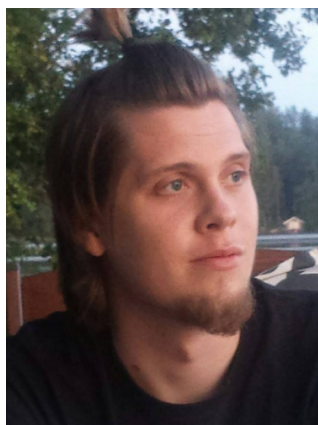




Post-Newtonian dynamical modelling of super-massive black holes in global large-scale simulations: **First applications of the KETJU code**

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Currently applying for postdoc positions!

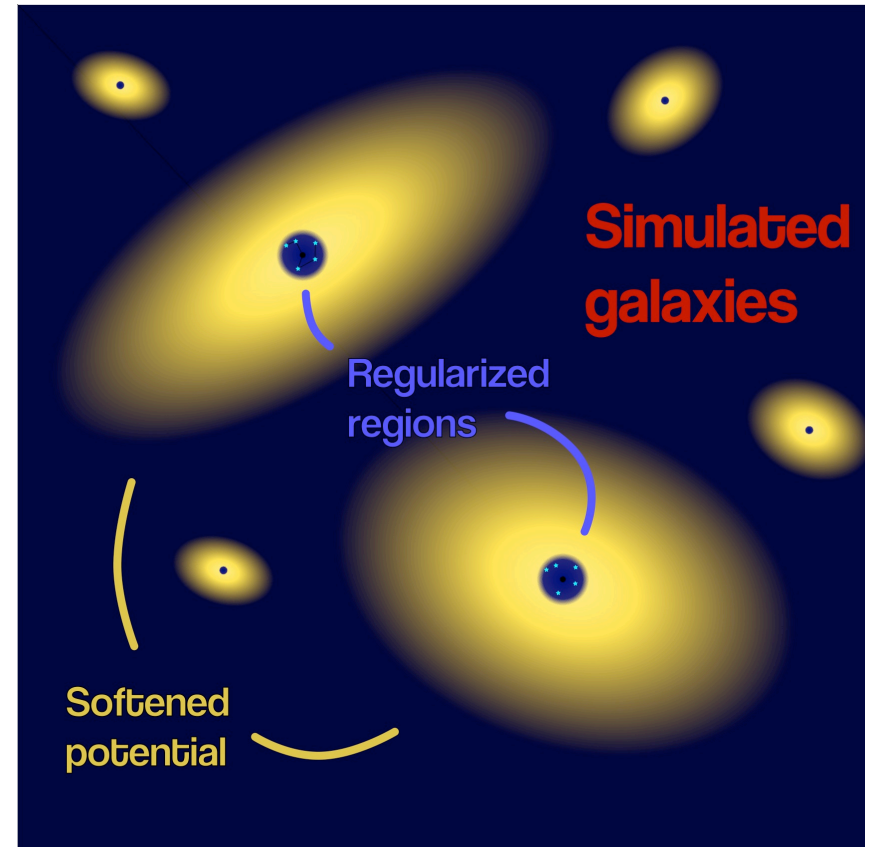
Rantala, Johansson, Naab, Thomas, Frigo, 2018, ApJL submitted, ArXiv: 1812.02732

Rantala, Johansson, Naab, Thomas, Frigo, 2018, ApJ, 864, 113

Rantala, Pihajoki, Johansson, Naab, Lahén, Sawala, 2017, ApJ, 840, 53

1. Motivation

- The dynamics of black holes in have traditionally been studied either with 10-100 million particle **softened simulations** (e.g. **Gadget-3**) or by **unsoftened direct** N-body simulations restricted to **~ 1 million particles** (e.g. Nbody-7).
- We have added a submodule to GADGET-3 that employs **algorithmically regularized dynamics** and resolves the dynamics accurately near SMBHs with no softening.
- We can now **resolve the dynamical friction, three-body interaction and gravitational wave phase** self-consistently in one single simulation.



KETJU - Regularized Gadget: Main features

1. **KETJU** (chain in Finnish): An extension of Gadget-3, which includes **an algorithmically regularized chain** (Mikkola & Merritt 2008) module that makes two-body collisions integrable by a **simple leapfrog integrator**.
2. Supports **multiple regularized chains**, where high-resolution regularized regions can be included around every BH in the simulation.
3. Includes **Post-Newtonian corrections** up to order 3.5 PN (or c^{-7}). Includes an explicit leapfrog that account for the fact that the PN correction terms depend on the particle velocities, and possibly spins, in addition to the particle coordinates. **The PN approach is valid up to ~ 10 Schwarzschild radii**.

$$\vec{a}_{2\text{-body}} = \vec{a}_{\text{New}} + \sum_{k=2}^7 c^{-k} \vec{a}_{k/2PN} + \vec{a}_S$$



Algorithmic chain regularization

1. The dynamics in the high-resolution region is regularized through a **time transformation that avoids force divergences** and allows even for particle collisions (Mikkola & Tanikawa 1999, Preto & Tremaine 1999).
2. The particles are organized into a **chain** and in the calculation inter-particle vectors are used which significantly **reduces round-off errors**.
3. Particles in the chain are integrated using the **Bulirsch-Stoer extrapolation** method, in which a large number (~ 100) substeps are taken during a full Gadget timestep resulting in good convergence.

Define $t \mapsto s$ by

$$ds = [\alpha(T + B) + \beta\omega + \gamma] dt \\ = (\alpha U + \beta\Omega + \gamma) dt,$$

where $\alpha, \beta, \gamma \in \mathbb{R}$, and

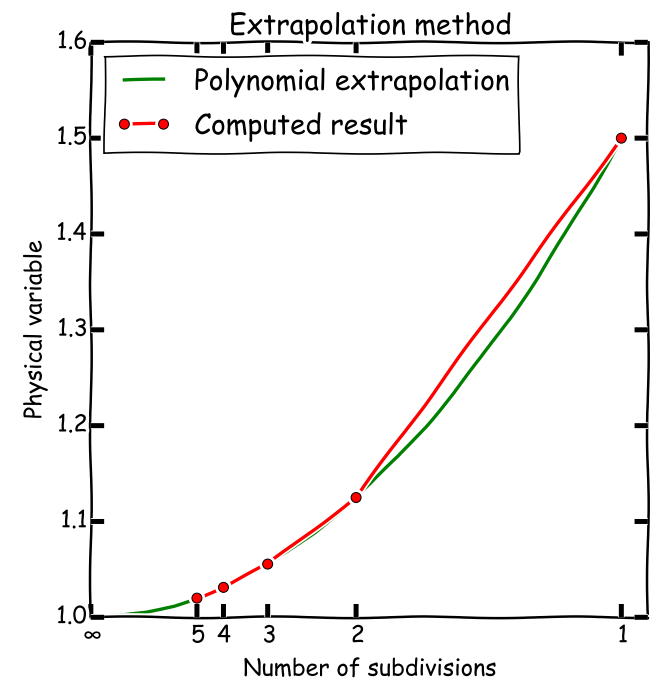
$$T = \sum_i \frac{1}{2} m_i \|\vec{v}_i\|^2 \quad \text{kinetic energy,}$$

$$U = \sum_i \sum_{j>i} \frac{Gm_i m_j}{\|\vec{r}_{ij}\|} \quad \text{force function,}$$

$$B = -T + U \quad \text{binding energy,}$$

$$\Omega = \text{arbitrary function of } \vec{r}_i,$$

$$\dot{\omega} = \sum \nabla_{\vec{r}_i} \Omega \cdot \vec{v}_i.$$



Chain construction

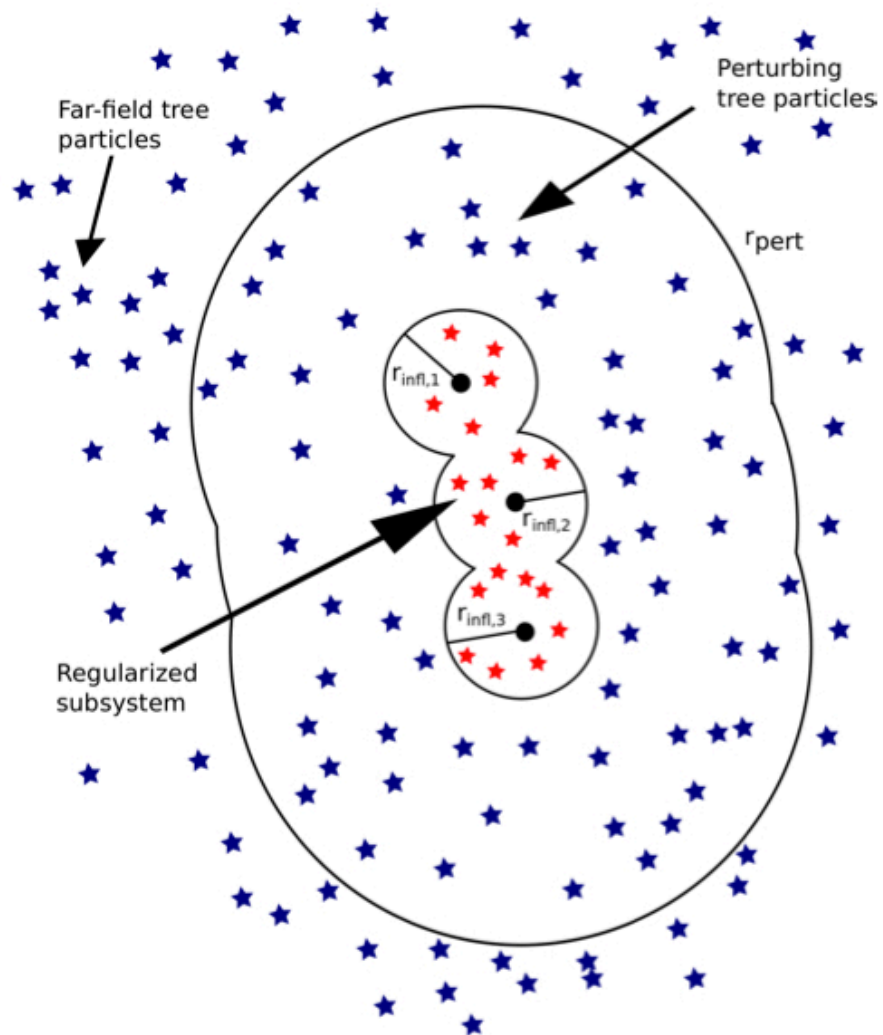
1. **Chain particles:** All the SMBH particles and stellar particles that lie within the influence radius of the SMBHs. Typically $r_{\text{infl}} \sim 10\text{-}30$ pc.

$$r_{\text{infl}} = \lambda \times \frac{M_{\text{BH}}}{10^{10} M_{\odot}} \text{kpc}$$

2. **Perturber particles:** Simulation particles, which induce strong tidal perturbations on a chain system. Typically $r_{\text{pert}} = 2 \times r_{\text{infl}}$

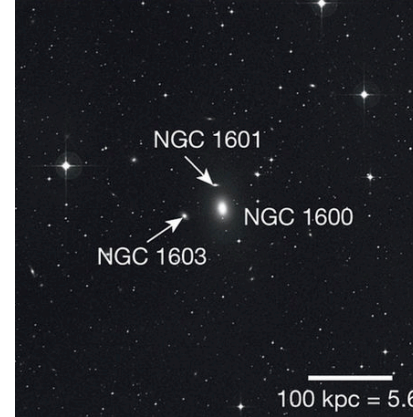
$$r < r_{\text{pert}} = \gamma \times r_{\text{infl}} \left(\frac{m}{M_{\text{BH}}} \right)^{1/3}$$

3. **Tree particles:** Other particles that do not reside near any of the SMBHs act as ordinary GADGET-3 particles.



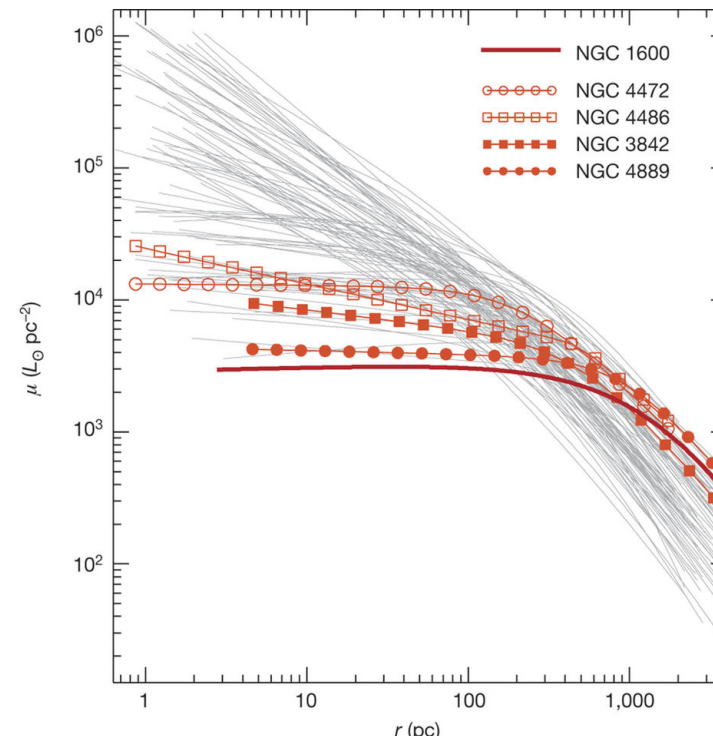
2. Formation of cored galaxies

- **Core ellipticals** exhibit large cores with nearly constant surface brightness. Typically very massive, slowly rotating and have boxy isophotes.
- Probably formed through a dry (gas-poor) merger between two massive early-type galaxies and **scouring of the core by the dynamical evolution of a SMBH binary.**



- **NGC 1600** is an extreme example of a cored galaxy. (Thomas et al. 2016).

$$M_{\text{BH}} = (1.7 \pm 0.15) \times 10^{10} M_{\odot}$$



Initial conditions and simulations

- The collisionless initial conditions are modelled using isotropic **Dehnen profiles** ($\gamma=1.5$ or $\gamma=1.0$) for the stars and $\gamma=1.0$ for the dark matter, including a central SMBH.

$$\rho(r) = \frac{(3 - \gamma)M}{4\pi} \frac{a}{r^\gamma(r + a)^{4-\gamma}}$$

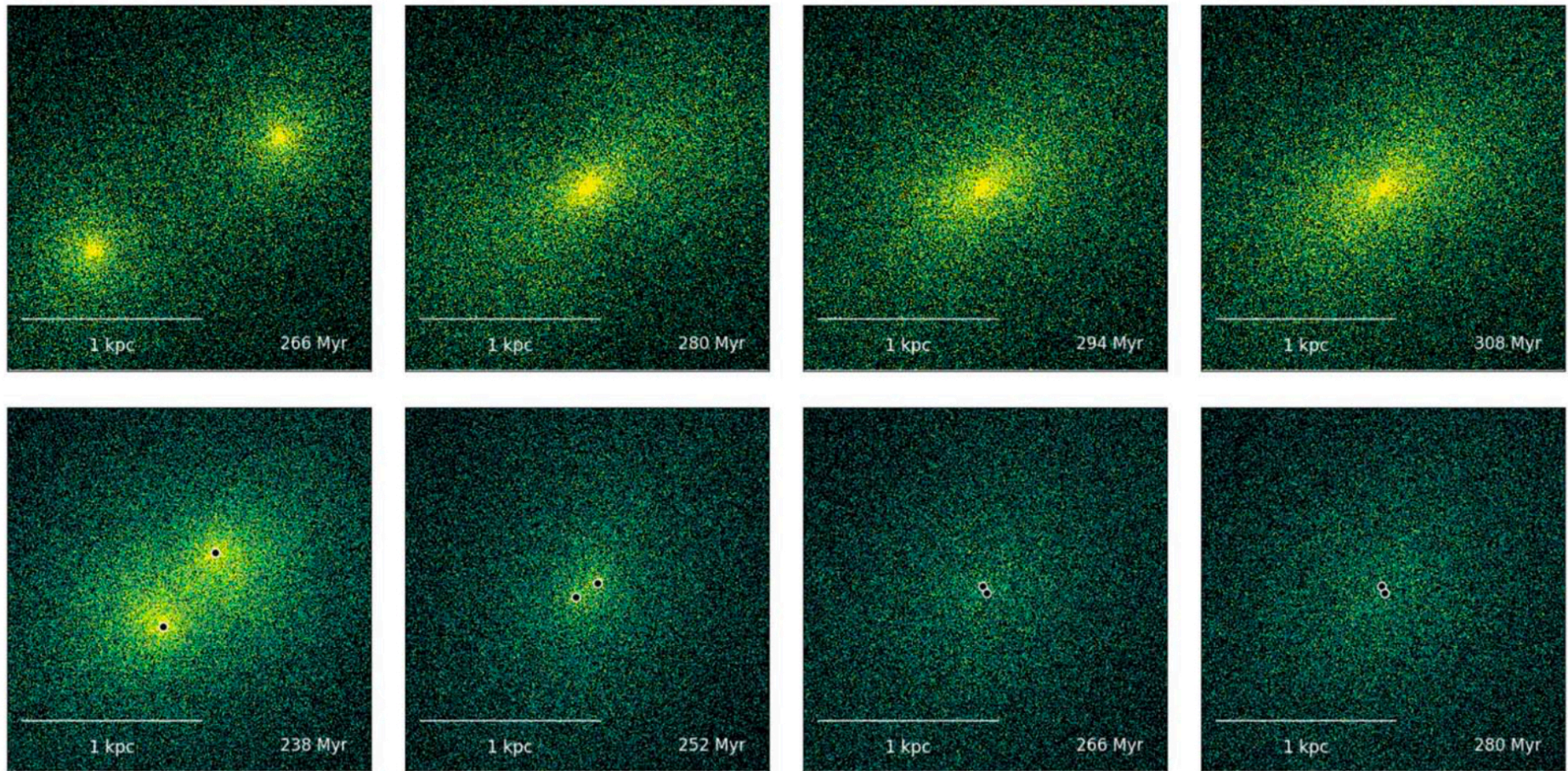
- We simulate major mergers to describe the **final dry major merger** that NGC 1600 likely experienced.
- High numerical resolution for N-body type of simulation.**

Parameter	Symbol	Value
Stellar mass	M_\star	$4.15 \times 10^{11} M_\odot$
Effective radius	R_e	7 kpc
DM halo mass	M_{DM}	$7.5 \times 10^{13} M_\odot$
DM fraction	f_{DM}	0.25
Number of stellar particles	N_\star	4.15×10^6
Number of DM particles	N_{DM}	1.0×10^7

Progenitor	γ	M_\bullet	Progenitor	γ	M_\bullet
γ -1.0-BH-0	1.0	-	γ -1.5-BH-0	1.5	-
γ -1.0-BH-1	1.0	$8.5 \times 10^8 M_\odot$	γ -1.5-BH-1	1.5	$8.5 \times 10^8 M_\odot$
γ -1.0-BH-2	1.0	$1.7 \times 10^9 M_\odot$	γ -1.5-BH-2	1.5	$1.7 \times 10^9 M_\odot$
γ -1.0-BH-3	1.0	$3.4 \times 10^9 M_\odot$	γ -1.5-BH-3	1.5	$3.4 \times 10^9 M_\odot$
γ -1.0-BH-4	1.0	$5.1 \times 10^9 M_\odot$	γ -1.5-BH-4	1.5	$5.1 \times 10^9 M_\odot$
γ -1.0-BH-5	1.0	$6.8 \times 10^9 M_\odot$	γ -1.5-BH-5	1.5	$6.8 \times 10^9 M_\odot$
γ -1.0-BH-6	1.0	$8.5 \times 10^9 M_\odot$	γ -1.5-BH-6	1.5	$8.5 \times 10^9 M_\odot$

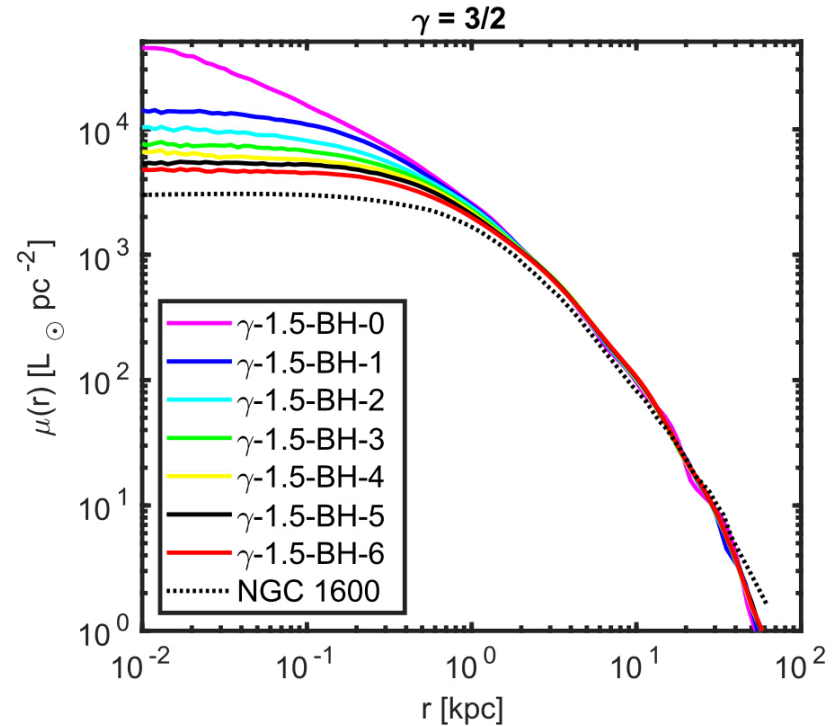
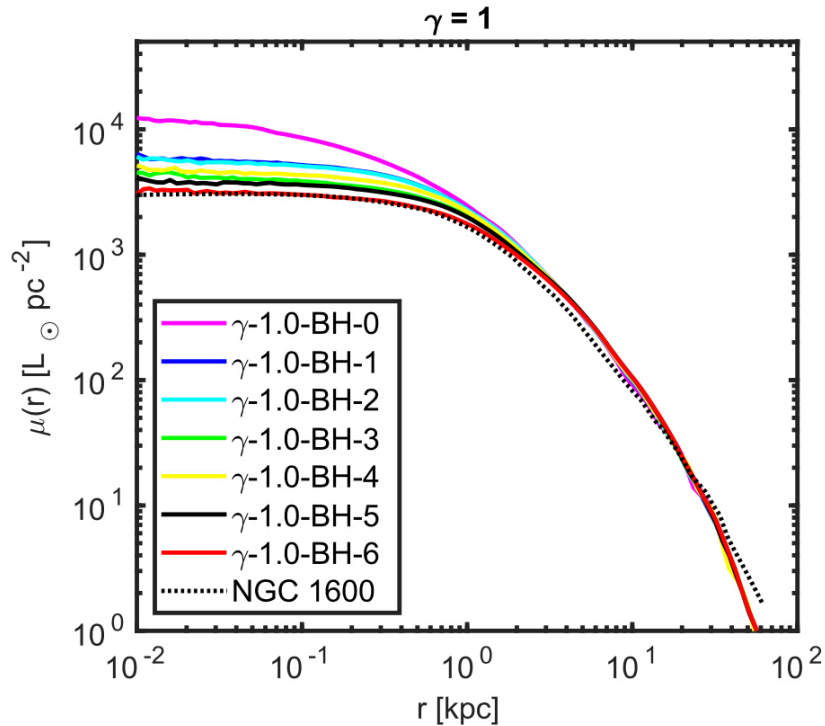
BH-0: no SMBHs and BH-6: Observed SMBHs

Stellar surface densities



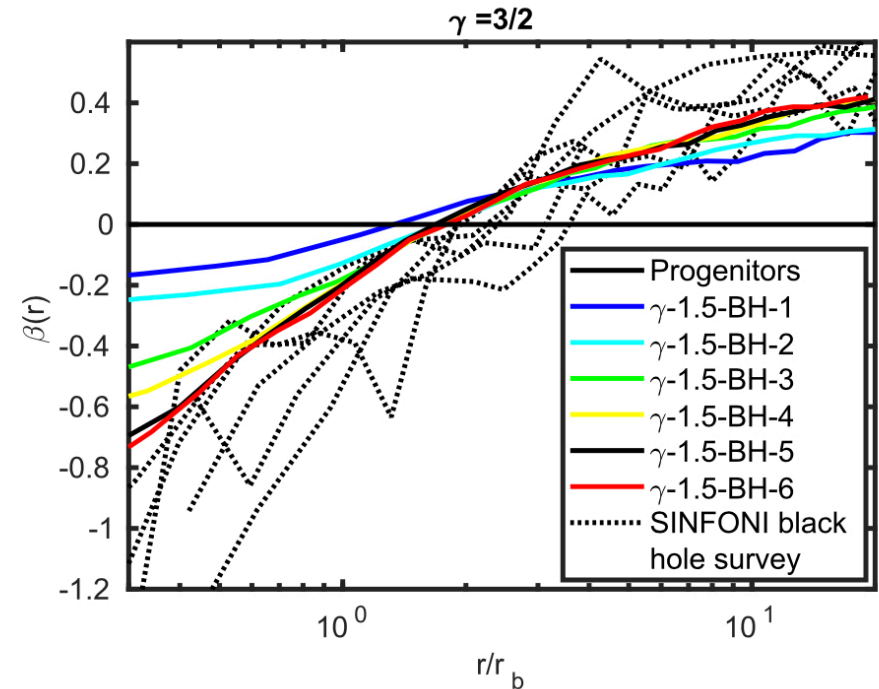
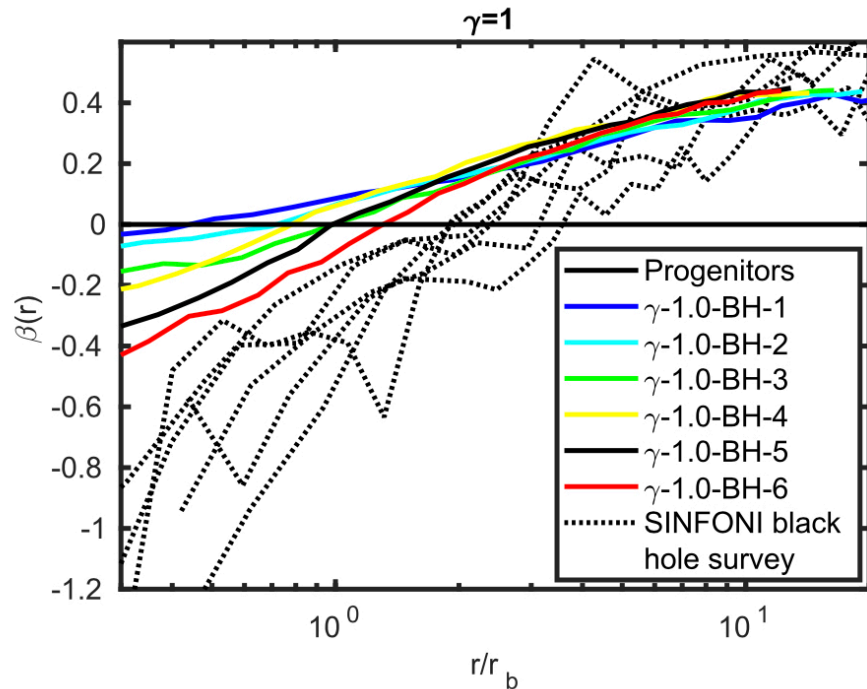
- Top: Merger without SMBHs. Bottom: Merger with massive SMBHs.
- The effect of core scouring by the SMBHs can clearly be seen in the surface density plot.

Surface brightness profiles



- Similarly to the Thomas et al. (2016) observations we assume a constant mass-to-light ratio of $M_*/L=4.0$.
- As expected we find a **systematic decrease in the surface brightness** as a function of increasing BH mass (e.g. Merritt 2006).

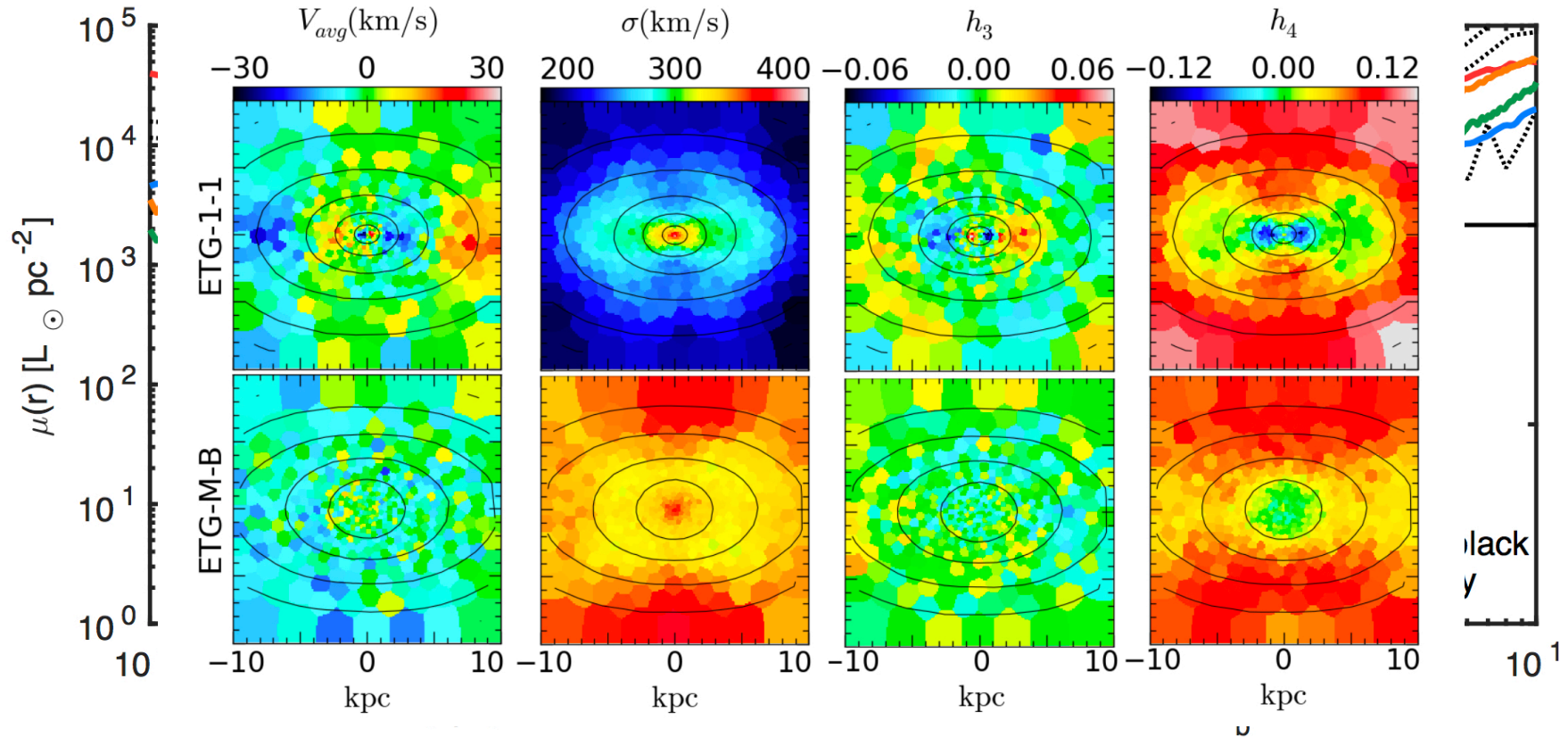
Velocity anisotropy profiles



- We find a **monotonic decrease in the central β -parameter**, meaning an increasingly more tangentially biased stellar population in the core region.
- More massive BHs have larger spheres of influence -> lower β .**

$$\beta = 1 - \frac{\sigma_{\theta}^2 + \sigma_{\phi}^2}{2\sigma_r^2} = 1 - \frac{\sigma_t^2}{\sigma_r^2}.$$

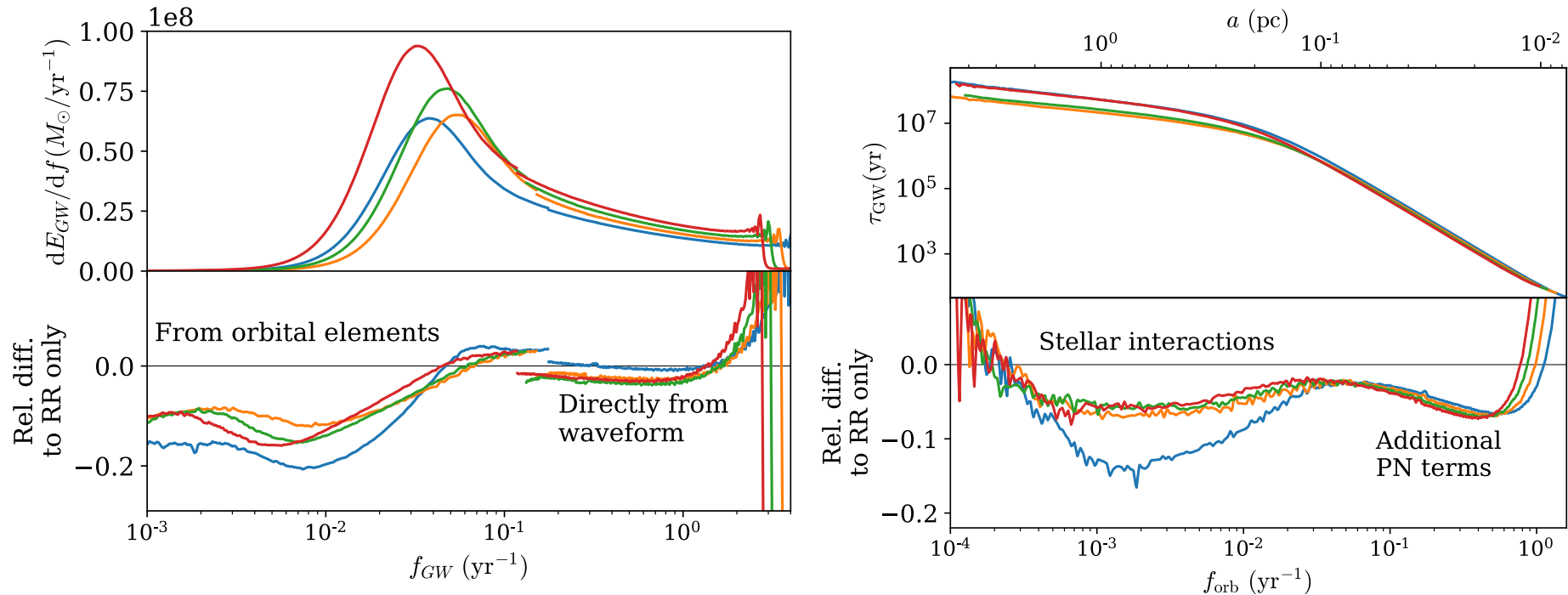
3. Multiple merger generations and remergers



- ETG-1-1-nobh no SMBH, ETG-1-1 merger, ETG-M-A remerger of ETG-1-1 merger remnants, ETG-M-B 5 generations of 5:1 mergers.
- Repeated mergers (green and orange lines) render the central velocity dispersion closer to isotropic, confirmed also by velocity maps.
- Very large cores are probably formed in multiple minor mergers.

4. First GW calculations with KETJU

4 merger generations: 5:1, 6:1, 7:1, 8:1, each mergers adds the same SMBH mass.



- The emitted **total gravitational energy (left)** and **GW timescale (right)** as a function of frequency can be calculated from the **KETJU orbital elements** and during the final KETJU timestep **directly from the SMBH positions and velocities** using a fast Fourier transform algorithm.
- Differences to the Peters & Mathews (1963) PN2.5 radiation reaction only term are given in the bottom panels.
$$\left| \frac{da}{dt} \right| = \frac{64}{5} \frac{G^3 M_1 M_2 (M_1 + M_2)}{c^5 a^3} \frac{1 + \frac{73}{24} e^2 + \frac{37}{96} e^4}{(1 - e^2)^{7/2}}$$

Summary/Connection to LISA

- The KETJU code is a version of Gadget includes an **algorithmically regularized chain module** that makes two-body collisions integrable by a simple leapfrog integrator.
- **Cores form rapidly** on the order of the crossing timescale by SMBH binary evolution and the **velocity distribution becomes increasingly more tangential over a longer timescale**.
- Thus far all published KETJU papers have been used **simulations without a gas component**, but as KETJU is built on Gadget running simulations with gas and feedback should be relatively straightforward.
- LISA will be most sensitive to GW signals from SMBHs with masses in range 10^6 - $10^7 M_{\odot}$, **thus modelling the accurate small-scale dynamics simultaneously with the gas physics will be important**.
- We are planning to make the **KETJU code publicly available**, not just for Gadget, but also other simulation platforms such as **AREPO, RAMSES and GASOLINE**.
- In Helsinki a **KETJU related ERC project** will start next summer and we are looking to hire dynamics/GW experts also from within the larger LISA community.

