Backreaction of the infrared modes of scalar fields on de Sitter geometry

Gabriel Moreau, Julien Serreau

APC, AstroParticule et Cosmologie, Université Paris Diderot, CNRS/IN2P3, CEA/Irfu, Observatoire de Paris, Sorbonne Paris Cité, 10, rue Alice Domon et Léonie Duquet, 75205 Paris Cedex 13, France.

Based on arXiv:1809.03969, arXiv:1808.00338

Table of Contents

Quantum fields in curved spacetime

We do a semi-classical treatment with

- a classical background metric
- quantum fields as a content

$\rightarrow$ **No graviton loops**

We study the effects of non trivial backgrounds on the dynamic of quantum fields.

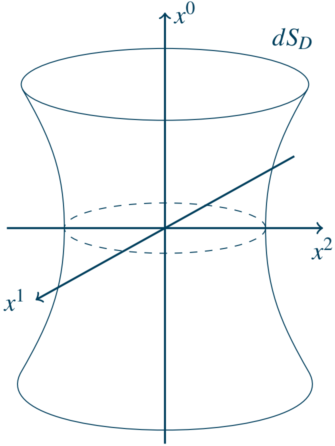
A well known result : Unruh-Hawking radiation for black hole physics

De Sitter spacetime

It is **maximally symmetric**

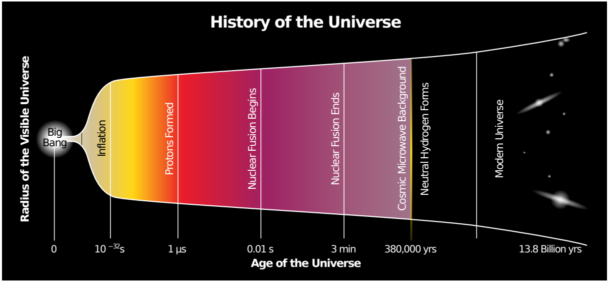
We will consider the **Expanding Poincaré patch**

- $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\vec{X}^2$, $a(t) = e^{Ht}$ with constant H.
- Conformal time, $\mathrm{d}\eta = \frac{\mathrm{d}t}{a(t)}$, $\mathrm{d}s^2 = a^2(\eta)\left(-\mathrm{d}\eta^2 + \mathrm{d}\vec{X}^2\right)$ $\rightarrow$ spatially homogeneous.
- Lemaitre-Painlevé-Gullstrand $\mathrm{d}s^2 = -(1-\vec{x}^2)\mathrm{d}t^2 - 2\vec{x}\cdot\mathrm{d}\vec{x}\mathrm{d}t + \mathrm{d}\vec{x}^2$, $\rightarrow$ stationary.



Why de Sitter ?

It is relevant for **inflation**



Inflation is a postulated phase in the history of the univers which answers several fine-tuning problems of the cosmological standard model

- the horizon problem
- the flatness problem

Particle creation and backreaction

Gravitational effects in de Sitter : **particle creation**

Similar effects when you put a quantum field with a constant background field

- Schwinger effect : pair creation from vacuum because of an electric field $\vec{E}$
- Unruh-Hawking radiation : pair creation because of the horizon of a black hole

In both cases : the creation of pairs **draws energy from the background source**

Stability of de Sitter

For scalar field in dS,

- Large gravitational effects in the infrared (superhorizon scales)
- Infrared modes are amplified
- Interactions cannot be treated perturbatively

A. A. Starobinsky, J. Yokoyama '94 ; C. P. Burgess et al. '10 ; N. C. Tsamis, R. P. Woodard '05

It is interesting to study the **backreaction** of these infrared modes fluctuations to test whether de Sitter space is stable under their effects.

A. M. Polyakov '10, '12 ; E. Mottola '85 ; I. Antoniadis et al. '86 ; R. H. Brandenberger et al. '96 ; Unruh '98

Table of Contents

Free scalar field

With the action

$$S = \int \mathrm{d}^D x \sqrt{-g} \left(\frac{1}{2} \varphi \Box \varphi - \frac{m^2}{2} \varphi^2 \right)$$

We get the Klein Gordon equation $\left(-\Box + m^2\right)\varphi = 0$ where

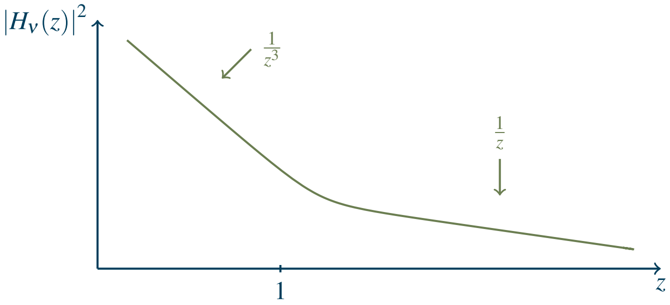
$$\Box = \frac{1}{a(\eta)} \left(-\partial_\eta^2 + \frac{d-1}{\eta} \partial_\eta + \vec{\partial}_X^2 \right)$$

It gives for the mode decomposition of φ

$$\varphi(\eta, \vec{X}) \sim \int \frac{\mathrm{d}^d k}{(2\pi)^d} \left(e^{i\vec{k}\cdot\vec{X}} H_\nu \left(\frac{k}{a(\eta)} \right) a_k + \mathrm{h.c.} \right)$$

with $\nu = \sqrt{\frac{d^2}{4} - \frac{m^2}{H^2}} \approx \frac{d}{2}$

Free scalar field 2



In the case of light scalar fields $m \ll H$,

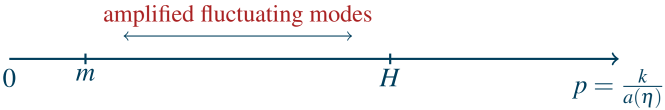


Table of Contents

Non perturbative renormalization group

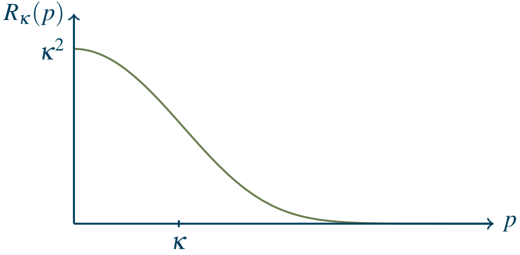
A. Kaya '13 ; J. Serreau '13 ; M. Guilleux, J. Serreau '15

$$e^{i\mathscr{W}[j,g]} = \int \mathscr{D}\hat{\varphi}\, e^{iS[\hat{\varphi},g]+i\int j\hat{\varphi}}, \quad \Gamma[\varphi,g] = \mathscr{W}[j,g] - j\cdot\varphi$$

with $g_{\mu\nu}$ the background metric and S the action for an $O(N)$ theory.

Add a regulator : it defines a **continuum of coarse grained theories**

$$i\Delta S_\kappa[\hat{\varphi},g] = i\int_{x,y} R_\kappa(x,y)\hat{\varphi}(x)\hat{\varphi}(y), \quad \Gamma_\kappa[\varphi,g] = \mathscr{W}_\kappa[j,g] - j\cdot\varphi - \Delta S_\kappa[\varphi,g]$$



$$\Gamma_{\kappa\to\infty} = S$$

$$\Gamma_{\kappa\to 0} = \Gamma$$

Non perturbative renormalization group 2

We want to solve the flow of Γ_κ : it obeys the **Wetterich equation**, which is IR and UV finite

$$\dot{\Gamma}_\kappa = \frac{1}{2}\,\mathrm{tr}\,\dot{R}_\kappa(\Gamma_\kappa^{(2)} + R_\kappa)^{-1}.$$

C. Wetterich ’93

The **physical values** for g and φ are simultaneously determined at each scale κ through

$$\frac{\delta\Gamma_\kappa}{\delta\varphi} = 0, \quad \frac{\delta\Gamma_\kappa}{\delta g^{\mu\nu}} = 0 \quad \text{or} \quad G^\kappa_{\mu\nu} = \langle T^\kappa_{\mu\nu}\rangle$$

We take constant values of φ and de Sitter spacetime : it gives the **flow of the Hubble constant**

Table of Contents

Zero dimensional theory

Under the following assumptions,

- Infrared regime ($\to$ local potential approximation) : $\kappa \ll H_\kappa$
- Small curvature : $m^2_{t/l,\kappa} \ll H^2_\kappa$

The theory flows towards a zero dimensional theory

$$e^{H^{-D}\Omega_{D+1}\mathscr{W}_\kappa(j,h)} = \int \mathrm{d}^N\hat{\varphi}\, e^{-H^{-D}\Omega_{D+1}\left(V_{in}(\hat{\varphi},h)+\frac{\kappa^2}{2}\hat{\varphi}^2-j\cdot\hat{\varphi}\right)}$$

with the initial conditions V_{in} that match the microscopic potential,

- It coincides with the equilibrium probability distribution in the stochastic formalism

  A. A. Starobinsky, J. Yokoyama '94
- It is the **effective theory for the scalar field averaged over a Hubble patch** at constant values of the field

Flow of the physical quantities

Taking as initial conditions

$$V_{in}(\hat{\varphi}, h) = N\left(\alpha - \frac{\beta}{2}H^2\right) + \frac{\lambda}{8N}(\hat{\varphi}_a^2)^2.$$

where $\alpha \propto \Lambda$ and $\beta H^2 \propto R$ the cosmological constant and Einstein-Hilbert term in a de Sitter geometry.

The minimization of the effective action gives

$$\begin{cases} \varphi_\kappa = \langle \hat{\varphi} \rangle \\ H_\kappa^2 = \dfrac{4\alpha}{\beta} + \dfrac{2\kappa^2}{N\beta}\left(\langle \hat{\varphi}^2 \rangle - \varphi^2\right) + \dfrac{\lambda}{2\beta N^2}\langle \hat{\varphi}^4 \rangle \end{cases}$$

The expectation values are to be computed in the zero dimensional theory.

Table of Contents

Large N case

In the large N regime, we can **solve everything analytically** while keeping the main effects.

Define

$$\rho = \frac{\varphi_a^2}{2N} \quad \text{and} \quad NU_\kappa(\rho, H) = V_\kappa(\phi_a, H)$$

We wish to solve

$$\partial_\rho U_\kappa(\rho, H)\Big|_{\rho_\kappa, H_\kappa} = 0 \quad \text{and} \quad \partial_H \frac{U_\kappa(\rho, H)}{H^4}\Big|_{\rho_\kappa, H_\kappa} = 0$$

Large N case 2

We get $\rho_\kappa = 0$,

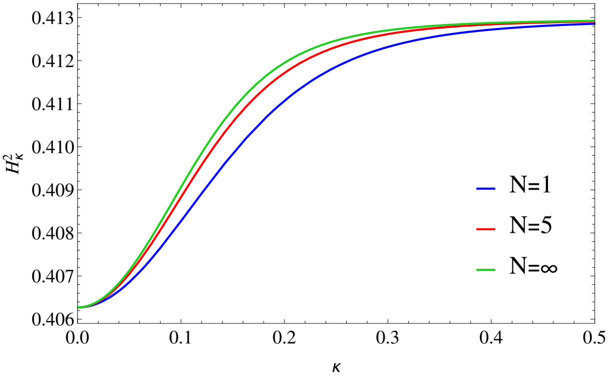
$$m_{t,\kappa}^2 = -\frac{\kappa^2}{2} + \sqrt{\frac{\kappa^4}{4} + \frac{\lambda H^4}{2\Omega}}, \quad H_\kappa^2 = \frac{4\alpha}{\beta} + \frac{H_\kappa^4}{\beta\Omega}\left(1 + \frac{\kappa^2}{m_{t,\kappa}^2 + \kappa^2}\right)$$

We have **finite asymptotic values** which we can compute exactly

$$H_\infty^2 = \frac{\beta\Omega}{4}\left(1 - \sqrt{1 - \frac{32\alpha}{\beta^2\Omega}}\right) \approx \frac{4\alpha}{\beta} + \frac{2}{\beta\Omega}\left(\frac{4\alpha}{\beta}\right)^2 + \cdots$$

$$H_0^2 = \frac{\beta\Omega}{2}\left(1 - \sqrt{1 - \frac{16\alpha}{\beta^2\Omega}}\right) \approx \frac{4\alpha}{\beta} + \frac{1}{\beta\Omega}\left(\frac{4\alpha}{\beta}\right)^2 + \cdots$$

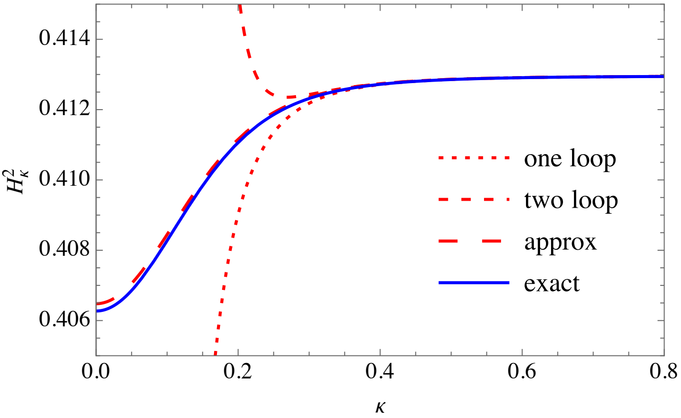
Large N case 3



- The superhorizon modes of the massless scalar fields are greatly enhanced, drawing energy from the gravitational field
- The dynamical generation of a mass screens this effect, leading to a finite renormalization of the Hubble constant

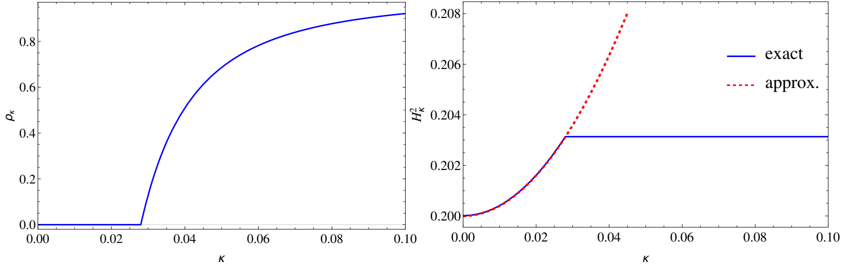
Perturbation theory

Expansion parameter is $\frac{\lambda H^4}{\kappa^4}$: perturbation theory breaks down when κ decreases



Broken phase

$$\rho_\kappa = -\frac{H_\kappa^4}{2\Omega\kappa^2} - \frac{m^2+\zeta H_\kappa^2}{\lambda}, \quad 4\alpha - \beta H_\kappa^2 - \frac{2m^2\left(m^2+\zeta H_\kappa^2\right)}{\lambda} + \frac{2H_\kappa^4}{\Omega} = 0$$



- the symmetry is always restored
- **the Goldstone bosons do not renormalize** h_κ**!**

Broken phase 2

The backreaction from the longitudinal mode is more and more important for decreasing N

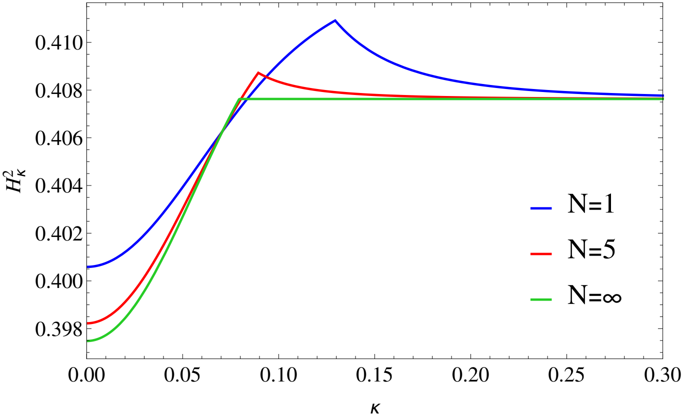


Table of Contents

Conclusion

The backreaction we studied is influenced by several phenomena :

- The mass generation screens the renormalization of the Hubble parameter
- Non minimal coupling between the scalar fields and gravitational field has a non trivial effect on the flow
- Goldstone modes do not contribute

A full non perturbative treatment is needed as the perturbative approach breaks down

Perspectives :

- Work in a more general FLRW spacetime