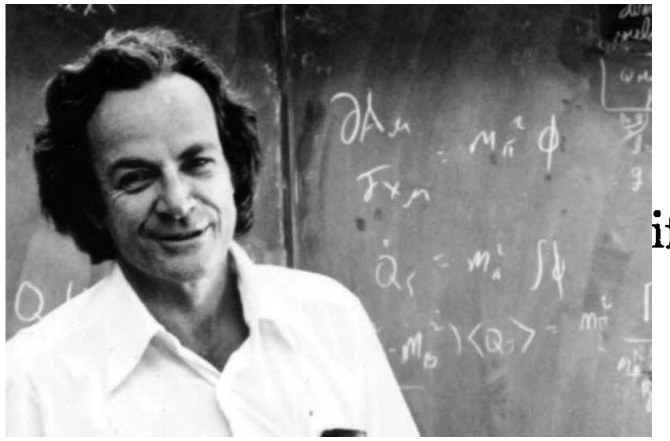


Fig. 2.

A mass situated between these rays bends the latter in such a way that Θ is increased (Fig. 2). What we have in mind is the bending of light rays by the gravitational field predicted by Einstein; this bending amounts to 1.75" for a light ray passing near the limb of the solar disc and has been confirmed by observation.

ON THE PROPAGATION OF LIGHT IN INHOMOGENEOUS COSMOLOGIES. I. MEAN EFFECTS



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Received February 23, 1967; revised May 23, 1967

ABSTRACT

The statistical effects of local inhomogeneities on the propagation of light are investigated, and deviations (including rms fluctuations) from the idealized behavior in homogeneous universes are investigated by a perturbation-theoretic approach. The effect discussed by Feynman and recently by Bertotti of the density of the intergalactic medium being systematically lower than the mean mass density is examined, and expressions for the effect valid at all redshifts are derived.

I. INTRODUCTION

In an unpublished colloquium given at the California Institute of Technology in 1964, Feynman discussed the effect on observed angular diameters of distant objects if the intergalactic medium has lower density than the mean mass density, as would be the case if a significant fraction of the total mass were contained in galaxies. It is an obvious extension of the existence of this effect that luminosities will also be affected, though this was apparently not realized at the time. This realization prompted the conviction that the effect of known kinds of deviations of the real Universe from the homogeneous isotropic models (upon which predictions had been based in the past) upon observable quantities like luminosity and angular diameter should be investigated. The author (1967) has recently made such a study for angular diameters; the present work deals primarily with mean statistical effects upon luminosity. A third paper will deal with possible extreme effects one may expect to encounter more rarely. Some of the results discussed here have been discussed independently by Bertotti (1966) and Zel'dovich (1965).

Kantowski '69

CORRECTIONS IN THE LUMINOSITY-REDSHIFT RELATIONS OF THE HOMOGENEOUS FRIEDMANN MODELS

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ABSTRACT

In this paper the bolometric luminosity-redshift relations of the Friedmann dust universes ($\Lambda = 0$) are corrected for the presence of inhomogeneities. The "locally" inhomogeneous Swiss-cheese models are used, and it is first shown that the introduction of clumps of matter into Friedmann models does not significantly affect the $R(z)$ or $R(v)$ relations (Friedmann radius versus the redshift or affine parameter) along a null ray. Then, by the use of the optical scalar equations, a linear third-order differential equation is arrived at for the mean cross-sectional area of a light beam as a function of the affine parameter. This differential equation is confirmed by rederiving its small redshift solution from an interesting geometrical point of view. The geometrical argument is then extended to show that "mild" inhomogeneities of a transparent type have no effect on the mean area of a light beam.

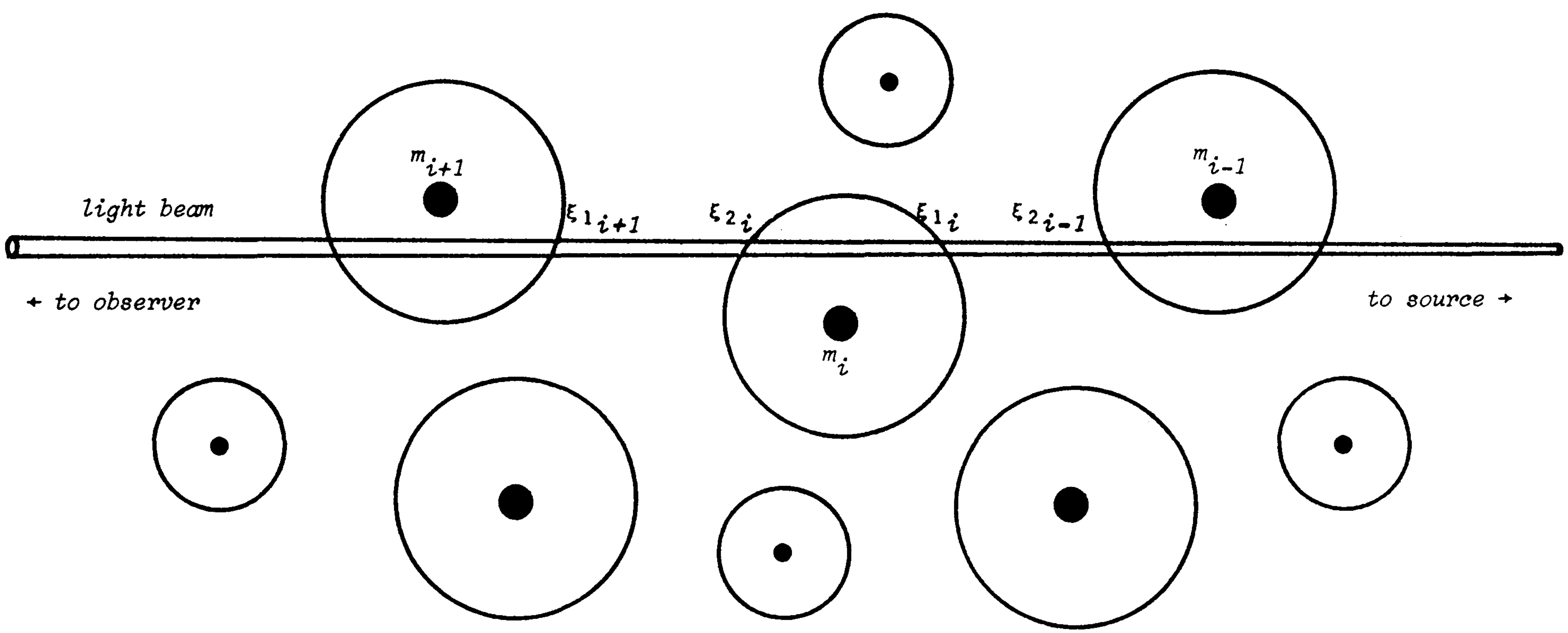

FIG. 1.—Spacelike section of a typical Swiss-cheese universe

Dyer & Roeder '72

THE DISTANCE-REDSHIFT RELATION FOR UNIVERSES WITH NO INTERGALACTIC MEDIUM

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Received 1972 April 19

ABSTRACT

The distance-redshift relation is derived for model universes in which there is negligible intergalactic matter and in which the line of sight to a distant object does not pass close to intervening galaxies. When fitted to observations, this relation yields a higher value of q_0 than does a homogeneous model.

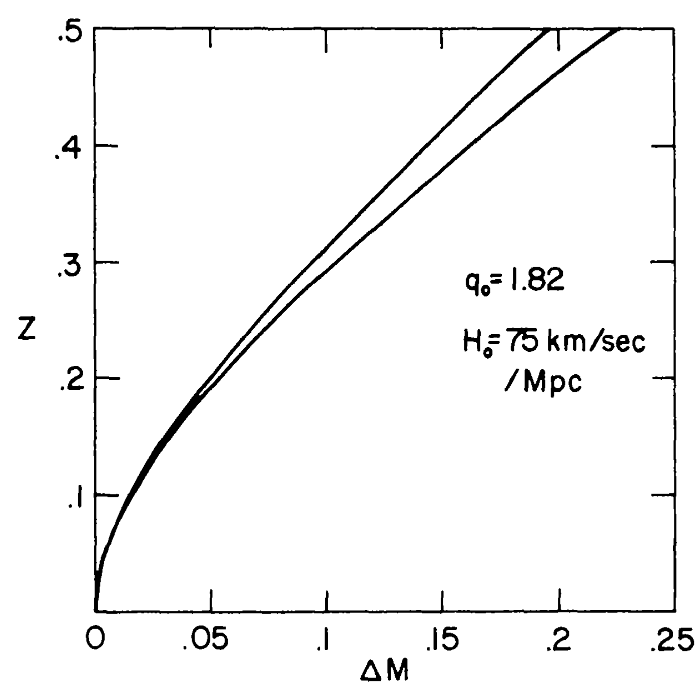
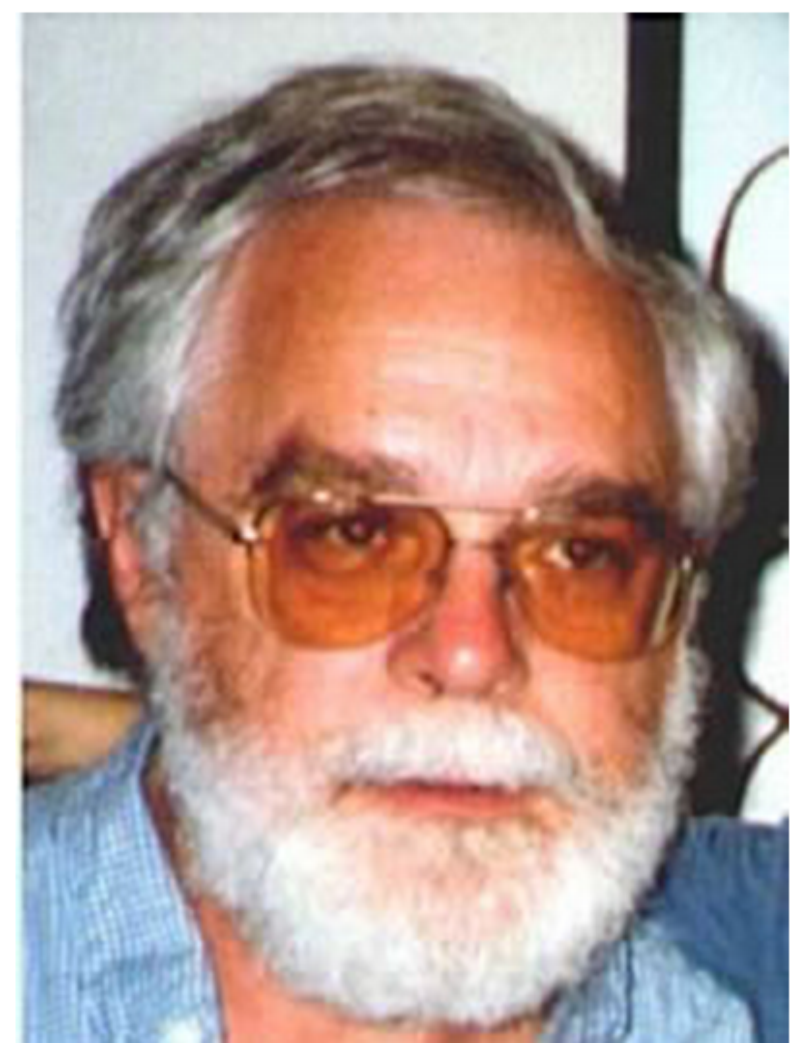


FIG. 1.—The dimming, relative to the homogeneous model, assuming that the beam passes far from any intervening galaxies (*lower curve*) and assuming that the beam passes no closer than 2 kpc to the center of galaxies similar to our own (*upper curve*).



Weinberg 1976 - no effect (flux conservation)

APPARENT LUMINOSITIES IN A LOCALLY INHOMOGENEOUS UNIVERSE

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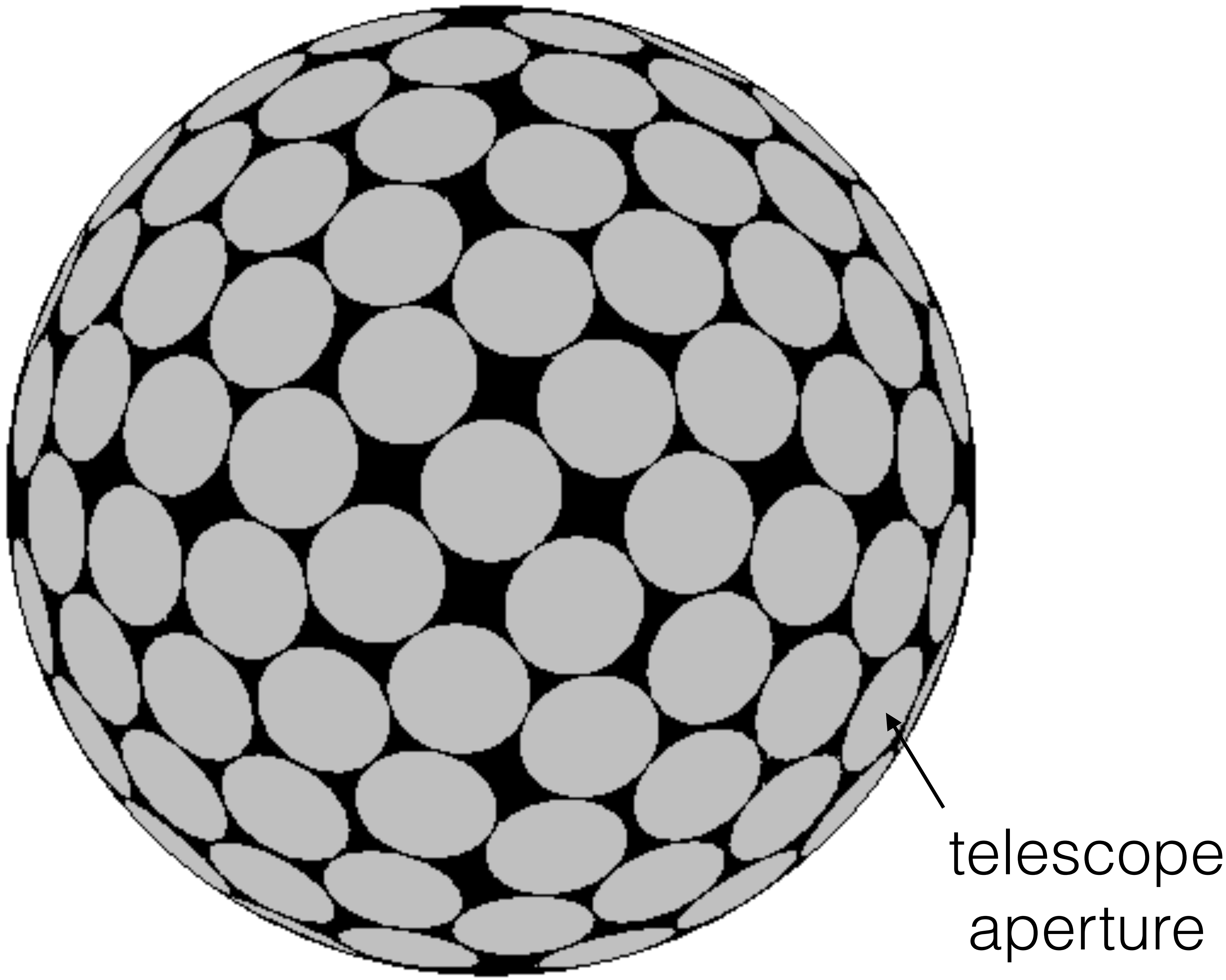
ABSTRACT

Apparent luminosities are considered in a locally inhomogeneous universe, with gravitational deflection by individual clumps of matter taken into account. It is shown that as long as the clump radii are sufficiently small, gravitational deflection by the clumps will produce the same average effect as would be produced if the mass were spread out homogeneously. The conventional formulae for luminosity distance as a function of redshift consequently remain valid, despite the presence of any local inhomogeneities of less than galactic dimensions. For clumps of galactic size, the validity of the conventional formulae depends on the selection procedure used and the redshift of the object studied.
Subject headings: cosmology — galaxies: redshifts — gravitation

Weinberg's argument (that <magnification> = 1)



Seitz, Schneider & Ehlers (1994)

Finally, we have derived an equation for the size of a light beam in a clumpy universe, relative to the size of a beam which is unaffected by the matter inhomogeneities. If we require that this second-order differential equation contains only the contribution by matter clumps as source term, the independent variable is uniquely defined and agrees with the χ-function previously introduced [see SEF, eq. (4.68)] for other reasons. This relative focusing equation immediately yields the result that a light beam cannot be less focused than a reference beam which is unaffected by matter inhomogeneities, prior to the propagation through its first conjugate point. In other words, no source can appear fainter to the observer than in the case that there are no matter inhomogeneities close to the line-of-sight to this source, a result previously demonstrated for the case of one (Schneider 1984) and several (Paper I, Seitz & Schneider 1994) lens planes.

Seitz, Schneider & Ehlers 94

1992). Taking a somewhat different approach, Seitz, Schneider & Ehlers (1994) have used the optical scalars formalism of Sachs (1961) to show that the square root of the proper area of a narrow bundle of rays $D = \sqrt{A}$ obeys the 'focusing equation':

$$\ddot{D}/D = -(R + \Sigma^2). \quad (1)$$

Here $\ddot{D}$ is the second derivative of D with respect to affine distance along the bundle; $R = R_{\alpha\beta}k^\alpha k^\beta/2$ is the local Ricci focusing from matter in the beam, which for non-relativistic velocities is just proportional to the matter density; and Σ^2 is the squared rate of shear from the integrated effect of up-beam Weyl focusing – i.e. the tidal field of matter outside the beam. The resulting *focusing theorem* is that the RHS of (1) is non-positive, so that beams are always focused to smaller sizes, at least as compared to empty space-time,

Kibble & Lieu (2005)

AVERAGE MAGNIFICATION EFFECT OF CLUMPING OF MATTER

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ABSTRACT

The aim of this paper is to reexamine the question of the average magnification in a universe with some inhomogeneously distributed matter. We present an analytic proof, valid under rather general conditions, including clumps of any shape and size and strong lensing, that as long as the clumps are uncorrelated, the average "reciprocal" magnification (in one of several possible senses) is precisely the same as in a homogeneous universe with an equal mean density. From this result, we also show that a similar statement can be made about one definition of the average "direct" magnification. We discuss, in the context of observations of discrete and extended sources, the physical significance of the various different measures of magnification and the circumstances in which they are appropriate.

Subject headings: cosmology: miscellaneous — distance scale — galaxies: distances and redshifts — gravitational lensing

Kibble & Lieu 2005

There is another important distinction to be made. We may choose at random one of the sources at redshift *z*, or we may choose a random direction in the sky and look for sources there. These are not the same; the choices are differently weighted. If one part of the sky is more magnified, or at a closer angular-size distance, the corresponding area of the constant-*z* surface will be smaller, so fewer sources are likely to be found there. In other words, choosing a source at random will give on average a smaller magnification or larger angular-size distance.

- Weinberg: $<\mu> = 1$ when averaged over *sources*
- Kibble & Lieu: $<1/\mu> = 1$ when averaged over *directions on the sky*
  - latter is more relevant for CMB observations
    - strictly only valid in weak lensing regime

Recent developments...

- Backreaction: "have cosmologists erred in failing to take into account the inherent non-linearity of Einstein's equations?"
  - cosmologists tend to do linear theory calculations
  - but Einstein's equations (metric <-> matter) are non-linear
  - averaging and non-linearity "do not commute"
  - so is *dark energy* a mirage?
- requires calculations in 2nd order perturbation theory (v. technical)
- now mostly accepted that effects are too small to explain acceleration
- but maybe there are still interesting percent level effects:
  - Clarkson, Ellis++ '12 - large ($O(\kappa^2)$) source magnification
  - Clarkson++ '14 - similarly large z-surface *area* increase
    - violates Weinberg's assumption
  - "backreaction" strikes back?
  - and the size of the effect is qualitatively consistent with expectation of the *focusing theorem* (Seitz, Schneider & Ehlers)




What is the distance to the CMB? How relativistic corrections remove the tension with local H_0 measurements

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The success of precision cosmology depends not only on accurate observations, but also on the theoretical model – which must be understood to at least the same level of precision. Subtle relativistic effects can lead to biased measurements if they are neglected. One such effect gives a systematic shift in the distance-redshift relation away from its background value, due to the accumulation of all possible lensing events. We estimate the expectation value of this aggregated lensing using second-order perturbations about a concordance background, and show that the distance to last scattering is shifted by several percent. Neglecting this shift leads to significant bias in the background cosmological parameters. We show that this removes the tension between local measurements of H_0 and those measured through the CMB and favours a closed universe.

Clarkson et al. 2014

$$\langle \Delta \rangle \simeq \frac{3}{2} \left\langle \left(\frac{\delta d_A}{\chi_s} \right)^2 \right\rangle = \frac{3}{2} \left\langle \kappa^2 \right\rangle \,, \tag{1.5}$$

where κ is the usual linear lensing convergence. This is actually the leading contribution to the expected change to large distances. We prove this remarkably simple and important result in a variety of ways in several appendices. It implies that the total area of a sphere of constant redshift will be larger than in the background. Physically this is because a sphere about us in redshift space is not a sphere in real space — lensing implies that this 'sphere' becomes significantly crumpled in real space, and hence has a larger area. When interpreted

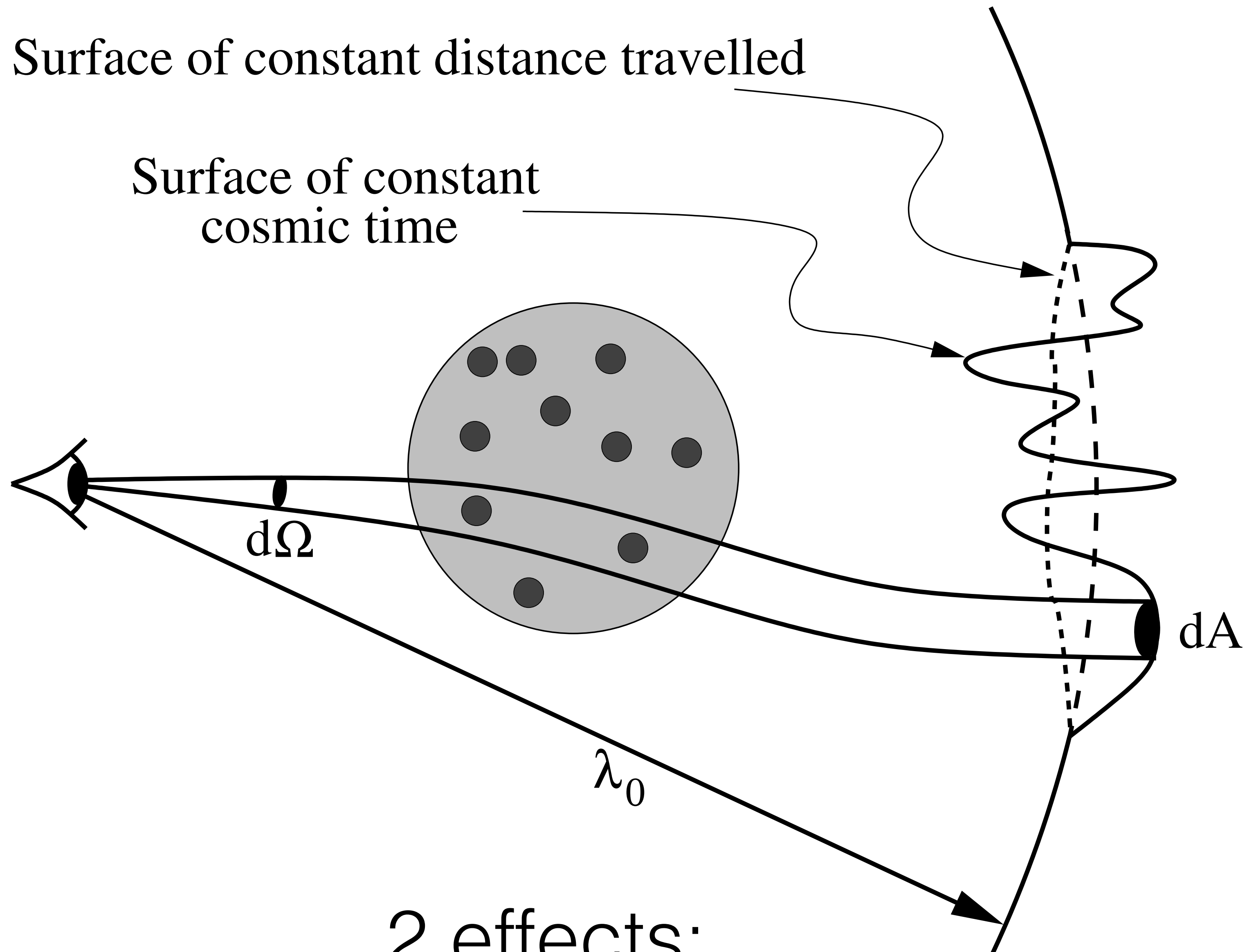
4 Conclusions

We have demonstrated an important overall shift in the distance redshift relation when the aggregate of all lensing events is considered, calculated by averaging over an ensemble of universes. This result is a consequence of flux conservation at second-order in perturbation theory. This is a purely relativistic effect with no Newtonian counterpart — and it is the first quantitative prediction for a significant change to the background cosmology when averaging over structure [21]. The extraordinary amplification of aggregated lensing comes mainly from the integrated lensing of structure on scales in the range 1–100 Mpc, although structure down to 10kpc scales contributes significantly. We have estimated the size of the effect using

NK + Peacock 2015

- Weinberg *assumes* that the area of a surface of constant redshift is unperturbed by lensing by intervening structures
  - same assumption is made by Kibble & Lieu
  - seems reasonable since *static* lenses do not affect redshift
  - and leads to conservation of e.g. source-averaged flux density
    - but not strictly true and breaks down at some level
- What *is* the change in the area of the constant-z surface (or cosmic photosphere) caused by structure?

KP2015: closing the loophole in Weinberg's argument



2 effects:

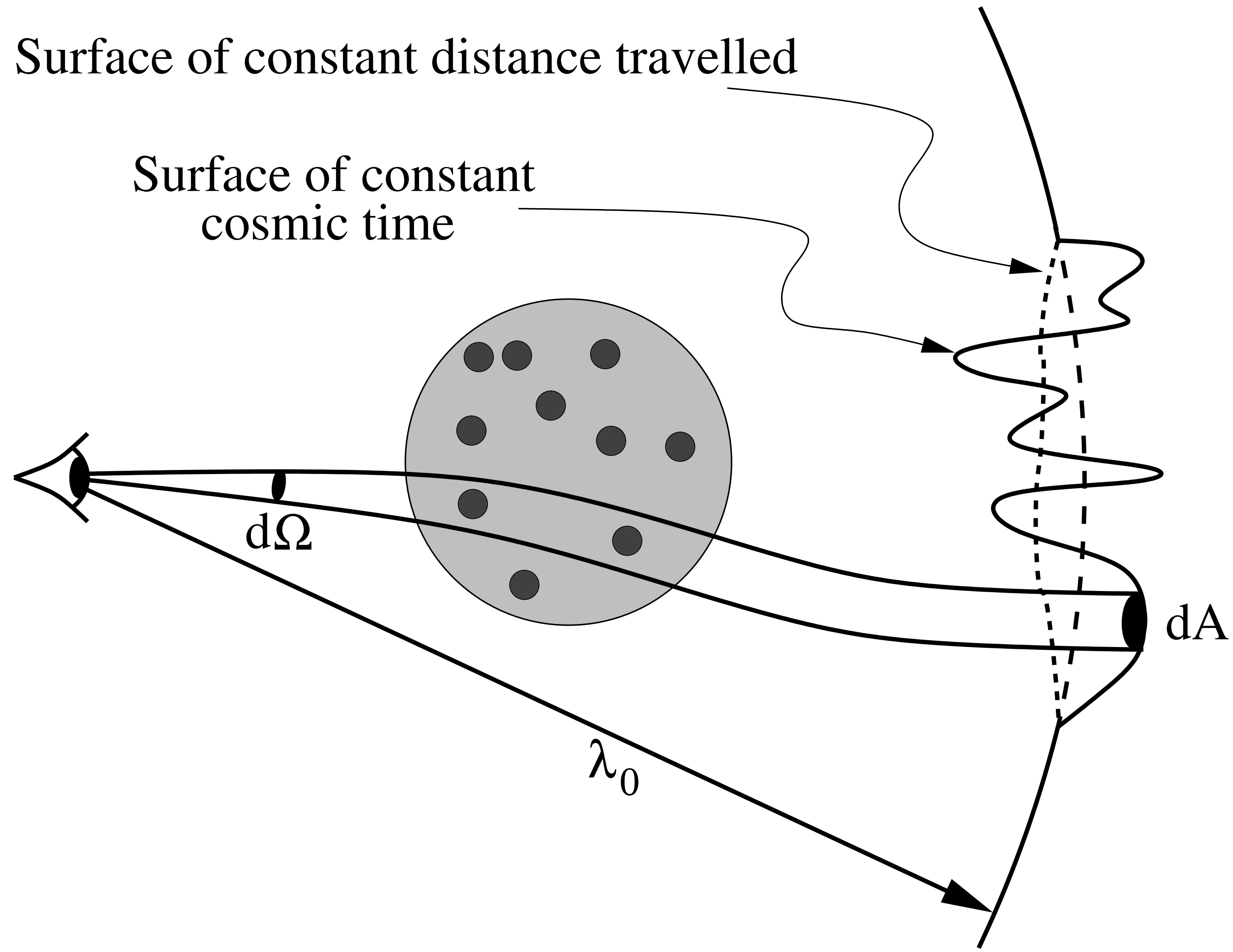
1) wiggly lines don't get as far as straight lines
Estimate from phythagoras

2) wrinkly surface has more area than a smooth one

What is the area of a wavy surface?

Or a lumpy sphere?

KP2015: closing the loophole in Weinberg's argument



2 effects:

1) wiggly lines don't get as far as straight lines

2) wrinkly surface has more area than a smooth one

but both effects are ~(bending angle)^2 ~ 10^{-6}

Key features of KP15 calculation of area of photosphere

- Calculations are rather technical, some key features are:
  - Weak field assumption:
    - we model the metric as weak field limit of GR
      - but we allow for non-rel motion of sources
        - these have negligible effects
      - similarly for gravitational waves
        - "photons can't surf a gravitational wave"
      - going beyond 1st order can be estimated and is tiny effect
    - *the problem is isomorphic to light propagation in "lumpy glass"*
  - Boundary conditions:
    - Perturbation theory calculations assume photosphere is constant z
    - Not true. It is more realistically a surface of constant cosmic time
    - Pert. theo. results may be qualitatively OK, but fail quantitatively
  - Final result for perturbation to the area of the photosphere is

$$\langle \Delta A \rangle / A_0 = \frac{1}{\lambda_0^2} \int_0^{\lambda_0} d\lambda \, (2\lambda(\lambda_0 - \lambda) + \lambda^2) J(\lambda).$$

where

$$J \equiv -8 \int_{-\infty}^{0} dy \, \xi'_\phi(y)/y = 2\pi \int k \, \Delta_\phi^2(k) \, d \ln k,$$

but J = d<θ^2>/dλ and Jλ is on the order of 10^{-6}

NK + Peacock 2015 - 2nd point

- Perturbation to the *area* is on the order of the mean squared cumulative deflection angle
- This is a one-part-in-a-million effect
  - dominated by large-scale structure
- Relativistic perturbation theory, *focussing theorem* etc. give perturbation to the distance that is on the order of the mean squared convergence
  - much larger
  - dominated by small-scale structure (possibly divergent)
- These large effects are correct, but are *purely statistical effects*:
  - The mean flux magnification μ of a source is unity
    - so $\langle \Delta\mu \rangle_{source} = 0$
  - but μ is a fluctuating quantity
  - so any non-linear function of μ (e.g. $D/D_0 = 1 / \sqrt{\mu}$) will *not* average to unity

KP15: Statistical biases...

- Example: Source averaged distance bias:
  - $D/D_0 = \mu^{-1/2} = (1 + \Delta\mu)^{-1/2} = 1 - \Delta\mu / 2 + 3(\Delta\mu)^2/8 + \ldots$
  - so $\langle D/D_0 \rangle_{source} = 1 + 3\langle(\Delta\mu)^2\rangle/8 + \ldots = 1 + 3\langle\kappa^2\rangle/2 + \ldots$
- Similarly for source averaged mean inverse magnification
  - $\langle D^2/D_0^2 \rangle_{source} = 1 + 4 \langle\kappa^2\rangle + \ldots$
- *These are the results found from 2nd order perturbation theory*
- But e.g. the latter is not the perturbation to the constant z surface area
  - that would be the average over *directions* rather than over sources
- Similarly, Clarkson et al. 2012 claim mean source averaged flux magnification is $\langle\mu\rangle = 1 + \langle 3\kappa^2 + \gamma^2 \rangle + \ldots = 1 + \langle 4\kappa^2 \rangle + \ldots.$
  - but this is the *direction* averaged magnification
- *These come from non-commutativity of averaging and non-linearity*
  - $\langle f(x) \rangle \neq f(\langle x \rangle)$ if x is a fluctuating quantity
  - and have nothing to do with the non-linearity of Einstein's equations

What about the "*focusing theorem*"? $\langle \ddot{D} \rangle / D_0 = -\langle \Sigma^2 \rangle < 0.$

- We have developed the optical scalar transport equations in a form appropriate when one wishes to specify the metric fluctuations as a stochastic random field (with zero mean for k=0 component)
  - interesting subtlety: one should *not* assume $\langle \delta R \rangle = 0$
  - in inflationary context, small scale space-time curvature fluctuations have to accommodate themselves within the (flat-space) boundary conditions imposed when the larger regions accelerate outside of horizon
- We have solved these to obtain the ensemble average of the perturbation to the *area* of a beam of specified solid angle fired off from the observer and propagating back to the source surface.
- Cancellations: Not just "Born level", but 1st "beyond Born" also
- We were only able to solve for the case where metric fluctuations are non-evolving (like in Einstein - de Sitter) but were able to obtain the "un-focusing theorem": $\langle \Delta A/A \rangle = - 2J\lambda/3 + ...$
  - this is consistent with the more general result (variable J) found by more straightforward approach.
- An exactly analogous calculation for $\langle \Delta D/D \rangle$ does not show cancellation and results in much larger ($O(\varkappa^2)$) result. *But just the statistical bias*. QED

Optical scalars (in weak-field GR or lumpy glass)

$\ddot{\mathbf{r}} = \boldsymbol{\nabla}_\perp \tilde{n}$ Geodesic equation

$$n = [(1 - 2\phi(\mathbf{r})/c^2)/(1 + 2\phi(\mathbf{r})/c^2)]^{1/2}$$

Optical *tensor* transport equation:

$$\dot{\mathbf{K}} = (\boldsymbol{\nabla}_\mathbf{x}\boldsymbol{\nabla}_\mathbf{x} - \mathbf{K}\partial_z)\tilde{n} - \boldsymbol{\nabla}_\mathbf{x}\tilde{n}\boldsymbol{\nabla}_\mathbf{x}\tilde{n} - \mathbf{K}\cdot\mathbf{K}$$

Optical scalar transport equations:

$$\dot{\theta} = \left(\frac{{\nabla_\perp}^2}{2} - \theta\partial_\lambda\right)\tilde{n} - |\boldsymbol{\nabla}_\perp\tilde{n}|^2/2 - \theta^2 - \Sigma^2$$

$$\dot{\boldsymbol{\Sigma}} = (\{\boldsymbol{\nabla}_\perp\boldsymbol{\nabla}_\perp\} - \boldsymbol{\Sigma}\partial_\lambda)\tilde{n} - \{\boldsymbol{\nabla}_\perp\tilde{n}\boldsymbol{\nabla}_\perp\tilde{n}\} - 2\theta\boldsymbol{\Sigma}$$

Solve for θ

The solution of $\dot{A}/2A = \theta(\lambda) = \lambda^{-1} + \Delta\theta(\lambda)$ is

$$A = \Omega\lambda^2 \exp\left(2\int_0^\lambda d\lambda'\,\Delta\theta(\lambda')\right)$$

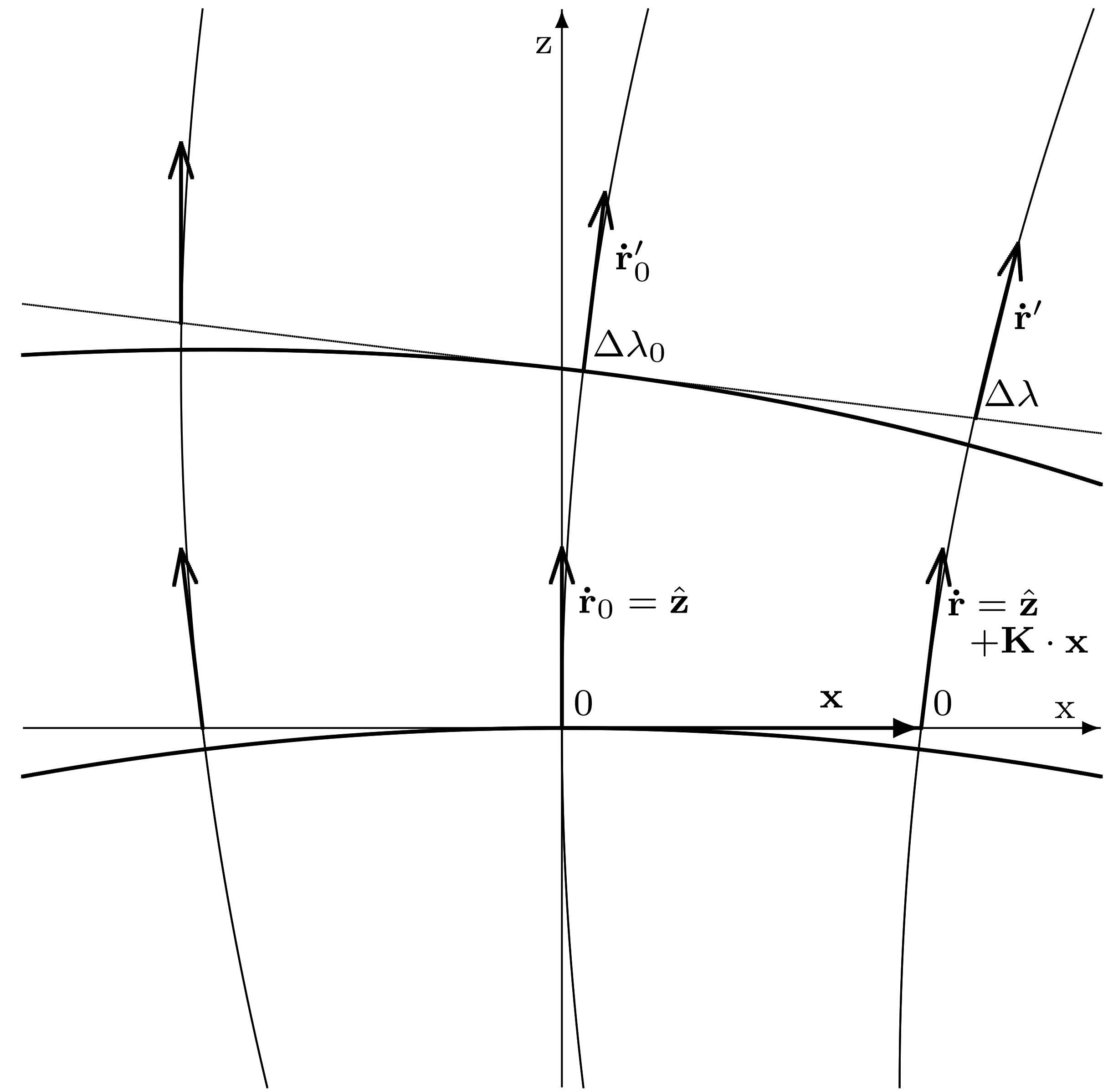
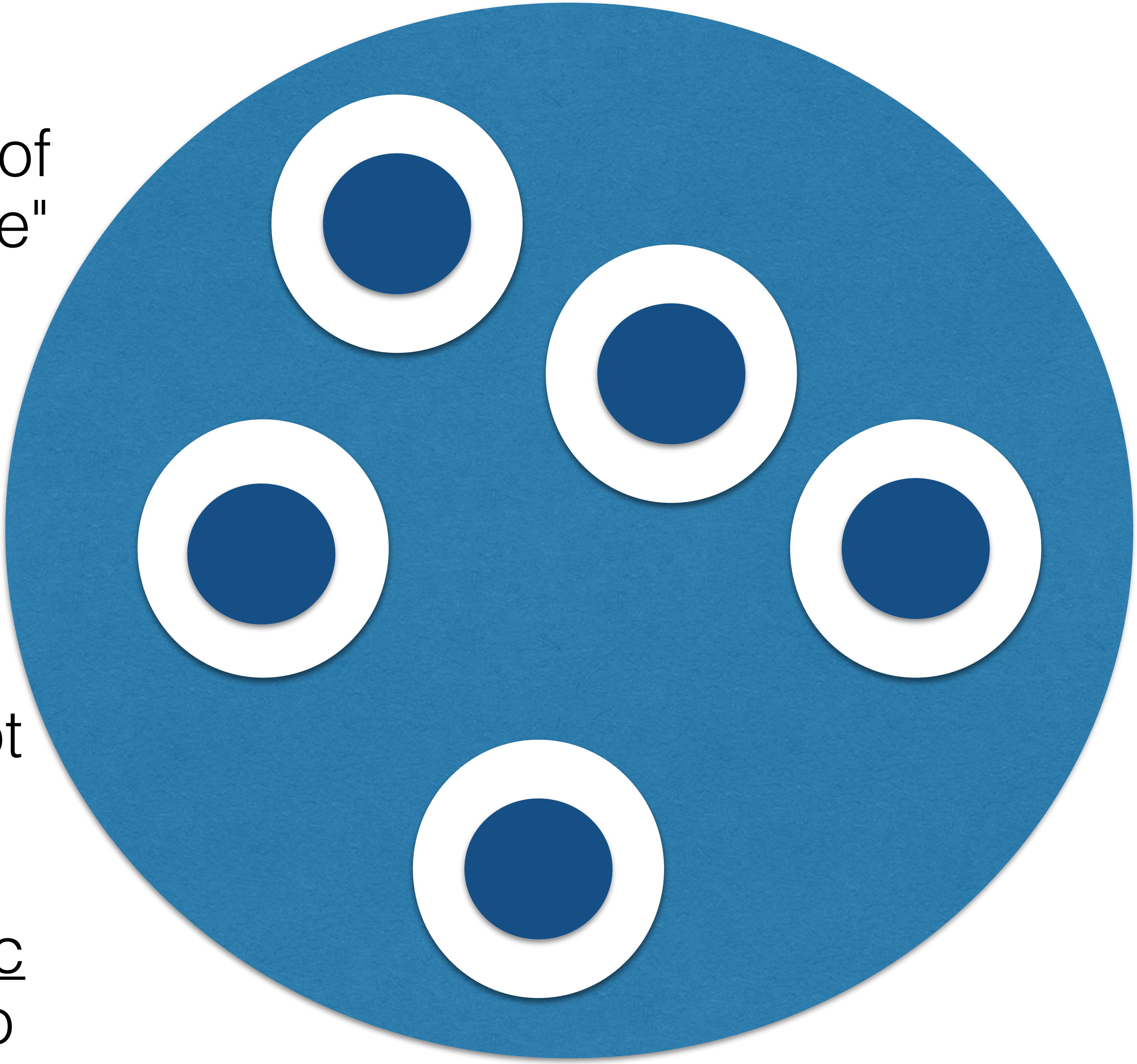


Figure D1. Illustration of a bundle of rays (thin curves) and associated wave-fronts (thick curves) and ray direction vectors $\dot{\mathbf{r}} = d\mathbf{r}/d\lambda$ (arrows). The base of each arrow is labelled by distance (physical for lumpy glass, background conformal for perturbed FRW) along the path. Close to the guiding ray the ray vectors will vary linearly with transverse displacement. The optical tensor $\mathbf{K}$ is the derivative of the ray direction with respect to coordinates $\mathbf{x}$ on the plane that is tangent to the wavefront at the location of the guiding ray. The optical tensor transport equation tells us how $\mathbf{K}$ evolves as the bundle propagates through any metric or refractive index fluctuations. Since rays are perpendicular to the

- Einstein-Straus '45
  - "What is the effect of expansion of space"
- -> Swiss-cheese
- Fully non-linear
- proper mass perturbation does not average to zero
- Need to model metric perturbations as zero mean process



Why there is no Newtonian backreaction

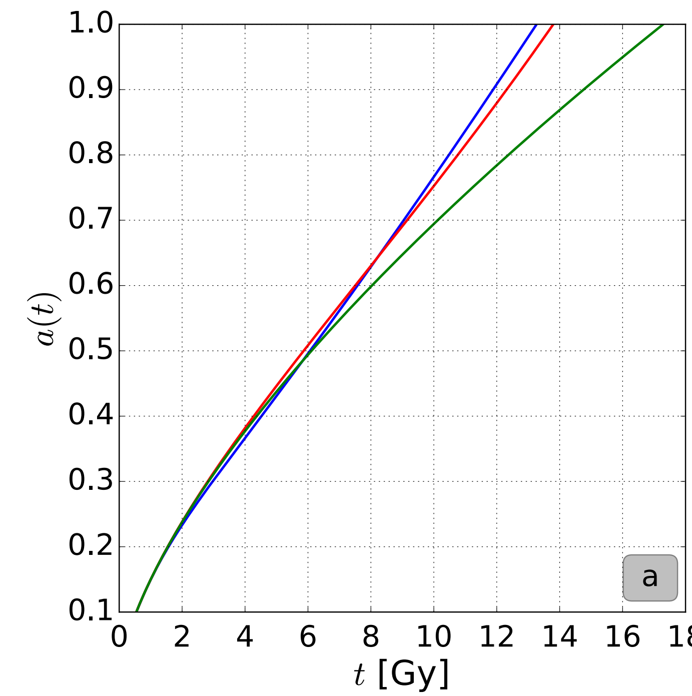
arXiv:1703.08809

Conventional Framework for Cosmological Dynamics

- Homogeneous background with scale factor a(t)
  - $a'' = -(4\pi/3)\, G\, \rho_b\, a$ $(' = d/dt)$ Friedmann eq
- Structure (in e.g. N-body calc.) obeys
  - $\mathbf{x}'' + 2\,(a'/a)\,\mathbf{x}' + \boldsymbol{\nabla}\phi \,/\, a^2 = 0$ where
    - $\mathbf{x} = \mathbf{r} \,/\, a(t)$ are "conformal" coords, and
    - $\nabla^2\phi = 4\pi\, G\, (\rho - \rho_b)\, a^2$
- No feedback (or "backreaction") of $\delta\rho$ on evolution of a(t)
- G.F.R. Ellis (1984...): is this legitimate?
  - explored by Buchert & Ehlers '97 plus many others

Racz et al 2017: Modified N-body calculations

- They assume the conventional structure equations:
  - $\mathbf{x}'' + 2\,(a'/a)\,\mathbf{x}' + \boldsymbol{\nabla}\phi\,/\,a^2 = 0$
  - $\nabla^2\phi = 4\pi\,G\,(\rho - \rho_b)\,a^2$
- but evolve a(t) according to $a \rightarrow a + a'\delta t$



  - with a' obtained by averaging local expansion: <a'/a> invoking "separate universe" approximation
  - "Strong backreaction" based on Newtonian physics
- Big effect: a(t) very similar to ΛCDM concordance model
  - "concordance cosmology without dark energy"

Racz et al. world view

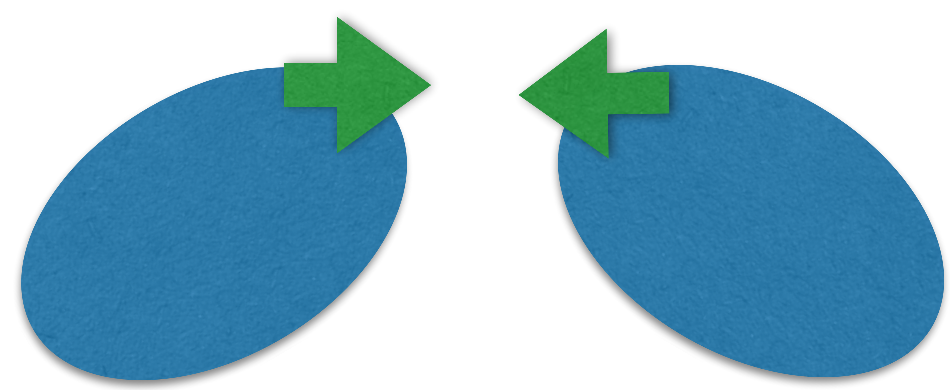
- *"N-body simulations integrate Newtonian dynamics with a changing GR metric that is calculated from averaged quantities"*
- *"changing GR metric"*: FRW metric: expansion factor a(t)
  - a(t) comes from strong-field GR physics
    - so we don't really understand it except in highly idealised (e.g. homogeneous) situations
    - hence legitimate to propose alternative ansatz?
  - a(t) - the "expansion of space" - affects the small-scale dynamics of structure

Is it legitimate to modify the Friedmann equation?

- Does emergence of structure really "backreact" on a(t)?
- Can address this in Newtonian gravity. Relevant as:
    - Accurate description of the local universe ($v << c$)
        - aside from effects from BHs
    - this is where we observe e.g. H_0 = 70 km/s/Mpc!
        - not H_0 ~ 35 km/s/Mpc expected w/o dark energy, Ω_k
    - At $z = 0.1$ relativistic corrections ~ 0.01
- If backreaction is important at > 1% level Newtonian analysis should show it

Why we might expect backreaction - tidal torques

- Neighbouring structures exert torques on each other
  - happens as structures reach $\delta \sim 1$
  - a non-linear (2nd order) effect
  - purely Newtonian
  - explains spin of galaxies
- can this affect expansion?
  - it does in the local group
  - do internal degrees of freedom couple to (i.e. exchange energy with) universal expansion



Inhomogeneous Newtonian cosmology

- Lay down particles on a uniform grid in a big uniformly expanding sphere ($\mathbf{v} = H\mathbf{r}$)
- Perturb the particles off the grid $\mathbf{r} \rightarrow \mathbf{r} + \delta\mathbf{r}$
  - plus related velocity perturbations to generate "growing mode" of structure
- $\mathbf{g}(\mathbf{r})$ can be decomposed into:
  - homogenous field sourced by mean density ρ
  - inhomogeneous field sourced by $\delta\rho$ (little dipoles)
- equations of motions $\mathbf{r}'' = \mathbf{g}$ can be re-scaled
  - gives the equations that are solved in N-body codes

Newtonian gravity in re-scaled coordinates

N-particles of mass m: $$\ddot{\mathbf{r}}_i = Gm \sum_{j \neq i} \frac{\mathbf{r}_j - \mathbf{r}_i}{|\mathbf{r}_j - \mathbf{r}_i|^3}.$$

With **r** = a(t) **x** for arbitrary a(t)

$$\ddot{\mathbf{x}}_i + 2\frac{\dot{a}}{a}\dot{\mathbf{x}}_i = \frac{Gm}{a^3} \sum_{j \neq i} \frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|^3} - \frac{\ddot{a}}{a}\mathbf{x}_i.$$

initial conditions: $\mathbf{x} = \mathbf{r}/a$ and $\dot{\mathbf{x}} = ((H - \dot{a}/a)\mathbf{r} + \delta\dot{\mathbf{r}})/a$

Defining $n(\mathbf{x}) \equiv \sum_i \delta(\mathbf{x} - \mathbf{x}_i)$ and $\delta n \equiv n - \overline{n}$

$$\ddot{\mathbf{x}}_i + 2\frac{\dot{a}}{a}\dot{\mathbf{x}}_i - \frac{Gm}{a^3} \int d^3x \; \delta n(\mathbf{x}) \frac{\mathbf{x} - \mathbf{x}_i}{|\mathbf{x} - \mathbf{x}_i|^3} = -\left(\frac{\ddot{a}}{a} + \frac{4\pi G m \overline{n}}{3a^3}\right)\mathbf{x}_i.$$

Exactly equivalent to the usual equations in **r**-coords

Newtonian cosmology:

$$\ddot{\mathbf{x}}_i + 2\frac{\dot{a}}{a}\dot{\mathbf{x}}_i - \frac{Gm}{a^3}\int d^3x\ \delta n(\mathbf{x})\frac{\mathbf{x}-\mathbf{x}_i}{|\mathbf{x}-\mathbf{x}_i|^3} = -\left(\frac{\ddot{a}}{a} + \frac{4\pi G m\bar{n}}{3a^3}\right)\mathbf{x}_i.$$

with ICs

$$\mathbf{x} = \mathbf{r}/a \quad \text{and} \quad \dot{\mathbf{x}} = ((H - \dot{a}/a)\mathbf{r} + \delta\dot{\mathbf{r}})/a$$

- 3N equations for N particles
  - there is no extra equation of motion for a(t)
- But we may choose a(t) to obey Friedmann equation
  - an "auxiliary relation"
- Gives conventional expansion + structure equations
  - a(t) suffers no backreaction from structure emergence
  - a(t) is just a "book-keeping" factor - no physical effect

Part 1: summary/conclusions

- A different perspective on the conventional equations for structure growth (Dmietriev & Zel'dovich '63)
    - fully non-linear & exact (but Newtonian) description
- a(t) is arbitrary, but extra terms appear in equations of motion if a(t) does not obey Friedmann's equation
    - physical quantities invariant under choice of a(t)
- No coupling of expansion to internal structure via tidal torques
    - can also be understood from scaling with radius/mass

Relation to Buchert & Ehlers '97 "kinematic BR"

- Matter modelled as pressure-free Newtonian fluid
  - unrealistic, but maybe a useful "toy model"
- Consider a specific volume $V = a^3$ containing mass M
- Raychaudhuri equation (expansion θ, vorticity ω, shear σ)
  - $a''/a + (4\pi/3)\, GM/a^3 = Q$
    - with $Q = 2(\langle\theta^2\rangle - \langle\theta\rangle^2)/3 + 2\langle\omega^2 - \sigma^2\rangle$
    - 2nd order - no linear effect!
- Naively a big effect (individual terms in $Q \sim G\rho$)
  - but...

Buchert & Ehlers '97

- "Generalised Friedmann equation": $\ddot{a}/a + GM/a^3 = Q$
  - $Q = 2(\langle\theta^2\rangle - \langle\theta\rangle^2)/3 + 2\langle\omega^2-\sigma^2\rangle$
  - Q=0 is *"conspiracy assumption"*
- But *"Q is a divergence"*: $Q = V^{-1}\int \mathbf{dA}.(\mathbf{u}(\nabla.\mathbf{u})-(\mathbf{u}.\nabla)\mathbf{u})$
    - so no global effect for periodic BCs - *"by construction"*
- No surprise that $a''/a \neq -GM/a^3$ for an individual region
  - fluctuations affect acceleration a'' and M
  - but local, not "backreaction of $\delta\rho$ on global expansion"
- If $\langle Q\rangle_{V\to\infty}$!= 0 would imply a conflict - this is not the case

Do B&E claim Newtonian backreaction?

- Q = 0 requires *"conspiracy"* - but *"the average motion may be approximately given by the Friedmann equation on a scale which is larger than the largest existing inhomogeneities"*

- Later works: E.g. Buchert & Rasanen 2011 review

  - *"..linear theory ... effect vanishes by construction ... in Newtonian ... true also in non-perturbative regime"*

  - *"When we impose periodic BCs .... Q is strictly zero"*

  - but *"If backreaction is substantial then current Newtonian simulations (and analytic studies) are inapplicable".*

- So the absence of backreaction is a consequence of assuming (falsely, one presumes) periodicity.

- How big is Q in reality?

How large is $Q = (3 a''/a + 4\pi GM/a^3)$?

- $Q = Q_1 + Q_2 = V^{-1}\int \mathbf{dA}.(\mathbf{u}(\nabla.\mathbf{u})-(\mathbf{u}.\nabla)\mathbf{u}) - (3/2V^2)(\int \mathbf{dA}.\mathbf{u})^2$
  - **u** is peculiar velocity wrt <u>global</u> H
- If structure is a stat. homog. and isotropic random process (i.e. random vector field)
  - $\langle \mathbf{u}(\nabla.\mathbf{u})-(\mathbf{u}.\nabla)\mathbf{u}\rangle_{\text{ensemble}} = 0$ (Monin and Iaglom, 1975)
  - so Q_1 is pure fluctuation
  - $|Q_1| \sim \langle u^2\rangle/r^2$ independent of coherence length λ
- Second term is systematic: $Q_2 \sim \langle u^2\rangle \lambda^2 / r^4$
- Both are <u>very small</u> ($\ll H^2$) for large V

Is there relativistic backreaction?

- Claims: *"GR backreaction"* is non-zero - and large
- But local universe should be accurately Newtonian
    - errors ~ $v^2/c^2 \rightarrow$ ~1% accuracy within $z = 0.1$
    - and that's where we measure H_0
    - so very hard to believe there are >> 1% effects
- Q: Are there even very small effects on expansion history coming from non-relativistic effects?

Is there *relativistic* backreaction?

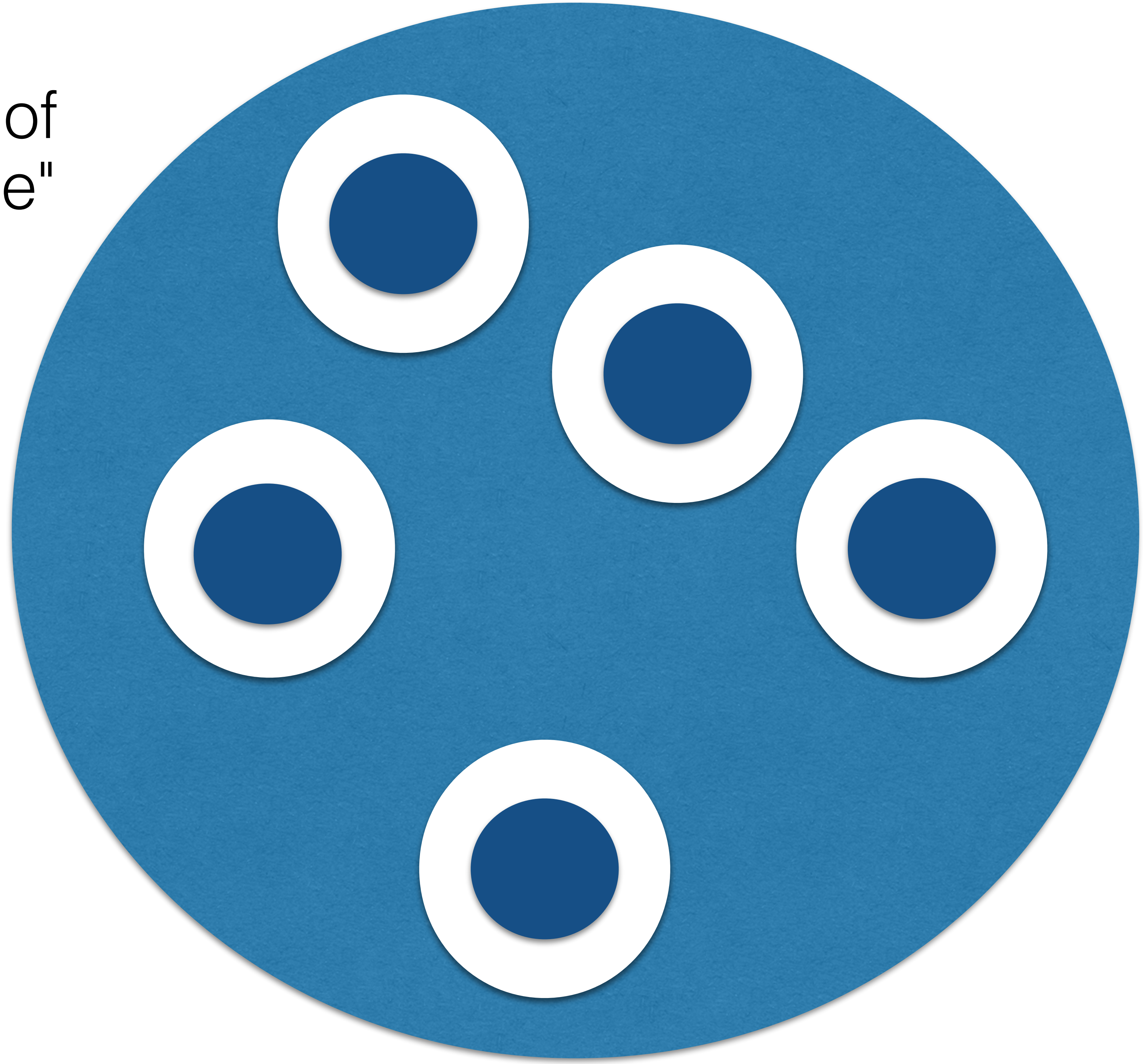
- Averaging of Einstein equations: **G** = **T**
- FRW: metric **g** -> **G** and **T** = diag(ρ, P, P, P) are diagonal
  - **G** = **T** and **∇.T** = 0 -> Friedmann equations
- with inhomogeneity < **G** > = < **T** >?
  - *"averaging problem"* widely discussed in BR literature
- what about internal pressure P of clusters?
  - or internal pressure in stars, other compact objects
- Do those give Friedmann equations with non-zero P?
  - and hence deviation from Newtonian expansion law?

Averaging of Einstein equations: < **G** > = < **T** >?

- Consider e.g. stars with internal pressure P
  - does that give Friedmann equations with non-zero P?
- No. Stars have Schwarzschild exterior with mass m
  - space integral of the stress pseudo-tensor
    - includes rest mass, motions, P, binding energy
    - but is independent of time
- Conservation of stars implies $\rho \sim a^{-3}$
  - which demands P = 0 in the Friedmann equations

Relativistic BR from large-scale structure?

- Einstein-Straus '45
  - "What is the effect of expansion of space"
- -> Swiss-cheese
- Fully non-linear
- Interesting pert^n to e.g. proper mass
- but background expansion is exactly unperturbed
- small effects on D(z)



Backreaction from inter-galactic pressure

- Stars & DM ejected from galaxies by merging SMBHs
  - intergalactic pressure $P = n\ m\ \sigma_v^2$
  - and P in the background of GWs emitted
- Homogeneous (in conformal coords) pressure is a <u>flux of energy</u> with non-zero divergence in real space
  - 1st law ... PdV work .... : $\rho' = -(\rho + P/c^2)\ V' / V$
    - but a <u>very small effect</u>
- relies on pressure being extended throughout space
  - no effect from internal pressure in bound systems that are surrounded by empty space

Summary

- A different perspective on the DZ equations. There is no dynamical equation for a(t). a(t) is arbitrary. But there is no freedom to modify F-equation w/o changing structure eqs. Conventional system of equations is exact.

- Clarification of "generalised Friedmann equation". Periodic BCs is not the issue. $|Q_1| \sim <v^2>/r^2$ and $<Q_1> = 0$ (Monin and Iaglom). $<Q_2> \sim <v^2>\lambda^2/r^4$. Both are v. small and tend to zero for large r.

- Discussion of relativistic backreaction. Averaging of stress-energy for systems with internal pressure does not introduce non-zero P in Freidmann equations. Exact non-linear solutions show no backreaction. Intergalactic P does backreact, but P is weak and positive.