# Massive black hole triplets as LISA sources

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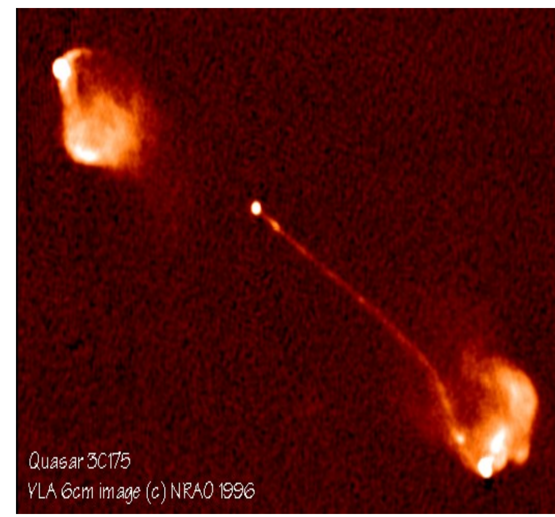
# Massive black hole binaries

Galaxies merge

Galaxies host MBHs

Formation of Massive Black Hole Binaries (MBHBs)

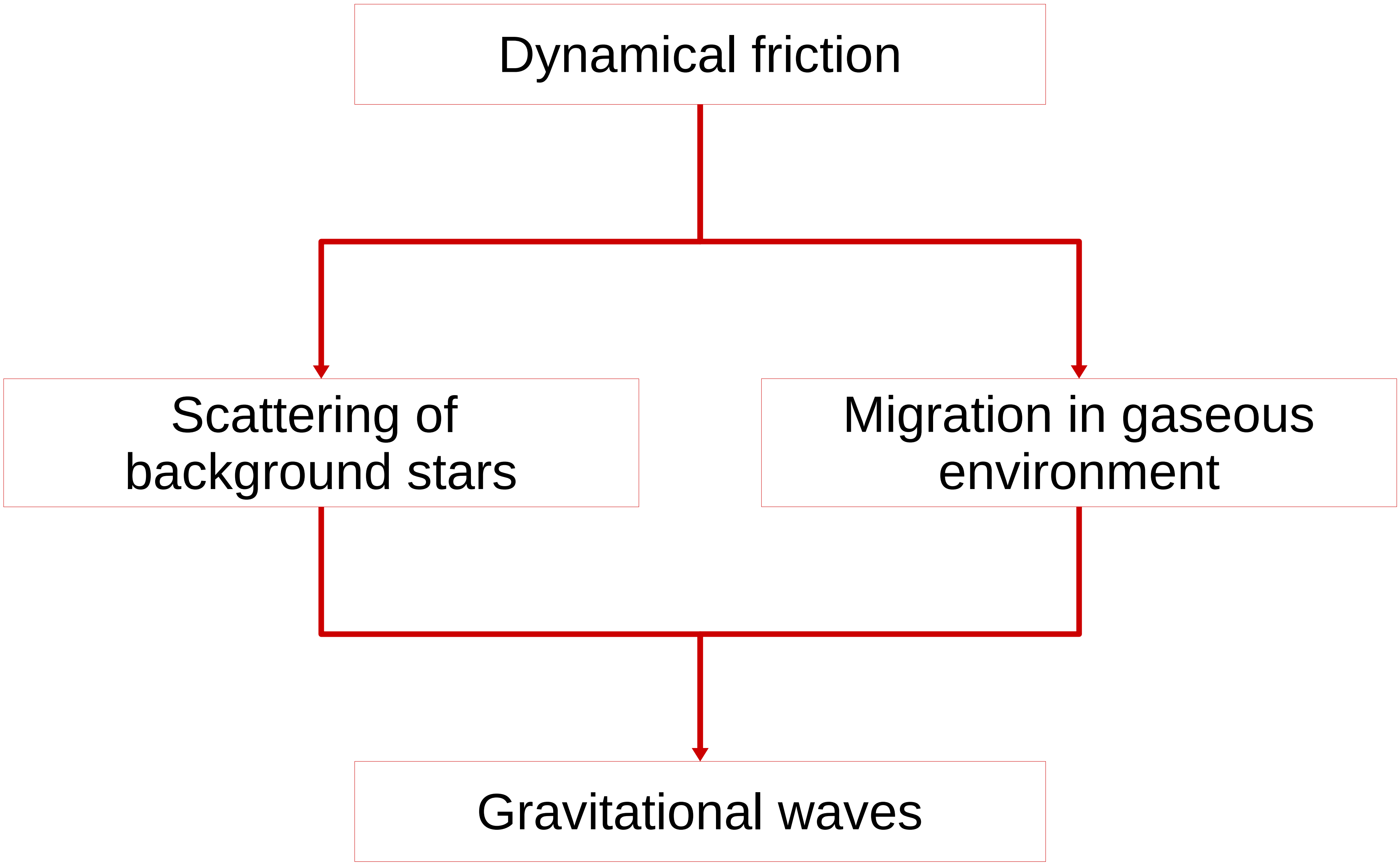



**Source of GWs**

# Standard scenario

Two MBHs in order to get close enough to emit detectable GWs have to cover a **huge spatial range**.
They must rely on several astrophysical processes

# Standard scenario



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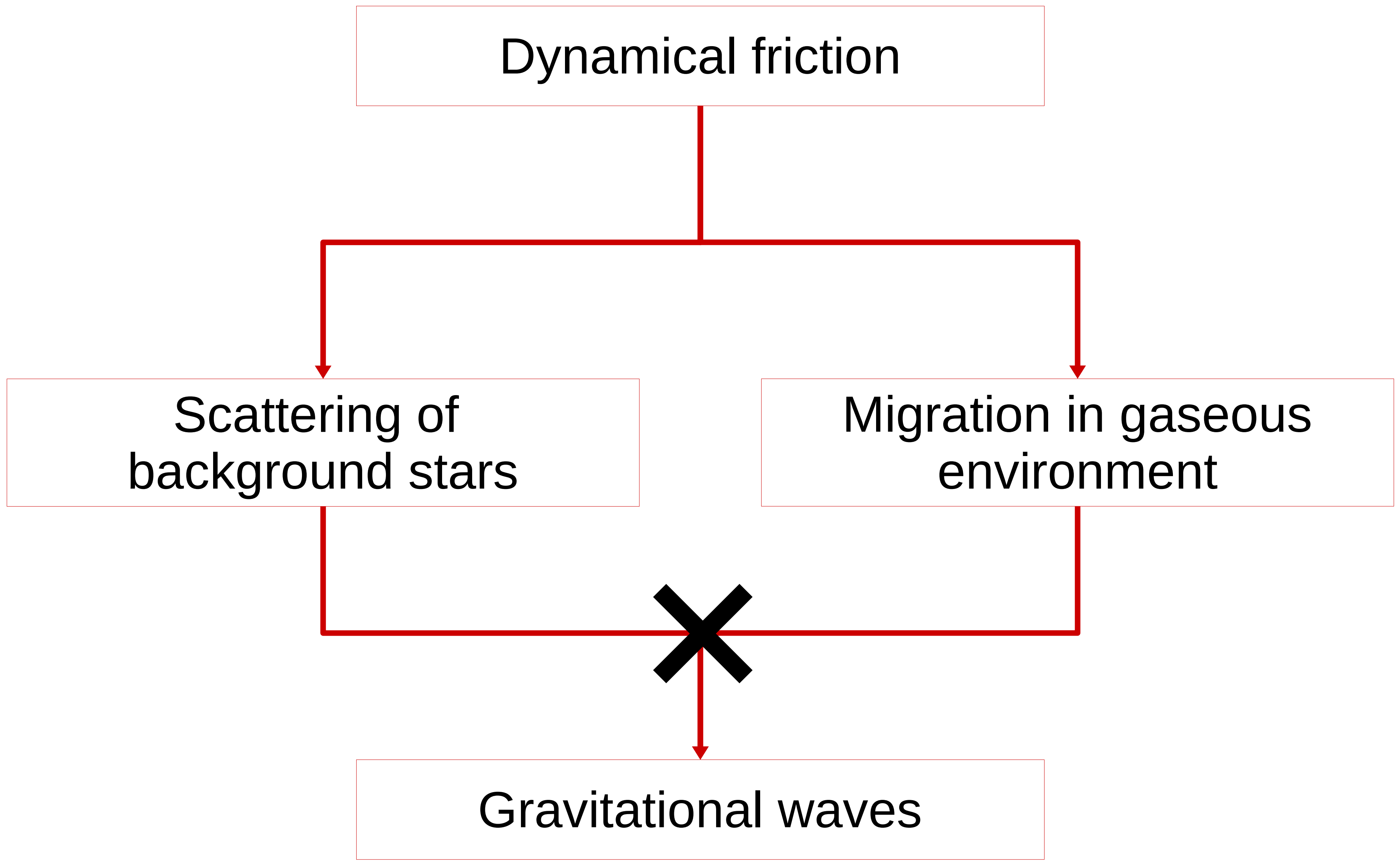

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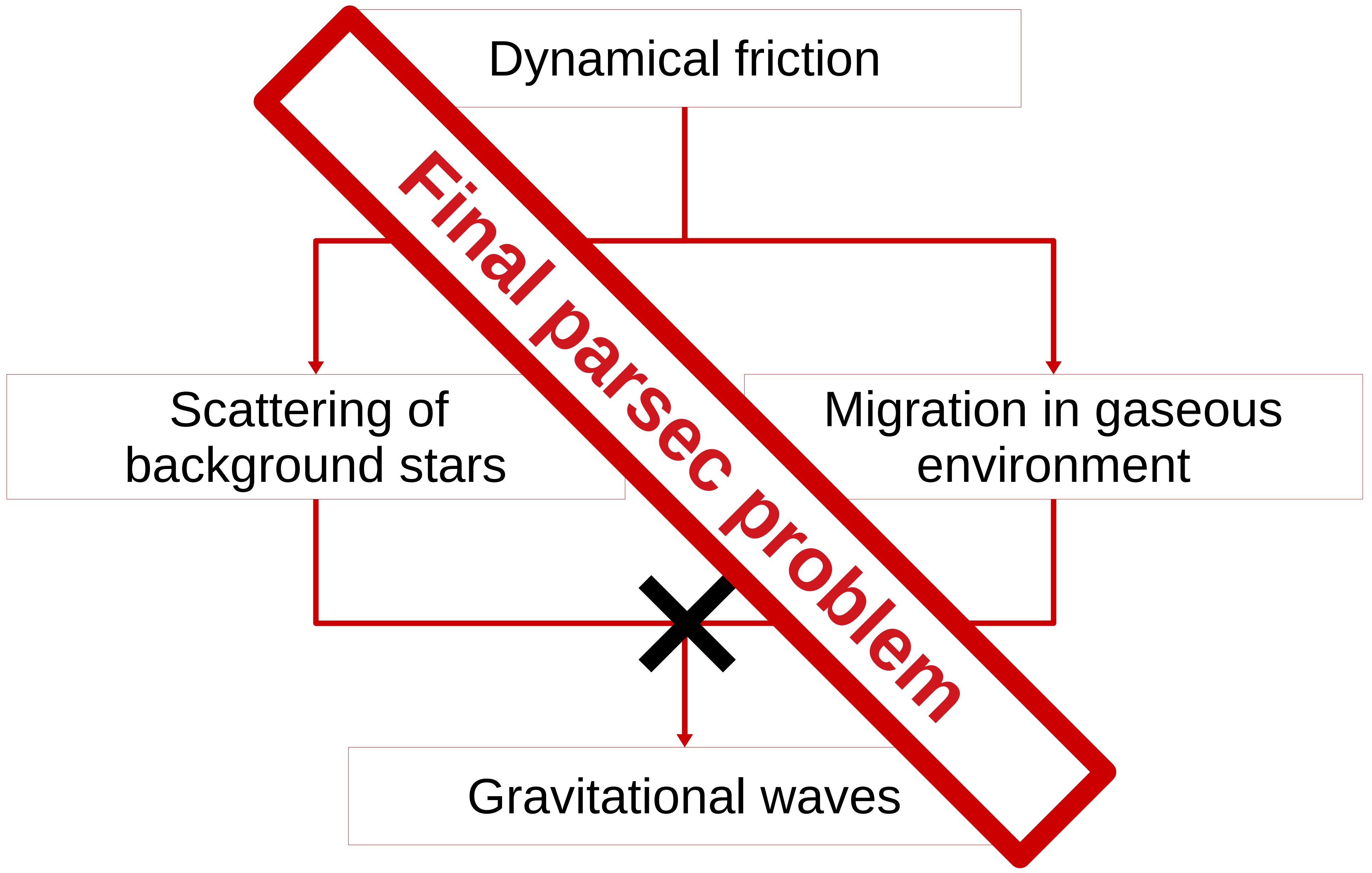

# What if...

Hierarchical growth of cosmic structure

Binary stalling

- Successive galaxy mergers
- Galaxies host at least one MBH

Formation of triplets of MBHs

# Our Goal

Assess the implications of a sizeable population of MBH triplets

How?

Simulating MBH triplets in galactic nuclei with astrophysically and cosmologically motivated initial conditions

Part 1:
Integrate equations of motion of a wide set of triplets with different initial conditions.
Always a stalled binary + a third body

Part 2:
Embed the results in a cosmological framework

# Part 1: setup

Simulate MBH triplets in galactic nuclei with astrophysically and cosmologically motivated initial conditions

3-body Newtonian dynamics + GR corrections up to 2.5PN

# Part 1: Hamiltonians

## Newtonian

$$H_0 = \frac{1}{2}\sum_\alpha \frac{|\vec{p}_\alpha|^2}{m_\alpha} - \frac{G}{2}\sum_\alpha\sum_{\beta\neq\alpha}\frac{m_\alpha m_\beta}{r_{\alpha\beta}}$$

## 1PN

$$\begin{aligned}
H_1 = &-\frac{1}{8}\sum_\alpha m_\alpha\left(\frac{|\vec{p}_\alpha|^2}{m_\alpha^2}\right)^2 \\
&-\frac{G}{4}\sum_\alpha\sum_{\beta\neq\alpha}\frac{1}{r_{\alpha\beta}}\left[6\frac{m_\beta}{m_\alpha}|\vec{p}_\alpha|^2 - 7\vec{p}_\alpha\cdot\vec{p}_\beta - (\vec{n}_{\alpha\beta}\cdot\vec{p}_\alpha)(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta)\right] \\
&+\frac{G^2}{2}\sum_\alpha\sum_{\beta\neq\alpha}\sum_{\gamma\neq\alpha}\frac{m_\alpha m_\beta m_\gamma}{r_{\alpha\beta}r_{\alpha\gamma}}
\end{aligned}$$

## 2.5PN

$$H_{2.5} = \frac{G}{45}\dot{\chi}_{(4)ij}(\vec{x}_{\alpha'},\vec{p}_{\alpha'};t)\chi_{(4)ij}(\vec{x}_\alpha,\vec{p}_\alpha)$$

## 2PN

$$\begin{aligned}
H_2 = &\frac{1}{16}\sum_\alpha m_\alpha\left(\frac{|\vec{p}_\alpha|^2}{m_\alpha^2}\right)^3 + \frac{G}{16}\sum_\alpha\sum_{\beta\neq\alpha}\frac{(m_\alpha m_\beta)^{-1}}{r_{\alpha\beta}}\left[10\left(\frac{m_\beta}{m_\alpha}|\vec{p}_\alpha|^2\right)^2 - 11|\vec{p}_\alpha|^2|\vec{p}_\beta|^2 - 2(\vec{p}_\alpha\cdot\vec{p}_\beta)^2\right. \\
&\left.+10|\vec{p}_\alpha|^2(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta)^2 - 12(\vec{p}_\alpha\cdot\vec{p}_\beta)(\vec{n}_{\alpha\beta}\cdot\vec{p}_\alpha)(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta) - 3(\vec{n}_{\alpha\beta}\cdot\vec{p}_\alpha)^2(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta)^2\right] \\
&+\frac{G^2}{8}\sum_\alpha\sum_{\beta\neq\alpha}\sum_{\gamma\neq\alpha}\frac{1}{r_{\alpha\beta}r_{\alpha\gamma}}\left[18\frac{m_\beta m_\gamma}{m_\alpha}|\vec{p}_\alpha|^2 + 14\frac{m_\alpha m_\gamma}{m_\beta}|\vec{p}_\beta|^2 - 2\frac{m_\alpha m_\gamma}{m_\beta}(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta)^2\right. \\
&-50m_\gamma(\vec{p}_\alpha\cdot\vec{p}_\beta) + 17m_\alpha(\vec{p}_\beta\cdot\vec{p}_\gamma) - 14m_\gamma(\vec{n}_{\alpha\beta}\cdot\vec{p}_\alpha)(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta) \\
&\left.+14m_\alpha(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta)(\vec{n}_{\alpha\beta}\cdot\vec{p}_\gamma) + m_\alpha(\vec{n}_{\alpha\beta}\cdot\vec{n}_{\alpha\gamma})(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta)(\vec{n}_{\alpha\gamma}\cdot\vec{p}_\gamma)\right] \\
&+\frac{G^2}{8}\sum_\alpha\sum_{\beta\neq\alpha}\sum_{\gamma\neq\alpha}\frac{1}{r_{\alpha\beta}^2}\left[2m_\beta(\vec{n}_{\alpha\beta}\cdot\vec{p}_\alpha)(\vec{n}_{\alpha\gamma}\cdot\vec{p}_\gamma) + 2m_\beta(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta)(\vec{n}_{\alpha\gamma}\cdot\vec{p}_\gamma)\right. \\
&\left.+\frac{m_\alpha m_\beta}{m_\gamma}\left(5(\vec{n}_{\alpha\beta}\cdot\vec{n}_{\alpha\gamma})|\vec{p}_\gamma|^2 - (\vec{n}_{\alpha\beta}\cdot\vec{n}_{\alpha\gamma})(\vec{n}_{\alpha\gamma}\cdot\vec{p}_\gamma)^2 - 14(\vec{n}_{\alpha\beta}\cdot\vec{p}_\gamma)(\vec{n}_{\alpha\gamma}\cdot\vec{p}_\gamma)\right)\right] \\
&+\frac{G^2}{4}\sum_\alpha\sum_{\beta\neq\alpha}\frac{m_\alpha}{r_{\alpha\beta}^2}\left[\frac{m_\beta}{m_\alpha}|\vec{p}_\alpha|^2 + \frac{m_\alpha}{m_\beta}|\vec{p}_\beta|^2 - 2(\vec{p}_\alpha\cdot\vec{p}_\beta)\right] \\
&+\frac{G^2}{2}\sum_\alpha\sum_{\beta\neq\alpha}\sum_{\gamma\neq\alpha,\beta}\frac{(n^i_{\alpha\beta}+n^i_{\alpha\gamma})(n^j_{\alpha\beta}+n^j_{\gamma\beta})}{(r_{\alpha\beta}+r_{\beta\gamma}+r_{\gamma\alpha})^2}\left[8m_\beta(p_{\alpha i}p_{\gamma j}) - 16m_\beta(p_{\alpha j}p_{\gamma i})\right. \\
&\left.+3m_\gamma(p_{\alpha i}p_{\beta j}) + 4\frac{m_\alpha m_\beta}{m_\gamma}(p_{\gamma i}p_{\gamma j}) + \frac{m_\beta m_\gamma}{m_\alpha}(p_{\alpha i}p_{\alpha j})\right] \\
&+\frac{G^2}{2}\sum_\alpha\sum_{\beta\neq\alpha}\sum_{\gamma\neq\alpha,\beta}\frac{m_\alpha m_\beta m_\gamma}{(r_{\alpha\beta}+r_{\beta\gamma}+r_{\gamma\alpha})r_{\alpha\beta}}\left[8\frac{\vec{p}_\alpha\cdot\vec{p}_\gamma - (\vec{n}_{\alpha\beta}\cdot\vec{p}_\alpha)(\vec{n}_{\alpha\beta}\cdot\vec{p}_\gamma)}{m_\alpha m_\gamma}\right. \\
&\left.-3\frac{\vec{p}_\alpha\cdot\vec{p}_\beta - (\vec{n}_{\alpha\beta}\cdot\vec{p}_\alpha)(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta)}{m_\alpha m_\beta} - 4\frac{|\vec{p}_\gamma|^2 - (\vec{n}_{\alpha\beta}\cdot\vec{p}_\gamma)^2}{m_\gamma^2} - \frac{|\vec{p}_\alpha|^2 - (\vec{n}_{\alpha\beta}\cdot\vec{p}_\alpha)^2}{m_\alpha^2}\right] \\
&-\frac{G^3}{2}\sum_\alpha\sum_{\beta\neq\alpha}\left(\sum_{\gamma\neq\alpha,\beta}\frac{m_\alpha^2 m_\beta m_\gamma}{r_{\alpha\beta}^2 r_{\beta\gamma}} + \frac{1}{2}\sum_{\gamma\neq\beta}\frac{m_\alpha^2 m_\beta m_\gamma}{r_{\alpha\beta}^2 r_{\beta\gamma}}\right) \\
&-\frac{3G^3}{8}\sum_\alpha\sum_{\beta\neq\alpha}\left(\sum_{\gamma\neq\alpha}\frac{m_\alpha^2 m_\beta m_\gamma}{r_{\alpha\beta}^2 r_{\alpha\gamma}} + \sum_{\gamma\neq\alpha,\beta}\frac{m_\alpha^2 m_\beta m_\gamma}{r_{\alpha\beta}^2 r_{\beta\gamma}}\right) \\
&-\frac{3G^3}{8}\sum_\alpha\sum_{\beta\neq\alpha}\sum_{\gamma\neq\alpha,\beta}\frac{m_\alpha^2 m_\beta m_\gamma}{r_{\alpha\beta}r_{\alpha\gamma}r_{\beta\gamma}} \\
&-\frac{G^3}{64}\sum_\alpha\sum_{\beta\neq\alpha}\sum_{\gamma\neq\alpha,\beta}\frac{m_\alpha^2 m_\beta m_\gamma}{r_{\alpha\beta}r_{\alpha\gamma}^3 r_{\beta\gamma}}\left[18r_{\alpha\gamma}^2 - 60r_{\beta\gamma}^2 - 24r_{\alpha\gamma}(r_{\alpha\beta}+r_{\beta\gamma})\right. \\
&\left.+60\frac{r_{\alpha\gamma}r_{\beta\gamma}^2}{r_{\alpha\beta}} + 56r_{\alpha\beta}r_{\beta\gamma} - 72\frac{r_{\beta\gamma}^3}{r_{\alpha\beta}} + 35\frac{r_{\beta\gamma}^4}{r_{\alpha\beta}^2} + 6r_{\alpha\beta}^2\right] - \frac{G^3}{4}\sum_\alpha\sum_{\beta\neq\alpha}\frac{m_\alpha^2 m_\beta^2}{r_{\alpha\beta}^3}.
\end{aligned}$$

# Part 1: setup

Simulate MBH triplets in galactic nuclei with astrophysically and cosmologically motivated initial conditions

3-body Newtonian dynamics + GR corrections up to 2.5PN
+ interaction with stellar environment, i.e.,

- bulge potential (spherically symmetric)
- Dynamical Friction + stellar hardening (dissipative forces)

# Part 1: exploration through simulations

Systematic survey of the parameter space

$m_1 = 10^5 - 10^{10} \mathrm{M}_\odot$

$\log q_{\rm in} = -1.5, -1.0, -0.5, 0.0$

$\log q_{\rm out} = -1.5, -1.0, -0.5, 0.0$

$e_{\rm in} = 0.2, 0.4, 0.6, 0.8$

$e_{\rm out} = 0.3, 0.6, 0.9$

$\cos \iota =$ 13 values equally spaced in (-1,1)

14976 simulations

| $\log m_1$ $[\mathrm{M}_\odot]$ | % Mergers $m_1$-$m_2$ | $m_1$-$m_3$ | $m_2$-$m_3$ | Total |
|---|---|---|---|---|
| 5 | 16.8 | 0.9 | 0.8 | 18.5(1.6) |
| 6 | 16.2 | 1.4 | 1.0 | 18.5(1.9) |
| 7 | 15.4 | 2.5 | 1.4 | 19.4(4.4) |
| 8 | 14.7 | 4.0 | 2.5 | 21.2(6.3) |
| 9 | 15.2 | 4.1 | 3.2 | 22.5(11.2) |
| 10 | 21.1 | 7.6 | 3.3 | 31.9(12.7) |

We found that ~20-30% of otherwise stalled binaries actually undergo a merger. Main driver is the high eccentricity.

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- Secular 3-body dynamics (Kozai-Lidov mechanism)
- Chaotic 3-body dynamics (ejections, exchanges, strong encounters)

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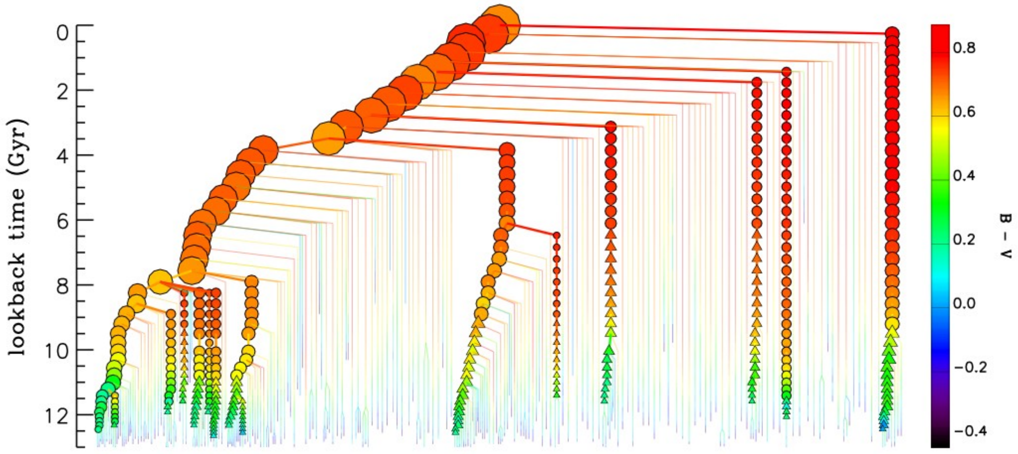
# Part 2: SAM

Systematic survey of the parameter space

Coupling the results to a cosmological merger tree + SAM




Infer the "cosmological weight" of each combination of surveyed parameters

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Systematic survey of the parameter space

↓

Coupling the results to a cosmological merger tree + SAM

SAM of Barausse (2012) and later expansions

+

Ad-hoc recipe to include triple interactions in a cosmological framework

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Ad-hoc recipe to include triple interactions in a cosmological framework

Catalogues of coalescing MBHBs,
keeping track of the mechanism that forced the coalescence

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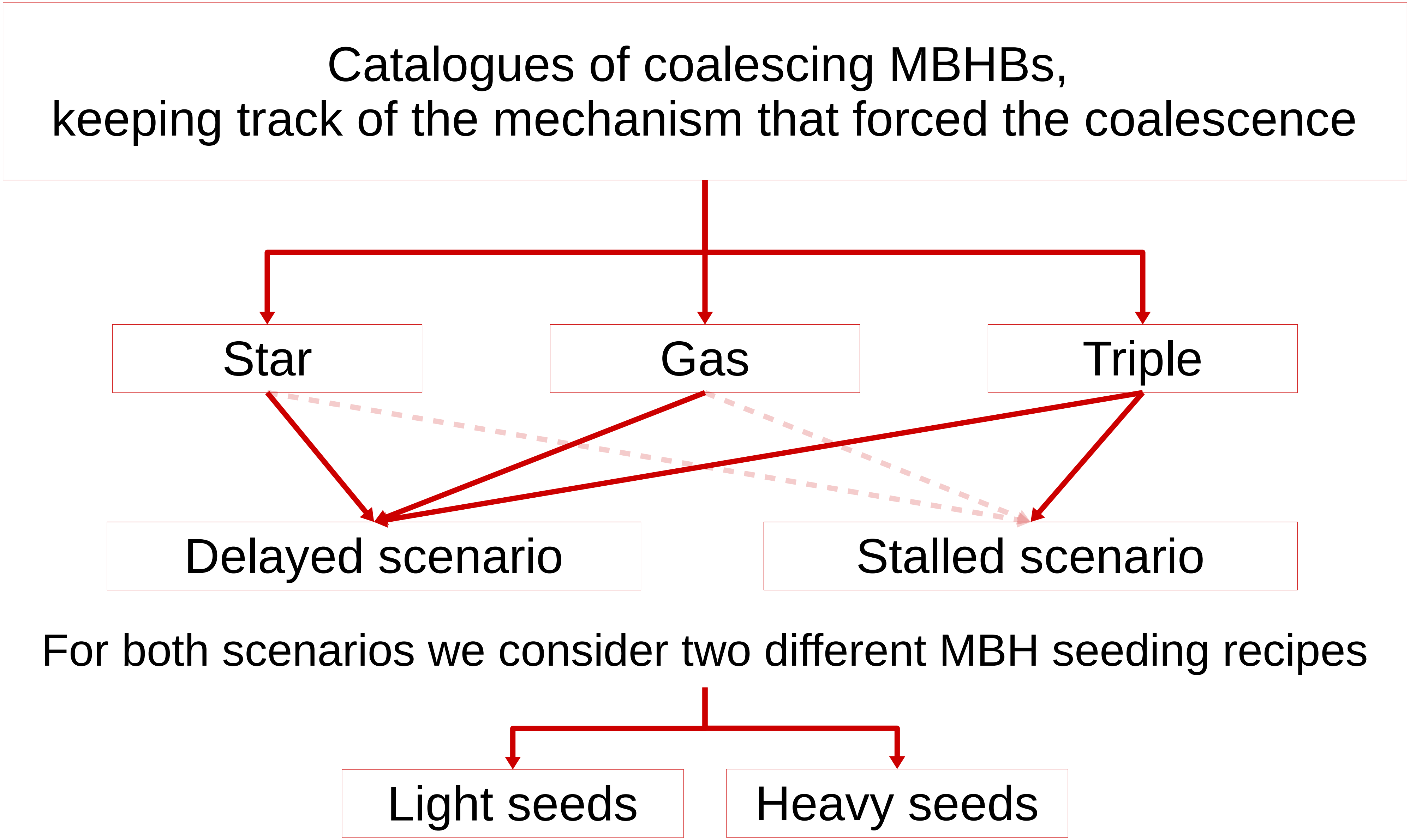
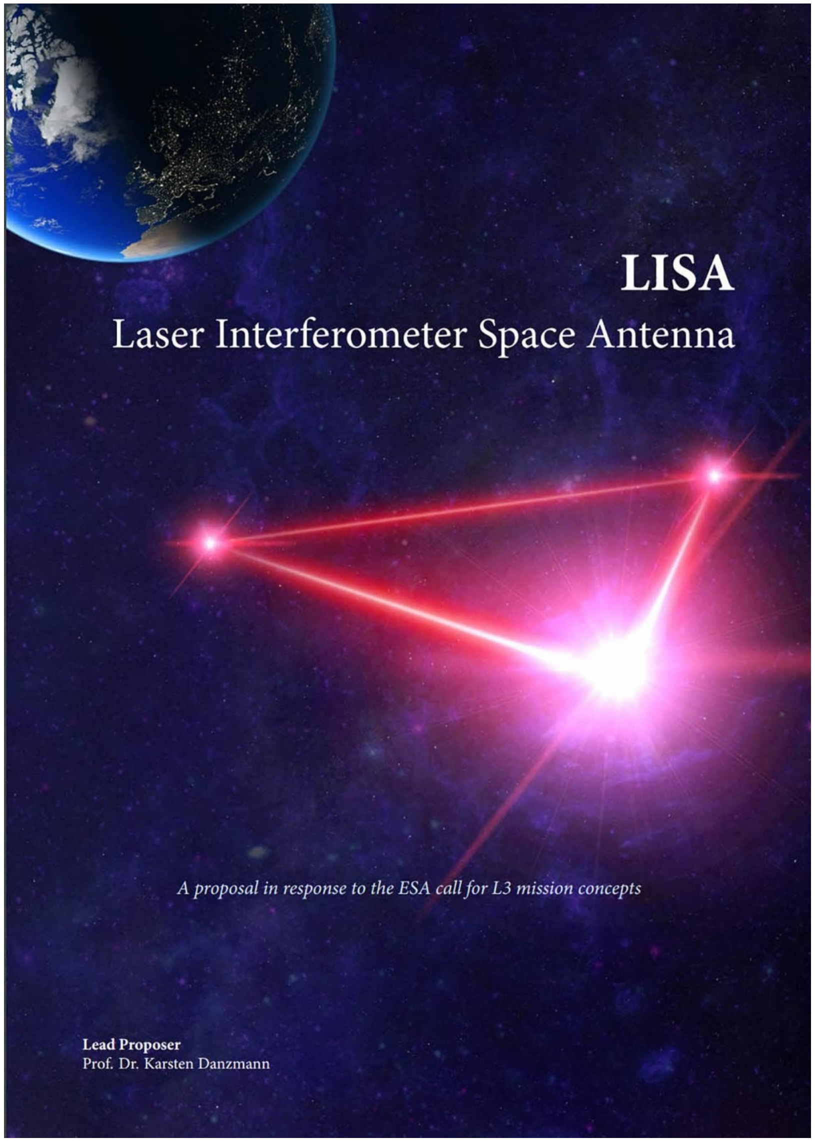
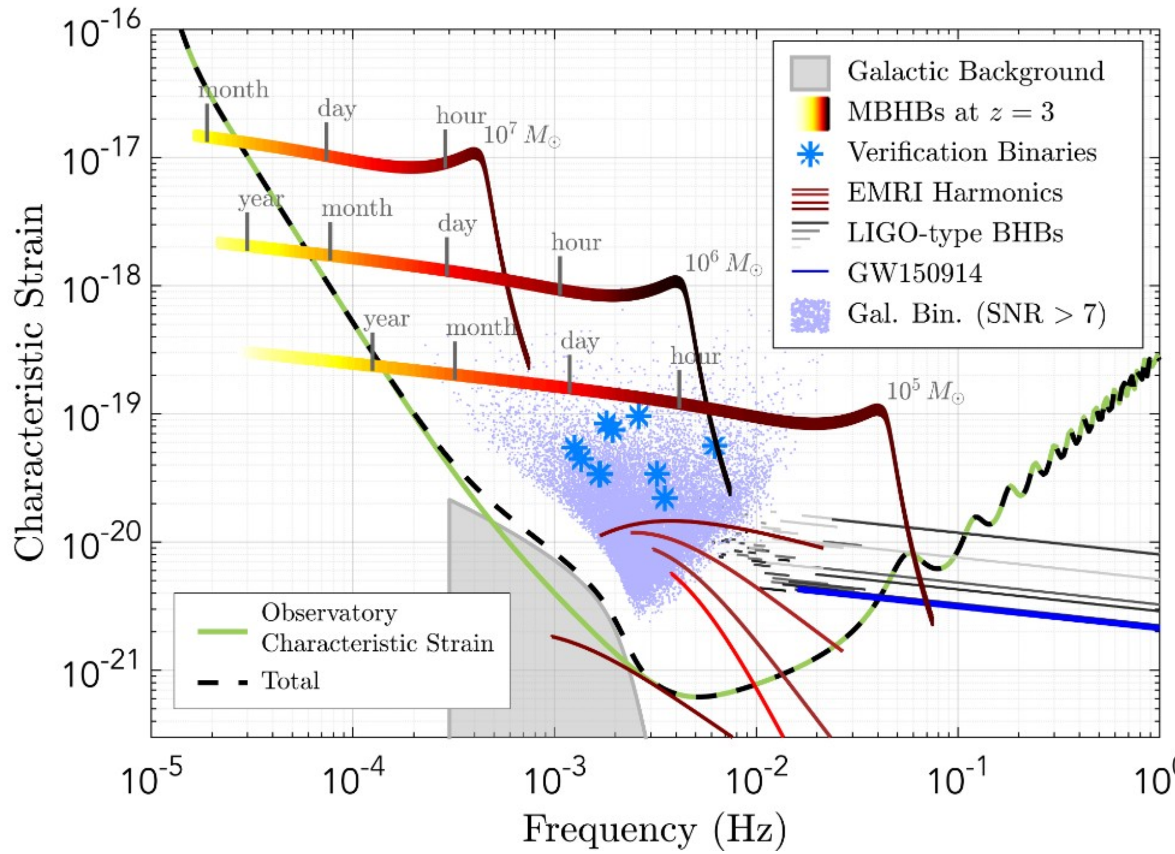



**4 different Universe realisations**

# Implications for LISA



# Implications for LISA: mergers

We are mainly interested in single source detection

From the SAM we can infer the merger rate

$$\frac{\mathrm{d}N}{\mathrm{d}t} \times 4\ \mathrm{yr}$$

| Average number of mergers in 4 yr | | | | |
|---|---|---|---|---|
| | HS-stalled | LS-stalled | HS-delayed | LS-delayed |
| Triple | ~42 | ~86 | ~36 | ~15 |
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How many events are effectively observable with LISA?

Average number of mergers in 4 yr

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|---|---|---|---|---|
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# Implications for LISA: detections

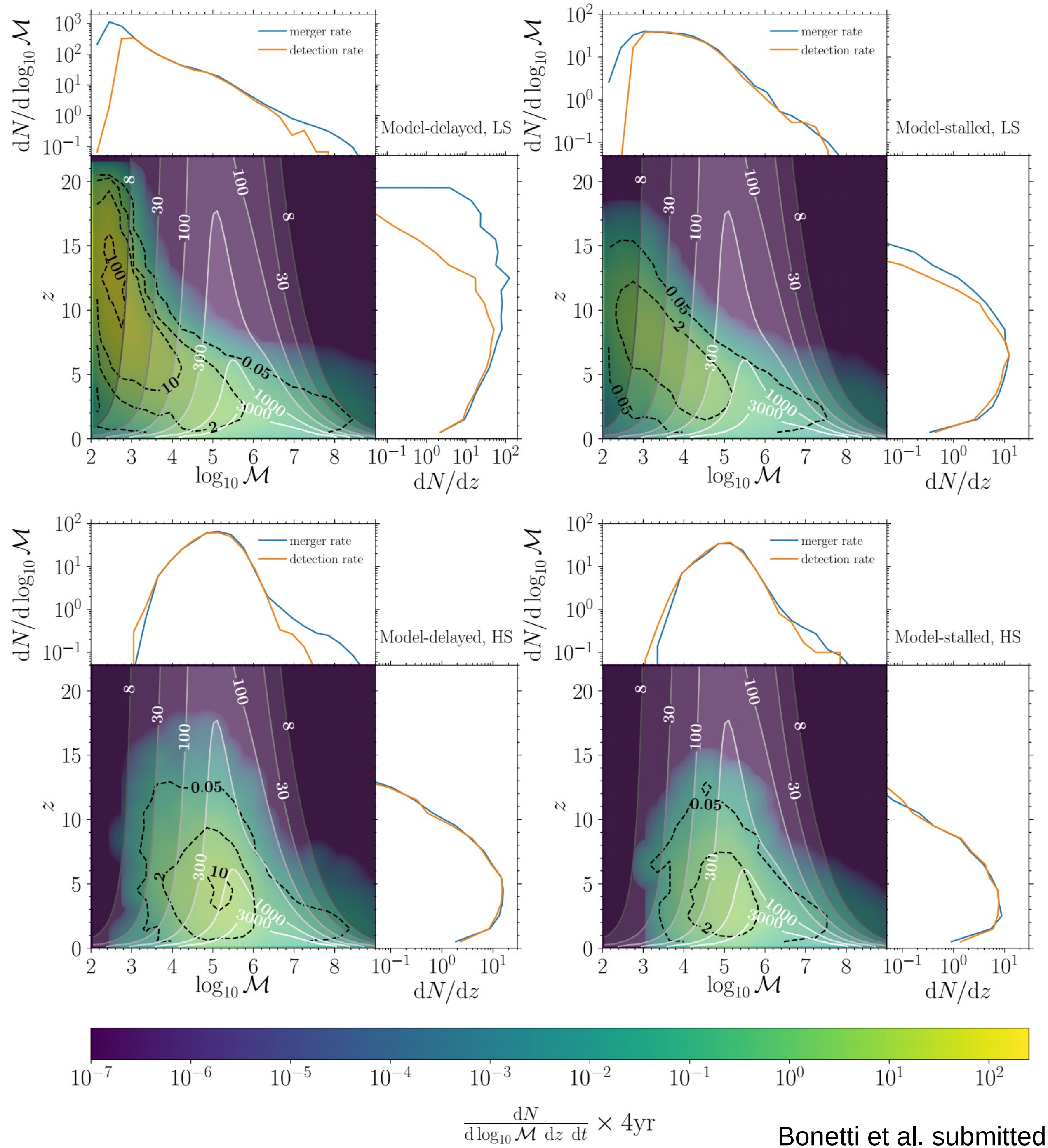
**<u>Qualitative answer</u>**

Note the difference according to seeding model

HS: all mergers are detected

LS: some mergers will be missed

$$\frac{d^3 N}{dz\, d\log_{10}\mathcal{M}\, dt} \times 4\ \mathrm{yr}$$

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# Implications for LISA: detections

**Quantitave answer**

To infer the detection rate we have to switch from a “time-domain” to a “frequency-domain” description

$$\frac{\mathrm{d}^3 N}{\mathrm{d}z\,\mathrm{d}\mathcal{M}\,\mathrm{d}t} \quad \Rightarrow \quad \frac{\mathrm{d}^4 N}{\mathrm{d}z\,\mathrm{d}\mathcal{M}\,\mathrm{d}f\,\mathrm{d}e}$$

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Time spent by binaries in each frequency bin

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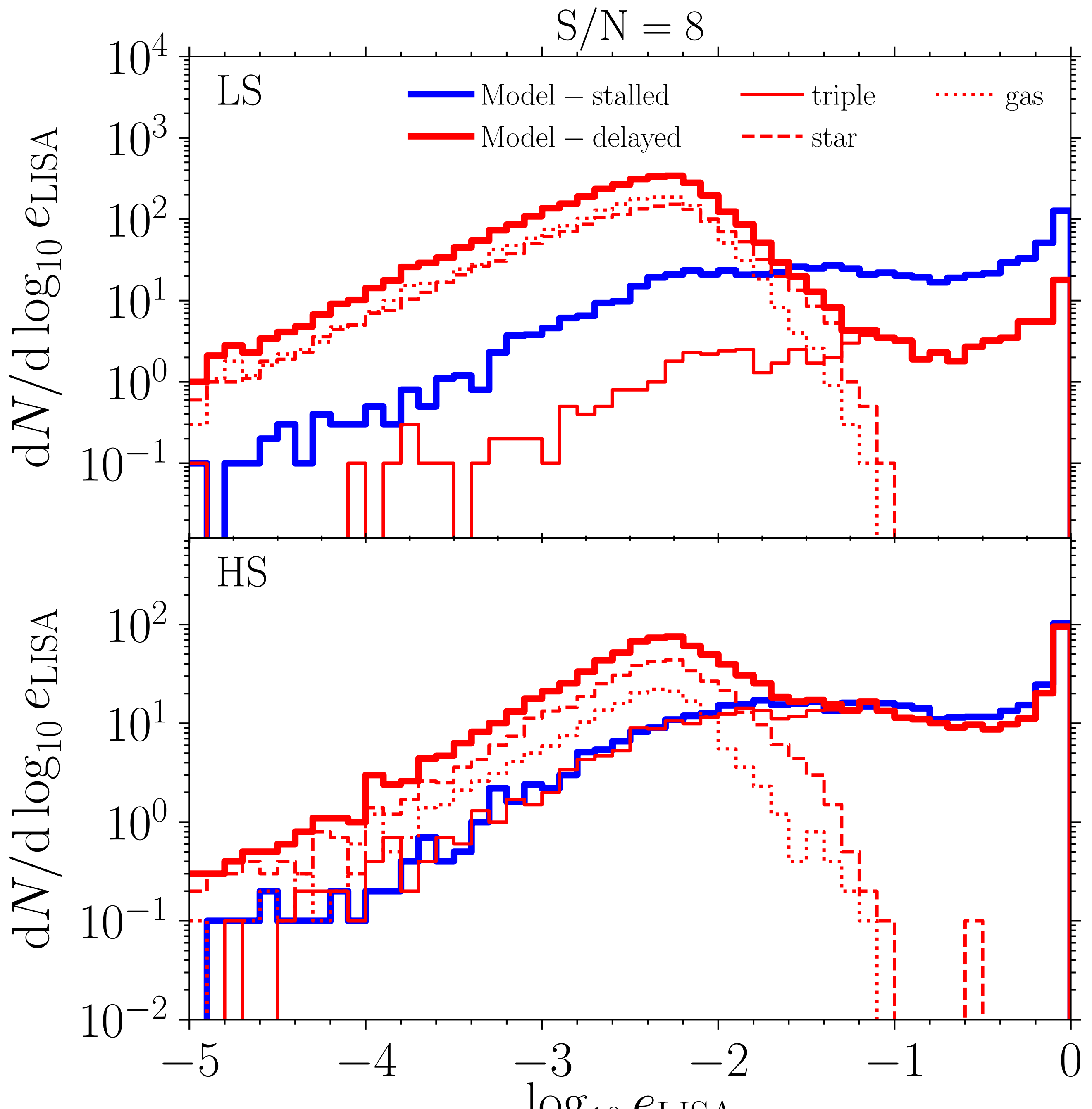
Through a Montecarlo sampling we obtain the total population of binaries emitting GW in the Universe

Time spent by binaries in each frequency bin

# Implications for LISA: eccentricity

**The eccentricity in the LISA band (i.e. when S/N = 8) can be quite high**

The dynamical channel is the only evolutionary path (apart form very extreme case) that can leave such imprint

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# Conclusions & main results

- **Triple interactions** can be a **viable evolutionary channel**

- Even in the **most pessimistic scenario** the merger rate is **not heavily suppressed**

- Triple induced mergers can enter the LISA band with **high eccentricity, requiring specific waveform templates**

**How to fit in the main LISA science target:**
Astrophysically motivated catalogues of GW sources for
  - Setting requirements for accurate waveforms
  - Informing data analysis people about additional possible sources