Vector mesons couplings to constituent quarks from QCD: a dynamical approach

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INCT-FNA

1- MOTIVATIONS

* This work is about the low energy behavior of Strongly Interacting systems

Experimentally only hadrons properties are available at low energies
From fundamental point of view there are quarks/gluons

* Relation between QCD degrees of freedom and hadrons

Different mechanism from instantons induced interactions ?
[would it yield double counting: instantons, condensates, polarization?]

* The method below was enough to produce a whole EFT
for pions and constituent quarks : S. Weinberg (2010) LARGE Nc effective theory
In F.L.B. EPJA 52, 134 (2016) and EPJA 54, 45 (2018)

* To some extend this method is parallel to SDE – rainbow ladder approximation

Mesons interactions with baryons can determine the nucleon and nuclear potentials

In particular for intermediary energies the light SU(2) vector mesons are relevant

And it is possible to identify chiral partners

$\rho(770)$ [h]

$I^G(J^{PC}) = 1^+(1^{--})$

Mass $m = 775.26 \pm 0.25$ MeV

$a_1(1260)$ [k]

$I^G(J^{PC}) = 1^-(1^{++})$

Mass $m = 1230 \pm 40$ MeV [l]

$\omega(782)$

$I^G(J^{PC}) = 0^-(1^{--})$

Mass $m = 782.65 \pm 0.12$ MeV (S = 1.9)

$f_1(1285)$

$I^G(J^{PC}) = 0^+(1^{++})$

Mass $m = 1281.9 \pm 0.5$ MeV (S = 1.8)

From QCD to effective quark interactions

$$Z = N \int \mathcal{D}[\psi, \overline{\psi}]\mathcal{D}[A_\mu]\mathcal{D}[\chi, \chi^*] e^{<\overline{\psi}(i\gamma_\mu \mathcal{D}^\mu - m_i)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{FP} + \mathcal{L}_{gf}>}$$

Dressed Gluon mediated quark interaction from QCD

$$S_{eff}[\bar{\psi}, \psi] = \int_x \left[\bar{\psi} \left(i\partial\!\!\!/ - m \right) \psi - \frac{g^2}{2} \int_y j_\mu^b(x)(\tilde{R}_{bc}^{\mu\nu})(x-y) j_\nu^c(y) \right] + S_A,$$

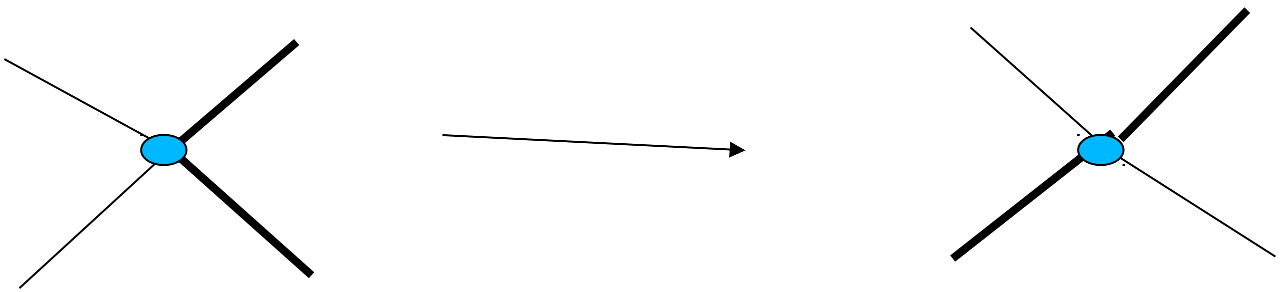
$$j_\mu^c = \bar{\psi}_i \gamma_\mu g \lambda_c \psi_i,$$

Interaction is of part of QCD-Seff: yields NJL=longwavelength/local

-It will be assumed strength is enough for DChSB and formation of condensate

-(dressed gluon propagator and g)

With a Fierz transformation: flavor structure
Color singlet terms are dominant (1/Nc)



$$j_s(x,y)=\bar{\psi}(x)\psi(y),$$

$$j_p(x,y)=\bar{\psi}(x)i\gamma_5\sigma_i\psi(y),\; j_{si}(x,y)=\bar{\psi}(x)\sigma_i\psi(y),$$

$$j_{ps}(x,y)= \bar{\psi}(x)i\gamma_5\psi(y), \quad j_V^\mu(x,y)=\bar{\psi}(x)\gamma^\mu\sigma_i\psi(y), \quad j_{as}^\mu(x,y)=\bar{\psi}(x)i\gamma_5\gamma^\mu\psi(y).$$

$$j_A^\mu(x,y)= \bar{\psi}(x)i\gamma_5\gamma^\mu\sigma_i\psi(y), \quad j_{vs}^\mu(x,y)=\bar{\psi}(x)\gamma^\mu\psi(y),$$

Flavor Currents will have the corresponding light mesons

Quarks = sea + valence

$$\bar{\psi}\Gamma^q\psi \to t_2(\bar{\psi}\Gamma^q\psi)_2 + t_1(\bar{\psi}\Gamma^q\psi)_1,$$

Those for chiral condensate
And light mesons

Those that may form baryon states
(and heavier mesons)

Auxiliary fields for meson fields

From the bilocal auxiliary fields

The local mesons are obtained

SU(2) or SU(3) GROUND STATE in tl

With scalar qq condensate

$$\frac{\partial \mathcal{V}_{eff}}{\partial S_i}\bigg|_{S^i=S^i_0,\phi=\phi_0} = 0,$$

$$\frac{\partial \mathcal{V}_{eff}}{\partial P_i}\bigg|_{S^i=S^i_0,\phi=\phi_0} = 0,$$

$$M^* = M + S_i \lambda_i,$$

It can be calculated by NJL-type GAP equations

or by the corresponding SD equations

Usually Phenomenological

quark effective masses (u,d) or (u,d,s) M*

Sea quarks determinant

$$S_d = -i\, Tr \log \left[1 + \frac{1}{N_c} S_0^c \left(N_c \tilde{\Phi}_N + \frac{\tilde{g}^2}{N_c} \alpha t_1 N_c^2 \sum_q R_q \frac{1}{N_c} \Gamma_q \bar{\psi} \Gamma_q \psi \right) \right].$$

By neglecint quark field : ’t Hooft large Nc

Model of weakly interacting quark-antiquark states (mesons)

Expansion presents the structure of the general Expansion addressed by Manohar-Georgi:

$$I_{ABCD} \sim c_{A,B,C,D} \left(\frac{\pi}{f}\right)^A \left(\frac{\bar{\psi}\Gamma\psi}{f^2\Lambda}\right)^B \left(\frac{\mathcal{G}[A_\mu^{1,a}]}{\Lambda}\right)^C \left(\frac{p}{\Lambda}\right)^D$$

Light Vector and axial mesons couplings to constituent quarks

F.L.B.- EPJA- (2016)
EPJA (2018)

Here the scalar-pseudoscalar setor will be neglected...

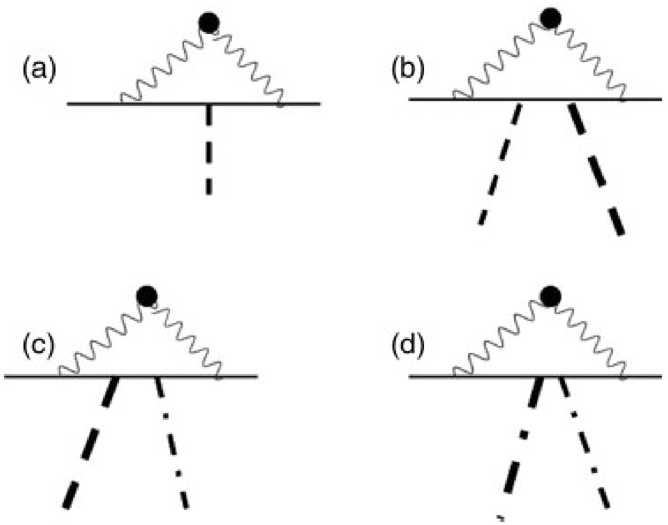
The leading terms from a large M* and Mg expansion

$$\mathcal{L}_{v-q} = g_{r1}(V_i^{\mu}(x) j_{\mu}^{V,i}(x) + \bar{A}_i^{\mu}(x) j_{\mu}^{A,i}(x)) + g_{v1}(V^{\mu}(x) j_{\mu}(x) \quad + \bar{A}_{\mu}(x) j_A^{\mu}(x)),$$

$$g_{r1} = g_{v1} = 4iN_c d_1 (\alpha g^2) \mathrm{Tr}'((S_0(k) S_0(k) \bar{\bar{R}}(k))),$$

Gauge type couplings , dimensionless

All equal = it goes along universality idea



(a)
(b)
(c)
(d)

Next Leading terms:

$$
\begin{aligned}
\mathcal{L}_{2v-j} = & \, g_{va-j}([V_\mu \bar{A}_i^\mu + V_\mu^i \bar{A}^\mu] j_{ps}^i + [V_\mu^i \bar{A}_i^\mu + V_\mu \bar{A}^\mu] j_{ps}) \\
& + g_{va-j}([(V_\mu^2 + \bar{A}_\mu^2) + (V_\mu^{i\,2} + (\bar{A}_i^\mu)^2)] j_s + [V^\mu V_\mu^i + \bar{A}_\mu \bar{A}_i^\mu] j_s^i) \\
& + g_{Fjs}[((\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \mathcal{F}_{\mu\nu}^A \mathcal{F}_A^{\mu\nu}) + (\mathcal{F}_{\mu\nu}^i \mathcal{F}_i^{\mu\nu} + \mathcal{F}_{\mu\nu}^{A,i}\mathcal{F}_{A,i}^{\mu\nu})) j_s \\
& + (\mathcal{F}_{\mu\nu}^i \mathcal{F}_A^{\mu\nu} + \mathcal{F}_{\mu\nu}\mathcal{F}_{A,i}^{\mu\nu}) j_p^i + (\mathcal{F}_{\mu\nu}^i \mathcal{F}^{\mu\nu} + \mathcal{F}_{\mu\nu}^{A,i}\mathcal{F}_A^{\mu\nu}) j_s^i + (\mathcal{F}_{\mu\nu}\mathcal{F}_A^{\mu\nu} + \mathcal{F}_{\mu\nu}\mathcal{F}_A^{\mu\nu}) j_p] \\
& + g_{\epsilon v} i\epsilon_{ijk}(\mathcal{F}_{\mu\nu}^i V_j^\nu + \mathcal{F}_{\mu\nu}^{i,A}\bar{A}_j^\nu) j_{k,V}^\mu + g_{\epsilon v} i\epsilon_{ijk}(\mathcal{F}_{\mu\nu}^i \bar{A}_j^\nu + \mathcal{F}_{\mu\nu}^{i,A}\bar{V}_j^\nu) j_{k,A}^\mu,
\end{aligned}
$$

Very large quark mass limit M*

$$
g_{\epsilon v} = -\frac{g_{va-j}}{M^*}, \qquad \frac{g_{va-j}}{g_{r1}} \sim \frac{3}{2M^*}, \qquad \frac{g_{Fjs}}{g_{r1}} \sim \frac{3}{2M^{*3}}. \qquad \frac{g_{\epsilon v}}{g_{va-j}} = -\frac{1}{M^*}.
$$

3 independent coupling constants for more than 20 couplings

All written in terms of components of gluon and propagators

TABLE I. In the first column, the quark effective masses are displayed with the values of the factor h_a that were chosen to fix the coupling constant $g_{v1} = g_{r1} = 12$ as a typical numerical value considered in nucleon-nucleon potential [15]. In the second column, the gluon propagators are indicated: $D_I(k)$ and $D_{II}(k)$ are the gluon propagators, respectively, from Refs. [35,43] and Ref. [38]. In the other columns, results, which depend on the gluon propagator, from the expressions (14), (17), and (19) are displayed, and also parameters do not depend on the gluon propagators (21) and (22). The momentum cutoff used for the integrations (21), (22) is indicated together with $g_f^{(0)}$.

M^*, h_a (GeV)	$D_i(k)$	$g_{r1}h_a$	$g_{vaj}h_a$ (GeV^{-1})	$g_{Fjs}h_a$ (GeV^{-3})	$g_f^{(0)}$ (Λ) (GeV)	$M_v^{(0)}$ (GeV)
0.33, $\frac{12}{9.3}$	D_I	12	5.8	111	0.10 (2.0)	0.479
0.33, $\frac{12}{0.67}$	D_{II}	12	5.7	107	...	...
0.28, $\frac{12}{10.5}$	D_I	12	6.4	165	0.11 (2.0)	0.495
0.28, $\frac{12}{0.75}$	D_{II}	12	6.4	163	...	...
0.22, $\frac{12}{12.7}$	D_I	12	6.8	279	0.13 (2.0)	0.512
0.22, $\frac{12}{0.9}$	D_{II}	12	6.7	285	...	...
0.07, $\frac{12}{20.3}$	D_I	12	7.2	4944	0.22 (2.0)	0.545
0.07, $\frac{12}{1.5}$	D_{II}	12	7.6	3228	...	...

$$D_I(k) = \frac{8\pi^2}{\omega^4} D e^{-k^2/\omega^2} + \frac{8\pi^2 \gamma_m E(k^2)}{\ln[\tau + (1 + k^2/\Lambda_{\rm QCD}^2)^2]},$$

$$D_{II}(k) = K_F/(k^2 + M_k^2)^2,$$

I – Tandy-Maris

II – Cornwall - effective confining

$$\mathcal{L}_{2v-j} = g_{va-j}([V_\mu \bar{A}^\mu_i + V^i_\mu \bar{A}^\mu] j^i_{ps} + [V^i_\mu \bar{A}^\mu_i + V_\mu \bar{A}^\mu] j_{ps}) + g_{va-j}([(V^2_\mu + \bar{A}^2_\mu) + (V^{i\,2}_\mu + (\bar{A}^\mu_i)^2)] j_s + [V^\mu V^i_\mu + \bar{A}_\mu \bar{A}^\mu_i] j^i_s)$$

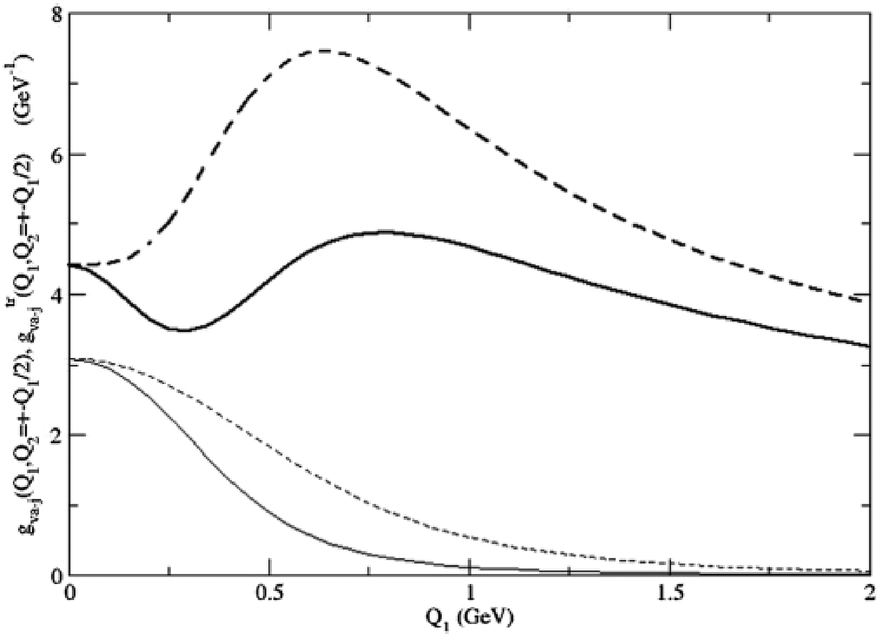


FIG. 6. By considering the gluon propagator I from Refs. [35,43], the expressions for $g^{\rm com}_{va-j}(Q_1, Q_2 = +Q_1/2)$ and $g^{\rm com}_{va-j}(Q_1, Q_2 = -Q_1/2)$ are plotted, respectively, in the solid thick line and dashed thick line; $g^{\rm tr}_{va-j}(Q_1, Q_2 = +Q_1/2)$ and $g^{\rm tr}_{va-j}(Q_1, Q_2 = -Q_1/2)$ are plotted, respectively, in the solid line and dashed line.

Form factor with truncated expression (quark-kernel)

$$\mathcal{L}_{v-q} = g_{r1}(V_i^\mu(x) j_\mu^{V,i}(x) + \bar{A}_i^\mu(x) j_\mu^{A,i}(x)) + g_{v1}(V^\mu(x) j_\mu(x) \quad + \bar{A}_\mu(x) j_A^\mu(x)),$$

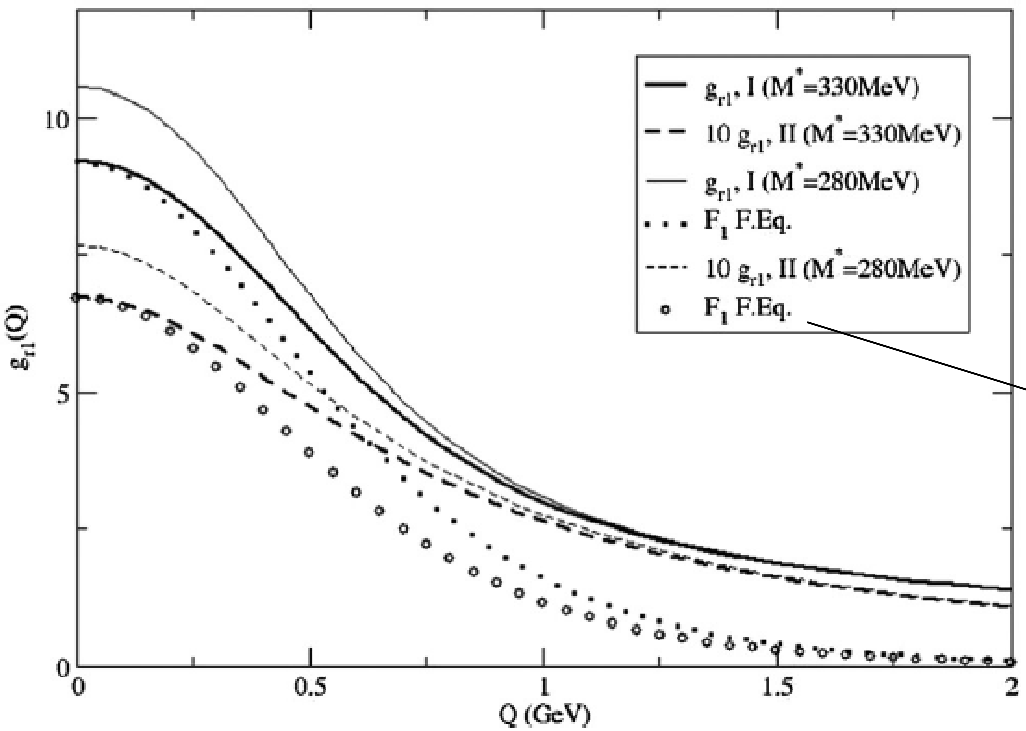


FIG. 8. The strong vector mesons form factor $g_{r1}(Q)$ given in expression (29) is presented for the two gluon propagators *I* [35,43], solid lines, and *II* [38], dashed lines. The quark effective mass was considered $M^* = 330$ MeV for thick lines and $M^* = 280$ MeV for thin lines. The normalized vector meson–nucleon form factor from Ref. [46] is also presented for further comparison with corresponding normalization with results of the two propagators above: $F_1^\rho(0) = 9.2$ (propagator *I*, thick dotted line) and $F_1^\rho(0) = 6.7$ (propagator *II*, lines with circles).

Fit from Faddeev equation
Nucleon- rho coupling (Bloch et al)

$$F_1^\rho(Q^2) = \frac{F_1^\rho(0)}{(1+\frac{Q^2}{\Lambda_{1,\rho}^2})^3},$$

$$\Lambda_{1,\rho} = 1.12 \text{ GeV} \quad \text{and} \quad F_1^\rho(0) = g_{\rho NN}$$

Strong quadratic radius of RHO and the other vector mesons

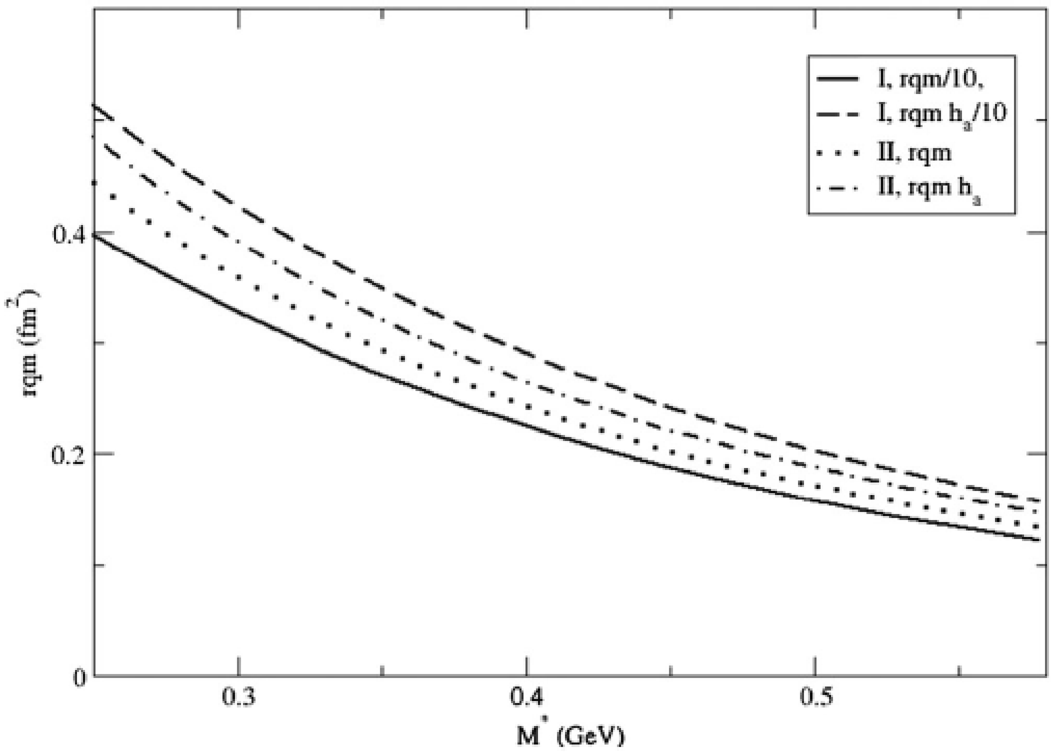


FIG. 9. By considering propagators I and II, the vector mesons strong quadratic radius is exhibited as a function of the quark effective mass M^*. Results are shown with and without the factor h_a explained in the table, with thick solid and thick dashed lines with $D_I(k)$, and they are multiplied by $1/10$ to be kept in the scale of the figure. The cases for gluon propagator D_{II} are represented in dotted and dotted-dashed lines, respectively, with and without h_a.

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$$\langle r_\rho^2 \rangle_s = -6 \frac{d g_{r1}(Q)}{dQ^2}\bigg|_{Q=0}.$$

The freedom to choose the

Quark gluon coupling constant

And the gluon propagator

Yield different absolute values...

Couplings to background photons

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$$Z = N \int \mathcal{D}[\bar{\psi},\psi] \exp i \int_x \Big[\bar{\psi}(i\not{D} - m)\psi - \frac{g^2}{2}\int_y j_\mu^b(x)\tilde{R}_{bc}^{\mu\nu}(x-y)j_\nu^c(y) + \bar{\psi}J + J^*\psi \Big],$$

$$D_\mu = \partial_\mu \delta_{ij} - ieQ_{ij}A_\mu$$

Leading and next leading terms from the expansion:

Phoron couplings to constituent quark currents and vector-axial mesons

$$I_{AV} = g_{qA}\left(A_\mu(x) j_3^\mu(x) + \frac{1}{3} A_\mu(x) j^\mu(x)\right),$$

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$$I_{FF} = 2 g_1 i \epsilon_{ij3} (F_{\mu\nu} \mathcal{G}_i^{\mu\nu} j_p^j + F_{\mu\nu} \mathcal{F}_i^{\mu\nu} j_s^j), \qquad (12)$$

$$I_{FFm} = g_1 [F_{\mu\nu} \mathcal{F}_3^{\mu\nu} j_s + F_{\mu\nu} \mathcal{F}^{\mu\nu} j_s^3 + F_{\mu\nu} \mathcal{G}_3^{\mu\nu} j_p + F_{\mu\nu} \mathcal{G}^{\mu\nu} j_p^3 + \frac{1}{3}(F_{\mu\nu} \mathcal{G}^{\mu\nu} j_p + F_{\mu\nu} \mathcal{F}^{\mu\nu} j_s + F_{\mu\nu} \mathcal{G}_i^{\mu\nu} j_p^i + F_{\mu\nu} \mathcal{F}_i^{\mu\nu} j_s^i)],$$

Couplings with Abelian tensors

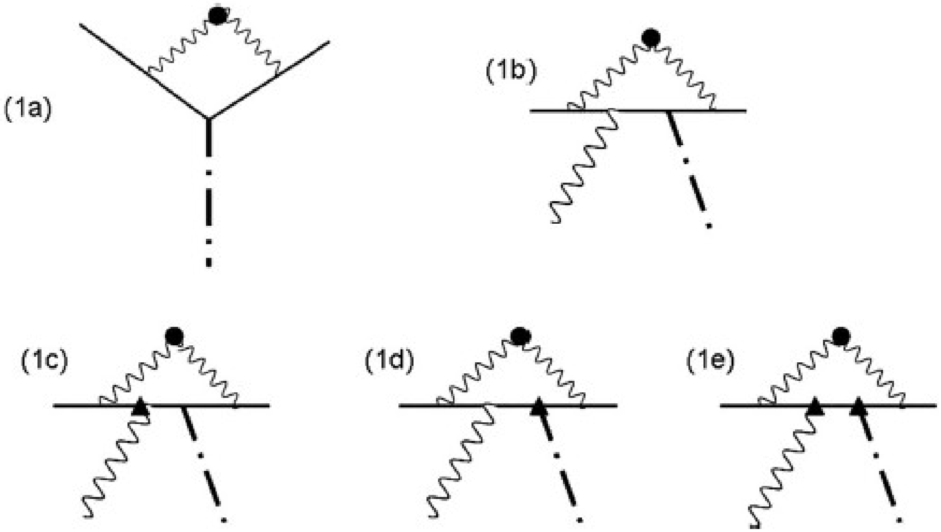
Couplings with vector-axial mesons

$$I_{VA} = 2 g_2 i \epsilon_{ij3} (A_\rho V_i^\rho j_s^j + A_\rho \bar{A}_i^\rho j_p^j), \qquad (14)$$

$$I_{VAm} = g_2 \left[A^\rho \bar{A}_\rho^3 j_p + A^\mu \bar{A}_\mu j_p^3 + A^\nu V_\nu^3 j_s + A^\nu V_\nu j_s^3 + \frac{1}{3}(A^\nu V_\nu j_S + A^\mu \bar{A}_\mu j_p + A_\rho V_i^\rho j_S^i + A^\mu \bar{A}_\mu^i j_p^i) \right],$$

$$I_{VF} = g_3 i \epsilon_{ij3} (F_{\rho\delta} V_i^\rho j_{j,V}^\delta + F_{\rho\delta} \bar{A}_i^\rho j_{j,A}^\delta),$$

$$I_{AF} = g_3 i \epsilon_{ij3} (\mathcal{F}_{\rho\delta}^i A^\rho j_{j,V}^\delta + \mathcal{G}_{\rho\delta}^i A^\rho j_{j,A}^\delta)),$$



M^* (GeV)	$(\frac{eB_0}{M^{*2}}, \frac{eB_0}{m_\pi^2})$	$D_i(k)$ ...	g_{r1} ...	g_2 (GeV^{-1})	$g_3^{B_1}/(\frac{eB_0}{M^{*2}})$ ...
0.33	(0, 0)	D_I	9.3	5.6	1.7
0.38	(0.2, 1.5)	D_I	8.2	4.7	1.8
0.45	(0.5, 5.2)	D_I	6.8	3.6	1.8
0.33	(0, 0)	D_{II}	1.1	0.3	0.19
0.38	(0.2, 1.5)	D_{II}	0.9	0.3	0.19
0.45	(0.5, 5.2)	D_{II}	0.8	0.2	0.19
0.22	(0, 0)	D_I	12.7	8.6	1.4
0.25	(0.2, 0.6)	D_I	11.6	7.7	1.6
0.28	(0.5, 2)	D_I	10.7	6.8	1.6
0.22	(0, 0)	D_{II}	1.5	0.6	0.19
0.25	(0.2, 0.6)	D_{II}	1.3	0.5	0.19
0.28	(0.5, 2)	D_{II}	1.2	0.4	0.19
0.07	(0, 0)	D_I	20.3	14.7	0.5
0.085	(0.2, .08)	D_I	19.4	14.0	0.6
0.10	(0.5, 0.3)	D_I	18.5	13.3	0.7
0.07	(0, 0)	D_{II}	2.4	0.9	0.06
0.085	(0.2, .08)	D_{II}	2.3	0.9	0.08
0.10	(0.5, 0.3)	D_{II}	2.1	0.9	0.1

$$I_{FF} = 2g_1 i\epsilon_{ij3}(F_{\mu\nu}\mathcal{G}_i^{\mu\nu} j_p^j + F_{\mu\nu}\mathcal{F}_i^{\mu\nu} j_s^j), \qquad (12)$$

$$I_{FFm} = g_1[F_{\mu\nu}\mathcal{F}_3^{\mu\nu} j_s + F_{\mu\nu}\mathcal{F}^{\mu\nu} j_s^3 + F_{\mu\nu}\mathcal{G}_3^{\mu\nu} j_p + F_{\mu\nu}\mathcal{G}^{\mu\nu} j_p^3$$

$$I_{VA} = 2g_2 i\epsilon_{ij3}(A_\rho V_i^\rho j_s^j + A_\rho \bar{A}_i^\rho j_p^j), \qquad (14)$$

$$I_{VAm} = g_2\Big[A^\rho \bar{A}_\rho^3 j_p + A^\mu \bar{A}_\mu j_p^3 + A^\nu V_\nu^3 j_s + A^\nu V_\nu j_s^3$$

$$I_{VF} = g_3 i\epsilon_{ij3}(F_{\rho\delta} V_i^\rho j_{j,V}^\delta + F_{\rho\delta}\bar{A}_i^\rho j_{j,A}^\delta),$$

$$I_{AF} = g_3 i\epsilon_{ij3}(\mathcal{F}_{\rho\delta}^i A^\rho j_{j,V}^\delta + \mathcal{G}_{\rho\delta}^i A^\rho j_{j,A}^\delta)),$$

If these photons are due to a weak magnetic field

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$$\langle r_\rho^2\rangle(B_0) = -6\frac{dF^E_{1,\rho}(Q^2)}{dQ^2}\Big|_{Q=0}$$

$$= -6\frac{d}{dQ^2}\left(g_{qA}(Q) + \frac{2}{3}g_3^{B_1}(Q)\right)\Big|_{Q=0}$$

$$\equiv \langle r_\rho^2(M^*)\rangle + \frac{eB_0}{M^{*2}}\Delta\langle r_\rho^2(M^*)\rangle.$$

Anysotropic correction

In the plane perpendicular to
The weak magnetic field (z)

$$I_{VF} = g_3 i\epsilon_{ij3}(F_{\rho\delta}V^\rho_i j^\delta_{j,V} + F_{\rho\delta}\bar{A}^\rho_i j^\delta_{j,A}),$$

$$I_{AF} = g_3 i\epsilon_{ij3}(\mathcal{F}^i_{\rho\delta}A^\rho j^\delta_{j,V} + \mathcal{G}^i_{\rho\delta}A^\rho j^\delta_{j,A})),$$

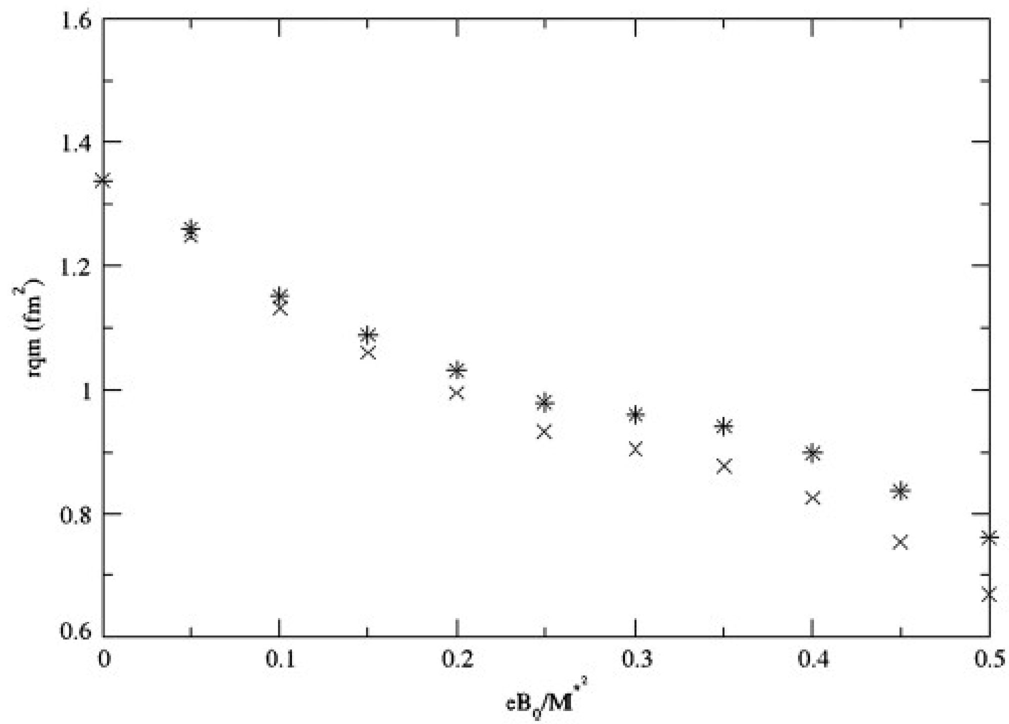

FIG. 4. The rho squared electromagnetic radius (rqm) with the gluon propagator $D_I(k)$ from Refs. [37,38] for $M^*(B_0 = 0) = 330$ MeV is shown as a function of the magnetic field in terms of the effective mass $(eB_0)/M^{*2}$. The symbol $\times$ indicates the leading correction $\langle r_\rho^2(M^*)\rangle = -6\frac{d}{dQ^2}g_{qA}(Q)$ in expression (41) and the symbols $*$ represent the total values of $\langle r_\rho^2\rangle(B_0)$ with the anisotropic correction $(eB_0)/M^{*2}$.

Approximation: very weak magnetic field
first Landau orbit

Summary : few remarks

By analizing structureless mesons limit

- Large variety of effective couplings and small number of
  effective coupling constants (universality)

- Need to test further the behavior of form factors/coupling constants
  with gluon/quark propagators and q-g running coupling constant
- Only color singlet currents and interactions

- In a related work: vector mesons mixings due to weak magnetic field
  Vector meson dominance under weak magnetic field

- To understand the strict relation between
  coupling to constituent quark and coupling to nucleon/baryon
  (Faddeev equation?)