Modeling gravitational waveforms from extended spinning test bodies in EMRIs

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EMRI of a non-spinning particle

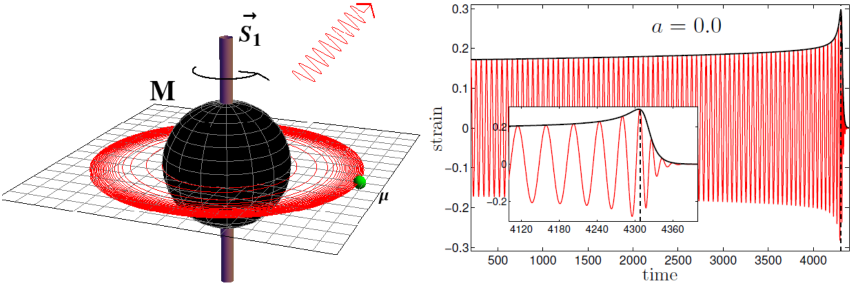

Figure: EMRI of a non-spinning particle on the equatorial plane on the left and on the right the respective waveform (Harms , 2016).

EoM of an extended spinning test body

A pole-dipole (mass-spin) approximation for an extended test body (Mathisson, 1937; Papapetrou, 1951).

The Equations of Motion as revised by Dixon (1970) (MPD):

$$\frac{\mathrm{D}P^{\mu}}{\mathrm{d}\lambda} = -\frac{1}{2} R^{\mu}{}_{\nu\kappa\lambda} U^{\nu} S^{\kappa\lambda} \quad \text{(\textbf{spin-curvature coupling})}$$

$$\frac{\mathrm{D}S^{\alpha\beta}}{\mathrm{d}\lambda} = P^{\alpha} U^{\beta} - U^{\alpha} P^{\beta}$$

- In general $U^{\mu} = \frac{dz^{\mu}}{d\lambda} \nparallel P^{\mu}$.
- λ is an affine parameter.
- MPD eq. need a Spin Supplementary Condition (SSC) $V_{\mu} S^{\mu\nu} = 0$, $V^{\mu} V_{\mu} = -1$.

Hamiltonians reproducing the MPD for certain SSCs

- For the Ohashi (2003); Kyrian & Semerák (2007) SSC independent of λ choice:

$$H_{\rm OKS} = \frac{1}{2m} g^{\mu\nu} P_\mu P_\nu \cong -\frac{m}{2} \,.$$

- For the Tulczyjew (1959); Dixon (1970) SSC independent of λ choice:

$$H_{\rm TD} = \frac{m_U}{2m^2} \left[\left(g^{\mu\nu} - \frac{4 S^{\nu\gamma} R^{\mu}{}_{\gamma\kappa\lambda} S^{\kappa\lambda}}{4m^2 + R_{\chi\eta\omega\xi} S^{\chi\eta} S^{\omega\xi}} \right) P_\mu P_\nu + m^2 \right] \cong 0 \,,$$

- For the Mathisson (1937); Pirani (1956) SSC only for proper time:

$$H_{\rm MP} = \frac{1}{2m_U} \left(g^{\mu\nu} - \frac{1}{S^2} S^{\mu\kappa} S^{\nu}{}_{\kappa} \right) P_\mu P_\nu \cong -\frac{m_U}{2} = \frac{P_\nu U^\nu}{2} \,. \qquad (1)$$

OKS and TD can be expressed in variables with canonical Poisson brackets (Witzany, Steinhoff & LG , 2018).

Different observers seeing different centers of mass

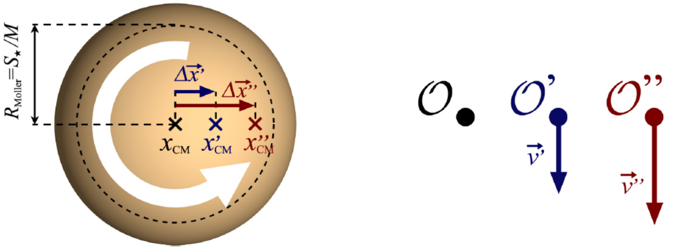

Figure: The centers of mass of an extended spinning body as seen by different observers in the flat spacetime. Picture taken from Costa & Natario (2015).

R_{Moller} the upper bound of separation for different center of masses (centroids) in order to belong to the same physical body (Möller , 1949). Minimal radius of a body not rotating with superluminous speed.

An example of different centroids (flat and Schwarzschild)

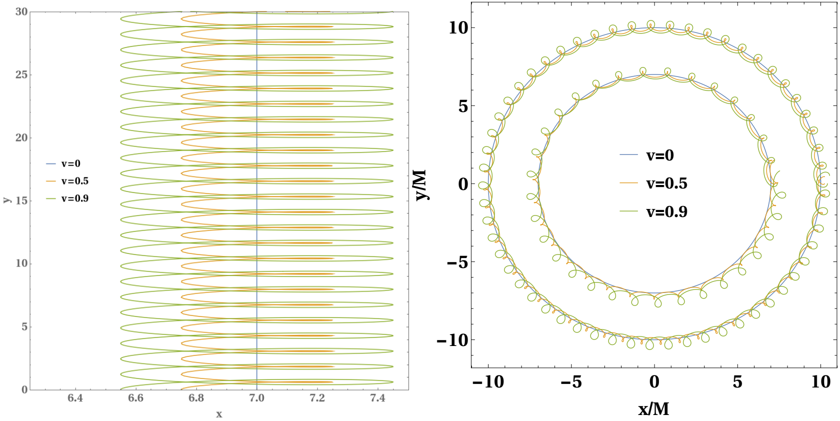

Figure: Orbits belonging to the same body, but for different SSCs (Costa, LG & Semerák , 2018). Left panel: Orbits in the flat spacetime. Right panel: Circular orbits in a Schwarzschild with different centroids for two radial distances.

Dimensionless spin measure

$$S^2 = \frac{1}{2} S^{\mu\nu} S_{\mu\nu}$$

If M the mass of the supermassive black hole and m the mass of the stellar compact object, then:

$$\sigma = \frac{S}{mM} \overset{S=m^2}{=} \frac{m}{M} = \nu \ll 1$$

$S = m^2$ holds for extreme black holes. For details see Hartl (2003).

Gravitational wave fluxes from Circular Equatorial Orbits

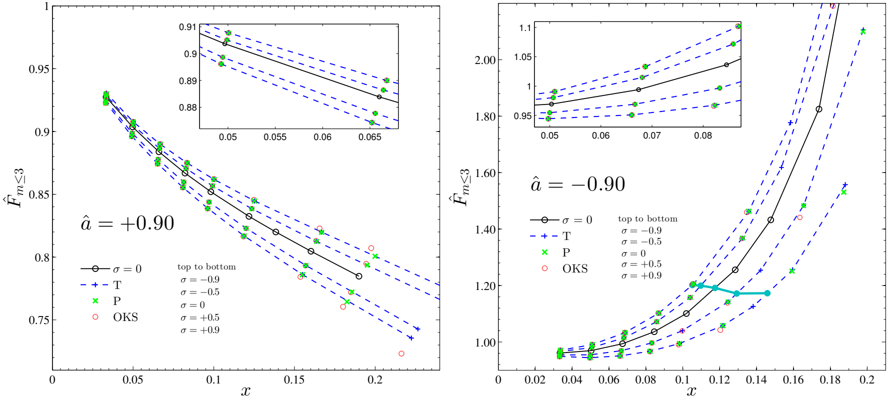

Figure: The gravitational wave fluxes as a function of the frequency parameter. In the left panel the dimensionless Kerr parameter is $\hat{a} = 0.9$, while in the right panel is $\hat{a} = -0.9$ (LG, Harms, Bernuzzi & Nagar , 2017; Harms, LG, Bernuzzi & Nagar , 2016).

Chaos in Gravitational Waves

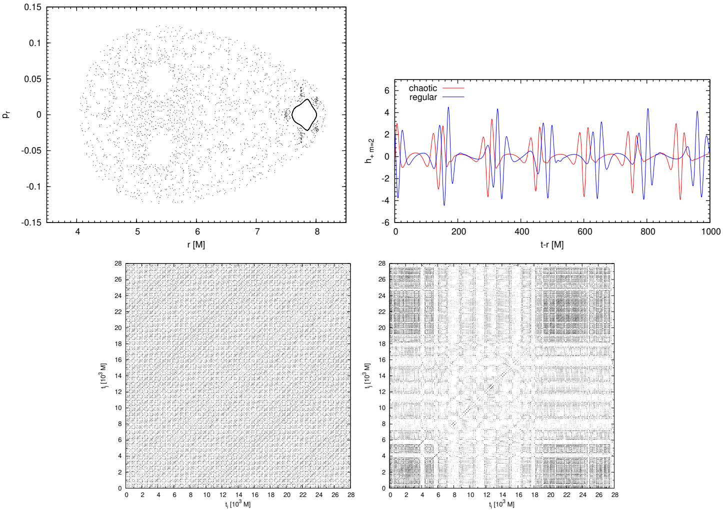

Figure: Recurrence analysis of the strains with multipole $m = 2$ computed with Teukode (Zelenka, LG, et al., in preparation)

Summary

- The SSC choice seems not to be "just" a gauge for the pole-dipole approximation in a curved spacetime. The reason comes from the multipole approximation itself.
- For astrophysical relevant spin values of the inspiraling body, the CEOs under different SSC modeling show no practical discrepancies.
- There are Hamiltonian functions reproducing the MPD equations on-shell, which might lead to a new EOB. For TD and OKS SSCs, we can also have a set of canonical variables.
- Orbital chaos is reflected in GWs. However, is chaos relevant in EMRIs? What is going on with the resonances?

Thank you for your attention!

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Quadrupole force vs. shift spin curvature force

Quadrupole force

$$F_{\mathrm{Q}} \sim m \| R_{\alpha\beta\gamma\delta,\lambda} \| R^2_{\mathrm{Moller}} \sim m M R^2_{\mathrm{Moller}} / r^4$$

Change in the spin-curvature force due centroid shifting

$$\Delta F \sim m M R_{\mathrm{Moller}} / r^3$$

$$\frac{F_{\mathrm{Q}}}{\Delta F} \sim \frac{R_{\mathrm{Moller}}}{r} \quad \left(\sim \frac{S}{mr} \right)$$

(Costa, LG & Semerák , 2018)

Canonical Poisson brackets

$$S^{AB} = S^{\mu\nu} e^A_\mu e^B_\nu \qquad p_\mu = P_\mu + \frac{1}{2} e_{\nu A;\mu} e^\nu_B S^{AB} \,,$$

$$A = S^{12} - \sqrt{(S^{12})^2 + (S^{23})^2 + (S^{31})^2} \,,$$

$$B = \sqrt{(S^{12})^2 + (S^{23})^2 + (S^{31})^2} - S \,,$$

$$\phi = -\arctan\left(\frac{S^{23}}{S^{31}}\right) \,,$$

$$\psi = -\arctan\left(\frac{S^{23}}{S^{31}}\right) - \arccos\left(S^{03}\sqrt{\mathcal{C}}\right) \,,$$

$$\mathcal{C} = \frac{(S^{12})^2 + (S^{23})^2 + (S^{31})^2}{[(S^{13})^2 + (S^{32})^2]\,[(S^{01})^2 + (S^{02})^2 + (S^{03})^2]} \,.$$

The brackets are $\{x^\mu, p_\nu\} = \delta^\mu_\nu$, $\{\phi, A\} = \{\psi, B\} = 1$ and 0 otherwise (Witzany, Steinhoff & LG , 2018)