Gravitational waves implications on dark energy and modified gravity

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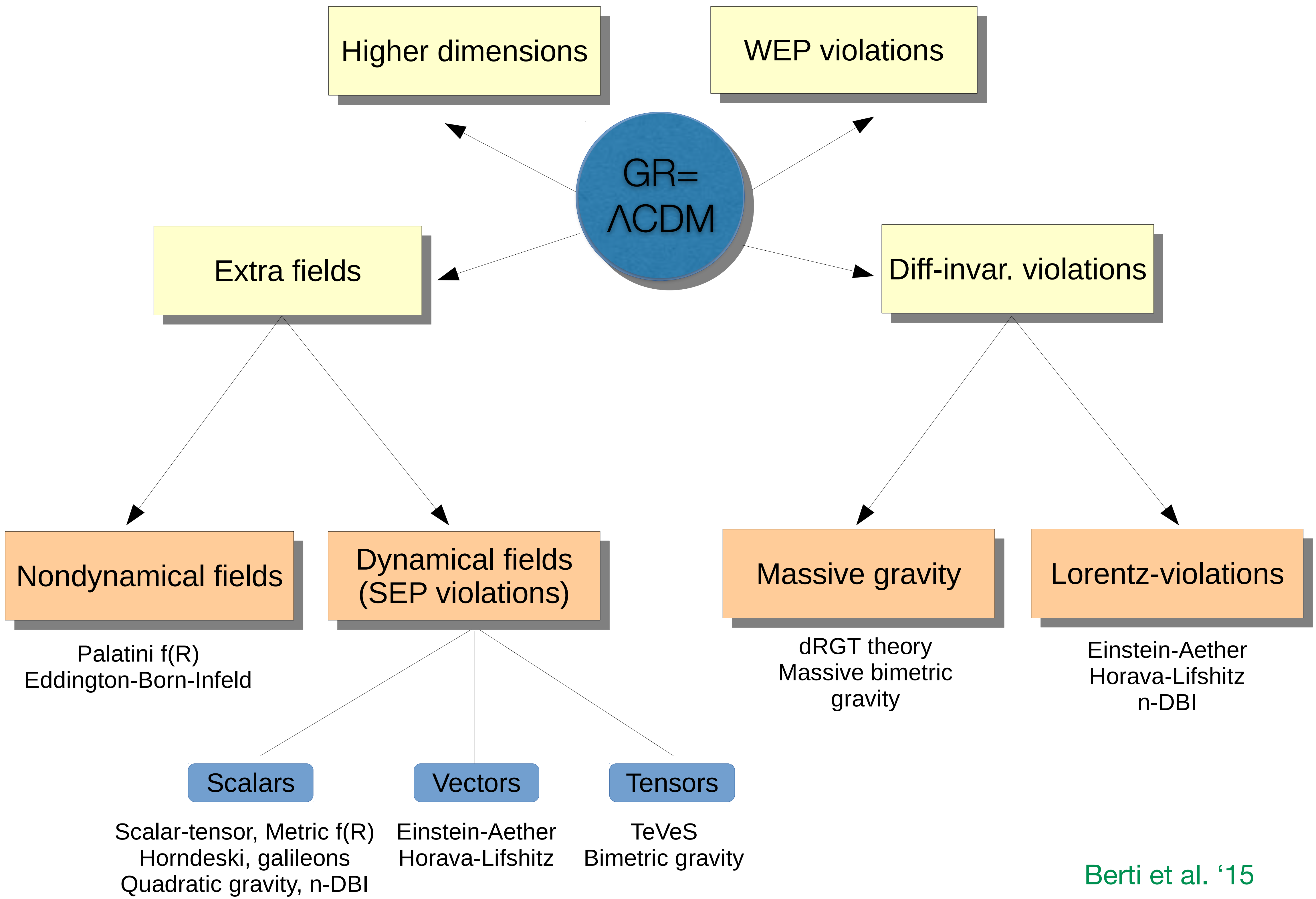
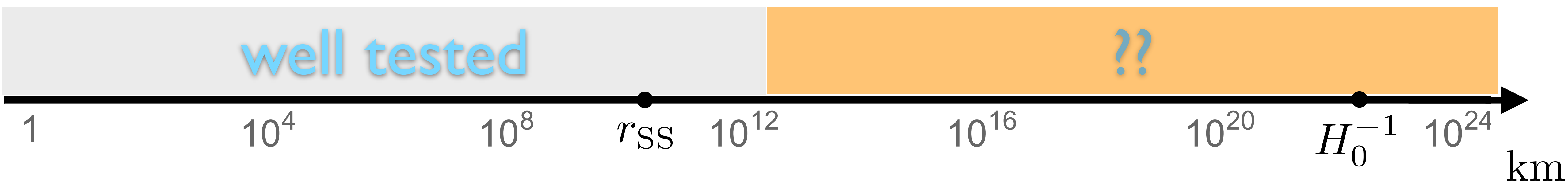
Dark energy and modified gravity

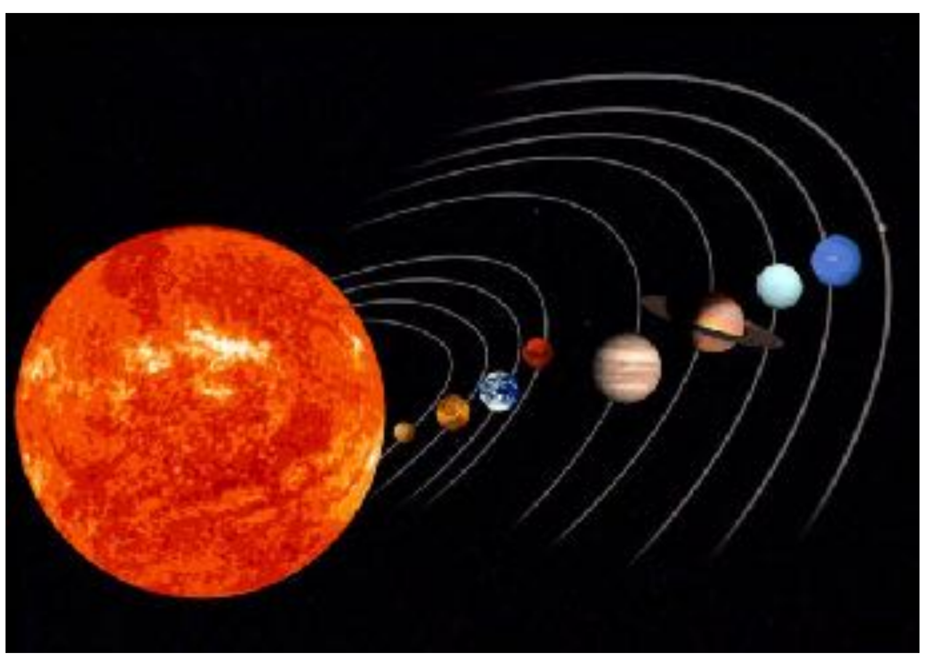


Berti et al. ‘15

Motivations

General relativity tested over special ranges of scales and masses. Cosmology is a window for testing it on very large distances. Distinguish among models and discover new physics. Cosmological precision tests of ΛCDM (precision tests of the Standard Model at the LHC)





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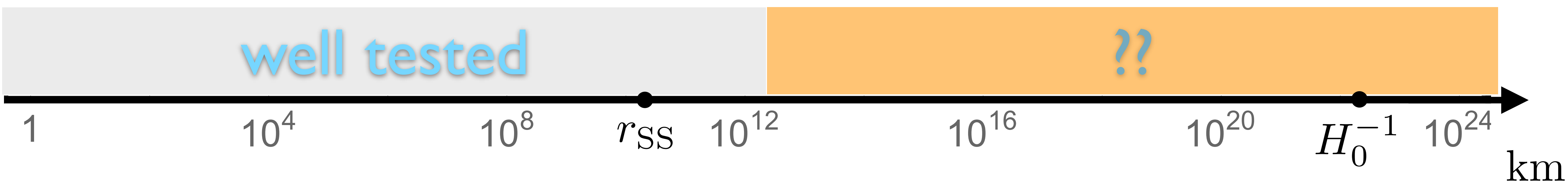
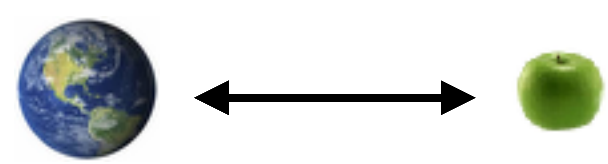
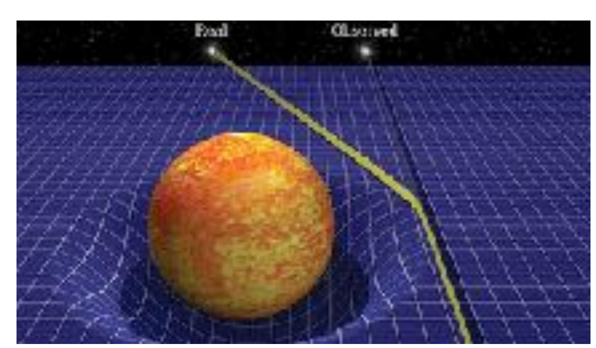


$$\nabla^2\Phi = 4\pi G\,\mu\,\delta\rho_{\rm m}$$
$$\nabla^2(\Phi+\Psi) = 8\pi G\,\Sigma\,\delta\rho_{\rm m}$$



fifth force



anomalous light bending

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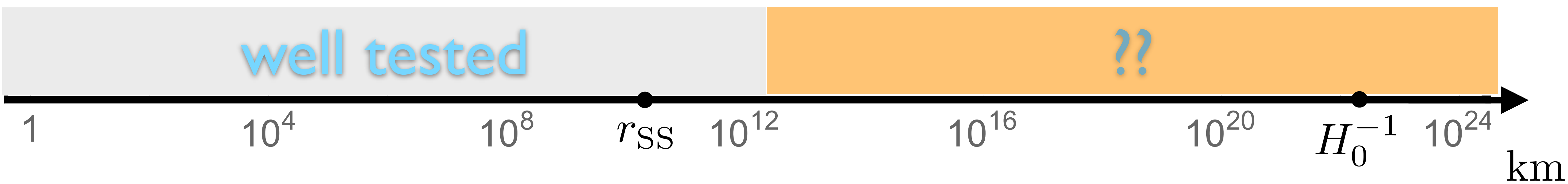


Will '14 $|\mu - 1| < 10^{-3} \div 10^{-6}$

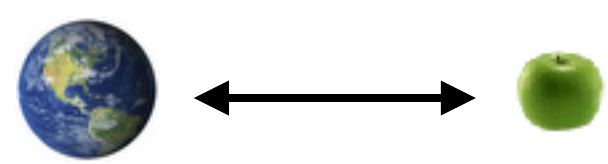
$|\Sigma - 1| < 10^{-5}$

Solar System scales

DES '18 $|\mu - 1| < 8 \times 10^{-2}$

$|\Sigma - 1| < 4 \times 10^{-1}$

Cosmological scales



fifth force

$$\nabla^2 \Phi = 4\pi G \, \mu \, \delta\rho_{\rm m}$$

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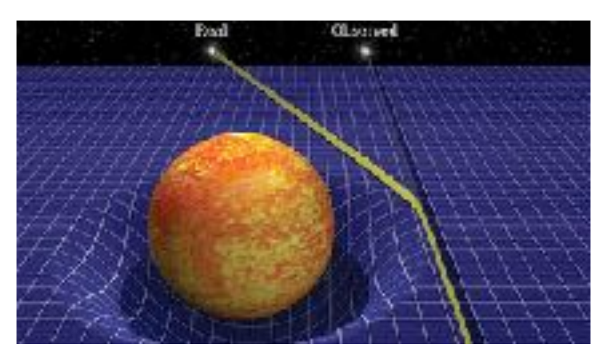
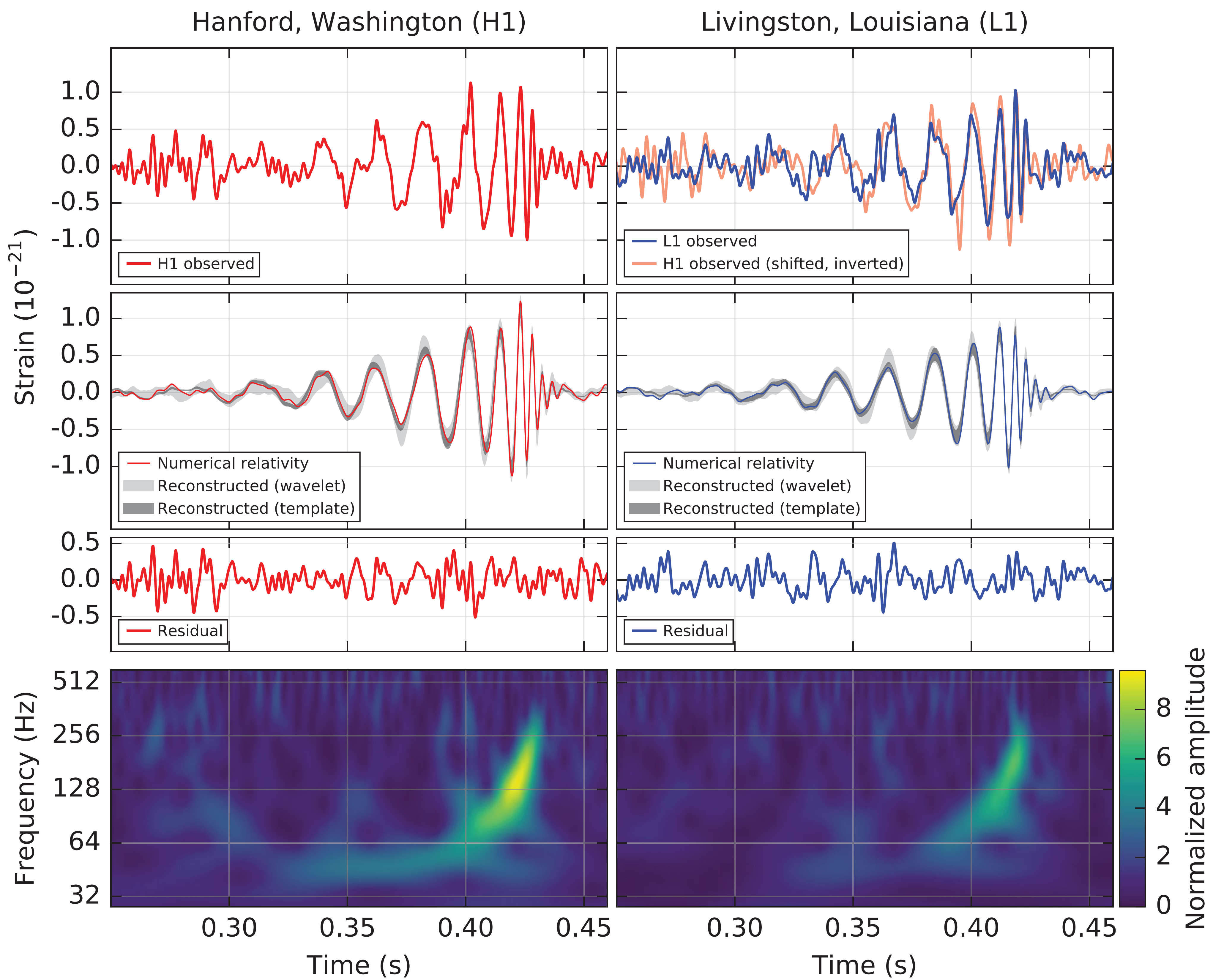


anomalous light bending

Gravitational Waves

Abbott et al. '16



Wave equation

Gravitational wave equation:

$$ds^2 = -dt^2 + a^2(t)\left[\delta_{ij} + \gamma_{ij}\right] d\vec{x}^i d\vec{x}^j \ , \qquad \gamma_{ii} = 0 = \partial_i \gamma_{ij} \ , \qquad H = \dot{a}/a$$

$$\ddot{\gamma}^{\lambda}_{ij} + 3H\dot{\gamma}^{\lambda}_{ij} + k^2 \gamma^{\lambda}_{ij} = 16\pi G\, S^{\lambda}_{ij} \ , \qquad \lambda = +, \times$$

Modified wave equation

Gravitational wave equation:

$$ds^2 = -dt^2 + a^2(t)\left[\delta_{ij} + \gamma_{ij}\right] d\vec{x}^i d\vec{x}^j \ , \qquad \gamma_{ii} = 0 = \partial_i \gamma_{ij} \ , \qquad H = \dot{a}/a$$

damping dispersion source polarizations

$$\ddot{\gamma}^{\lambda}_{ij} + H\left[3 + \ldots\right]\dot{\gamma}^{\lambda}_{ij} + \left[c_T^2 k^2 + \ldots\right]\gamma^{\lambda}_{ij} = 16\pi G\, S^{\lambda}_{ij} \ , \qquad \lambda = +, \times$$

Modified wave equation

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damping   dispersion   source   polarizations

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Modifications in the wave equation are related to modifications of gravity in the LSS:

$$\mu = \mu(\ldots) \ , \qquad \Sigma = \Sigma(\ldots)$$

$$\nabla^2 \Phi = 4\pi G\, \mu\, \delta\rho_{\mathrm{m}}$$

$$\nabla^2 (\Phi + \Psi) = 4\pi G\, \Sigma\, \delta\rho_{\mathrm{m}}$$

Source

Modified gravity can change the motion of inspiral objects and thus the production mechanism. Affects GW phase (and amplitude)

$$\ddot{\gamma}_{ij}^{\lambda} + 3H\dot{\gamma}_{ij}^{\lambda} + k^2\gamma_{ij}^{\lambda} = 16\pi G\, S_{ij}^{\lambda}\ , \qquad \lambda = +, \times$$

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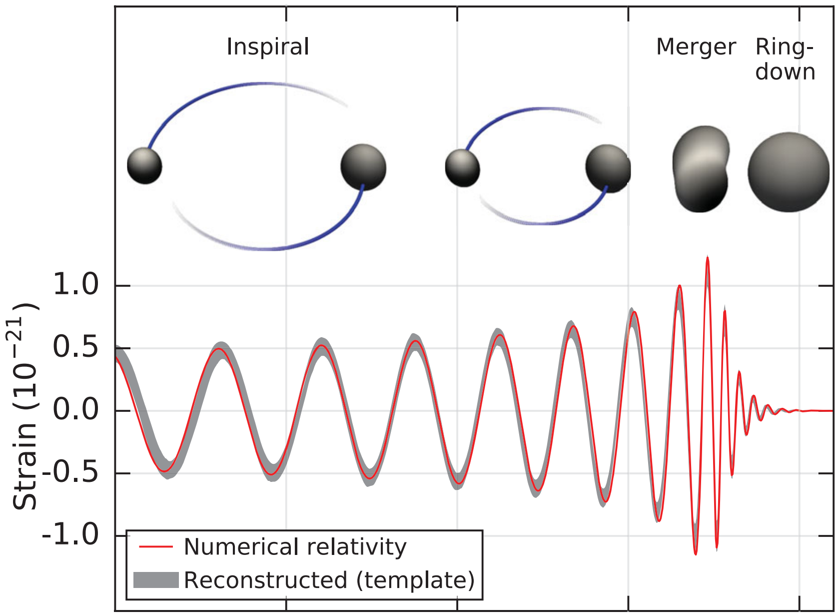

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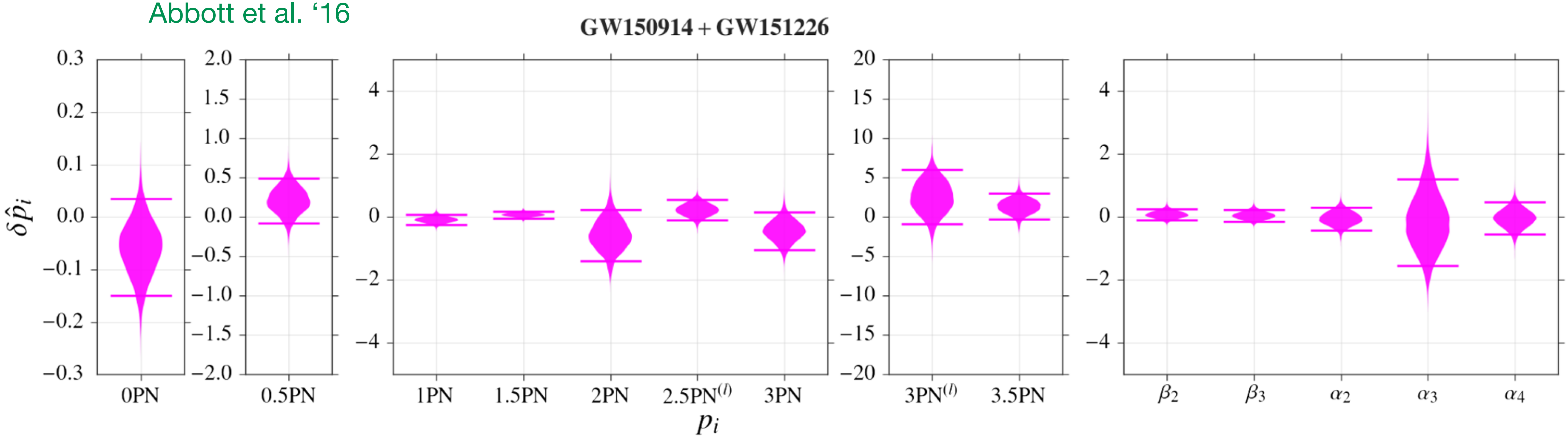
Abbott et al. '16

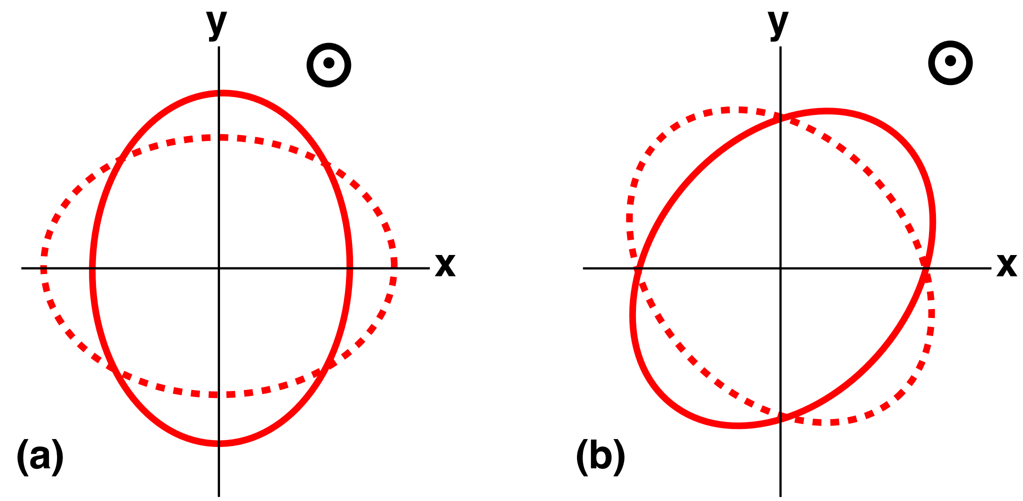
Consistency check of general relativity:



Polarizations

Modified gravity can treat differently the two polarizations in both the production mechanism and the propagation (ex. Cherns-Simons gravity) Jackiw & Pi ‘03

$$\ddot{\gamma}_{ij}^{\lambda} + 3H\dot{\gamma}_{ij}^{\lambda} + \left[k^2 + f^{\lambda}(k)\right]\gamma_{ij}^{\lambda} = 16\pi G\, S_{ij}^{\lambda}\ , \qquad \lambda = +, \times$$



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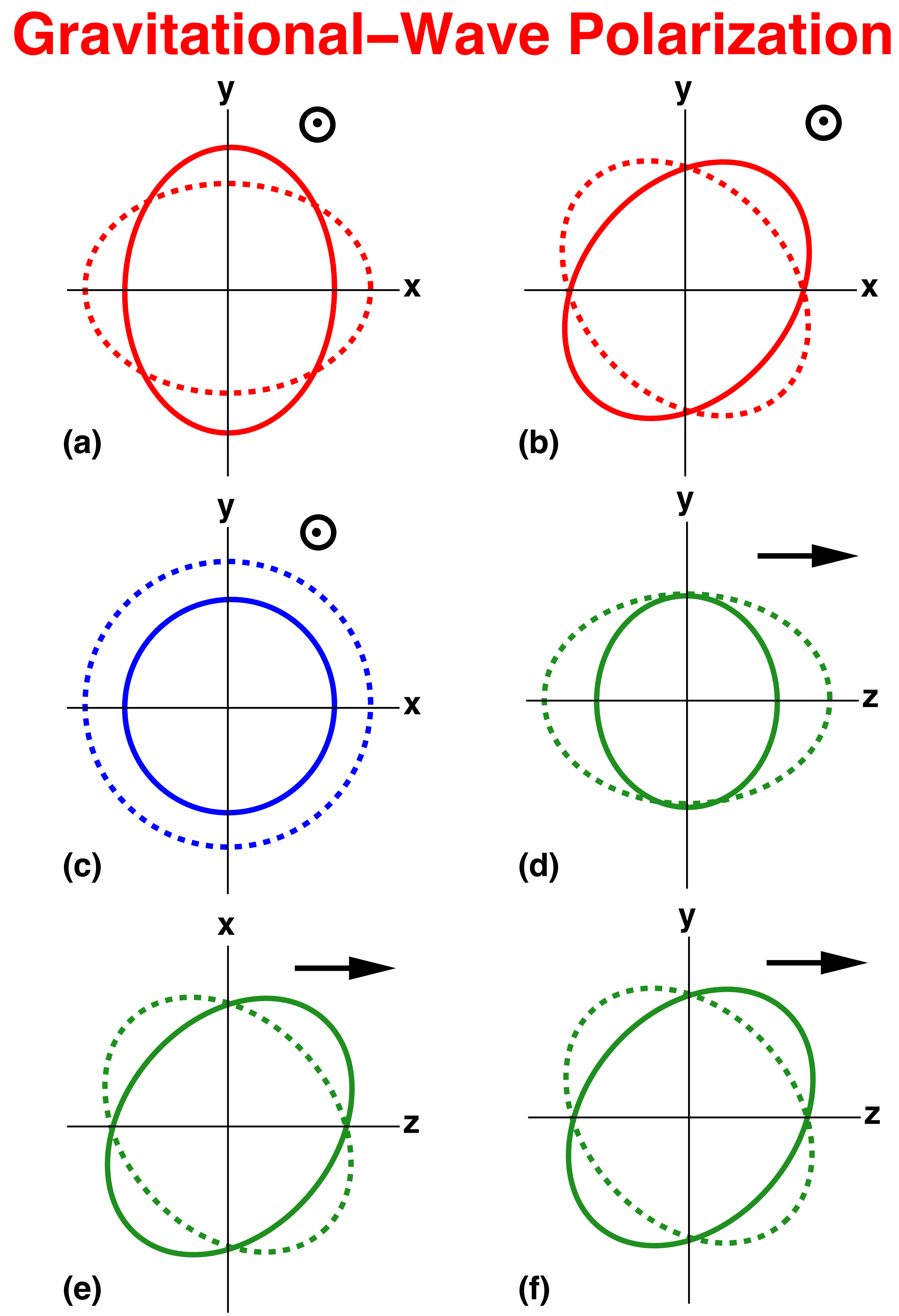
Modified gravity can involve different degrees of freedom: scalar, vectors, extra tensors

Polarizations

Will '14

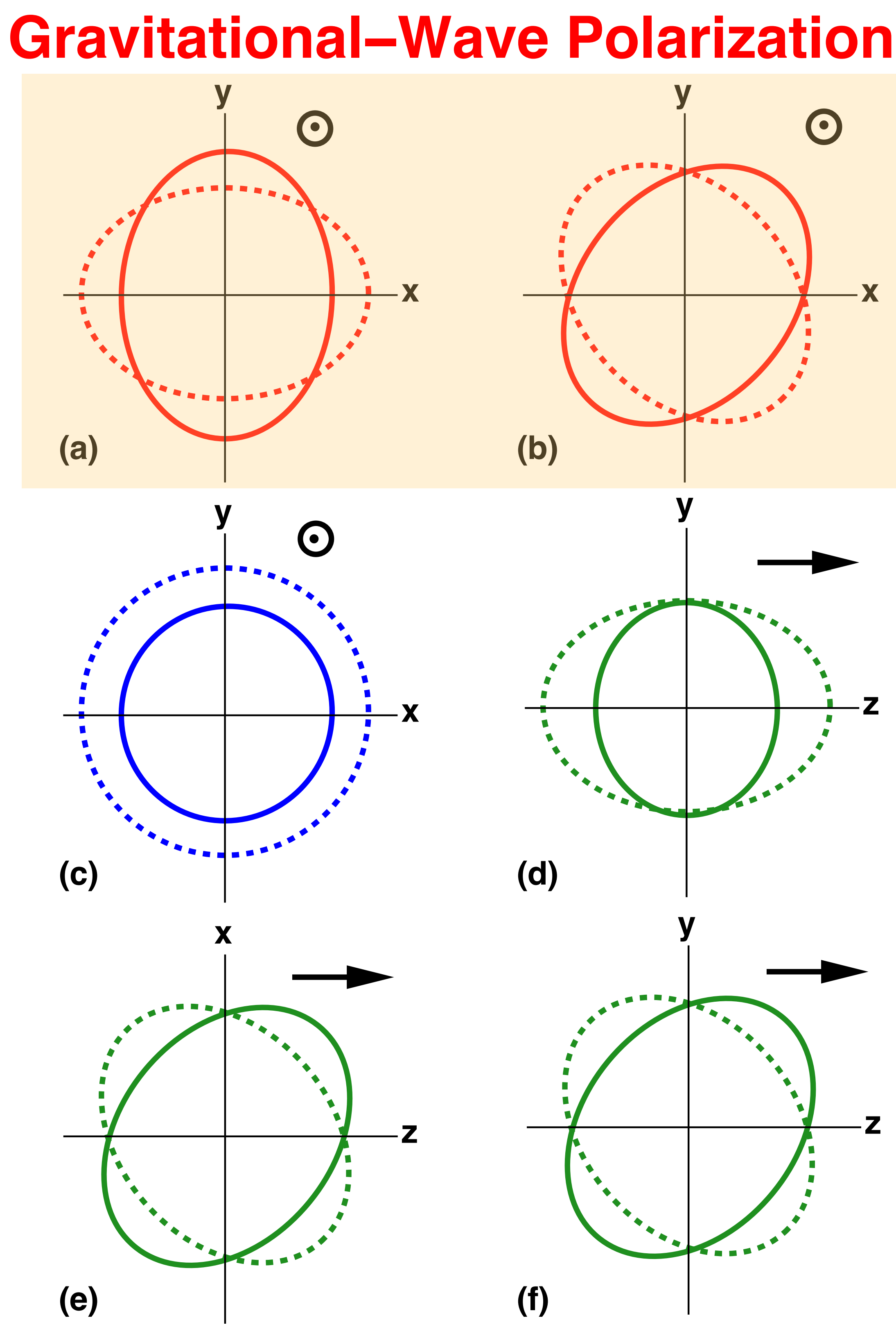
$$\begin{pmatrix} A_{\mathrm{S}} + A_{+} & A_{\times} & A_{\mathrm{V1}} \\ A_{\times} & A_{\mathrm{S}} - A_{+} & A_{\mathrm{V2}} \\ A_{\mathrm{V1}} & A_{\mathrm{V2}} & A_{\mathrm{L}} \end{pmatrix}$$



Polarizations

Will ‘14

Standard GR: 2 transverse modes (spin-2)



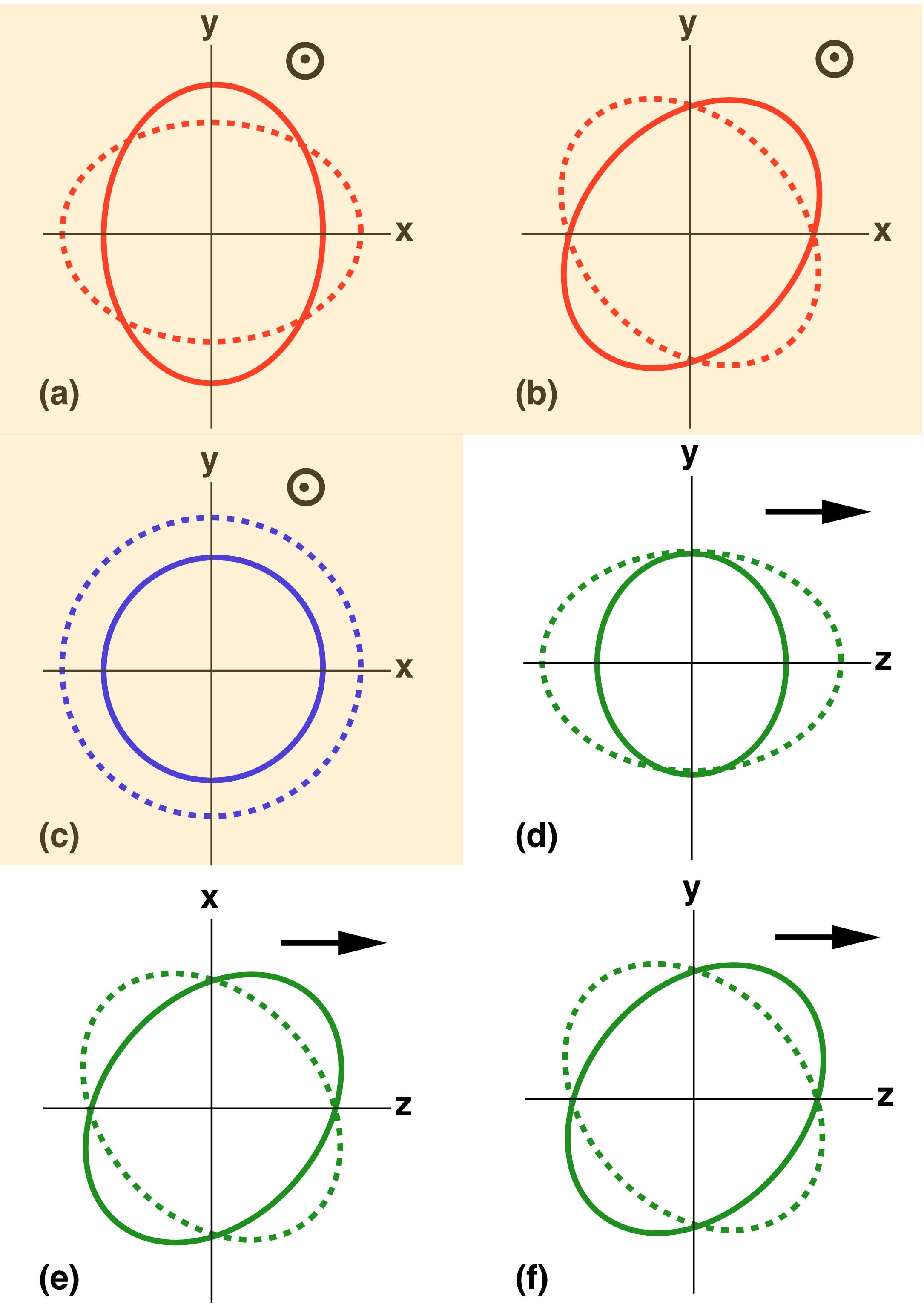
Polarizations

Will '14

Scalar-Tensor: 2 transverse modes (spin 2) + 1 transverse mode (spin 0)

Cf. Adrien Kuntz's talk

Gravitational–Wave Polarization

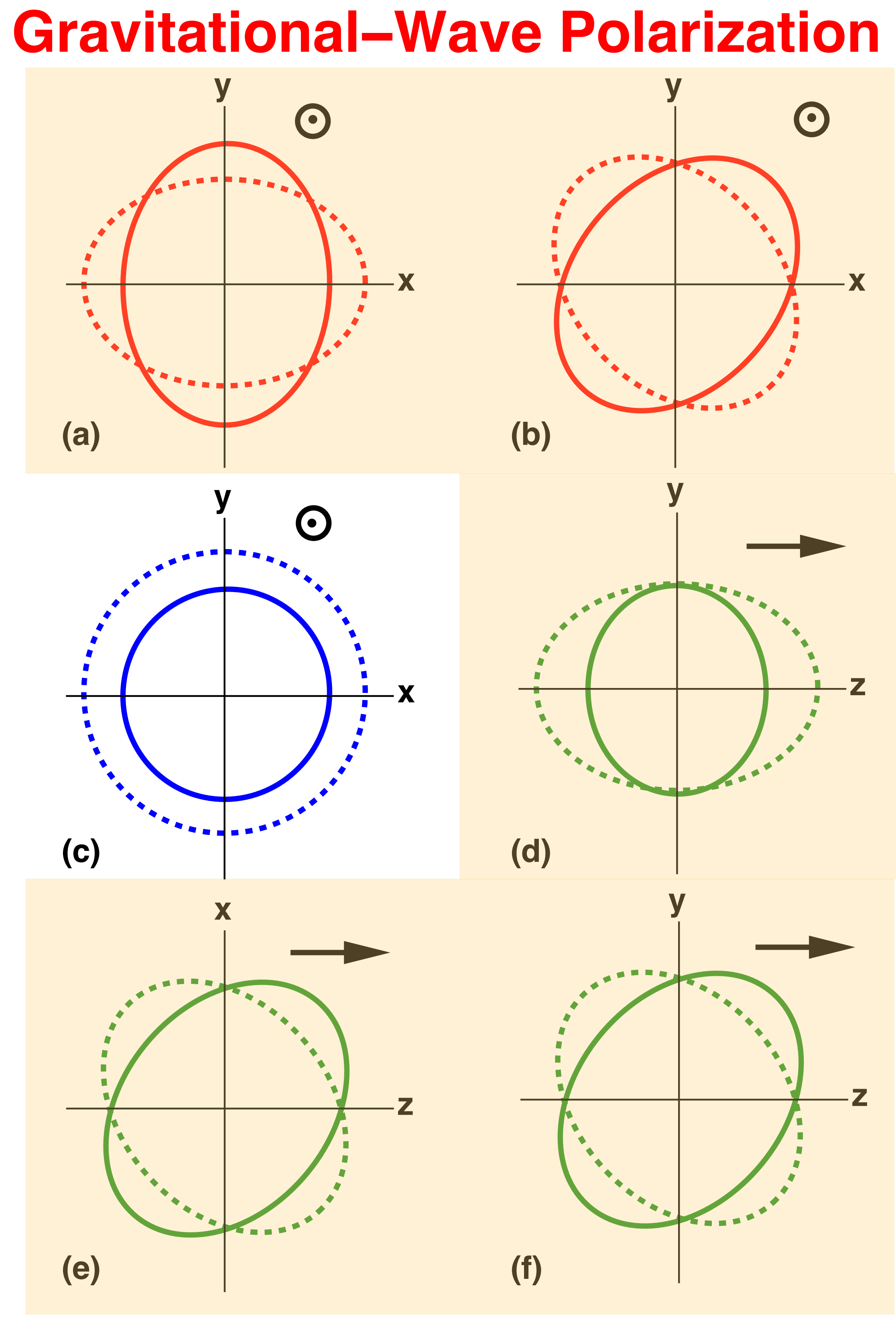


Polarizations

Will '14

Massive graviton: 2 transverse modes (spin-2) + 2 longitudinal modes (spin 1) + 1 longitudinal mode (spin 0)

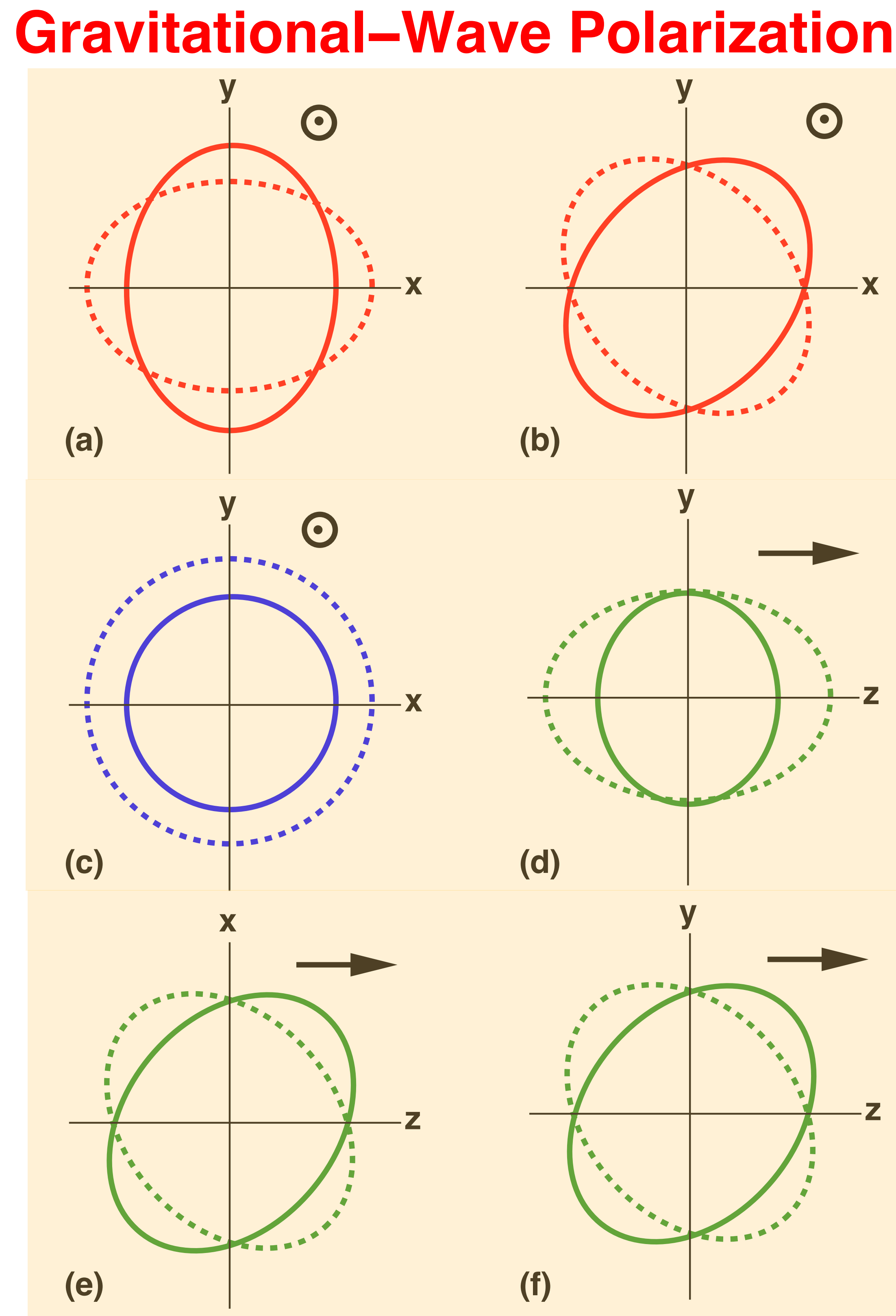
Cf. Philippe Brax's talk



Polarizations

Will '14

Scalar-Vector-Tensor: 2 transverse modes (spin-2) + 1 transverse mode (spin 0) + 2 longitudinal modes (spin 1) + 1 longitudinal mode (spin 0)

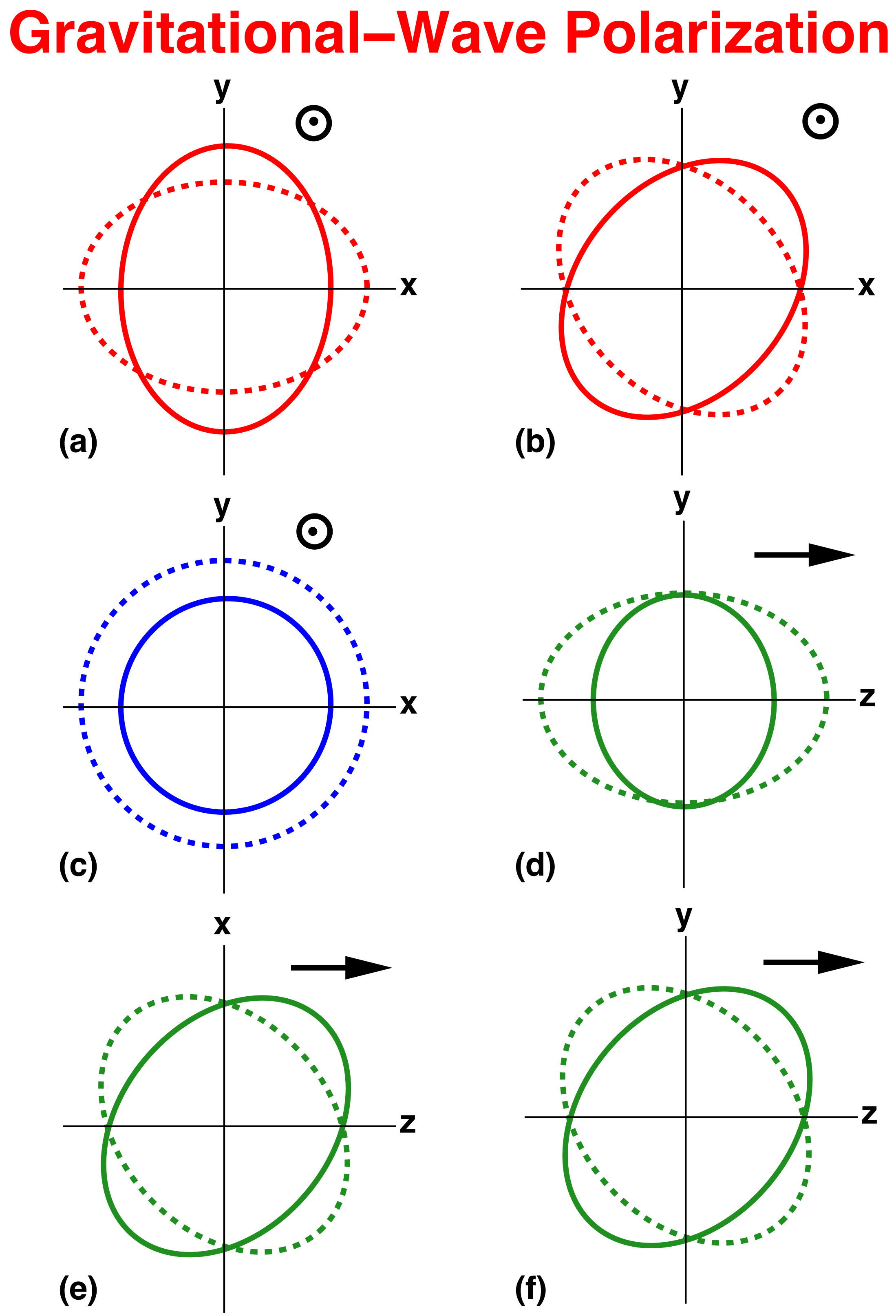


Polarizations

Will '14

6 polarizations + 2 directions = 8 unknowns

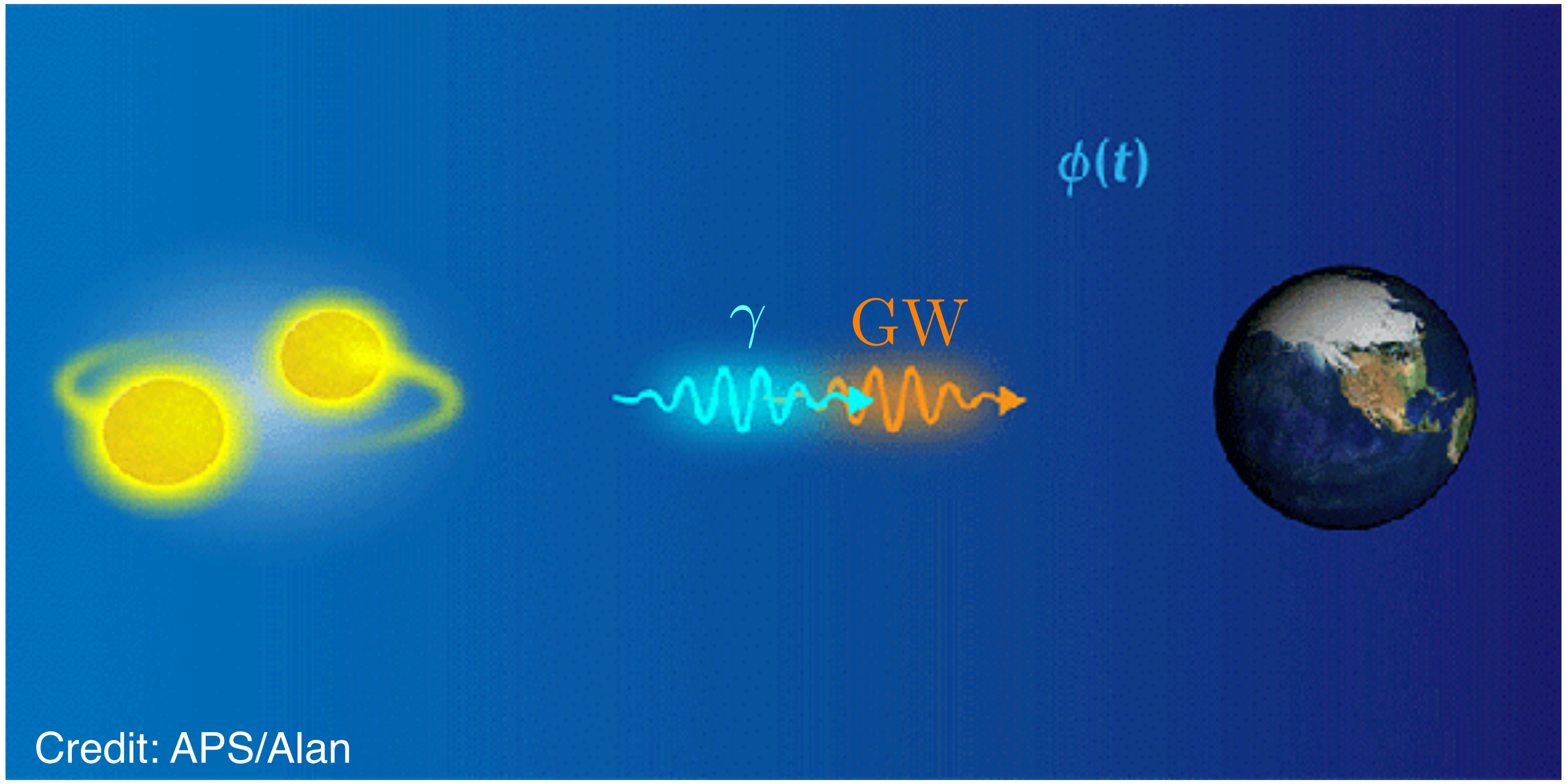
Assuming only transverse polarizations and known positions = 3 detectors enough



Gravitational Waves propagation

Modified gravity spontaneously breaks Lorentz Invariance. Acts like a medium, where gravitons are absorbed and dispersed. Effects accumulate on long time-scale.

$$\ddot{\gamma}_{ij}^{\lambda} + H\left[3 + \ldots\right]\dot{\gamma}_{ij}^{\lambda} + \left[c_T^2 k^2 + \ldots\right]\gamma_{ij}^{\lambda} = 0$$



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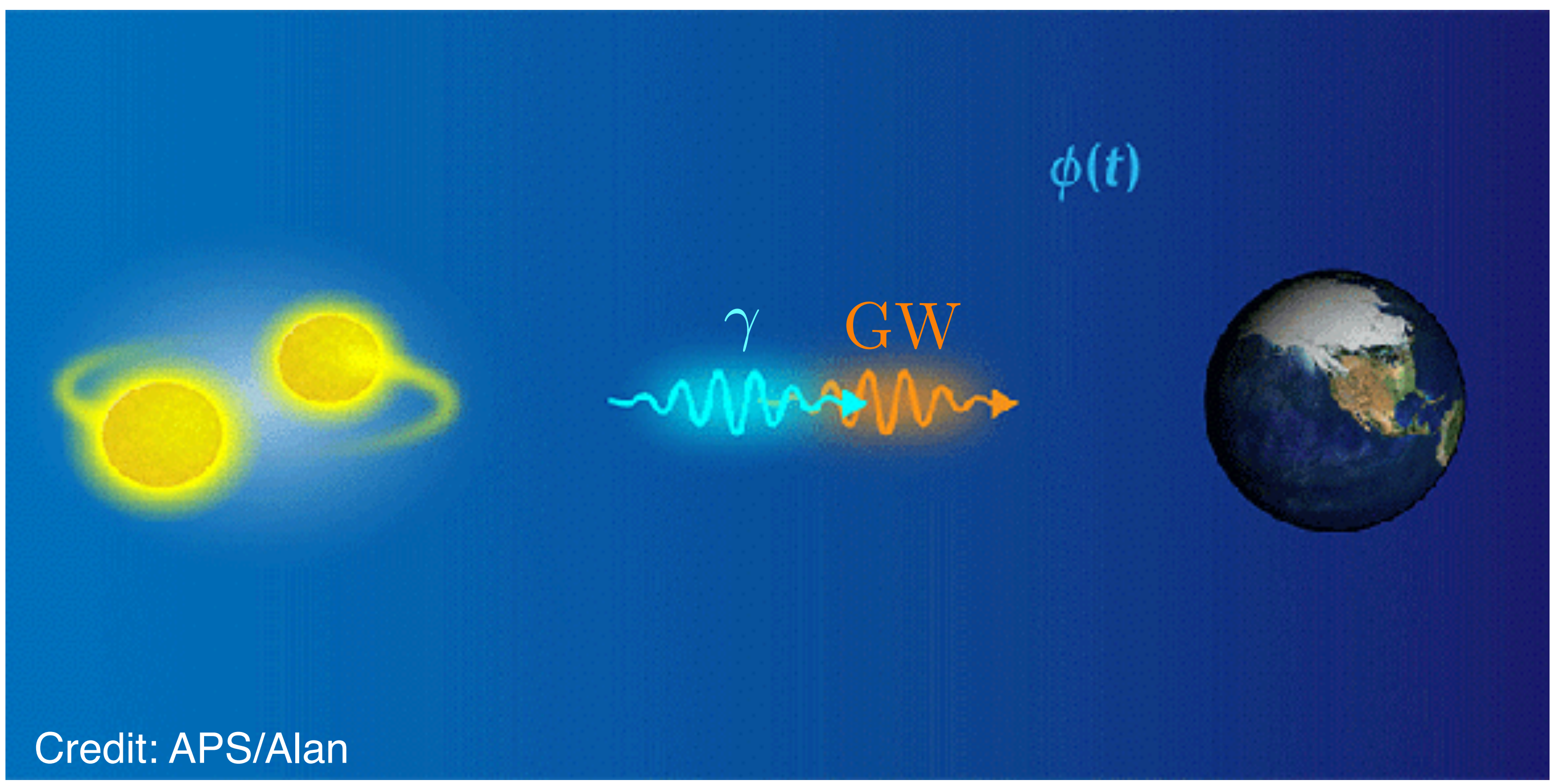

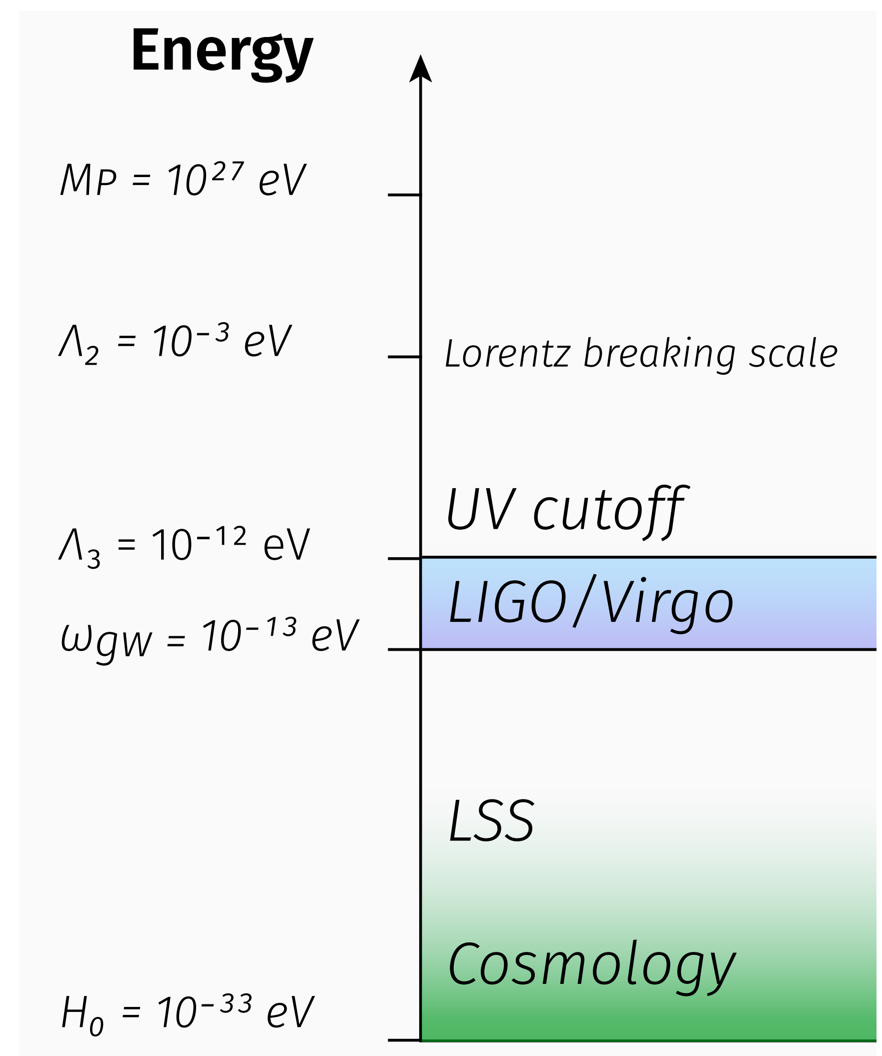

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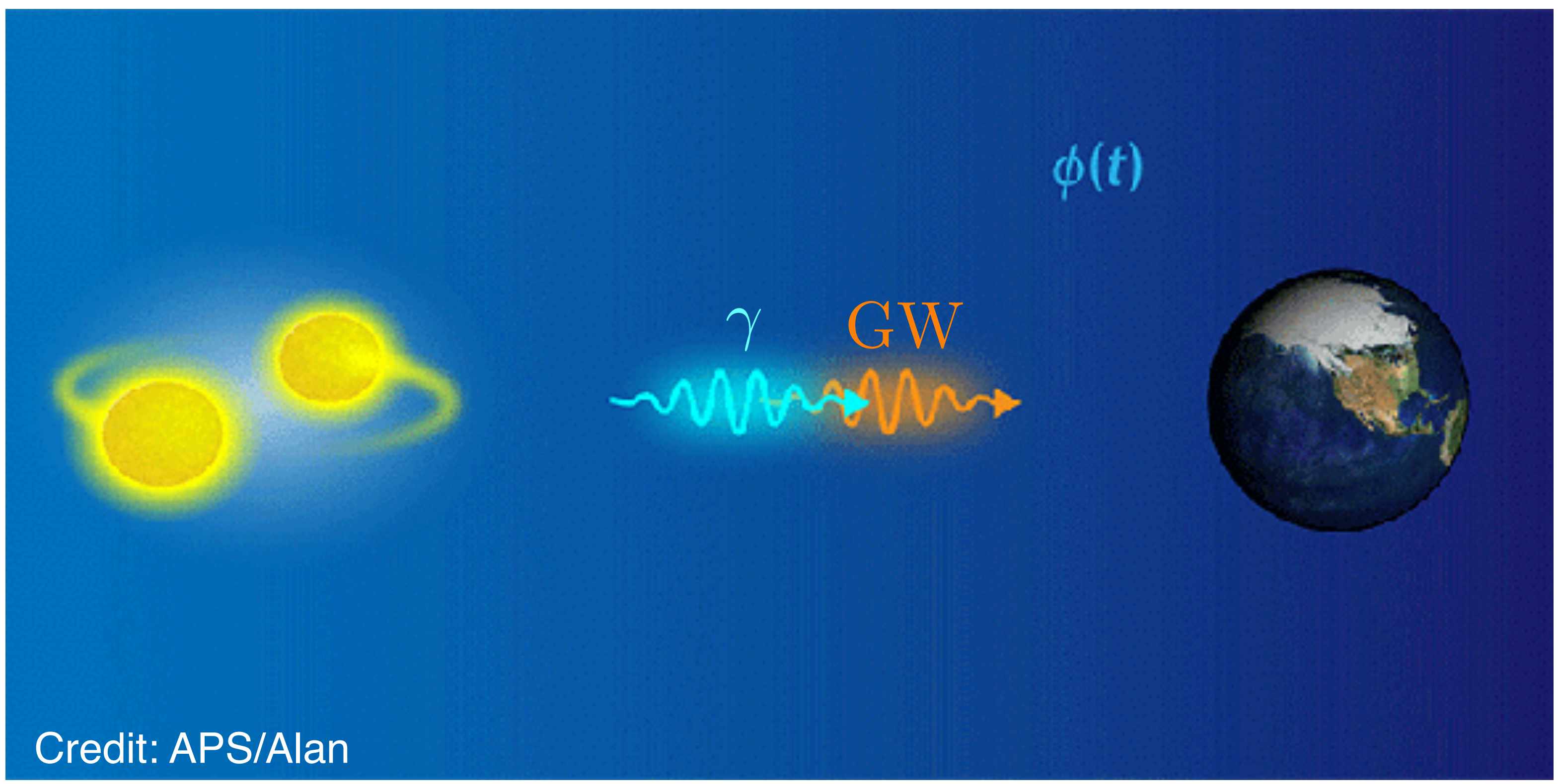
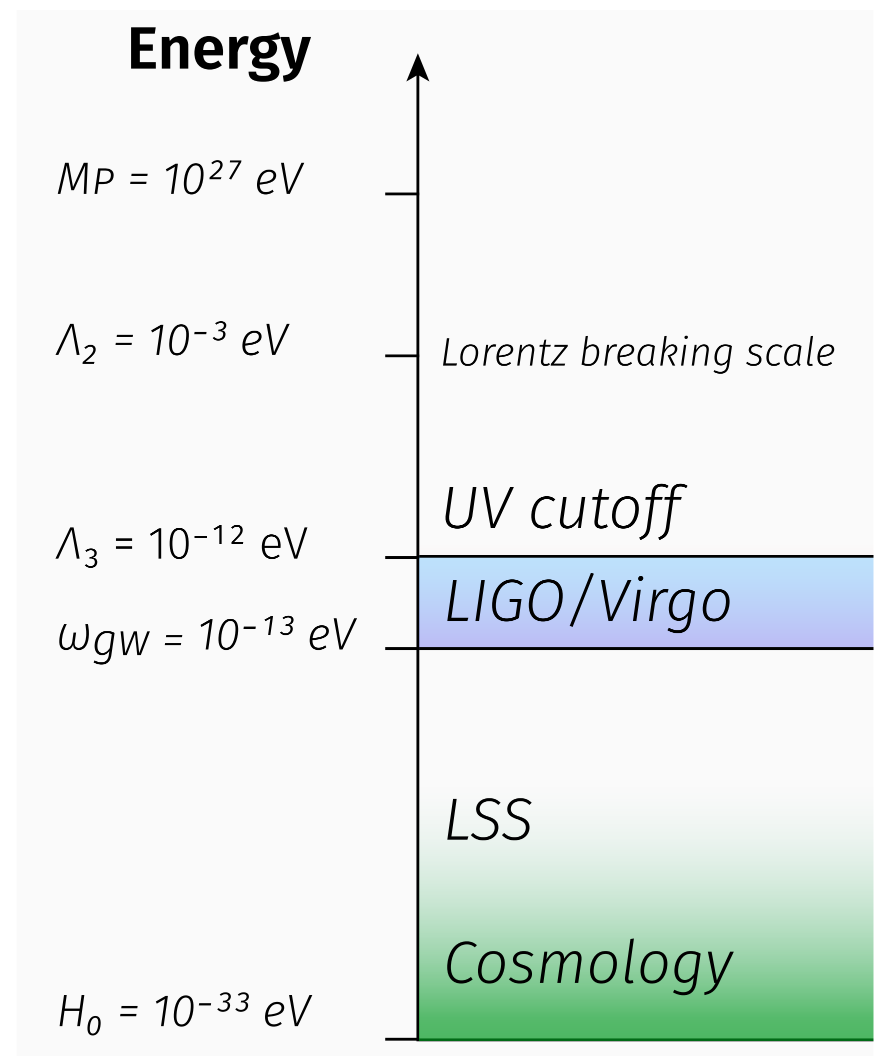


Credit: APS/Alan



Similar to GW damping by neutrinos after or damping and modification of the propagation speed by CDM

Weinberg ’03; Flauger & Weinberg ‘18

GW propagation

Modified gravity spontaneously breaks Lorentz Invariance. Acts like a medium, where gravitons are absorbed and dispersed. Effects accumulate on long time-scale.

Frequency independent effects:

$$\ddot{\gamma}_{ij} + H(3 + \alpha_M)\dot{\gamma}_{ij} + c_T^2 k^2 \gamma_{ij} = 0$$

damping (→ α_M)

speed of propagation (→ c_T^2)

$$\mu = \mu(\alpha_M, c_T^2, \ldots) \;, \qquad \Sigma = \Sigma(\alpha_M, c_T^2, \ldots)$$

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damping   speed of propagation

$d_L^{\rm gw} \neq d_L^{\rm em}$ different luminosity distances

Deffayet, Menou '07;
Calabrese, Battaglia, Spergel, '16;
Amendola et al. '17, Belgacem et al. '17,
etc…

LISA: $\sigma_{\alpha_M} \approx 0.03 - 0.1$

Amendola, Sawicki, Kunz, Saltas '18

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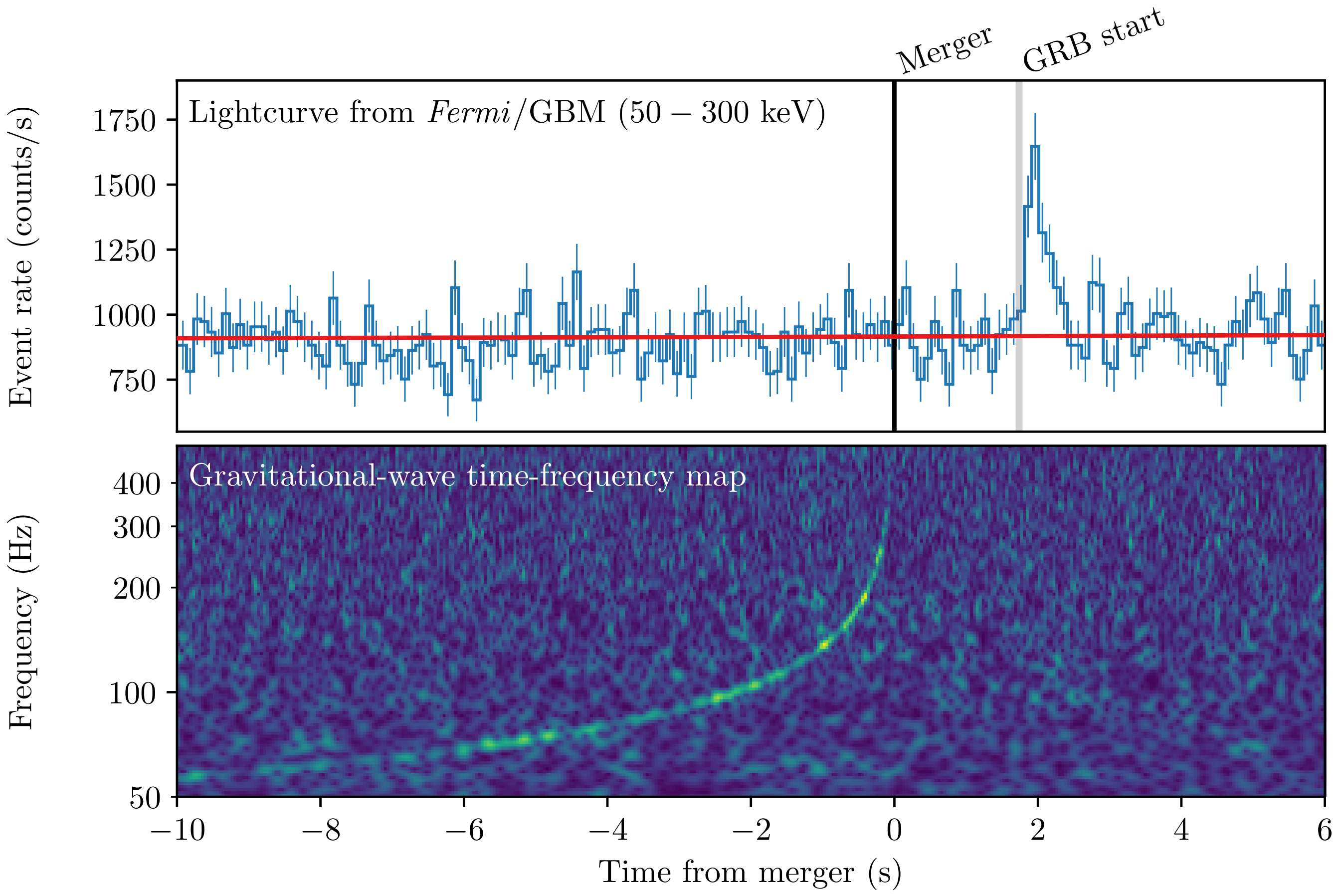
damping

speed of propagation

$$-3 \times 10^{-15} \leq \frac{c_g - c}{c} \leq 7 \times 10^{-16}$$



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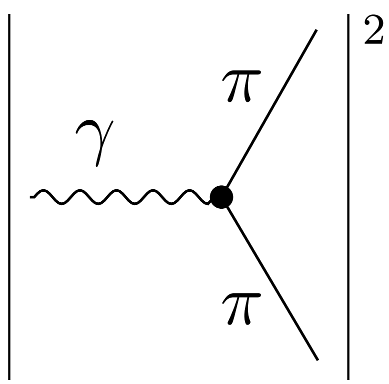
Frequency dependent effects:

Creminelli, Lewandowski, Tambalo, FV ‘18

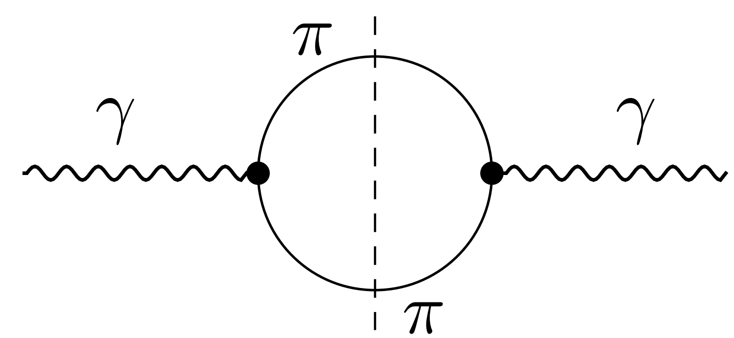
$$\ddot{\gamma}_{ij} + [(3+\alpha_{\rm M})H + \Gamma(k)]\,\dot{\gamma}_{ij} + \left[c_T^2 k^2 + f(k)\right]\gamma_{ij} = 0$$

decay



Similar to Cherenkov radiation

dispersion



Similar to a refractive material

Related by optical theorem: $\Gamma(k)\omega(k) = \mathrm{Im}\left[f(k)\right]$

GW propagation

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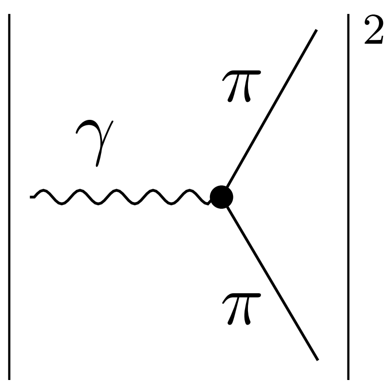

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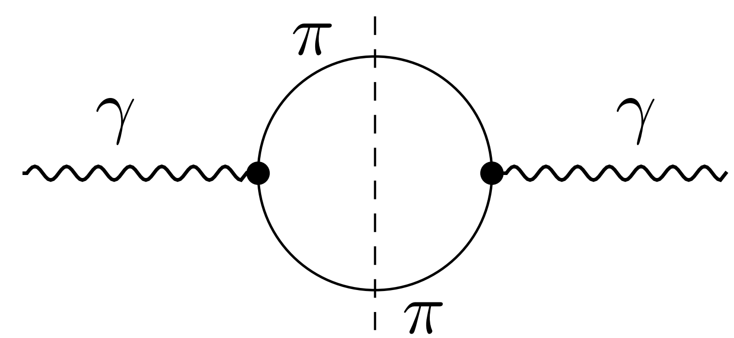

Similar to a refractive material

$$\frac{\Gamma(k)}{\omega} \lesssim \frac{1}{d_S\,\omega}$$

$$\frac{f(k)}{\omega^2} \lesssim \frac{1}{d_S\,\omega} \sim 10^{-18} \times \frac{2\pi \times 100\,{\rm Hz}}{\omega}\,\frac{40\,{\rm Mpc}}{d_S}$$

See e.g. Yunes, Yagi, Pretorius '16; Abbott et al. '17

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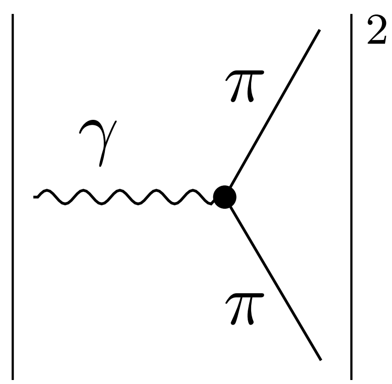
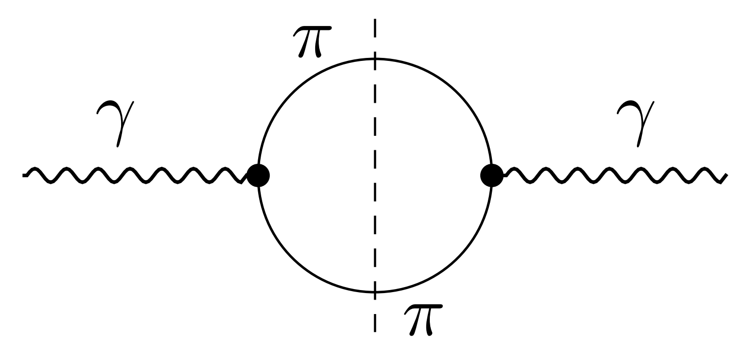


dispersion



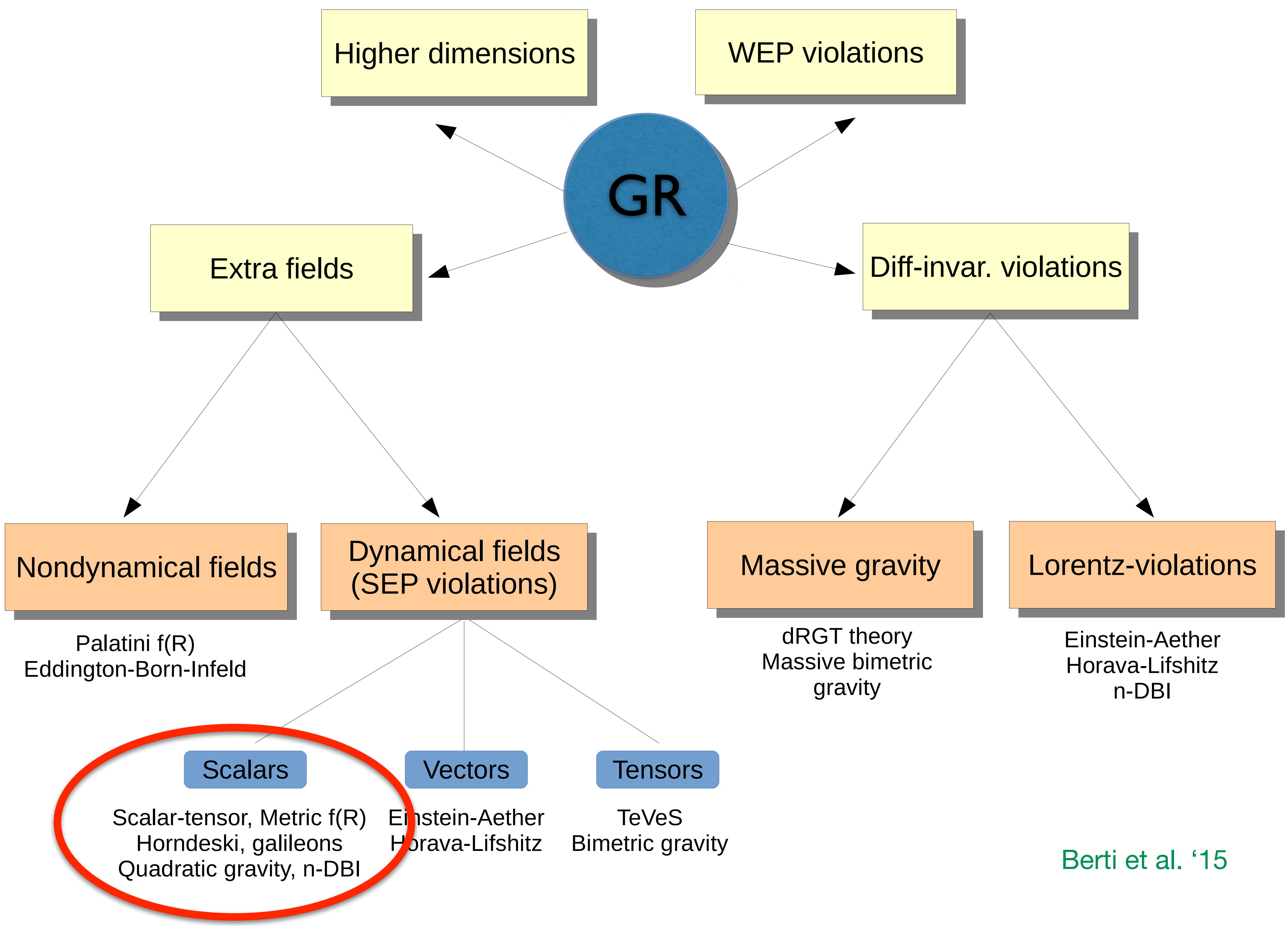
Similar to Cherenkov radiation

Similar to a refractive material

Simplest case: $f(k) \equiv m_\gamma^2\,,\qquad m_\gamma^2/\omega^2 < (d_S\omega)^{-1} \quad \Rightarrow \quad m_\gamma < 10^{-22}\ \mathrm{eV}$

See e.g. Yunes, Yagi, Pretorius ’16; Abbott et al. ‘17

Dark energy and modified gravity



Berti et al. '15

Scalar-tensor theories

Simplest models of modified gravity are base on single scalar field (universal coupling)

Ex: $\mathcal{L} = R + V(\phi) - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ quintessence

$$G_{\mu\nu} = 8\pi G \left(T_{\mu\nu}^{(\mathrm{matter})} + T_{\mu\nu}^{(\phi)} \right)$$

w ≠ 1

Scalar-tensor theories

Simplest models of modified gravity are base on single scalar field (universal coupling)

Ex: $$\mathcal{L} = R + G_2(\phi, X) \ , \quad X \equiv g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$ k-essence

$$G_{\mu\nu} = 8\pi G\left(T_{\mu\nu}^{(\mathrm{matter})} + T_{\mu\nu}^{(\phi)}\right)$$

$c_s^2 \neq 1$: clustering

Scalar-tensor theories

Simplest models of modified gravity are base on single scalar field (universal coupling)

Ex: $$\mathcal{L} = f(\phi)R + G_2(\phi, X)\ , \quad X \equiv g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$ scalar-tensor gravity

$$G_{\mu\nu}^{(\mathrm{modified})} = 8\pi G \left(T_{\mu\nu}^{(\mathrm{matter})} + T_{\mu\nu}^{(\phi)} \right)$$

self-acceleration

Modified gravity

Simplest models of modified gravity are base on single scalar field (universal coupling)

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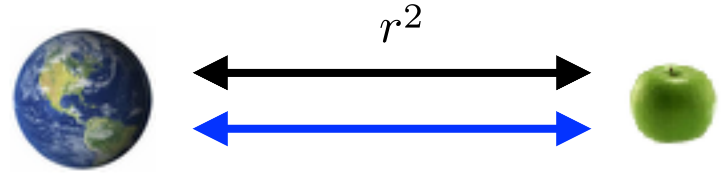
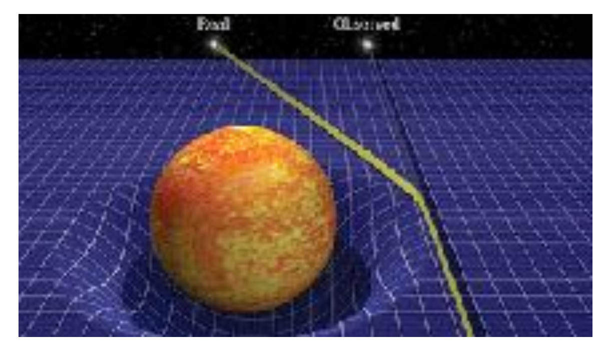
$$G_{\mu\nu}^{(\mathrm{modified})} = 8\pi G\left(T_{\mu\nu}^{(\mathrm{matter})} + T_{\mu\nu}^{(\phi)}\right)$$

$$\frac{Gm_1m_2}{r^2}$$



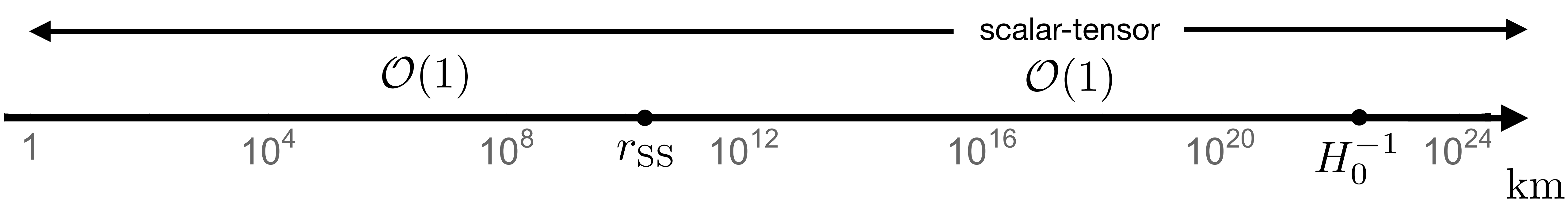
$$\alpha^2\frac{Gm_1m_2}{r^2}$$

fifth force



$\Psi \neq \Phi$

anomalous light bending



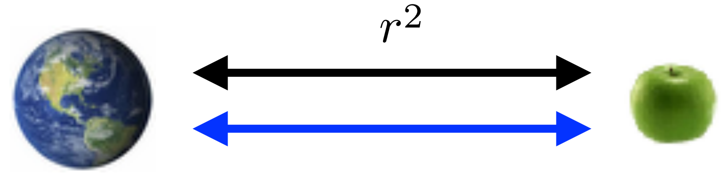
Screening

Simplest models of modified gravity are base on single scalar field (universal coupling)

Ex: $$\mathcal{L} = f(\phi)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi \qquad \Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$$

$$\frac{\Box\phi}{\Lambda_3^3} \gg 1$$ Vainshtein screening: large classical scalar field nonlinearities

$$\frac{Gm_1 m_2}{r^2}$$



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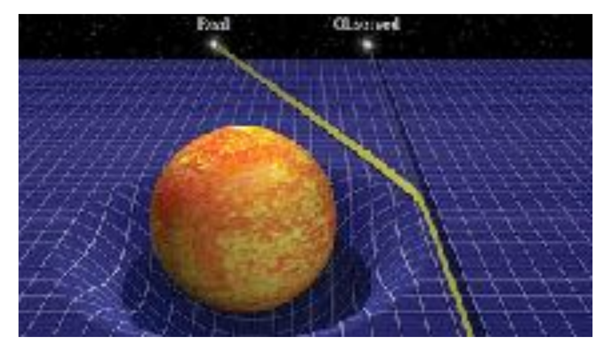

$$\Psi \neq \Phi$$

anomalous light bending

$$\Lambda_3 \sim (M_{\rm Pl} H_0^2)^{1/3} \sim (1000\ \mathrm{km})^{-1}$$

scalar-tensor

$\mathcal{O}(1)$ $\mathcal{O}(1)$

1 Λ_3^{-1} 10^4 10^8 $r_{\rm SS}$ 10^{12} 10^{16} 10^{20} H_0^{-1} 10^{24} km

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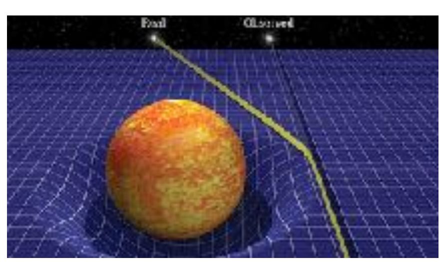
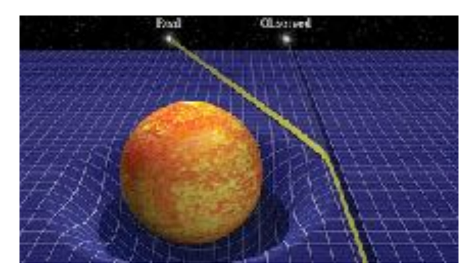
$\frac{Gm_1m_2}{r^2}$



$\Psi = \Phi$

$\frac{Gm_1m_2}{r^2}$

$\alpha^2\frac{Gm_1m_2}{r^2}$



$\Psi \neq \Phi$

$$\Lambda_3 \sim (M_{\rm Pl}H_0^2)^{1/3} \sim (1000\ {\rm km})^{-1}$$

almost GR: $\ll \mathcal{O}(10^{-2})$ | scalar-tensor: $\mathcal{O}(1)$

1, Λ_3^{-1}, 10^4, 10^8, $r_{\rm SS}$, 10^{12}, 10^{16}, 10^{20}, H_0^{-1}, 10^{24} km

$$\mathcal{L} = f(\phi)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi$$

Is this the end of the story?

Generalized theories

Horndeski 73
Deffayet et al. 11

Most general Lorentz-invariant scalar-tensor theory with 2nd-order EOM.

$$\Box\phi \equiv \phi^{;\mu}_{;\mu} \qquad X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu}$$

$$\begin{aligned}\mathcal{L} = {} & G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi \\ & - 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - (\phi_{;\mu\nu})^2\right] \\ & + G_5(\phi, X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3\right]\end{aligned}$$

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Degenerate theories: most general stable theory.

See Karim Noui's talk

Langlois, Noui '15; Crisostomi, Koyama, Tasinato '16
Zumalacarregui, Garcia-Bellido '13

Beyond Horndeski theories:

Gleyzes, Langlois, Piazza, FV '14

$$- F_4(\phi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'}$$

$$- F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'}\phi_{;\sigma\sigma'}$$

$$X G_{5,X} F_4 = 3F_5\left[G_4 - 2XG_{4,X} - (X/2)G_{5,\phi}\right]$$

Setting c_T=1

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Scalar field play with gravity through higher derivatives:

$$\nabla_\mu\nabla_\nu\phi \supset \Gamma^\rho_{\mu\nu}\partial_\rho\phi \qquad \Rightarrow \qquad \Gamma^0_{ij}\,\dot{\phi} \supset \dot{\gamma}_{ij}\,\dot{\phi}$$

$$\mathcal{L}_\gamma \sim (\dot{\gamma}_{ij})^2 - c_T^2(\partial_k\gamma_{ij})^2$$

$$c_T^2 - 1 \propto -2G_{4,X} - G_{5,\phi} - (H\dot{\phi} - \ddot{\phi})G_{5,X} + XF_4 - 3HX\dot{\phi}F_5$$

Expected from LSS: $|c_T^2 - 1| \lesssim \text{few} \times 0.01$

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Most general theory compatible with c_T=1: $\quad G_5 = F_5 = 0\ , \qquad XF_4 = 2G_{4,X}$

Creminelli, FV ’17; Sakstein, Jain ’17 ; Ezquiaga, Zumalacarregui ’17 ; Baker+ ’17

What remains

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$$XF_4 = 2G_{4,X}$$

$$\alpha_H \equiv -\frac{X^2F_4}{G_4}$$

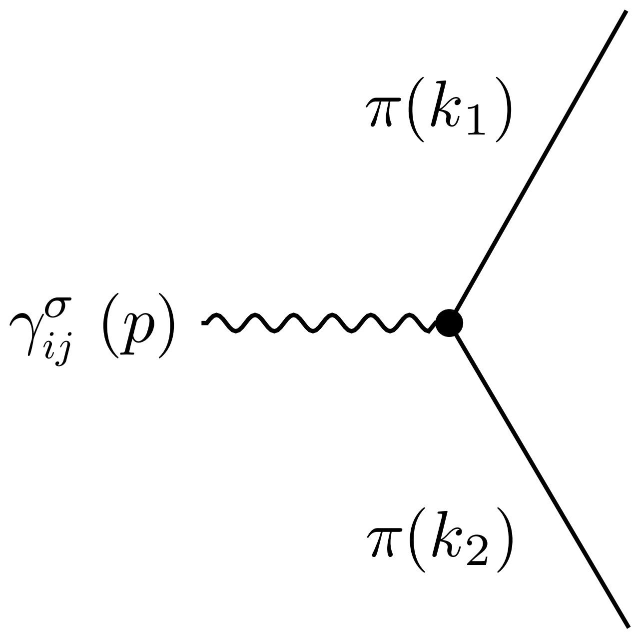
The decay of GW

Creminelli, Lewandowski, Tambalo, FV '18

Beyond Horndeski implies cubic interactions between GW and scalar fluctuations π

$$\mathcal{L}_{\gamma\pi\pi} = \frac{\alpha_H}{\Lambda_3^3}\ddot{\gamma}^c_{ij}\partial_i\pi_c\partial_j\pi_c$$

$$\Lambda_3 \equiv (M_{\rm Pl}H_0^2)^{1/3} \qquad \pi \equiv \delta\phi/\dot{\phi}_0$$



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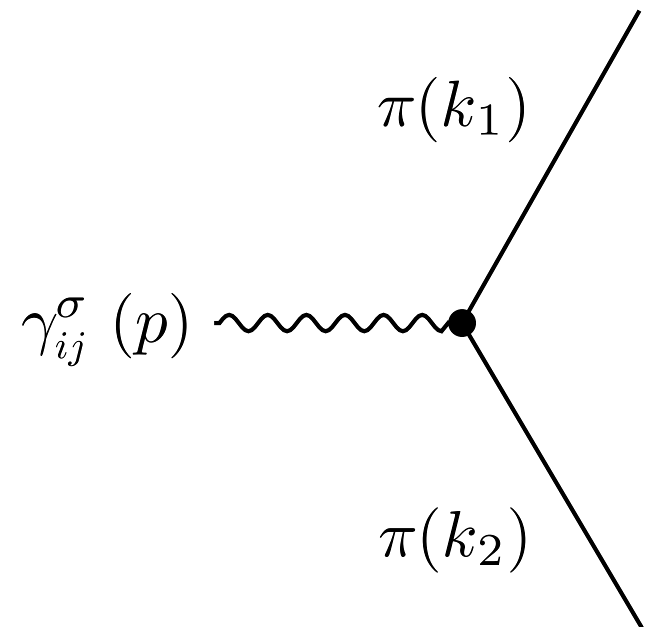

For $c_s < 1$ (c_s = sound speed of π fluctuations) GWs can decay into into dark energy fluctuations πs. Analogous to Cherenkov radiation

$$\Gamma \simeq \left(\frac{\alpha_H}{\Lambda_3^3}\right)^2 \frac{\omega_{\rm gw}^7(1-c_s^2)^2}{c_s^7}$$

decay rate

$$d_S\Gamma \ll 1 \qquad \Rightarrow \qquad \alpha_H \ll 10^{-8}$$

GW dispersion

Creminelli, Lewandowski, Tambalo, FV '18

Spontaneous Lorentz-breaking implies modifications of the dispersion relation

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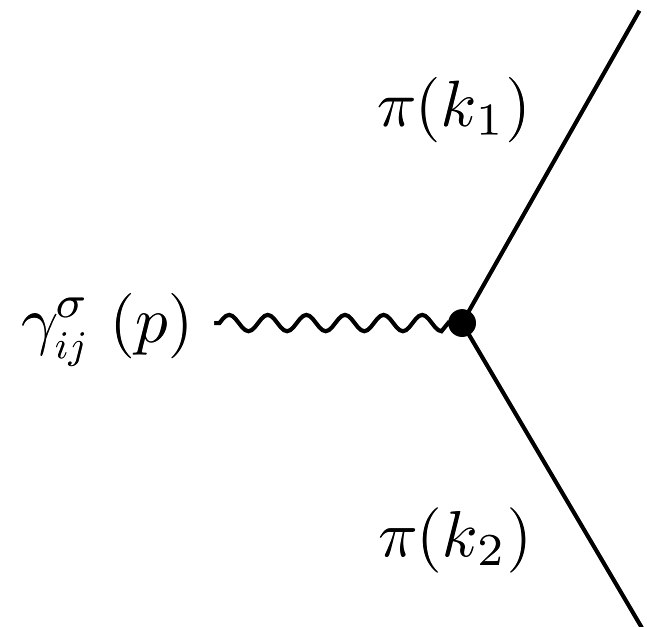
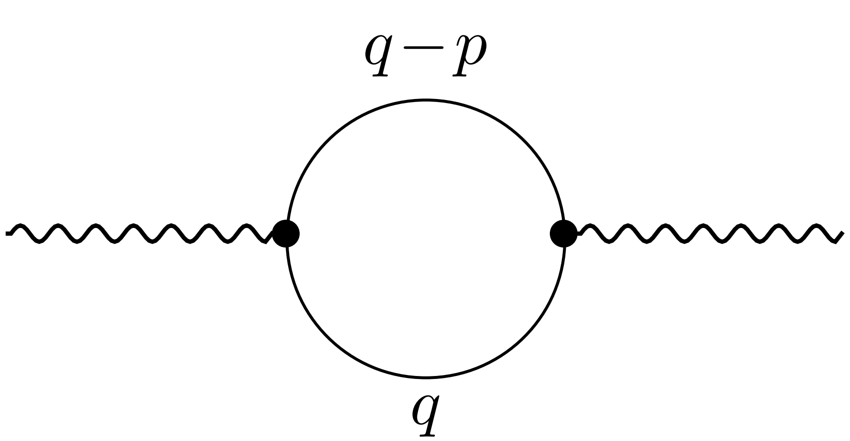


Graviton self-energy:



$$\omega^2 = k^2 - \left(\frac{\alpha_H}{\Lambda_3^3}\right)^2 \frac{k^8(1-c_s^2)^2}{\pi c_s^7}\log\left(-(1-c_s^2)\frac{k^2}{\mu^2}\right)$$

Strong constraints on α_H, also for $c_s > 1$

Optical theorem: $\Gamma(k)\omega(k) = \mathrm{Im}\left[f(k)\right]$

What remains

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$$XF_4 = 2G_{4,X} = 0$$

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Is this the end of the story?

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Yes.

Suppressed GW decaying rate and modification of the dispersion relation

$$\mathcal{L}_{\gamma\pi\pi} \simeq \frac{1}{\Lambda_2^2}\dot{\gamma}^c_{ij}\partial_i\pi_c\partial_j\pi_c$$

$$\Lambda_2 \equiv (M_{\rm Pl}H_0)^{1/2} \sim 10^{10}\Lambda_3$$

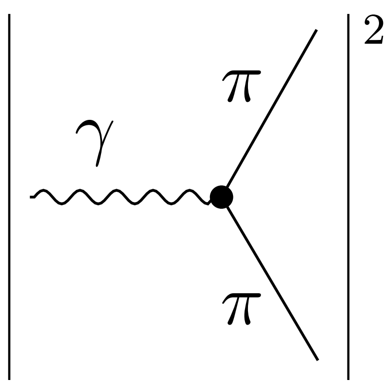
Here decay is perturbative but the high occupation number ($\sim 10^{40}$) of the GW will (Bose) enhance the decay. Can we rule out G_3 as well?

Gravitational Waves propagation

Modified gravity spontaneously breaks Lorentz Invariance. Acts like a medium, where gravitons are absorbed and dispersed.

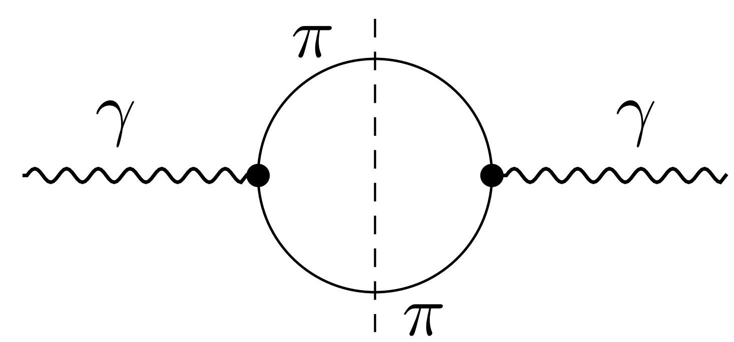
$$\ddot{\gamma}_{ij} + [(3+\alpha_{\mathrm{M}})H + \Gamma(k)]\,\dot{\gamma}_{ij} + \left[c_T^2 k^2 + f(k)\right]\gamma_{ij} = 0$$

decay



Similar to Cherenkov radiation

dispersion



Similar to a refractive material

$$\mu = \mu(\alpha_M, c_T^2 = 1, \alpha_H = 0, \ldots)\ , \qquad \Sigma = \Sigma(\alpha_M, c_T^2 = 1, \alpha_H = 0, \ldots)$$

Phenomenological parameters

Commonly used, very general, approach:

$$\Psi = -4\pi G \frac{\mu(t,k)}{k^2}\delta\rho$$

$$\Phi + \Psi = -8\pi G \frac{\Sigma(t,k)}{k^2}\delta\rho$$

- Infinite parameters at fixed redshift: needs binning
- Agnostic binning may miss physical ingredients (locality, causality, stability, etc.)
- Misses connection with other observables (e.g. gravitational waves)

Effective approach

Gubitosi, Piazza, FV '13
Gleyzes, Langlois, Piazza, FV '14
+ many refs and authors

Bridge models and observations
in a minimal and systematic way

Space of theories

$$G_4(\phi,X)R + G_2(\phi,X) + G_3(\phi,X)\Box\phi$$
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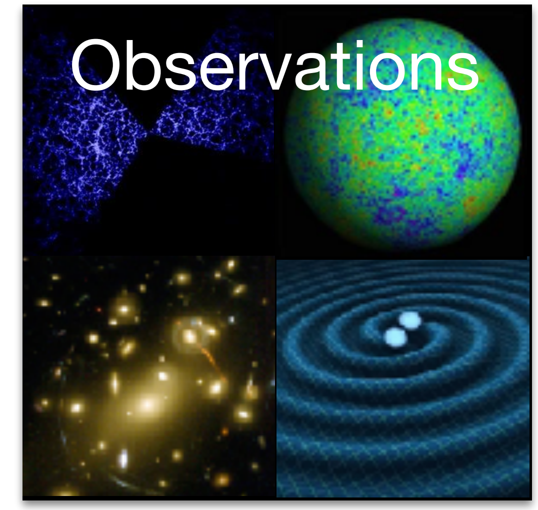
$$+\dots$$

EFT of DE

$$\alpha_1(t),\ \alpha_2(t),\ \dots$$



$$\mu = \mu\big(k; \alpha_1(t), \alpha_2(t), \dots\big)$$
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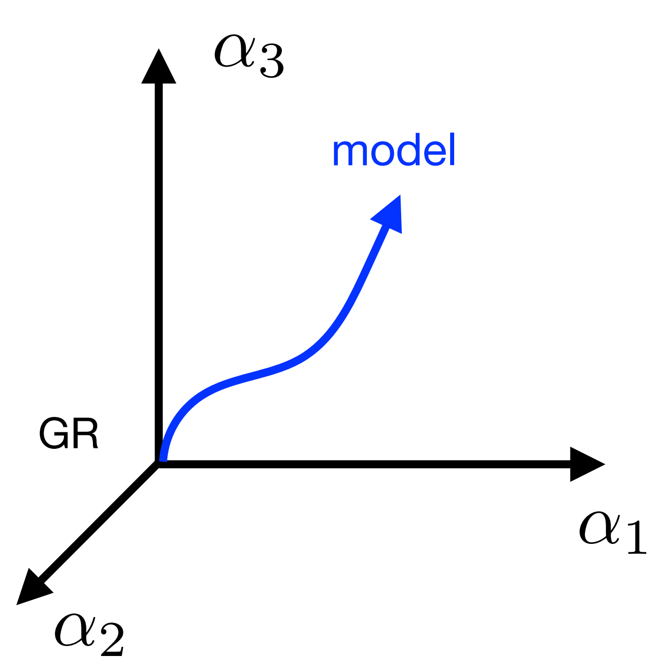

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←

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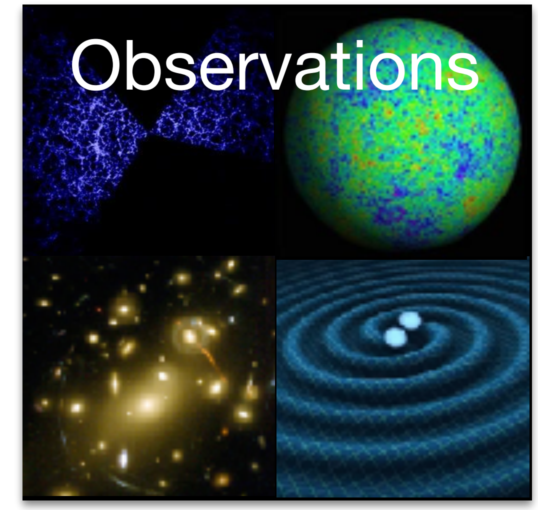
$$\alpha_1(t),\ \alpha_2(t),\ \dots$$

→



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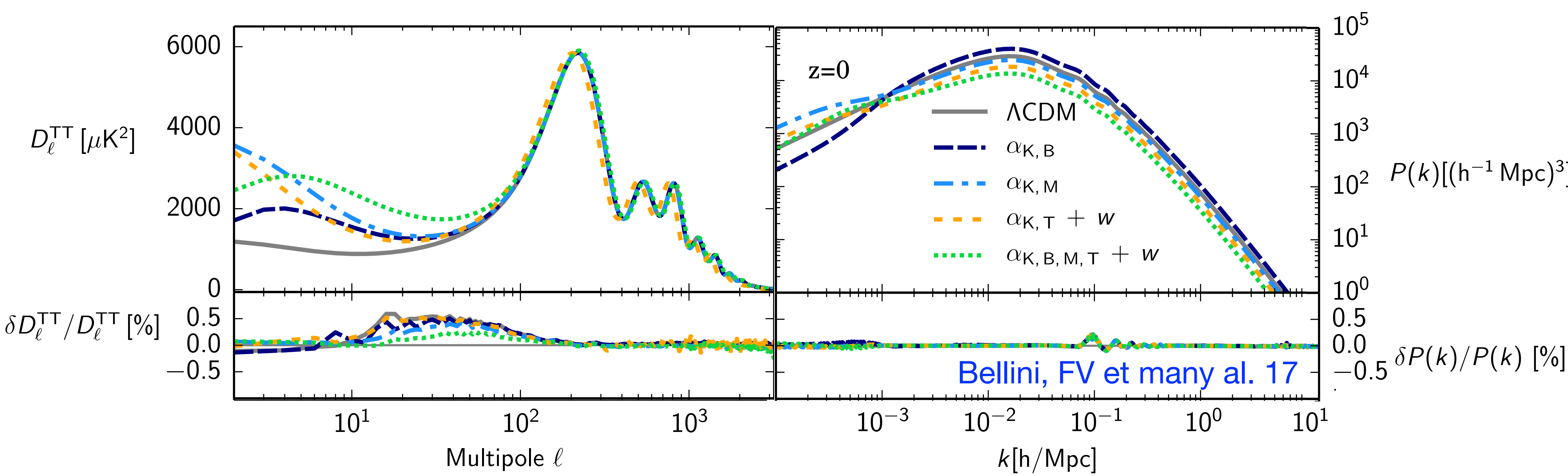
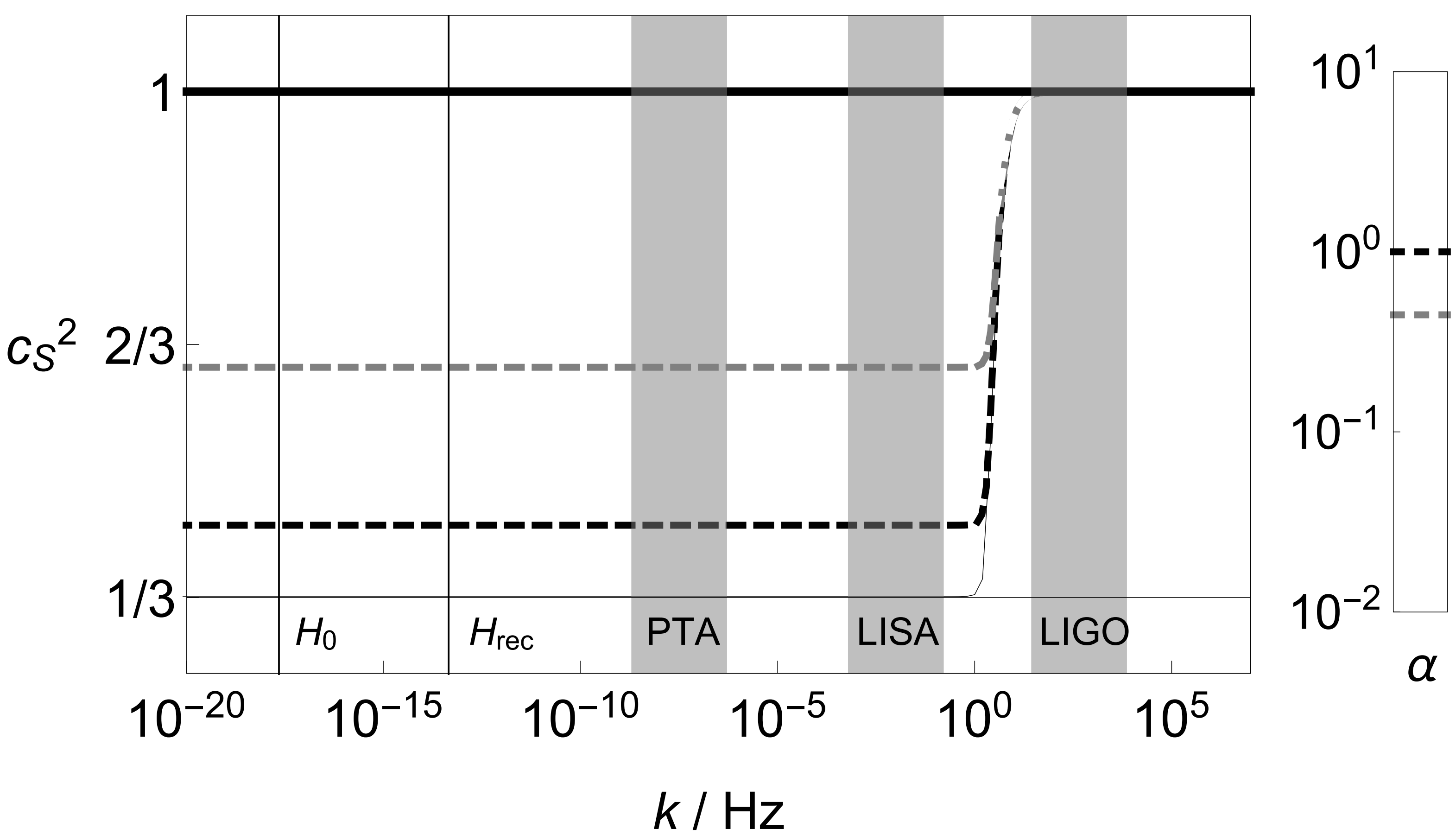
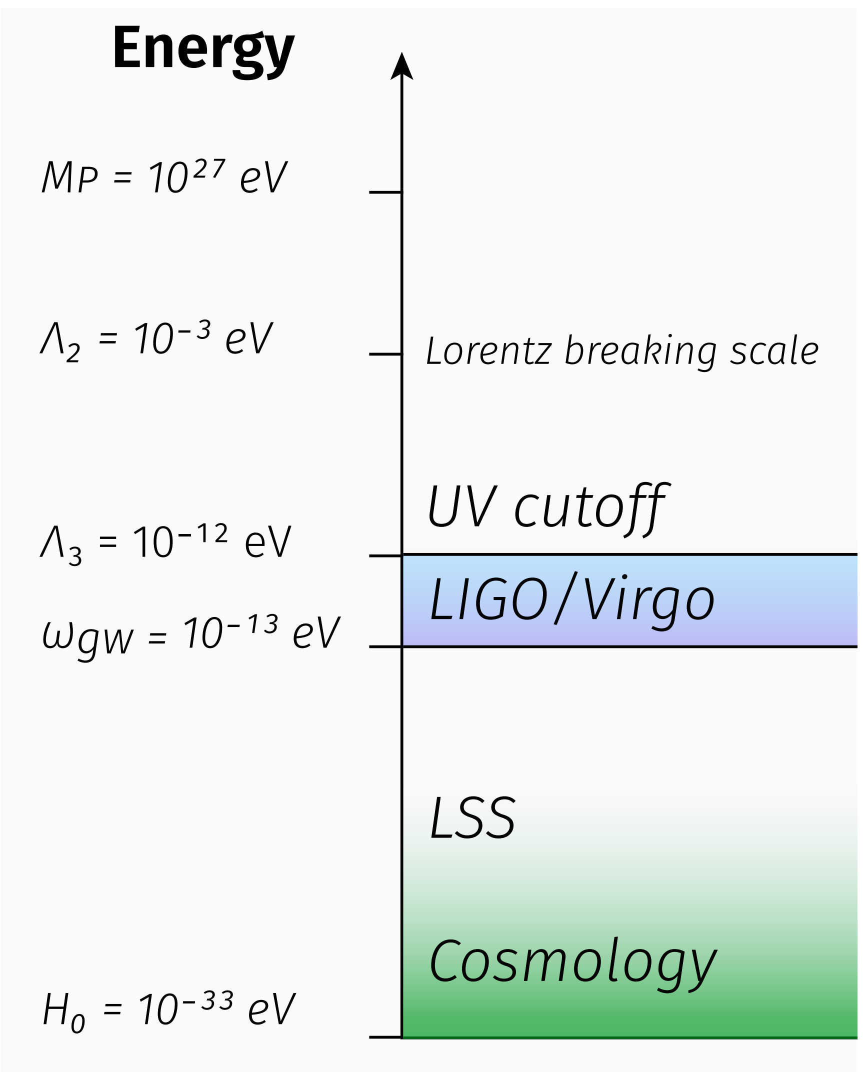


Caveat: recovering Lorentz invariance

$$\omega^2 = c_T^2(\alpha)k^2 + \frac{k^4}{M^2} + \ldots = k^2\left(1 + \mathcal{O}(M^2/k^2)\right)$$

de Rham, Melville ’18





Conclusion

- GWs dramatically change the prospect for LSS: huge cut in available models
- Breaking of Lorentz allow gravitons to decay
- Many theories are ruled out by $c_T=1$ and by the absence of GW decay and modifications in the graviton dispersion relation: G_4, G_5, F_4, F_5 are absent.

Future:

- Are there other models constrained by the same effect?
- Can we rule out G_3 as well?