

# ORIGIN OF THE MOST VISIBLE MATTER IN THE UNIVERSE



IFT - UNESP  
INSTITUTO DE FÍSICA  
TEÓRICA - UNESP

Gastão Krein  
Instituto de Física Teórica, São Paulo



1st Meeting of LIA - Subatomic Physics: from theory to  
applications

12-13 June 2018  
Instituto Tecnológico de Aeronáutica

# This talk based on:

G.K., A.W. Thomas & K. Tsushima

— Prog. Part. Nucl. Phys. 100, 161 (2018)

N. Brambilla, G.K., J. Tarrús-Castellà & A. Vairo

— Phys. Rev. D 93, 054002 (2016)

J. Tarrús-Castellà & G.K.

— arXiv: 1803.05412

# Motivation

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Nothing really ambitious

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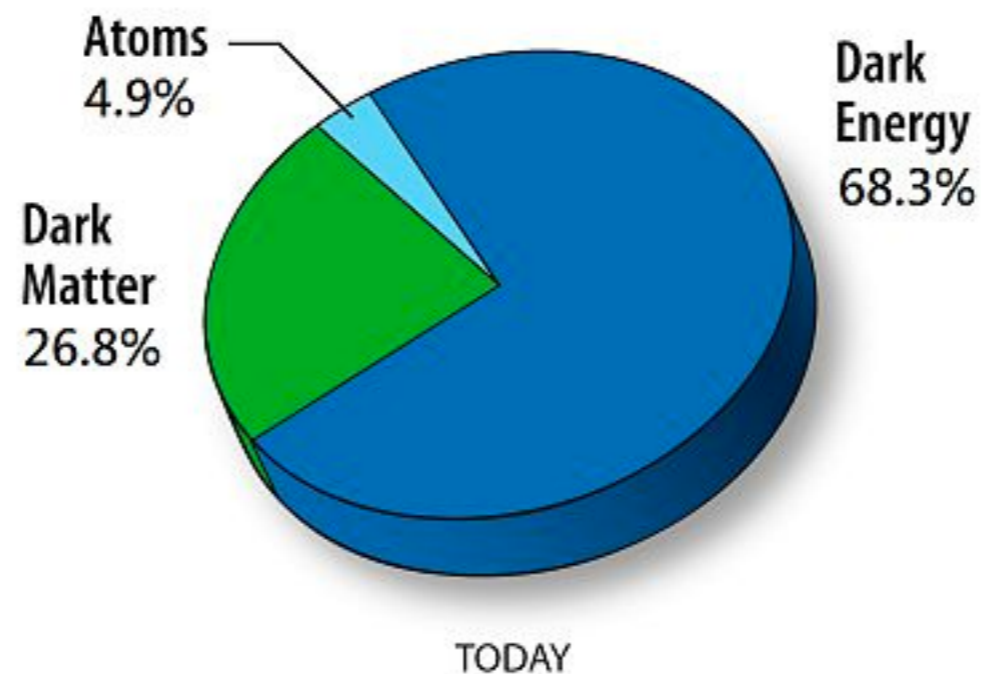
Nothing really ambitious

We want to understand matter that amounts to 5% of the mass of the universe

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We want to understand matter that amounts to 5% of the mass of the universe





Actually we are even less  
ambitious than that

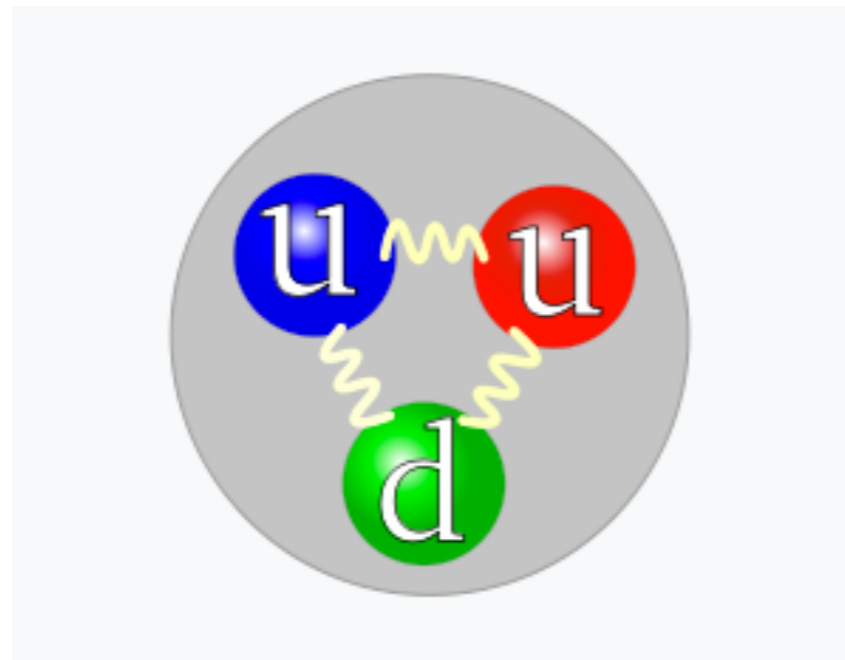


Actually we are even less  
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We want to understand the  
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# Starting point ?

Seems to be

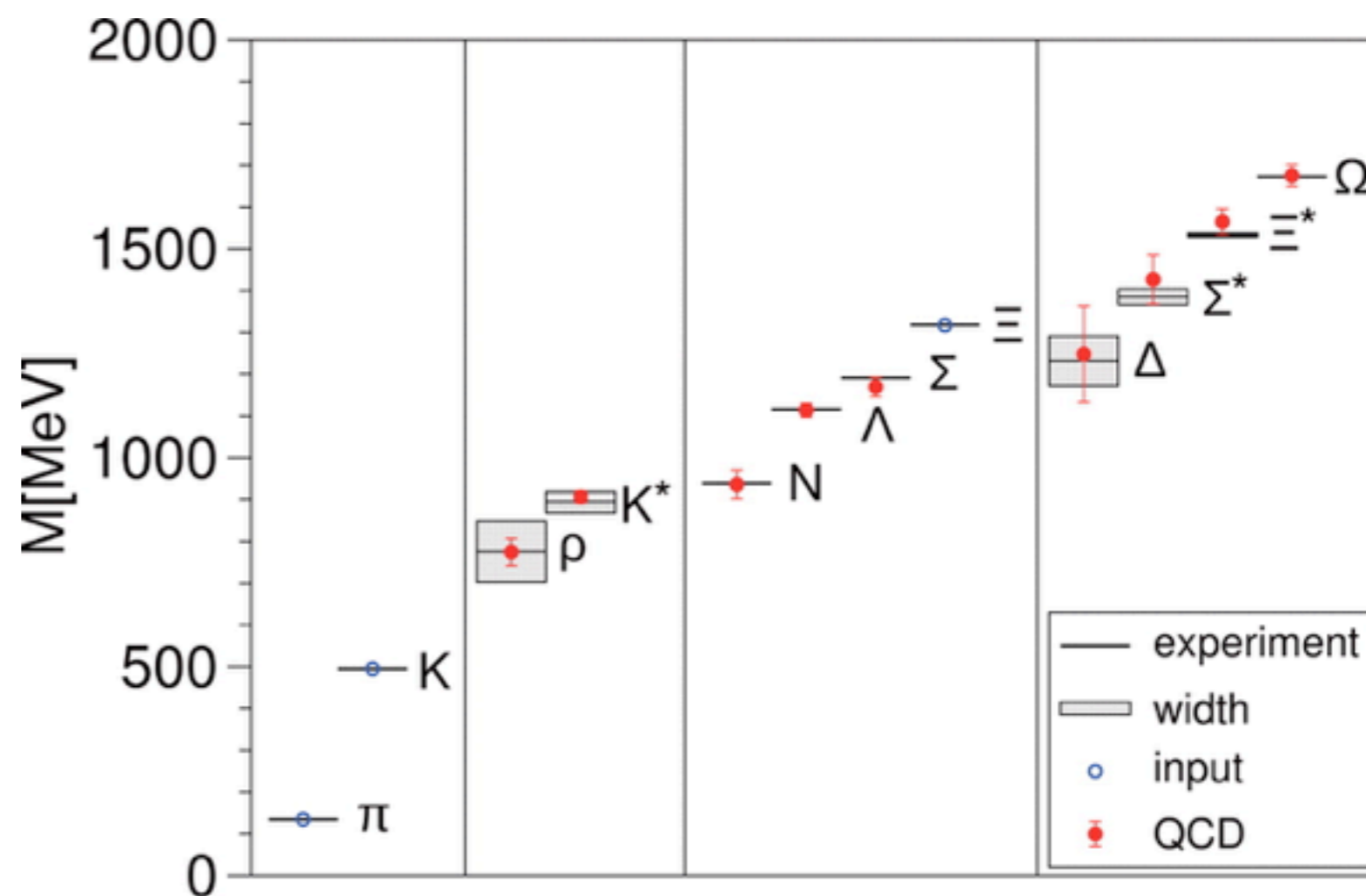
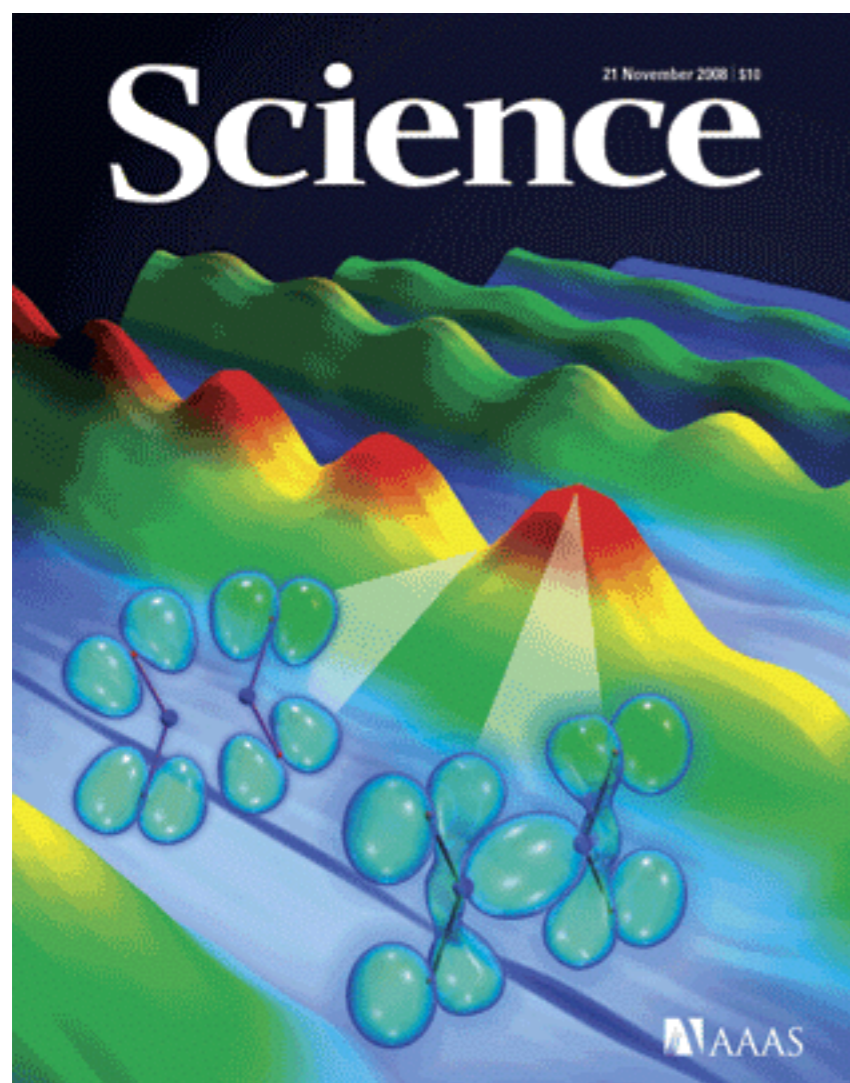
Q C D

Quantum Chromodynamics

## Ab Initio Determination of Light Hadron Masses

S. Dürr, Z. Fodor, J. Frison, C. Hoelbling, R. Hoffmann, S. D. Katz, S. Krieg, T. Kurth, L. Lellouch, T. Lippert, K. K. Szabo and G. Vulvert

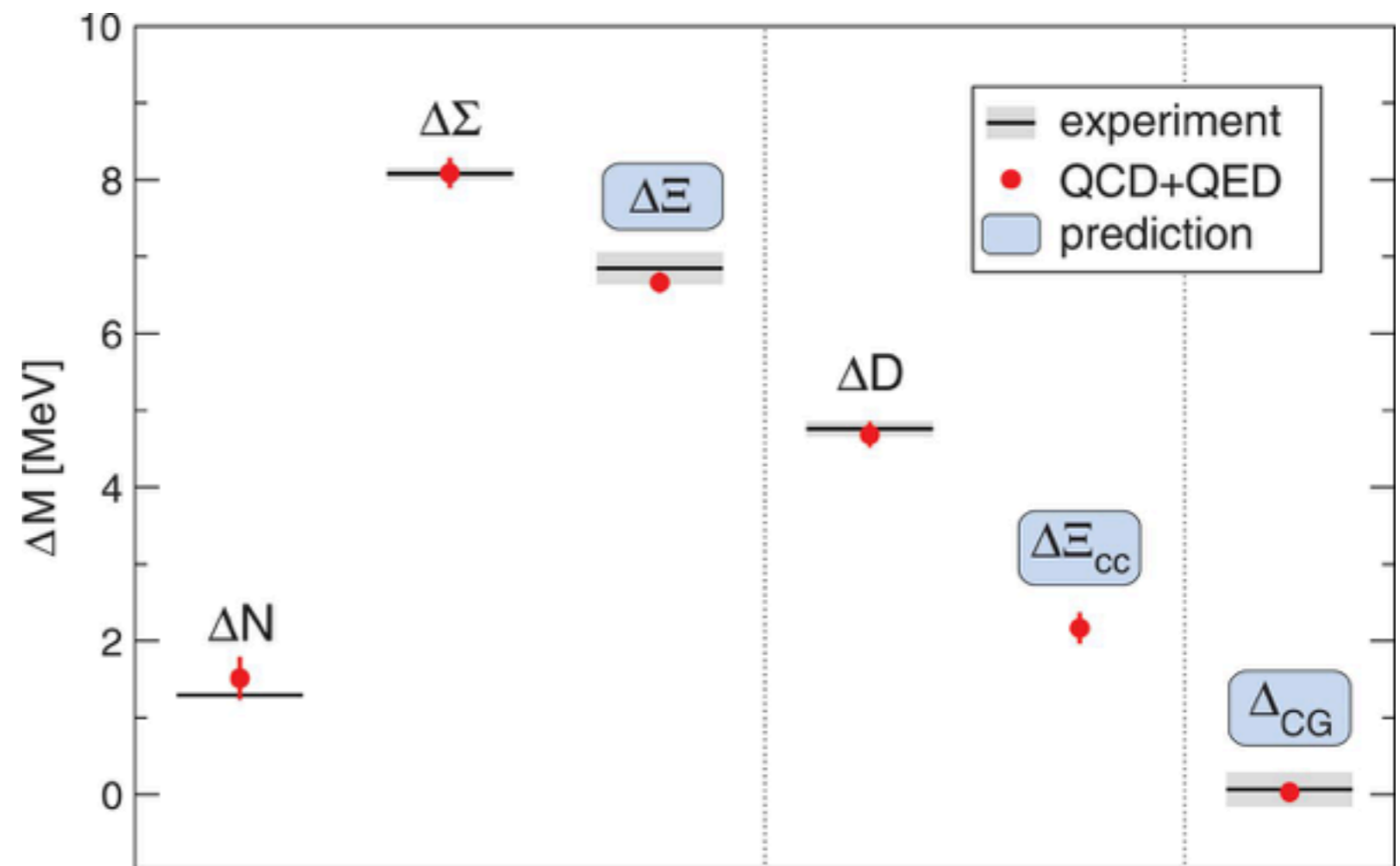
*Science* **322** (5905), 1224-1227.  
DOI: 10.1126/science.1163233



## Ab initio calculation of the neutron-proton mass difference

Sz. Borsanyi, S. Durr, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, L. Lellouch, T. Lippert, A. Portelli, K. K. Szabo and B. C. Toth

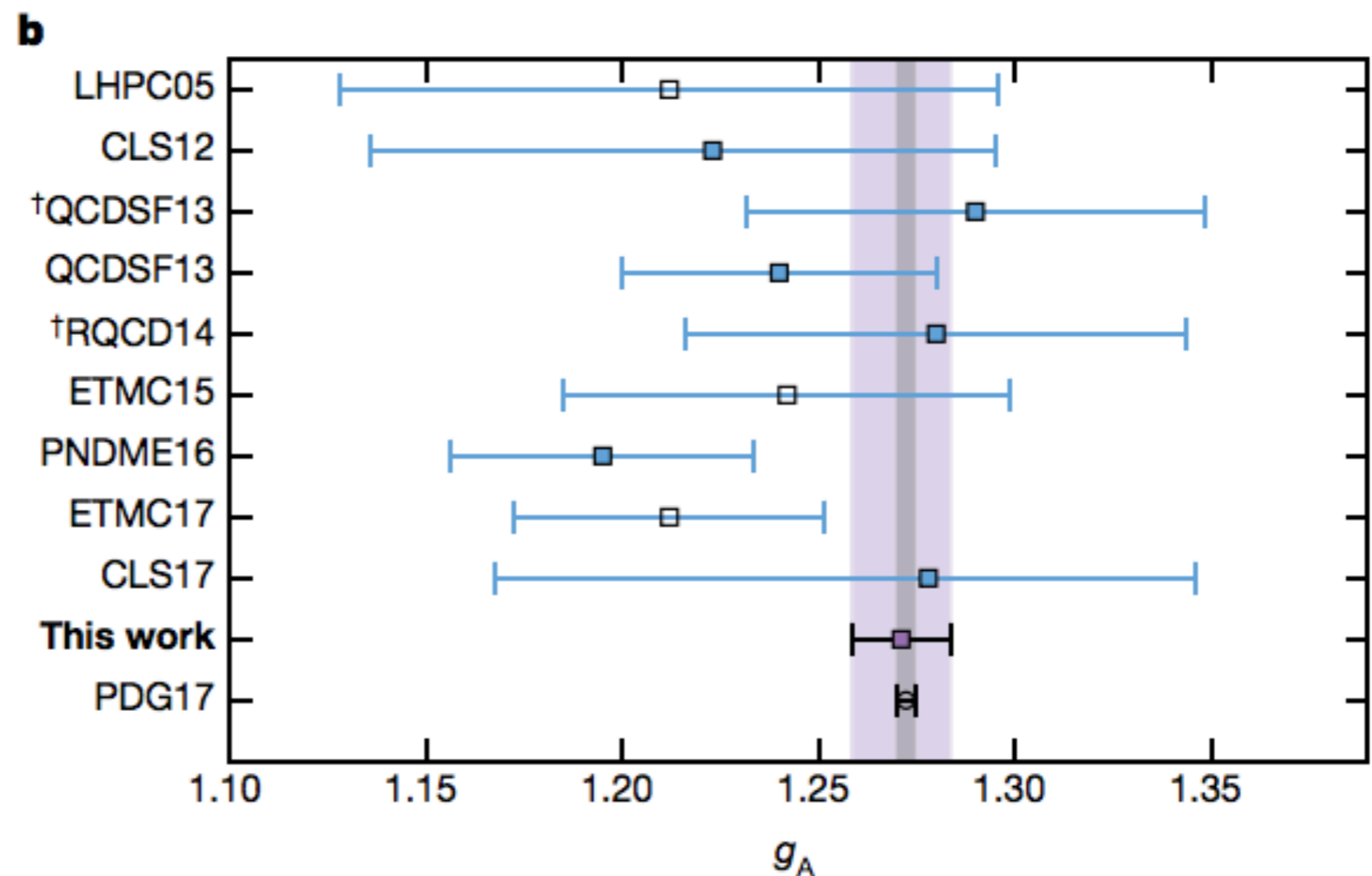
*Science* **347** (6229), 1452-1455.  
DOI: 10.1126/science.1257050



# A per-cent-level determination of the nucleon axial coupling from quantum chromodynamics

C. C. Chang, A. N. Nicholson, E. Rinaldi, E. Berkowitz, N. Garron, D. A. Brantley, H. Monge-Camacho, C. J. Monahan, C. Bouchard, M. A. Clark, B. Joó, T. Kurth, K. Orginos, P. Vranas & A. Walker-Loud ✉

*Nature* **558**, 91–94 (2018)



# Computing from first principles!



# Computation of the masses

$h(x)$ : hadron interpolating field, e.g.  $\pi^+(x) = \bar{u}(x)\gamma_5 d(x)$

$$\langle h(x)h(x+T) \rangle = \frac{\int [\mathcal{D}\psi\bar{\psi}A_\mu] h(x)h(x+T) e^{-\int d^4x \mathcal{L}_{\text{QCD}}}}{\int [\mathcal{D}\psi\bar{\psi}A_\mu] e^{-\int d^4x \mathcal{L}_{\text{QCD}}}}$$

$$\lim_{T \rightarrow \infty} \langle h(x)h(x+T) \rangle \sim e^{-M_h T}$$



**Great, Impressive ...**

# Great, Impressive ...

**BUT,** how precisely those numbers  
come out from  
the QCD Lagrangian ?

# Big questions

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- how precisely are masses generated from almost nothing?
- why are quarks and gluons not seen in isolation?
- dozens of new hadrons (X,Y,Z) have been discovered in the last 10 years. What they are?

# Big questions

- how precisely are masses generated from almost nothing?
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**Our ignorance is profound!**

# Trace anomaly

Take  $m_q = 0$  &  $m_Q = \infty$

$$x^\mu \rightarrow x'^\mu = \lambda x^\mu$$

$$q(x) \rightarrow q'(x) = \lambda^{3/2} q(\lambda x)$$

$$A_\mu(x) \rightarrow A'_\mu(x) = \lambda A_\mu(\lambda x)$$

$$S'_{\text{QCD}} = \int d^4x \lambda^4 \mathcal{L}_{\text{QCD}}(\lambda x) = \int d^4x' \mathcal{L}_{\text{QCD}}(x') = S_{\text{QCD}}$$

**Classical action is invariant**

# Hadron masses

$|h\rangle$ : hadron state  $m_h = \langle h | T_\mu^\mu(x) | h \rangle$

From classical Lagrangian:

$$\frac{\delta S_{\text{QCD}}}{\delta \lambda} = - \int d^4x T_\mu^\mu(x) = 0$$

$$\langle h | T_\mu^\mu | h \rangle = m_h \rightarrow 0$$

# Quantum theory

$$g = g(\mu)$$

$$\delta S_{\text{QCD}} = \delta \left( -\frac{1}{4\pi\alpha_s} \frac{1}{4} \int d^4x \bar{G}_{\mu\nu}^a(x) \bar{G}^{a\mu\nu}(x) \right) = -\frac{2\beta(\alpha_s)}{\alpha_s} S_{\text{QCD}} \delta\lambda$$

$$\begin{aligned} T_{\mu}^{\mu}(x) &= \frac{2\beta(\alpha_s)}{\alpha_s} \frac{1}{4} G_{\mu\nu}^a(x) G^{a\mu\nu}(x) = -\frac{1}{2} b_0 \alpha_s G_{\mu\nu}^a(x) G^{a\mu\nu}(x) \\ &= -\frac{9}{32\pi^2} g^2 G_{\mu\nu}^a(x) G^{a\mu\nu}(x) \end{aligned}$$

- this is the trace anomaly
- no scale invariance
- trace of  $T^{\mu\nu}$  is nonzero

$$m_h = -\frac{9}{32\pi^2} \langle h | g^2 G_{\mu\nu}^a G^{a\mu\nu} | h \rangle$$

The entire mass  
comes from gluons



# Contribution from quark masses

$$m_h = \frac{\beta(\alpha_s)}{2\alpha_s} G_{\mu\nu}^a(x) G^{a\mu\nu}(x) + \langle h | \bar{q} m_q q | h \rangle$$

↑  
small

# Why is this interesting ?

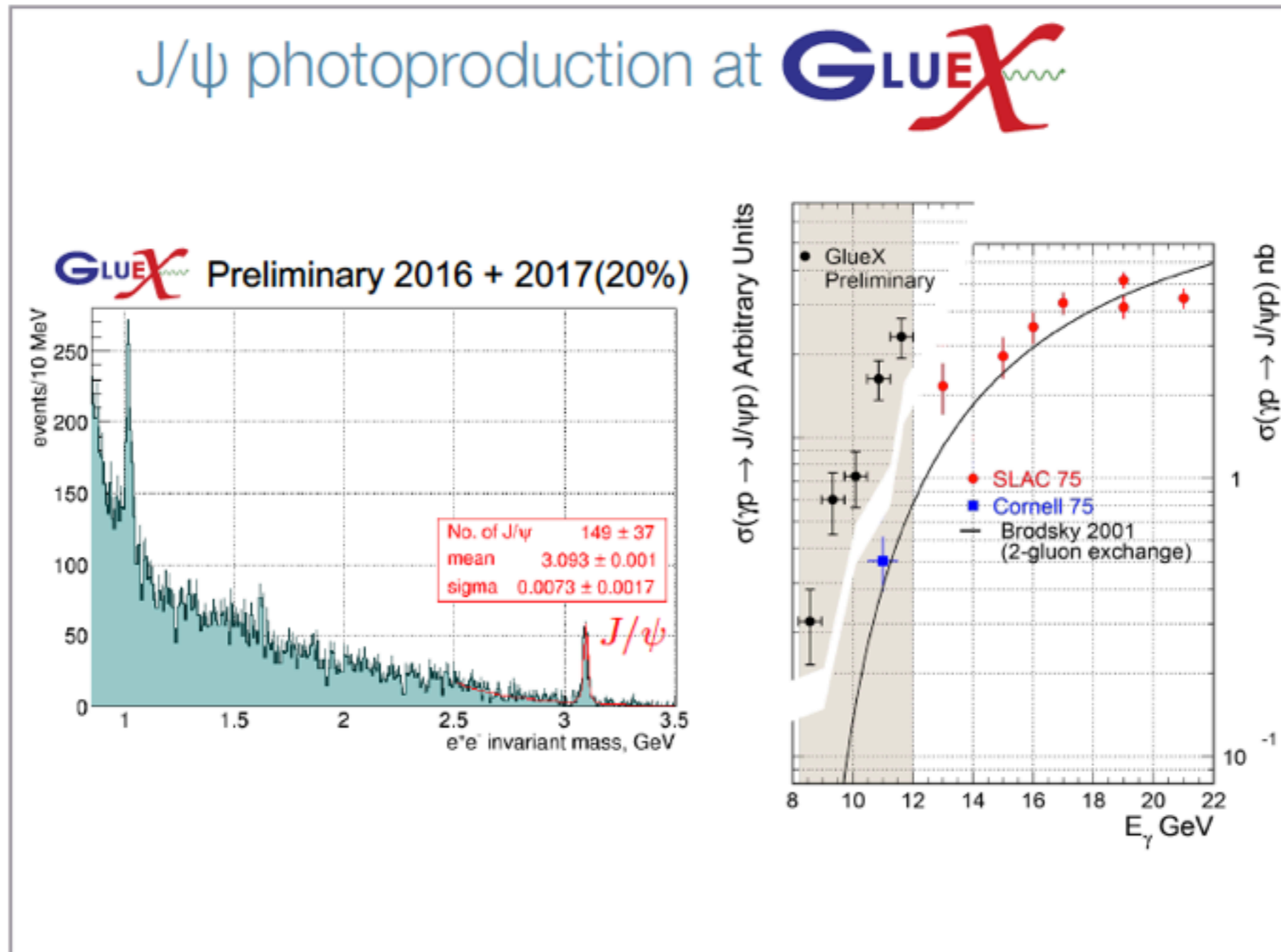
Because

$$\langle h | g^2 G_{\mu\nu}^a G^{a\mu\nu} | h \rangle$$

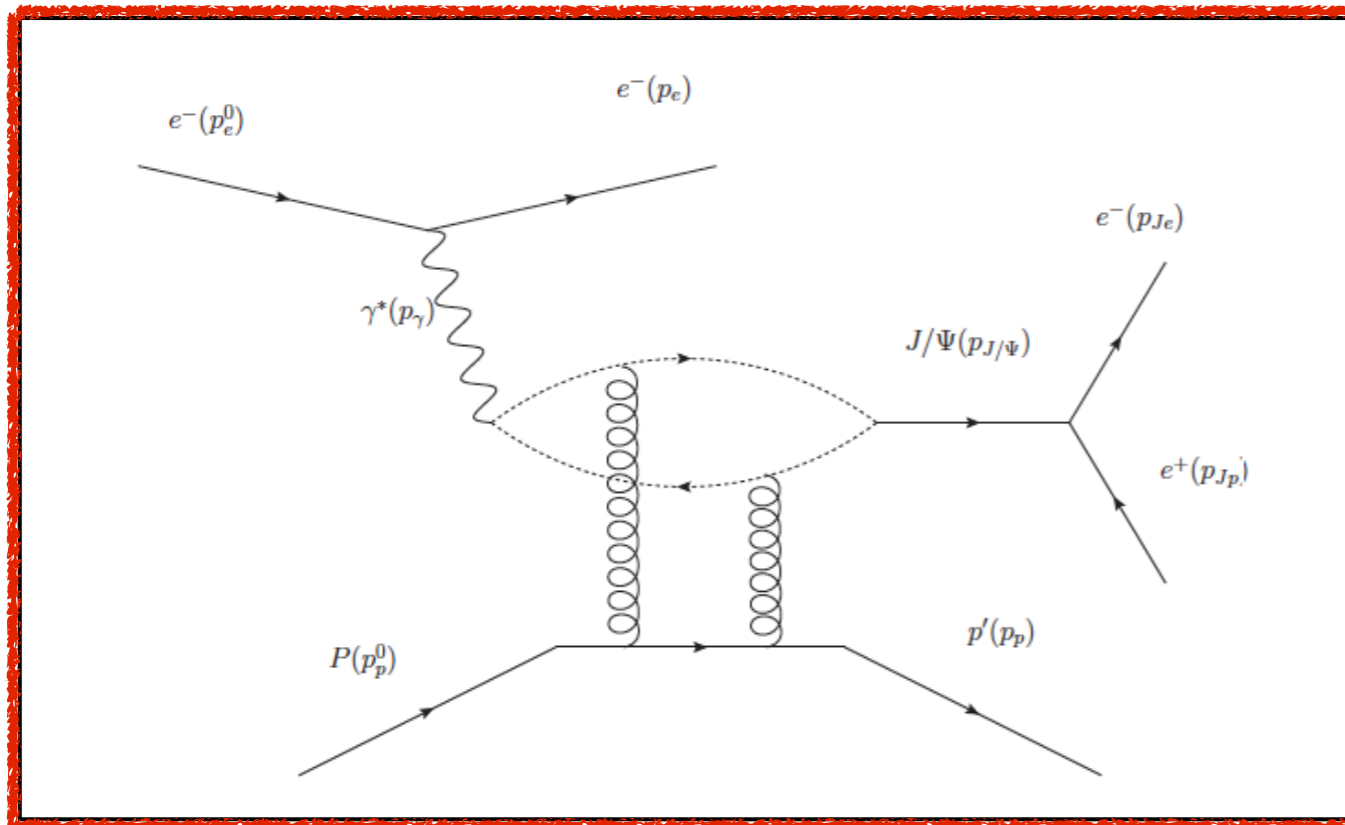
contributes to threshold  
quarkonium-nucleon scattering

# Experiments

## — JLab



# ATHENNA\* collaboration JLab @ 12 GeV



Hall A — E12-12-006

K. Hafidi, Z.-E. Meziani, N. Sparveris, Z.W. Zhao

\*A  $J/\psi$  Threshold Electroproduction on the Nucleon and Nuclei Analysis

Hall C — E-12-16-007 (Pentaquarks)

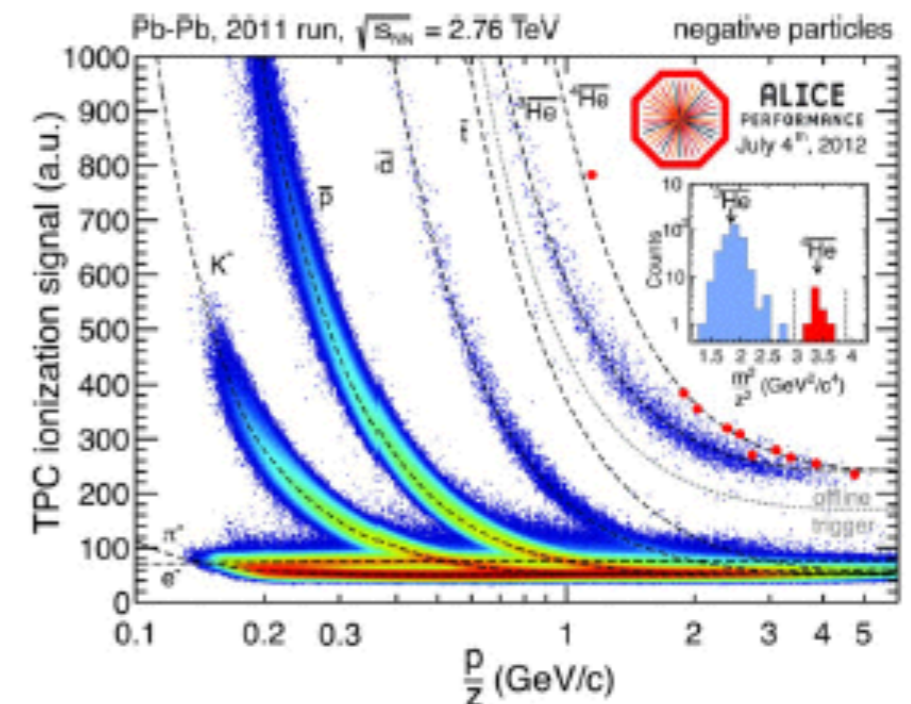
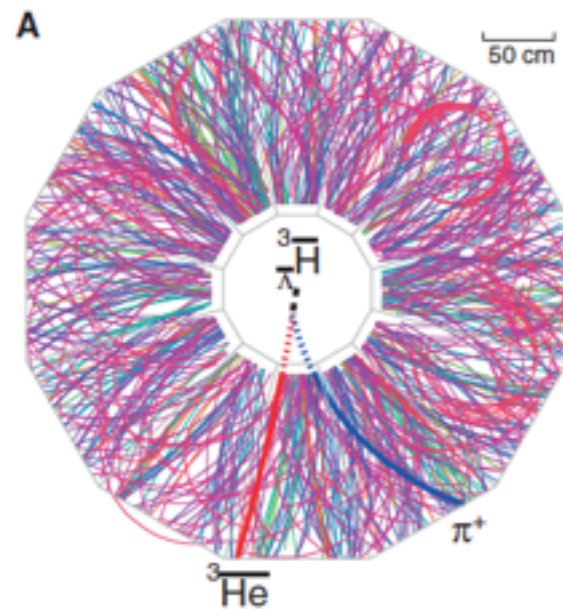
Z.-E. Meziani, S. Joosten, et al.

# Coalescence at the LHC

- Chances of a charmed hadron meeting one or two nucleons **not smaller** than of two antinucleons and one antihyperon meeting to form an antihypernucleus

**Science**  
AAAS

Observation of an Antimatter Hypernucleus  
The STAR Collaboration  
*Science* 328, 58 (2010);  
DOI: 10.1126/science.1183980



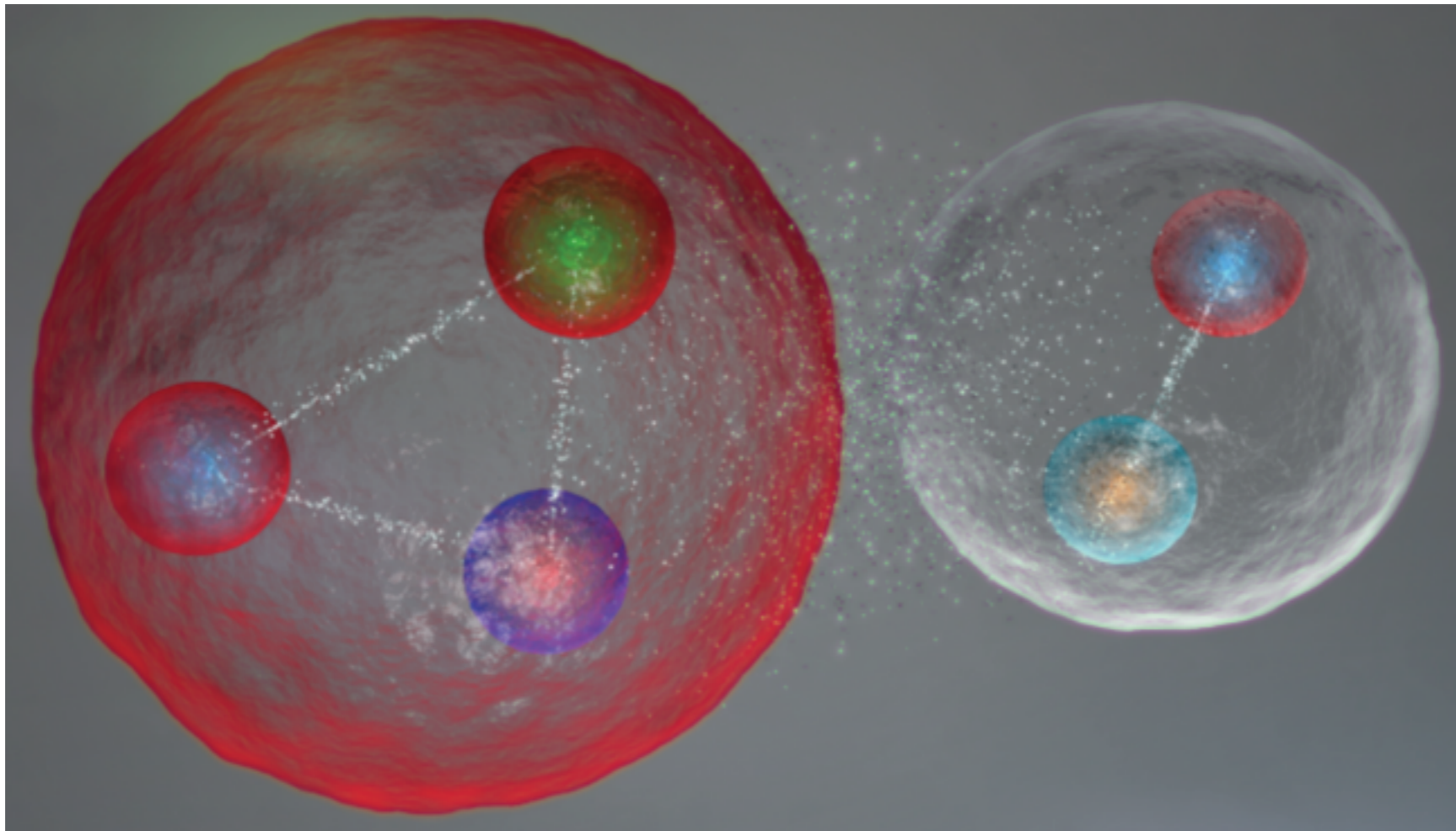
Need to detect in coincidence  
the decay products

# Experiments

## PANDA @ FAIR



# Quarkonium-nucleon



**Quarkonium:**  $\phi(s\bar{s})$ ,  $\eta_c(c\bar{c})$ ,  $J/\Psi(c\bar{c})$ ,  $\eta_b(b\bar{b})$ ,  $\Upsilon(b\bar{b})$

# Quarkonium-nucleon scattering

$$\varphi = \phi(s\bar{s}), \quad \eta_c(c\bar{c}), \quad J/\Psi(c\bar{c}), \quad \eta_b(b\bar{b}), \quad \Upsilon(b\bar{b})$$

## Forward amplitude

$$\mathcal{A}_{\varphi N} = \frac{1}{2} \alpha_{\varphi} \langle N | (g\vec{E})^2 | N \rangle$$

$\alpha_{\varphi}$  : color polarizability  
(property of the quarkonium)



$$\mathcal{A}_{\varphi N} = \frac{1}{2} \alpha_{\varphi} \langle N | (g\vec{E})^2 | N \rangle$$

Measure scattering length:

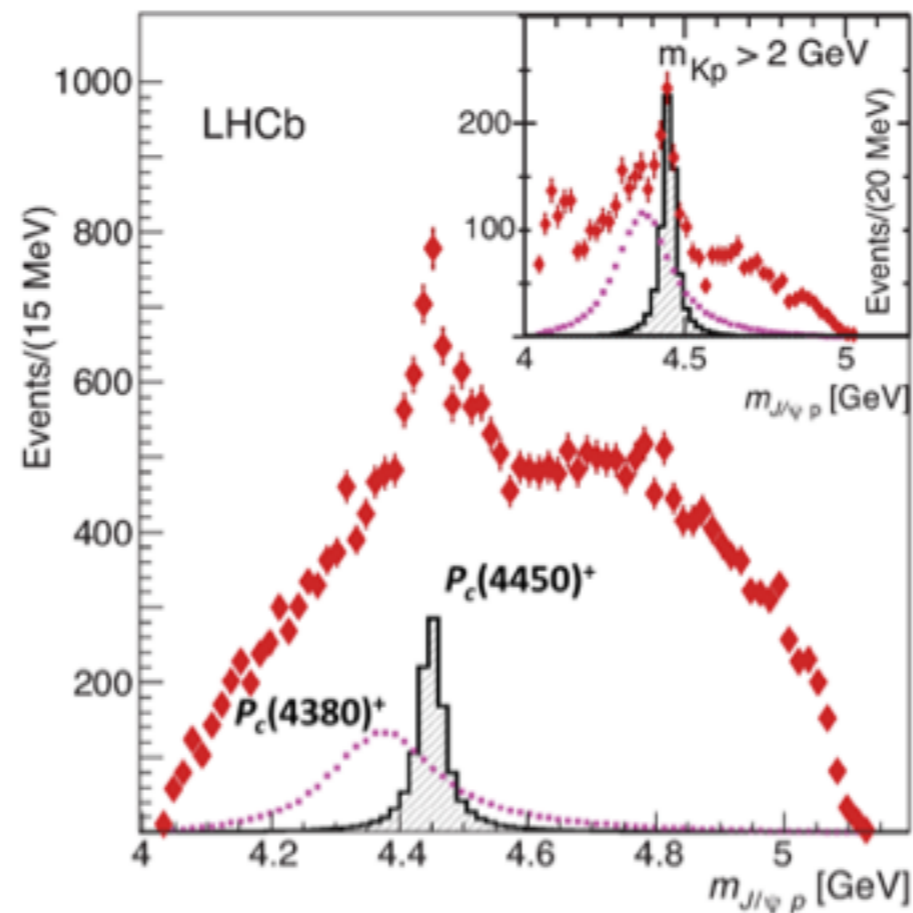
$$a_{\varphi N} = - \left( \frac{\mu_{\varphi N}}{2\pi} \right) \mathcal{A}_{\varphi N} = - \left( \frac{\mu_{\varphi N}}{4\pi} \right) \alpha_{\varphi} \langle N | (g\vec{E})^2 | N \rangle$$

Bound from trace anomaly:

$$\langle N | \left[ (g\vec{E})^2 - (g\vec{B})^2 \right] | N \rangle = -\frac{1}{2} \langle N | g^2 G_{\mu\nu}^a G^{a\mu\nu} | N \rangle = \frac{16\pi^2}{9} m_N \leq \langle N | (g\vec{E})^2 | N \rangle$$

$$a_{\varphi N} \leq - \left( \frac{\mu_{\varphi N}}{4\pi} \right) \frac{16\pi^2}{9} m_N \alpha_{\varphi} = -\frac{4\pi m_N}{9} \mu_{\varphi N} \alpha_{\varphi}$$

# Renewed interest in quarkonium-nucleon



# Quarkonium in nuclei:

- scattering amplitude is enhanced
- formation of a new exotic nuclear state
- adds a new flavor axis in the nuclear e.o.s.

# Scales in nuclei

$$\rho \sim \rho_0 = 0.16 \text{ fm}^{-3}$$

baryon density (center nucleus)

$$d_{\text{av}} \sim \rho^{-1/3} \sim 1.8 \text{ fm}$$

internucleon distance

$$\langle T_N \rangle \sim \frac{3}{5} \frac{k_F^2}{2m_N} \sim 20 \text{ MeV} \ll \Lambda_{\text{QCD}}$$

nucleon kinetic energy

Recall that:

$$r_N \sim \sqrt{\langle r_{\text{ch}}^2 \rangle} \simeq 0.88 \text{ fm} \sim \Lambda_{\text{QCD}}^{-1}$$

nucleon (charge) radius in free space

$$r_{\text{NN}}^{\text{hard-core}} \sim 0.2 \text{ fm}$$

hard-core NN force

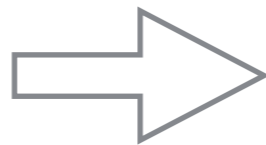
# Scales in nuclei

$$d_{av} \sim 2 r_N + \text{hard-core NN force}$$



Little (or no) superposition of nucleons in nuclei

+ Pauli principle  
(among nucleons)



- Independent-particle model
- Mean-field model
- Nuclear shell model



# Low-momentum quarkonium in a nucleus

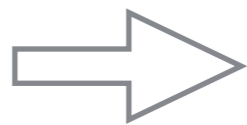
- Quarkonium interacts with light quarks in nucleons by exchanging gluons with wavelengths

$$\lambda \sim r_N$$

- Size of quarkonium

$$r_{J/\Psi} \sim 0.35 \text{ fm}$$

$$\lambda \geq 2 r_{J/\Psi}$$



Quarkonium behaves as  
a small color dipole  
immersed in a uniform gluon field

# Low-momentum quarkonium in a nucleus

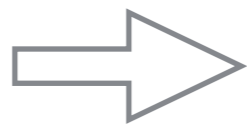
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$$\langle T_N \rangle \sim \frac{3}{5} \frac{k_F^2}{2m_N} \sim 20 \text{ MeV} \ll \Lambda_{\text{QCD}}$$



# Low-momentum quarkonium in a nucleus

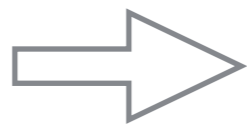
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# Embedding quarkonium-nucleon into a **Nonrelativistic** nuclear many-body problem

$$H = H_N + H_{\varphi N}$$

$$H_{\varphi N} = \int d^3r \varphi^\dagger(t, \vec{r}) \left( -\frac{1}{2m_\varphi} \nabla^2 \right) \varphi(t, \vec{r})$$
$$+ \int d^3r d^3r' N^\dagger(t, \vec{r}) \varphi^\dagger(t, \vec{r}') W_{\varphi N}(\vec{r} - \vec{r}') \varphi(t, \vec{r}') N(t, \vec{r})$$

↑  
**quarkonium-nucleon**

# Hartree-Fock equation

— for quarkonium in a nucleus

$$-\frac{1}{2m_\varphi}\nabla^2\varphi_\alpha(\vec{r}) + W_{\varphi A}(\vec{r})\varphi_\alpha(\vec{r}) = \epsilon_\alpha\varphi_\alpha(\vec{r})$$

$$W_{\varphi A}(\vec{r}) = \int d^3r' W_{\varphi N}(\vec{r} - \vec{r}') \rho_A(\vec{r}') \quad \text{quarkonium-nucleus potential}$$

$$\rho_A(\vec{r}) = \langle A|N^\dagger(\vec{r})N(\vec{r})|A\rangle = \sum_{n=1}^A N_n^*(\vec{r})N_n(\vec{r}) \quad \text{nuclear density functional}$$

Neglecting back reaction of quarkonium on nucleons,  
take density from a model or experiment

# Need quarkonium-nucleon potential

$$W_{\varphi A}(\vec{r}) = \int d^3 r' W_{\varphi N}(\vec{r} - \vec{r}') \rho_A(\vec{r}')$$

From the forward amplitude:

$$W_{\varphi N}^{\text{pol}}(\vec{r}) = \frac{4\pi}{2\mu_{\varphi N}} a_{\varphi N} \delta(\vec{r}) = -\frac{8\pi^2}{9} m_N \alpha_{\varphi} \delta(\vec{r}).$$

$$W_{\varphi A}^{\text{pol}}(\vec{r}) = \frac{4\pi}{2\mu_{\varphi N}} a_{\varphi N} \rho_A(\vec{r}) = -\frac{8\pi^2}{9} m_N \alpha_{\varphi} \rho_A(\vec{r}).$$

$$k \cotan \delta(k) = -\frac{1}{a} + \frac{1}{2} r_e k^2 + \dots$$

# $J/\Psi$ in nuclei

— nuclear potentials

Nuclear density from QMC model:  
P.A.M. Guichon et al.

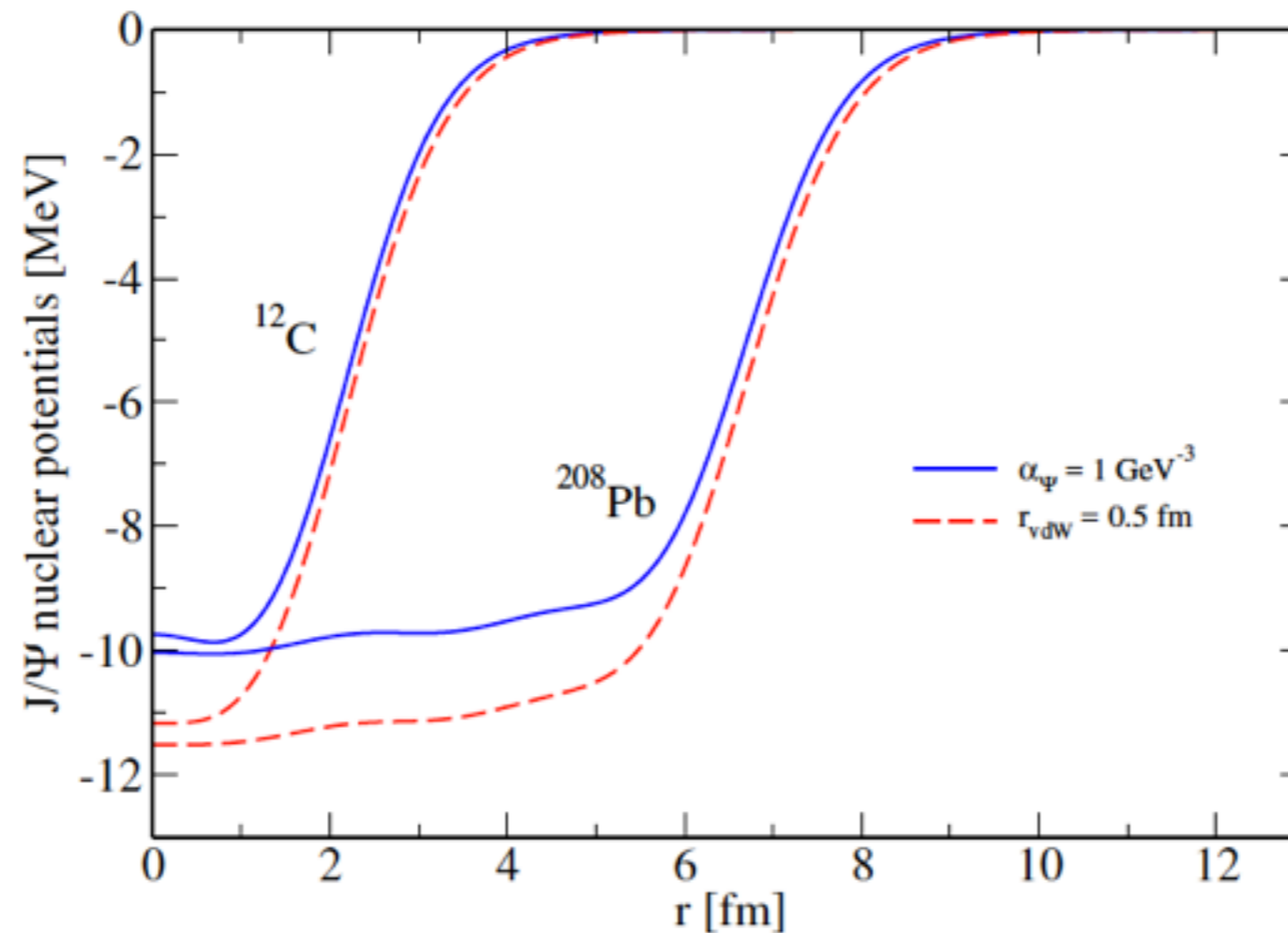


Figure 8:  $J/\Psi$  nuclear potentials  $W_{J/\Psi A}^{\text{pol}}(\vec{r})$  (solid line) for a polarizability  $\alpha_{J/\Psi} = 1 \text{ GeV}^{-3}$  and  $W_{J/\Psi A}^{\text{latt}}(\vec{r})$  (dashed line) from a fit to the lattice data with a cutoff  $r_{\text{vdW}} = 0.5 \text{ fm}$ .

# Quarkonium in nuclei

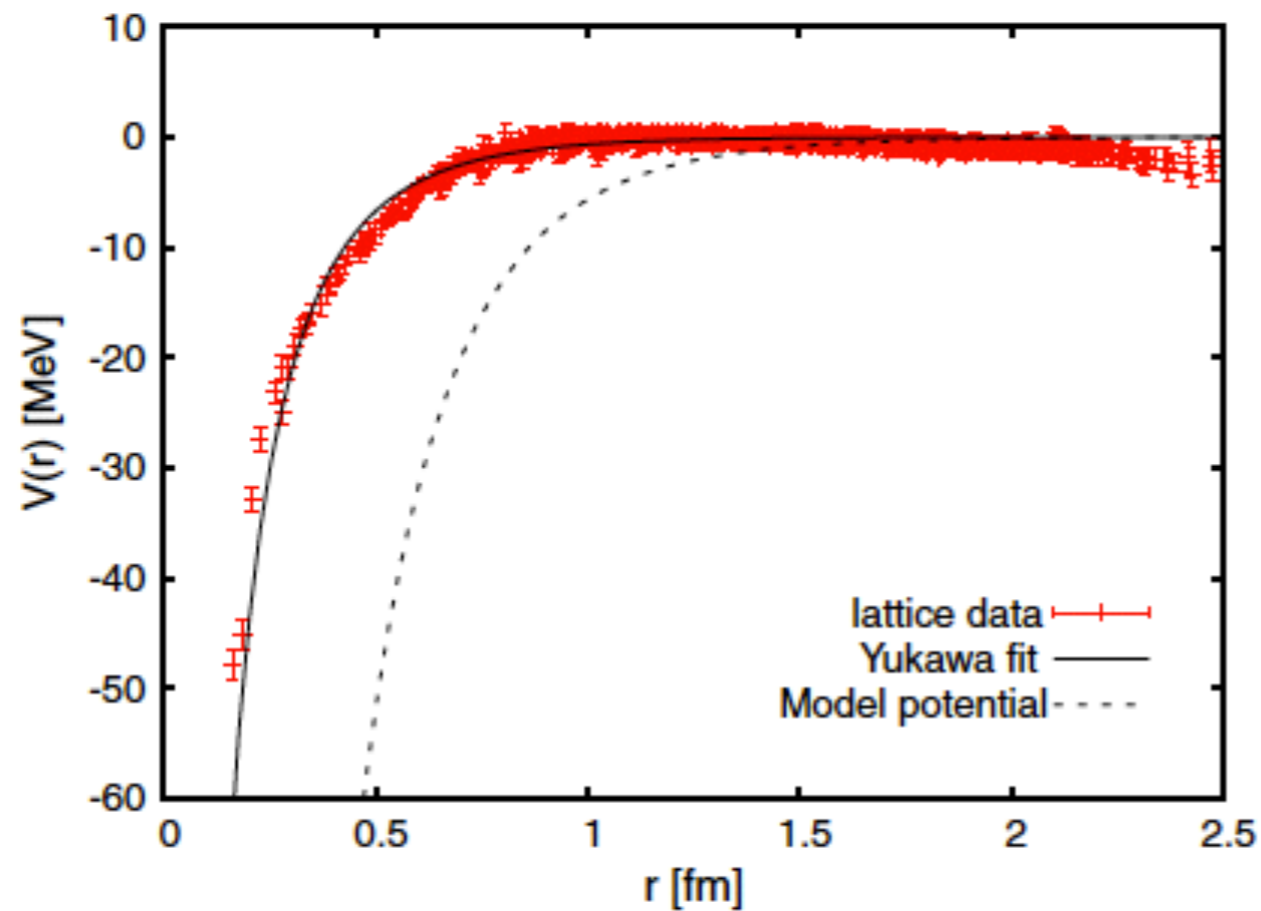
— use scattering length only

Table 7: Predictions for  $J/\Psi$  single-particle energies in several nuclei obtained with the polarization potential  $W_{J/\Psi A}^{\text{pol}}(\vec{r})$ , defined in Eq. (105).

	${}^4_{J/\Psi}\text{He}$	${}^{12}_{J/\Psi}\text{C}$	${}^{16}_{J/\Psi}\text{O}$	${}^{40}_{J/\Psi}\text{Ca}$	${}^{48}_{J/\Psi}\text{Ca}$	${}^{90}_{J/\Psi}\text{Zr}$	${}^{208}_{J/\Psi}\text{Pb}$
$\alpha_{J/\Psi} = 1 \text{ GeV}^{-3} \leftarrow a_{J/\Psi N} = -0.18 \text{ fm}$							
1s	n	-3.36	-4.41	-6.77	-6.84	-7.91	-8.38
1p	n	n	-0.39	-3.47	-3.95	-5.71	-7.05
2s	n	n	n	-0.26	-0.59	-2.70	-5.01
2p	n	n	n	n	n	-0.21	-2.94
3s	n	n	n	n	n	n	-0.70
$\alpha_{J/\Psi} = 2 \text{ GeV}^{-3} \leftarrow a_{J/\Psi N} = -0.36 \text{ fm}$							
1s	-4.49	-10.76	-12.62	-16.41	-16.16	-17.70	-17.27
1p	n	-3.98	-6.54	-11.95	-12.44	-14.95	-16.30
2s	n	n	-0.54	-6.74	-7.50	-11.07	-13.95
2p	n	n	n	-1.62	-2.52	-7.33	-11.41
3s	n	n	n	n	n	-2.71	-8.28

# Lattice\*

— quenched,  $m_\pi = 640$  MeV



Yukawa fit

$$V_{N\eta_c} = -\gamma \frac{e^{-\alpha r}}{r}$$

$$\gamma = 0.1$$

$$\alpha = 3 \text{ fm}^{-1}$$

# Pion mass dependence

— quenched x unquenched

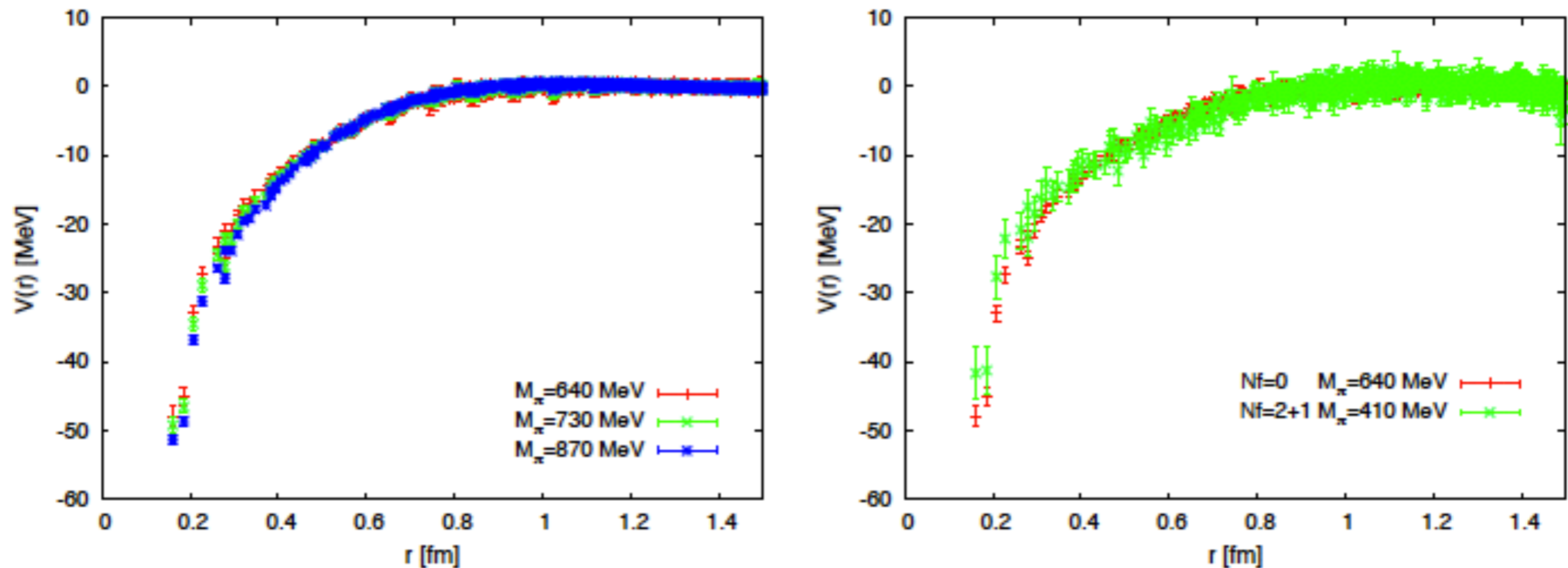


Figure 2: The quark-mass dependence of the  $\eta_c$ - $N$  potential (left) and a comparison between quenched and dynamical simulations (right)



# Scattering length & effective range

— unquenched

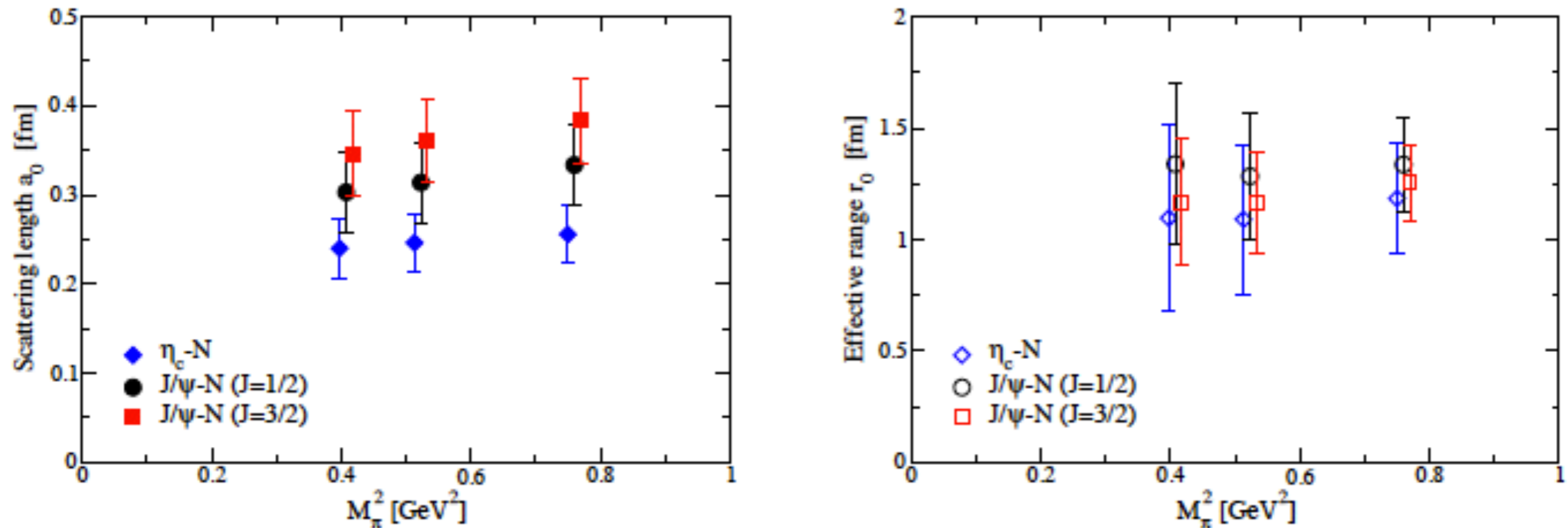


Figure 5: The scattering length  $a_0$  (left) and effective range  $r_0$  (right) as a function of  $M_\pi^2$ . The squared (diamond) symbols have been moved slightly in the plus (minus) x-direction.

# Fit to lattice results

Reproduce scattering length,  
leave effective range free

$$W_{\varphi N}^{\text{latt}}(r) = -W_0 [1 - f(r, r_{\text{vdW}})] + V_{\eta c N}^{\text{fit}}(r) f(r, r_{\text{vdW}})$$

$$f(r, r_{\text{vdW}}) = \frac{1}{1 + (r_{\text{vdW}}/r)^{10}}.$$

$$W_{\varphi A}^{\text{latt}}(\vec{r}) = \int d^3 r' W_{\varphi N}^{\text{latt}}(\vec{r} - \vec{r}') \rho_A(\vec{r}').$$

# Fit to lattice results

$$(a_{J/\Psi N})_{\text{SAV}} \sim 0.35 \text{ fm} > a_{\eta_c N} \sim 0.25 \text{ fm}$$

$$r_e \sim 1.0 \text{ fm}$$

$r_{\text{vdW}}$	$\eta_c N$		$J/\Psi N$	
	$W_0$	$r_e$	$W_0$	$r_e$
0.3	252	1.4	288	1.2
0.5	74	1.7	95	1.4

# $J/\Psi$ in nuclei

— nuclear potentials

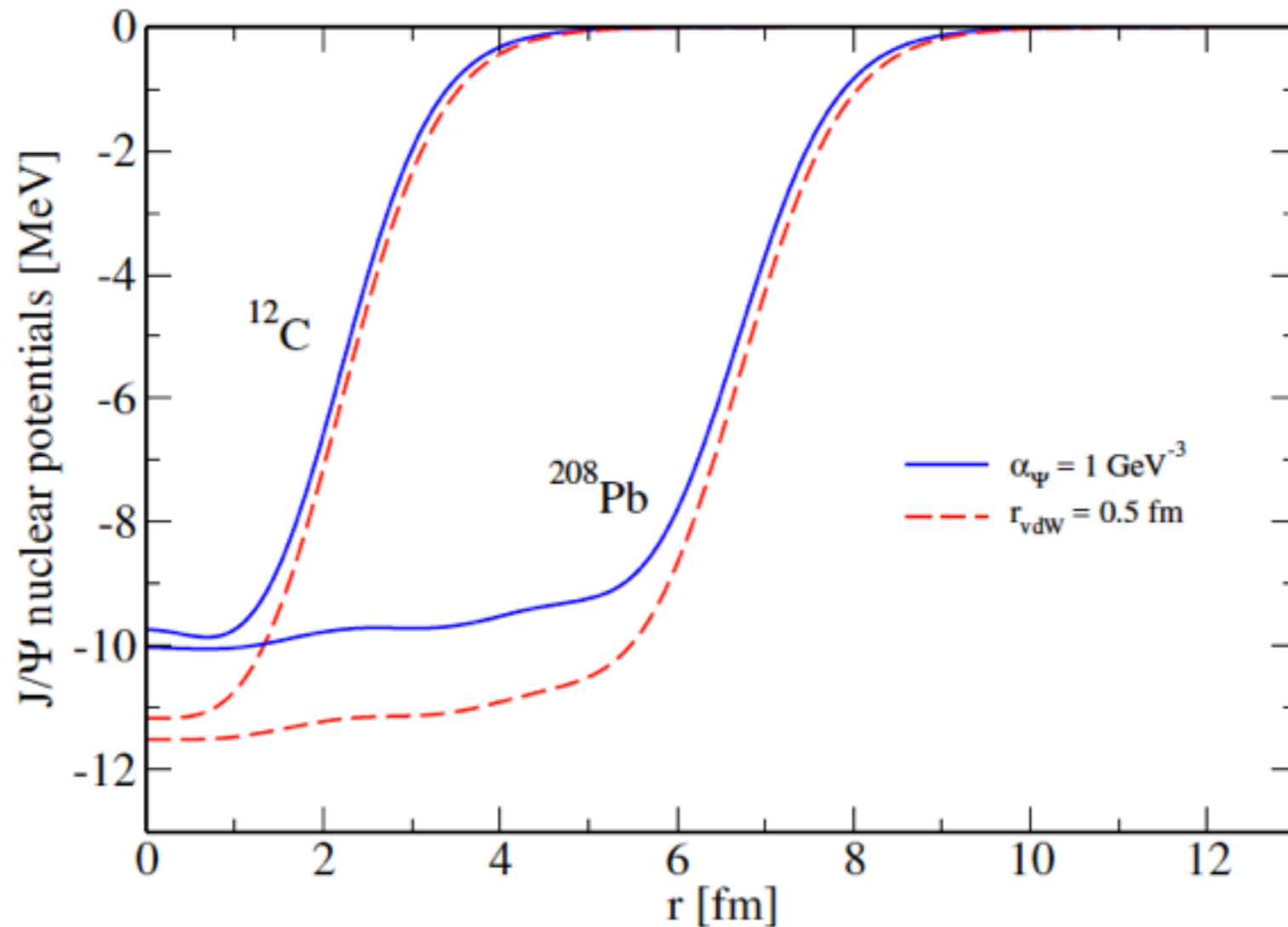


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# Quarkonium in nuclei

## — use of lattice potential

Table 8: Single-particle energies of  $\eta_c$  and  $J/\Psi$  in selected nuclei. The  $\eta_c N$  and  $J/\Psi N$  potentials fit the lattice scattering lengths and incorporate the Yukawa tail from the fit from lattice data.

	$^{16}\text{O}_{\eta_c}$	$^{40}\text{Ca}_{\eta_c}$	$^{90}\text{Zr}_{\eta_c}$	$^{290}\text{Pb}_{\eta_c}$	$^{16}\text{O}_{J/\Psi}$	$^{40}\text{Ca}_{J/\Psi}$	$^{90}\text{Zr}_{J/\Psi}$	$^{290}\text{Pb}_{J/\Psi}$
	$r_{\text{vdW}} = 0.3 \text{ fm}$							
1s	-2.92	-5.15	-6.32	-6.88	-3.62	-5.92	-7.10	-7.62
1p	n	-2.06	-4.17	-5.55	n	-2.74	-4.93	-6.29
2s	n	n	-1.40	-3.53	n	n	-2.06	-4.29
2p	n	n	n	-1.50	n	n	n	-2.30
	$r_{\text{vdW}} = 0.5 \text{ fm}$							
1s	-3.62	-5.99	-7.23	-7.79	-5.23	-7.95	-9.24	-9.74
1p	n	-2.72	-4.99	-6.41	-0.87	-4.41	-6.90	-8.33
2s	n	n	-2.04	-4.33	n	-0.82	-3.71	-6.20
2p	n	n	n	-2.28	n	n	-0.92	-4.03

## Quarkonium-nucleus bound states from lattice QCD

S. R. Beane,<sup>1</sup> E. Chang,<sup>1,2</sup> S. D. Cohen,<sup>2</sup> W. Detmold,<sup>3</sup> H.-W. Lin,<sup>1</sup> K. Orginos,<sup>4,5</sup> A. Parreño,<sup>6</sup> and M. J. Savage<sup>2</sup>  
(NPLQCD Collaboration)

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Cambridge, Massachusetts 02139, USA*

<sup>4</sup>*Department of Physics, College of William and Mary, Williamsburg, Virginia 23187-8795, USA*

<sup>5</sup>*Jefferson Laboratory, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*

<sup>6</sup>*Departament d'Estructura i Constituents de la Matèria and Institut de Ciències del Cosmos, Universitat  
de Barcelona, Martí i Franquès 1, Barcelona, 08028, Spain*

(Received 9 November 2014; published 11 June 2015)

Quarkonium-nucleus systems are composed of two interacting hadronic states without common valence quarks, which interact primarily through multigluon exchanges, realizing a color van der Waals force. We present lattice QCD calculations of the interactions of strange and charm quarkonia with light nuclei. Both the strangeonium-nucleus and charmonium-nucleus systems are found to be relatively deeply bound when the masses of the three light quarks are set equal to that of the physical strange quark. Extrapolation of these results to the physical light-quark masses suggests that the binding energy of charmonium to nuclear matter

is  $B_{\text{phys}}^{\text{NM}} \lesssim 40 \text{ MeV}$ .

# Models

TABLE I. Estimates for the binding energies of charmonium to light nuclei and nuclear matter (in MeV) from selected models. A “\*” indicates the system is predicted to be unbound, while entries with center dots indicate that the system was not addressed

Ref.	Binding energy (MeV)			Binding energy (MeV)	
	${}^3\text{He } \eta_c$	${}^4\text{He } \eta_c$	NM $\eta_c$	${}^4\text{He } J/\psi$	NM $J/\psi$
[1]	19	140	*		
[2]	0.8	5	27		
[3]			10		10
[5]	*	*	9		
[6]					5
[7]				5	18
[8]				15.7	

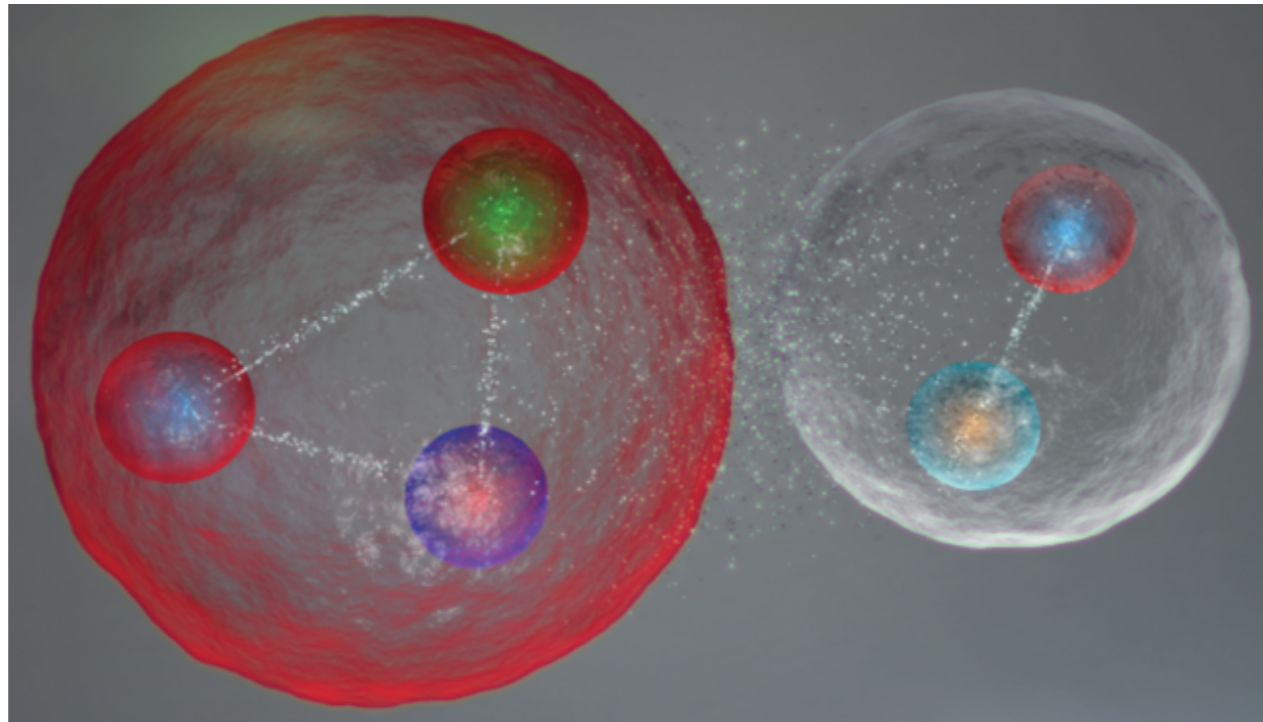


TABLE V. The binding energies (in MeV) of charmonium-nucleus systems calculated on the  $L = 24$  and  $32$  ensembles. The rightmost column shows the infinite-volume estimate, which, without results on the  $L = 48$  ensemble, is taken to be the binding calculated on the  $L = 32$  ensemble. The first and second sets of parentheses shows the statistical and quadrature-combined statistical plus systematic uncertainties, respectively.

System	$24^3 \times 64$	$32^3 \times 64$	$L = \infty$
$N\eta_c$	17.9(0.4)(1.5)	19.8(0.7)(2.6)	19.8(2.6)
$d\eta_c$	39.3(1.3)(4.8)	42.4(1.1)(7.9)	42.4(7.9)
$pp\eta_c$	37.8(1.1)(4.5)	41.5(1.0)(7.5)	41.5(7.6)
${}^3\text{He}\eta_c$	57.2(1.3)(8.3)	56.7(2.0)(9.4)	56.7(9.6)
${}^4\text{He}\eta_c$	70(02)(13)	56(06)(17)	56(18)
${}^4\text{He}J/\psi$	75.7(1.9)(9.4)	53(07)(18)	53(19)



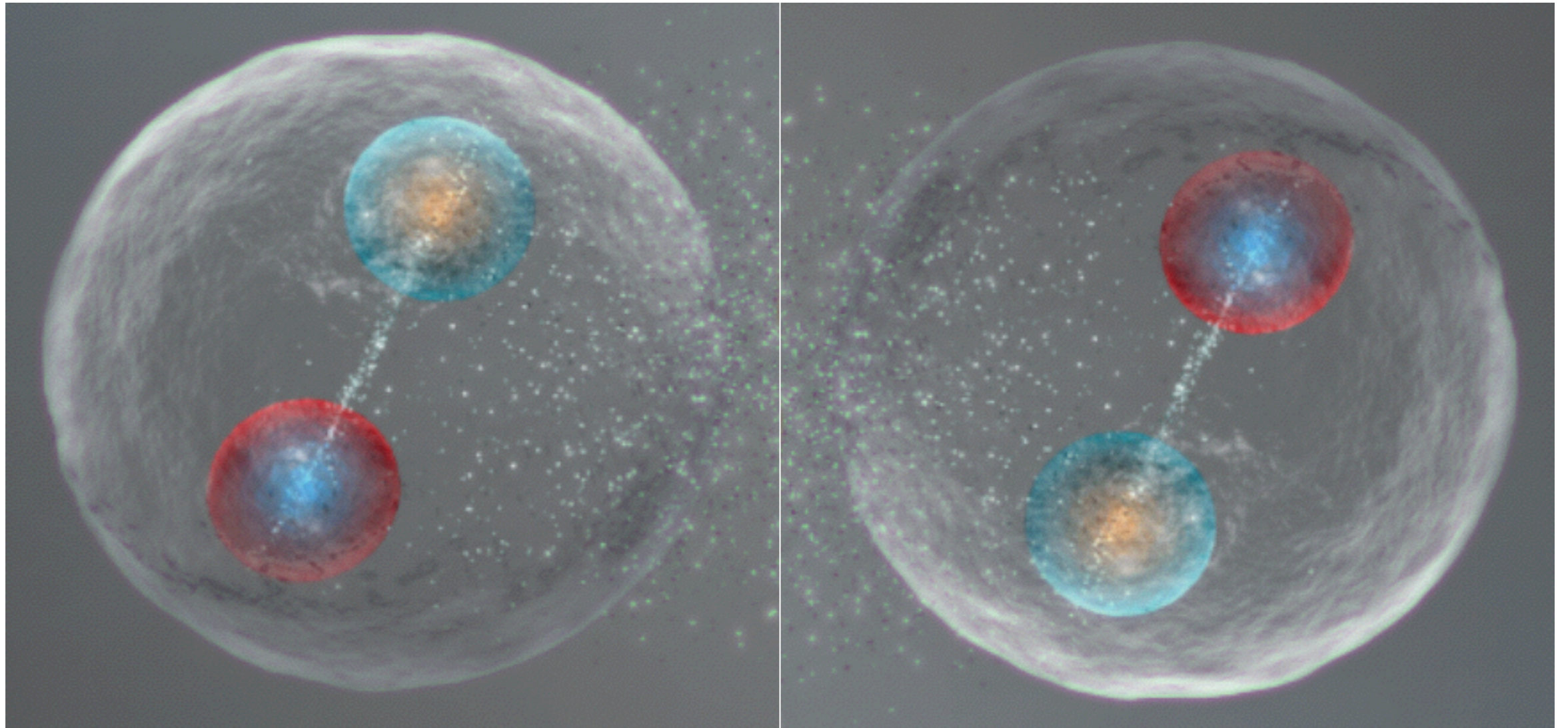
# NPLQCD



**Can one do better?**



$$\eta_b - \eta_b$$



# Chromopolarizability & color van der Waals forces — an EFT perspective

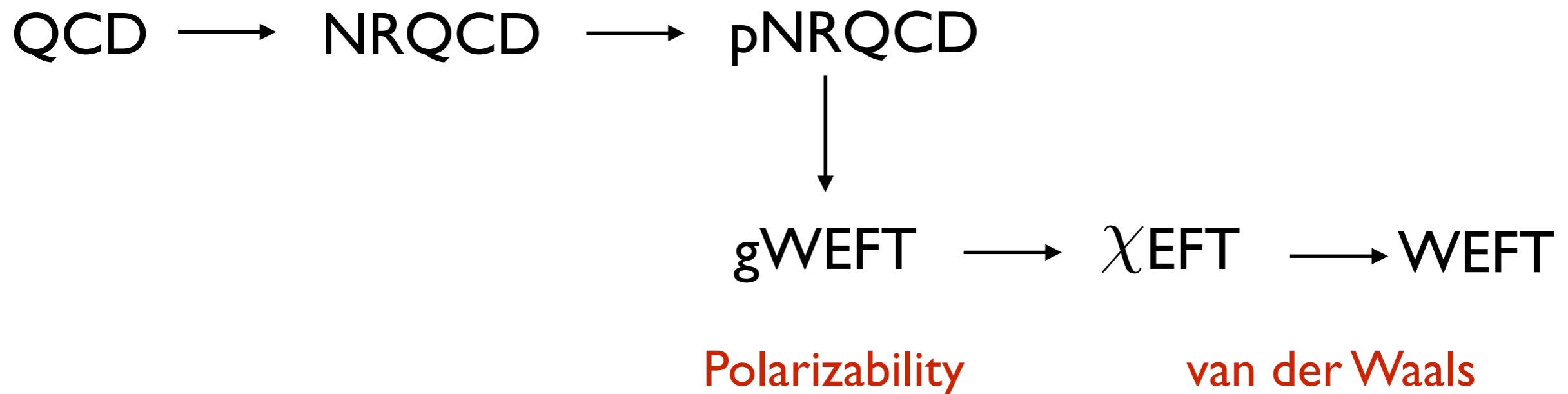
## Interactions between color neutral objects:

Via creation of instantaneous color dipole moments &  
gluon transitions in virtual color-octet intermediate state

— Polarizability—

# EFT approach

- **Chromopolarizability** of IS bottomonium;  
use pNRQC (potential Nonrelativistic QCD)
- **van der Waals force** between two bottomonia;  
use QCD trace anomaly to match pNRQC to a chiral EFT



# Scales

$m$  : bottom mass,  $v$  : relative velocity

$$m \gg mv \gg mv^2 \gg \Lambda_{\text{QCD}}$$

QCD  $\longrightarrow$  NRQCD  $\longrightarrow$  pNRQCD  $\longrightarrow$  gWEFT

$m_\phi$  : mass bottomonium,  $r_{\phi\phi} \sim 1/m_\pi$  : relative distance

$$\mathbf{k}_{\phi\phi}^2/m_\phi = m_\pi^2/m_\phi \ll m_\pi$$

gWEFT  $\longrightarrow$   $\chi$ EFT  $\longrightarrow$  WEFT

# pNRQCD\*

— obtain e.g. bound states

S,O: singlet, octet  $Q\bar{Q}$  states

$$\mathcal{L}_{\text{pNRQCD}} = \mathcal{L}_{\text{light}} + \int d^3r \left\{ \text{Tr} \left[ S^\dagger (i\partial_0 - h_s) S + O^\dagger (iD_0 - h_o) O \right] \right. \\ \left. + gV_A(r) \text{Tr} \left[ O^\dagger(\vec{r} \cdot \vec{E})S + S^\dagger(\vec{r} \cdot \vec{E})O \right] + \frac{g}{2}V_B(r) \text{Tr} \left[ O^\dagger(\vec{r} \cdot \vec{E})O + O^\dagger O(\vec{r} \cdot \vec{E}) \right] \right\}$$

$$h_s = -\frac{\nabla_r^2}{m_Q} - \frac{\nabla_R^2}{4m_Q} + V_s(r)$$

$$h_o = -\frac{\nabla_r^2}{m_Q} - \frac{D_R^2}{4m_Q} + V_o(r)$$

$$V_s(r) = -C_F \frac{\alpha_s(r)}{r},$$

$$V_o(r) = \left( \frac{C_A}{2} - C_F \right) \frac{\alpha_s(r)}{r},$$

$$V_A(r) = 1,$$

$$V_B(r) = 1,$$

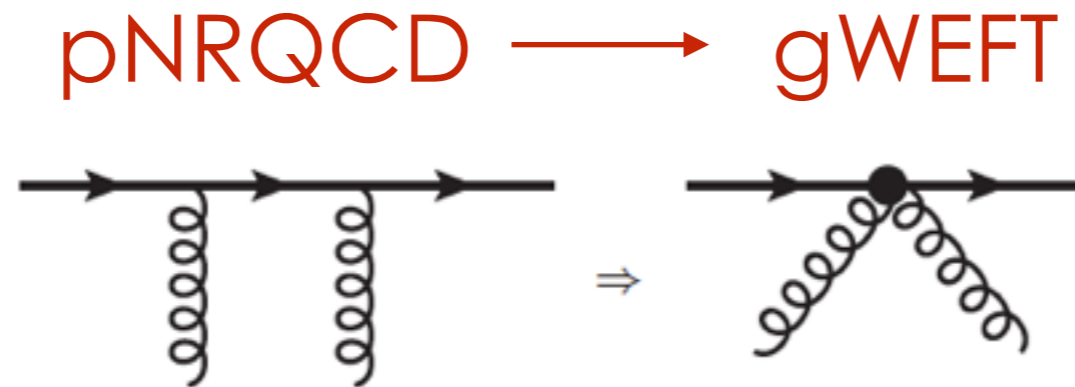
$$C_A = N_c = 3, C_F = (N_c^2 - 1)/(2N_c) \text{ and } T_F = 1/2$$

\* A. Pineda and J. Soto, Nucl. Phys. B, Proc. Suppl. 64, 428 (1998)

N. Brambilla, A. Pineda, J. Soto, and A. Vairo, Nucl. Phys. B 566, 275 (2000)

# gWEFT

— hadronization, chromopolarizability



$$\mathcal{L}_{\text{gWEFT}} = \mathcal{L}_{\text{light}} + \varphi^\dagger(t, \vec{R}) \left( i\partial_0 - E_\varphi + \frac{\nabla_{\vec{R}}^2}{4m} + \frac{1}{2}\alpha_\varphi g^2 \vec{E}^2 + \dots \right) \varphi(t, \vec{R})$$

Chromopolarizability

$$\alpha_\varphi = -\frac{2V_A^2 T_F}{3N_c} \langle \varphi | r^i \frac{1}{E_\varphi - h_0} r^i | \varphi \rangle$$

# Results: polarizability

$$E_\varphi = -m_Q \frac{(C_F \alpha_s)^2}{4} = -\frac{1}{m_Q a_0}$$

$$m_\varphi = 9.4454 \text{ GeV} \left\{ \begin{array}{l} \text{average of} \\ \eta_b \text{ \& } \Upsilon_b(1S) \end{array} \right.$$

$$\alpha_s(1 \text{ GeV}) \approx 0.5$$

$$m = 5 \text{ GeV}$$

$$\alpha_s(1.5 \text{ GeV}) \approx 0.35$$

$$\alpha_{\eta_b} = 0.50^{+0.42}_{-0.38} \text{ GeV}^{-3}$$

$$\alpha_s(2 \text{ GeV}) \approx 0.3$$

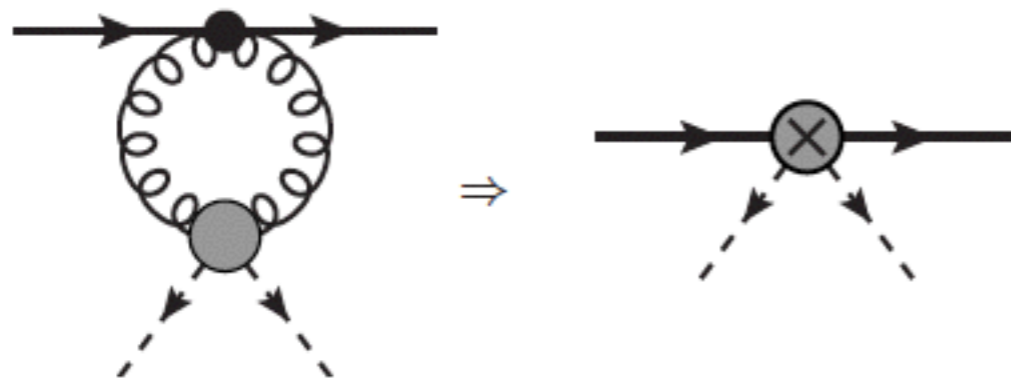
$$\Upsilon(2S) \rightarrow \Upsilon(1S)\pi\pi$$

$$\beta_{\Upsilon-\Upsilon'} = 0.66 \text{ GeV}^{-3}$$

# van der Waals force

$g\text{WEFT} \longrightarrow \chi\text{EFT}$

Nonperturbative matching



QCD trace anomaly

$$g^2 \langle \pi^+(p_1) \pi^-(p_2) | \mathbf{E}_a^2 | 0 \rangle = \frac{8\pi^2}{3b} \left( (p_1 + p_2)^2 \kappa_1 + m_\pi^2 \kappa_2 \right)$$

$$\kappa_1 = 1 - 9\kappa/4, \quad \kappa_2 = 1 - 9\kappa/2$$

$$b = \frac{11}{3}N_c - \frac{2}{3}N_f$$

$$\kappa = 0.186 \pm 0.003 \pm 0.006$$



$$\psi' \rightarrow J/\psi \pi^+ \pi^-$$



# Matching

$g\text{WEFT} \longrightarrow \chi\text{EFT}$

$$\mathcal{L}_{\chi\text{EFT}}^{\phi} = \phi^{\dagger} \left( i\partial_0 - \frac{\nabla^2}{2m_{\phi}} \right) \phi$$

$$\mathcal{L}_{\chi\text{EFT}}^{\pi} = \frac{F^2}{4} (\langle \partial_{\mu} U \partial^{\mu} U^{\dagger} \rangle + \langle \chi^{\dagger} U + \chi U^{\dagger} \rangle)$$

$$U = e^{i\phi/F} = u^2, \quad \phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

$$\chi = 2B\hat{m}\mathbf{1} \quad F = F_{\pi} = 92.419 \text{ MeV}$$

$$\mathcal{L}_{\chi\text{EFT}}^{\phi-\pi} = \phi^{\dagger} \phi \frac{F^2}{4} (c_{d0} \langle \partial_0 U \partial_0 U^{\dagger} \rangle + c_{di} \langle \partial_i U \partial^i U^{\dagger} \rangle + c_m \langle \chi^{\dagger} U + \chi U^{\dagger} \rangle)$$

$$c_{d0} = -\frac{4\pi^2 \alpha_{\phi}}{b} \kappa_1$$

$$c_{di} = -\frac{4\pi^2 \alpha_{\phi}}{b} \kappa_2$$

$$c_m = -\frac{12\pi^2 \alpha_{\phi}}{b}$$

# van der Waals force

$$r_{\phi\phi} \sim 1/m_\pi \qquad \mathbf{k}_{\phi\phi}^2/m_\phi = m_\pi^2/m_\phi \ll m_\pi$$

Relative motion at energies lower than pion mass

— integrate out the pion

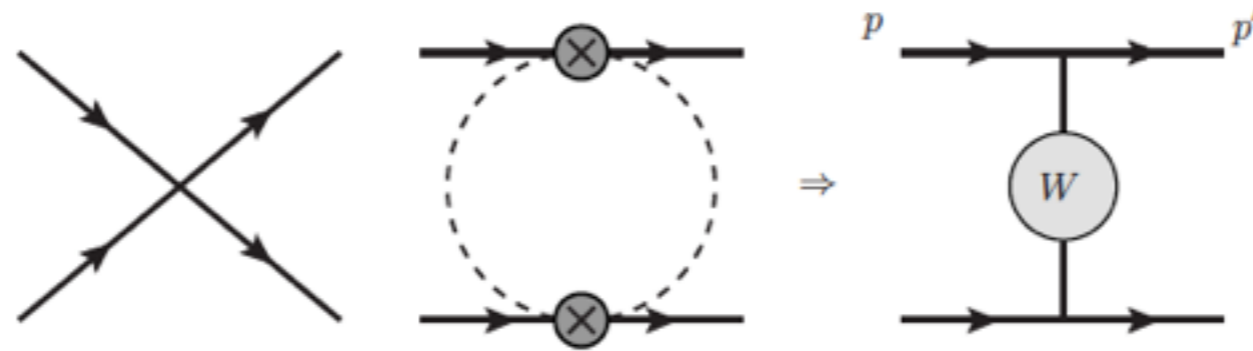
$\chi$ EFT  $\longrightarrow$  WEFT

# Matching

~~$\chi$~~  EFT



WEFT



$$L_{\varphi\varphi} = \int d^3R \varphi^\dagger(t, \vec{R}) \left( i\partial_0 + \frac{\nabla^2}{2m_\varphi} \right) \varphi(t, \vec{R}) - \frac{1}{2} \int d^3R d^3R' \varphi^\dagger(t, \vec{R}) \varphi^\dagger(t, \vec{R}') W_{\varphi\varphi}(\vec{R}, \vec{R}') \varphi(t, \vec{R}') \varphi(t, \vec{R})$$

# vdW potential

$$\begin{aligned}\widetilde{W}(\mathbf{k}^2) &= \text{contact terms} \\ &- \frac{3}{8} (\mathbf{k}^2 c_{di} + 2m_\pi^2 (c_{di} - c_m))^2 B [m_\pi^2, -\mathbf{k}^2] \\ &- \frac{3}{2} (c_{d0} - c_{di}) (\mathbf{k}^2 c_{di} + 2m_\pi^2 (c_{di} - c_m)) C_1 [m_\pi^2, -\mathbf{k}^2] \\ &- \frac{3}{2} (c_{d0} - c_{di})^2 C_2 [m_\pi^2, -\mathbf{k}^2]\end{aligned}$$



$$B [m_\pi^2, -\mathbf{k}^2] = \frac{1}{16\pi^2} \left( \lambda + 1 - \log \frac{m_\pi^2}{\nu^2} + \sqrt{1 + \frac{4m_\pi^2}{\mathbf{k}^2}} \log \left[ \frac{\sqrt{1 + \frac{4m_\pi^2}{\mathbf{k}^2}} - 1}{\sqrt{1 + \frac{4m_\pi^2}{\mathbf{k}^2}} + 1} \right] \right)$$

$$B [m_\pi^2, -\mathbf{k}^2] = \frac{1}{16\pi^2} \left( \lambda + 1 - \log \frac{m_\pi^2}{\nu^2} + \sqrt{1 + \frac{4m_\pi^2}{\mathbf{k}^2}} \log \left[ \frac{\sqrt{1 + \frac{4m_\pi^2}{\mathbf{k}^2}} - 1}{\sqrt{1 + \frac{4m_\pi^2}{\mathbf{k}^2}} + 1} \right] \right)$$

$$C_2 [m_\pi^2 - \mathbf{k}^2] = \frac{31\mathbf{k}^4 + 280\mathbf{k}^2 m_\pi^2 + 705m_\pi^4}{19200\pi^2} + \frac{1}{1280\pi^2} \left( (\mathbf{k}^4 + 10\mathbf{k}^2 m_\pi^2 + 30m_\pi^4) \right. \\ \left. \times \left( \lambda - \log \frac{m_\pi^2}{\nu^2} \right) + \mathbf{k}^4 \left( 1 + \frac{4m_\pi^2}{\mathbf{k}^2} \right)^{5/2} \log \left[ \frac{\sqrt{1 + \frac{4m_\pi^2}{\mathbf{k}^2}} - 1}{\sqrt{1 + \frac{4m_\pi^2}{\mathbf{k}^2}} + 1} \right] \right)$$

$$B [m_\pi^2, -\mathbf{k}^2] = \frac{1}{16\pi^2} \left( \lambda + 1 - \log \frac{m_\pi^2}{\nu^2} + \sqrt{1 + \frac{4m_\pi^2}{\mathbf{k}^2}} \log \left[ \frac{\sqrt{1 + \frac{4m_\pi^2}{\mathbf{k}^2}} - 1}{\sqrt{1 + \frac{4m_\pi^2}{\mathbf{k}^2}} + 1} \right] \right)$$

$$C_2 [m_\pi^2 - \mathbf{k}^2] = \frac{31\mathbf{k}^4 + 280\mathbf{k}^2 m_\pi^2 + 705m_\pi^4}{19200\pi^2} + \frac{1}{1280\pi^2} \left( (\mathbf{k}^4 + 10\mathbf{k}^2 m_\pi^2 + 30m_\pi^4) \times \left( \lambda - \log \frac{m_\pi^2}{\nu^2} \right) + \mathbf{k}^4 \left( 1 + \frac{4m_\pi^2}{\mathbf{k}^2} \right)^{5/2} \log \left[ \frac{\sqrt{1 + \frac{4m_\pi^2}{\mathbf{k}^2}} - 1}{\sqrt{1 + \frac{4m_\pi^2}{\mathbf{k}^2}} + 1} \right] \right)$$

$$C_1 [m_\pi^2 - \mathbf{k}^2] = \frac{5\mathbf{k}^2 + 24m_\pi^2}{576\pi^2} + \frac{1}{192\pi^2} \left( (\mathbf{k}^2 + 6m_\pi^2) \left( \lambda - \log \frac{m_\pi^2}{\nu^2} \right) + \mathbf{k}^2 \left( 1 + \frac{4m_\pi^2}{\mathbf{k}^2} \right)^{3/2} \log \left[ \frac{\sqrt{1 + \frac{4m_\pi^2}{\mathbf{k}^2}} - 1}{\sqrt{1 + \frac{4m_\pi^2}{\mathbf{k}^2}} + 1} \right] \right)$$



# vdW potential

$$\begin{aligned}
 W(r) &= \frac{1}{2\pi^2 r} \int_{2m_\pi}^{\infty} d\mu \mu e^{-\mu r} \text{Im} \left[ \widetilde{W}(\epsilon - i\mu) \right] \\
 &= -\frac{3\pi\beta^2 m_\pi^2}{8b^2 r^5} \left[ \left( 4(\kappa_2 + 3)^2 (m_\pi r)^3 + (3\kappa_1^2 + 43\kappa_2^2 + 14\kappa_1\kappa_2) m_\pi r \right) K_1(2m_\pi r) \right. \\
 &\quad \left. + 2 \left( 2(\kappa_2 + 3)(\kappa_1 + 5\kappa_2) (m_\pi r)^2 + (3\kappa_1^2 + 43\kappa_2^2 + 14\kappa_1\kappa_2) \right) K_2(2m_\pi r) \right]
 \end{aligned}$$



asymptotic

$$W(r) = -\frac{3(3 + \kappa_2)^2 \pi^{3/2} \beta^2 m_\pi^{9/2}}{4b^2 r^{5/2}} e^{-2m_\pi r}$$

Completed the calculation of  
H. Fujii and D. Kharzeev, PRD 60, 114039 (1999)

# Numerical result

— vdW potential

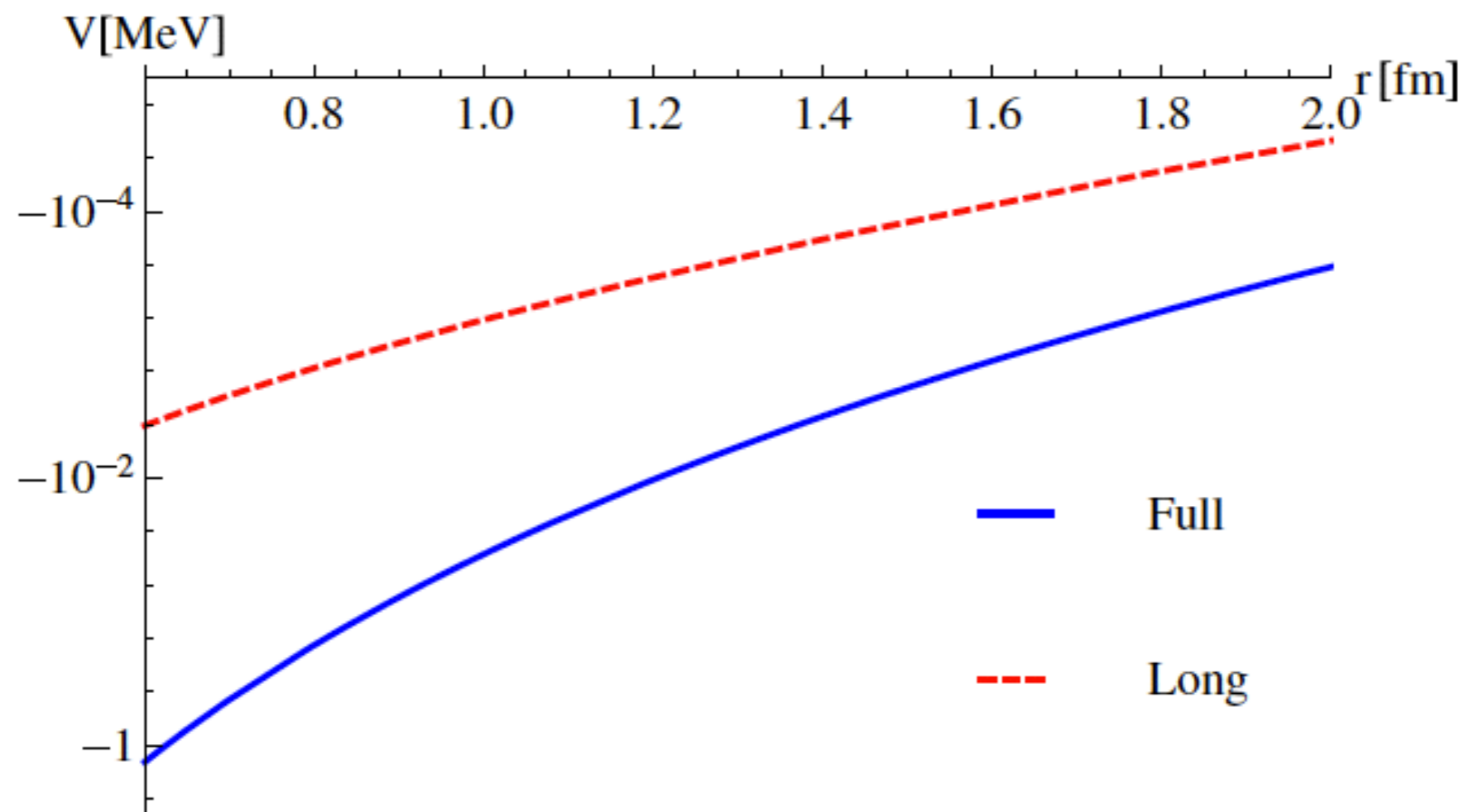


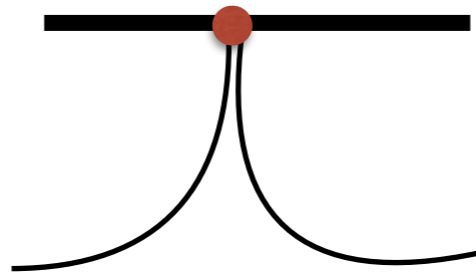
FIG. 9. Comparison of the van der Waals potential (40) (blue line) with its long-range expansion (41) (red line) for  $\beta = 0.92 \text{ GeV}^{-3}$  and other parameters like in Fig. 8.

# Are there $\eta_b \eta_b$ bound-states?

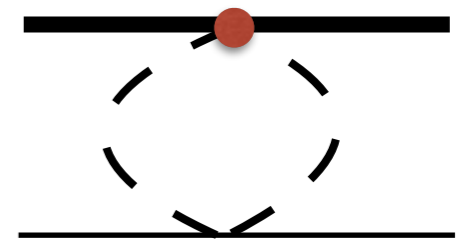
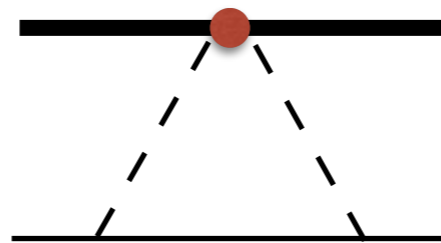
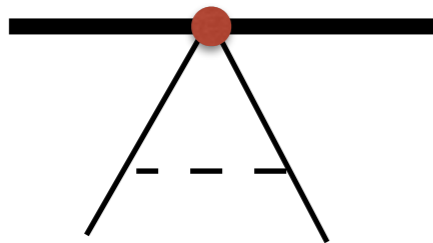
- It is likely, but depends somewhat on the medium- and short-range pieces

# Quarkonium-nucleon\*

Lattice



Chiral EFT



# Degrees of freedom & Scales & Power counting

DOF: nucleons, quarkonia, pions

Scales:  $E_N, E_\phi \sim m_\pi \ll \Lambda_\chi \sim 1\text{GeV}$

Power counting: terms of the effective  
(~ Weinberg for NN) Lagrangian organized  
in powers of

$$\frac{m_\pi}{\Lambda_\chi}$$

Loops: dimensional regularization

# Quarkonium-nucleon EFT

## — QNEFT

### Quarkonium

$$\mathcal{L}^\phi = \phi^\dagger \left( i\partial_0 + \frac{\nabla^2}{2\hat{m}_\phi} \right) \phi$$

### Nucleon-pion

$$u^2 = U = e^{i\Phi/F}, \quad \Phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

$$\mathcal{L}^N = N^\dagger \left( iD_0 + \frac{\mathbf{D}^2}{2\hat{m}_N} \right) N - \frac{g_A}{2} N^\dagger \mathbf{u} \cdot \boldsymbol{\sigma} N$$

$$u_\mu = i \{u^\dagger, \partial_\mu u\}$$

$$D_\mu N = \partial_\mu N + \Gamma_\mu N$$

$$\Gamma_\mu = \frac{1}{2} [u^\dagger, \partial_\mu u]$$

# Quarkonium-nucleon EFT

## — QNEFT

### Quarkonium-Nucleon

$$\begin{aligned}\mathcal{L}^{\phi-N} = & -c_0 N^\dagger N \phi^\dagger \phi - d_m \langle \chi_+ \rangle N^\dagger N \phi^\dagger \phi - d_1 \nabla (N^\dagger N) \cdot \nabla (\phi^\dagger \phi) \\ & - d_2 \left( N^\dagger \overleftrightarrow{\mathbf{D}} N \right) \cdot \left( \phi^\dagger \overleftrightarrow{\nabla} \phi \right) - d_3 \mathbf{D} N^\dagger \cdot \mathbf{D} N \phi^\dagger \phi \\ & - d_4 N^\dagger N \nabla \phi^\dagger \cdot \nabla \phi\end{aligned}$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$$

$$\chi = 2B\hat{m}\mathbb{1}$$

$$m_u = m_d \equiv \hat{m}$$

# Low-energy quarkonium-nucleon dynamics

Quarkonium-nucleon dynamics,  
e.g bound to nucleus occurs at energies

$$E_{\phi N} \sim \frac{k_{\phi N}^2}{2\mu_{\phi N}} < \frac{m_\pi^2}{\Lambda_\chi} \ll m_\pi$$

Integrate out the pion



# Low-energy quarkonium-nucleon dynamics

$$\langle T_N \rangle \sim \frac{3}{5} \frac{k_F^2}{2m_N} \sim 20 \text{ MeV} \ll \Lambda_{\text{QCD}}$$

Quarkonium-nucleon dynamics,  
e.g bound to nucleus occurs at energies

$$E_{\phi N} \sim \frac{k_{\phi N}^2}{2\mu_{\phi N}} < \frac{m_\pi^2}{\Lambda_\chi} \ll m_\pi$$

Integrate out the pion

# Low-energy quarkonium-nucleon dynamics

$$\langle T_N \rangle \sim \frac{3}{5} \frac{k_F^2}{2m_N} \sim 20 \text{ MeV} \ll \Lambda_{\text{QCD}}$$

Quarkonium-nucleon dynamics,  
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$$E_{\phi N} \sim \frac{k_{\phi N}^2}{2\mu_{\phi N}} < \frac{m_\pi^2}{\Lambda_\chi} \ll m_\pi$$

Integrate out the pion

# Quarkonium-nucleon potential

## — pQNEFT

Integrate out the pion

$$\begin{aligned}\mathcal{L}^{\text{pQNEFT}} = & N^\dagger \left( i\partial_0 + \frac{\nabla^2}{2m_N} \right) N + \phi^\dagger \left( i\partial_0 + \frac{\nabla^2}{2m_\phi} \right) \phi \\ & - C_0 N^\dagger N \phi^\dagger \phi - D_1 \nabla (N^\dagger N) \cdot \nabla (\phi^\dagger \phi) \\ & - D_2 \left( N^\dagger \overleftrightarrow{\nabla} N \right) \cdot \left( \phi^\dagger \overleftrightarrow{\nabla} \phi \right) - D_3 \nabla N^\dagger \cdot \nabla N \phi^\dagger \phi - D_4 N^\dagger N \nabla \phi^\dagger \cdot \nabla \phi \\ & - \int d^3r N^\dagger N(t, \mathbf{x}_1) V(\mathbf{x}_1 - \mathbf{x}_2) \phi^\dagger \phi(t, \mathbf{x}_2)\end{aligned}$$

# Matching



renormalization of couplings + van der Waals

# Renormalized couplings

$$C_0 = c_0 + 4m_\pi^2 d_m + \frac{9g_A^2 m_\pi^2 c_0}{64\pi^2 F^2} \left( \log \frac{m_\pi^2}{\nu^2} + \frac{2}{3} \right) + \frac{3g_A^2 m_\pi^3}{64\pi F^2} (5c_{di} - 3c_m)$$

$$D_1 = d_1 + \frac{g_A^2 m_\pi}{256\pi F^2} (23c_{di} - 5c_m)$$

$$D_j = d_j \quad \text{for } j = 2, 3 \text{ and } 4$$

# Long-distance part of QN potential

— **vdW force**

$$V(r) = \frac{3g_A^2 m_\pi^3}{128\pi^2 F^2 r^6} e^{-2m_\pi r} \{c_{di} [6 + m_\pi r(2 + m_\pi r)(6 + m_\pi r(2 + m_\pi r))] + c_m m_\pi^2 r^2 (1 + m_\pi r)^2\}$$

**No free parameters here:**

- trace anomaly
- chiral physics

First, model-independent derivation of a quarkonium-nucleon van der Waals force

For  $r \gg \frac{1}{2m_\pi}$  :

$$V(r) = \frac{3g_A^2 m_\pi^4 (c_{di} + c_m) e^{-2m_\pi r}}{128\pi^2 F^2 r^2}$$

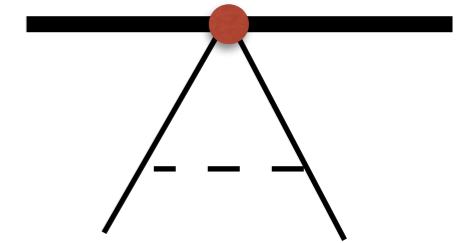
To extrapolate lattice data to physical quark masses, need:

$$m_N = \hat{m}_N - 4c_1 m_\pi^2 - \frac{3g_A^2 m_\pi^3}{32\pi F^2}$$

$$m_\phi = \hat{m}_\phi - F^2 c_m m_\pi^2$$

# Unknown contact couplings

— get them from lattice QCD



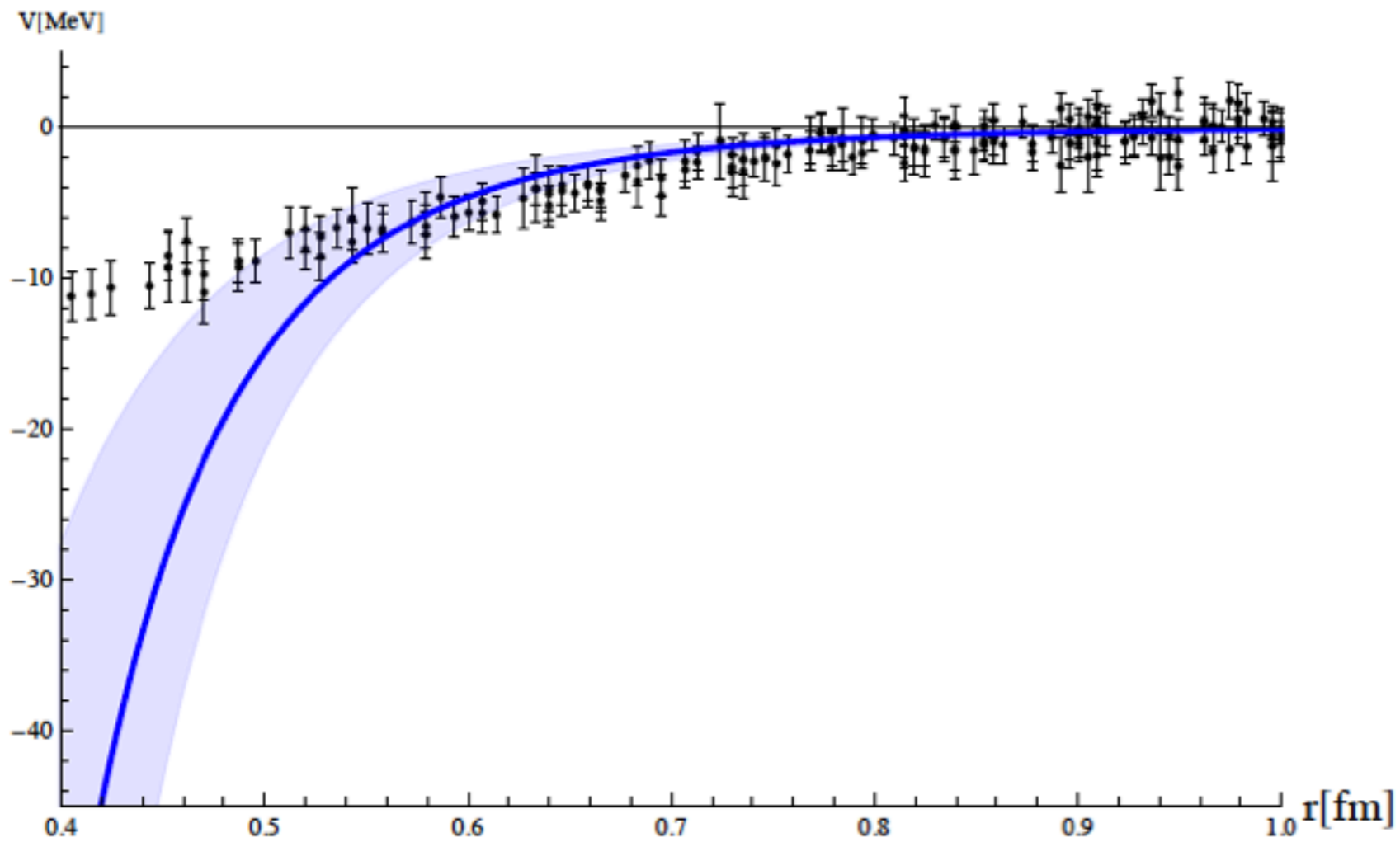
Reference		Channel	$a_0$ [fm]	$c_0$ [GeV <sup>-2</sup> ]	$d_m$ [GeV <sup>-2</sup> ]
[23]	PSF	$\eta_c$	-0.70(66)	-31(29)	Quenched
		$J/\psi$	-0.71(48)	-31(21)	
	LLE	$\eta_c$	-0.39(14)	-17(6)	
		$J/\psi$	-0.39(14)	-17(6)	
[25]		$\eta_c$	-0.25(5)	-8(2)	Quenched
		$J/\psi$	-0.35(6)	-12(3)	
[24]		$\eta_c$	-0.18(9)	-9.7(1.2)	14.7(4.8)
		$J/\psi$	-0.40(80)	-12(18)	-100(80)
[16]		$\beta$ [GeV <sup>-3</sup> ]			
		2	-0.37	-16.5	
		0.24	-0.05	-2.0	



Comparing long distance part  
with HAL lattice potential

vdW force

$$\eta_c - N$$

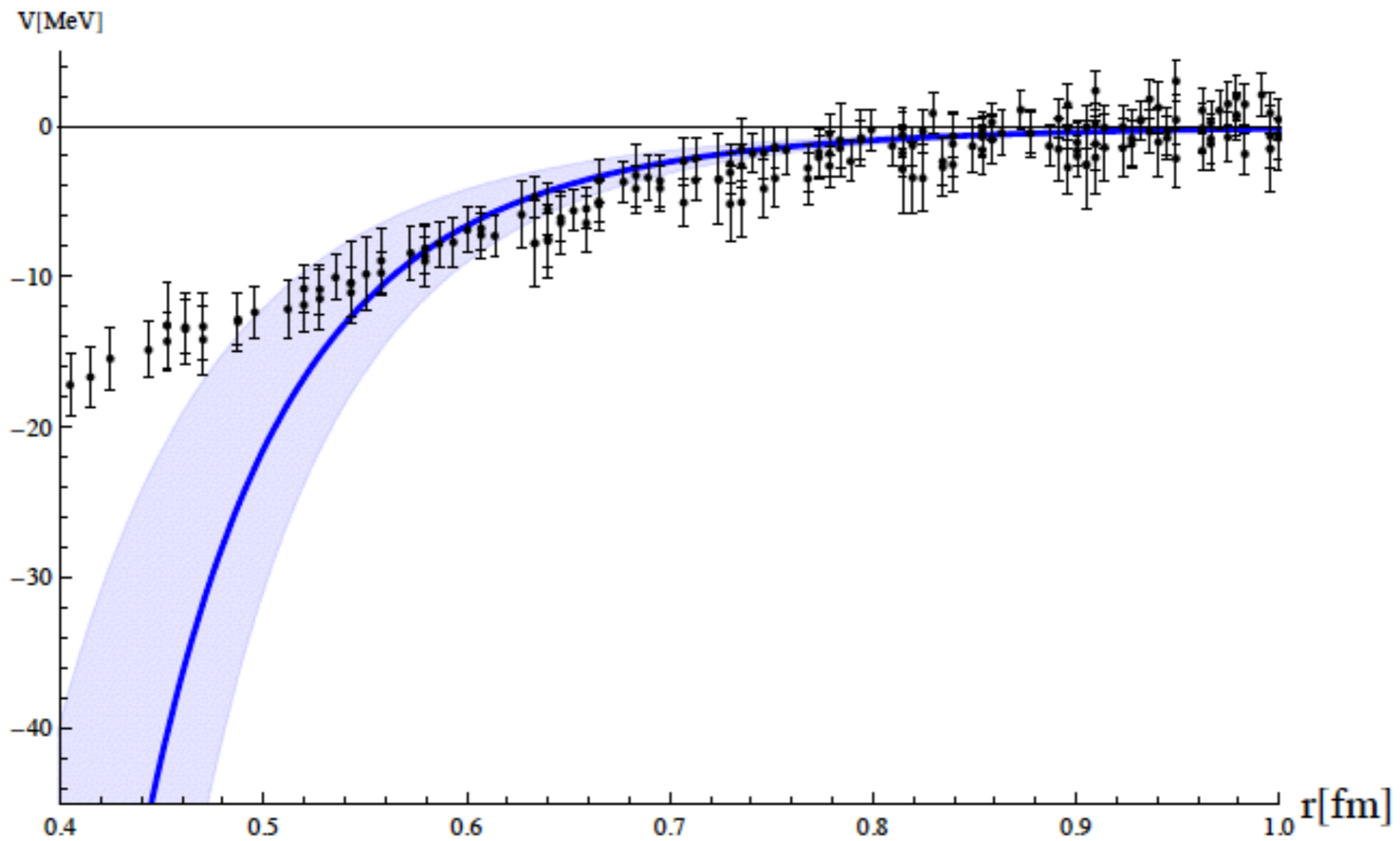


Lattice:

T. Kawanay & S. Sasaki, Pos (Lattice) 2010, 156 (2010)

vdW force

$$J/\Psi - N$$



Lattice:

T. Kawanay & S. Sasaki, Pos (Lattice) 2010, 156 (2010)

# Fits of the polarizabilities

	$c_{d0}$ [GeV <sup>-3</sup> ]	$c_{di}$ [GeV <sup>-3</sup> ]	$c_m$ [GeV <sup>-3</sup> ]
$\beta_{\eta_c} = 0.17 \text{ GeV}^{-3}$	-0.83	-1.71	-2.24
$\beta_{J/\psi} = 0.24 \text{ GeV}^{-3}$	-1.17	-2.42	-3.16

Table II. Values of the pion-quarkonium couplings according to the expressions in Eq. (5) for the values of the polarizabilities, in Eq. (29), obtained from the fit of the potential to the lattice data of Ref. Kawanai:2010ev.

# Are there quarkonium-nucleon bound states at this order in pQNEFT?

Scattering amplitude (s-wave)

$$\mathcal{A} = \frac{2\pi}{\mu_{\phi N}} \frac{1}{p \cot \delta - ip} = \frac{2\pi}{\mu_{\phi N}} \frac{1}{-\frac{1}{a_0} + \frac{1}{2}r_0 p^2 + \dots}$$

$$a_0 = \frac{\mu_{\phi N}}{2\pi} \left[ c_0 + 4d_m m_\pi^2 + \frac{9g_A^2 m_\pi^2 c_0}{64\pi^2 F^2} \left( \log \frac{m_\pi^2}{\nu^2} + \frac{2}{3} \right) + \frac{3g_A^2}{64\pi F^2} m_\pi^3 (5c_{di} - 3c_m) \right]$$

$$r_0 = \frac{8\pi}{\mu_{\phi N} c_0^2} \left[ (d_1 + d_2) + \frac{g_A^2}{256\pi F^2} m_\pi (23c_{di} - 5c_m) \right]$$

No quarkonium-nucleon bound states within the applicability of the present calculation:

$$|p_{\phi N}| \leq m_\pi$$

Are there quarkonium-nucleus  
bound states at this order in pQNEFT?

YES, for sufficiently large nuclei

# Potential for collaborations with colleagues in France

- Jaume Carbonell
- Jean-Marc Richard\*
- Pierre Guichon\*
- Bira van Kolck\*
- Matthieu Tissier
- Hervé Moutarde
- Julien Surreau

\* present talk

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