Introduction to Curci-Ferrari: Results and open questions

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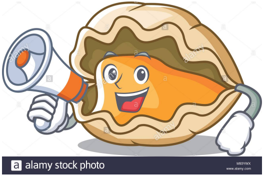
Outline

QCD before CF

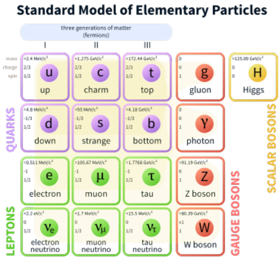
QCD with CF

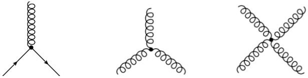
QCD after CF? - conclusions

QCD before CF



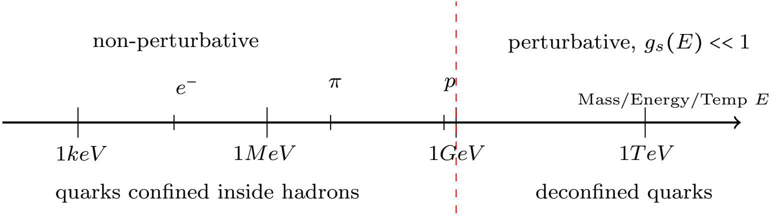
What is QCD?



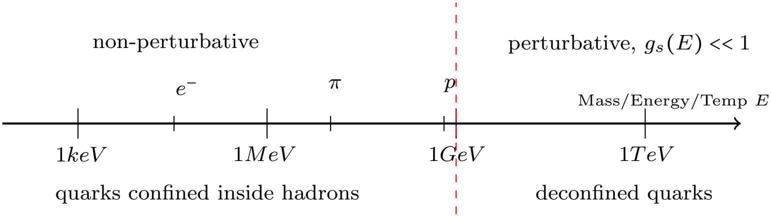


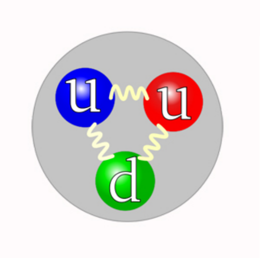
$g_s(E)$ $g_s(E)$ $g_s^2(E)$

A celebrated property:
Asymptotic freedom
$g_s(E) << 1$ for $E >> 1\text{GeV}$



What is QCD?


non-perturbative
perturbative, $g_s(E) << 1$
e^-
π
p
Mass/Energy/Temp E
$1keV$
$1MeV$
$1GeV$
$1TeV$
quarks confined inside hadrons
deconfined quarks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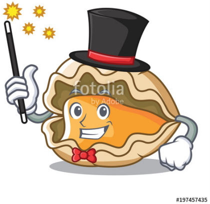
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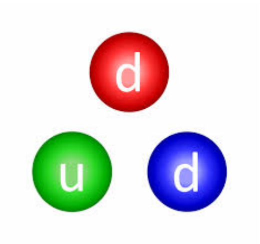
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QCD with CF

Gauge Fixing

$$S = \int_x \{\frac{1}{4}(F^a_{\mu\nu})^2 + \bar{\psi}(\not{D} + M + \mu\gamma_0)\psi\}$$

For computations in practice one has to gauge fix!

<u>QCD</u>:

$$(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})^{-1} \quad ??$$

because

$$(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})k_\nu = 0$$

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QCD:

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Picture:

$$\begin{pmatrix} 1 & 0 \\ -2 & 0 \end{pmatrix}^{-1} \quad ?? \quad \text{as } M\,\underline{v} = 0$$

Hence

$$\left[\begin{pmatrix} 1 & 0 \\ -2 & 0 \end{pmatrix} + \xi \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]^{-1}$$

Physics does not depend on the choice of ξ....(hopefully)...

Landau gauge fixing

Fix ξ s.t.

QCD:
covariant gauge:

$$\partial_\mu A_\mu = \omega$$

Landau gauge:

$$\boxed{\partial_\mu A_\mu = 0}$$

→ leads to Gribov copies due to non-complete gauge fixing.
Ie there are several configurations that are "physically degenerate".

Unsolved problem since 1978. [Singer]

→ Model via an effective theory!

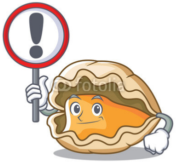
Picture:
If

$$\partial_t A = \omega \quad \text{or} \quad \partial_t A = 0,$$

then also

$$\partial_t (A + c) = \omega \quad \text{or} \quad \partial_t (A + c) = 0$$

Therefore A and $A + c$ are "physically degenerate".

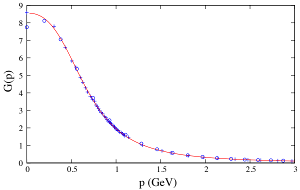


Curci-Ferrari and gluon mass term

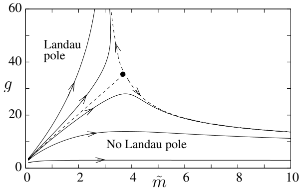
$$S = \int_x \left\{ \frac{1}{4}(F^a_{\mu\nu})^2 + \bar\psi(\not{D} + M + \mu\gamma_0)\psi \right\} + S_{FP} + \int_x \left\{ \frac{1}{2} m^2 (A^a_\mu)^2 \right\}$$

This gluon mass term can be motivated in several ways

- phenomenologically from lattice data of the Landau gauge gluon propagator saturating in the IR
- Residual ambiguity after non-complete gauge-fixing in Fadeev-Popov procedure due to presence of Gribov copies

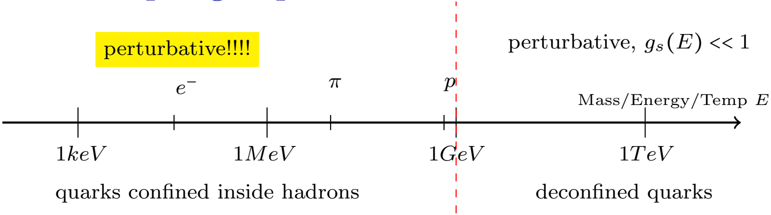


one-loop gluon propagator against lattice data,
from [Tissier, Wschebor (2011)]
[Bogolubsky et al. (2009), Dudal, Oliveira, Vandersickel (2010)]

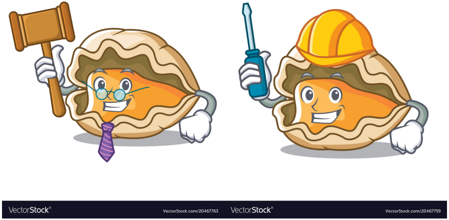


YM one-loop RG flow,
from [Serreau, Tissier (2012)]

Weak coupling expansion

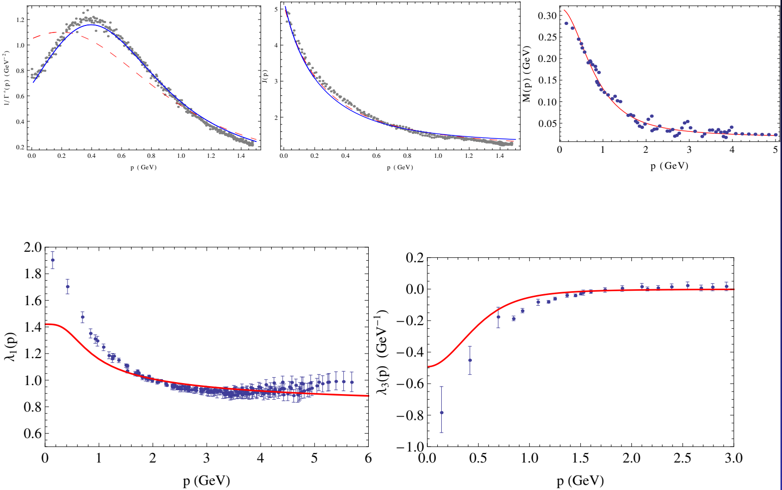


$$\mathcal{O} = \mathcal{O}_0 + g\,\mathcal{O}_1 + g^2\,\mathcal{O}_2 + g^3\,\mathcal{O}_3 + \dots$$

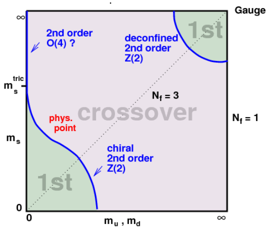


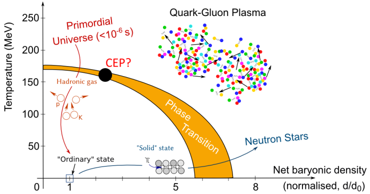
Some YM & QCD Correlation functions

[M. Pelaez, M. Tissier, N. Wschebor]



QCD Phase Diagram



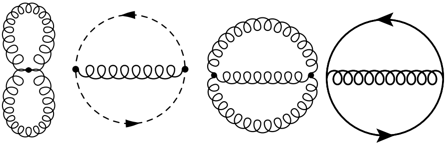


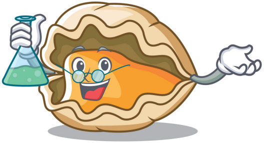
Several approaches on the market:

- Lattice QCD [de Forcrand, Philipsen, Rodriguez-Quintero, Mendes, ...]
- Dyson Schwinger Equations [Alkofer, Fischer, Huber, ...]
- Functional Renormalization Group [Pawlowski, Mitter, Schaefer...]
- Variational Approach [Reinhardt, Quandt, ...]
- Gribov-Zwanziger Action [Dudal, Oliveira, Zwanziger...]
- Matrix-, QM-, NJL-Model,... [Pisarski, Dumitru, Schaffner-B., Stiele, ...]
- Curci-Ferrari Model [Reinosa, Serreau, Tissier, Wschebor, ...]

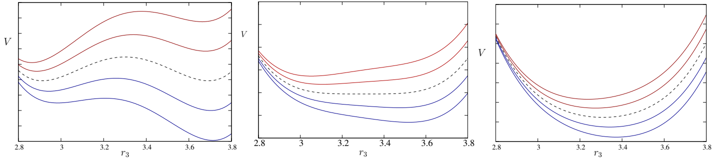
Loop Expansion

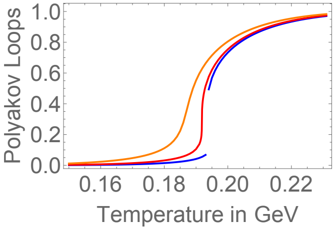
$$
\begin{aligned}
V(r_3, r_8) \quad = \quad & -\mathrm{Tr}\,\mathrm{Ln}\left(\not{\partial} + M + \mu\gamma_0 - ig\gamma_0 \bar{A}^k t^k\right) \\
& + \frac{3}{2}\mathrm{Tr}\,\mathrm{Ln}\left(\bar{D}^2 + m^2\right) - \frac{1}{2}\mathrm{Tr}\,\mathrm{Ln}\left(\bar{D}^2\right) \\
& +
\end{aligned}
$$



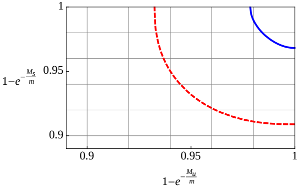


Qualitative Results





Quantitative Results 1



$$R_{N_f} \equiv \frac{M_c(N_f)}{T_c(N_f)}$$

$$\mathcal{O}(1): \quad M_{\text{bare}} = M_{\text{ren.}}$$
$$\mathcal{O}(g^2): \quad M_{\text{bare}} = Z_M\, M_{\text{ren.}} + C_M$$

$\longrightarrow$ hard to compare between different approaches!

However, Z_M, C_M are independent of N_f at $\mathcal{O}(g^2)$, and observing

$$\frac{T_c(N_f=3) - T_c(N_f=1)}{T_c(N_f=1)} \approx 0.2\%$$

allows for:

$$\overbrace{R_{N'_f}/R_{N_f} \approx M_c(N'_f)/M_c(N_f)}^{\text{if } C_M = 0} \qquad \overbrace{Y_{N_f} \equiv \frac{R_{N_f} - R_1}{R_2 - R_1}}^{\text{if } C_M \neq 0}$$

is scheme indep. & comparable to other approaches up to higher order corrections.

Quantitative Results 2

$\mu = 0$	R_1	R_2	R_3	R_2/R_1	R_3/R_1	Y_3
Matrix [1]	8.04	8.85	9.33	1.10	1.16	1.59
GZ1 [2]	7.09	7.92	8.40	1.12	1.19	1.58
GZ2 [2]	9.45	10.25	10.72	1.08	1.13	1.58
CF 1-loop [3]	6.74	7.59	8.07	1.13	1.20	1.58
CF 2-loop [2]	7.53	8.40	8.90	1.12	1.18	1.57
Lattice [4]	7.23	7.92	8.33	1.10	1.15	1.59
DSE [5]	1.42	1.83	2.04	1.29	1.43	1.51

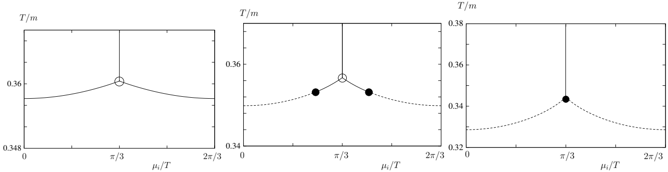
$\longrightarrow$ The overall good agreement seems to suggest that the underlying dynamics is well-described within (Curci-Ferrari) perturbation theory.

[1] Kashiwa, Pisarski, Skokov (2012) [2] JM, Reinosa, Serreau (2017+18)

[3] Reinosa, Serreau, Tissier (2015) [4] Fromm, Langelage, Lottini, Philipsen (2012)

[5] Fischer, Luecker, Pawlowski (2015)

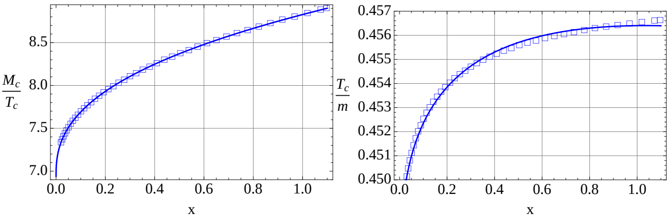
Imaginary chemical potential $\mu = i\mu_i$



The vicinity of the tricritical point is approximately described by the mean field scaling behavior

$$\frac{M_c(\mu_i)}{T_c(\mu_i)} = \frac{M_{\text{tric.}}}{T_{\text{tric.}}} + K\left[\left(\frac{\pi}{3}\right)^2 - \left(\frac{\mu_i}{T_c}\right)^2\right]^{\frac{2}{5}}$$

[de Forcrand, Philipsen (2010); Fischer, Luecker, Pawlowski (2015)]



$$x \equiv (\pi/3)^2 + (\mu/T_c)^2 = (\pi/3)^2 - (\mu_i/T_c)^2$$

Imaginary chemical potential $\mu = i\mu_i$

$\mu = i\pi T/3$	R_1	R_2	R_3	R_2/R_1	R_3/R_1	Y_3
Matrix [1]	5.00	5.90	6.40	1.18	1.28	1.56
GZ1 [2]	5.02	5.92	6.43	1.18	1.28	1.57
GZ2 [2]	7.51	8.34	8.82	1.11	1.17	1.58
CF 1-loop [3]	4.74	5.63	6.15	1.19	1.30	1.57
CF 2-loop [2]	5.47	6.41	6.94	1.17	1.27	1.57
Lattice [4]	5.56	6.25	6.66	1.12	1.20	1.59
DSE [5]	0.41	0.85	1.11	2.07	2.70	1.59

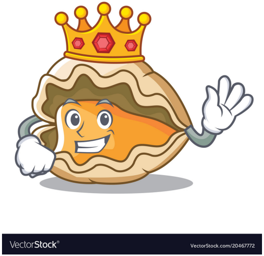
$\longrightarrow$ The Y_3 values are in overall very good agreement between all cases, one loop models and higher order ones.

[1] Kashiwa, Pisarski, Skokov (2012) [2] JM, Reinosa, Serreau (2017+18)

[3] Reinosa, Serreau, Tissier (2015) [4] Fromm, Langelage, Lottini, Philipsen (2012)

[5] Fischer, Luecker, Pawlowski (2015)

QCD after CF - conclusions



Conclusion

Done:

Implement Curci-Ferrari as an alternative method to non-pert. approaches in IR QCD

- Correlation functions at first orders
- chiral symmetry breaking
- robust perturbative description of the heavy quark phase diagram
- ...

Outlook:

We will keep pushing the model to see where it takes us!!

- chiral phase transition
- transport coefficients
- Off-equilibrium thermodynamics
- ...