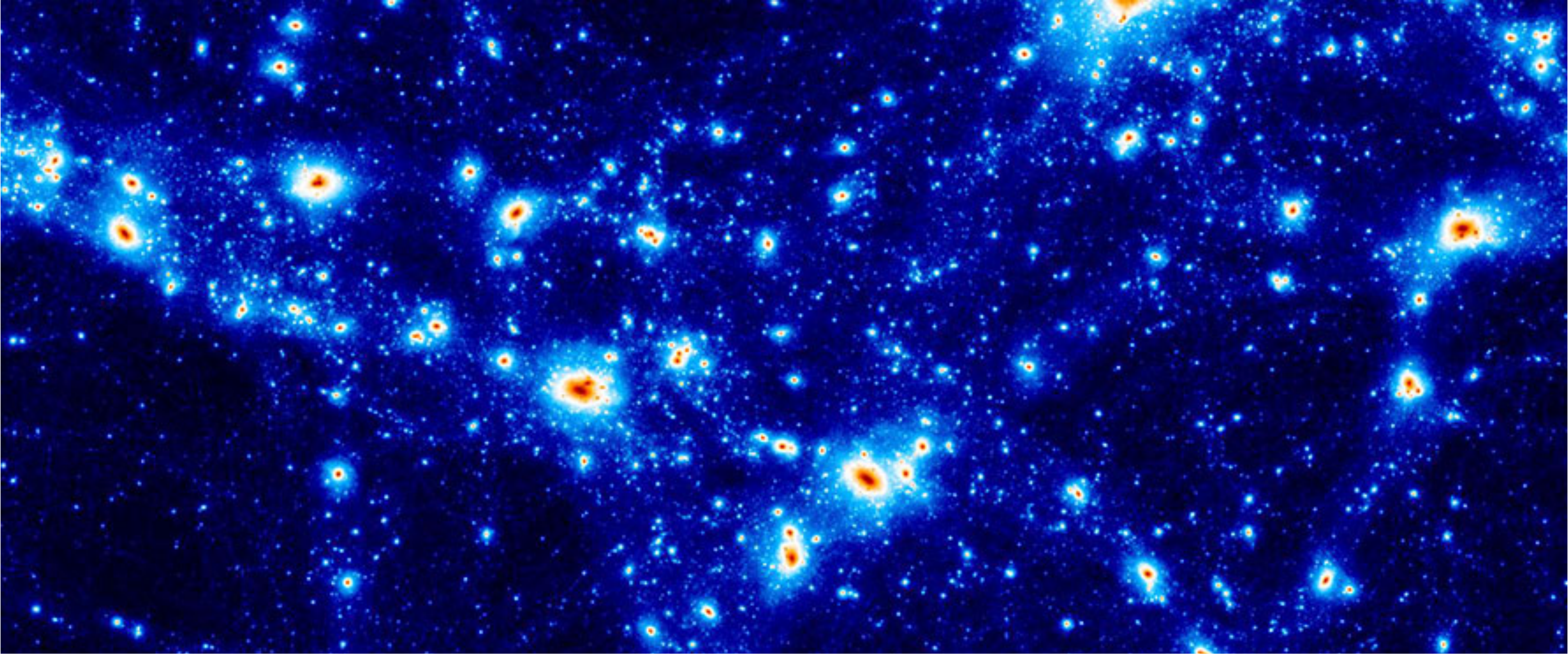
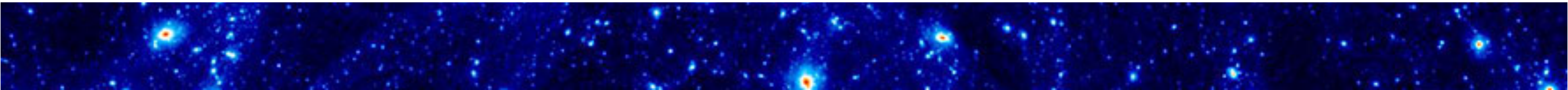
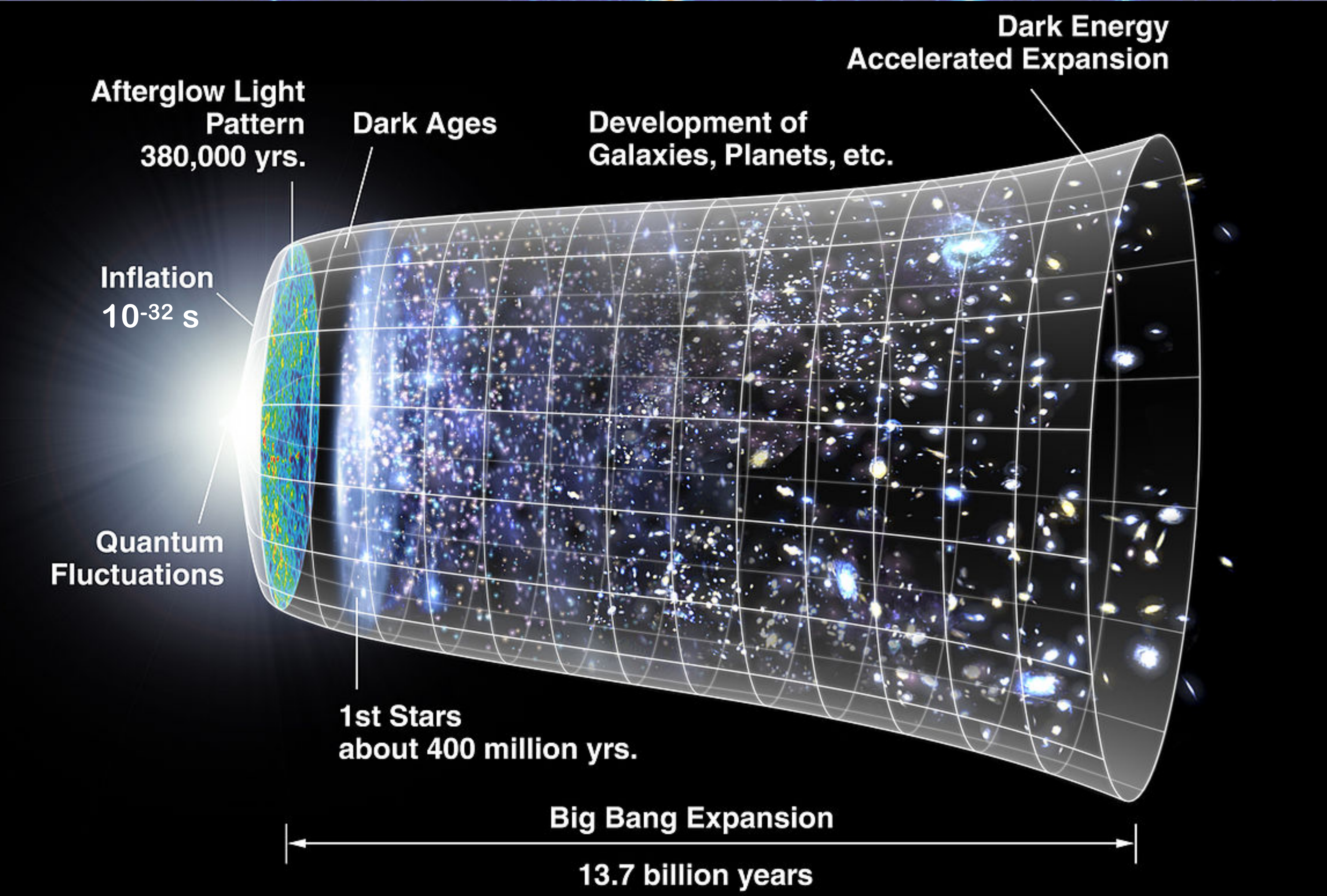


INTRODUCTION TO COSMOLOGY

针对两个无穷的物理研究 2018



A BRIEF HISTORY OF THE UNIVERS

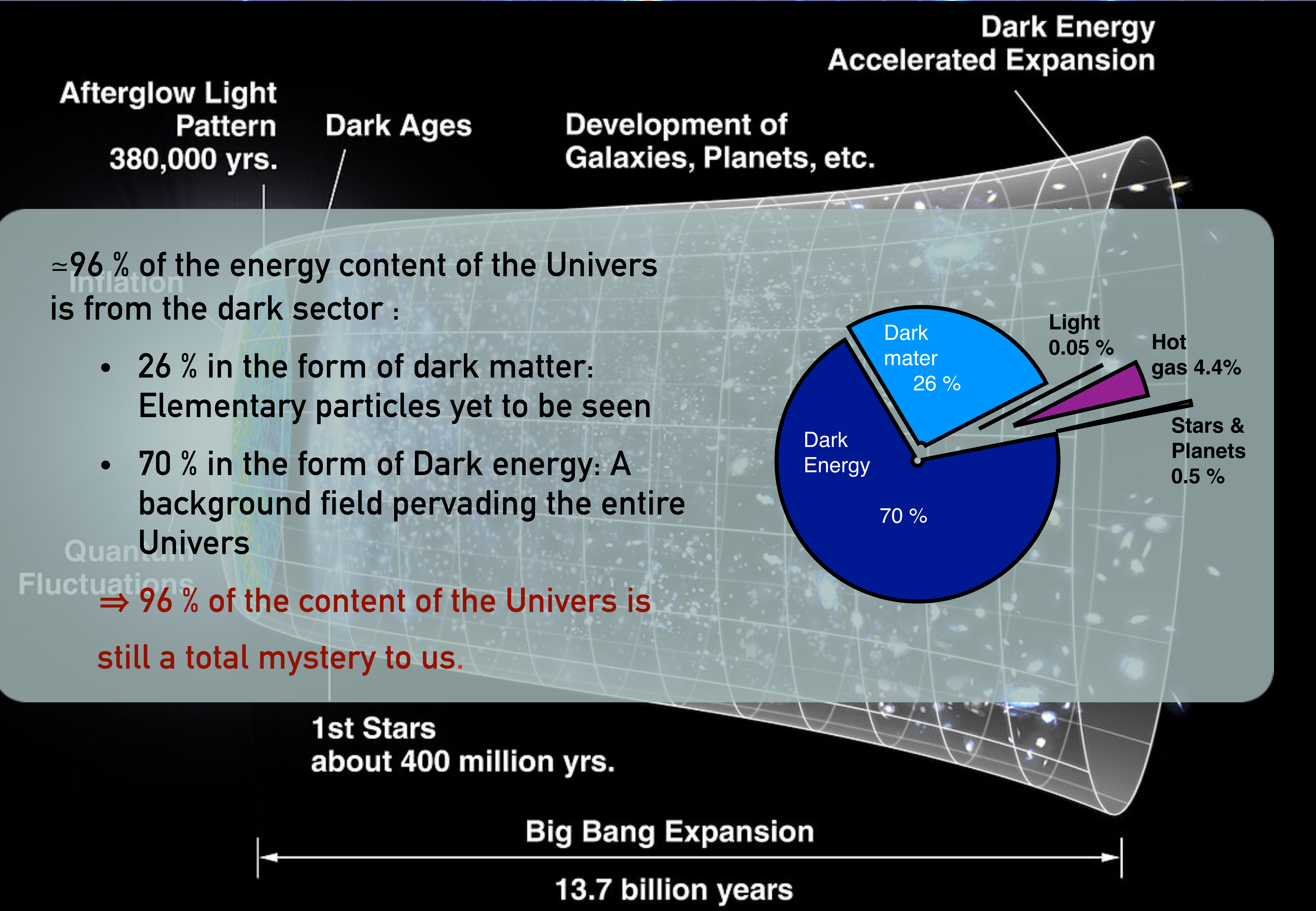


A BRIEF HISTORY OF THE UNIVERS

≃96 % of the energy content of the Univers is from the dark sector :

- 26 % in the form of dark matter: Elementary particles yet to be seen
- 70 % in the form of Dark energy: A background field pervading the entire Univers

⇒ 96 % of the content of the Univers is still a total mystery to us.



ORDER OF MAGNITUDE

Cosmology also goes down to the Planck scale …

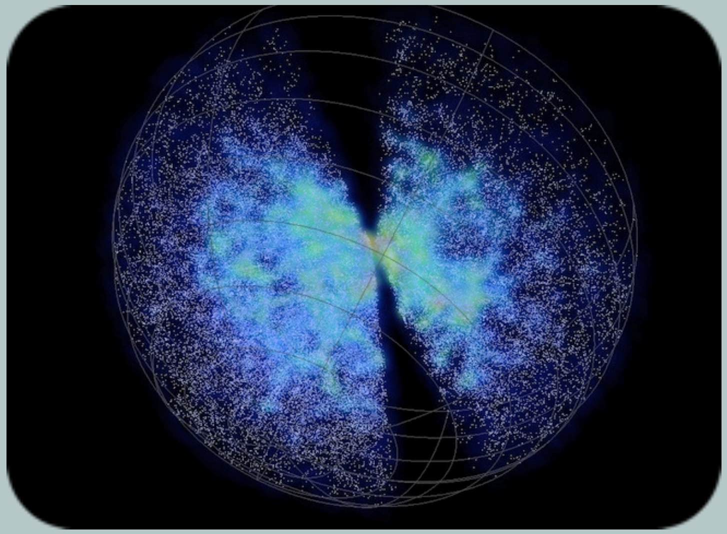
… but for now we are more interested in large scale !

Solar system:

- size: Billion of km (10^9 km)
- 1 Astronomical Unit (AU): 1.5×10^8 km
- Voyager reaches 128 AU

Galaxies:

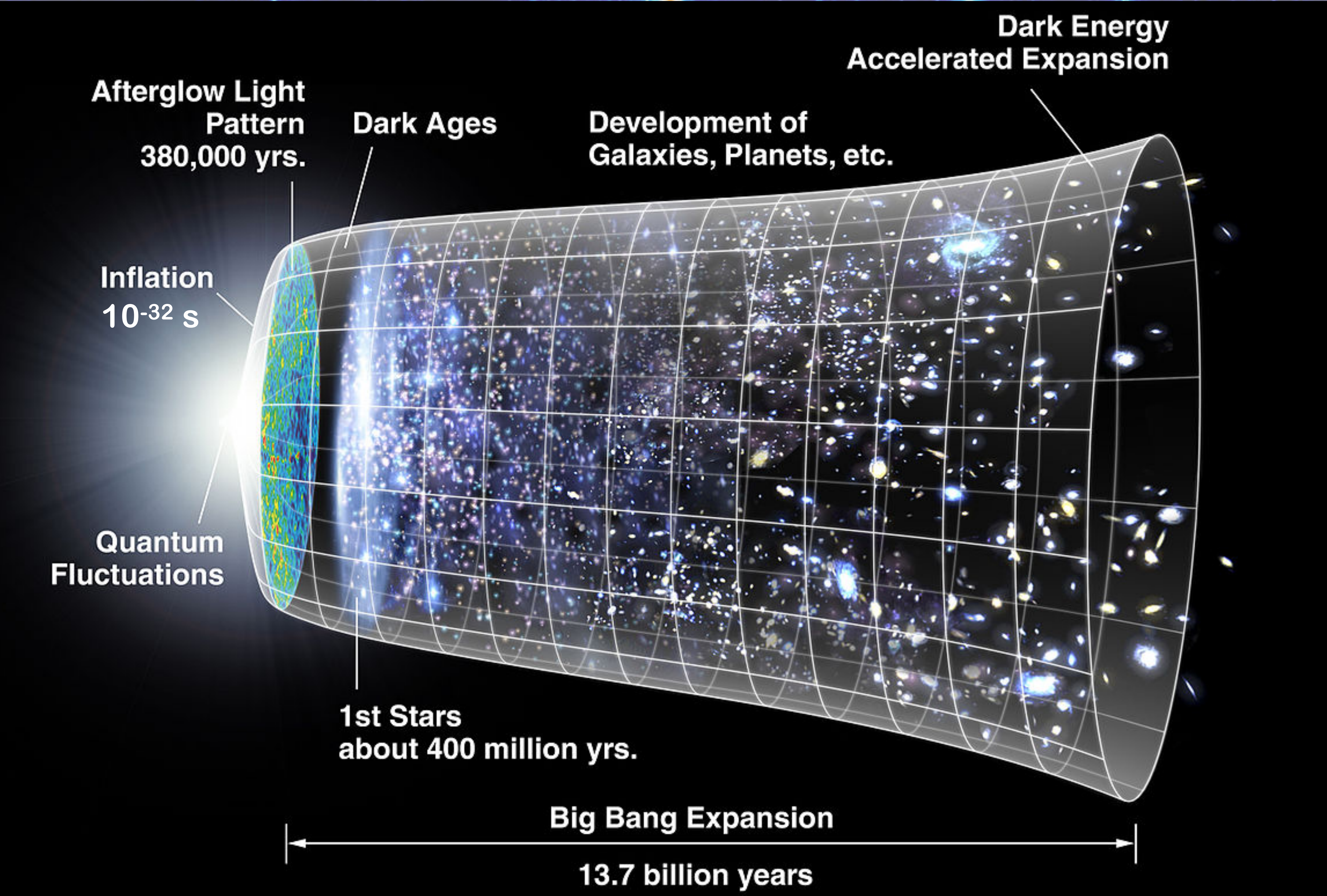
- size: Few 10 of kpc
- 1 parsec (pc) ≃ 3 lyrs ≃ 3×10^{13} km
- Contains billions of stars



Observable Univers:

- size: 10 Gpc ≃ 10^{23} km
- Contains ≃ 10^{11} galaxies

HOW TO DESCRIBE THE UNIVERS

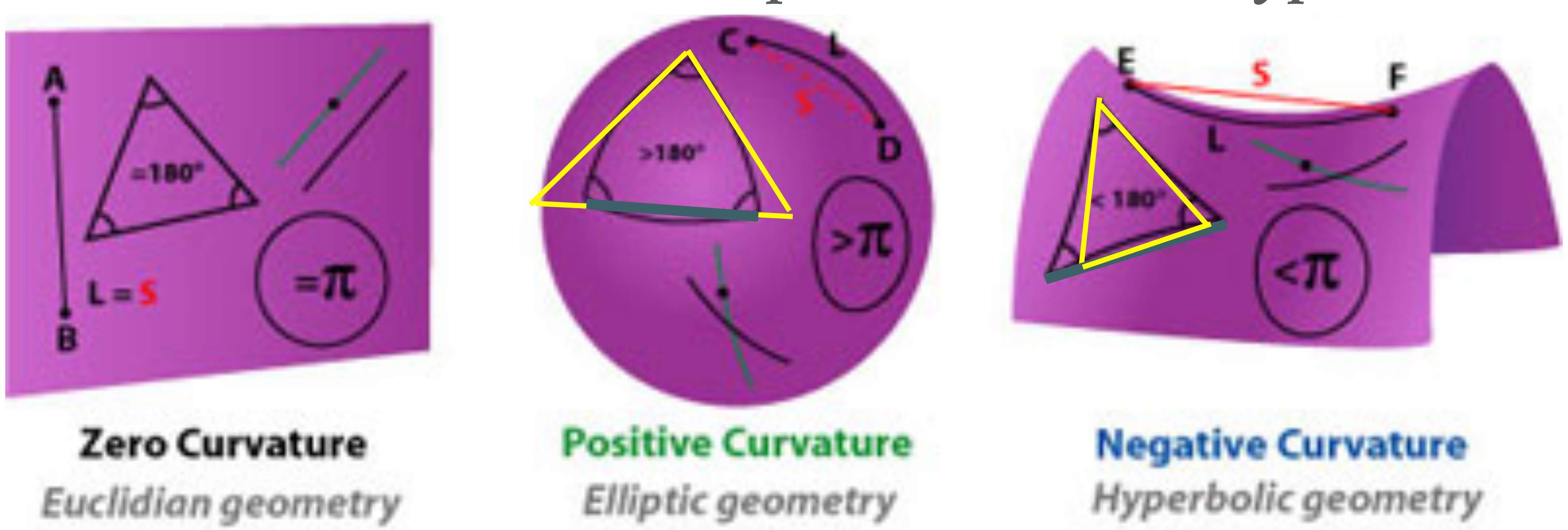


HOW TO DESCRIBE THE UNIVERS

The FLRW metric:

$$ds^2 = c^2 dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega \right)$$

k=0 : Plan *k>0 : Spheric* *k<0 : Hyperbolic*



HOW TO DESCRIBE THE UNIVERS

The FLRW metric:

$$ds^2 = c^2dt^2 - a(t)^2\left[d\chi^2 + S_k^2(\chi)d\Omega\right]$$

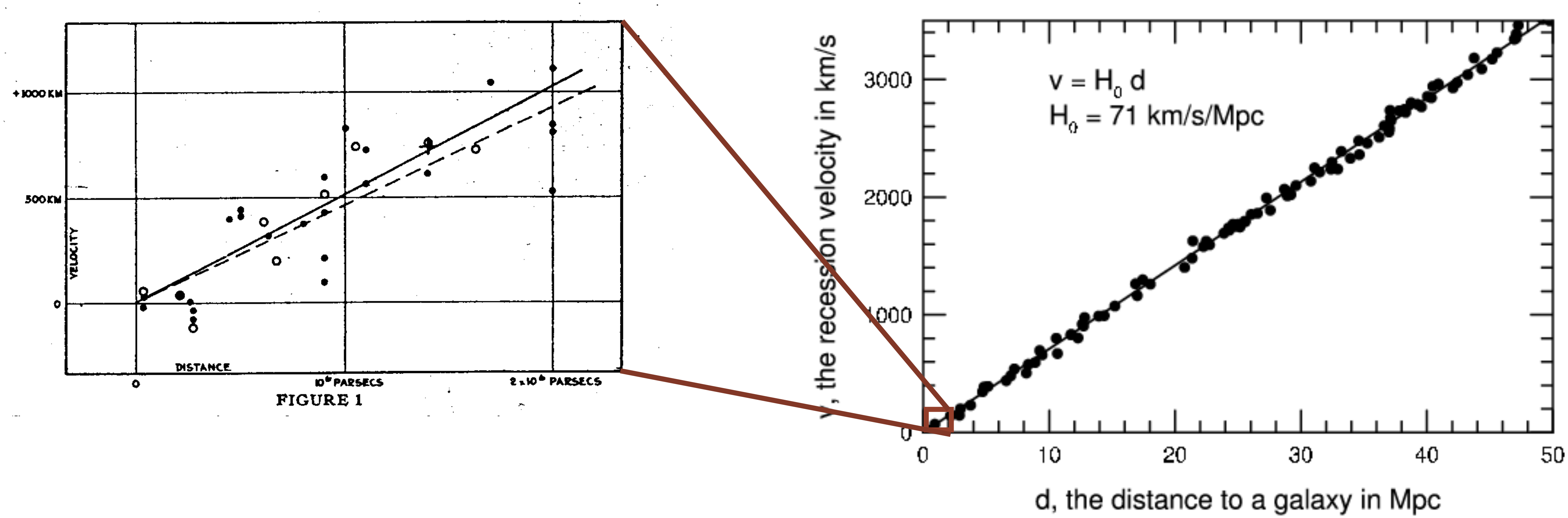
$$\begin{cases} S_k(\chi)=\sin\,\chi\,, k=+1 \\ S_k(\chi)=\chi \qquad\;\;, k=0 \\ S_k(\chi)=Sh\,\chi\,, k=-1 \end{cases}$$

$$\frac{dR}{dt} = H(t)\times R(t)\,\text{with}\,H(t) = \frac{\dot{a}(t)}{a(t)}$$

The univers is expending at a rate of H(t)

In real space, co-moving bodies move away from each other due to the expansion.

Hubble's Law



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The univers is expending at a rate of H(t)

In real space, co-moving bodies move away from each other due to the expansion.

$$c \int_0^{\tau} \frac{dt}{a(t)} = \chi$$

Distance and time are interchangeable

The horizon always grows with time

HOW TO DESCRIBE THE UNIVERS

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Distance and time are interchangeable

The horizon always grows with time

$$1 + z = \frac{a(t_o)}{a(t)} = \frac{\lambda_o}{\lambda_e}$$

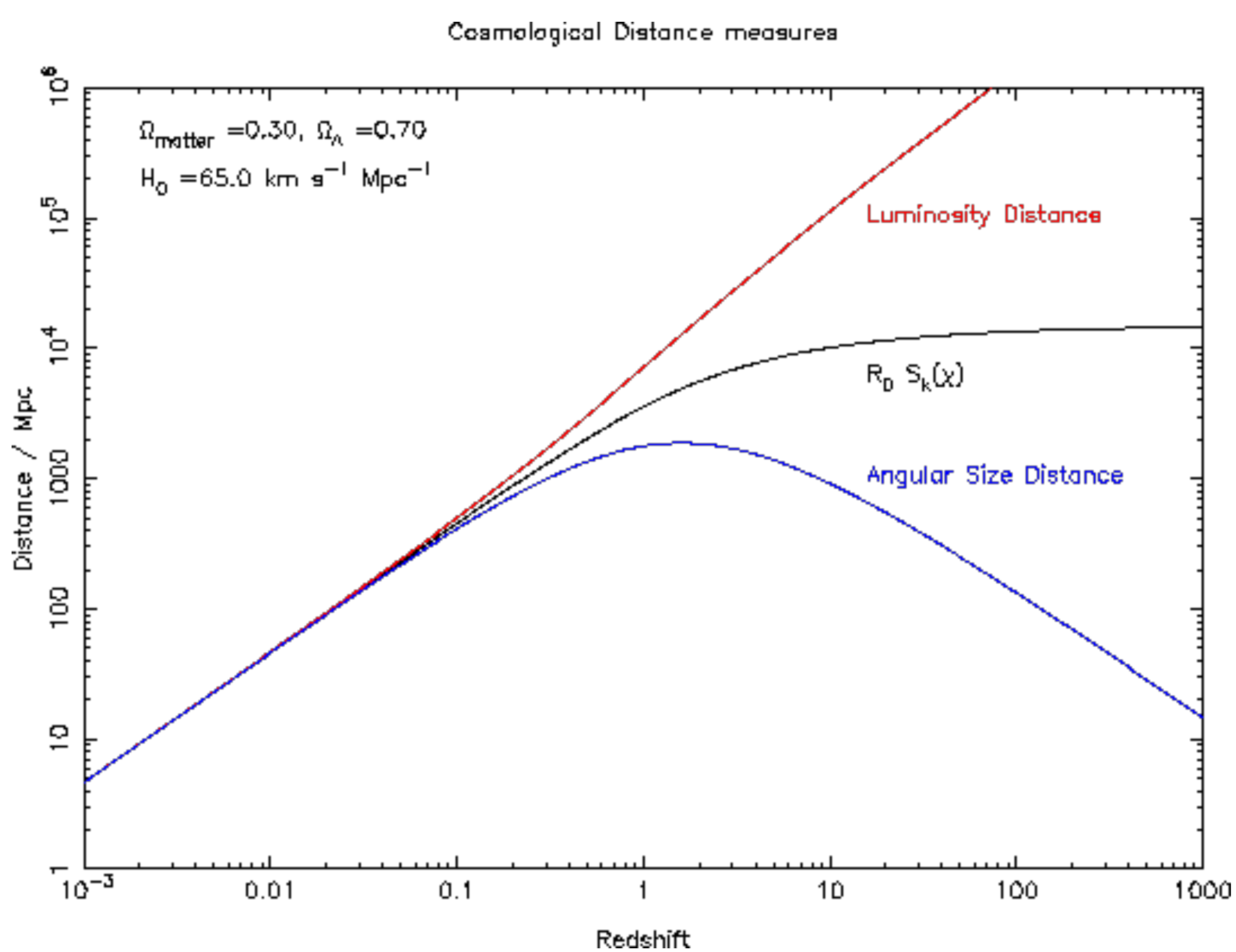
In an expanding (collapsing) Univers, photon wavelength is shifted toward the red (blue)

COSMOLOGICAL DISTANCES

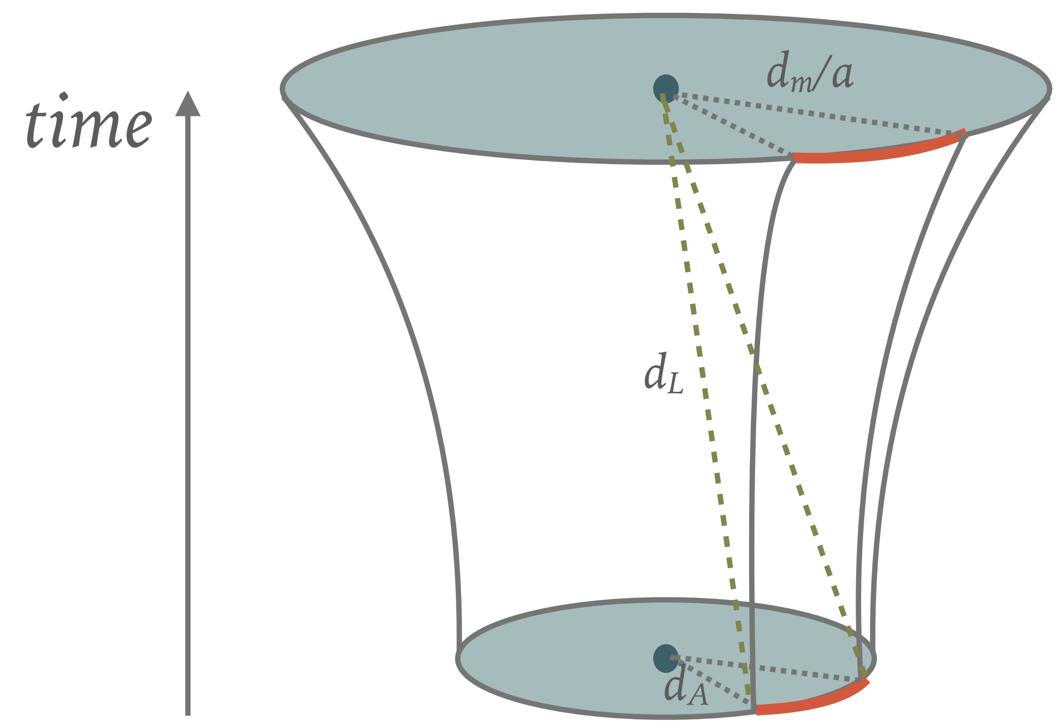
Comoving distances: $d_m = S(\chi)$ *with* $\chi = c\int \frac{dt}{a(t)}$ *for k=1*

Luminous distance: $d_L = (1+z)d_m$

Angular distance: $d_A = \dfrac{d_m}{1+z}$



$d_L = (1+z)^2 d_A$



SHAPING THE UNIVERS

So fare, we only considered geometry …

… and haven't yet use general relativity which states :

1- Gravitation can be described by a metric: $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g^{\mu\nu}R$

2- General relativity connect the metric to the matter/energy

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = 8\pi G T_{\mu\nu}$$

Geometry

Energy

- $G_{\mu\nu}$: The Einstein Tensor
- $R_{\mu\nu}$: The Ricci tensor
- $R = g^{\mu\nu}R_{\mu\nu}$: The Ricci Scalar
- $T_{\mu\nu}$: The Energy momentum tensor

THE FRIEDMAN EQUATION

After …

- Calculating the 64 partial derivatives of $g_{\mu\nu}$,
- Using those partial derivatives to calculate the 64 Christoffel symbols,
- Calculating the 256 partial derivatives of those Christoffel symbols,
- Using those partial derivatives to calculate the 16 components of the Ricci tensor,
- Calculating the Ricci scalar,
- Plug the results of those calculations into Einstein's field equations,
- Apply some straightforward calculus and algebra to those equations …

one can derived the generalised Friedman equation (taking c=1)

$$\frac{\dot{a}^2}{a^2} = \sum_i \frac{8\pi G\rho_i}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \quad (1)$$

$$\frac{\ddot{a}}{a} = -\sum_i \frac{4\pi G}{3}(\rho_i + 3P_i) + \frac{\Lambda}{3} \quad (2)$$

$$(1) \leftrightarrow (2) \quad \rightarrow \quad -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} + \Lambda - \frac{k}{a^2} = \sum_i 8\pi G P_i \quad (3)$$

$$\frac{d(1)}{dt} \quad \rightarrow \quad \sum_i \dot{\rho}_i = 3\frac{\dot{a}}{a}\sum_i (\rho_i + P_i) \quad (4)$$

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We could also consider the Dark Energy as a fluid of equation of state $\omega=P/\rho=-1$.
For cosmological constant, we verify: $\Lambda = 8\pi G\rho_\Lambda$ & $-4\pi G\rho_\Lambda/3(1+3\omega)=8\pi G\rho_\Lambda/3=\Lambda/3$

$$\frac{\dot{a}^2}{a^2} = \sum_i \frac{8\pi G\rho_i}{3} - \frac{k}{a^2} \quad (1)$$

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This equation gives hints on how the Univers expend with respect to its content.

$$(1) \leftrightarrow (2) \rightarrow -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{k}{a^2} = \sum_i 8\pi G P_i \qquad (3)$$

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This tell us how the expansion is accelerating or decelerating depending on the content of the Univers.
The expansion of the Univers accelerate if ω < -1/3, with ω=P/ρ the equation of state of the fluid.

THE FRIEDMAN EQUATION

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The fluid equation: Tell us how the energy density does vary with the expanding Univers.

$$\rho(a) \propto a^{-3(1+\omega)}$$

$$\frac{d(1)}{dt} \quad \rightarrow \quad \sum_i \dot{\rho}_i = 3\frac{\dot{a}}{a}\sum_i(\rho_i + P_i) \qquad (4)$$

THE COSMOLOGICAL PARAMETERS

Remember H(t) = ?

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

The Friedman equations become:

$$H^2 = \sum_i \frac{8\pi G \rho_i}{3} - \frac{k}{a^2} \qquad (1)$$

$$\dot{H} + H^2 = -\sum_i \frac{4\pi G}{3}(\rho_i + 3P_i) \qquad (2)$$

$$\dot{\rho}_i = -3H(\rho_i + P_i) \qquad (4)$$

And we define the critical density: the density of a matter dominated Univers

$$\rho_c = \frac{3H^2}{8\pi G}$$

The 1st Friedman equation becames:

$$1 = \sum_i \Omega_i + \Omega_k \qquad (1)$$

THE COSMOLOGICAL PARAMETERS

Radiation energy density parameter:

- $\Omega_\gamma = \rho_\gamma/\rho_c$ ← the fraction of radiation

Matter energy density parameter:

- $\Omega_m = \rho_m/\rho_c$ ← the fraction of matter

Curvature parameter:

- $\Omega_k = -k/(Ha)^2$

Dark Energy density parameter:

- $\Omega_{DE} = \rho_\Lambda/\rho_c = \Lambda/(3H^2)$ with $\Lambda = 8\pi G\rho\Lambda$

Radiation energy density parameter:

- $\Omega_\gamma{}^0$ ← the fraction radiation today

Matter energy density parameter:

- $\Omega_m{}^0$ ← the fraction of matter today

Curvature parameter:

- $\Omega_k{}^0$ ← the curvature today

Dark Energy density parameter:

- $\Omega_{DE}{}^0$ ← the fraction dark energy today

Lets assume we normalise the scale factor today at 1. The Friedman equation become:

$$H^2 = H_0^2 \left[\Omega_\gamma^0 (1+z)^4 + \Omega_m^0 (1+z)^3 + \Omega_{DE} + \Omega_k (1+z)^2\right] \quad (1)$$

Do you remember the cosmological distances ?

$$d_m = a(t) S_k(\chi) \qquad d_A = \frac{d_m}{1+z} \qquad d_L = (1+z) d_m$$

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Cosmological distances becomes

$$d_A = \frac{1}{(1+z)^2} S_k \left[\int_0^z \frac{dz}{H}\right] = \frac{1}{(1+z)^2} S_k \left[\int_0^z \frac{dz}{H_0 \sqrt{\Omega_\gamma^0 (1+z)^4 + \Omega_m^0 (1+z)^3 + \Omega_k^0 (1+z)^2 + \Omega_\Lambda^0}}\right]$$

$$d_L = \frac{1}{(1+z)^2} d_A = S_k \left[\int_0^z \frac{dz}{H_0 \sqrt{\Omega_\gamma^0 (1+z)^4 + \Omega_m^0 (1+z)^3 + \Omega_k^0 (1+z)^2 + \Omega_\Lambda^0}}\right]$$

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$$d_L = \frac{1}{(1+z)^2} d_A = S_k\left[\int_0^z \frac{dz}{H_0\sqrt{\Omega^0_\gamma(1+z)^4 + \Omega^0_m(1+z)^3 + \Omega^0_k(1+z)^2 + \Omega^0_\Lambda}}\right]$$

Measuring distances allows to infer constraints on the cosmological parameters.

At low redshift (z<<1) cosmological distance only depends on H_0

PECULIAR CASE: MATTER DOMINATED FLAT UNIVERS

$$H^2 = H_0 \frac{\Omega_m}{a^3} = \frac{H_0}{a^3} \qquad (1)$$

This equation has a strait-forward solution:

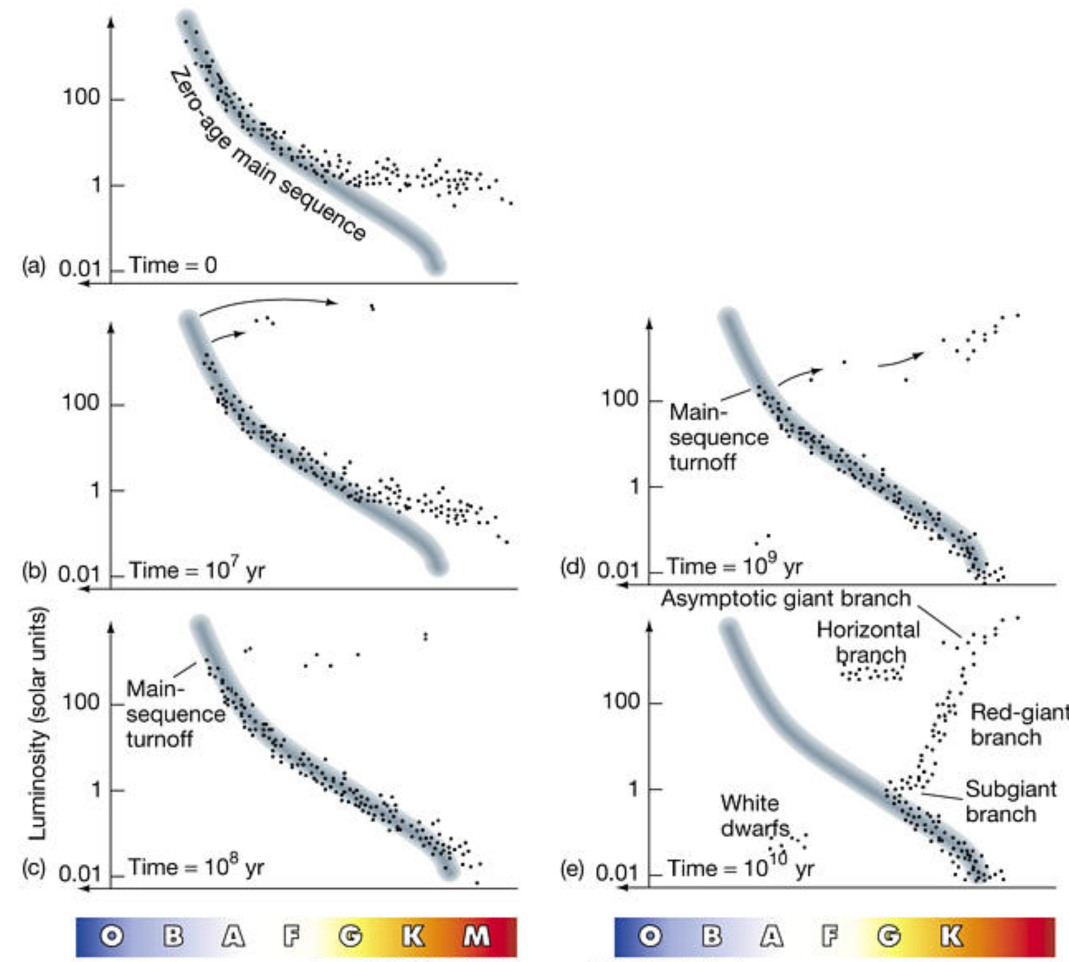
$$\frac{\dot{a}}{a} \propto a^{-3/2} \Rightarrow a(t) \propto t^{2/3} \Rightarrow \ddot{a} \propto \frac{3}{4\sqrt{t}}$$

(1) The Univers always expands itself.
(2) The expansion slow down with time.

The age of the Univers:

$$t_0 = \int_0^{t_0} dt = \int_0^1 \frac{da}{\dot{a}} = \int_0^\infty \frac{dz}{H(1+z)} = \frac{2}{3H_0}$$

$H_0 = 71.9$ (km/s)/Mpc $\simeq (13.6$ Gyr$)^{-1} \Rightarrow t_0 = 9.06$ Gyr



An age of 9 Gyr isn't compatible with the measured age of the oldest observed star which have been estimated to be more than 11 Gyr old ⇒ This simple calculation already implies that something is missing !

CONCLUSIONS

Modern cosmology is constructed upon cosmological principle and FLRW metrics …

- How strong are those assumptions ?
  This have to be tested at high precision level ← recent measurement verify those principle at 10^{-5} precision level
- We know at small scale the Univers isn't homogenous (but it still remains homogenous on average)
  Galaxy Clusters, galaxies and stars results from initial perturbations in the metric ant Energy-momentum tensor that have grown while the Universe expended. Thos fluctuation provide additional valuable information to test the cosmological models (see the lecture of J. Bell and A Pisani tomorrow)

Current observations of our Univers lead us to the so-called Λ-CDM model:

- So far no observations were able to rule-out this simplest model. However, many questions remain to be solved:
  - *96 % of the energy content of the Univers is still a mystery to us: What are made of the Dark Energy and the Dark Matter ?*
  - *Is Dark Energy a cosmological constant ? Why Cosmological constant so weak ?*
  - *Why does the Univers is so flat ?! $\Omega_K = 0.000 \pm 0.005$!*
  - *What generated the primordial energy density fluctuation that produces the small scale structures of the Univers*