Description of nuclear matter from hadronic mean-field models: some approaches and results.

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Subatomic Physics: from theory to applications

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- ***Many body systems***
- ***Thermodynamical approach***
- ***Some results***
- ***Studies in progress***

Many body nuclear systems

- ✔ Many body nuclear systems can be obtained basically from two different approaches:

  - Microscopic models where the nucleon-nucleon potential is constructed. Free parameters are found by fitting deuteron properties, for instance.

  - Effective mean-field models. Thermodynamical equations of state (EOS) built in order to reproduce known data from infinite nuclear matter and finite nuclei. Free parameters adjusted from that.

- ✔ Applications of nuclear matter EOS are many.

Many body nuclear systems

✔ At zero temperature regime.

- Low densities ($\rho << \rho_0$): description of crust of compact stars, for instance. Formation of clusters and structures known as pasta.

- High densities ($\rho >> \rho_0$): determination of neutron stars mass from the solution of TOV equations. Observational data.

✔ At finite temperature regime.

- Low temperatures (T < 20 MeV): liquid-gas phase transition of symmetric and asymmetric nuclear matter. Critical parameters. Caloric curves. Experimental data.

- High temperatures (T ~ 150 MeV): possibility of hadron-quark phase transitions (quark models also needed). Heavy ion collisions.

Thermodynamical approach

- Different kind of relativistic and nonrelativistic models are widely used.

- Nonrelativistic Skyrme model (widely used).

$$\mathcal{H} = \mathcal{K} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{eff}$$

$$\begin{aligned}
\mathcal{K} &= \frac{\hbar^2}{2M}(\tau_p + \tau_n) \\
\mathcal{H}_0 &= \frac{t_0}{2}\left[\left(1+\frac{x_0}{2}\right)\rho^2 - \left(\frac{1}{2}+x_0\right)(\rho_n^2+\rho_p^2)\right] \\
\mathcal{H}_3 &= \frac{1}{12}\sum_{i=1}^{3} t_{3i}\left[\left(1+\frac{x_{3i}}{2}\right)\rho^2 - \left(\frac{1}{2}+x_{3i}\right)(\rho_n^2+\rho_p^2)\right]\rho^{\sigma_i} \\
\mathcal{H}_{eff} &= \rho(\tau_n+\tau_p)\left[\frac{t_1}{4}\left(1+\frac{x_1}{2}\right)+\frac{t_2}{4}\left(1+\frac{x_2}{2}\right)+\frac{t_4}{4}\left(1+\frac{x_4}{2}\right)\rho^\beta+\frac{t_5}{4}\left(1+\frac{x_5}{2}\right)\rho^\gamma\right] \\
&+ (\rho_n\tau_n+\rho_p\tau_p)\left[\frac{t_2}{4}\left(\frac{1}{2}+x_2\right)-\frac{t_1}{4}\left(\frac{1}{2}+x_1\right)-\frac{t_4}{4}\left(\frac{1}{2}+x_4\right)\rho^\beta+\frac{t_5}{4}\left(\frac{1}{2}+x_5\right)\rho^\gamma\right]
\end{aligned}$$

Thermodynamical approach

$$
\begin{aligned}
\frac{\mathcal{E}}{\rho} &= \frac{3\hbar^2}{10M}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{2/3}H_{5/3} + \frac{t_0}{8}\rho\left[2(x_0+2)-(2x_0+1)H_2\right] \\
&+ \frac{1}{48}\sum_{i=1}^{3} t_{3i}\rho^{\sigma_i+1}\left[2(x_{3i}+2)-(2x_{3i}+1)H_2\right] + \frac{3}{40}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{5/3}\left(aH_{5/3}+bH_{8/3}\right) \\
&+ \frac{3}{40}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{5/3+\beta}\left[t_4(x_4+2)H_{5/3}-t_4\left(x_4+\frac{1}{2}\right)H_{8/3}\right] \\
&+ \frac{3}{40}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{5/3+\gamma}\left[t_5(x_5+2)H_{5/3}+t_5\left(x_5+\frac{1}{2}\right)H_{8/3}\right]
\end{aligned}
$$

$$
\begin{aligned}
a &= t_1(x_1+2)+t_2(x_2+2), \\
b &= \frac{1}{2}\left[t_2(2x_2+1)-t_1(2x_1+1)\right], \\
H_n(y) &= 2^{n-1}\left[y^n+(1-y)^n\right].
\end{aligned}
$$

$$y = Z/(Z+N)$$

- free parameters:

$t_0, t_1, t_2, t_{3i}, t_4, t_5$

$x_0, x_1, x_2, x_{3i}, x_4, x_5$

σ, β, γ

Thermodynamical approach

✔ Relativistic mean-field (RMF) model (Generalized Walecka one).

$$\mathcal{L} = \mathcal{L}_{nm} + \mathcal{L}_{\sigma} + \mathcal{L}_{\omega} + \mathcal{L}_{\rho} + \mathcal{L}_{\sigma\omega\rho},$$

$$\begin{aligned}
\mathcal{L}_{nm} &= \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - M)\psi - g_{\sigma}\sigma\overline{\psi}\psi - g_{\omega}\overline{\psi}\gamma^{\mu}\omega_{\mu}\psi - \frac{g_{\rho}}{2}\overline{\psi}\gamma^{\mu}\vec{\rho}_{\mu}\vec{\tau}\psi,\\
\mathcal{L}_{\sigma} &= \frac{1}{2}(\partial^{\mu}\sigma\partial_{\mu}\sigma - m_{\sigma}^{2}\sigma^{2}) - \frac{A}{3}\sigma^{3} - \frac{B}{4}\sigma^{4},\\
\mathcal{L}_{\omega} &= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_{\omega}^{2}\omega_{\mu}\omega^{\mu} + \frac{c}{4}(g_{\omega}^{2}\omega_{\mu}\omega^{\mu})^{2},\\
\mathcal{L}_{\rho} &= -\frac{1}{4}\vec{B}^{\mu\nu}\vec{B}_{\mu\nu} + \frac{1}{2}m_{\rho}^{2}\vec{\rho}_{\mu}\vec{\rho}^{\mu} \qquad \text{e}\\
\mathcal{L}_{\sigma\omega\rho} &= -g_{\sigma}g_{\omega}^{2}\sigma\omega_{\mu}\omega^{\mu}\left(\alpha_{1} - \frac{1}{2}\alpha_{1}{}'g_{\sigma}\sigma\right) - g_{\sigma}g_{\rho}^{2}\sigma\vec{\rho}_{\mu}\vec{\rho}^{\mu}\left(\alpha_{2} - \frac{1}{2}\alpha_{2}{}'g_{\sigma}\sigma\right)\\
&+ \frac{1}{2}\alpha_{3}{}'g_{\omega}^{2}g_{\rho}^{2}\omega_{\mu}\omega^{\mu}\vec{\rho}_{\mu}\vec{\rho}^{\mu}
\end{aligned}$$

Thermodynamical approach

$$\mathcal{E}_{\rm NL} = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{A}{3}\sigma^3 + \frac{B}{4}\sigma^4 - \frac{1}{2}m_\omega^2\omega_0^2 - \frac{C}{4}(g_\omega^2\omega_0^2)^2 - \frac{1}{2}m_\rho^2\bar{\rho}_{0(3)}^2 + g_\omega\omega_0\rho$$
$$+ \frac{g_\rho}{2}\bar{\rho}_{0(3)}\rho_3 + \frac{1}{2}m_\delta^2\delta_{(3)}^2 - g_\sigma g_\omega^2\sigma\omega_0^2\left(\alpha_1 + \frac{1}{2}\alpha_1' g_\sigma\sigma\right)$$
$$- g_\sigma g_\rho^2\sigma\bar{\rho}_{0(3)}^2\left(\alpha_2 + \frac{1}{2}\alpha_2' g_\sigma\sigma\right) - \frac{1}{2}\alpha_3' g_\omega^2 g_\rho^2\omega_0^2\bar{\rho}_{0(3)}^2 + \mathcal{E}_{\rm kin}^p + \mathcal{E}_{\rm kin}^n,$$

$$\mathcal{E}_{\rm kin}^{p,n} = \frac{\gamma}{2\pi^2}\int_0^{k_{F\,p,n}} k^2\left(k^2 + M_{p,n}^{*2}\right)^{1/2} dk$$

- free parameters: $g_\sigma,\ g_\omega,\ g_\rho,\ A,\ B,\ c,\ \alpha_1,\ \alpha_1',\ \alpha_2,\ \alpha_2',\ \alpha_3'$

✔ From the energy density $\mathcal{E}$ all remaining EOS are found.

Some results

✔ Searching for the "best" parametrizations of Skyrme and RMF models:

M. Dutra, O. Lourenço, J. S. Sá Martins, A. Delfino, J. R. Stone, P. D. Stevenson Phys. Rev. C 85, 035201 (2012).

- 240 parametrizations of the Skyrme model tested in 11 constraints from nuclear matter, pure neutron matter, experimental/observational data of some bulk quantities.

- 16 parametrizations simultaneously consistent:

GSkI, GSkII, KDE0v1, LNS, MSL0, NRAPR, Ska25s20, Ska35s20, SKRA, SkT1, SkT2, SkT3, Skxs20, SQMC650, SQMC700, SV-sym32

Some results

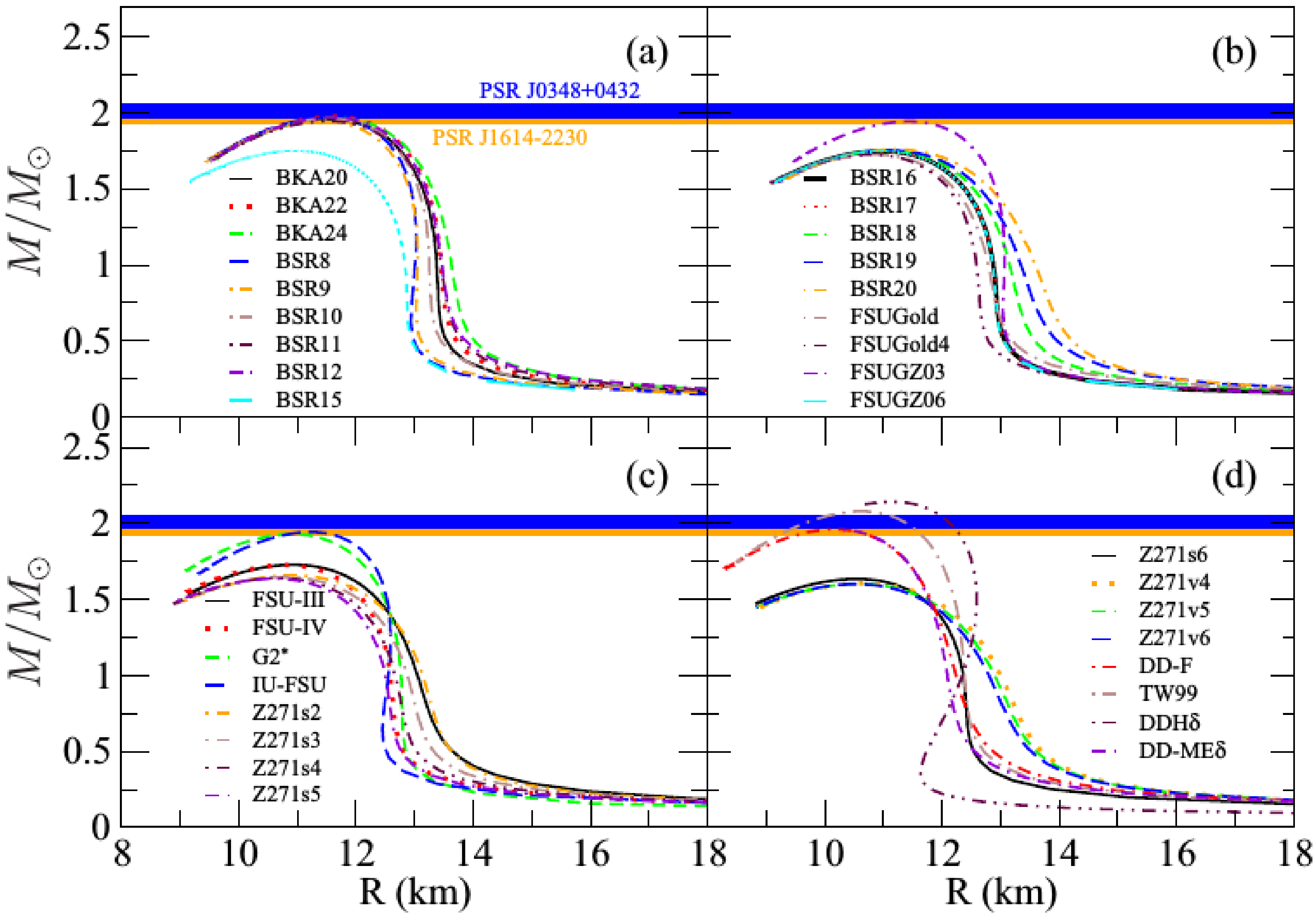
M. Dutra, O. Lourenço, S. S. Avancini, B. V. Carlson, A. Delfino, D. P. Menezes, C. Providência, S. Typel, and J. R. Stone,
Phys. Rev. C 90, 055203 (2014).

- 263 RMF parametrizations tested.
- 35 of them simultaneously consistent:

BKA20, BKA22, BKA24, BSR8, BSR9, BSR10, BSR11, BSR12, BSR15, BSR16, BSR17, BSR18, BSR19, DD-MEδ, DD-F, DDHδ, FSU-III, FSU-IV, FSUGold, FSUGold4, FSUGZ03, FSUGZ06, G2*, IU-FSU, TW99, BSR20, FA3, Z271s2, Z271s3, Z271s4, Z271s5, Z271s6, Z271v4, Z271v5 e Z271v6.

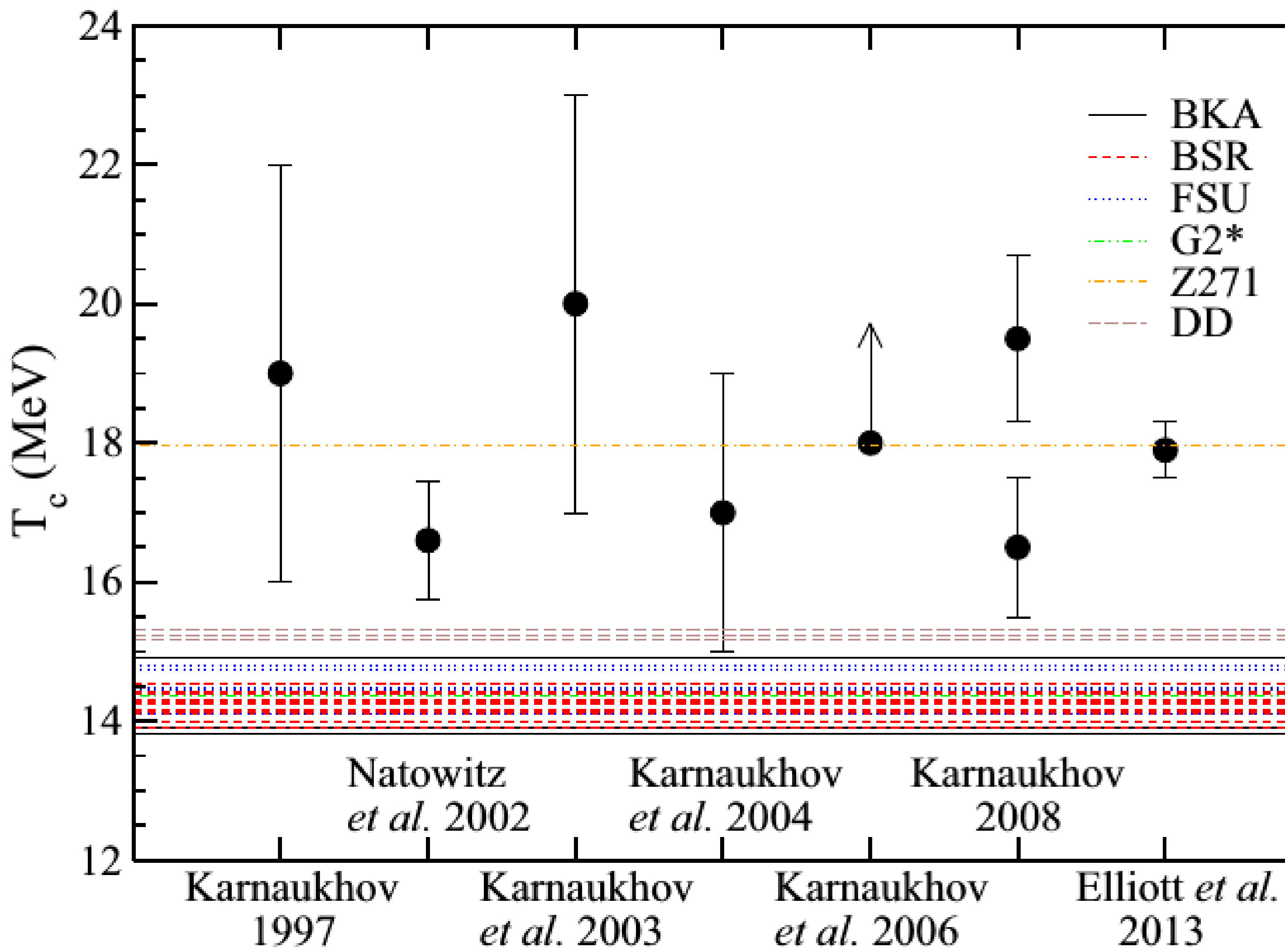
Some results

M. Dutra, O. Lourenço, and D. P. Menezes, Phys. Rev. C 93, 025806 (2016).



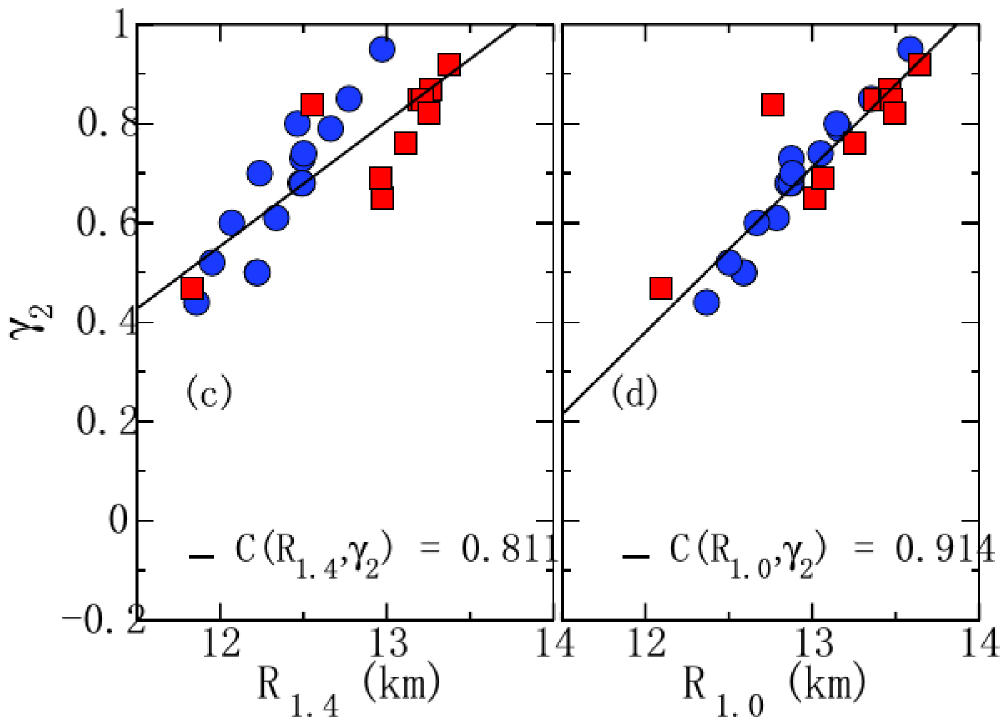
Some results

M. Dutra, O. Lourenço, and D. P. Menezes, Phys. Rev. C 95, 065212 (2017).



M. Dutra, O. Lourenço, O. Hen, E. Piasetzky and D. P. Menezes, Chinese Physics C 42, 064105 (2018).

$$\mathcal{S}(\rho) = \frac{1}{8}\frac{\partial^2(\mathcal{E}/\rho)}{\partial y^2}\bigg|_{\rho,y=1/2} = \mathcal{S}^{kin}(\rho) + \mathcal{S}^{\rm pot}(\rho), \qquad \mathcal{S}^{\rm pot}(\rho) = \mathcal{S}_0^{\rm pot}(\rho/\rho_0)^{\gamma}$$



Studies in progress

✔ Deconfinement effects in quark-meson-coupling (QMC) model.

Dyana C. Duarte, O. Lourenço, T. Frederico.

$$\mathcal{E} = \frac{1}{2}m_\sigma^2\sigma_0^2 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2 b_{03}^2 + \frac{\gamma}{(2\pi)^3}\sum_{N=p,n}\int^{k_{f,N}} d^3k\sqrt{k^2 + M_N^{*2}},$$

$$M_N^* = E_N^{*0} - \epsilon_{\text{c.m.}} \qquad V(r) = (ar^2 + V_0),$$

$$a \to a(1 - \Phi^2), \quad \text{and} \quad V_0 \to V_0(1 - \Phi^2)$$

Studies in progress

✔ Equations of state from a nonrelativistic lagrangian density.

O. Lourenço, M. Dutra, W. de Paula, A. Delfino and T. Frederico.

$$\mathcal{L} = \frac{i}{2}\Psi^*\dot{\Psi} - \frac{i}{2}\dot{\Psi}^*\Psi - \frac{1}{2m}\nabla\Psi^*.\nabla\Psi - V(\Psi^*,\Psi)$$

$$V_N(\Psi^\dagger,\Psi)|_{\vec{r_1}} = \frac{1}{N!}\sum_{s_i t_i}\sum_{s_i' t_i'}\prod_{i=2}^{N}\int d\vec{r_i}\prod_{i'=1}^{N}\int d\vec{r_i'}\,\Phi^{(N)\dagger}(\vec{r_i},s_i t_i) < \vec{r_i},s_i t_i|v^{(N)}|\vec{r_i'},s_i' t_i' > \Phi^{(N)}(\vec{r_i'},s_i' t_i'),$$

$$\Phi^{(N)}(\vec{r_i},s_i t_i) = \prod_{i=1}^{N}\Psi_{s_i t_i}(\vec{r_i}) = \Psi_{S_N t_N}(\vec{r}_N)\ldots\Psi_{S_1 t_1}(\vec{r}_1).$$

- analysis of the role played by 2, 3 and 4 body interactions.

Studies in progress

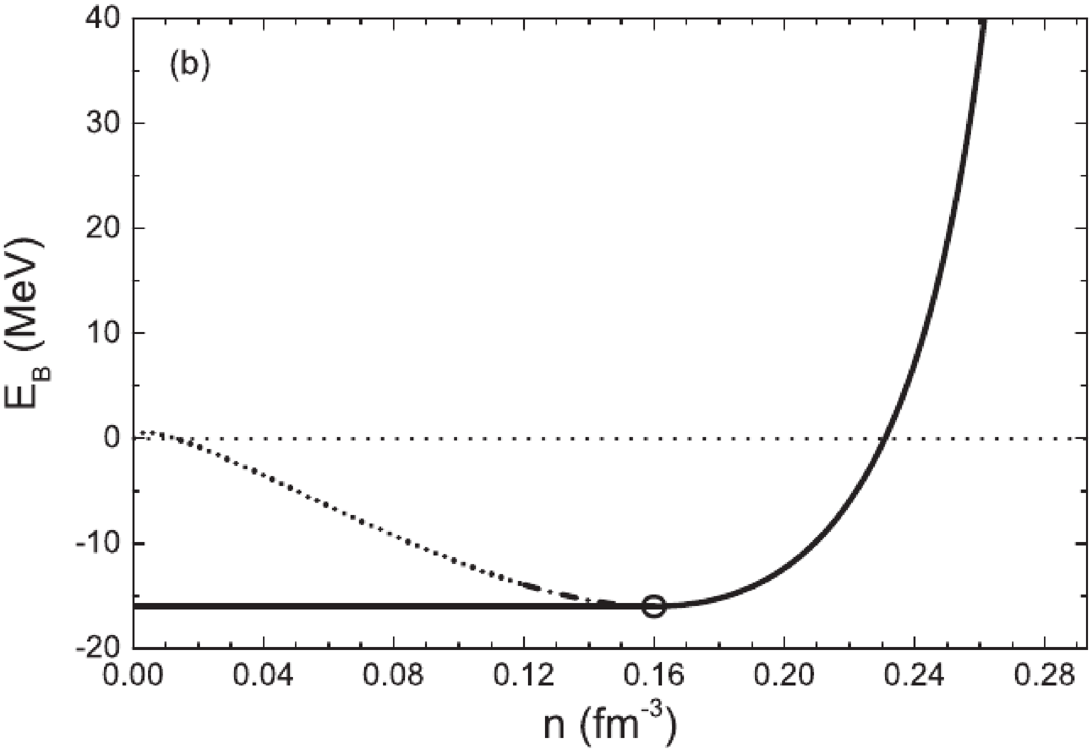
✔ van der Waals model applied to nuclear matter.

O. Lourenço, M. Dutra, B. M. Santos.

$$p(T,\mu) = p^{\mathrm{id}}(T,\mu^*) - an^2, \qquad \mu^* = \mu - bp(T,\mu) - abn^2 + 2an.$$

$$p^{\mathrm{id}}(T=0,\mu^*) =$$

$$\frac{d}{6\pi^2}\int_0^{\sqrt{\mu^{*2}-m^2}} dk \frac{k^4}{\sqrt{k^2+m^2}}$$



Thank you !