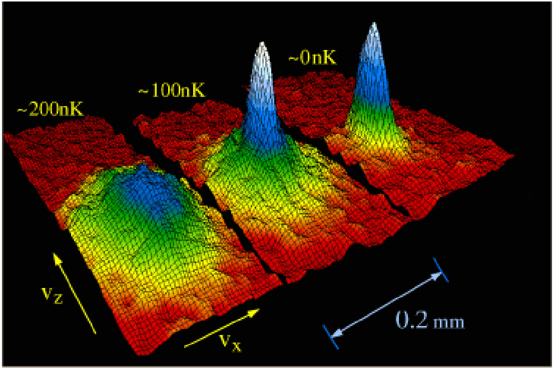
Clusters and matter made from unitary Bosons

Universal properties of Bosons at unitarity

Bose-Einstein condensates

2 D velocity distributions



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Work with

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The Hamiltonian

$$H = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk}$$

- Non-relativistic kinetic energy

$$V_{ij} = V_2^0 \frac{\hbar^2}{m}\mu_2^2 \exp[-(\mu_2 r_{ij})^2/2], \quad r_{ij} = r_i - r_j$$

- In the attractive V_{ij} the strength V_2^0 is tuned to unitarity

$$V_{ijk} = V_3^0 \frac{\hbar^2}{m}\left(\frac{\mu_3}{2}\right)^2 \exp[-(\mu_3 R_{ijk}/2)^2/2], \quad R_{ijk} = (r_{ij}^2 + r_{ik}^2 + r_{jk}^2)^{1/2}$$

- V_3^0 tuned to reproduce a weakly-bound three-particle (Efimov) state with energy E_3 in the repulsive 3-body interaction
- μ_2 and μ_3 are the two- and three-body ranges
- $\bar{R}_3 = (-\hbar^2/2mE_3)^{1/2}$ is a radius of the trimer

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Universality

- Universal parameters are directly related to the properties of the three-body system (energy and radius)
- The range of the two- and three-body interactions are kept much smaller than the size of the weakly bound trimer
- Details of the interactions are not relevant

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The trial-state wave functions

For clusters $\Psi_T = \prod_{i<j} f^{(2)}(r_{ij}) \prod_{i<j<k} f^{(3)}(R_{ijk}) \prod_{i} \exp(-\alpha r_i^2)$

For bulk matter $\Psi_T = \prod_{i<j} f^{(2)}(r_{ij}) \prod_{i<j<k} f^{(3)}(R_{ijk})$

$$f^{(2)}(r) = K \tanh(\mu_J r) \cosh(\gamma r)/r$$

$$f^{(3)}(R) = \exp[u_0 \exp(-R^2/(2r_0^2))]$$

- The parameters K and γ are chosen to have
  $$f^{(2)}(d) = 1 \text{ and } f^{(2)'}(d) = 0$$
  at the "healing distance" d
- α, μ_J, d, u_0 and r_0 are variational parameters
- Ground-state properties can be "exactly"solved with DMC

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Small clusters $N \leq 15$

- Cluster binding energy per particle

$$\frac{E_N}{N} = \xi_B(N)\frac{E_3}{3},$$

- $\xi_B(N)$ is a universal function of N for sufficiently small range
- Nonuniversal effects appear when the range of two- or three-particle interactions becomes significant compared to the average interparticle distance
- Our results also start to show nonuniversalality for large two-body interaction range for $N \simeq 15$

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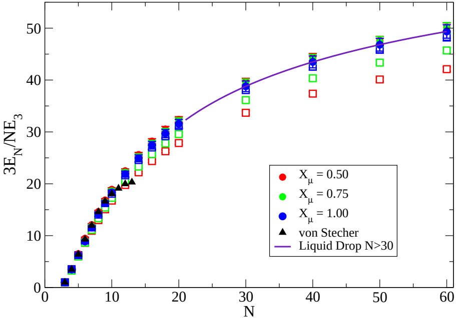
Clusters

Cluster Binding Energy



The binding energy per particle E_N/N normalized by the trimer binding energy per particle $E_3/3$ as a function of the number N of Bosons

- Liquid-drop extrapolation $E_N/N = E_B(N \to \infty)(1 + \eta N^{-1/3} + \ldots)$
- The surface energy scaled by the vol energy $\eta = -1.7 \pm 0.3$
- $\xi_B(N \to \infty) = 90 \pm 10$ fit for $N > 30$

- $X_\mu = \mu_3/\mu_2$ is the ratio between two and three-body interaction ranges
- Filled symbols $\mu_2 \bar{R}_3 = 65$ loosely bound trimers
- Open symbols $\mu_2 \bar{R}_3 = 46$ have large 2-body interaction range tightly bound trimers

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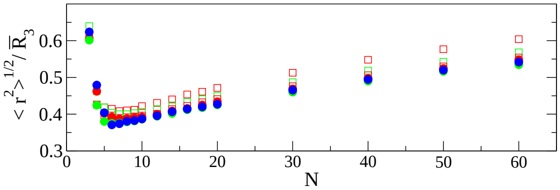
Root mean square radii of N-boson cluster



$X_\mu = \mu_3/\mu_2 = 0.5$, $X_\mu = 0.75$, $X_\mu = 1$

$\bar{R}_3$ is the radius associate to the three-body state

- The colors show different ratios between two and three-body interaction ranges $X = \mu_3/\mu_2$

- Open symbols have a larger two-body interaction range $\mu_2 \bar{R}_3 = 46$ than filled symbols when the particles are more loosely bound, $\mu_2 \bar{R}_3 = 65$

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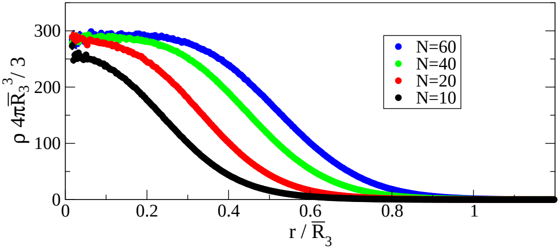
Cluster radial one-body density function



- $\mu_2 \bar{R}_3 = 65$ and $X_\mu = 1$
- The single-particle density near the center of the drops is consistent with the equilibrium density of matter ($\rho_0 4\pi \bar{R}_3^3/3 \simeq 275$)

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Properties of the bulk Bose liquid at unitarity

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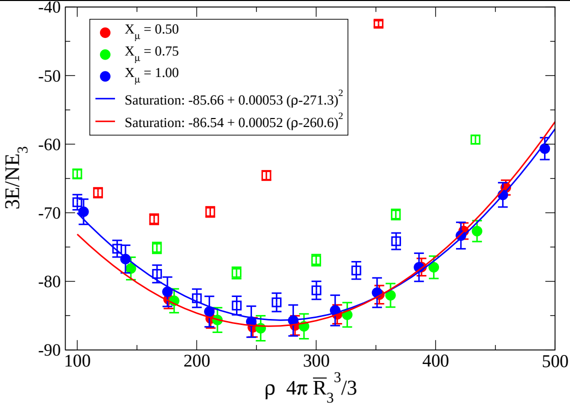
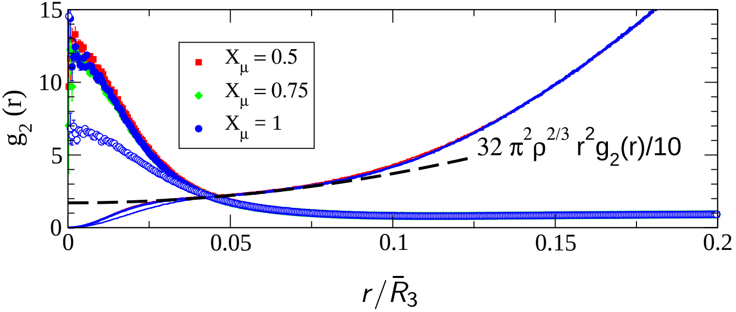
Equation of state for bulk matter



$$\left.\frac{3E_N(\rho)}{N|E_3|}\right|_{N\to\infty} = \xi_B(N\to\infty)\left[-1+\kappa\left(\frac{\rho-\rho_0}{\rho_0}\right)^2\right]$$

- Very small size effects
- $\xi_B(N\to\infty) = 87\pm 5$, compressibility $\kappa = 0.42\pm 0.05$
- Results for the liquid are consistent with those for clusters

Two-body radial distribution function



- Normalized to 1 at large separation
- Contact obtained by extrapolation to $r = 0$

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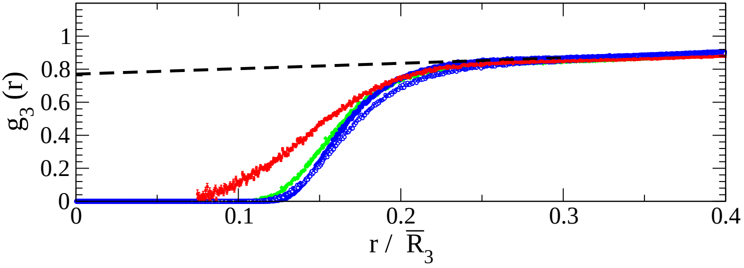
Three-body radial distribution function



$X_\mu = \mu_3/\mu_2 = 0.5$, $X_\mu = 0.75$, $X_\mu = 1$

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Contacts

- The contact is a universal parameter
- Central quantity for the many-body physics

$$C_2 = N\alpha_2\rho^{4/3}, \quad c_2 = \frac{C_2}{Vol} \propto \lim_{r\to 0} r^2 g_2(r)$$

- c_2 can be thought was a measurement of the local pair density

$$C_3 = N\beta_3\rho^{2/3}, \qquad C_3 \propto \left(\frac{\partial E}{\partial \bar{R}_3}\right)_a$$

DMC	$\alpha_2 = 17 \pm 3$	$\beta_3 = 0.9 \pm 0.1$
Exp	$\alpha_2 = 22 \pm 1$	$\beta_3 = 2.1 \pm 0.1$

Rapid quench experiments

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Condensate fraction

- High condensate fraction $n(k=0) = 0.93 \pm 0.01$
  - ^{4}He has about 7%
  - Similar cluster binding energy as a function of the number of atoms

$$\Downarrow$$

$\Rightarrow$ Suggests the possibility of investigating the Bose universal properties with the rapid quenching of a weakly interacting Bose condensate

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Thank you!