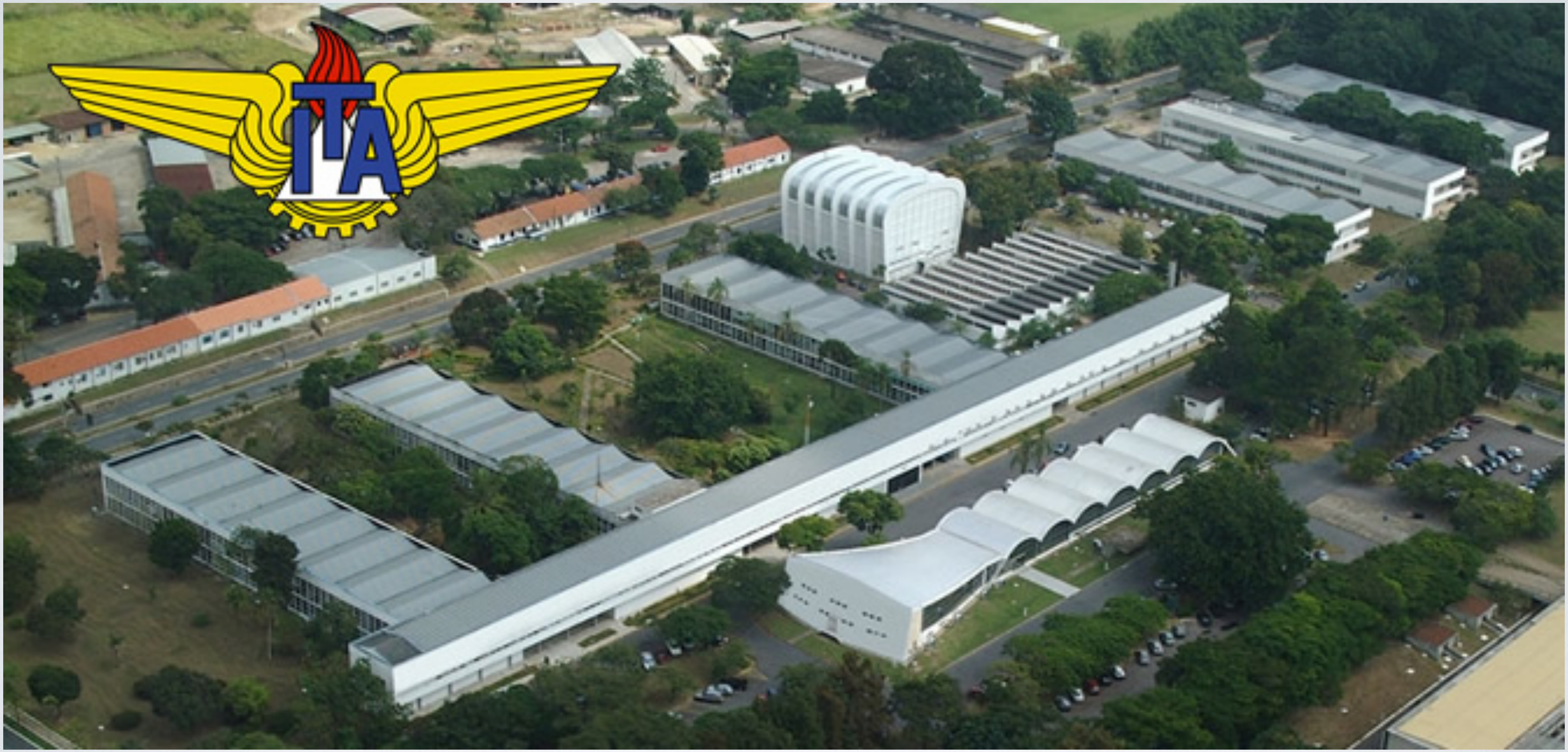
LIA SUBATOMIC PHYSICS - KICKOFF WORKSHOP

Effective models for hadronic systems

Varese S. Timóteo (FT / UNICAMP)



June 12 - 13, 2018 - São José dos Campos - SP

Outline

- Motivation: Bertsch problem and Unitary limit
- EFT: Implicit vs Explicit Renormalization
- Pions vs Contacts
- Neutron matter
- Final Remarks

E. Ruiz Arriola (UGR)

S. Szpigel (UPM)

Motivation

What is the ground state energy of a many-fermion system with zero range interactions and infinitely large scattering length ?

T. Papenbrock, G. F. Bertsch, Phys. Rev. C 59, 2052 (1999).

singlet state

$a_{np} = -23.7 \text{ fm}$

$a_{nn} = -18.7 \text{ fm}$

very large

small and negative scattering length

large and negative scattering length

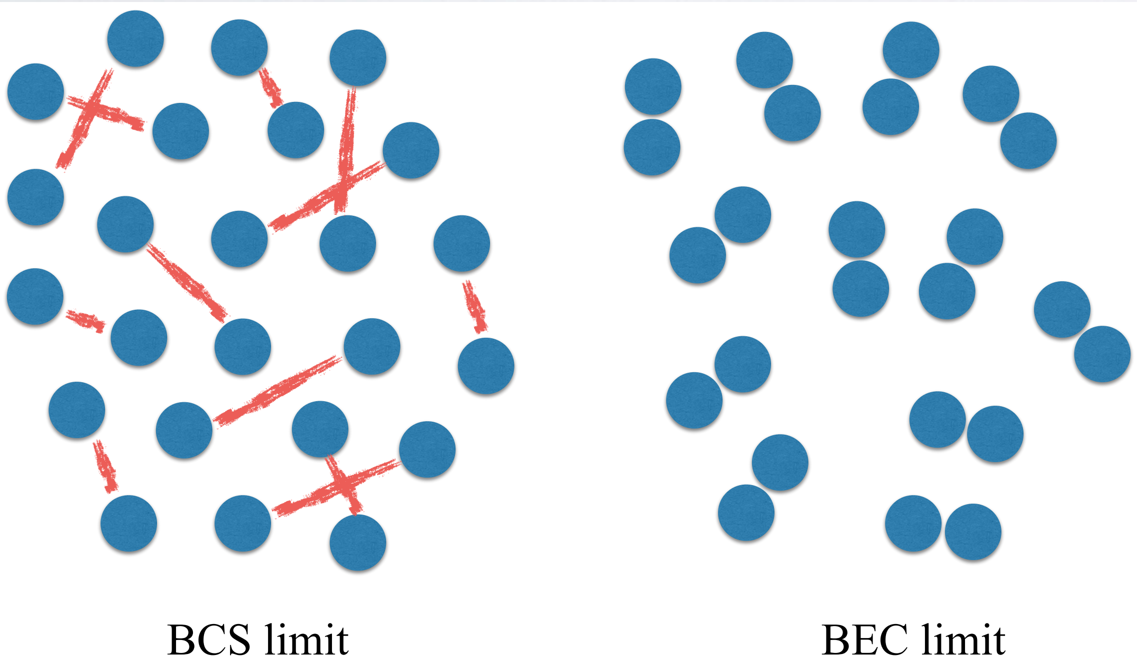
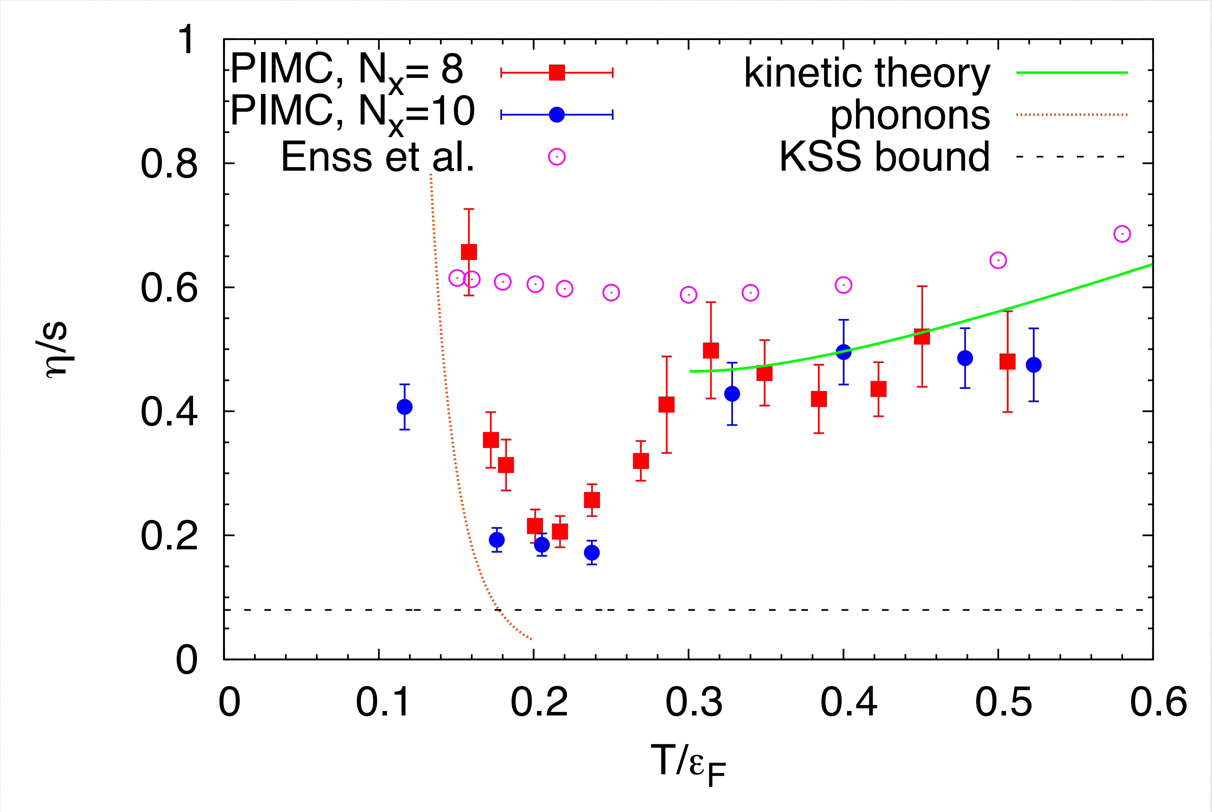


Figure 1. Pictorical representation of the BCS and BEC limits of a many-fermion system. The blurred red lines indicate the weak pairing. Adapted from Living Rev. Relativity **11**, 10 (2008).

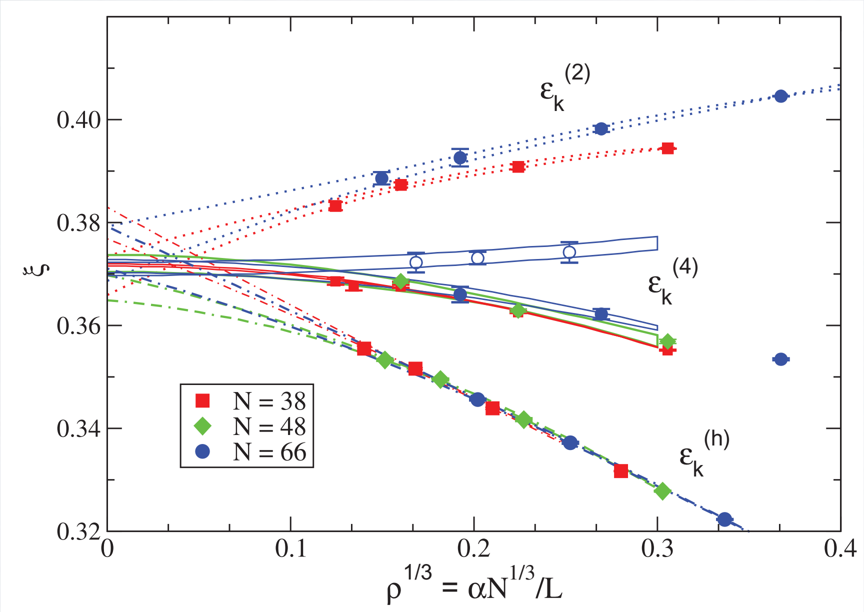
Journal of Physics: Conference Series **630** (2015) 012036

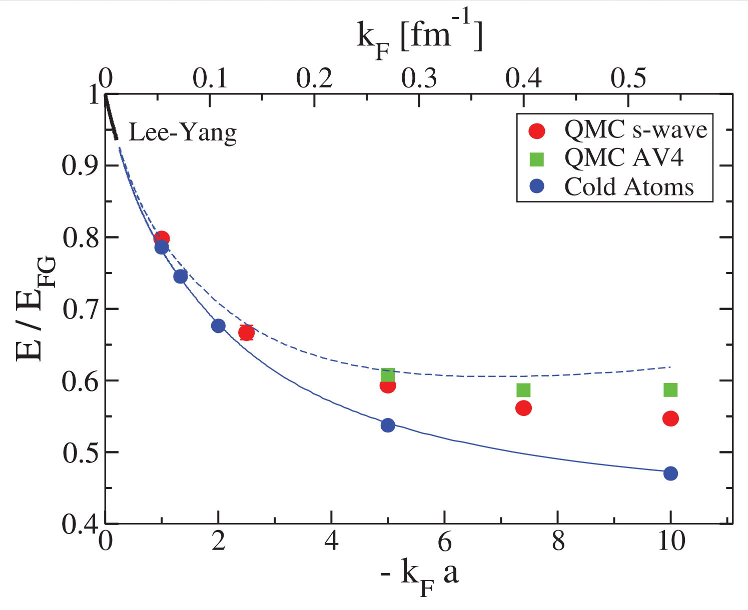


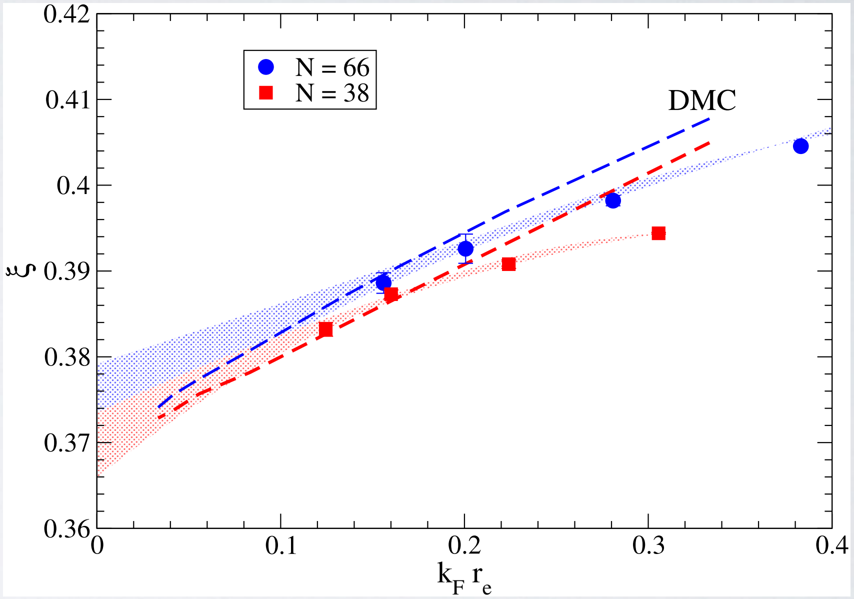
G. Wlazlowski, P. Magierski and J. E. Drut, Phys. Rev. Lett. 109, 020406 (2012)

Neutron matter & Cold Atoms

Monte Carlo simulations







J. Carlson, S. Gandolfi and A. Gezerlis, Prog. Theor. Exp. Phys., 01A209 (2012).

"We report quantum Monte Carlo calculations of superfluid Fermi gases with short-range two-body attractive interactions with infinite scattering length. The energy of such gases is estimated to be (0.44 ± 0.01) times that of the noninteracting gas, and their pairing gap is approximately twice the energy per particle."

J. Carlson, S. Y. Chang, V. R. Pandharipande and K. E. Schmidt, Phys. Rev. Lett. 91, 050401 (2003)

Unitary limit

$$\mathcal{T}_2(k,k) \propto \frac{1}{[k \cot \delta - ik]}$$

Effective Range Expansion

$$k \cot \delta = -\frac{1}{\alpha_0} + \frac{1}{2} r_0 k^2 - \frac{1}{4} v_0 k^4 + \cdots$$

Unitary limit

low energies

$$\alpha_0 \to \infty \text{ and } r_0 \to 0 \qquad \delta = \frac{\pi}{2}$$

Scale invariant T-matrix

$$\mathcal{T}_2(k,k) \propto \frac{i}{k} \qquad \sigma = 4\pi/k^2$$

Bertsch Parameter

$$\varepsilon = \xi \times \varepsilon_{FG} \qquad \varepsilon_{FG} = \frac{3}{5}\frac{\hbar^2 k_F^2}{2m} \qquad \xi = \frac{\varepsilon}{\varepsilon_{FG}}$$

IMPLICIT RENORMALIZATION

Contact theory in the continuum, regulated by a sharp cutoff

LO, NLO, NNLO

$$V_\Lambda(p',p) = C_0 + C_2(p^2+p'^2) + C_4(p^4+p'^4) + C_4' p^2 p'^2 + \dots .$$

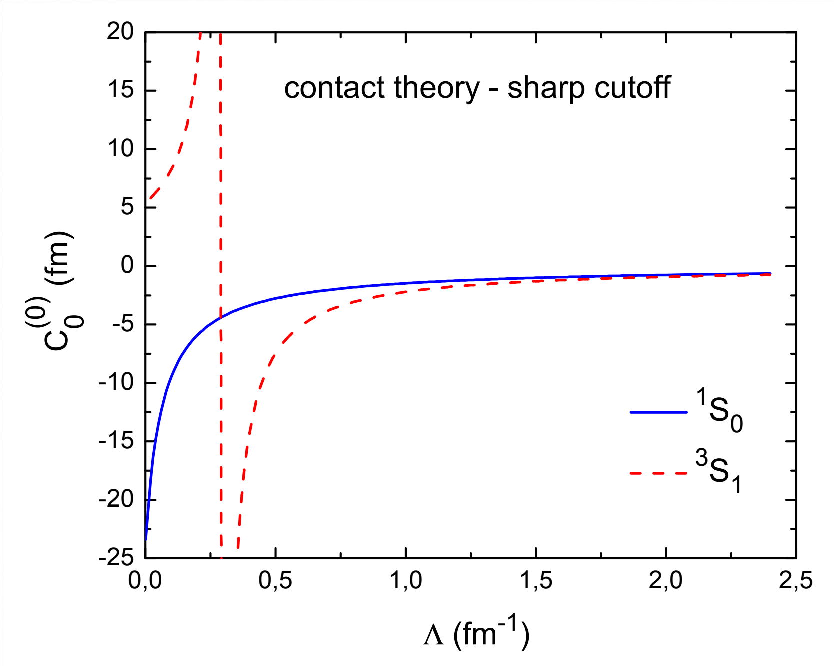
LO: $C_0 \to \alpha_0$

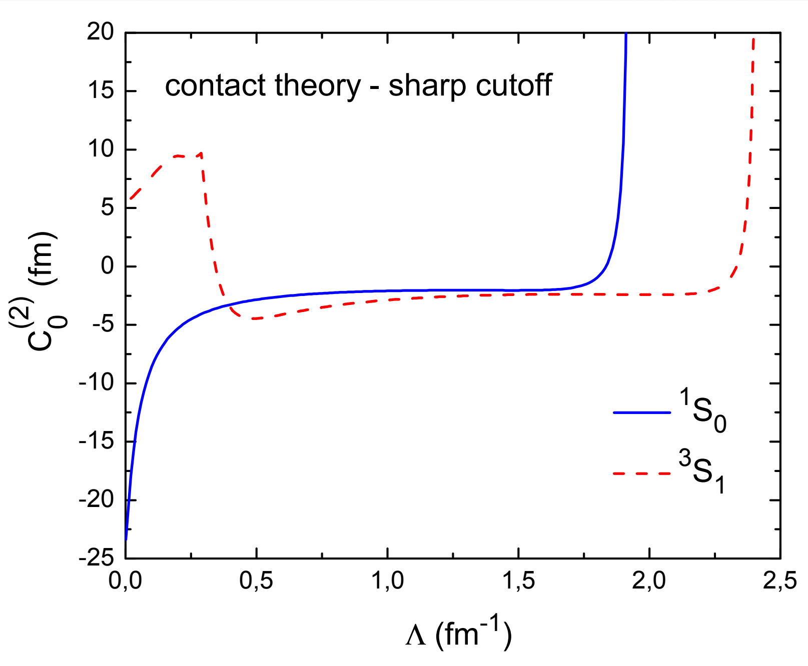
$$C_0(\Lambda) = \frac{\alpha_0}{1-\frac{2\Lambda\alpha_0}{\pi}}$$

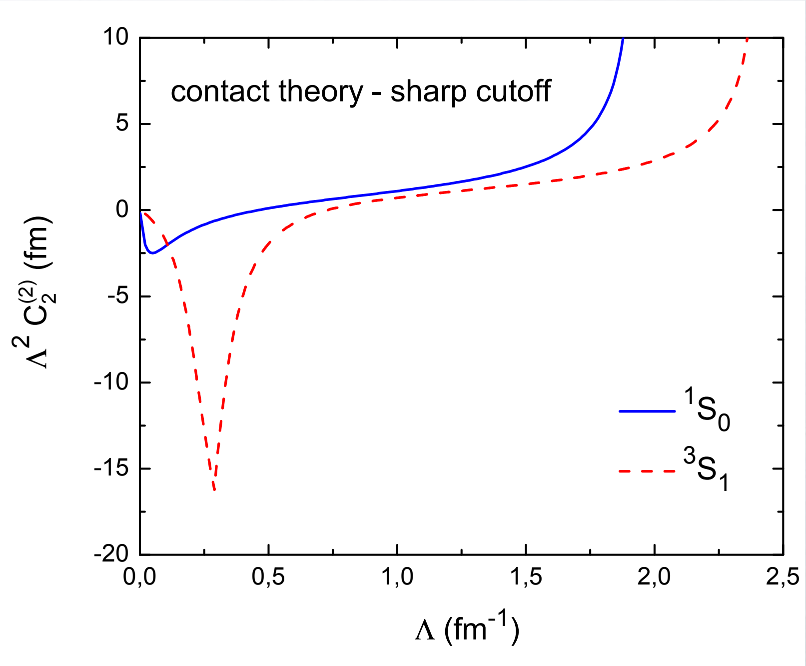
NLO: $(C_0, C_2) \to (\alpha_0, r_0)$

$$-\frac{1}{\alpha_0\Lambda} = \frac{4\left(-2c_2^2+90\pi^4+15(3c_0+2c_2)\pi^2\right)}{9\pi\left(c_2^2-10c_0\pi^2\right)}$$

$$r_0\Lambda = \frac{16\left(c_2^2+12\pi^2 c_2+9\pi^4\right)}{\pi\left(c_2+6\pi^2\right)^2} - \frac{12c_2\left(c_2+12\pi^2\right)}{\left(c_2+6\pi^2\right)^2}\frac{1}{\alpha_0\Lambda} + \frac{3c_2\pi\left(c_2+12\pi^2\right)}{\left(c_2+6\pi^2\right)^2}\frac{1}{\alpha_0^2\Lambda^2}$$

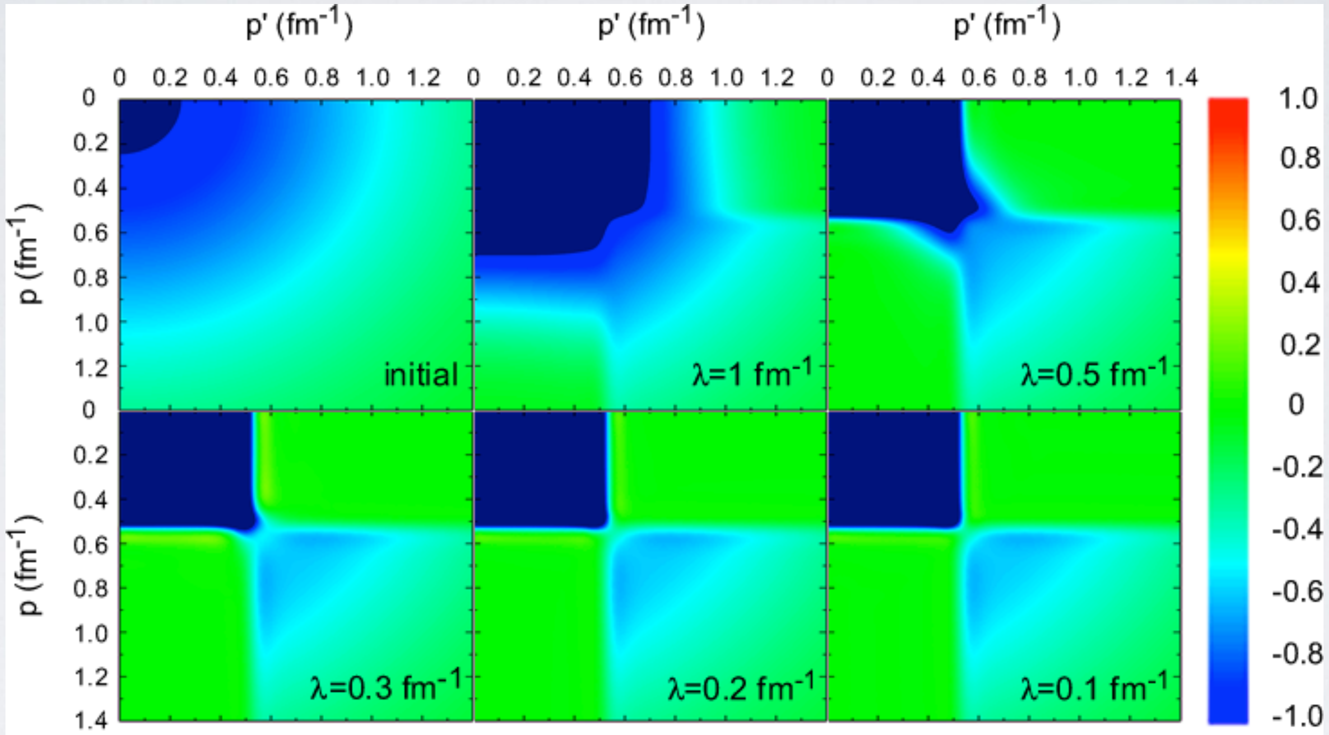


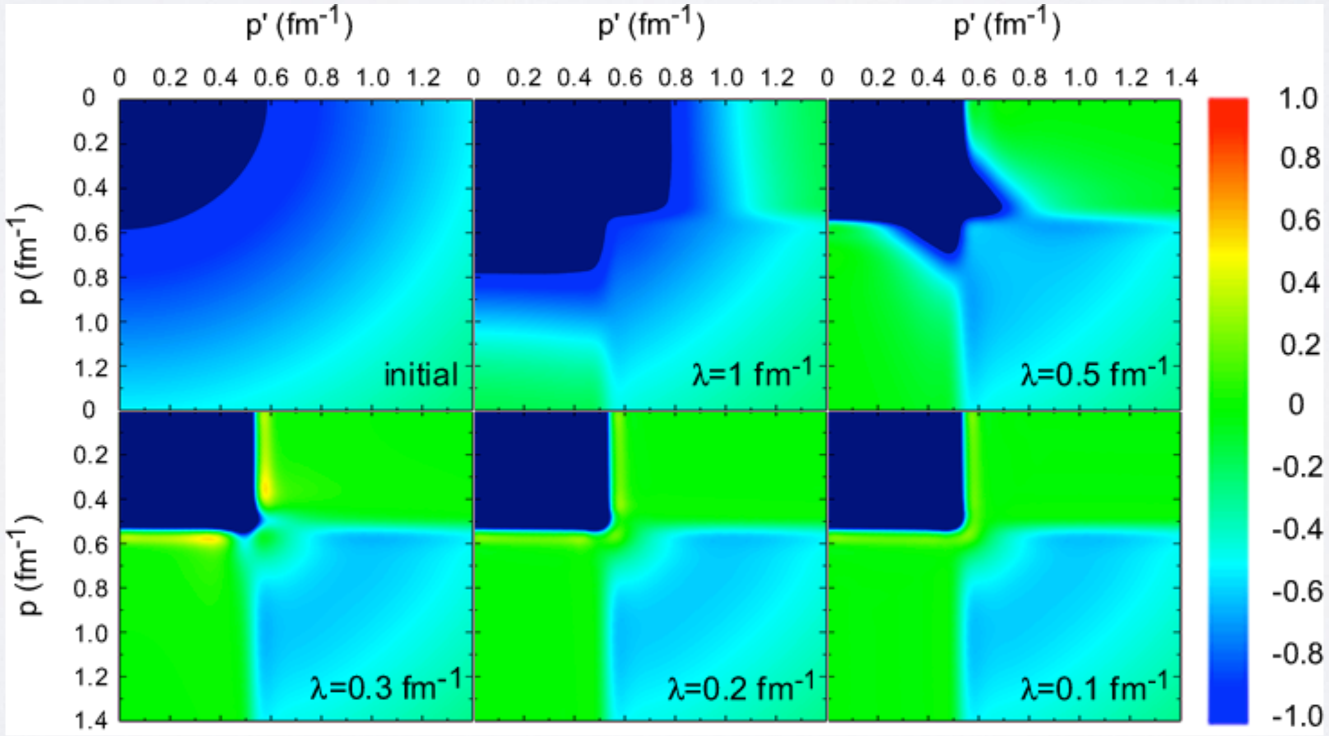




EXPLICIT RENORMALIZATION - 1S0 & 3S1

see works from Ohio group for a review on the SRG

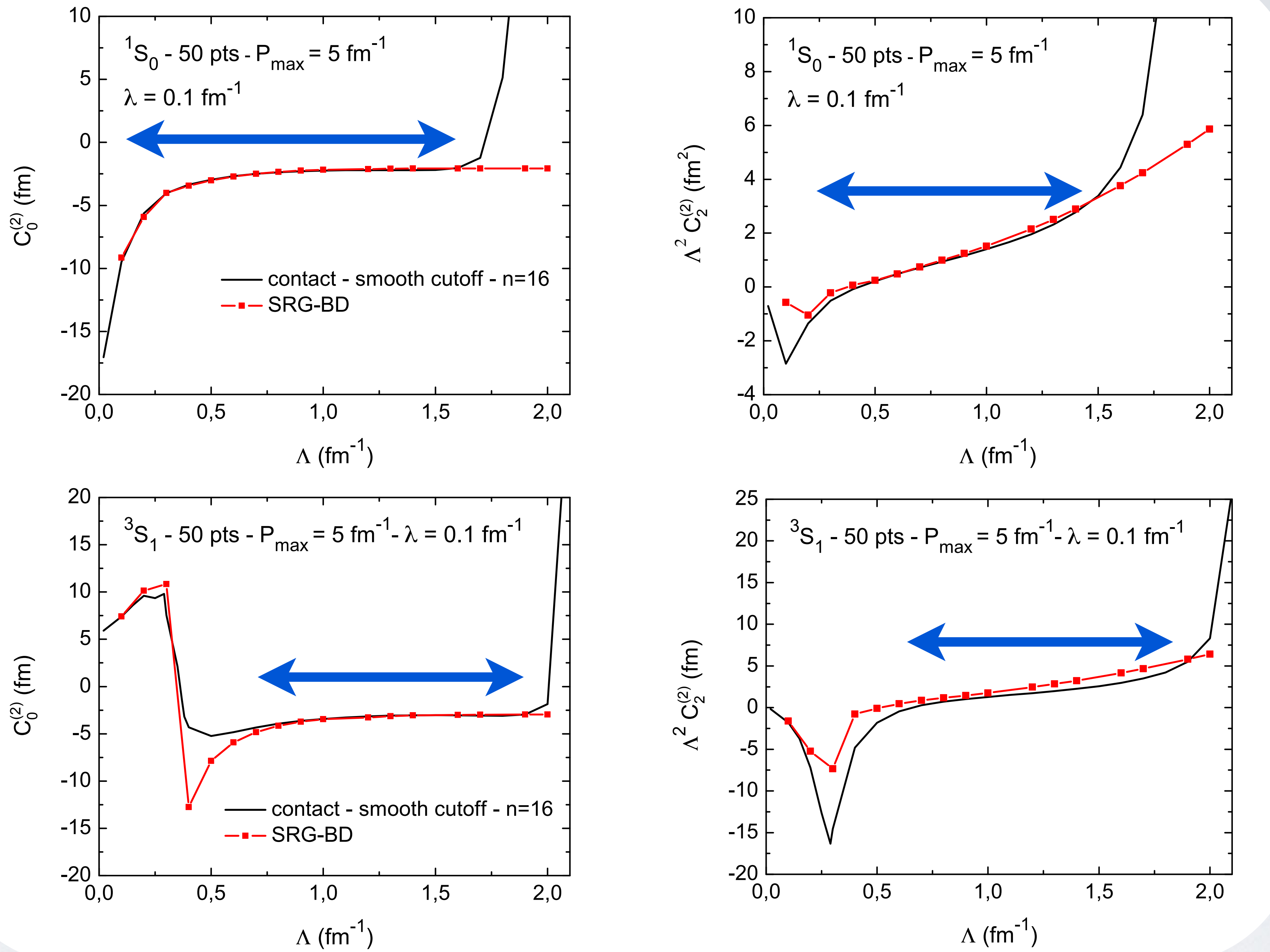




Szpigel, Ruiz Arriola, VST, Annals of Physics 353 (2015) 129–149

Implicit x Explicit - 1S0 & 3S1

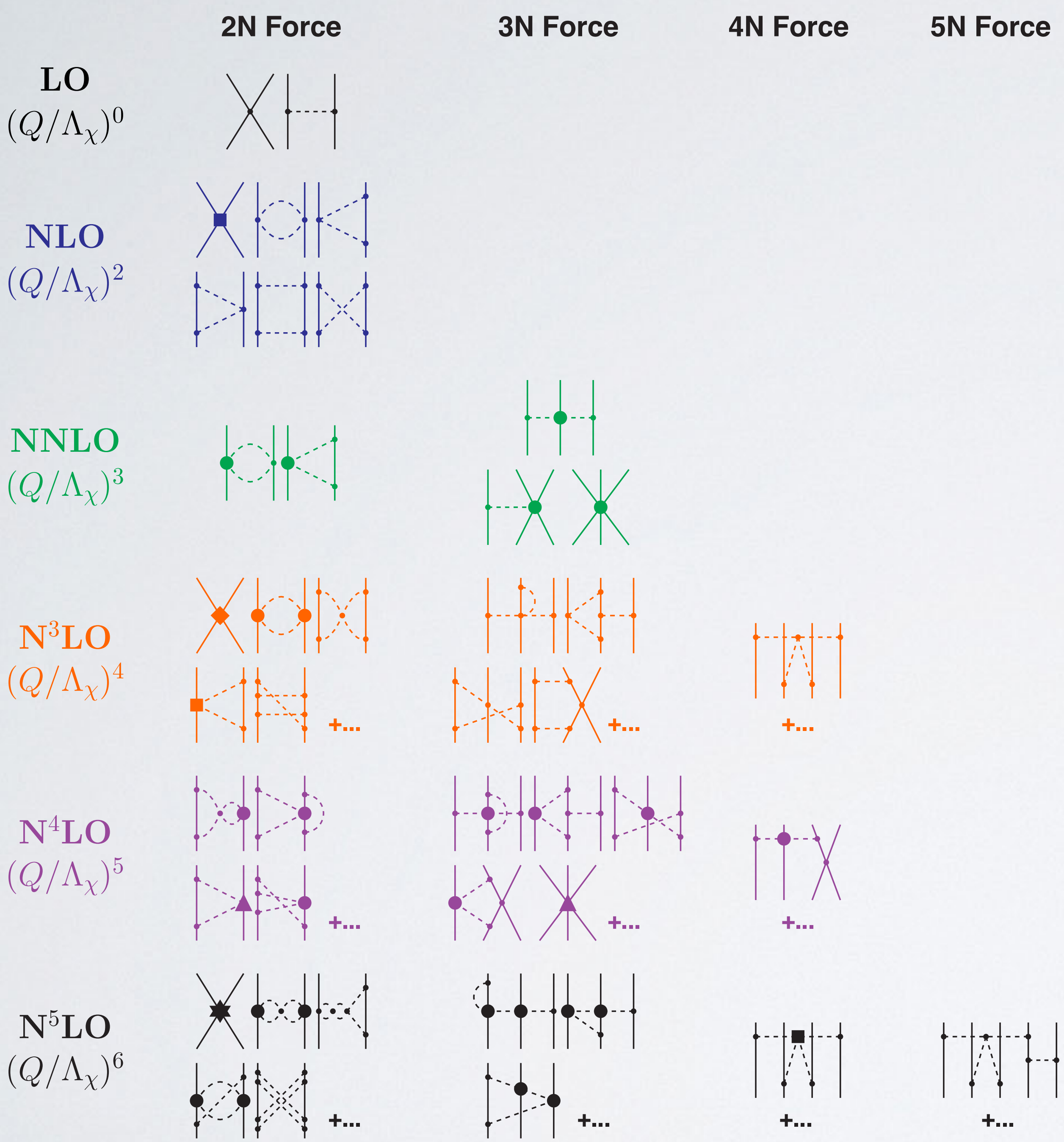
$$V_{\lambda,\Lambda}(p,p') = \tilde{C}_0 + \tilde{C}_2\,(p^2 + p'^2) + \cdots$$



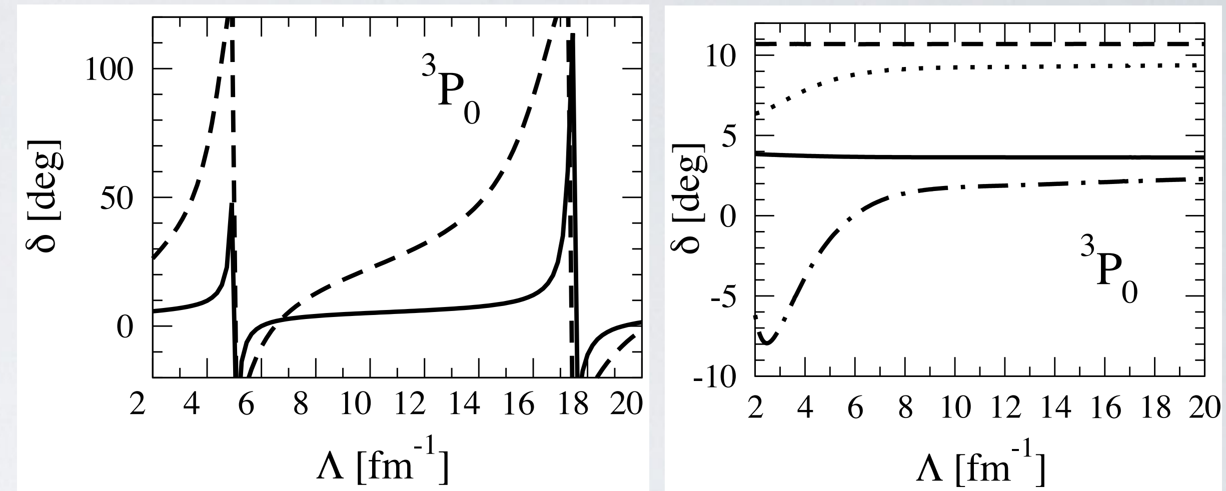
Szpigel, Ruiz Arriola, VST, Physics Letters B 728 (2014) 596–601

Chiral Forces with pions & nucleons as fundamental d.o.f.

see works from Machleidt et al. and Epelbaum et al.

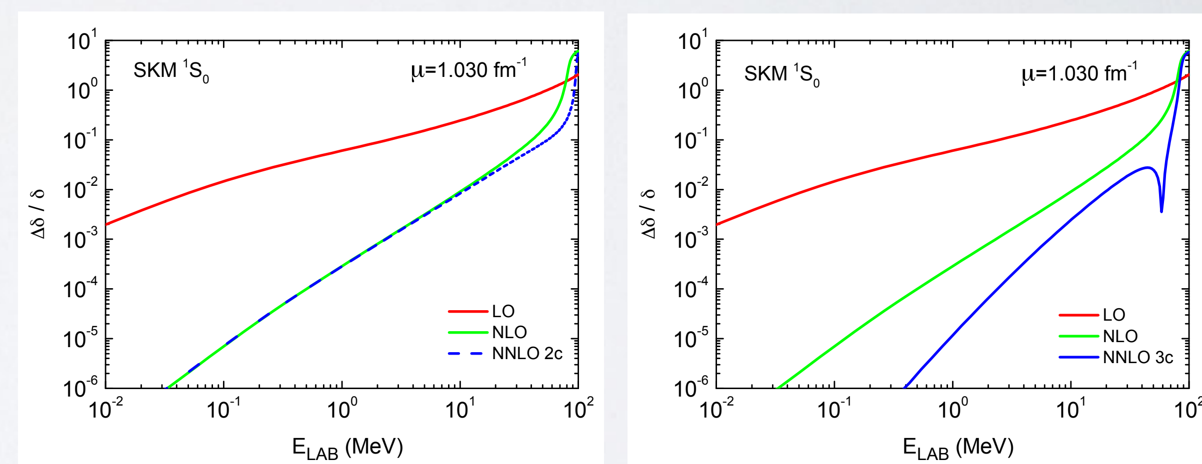


Nogga, Timmermans, van Kolck, Phys Rev C 72 (2005) 054006



LO

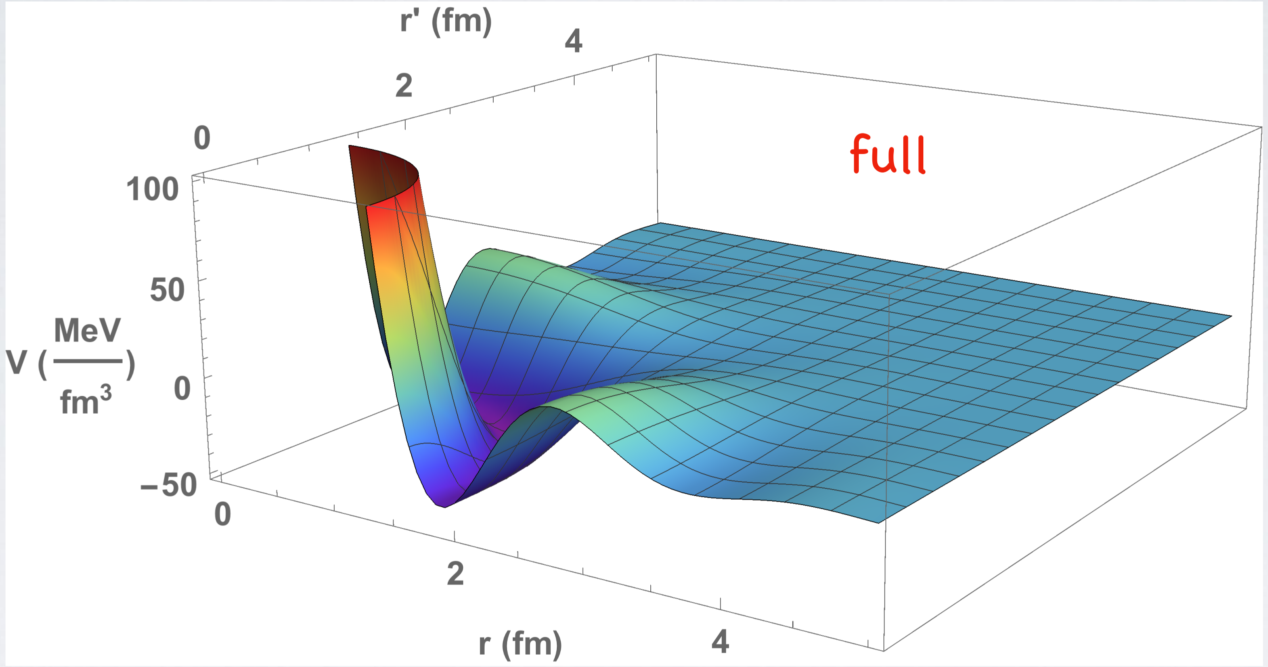
Szpigel & VST, J Phys G 39 (2012) 105102

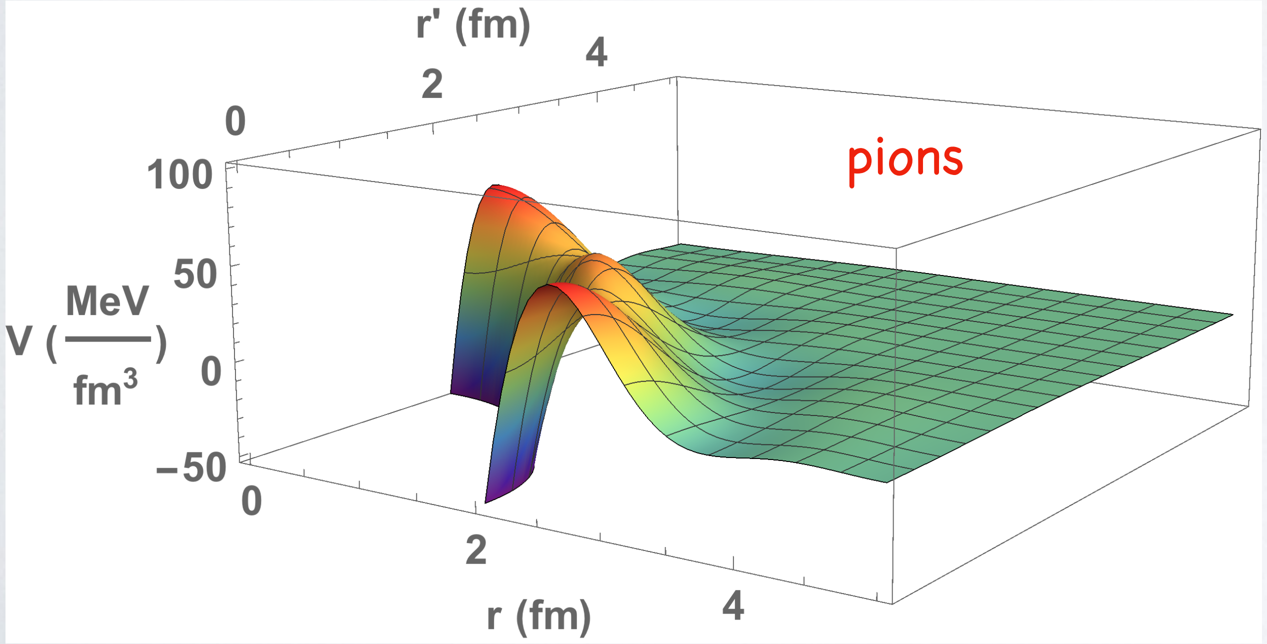


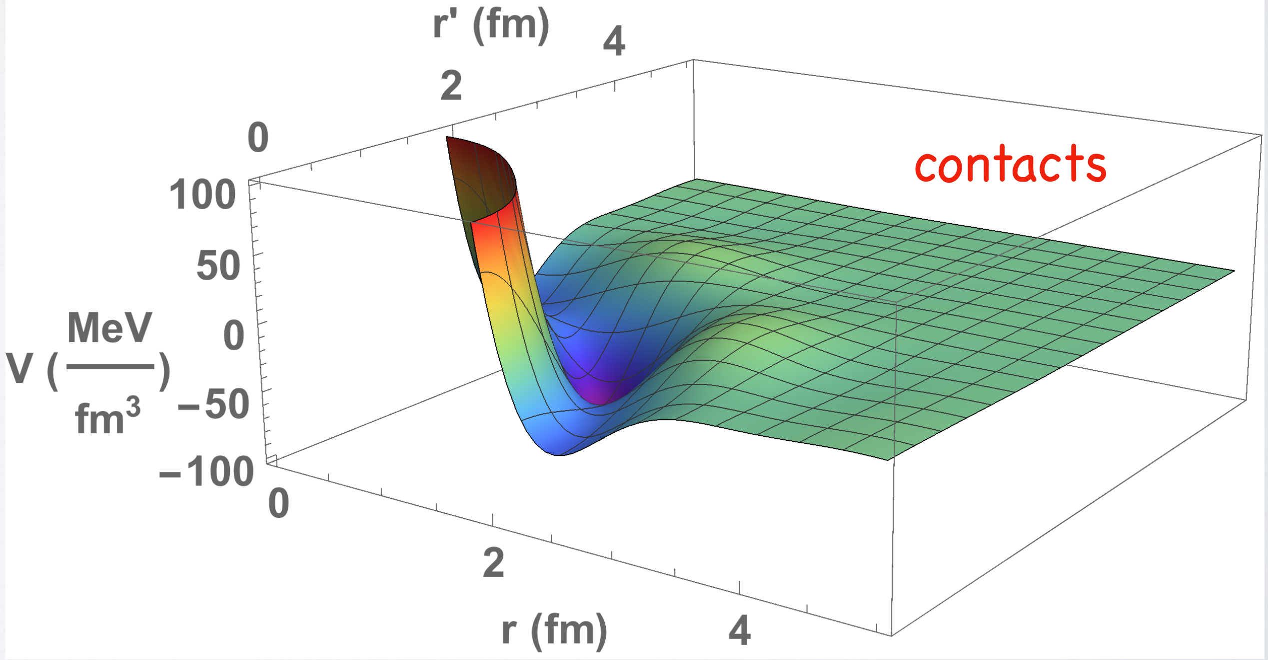
N2LO

N4LO force (1S0 channel)

configuration space







Neutron matter with only contact interactions

$$\xi_\lambda(k_F) = \frac{T(k_F) + V_\lambda(k_F)}{T(k_F)} = 1 + \frac{V_\lambda(k_F)}{T(k_F)}$$

$$T(k_F) = \frac{3k_F^2}{10m_n}$$

$$V_\lambda(k_F) = \frac{4}{m_n}\frac{2}{\pi}\int_0^{k_F} dk\ k^2 \left(1 - \frac{3k}{2k_F} + \frac{k^3}{2k_F^3}\right)\ V_\lambda^{^1S_0}(k,k)$$

$$V(p,p') = C_0 + C_2\ (p^2 + p'^2) + C_4\ (p^4 + p'^4) + C_4'\ p^2 p'^2 + \cdots$$

LO NLO NNLO

Constraining LECs to unitarity condition

$$V(p,p') = C_0 + C_2\ (p^2 + p'^2) + C_4\ (p^4 + p'^4) + C'_4\ p^2 p'^2 + \cdots$$

LO NLO NNLO

Compute two-body T-matrix with V(p,p')

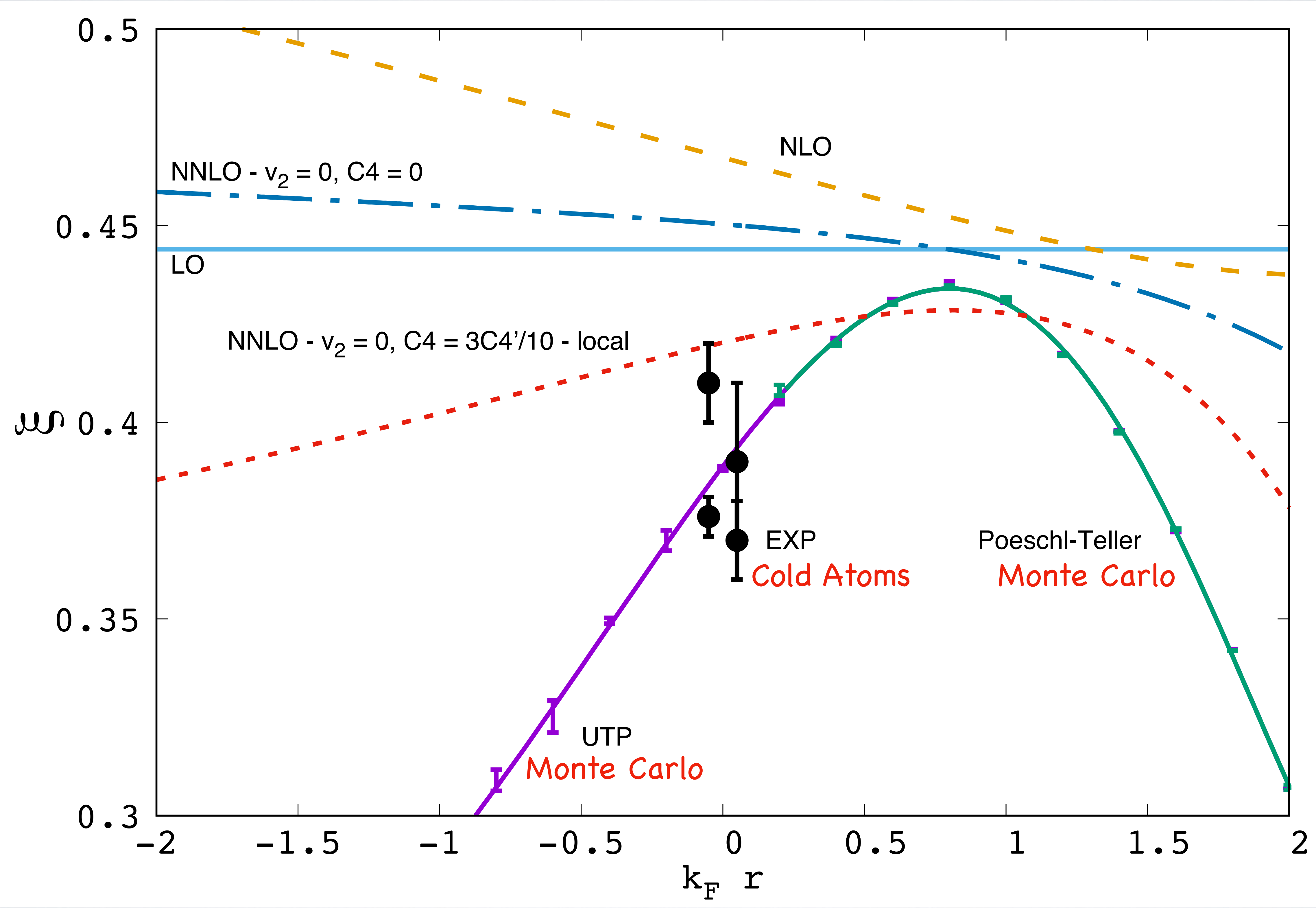
Match to Effective Range Expansion

Impose unitarity condition -1/a = 0 and r = 0

$$\xi_{\mathrm{LO}}(x) = \frac{4}{9} = 0.444\ldots \qquad \xi_{\mathrm{NLO}}(x) = \frac{\left(3\pi\, x - 6\sqrt{48 - 3\pi\, x} - 64\right)}{3\pi\, x - 48}$$

$$x = k_F r$$

Neutron Matter: Implicit Renormalization



Exact solution with separable potential

F. Tabakin, Phys. Rev. 177 (1969) 1443.

$$V_0(p',p) = \pm g(p')g(p)$$

$$[g(k)]^2 = \frac{\sin\delta(k)}{k}\exp\left[-⨍_{-\infty}^{\infty}\frac{\delta(k')}{k-k'}dk'\right]$$

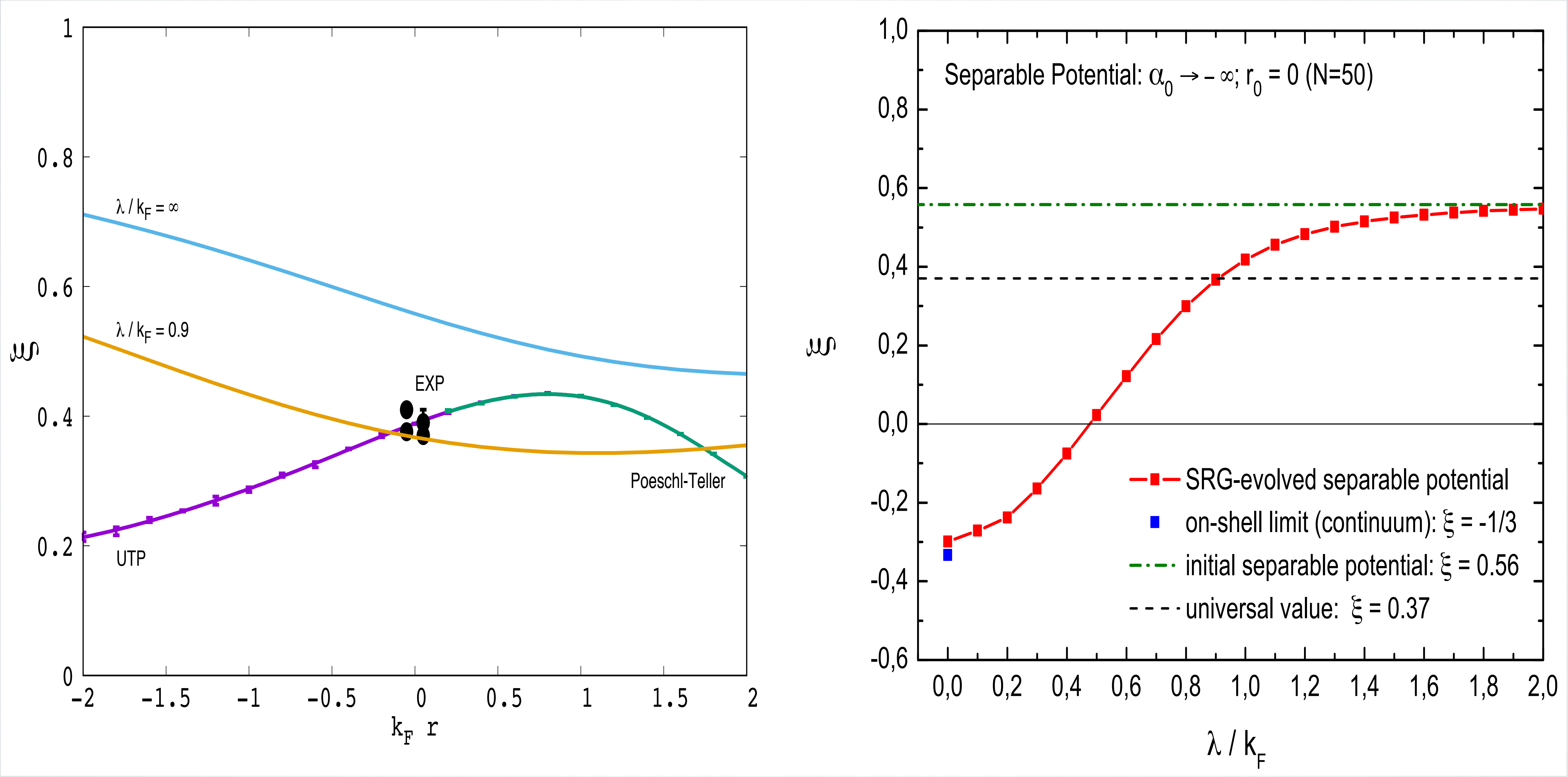
$$g(p) = \frac{\theta(\Lambda - p)}{\sqrt[4]{\Lambda^2 - p^2}}$$

$$V_{k_F}(k',k) = -\frac{\theta(k_F - k')}{\sqrt[4]{k_F^2 - k'^2}}\frac{\theta(k_F - k)}{\sqrt[4]{k_F^2 - k^2}}$$

$$\xi = 1 + \frac{80}{3\pi k_F^2}\int_0^{k_F} k^2 dk \left(1 - \frac{3k}{2k_F} + \frac{k^3}{2k_F^3}\right) V_{k_F}(k,k)$$

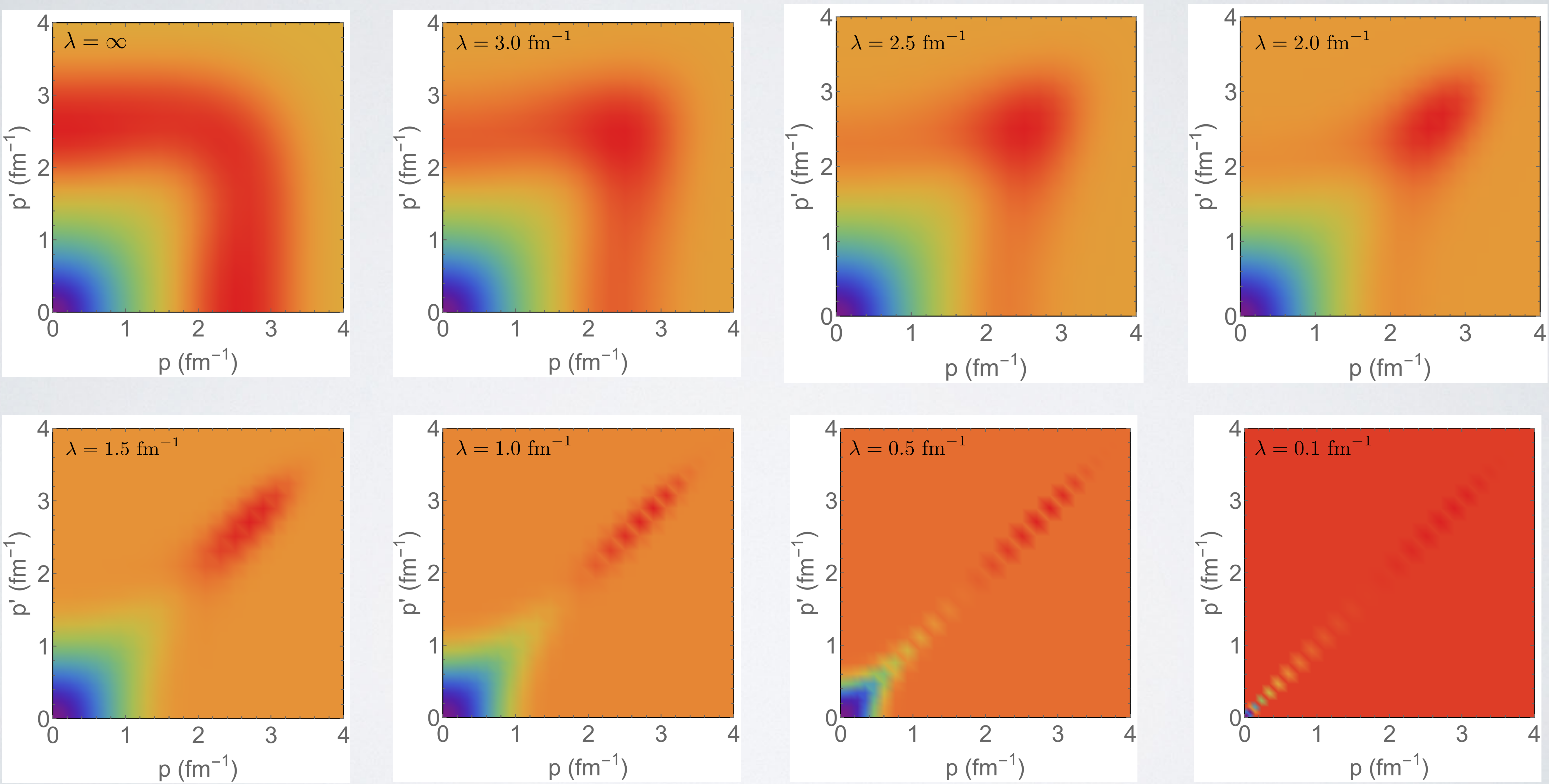
$$\xi = \frac{176}{9\pi} - \frac{17}{3} = 0.558$$

Neutron Matter: Explicit Renormalization



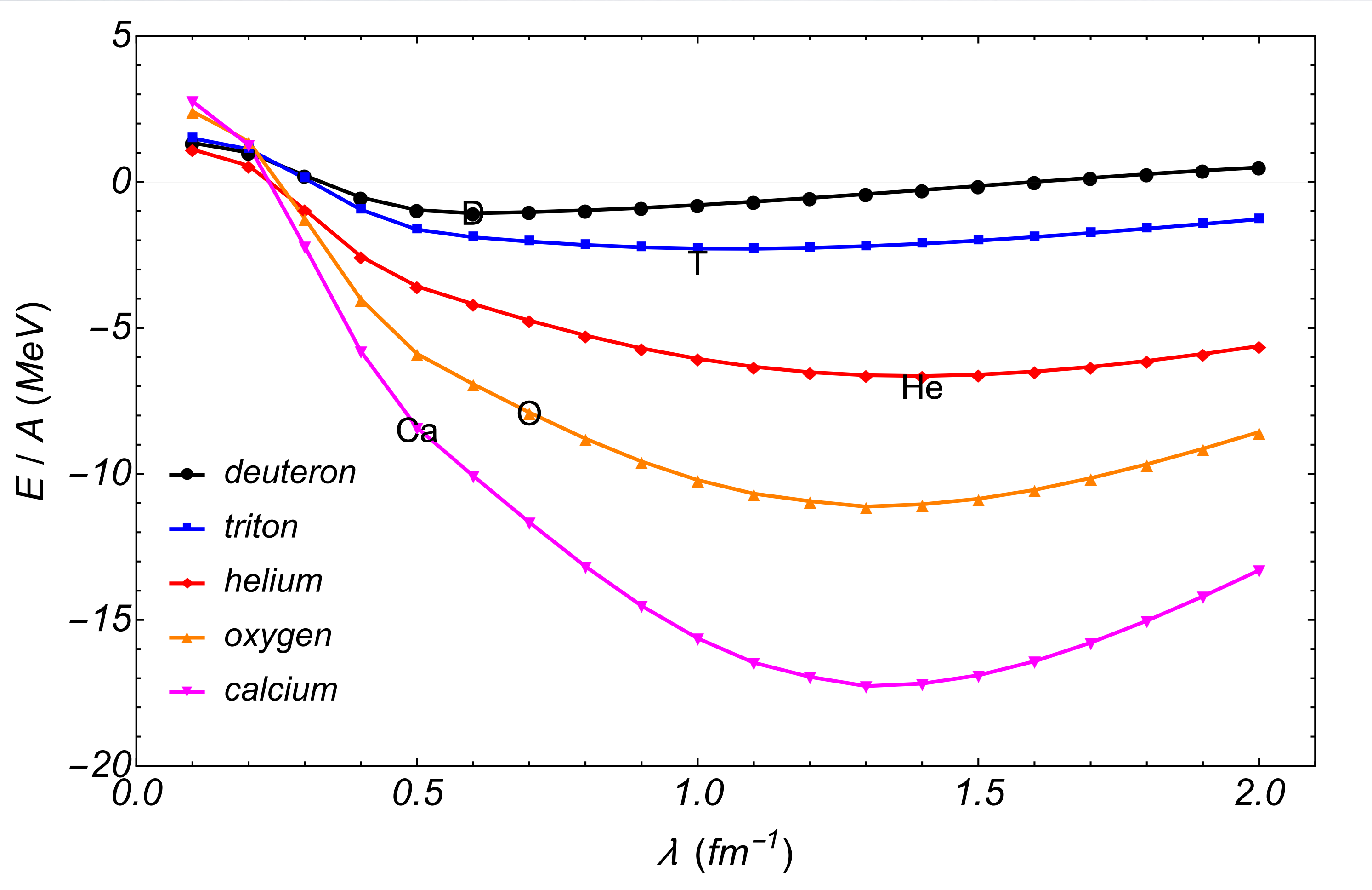
SRG evolution - Chiral N3LO - 1S0

for a review on applications of SRG to nuclear physics see
Furnstahl & Hebeler, Rept Prog Phys 76 (2013) 126301

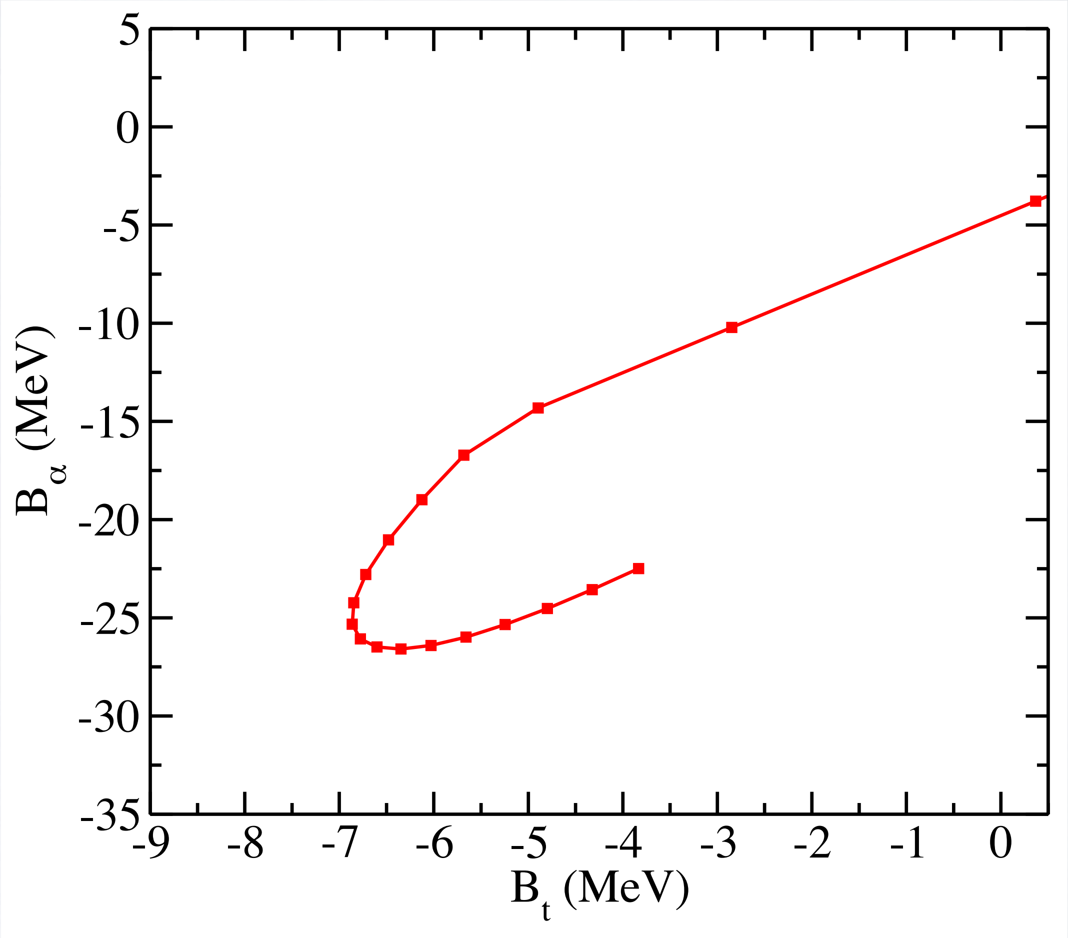


Finite Nuclei: Explicit Renormalization

Binding energies



Tjon line



Quantifying offshellness

The Frobenius norm:

$$\phi = ||V_\lambda|| = \sqrt{\mathrm{Tr}\ V_\lambda^2}$$

$$V_\lambda^2 = \frac{2}{\pi}\int_0^\infty dq\ q^2\ V_\lambda(p,q)\ V_\lambda(q,p')$$

Order parameter:

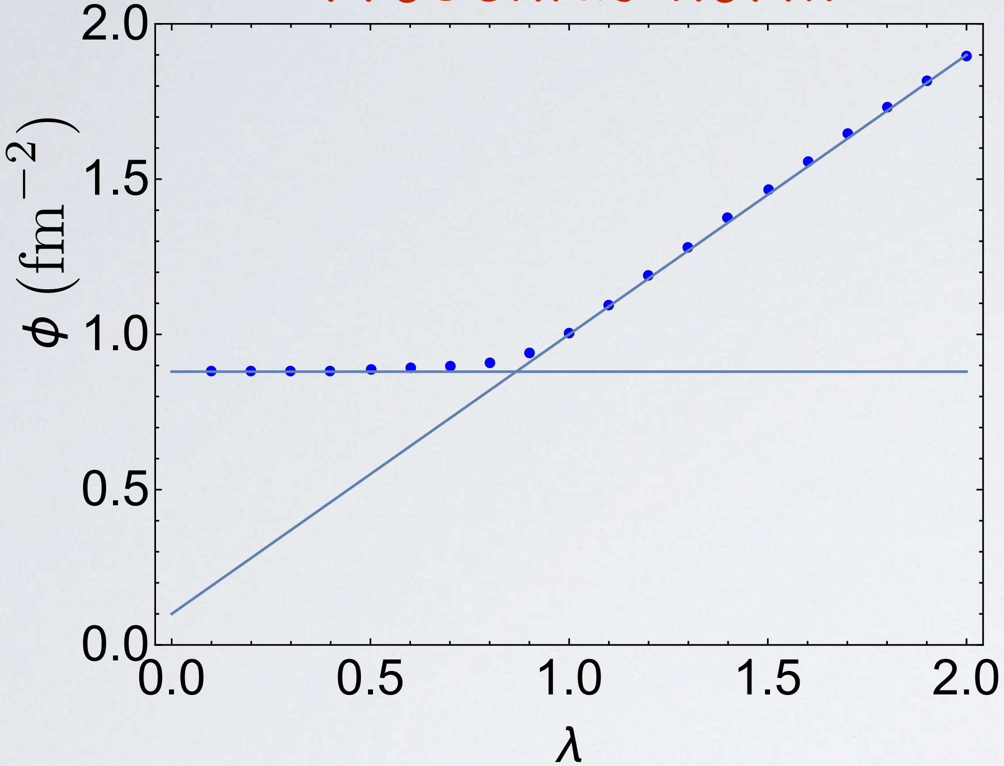
$$\beta = \frac{d\phi}{d\lambda}$$

Similarity susceptibility:

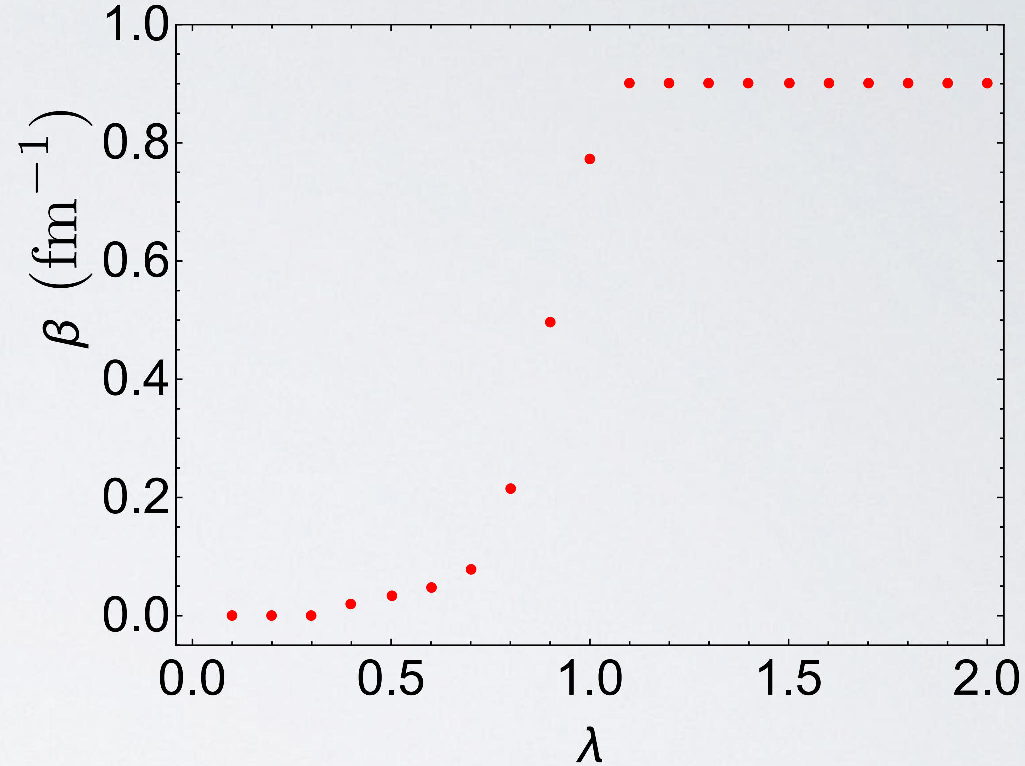
$$\eta = \frac{d\beta}{d\lambda} = \frac{d^2\phi}{d\lambda}$$

The on-shell transition - N3LO

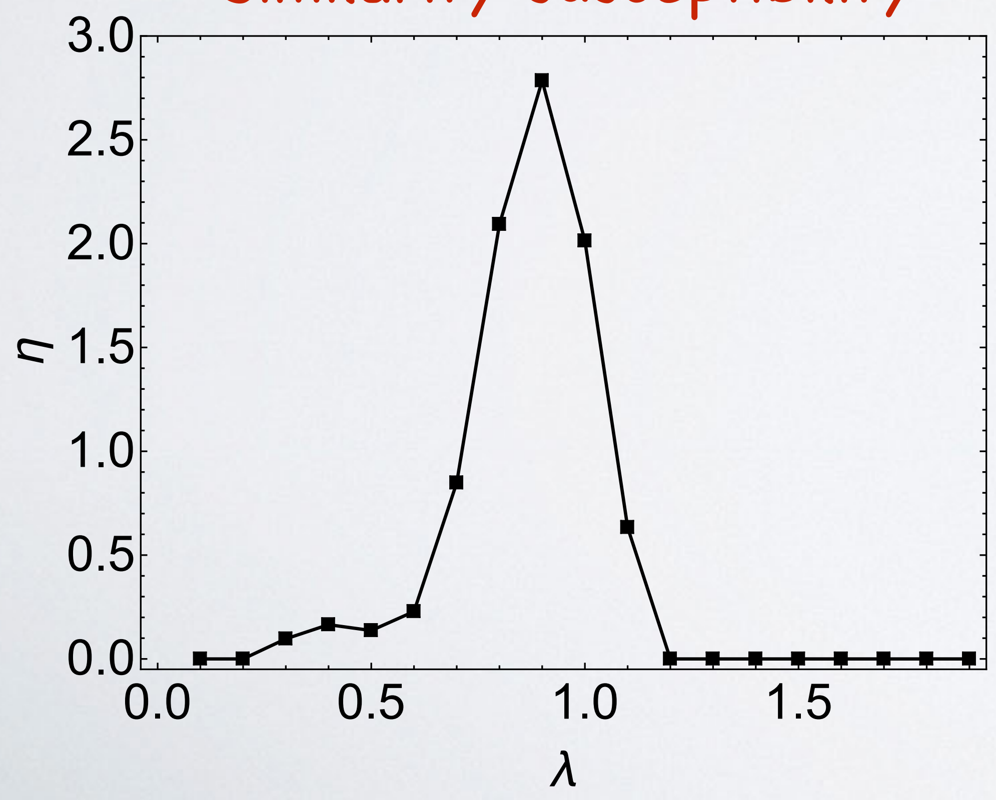
Frobenius norm



Order parameter



Similarity susceptibility



Critical λ

$$\lambda_c = 0.9\ \mathrm{fm}^{-1}$$

Final Remarks

- Equivalence between Implicit and Explicit renormalization
- S-wave completely dominated by most "unknown" part of the nuclear force
- Neutron matter in the unitary limit can be reasonably described by contact interactions
- N2LO results are close but indicate that N3LO terms are required
- Unitary transformations provide a range for ξ and matches the universal value at $\lambda_\xi = 0.9\, k_F = \lambda_c$