Le problème de l'énergie noire

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What Could Dark Energy (Really) Be ?

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Gravity is described by General Relativity (GR):

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}R$$

Uniqueness theorem (Weinberg 1965):

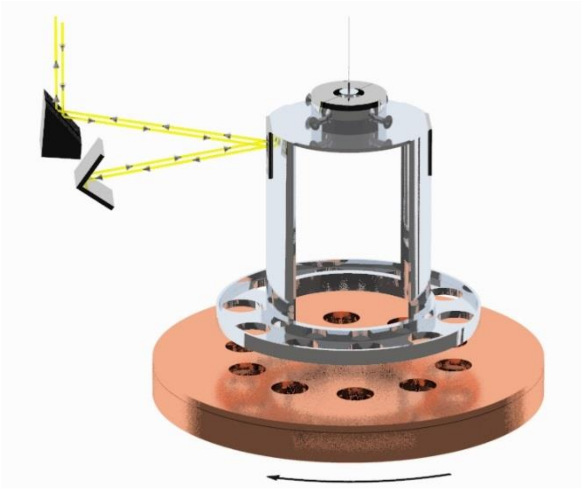
GR is the unique Lorentz invariant theory of massless helicity 2 fields

Lorentz invariance implies the weak equivalence principle (Weinberg 1965) for elementary particles.

$$S_m(\psi_i, g_{\mu\nu})$$

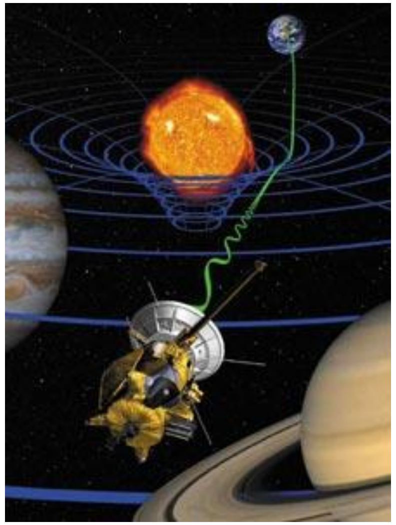
Particles couple to a unique metric.

GR has been wonderfully tested on many length scales:

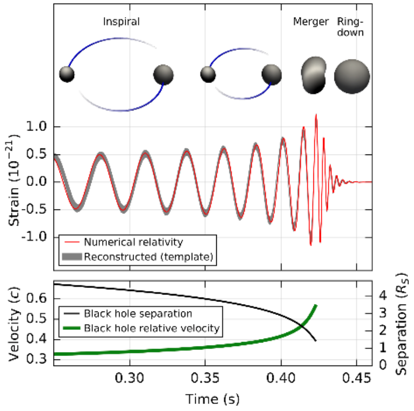


Laboratory experiments (Eotwash) tests of fifth forces and equivalence principle
0.1 mm

Cassini probe test of fifth forces
1 a.u., 150 million km.

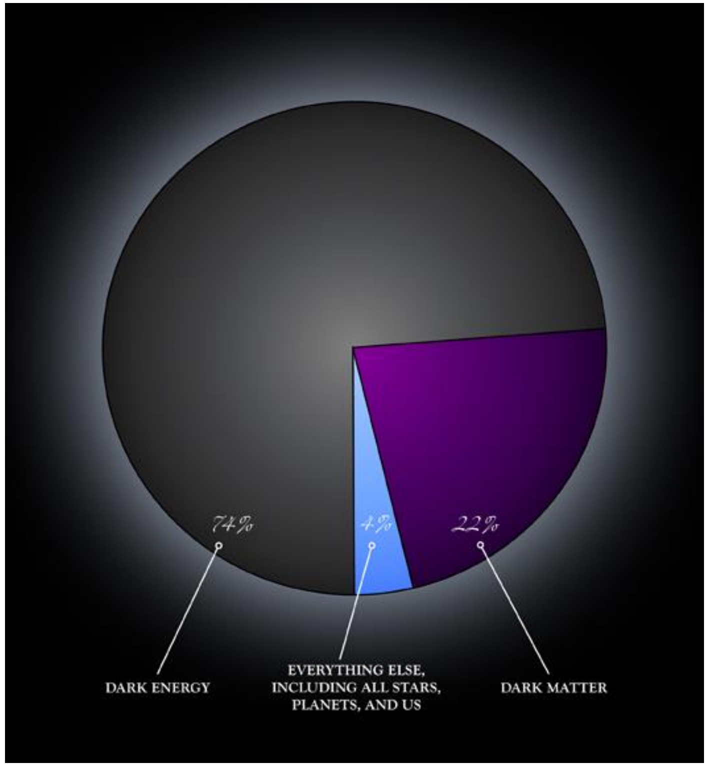


Lunar ranging tests of strong equivalence principle and time variation of Newton’s constant,
400 000 km



Gravitational wave emissions from black hole and neutron star mergers
50 Mpc

But no good understanding of dominant phenomena on cosmological scales where both dark matter and dark energy are necessary to explain Baryon Acoustic Oscillations (BAO) or the Cosmic Microwave Background (CMB).



$$S_{\Lambda\mathrm{CDM}} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}(R-2\Lambda)$$

Cosmological constant

The cosmological constant can be replaced by dynamical fields with more fundamental origins:

DARK ENERGY

The background cosmology of the Universe is governed by the Friedmann equation (1922)

$$H^2 = (\frac{\dot{a}}{a})^2 = \frac{8\pi G_N}{3}(\rho + \rho_\Lambda) - \frac{k}{a^2}$$

It involves matter and the cosmological constant as postulated by Einstein and corresponding to a fluid of negative pressure:

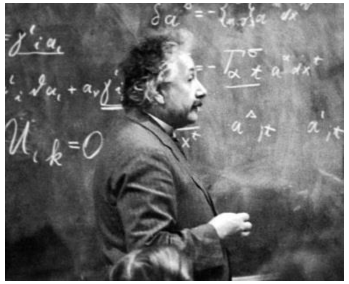
$$p_\Lambda = -\rho_\Lambda$$

Einstein introduced it in 1917 to describe his “Aristotelean” Universe:

Static and spherical Universe:

$$\rho_m = 2\rho_\Lambda$$

$$R = \sqrt{\frac{1}{8\pi G_N \rho_\Lambda}}$$



As early as 1916 with Nernst's hypothesis that the vacuum is filled with the zero point energy of radiation (always in interaction with matter) and the 1920's with Lenz, Pauli (and Jordan) linking the mysterious cosmological constant to the vacuum energy, we see that the role of the vacuum energy in cosmology begins to shape:

$$\rho_\Lambda = \frac{1}{2}\sum_i \omega_i = \frac{1}{8\pi^2}\int_0^{m_e} dk\ k^2\sqrt{k^2+m_e^2}$$

This is the zero point energy, i.e. the vacuum energy, of all the oscillators associated to the known particles: e.g. the electron in the 1920's. As the integral is divergent, Lenz (1926) and later Pauli (1928) cut off the integral at the highest energy envisageable then: the electron mass or the largest energy of known gamma-rays:

$$\rho_\Lambda \sim \frac{m_e^4}{32\pi^2}$$

This immediately leads to a Universe with a radius smaller than the distance to the moon (Lenz). This was long forgotten until, for instance, Zeldovich (1969) and also because by 1927 the Universe is observed to expand:

The calculation of the vacuum energy has a long and checkered history with many pitfalls, such as the confusing use of hard cut-offs in a relativistic context:

$$\rho_\Lambda = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\omega_k = \frac{1}{4\pi^2}\int_0^\Lambda dk\ k^2\ \sqrt{k^2+m^2} \sim \frac{\Lambda^4}{16\pi^2}$$

$$p_\Lambda = \frac{\rho_\Lambda}{3}$$

$$p_\Lambda = \frac{1}{6}\int \frac{d^3k}{(2\pi)^3}\frac{k^2}{\omega_k} = \frac{1}{12\pi^2}\int_0^\Lambda dk\ \frac{k^4}{\sqrt{k^2+m^2}} \sim \frac{\Lambda^4}{48\pi^2}$$

TOTALLY WRONG !!!

Why? Lorentz invariance is violated! Need to use a method like dimensional regularisation which preserves Lorentz invariance.

Very modern point of view: only matter particles contribute to the vacuum energy in flat space-time and the vacuum energy needs to be « renormalised »:

> Contrary to the eigen-oscillations in a crystal lattice (where theoretical as well as empirical reasons speak to the presence of a zero-point energy), for the eigen-oscillations of the radiation no physical reality is associated to this "zero-point energy" of ½*h*ν per degree of freedom. We are here doing with strictly harmonic oscillators, and since this "zero-point energy" can neither be absorbed nor reflected – and that includes its energy or mass – it seems to escape any possibility for detection. For this reason it is probably simpler and more satisfying to assume that for electromagnetic fields this zero-point radiation does not exist at all.[98]

Contradicted by Casimir effect.

Jordan and Pauli (1928)

The problem of the vacuum energy should always have been at the fore of cosmological research. Indeed one can learn a great deal from its examination:

- ✓ The result seems to depend on an arbitrary UV cut off.
- ✓ The result should take into account not only the electron but all the known particles.
- ✓ The sole contribution from the proton is larger than the energy at the formation of the elements (Big Bang Nucleosynthesis) preventing one from understanding the Universe’s dynamics since then.

The first two points have been gradually understood since the 1950’s with the advent of modern Quantum Field Theory:

$$\rho_\Lambda = \Lambda m_{\mathrm{Pl}}^2 + \rho_{\mathrm{transition}} + \sum (2j+1)(-1)^{2j} \frac{m_j^4}{64\pi^2} \ln \frac{\mu^2}{m_j^2}$$

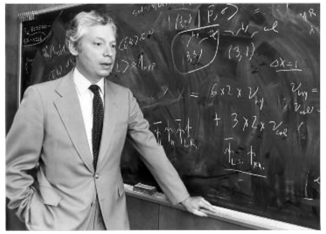
Cosmological constant

Phase transitions: QCD, electroweak …

Vacuum quantum fluctuations

Weinberg proved a "no-go" theorem which explains why such a tuning is contrived. Let us consider the simplest of all field theories and assume that its ground state represents a valid description of vacuum:

$$\mathcal{L} = \sum_i \frac{(\partial \phi_i)^2}{2} + V(\phi_i)$$



Let us also assume that no fundamental scale appears in this Lagrangian, to avoid any hidden fine-tuning. The ground state minimises the potential :

$$\phi_i = \lambda z_i, \quad V(\phi_i) = \lambda^4 \tilde{V}(z_i)$$

$$\begin{cases} \tilde{V}(z_i) = 0 \\ \partial_{z_i} \tilde{V}(z_i) = 0 \end{cases}$$

$$\tilde{V}(z_i) = y^{ijkl} z_i z_j z_k z_l$$

dilaton

Ground state , N equations for (N-1) fields!

$$g(y^{ijkl}) = 0$$

There must exist a relationship between the couplings for a ground state to be BUT the couplings vary with the scale you are probing and in general this evolution detunes the constraint between the couplings:

$$g(y^{ijkl}(\mu)) \neq 0$$

The *flat direction* of vanishing energy parameterised by the dilaton is “lifted” by quantum corrections.

The only ground state is then at the origin:

$$\phi_i = 0$$

Now some of the scalar gives a mass to the fermions by the Higgs mechanism:

$$\mathcal{L} = x^{ijk}\phi_i\bar{\psi}_j\psi_j \quad \rightarrow m_{ij} = x^{kij}\phi_k$$

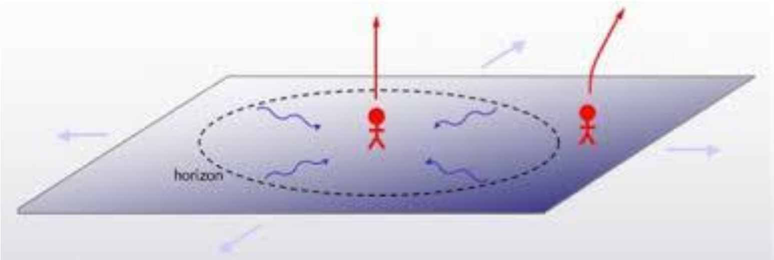
There would be no massive fermions in the Universe! ☐

One can easily violate the hypotheses of the “no-go” theorem:

- The field configurations are dynamical (see F. Vernizzi’s and K. Noui’s talks).
- A symmetry relates the couplings and a non-trivial vacuum exists: supersymmetry…
- Extra dimensions (Supersymmetric Large Extra Dimensions).
- The theory which would describe the vacuum energy could be a non-conventional field theory: global violation of causality (Sequestering).

Another possibility is to modify the gravitational physics:

- Massive gravity
- Multigravitons

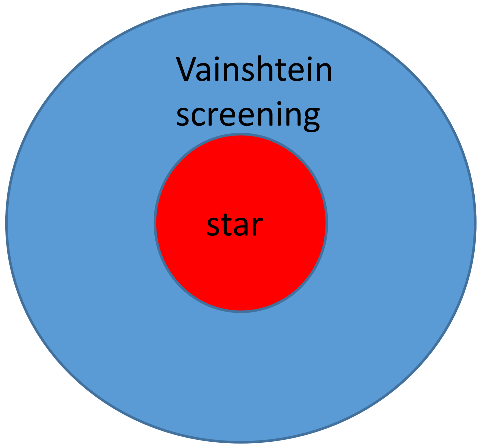


Time-dependent field configurations leading to the acceleration of the Universe can be sought for within the generalised Horndeski models (see Vernizzi's talk for more general settings):

$$\mathcal{L} = K(\phi, X) - G_3(\phi, X)D^2\phi + G_4(\phi, X)R + G_{4,X}((D^2\phi)^2 - (D_\mu D_\nu\phi)^2)$$
$$-\frac{1}{6}G_{5X}((D^2\phi)^3 - 3D^2\phi(D_\mu D_\nu\phi)^2 + 2D^\mu D_\alpha\phi D^\alpha D_\beta\phi D^\beta D_\mu\phi)$$

X is the kinetic term of the scalar field

The effects of the scalar field ϕ are screened locally by the Vainshtein mechanism whilst being full fledged on large scales (leading to acceleration)



Well inside *the Vainshtein radius*, Newtonian gravity is restored. Well outside gravity is modified.

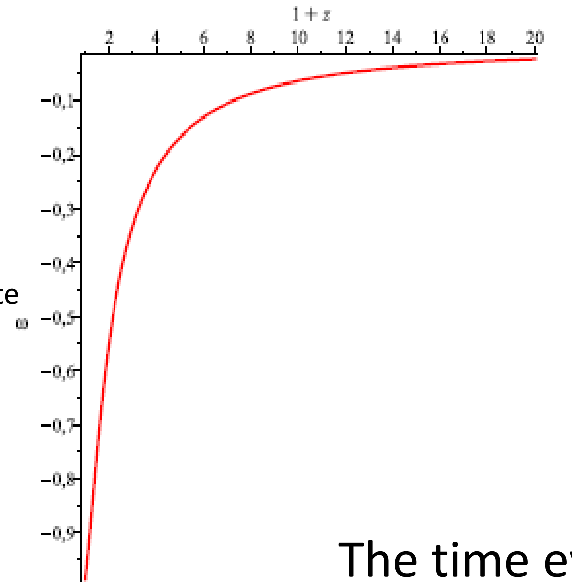
The Vainshtein radius is very large for stars, typically 0.1 kpc for the sun, and a mass for the graviton of the order of the Hubble rate .

The most famous example is provided by the Galileons whose coupling functions are not arbitrary but specified by 4 real coefficients:

$$K(\phi, X) = c_2 X, \; G_3(\phi, X) = -\frac{2c_3}{\Lambda^3} X, \; G_4(\phi, X) = \frac{2c_4}{\Lambda^6} X^2, \; G_5(\phi, X) = -\frac{6c_5}{\Lambda^9} X^2$$

$$\Lambda^3 = m_{\rm Pl} H_0^2$$

Strong coupling scale as large a 1000 km



Equation of state

These model give a convincing background cosmology and interesting features at the perturbation level… but suffer from one major flaw:

The time evolution of the scalar field induces a ***time dependent speed*** for the graviton

These models can have effects on the speed of gravity waves when the term Einstein-Hilbert term is modified in the significant way:

$$G_{4,X} \neq 0$$

Typically in astrophysical situations, we are interested in the emission of spherical waves in a time-dependent cosmological background. In this context the wave equation for gravitons takes the form:

$$\omega^2(G_4 - G_{4,X}\dot{\phi}^2) - 2\omega k\dot{\phi}\partial_r\phi - k^2(G_4 + G_{4,X}(\partial_r\phi)^2) = 0$$

The speed of gravity wave would be "screened", i.e. hardly modified, if the terms in $G_{4,X}$ could be neglected. Could it be that the "Vainshtein mechanism" plays a role here too and screens the speed of gravity waves?

Inside the Vainshtein radius of quartic Galileons, IF spatial gradients are larger than time derivatives:

$$c_T^2 = 1 - \frac{2XG_{4,X}}{G_4}$$

Where the gradient is essentially constant… and very small… implying that the speed of gravity waves would be screened…

$$X = -\frac{\Lambda^4}{2}\left(\frac{c_{0b}M}{8\pi m_{\rm Pl}c_4}\right)^{2/3}, \quad |\Delta c_T^2| \leq 10^{-30}$$

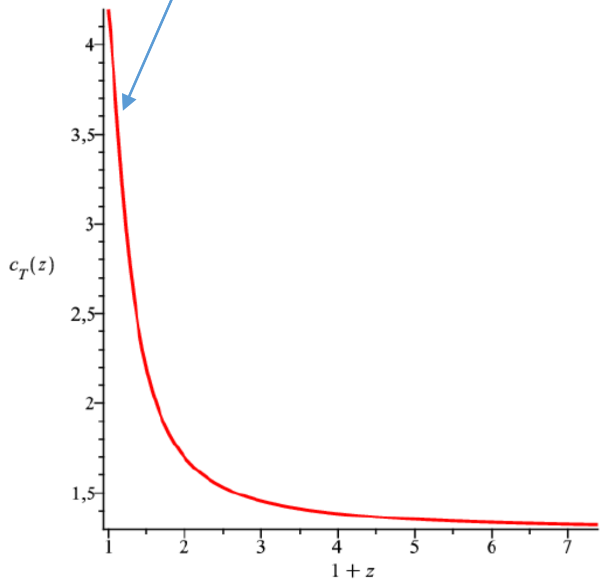
Unfortunately, the time derivatives are smaller than the gradients when the following condition is verified:

$$R_V H_0 \gg 1$$

This is violated for masses of around one solar mass as the Vainshtein radius is much smaller than the cosmological horizon!

$$R_V H_0 \sim 10^{-7}$$

Large values of the speed of gravitational waves for Galileons



The great massacre:

Simplest cases excluded cosmologically

$$\mathcal{L} = K(\phi, X) - G_3(\phi, X)D^2\phi + G_4(\phi, X)R + G_{4,X}((D^2\phi)^2 - (D_\mu D_\nu \phi)^2)$$
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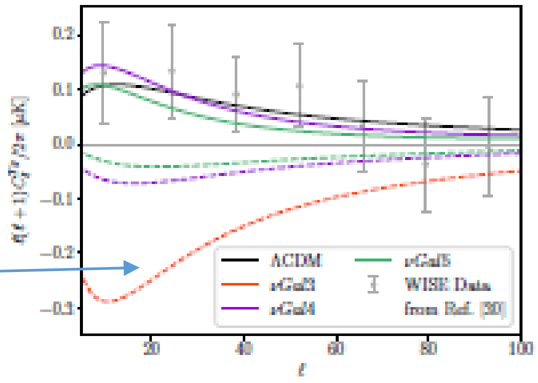


Cubic Galileon

Strong tension between cubic Galileons at the 7.8σ level in the temperature-galaxy cross correlation. ArXiv:170702263

Renewed interest in massive gravity after the discovery of the acceleration of the expansion of the Universe, originally expressed in terms of the graviton field at the quadratic order:

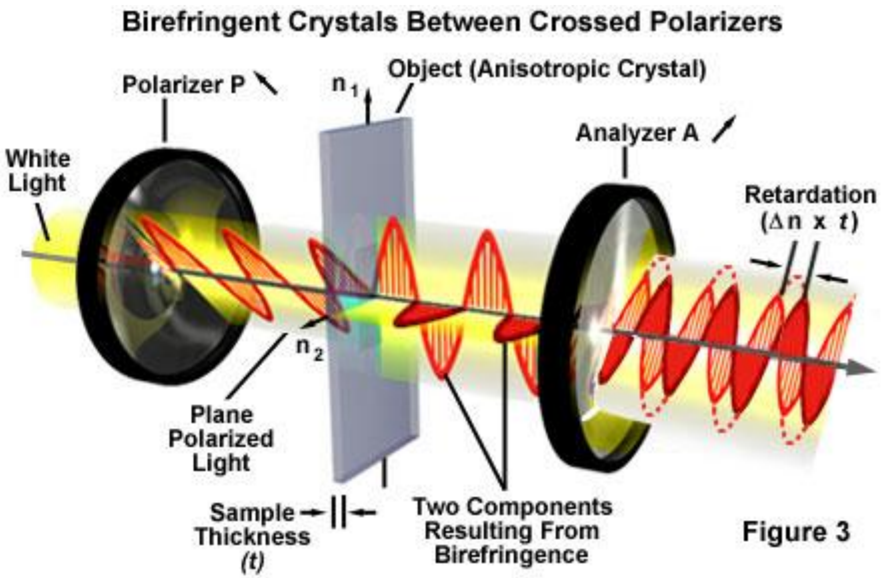
$$\mathcal{L}_{FP} = -\frac{m_G^2 m_{\rm Pl}^2}{8}\left(h_{\mu\nu}h^{\mu\nu} - (h^\mu_\mu)^2\right)$$

Fierz-Pauli (1939)

Unfortunately suffers from vDVZ discontinuity and has a ghost in curved space

$$h_{\mu\nu} = \frac{16\pi G_N}{p^2 + m_G^2}(T_{\mu\nu} - \frac{T}{3}\eta_{\mu\nu}) \longrightarrow h_{\mu\nu} = \frac{16\pi G_N}{p^2}(T_{\mu\nu} - \frac{\eta_{\mu\nu}}{2}T)$$

Discrepancy due to the scalar polarisation of ma massive graviton



Recent direct observation of GW have become a good testing ground for models beyond GR. Sensitive to the existence of multi-gravitons.

Bimetric Gravity

One way to construct a non-linear version of massive gravity involves two dynamical metrics:

$$S = \int d^4x \left(e_1 \frac{R_1}{16\pi G_N} + e_2 \frac{R_2}{16\pi G_N} + \Lambda^4 \sum_{ijkl} m_{ijkl} \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} e^{ai}_{\mu} e^{bj}_{\nu} e^{ck}_{\rho} e^{dl}_{\sigma}\right)$$

where the graviton mass is of order:

$$m_g^2 \sim \frac{\Lambda^4}{m_{\mathrm{Pl}}^2} \sim H_0^2$$

New link between *dark energy* and the *mass of the graviton*

Because of the presence of two dynamical metrics, there are two types of gravitational perturbations of space-time. In particular, in a Minkowski background, there is one massive and one massless graviton. In a cosmological FRW background, the two waves are coupled, where the two metrics are:

$$ds_1^2 = a_1^2(-d\eta^2 + d\vec{x}^2) \qquad ds_2^2 = a_2^2(-b^2 d\eta^2 + d\vec{x}^2)$$

$$\frac{d^2\bar{h}_1}{d\eta^2} - \Delta\bar{h}_1 + (M_{11}^2 - \frac{1}{a_1}\frac{d^2 a_1}{d\eta^2})\bar{h}_1 + M_{12}^2\bar{h}_2 = 0$$

$$\frac{d^2\bar{h}_2}{d\eta^2} - b^2\Delta\bar{h}_2 + (M_{22}^2 - \frac{b^{1/2}}{a_2}\frac{d^2(a_2 b^{-1/2})}{d\eta^2})\bar{h}_2 + M_{21}^2\bar{h}_1 = 0$$

Notice that one of the waves has a speed which is not unity when b is not equal to one.

The coupling to matter is determined by the combined vielbein:

$$e^a_\mu = \beta_1 e^a_{1\mu} + \beta_2 e^b_{2\mu}$$

From which one constructs the Jordan metric:

$$g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\mu$$

$$3H_1^2 m_{\rm Pl}^2 = \beta_1 \frac{a_J^3}{a_1^3}\rho + 24\Lambda^4 m^{1jkl}\frac{a_j a_k a_l}{a_1^3}$$

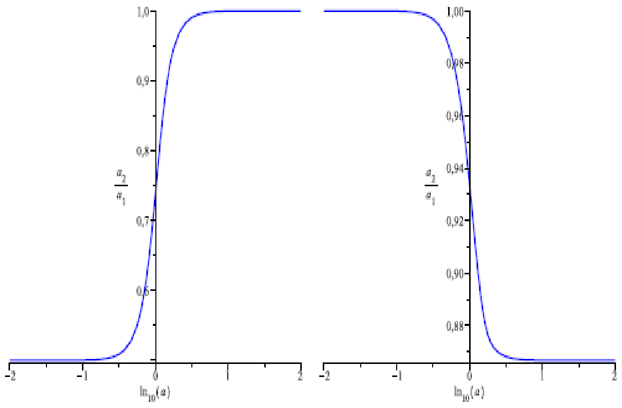
$$\frac{3H_2^2 m_{\rm Pl}^2}{b^2} = \beta_2 \frac{a_J^3}{a_2^3}\rho + 24\Lambda^4 m^{2jkl}\frac{a_j a_k a_l}{a_2^3}$$

The Raychaudhuri equations imply that there are two branches of solutions, one cosmological such that:

$$b = \frac{a_2 H_2}{a_1 H_1}$$

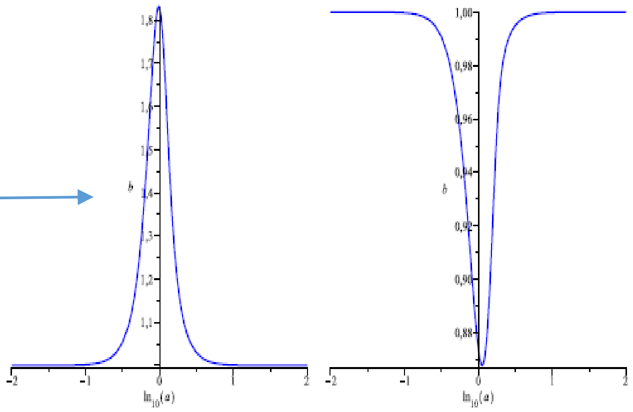
On this branch, the ratio between the two scale factors goes to a constant in the matter, radiation and dark energy eras, with **b=1 in each case**. This ratio determines the speed of the two gravitons.

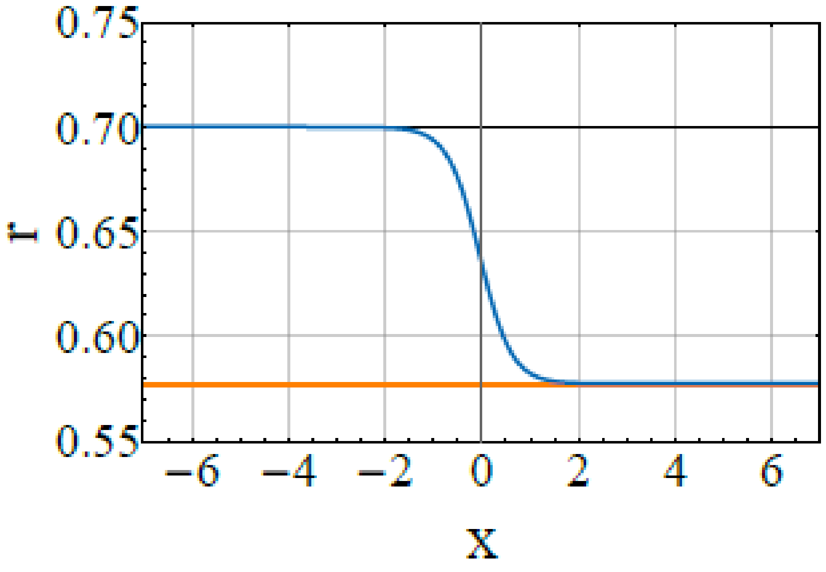
Variation of r=a2/a1 as a function of the Jordan frame redshift



Variation of b as a function of the Jordan frame redshift

Notice that b only differs from one in the recent Universe





Models with eithers all m’s=1 and different couplings or same coupling and one differing m.

In the matter and radiation eras, the ratio between the scale factors converges to

$$r = \frac{a_2}{a_1} \to \gamma = \frac{\beta_2}{\beta_1}$$

In the dark energy era, in the far future, the same ratio converges to a constant too

$$r = \frac{m^{2ijk} a_i a_j a_k}{m^{1ijk} a_i a_j a_k}$$

The time evolution of r is monotonic, implying that to maintain b=1 throughout the whole history of the Universe one must impose a constraint on the coefficients:

$$a_2 = \gamma a_1 \to \gamma = \frac{m^{2ijk} a_i a_j a_k}{m^{1ijk} a_i a_j a_k}$$

This is an algebraic equation for γ

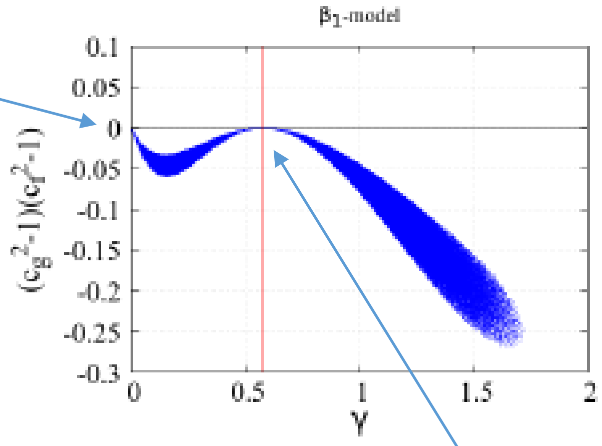
The detection of both the neutron star merger GW170817 and its electromagnetic counterpart gives us a much stronger constraint on the speed of gravitational waves. This should be compared to the speed of light as propagating along the Jordan metric:

$$c = \frac{\beta_1 a_1 + b\beta_2 a_2}{\beta_1 a_1 + \beta_2 a_2}$$

To be compared to the speed of the two gravitons 1 and b.

One can impose both the background cosmology constraints and the GW one. Typically c must be close to either 1 or b at the 10^{-15} level:

Singly coupled model

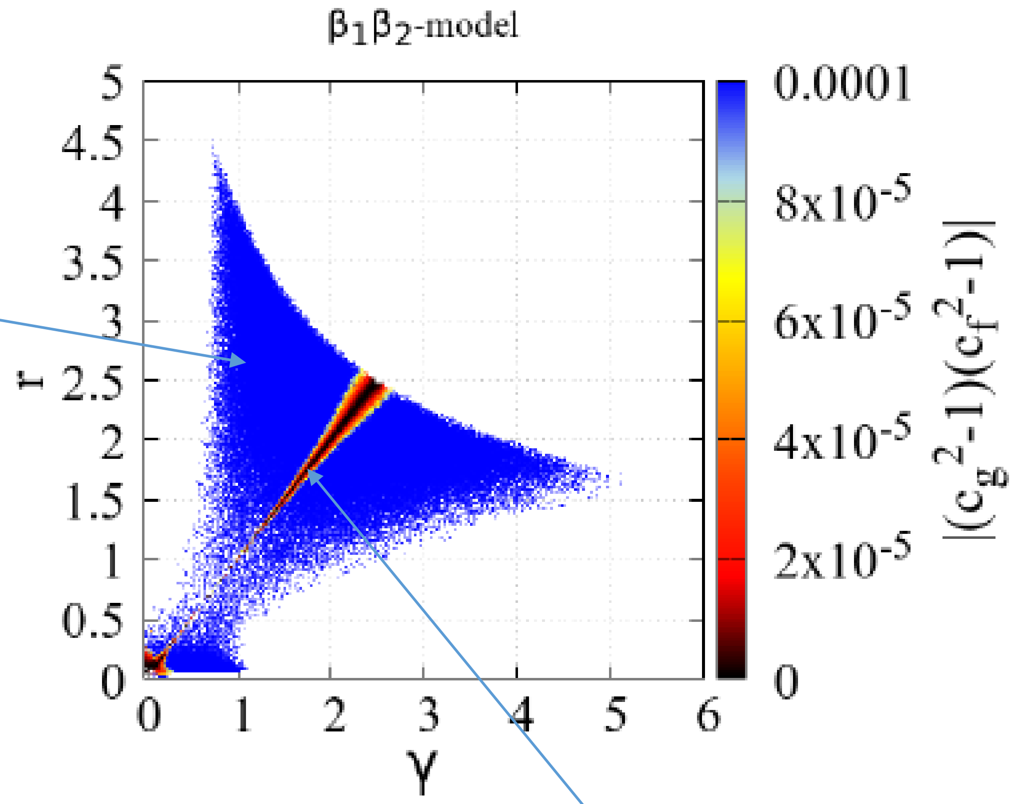


Here a model with only one $m_{1112} \neq 0$

Model with $r = \gamma = \frac{1}{\sqrt{3}}$

When more than one coupling are turned on, one can verify that the allowed models by GW accumulate along the $r = \gamma$ line.

Background cosmology



These models are equivalent to the concordance model at the background cosmology level and for the cosmological perturbations coupled to matter . Need to go non-linear to see differences….

arXiv:1803.09726

The GW constraint gives a line in the parameter space.

BACK TO THE DRAWING BOARD?

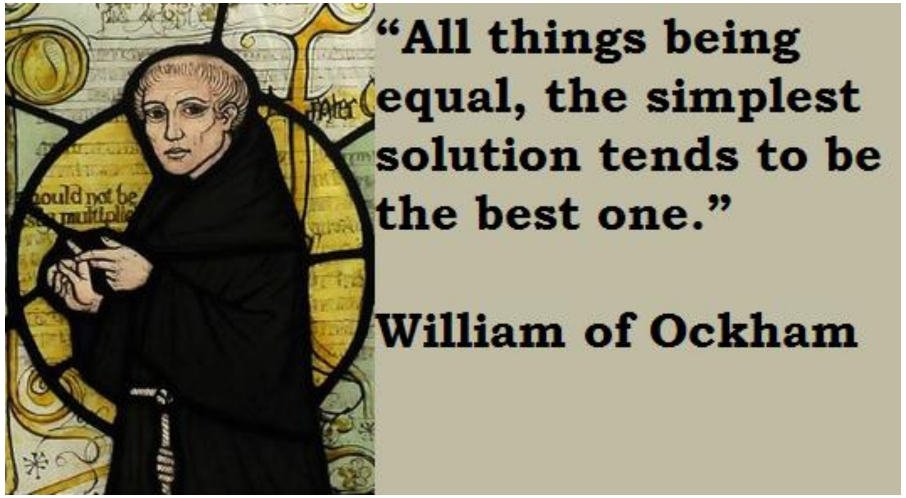
My Uncle Bob was a staunch Conservative, and voted straight Republican until the day he died in Chicago.
Since then he has voted Democrat.

The great massacre:

Quintessence and K-essence

Coupling to matter

$$\mathcal{L} = K(\phi, X) - G_3(\phi, X) D^2\phi + G_4(\phi, X) R + G_{4,X}\left((D^2\phi)^2 - (D_\mu D_\nu \phi)^2\right)$$
$$- \frac{1}{6} G_{5X}\left((D^2\phi)^3 - 3D^2\phi(D_\mu D_\nu\phi)^2 + 2D^\mu D_\alpha\phi D^\alpha D_\beta\phi D^\beta D_\mu\phi\right)$$

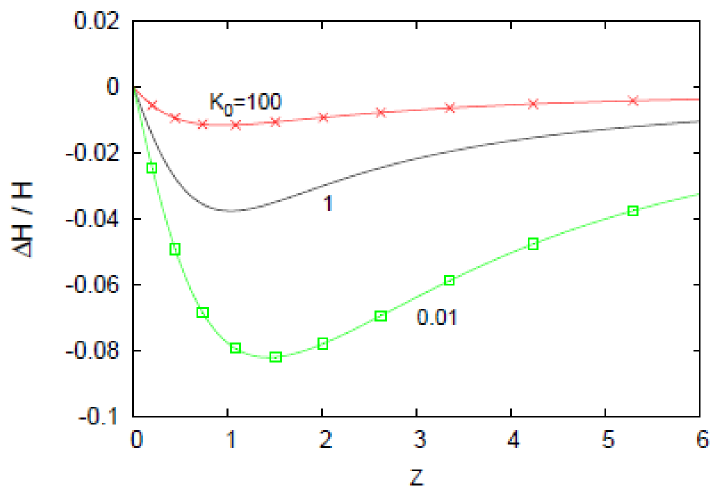


K-mouflage models

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N} + M^4K(\chi)\right) + S_m(\psi, A^2(\phi)g_{\mu\nu}) \qquad \chi = -\frac{(\partial\phi)^2}{2M^4}$$

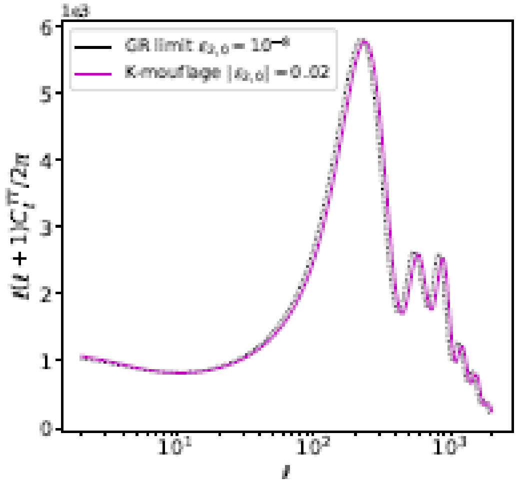
Not many choices left after the great massacre. Though like Bob, one might be staunchly conservative and be fine with tuning the scale M to the dark energy scale. This leads to many cosmologically interesting phenomena which are well worth investigating models.

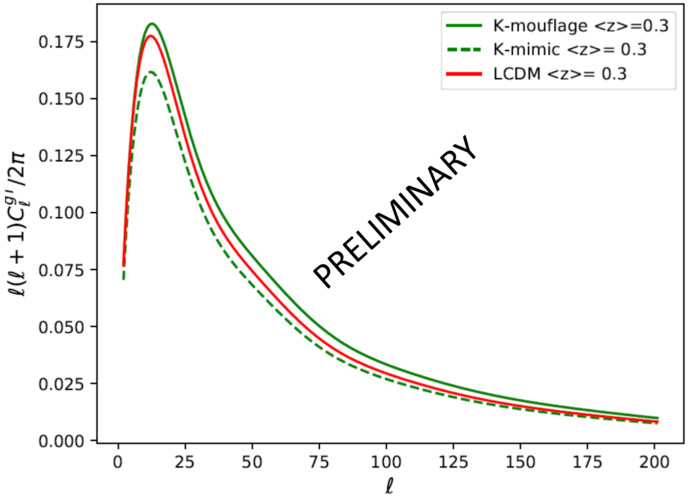
$$K(\chi) = -1 + \chi + K_0\chi^m$$



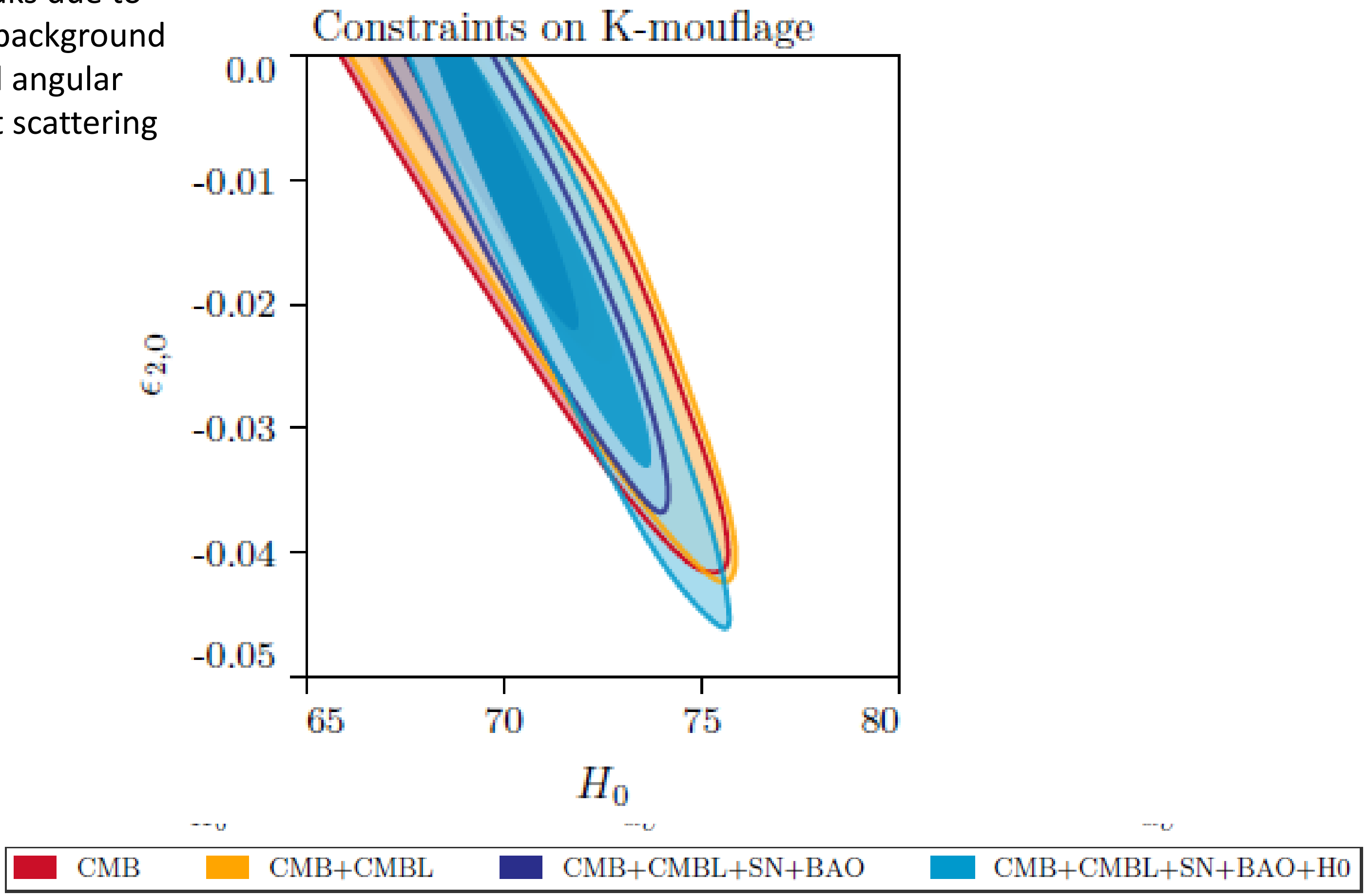
Non renormalisation theorem and validity of the model thoroughly studied in arXiv:1607.01129

Shift of the peaks due to the change of background and a modified angular distance to last scattering





No tension with cross correlation temperature-galaxy



Maybe eases the H0 tension because the A factor decreases with time implying that H0 must be increased. arXiv:1809.09958

LOOPY
IDEAS

SEQUESTERING

The motivation for this approach is Weinberg's "no-go" theorem and its loopholes. Let us start with the familiar action:

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G_N} + \lambda^4 \mathcal{L}_m(\lambda^2 g_{\mu\nu}, \psi^i) - 2\Lambda \right)$$

Matter action

where the dilaton λ and the vacuum energy Λ are constant in space-time.

In the absence of any new ingredient, then the dilaton vanishes and:

$$m_\psi = \lambda m_\psi^{(0)} \quad \to m_\psi = 0$$

Weinberg's no go theorem all over again.

To violate Weinberg's no-go theorem, add a *non-local* action (Kaloper-Padilla):

$$S_\sigma = \sigma(\frac{\Lambda}{\lambda\mu})$$

where σ vanishes at the origin and μ is a given scale. This contradicts locality in Quantum Field Theory but in a soft way, i.e. in a sector not involving matter or gravity. The field equations yield:

$$\int d^4x\sqrt{-g} = \frac{1}{2\lambda^4\mu^4}\sigma'(\frac{\Lambda}{\lambda^4\mu^4})$$

Finite volume of space-time: space must be finite and time must start with a bang and end in a crunch… This determines a non-vanishing value of the dilaton, hence evading Weinberg's no-go theorem.

The Einstein equations become:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N (T_{\mu\nu} - \frac{< T >}{4} g_{\mu\nu})$$

Einstein tensor describing curvature

Energy momentum tensor of matter and vacuum

Cosmological constant term

$$\Lambda = \frac{< T >}{4}$$

$$< T > = \frac{\int d^4x \sqrt{-g} T}{\int d^4x \sqrt{-g}}$$

Observed cosmological constant

Decomposing the energy-momentum tensor:

$$T_{\mu\nu} = T^m_{\mu\nu} - V_\Lambda g_{\mu\nu} \longrightarrow R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G_N (T^m_{\mu\nu} - \frac{< T^m >}{4} g_{\mu\nu})$$

Any quantum fluctuation, phase transition....

One finds that :

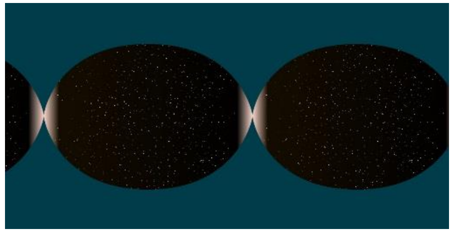
- ✓ The "old" problem of the cosmological constant has a solution coming from the stabilisation of the dilaton at a non-vanishing value.
- ✓ The dark energy problem is intimately linked to the violation of causality:

$$\rho_\Lambda = \frac{< T^m >}{4}$$

An explicit model with a scalar field:

Necessitates to know all the physics till the crunch… acausality in a global way

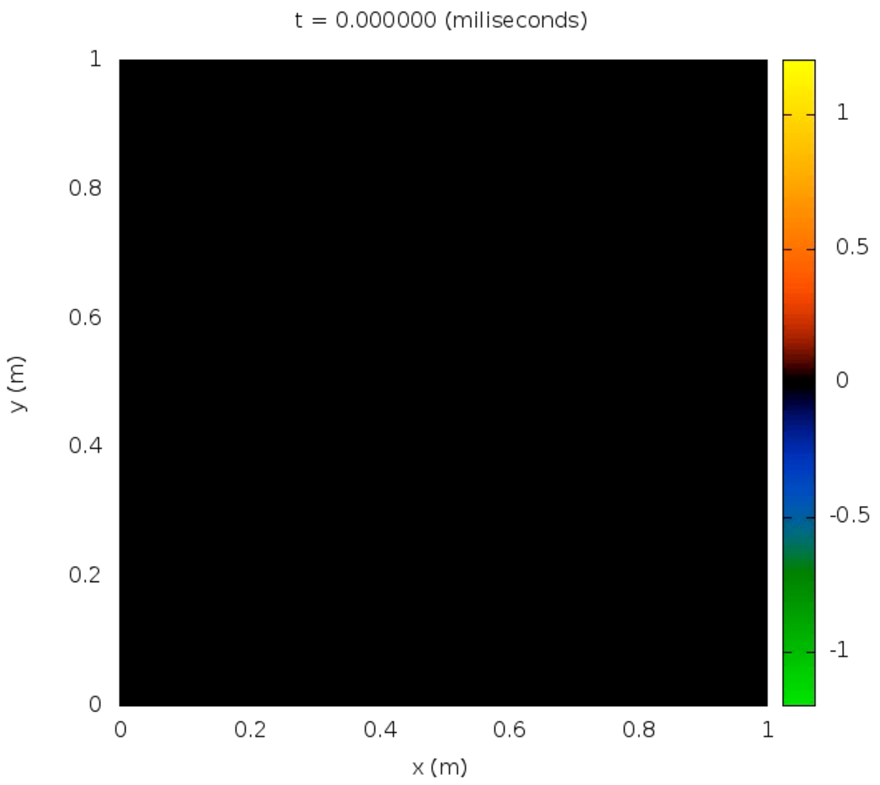
$$V(\phi) = m^3 \phi$$

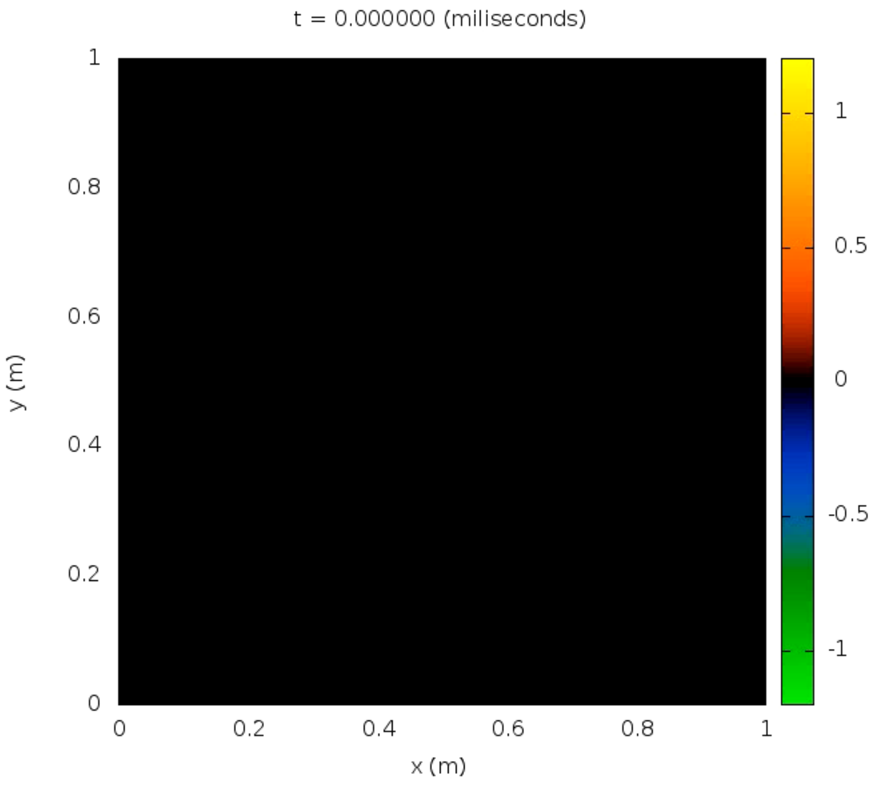


The field rolls down slowly until it becomes negative and then: *CRUNCH*


AWS

Maybe staring at the sky differently?



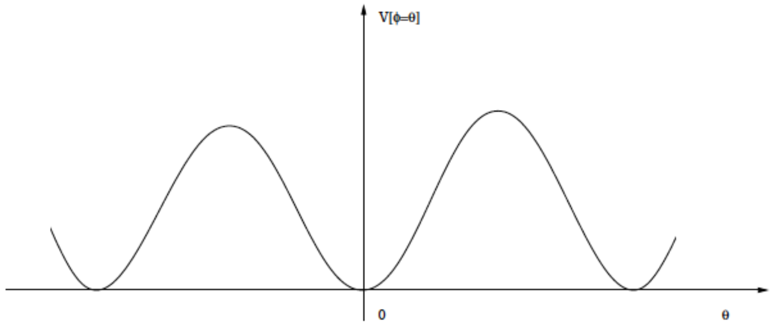


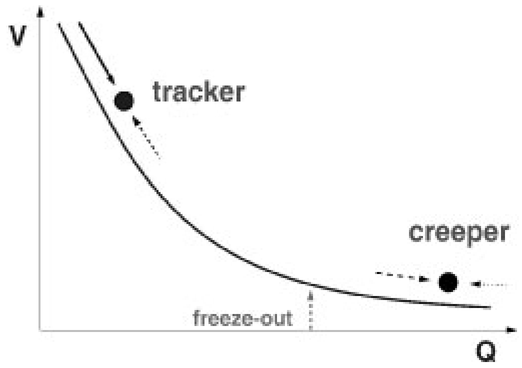
Simulation of the symmetron phase transition and the formation of stable domain walls pinned to matter overdensities.

C.Llinares numerical simulation
arXiv:1807.06870

So what is left?

- Quintessence models (and its coupled form to CDM) and K-mouflage (or K-essence if decoupled from matter)





- This may lead to many interesting phenomena both on cosmological scales and more exotically maybe in the laboratory.

- There is still plenty of room to tackle the cosmological constant problem, like sequestering, and one should be bold.