Searching for virtual Dark Matter

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Double holography in a slice of AdS

Sylvain Fichet (ICTP/SAIFR, Sao Paulo)

Based on : Just on-going work

AdS/CFT and Holography

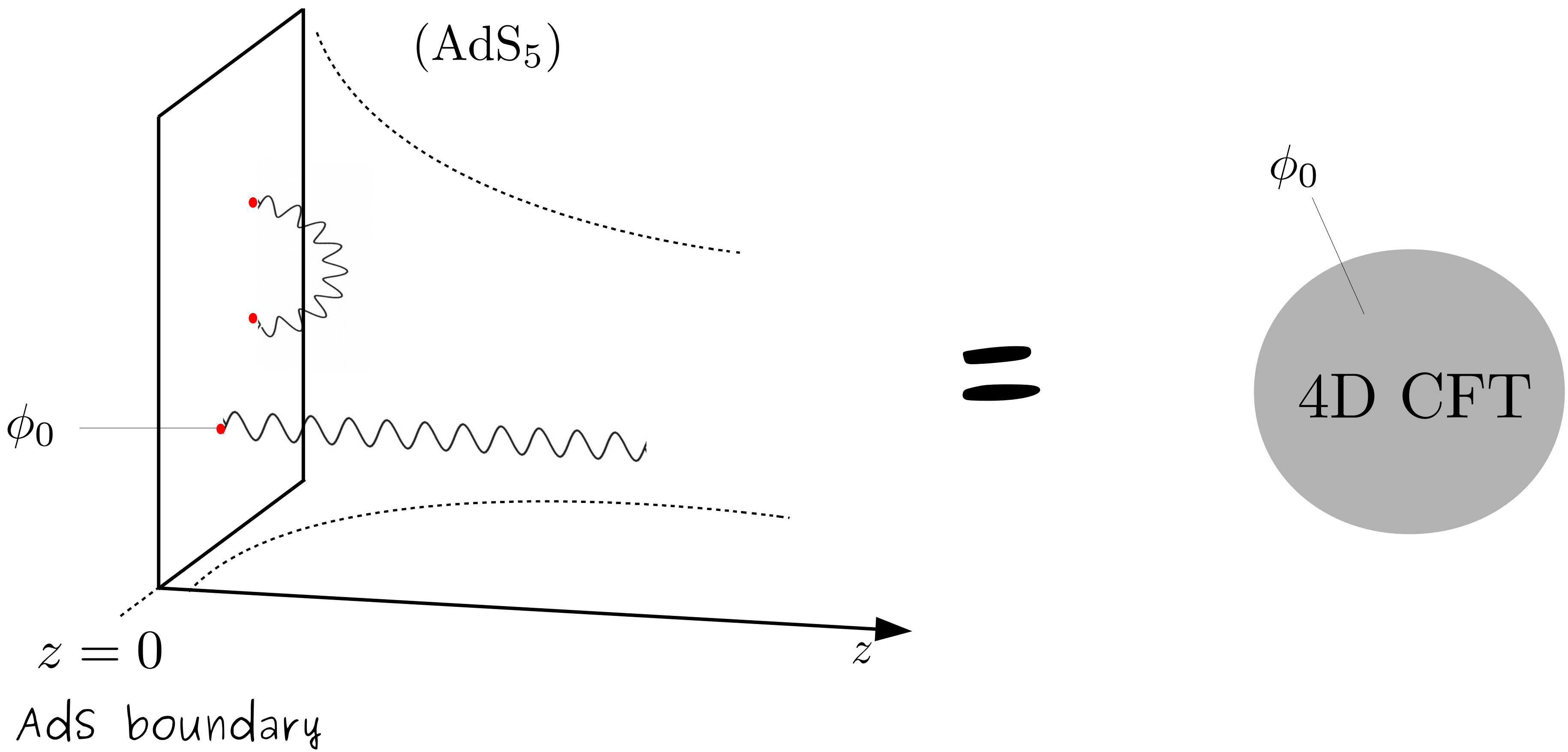
- AdS/CFT : Equivalence between type IIB string theory in AdS_5 x S_5 and N=4 SYM SU(N) gauge theory in 4D [Maldacena '97].

- For our purposes: assume large N, strong t'Hooft coupling, drop SUSY and S_5. Then we have equivalence between a QFT with gravity in AdS_5 and a strongly coupled CFT in 4D.

- Holographic formalism [Witten '98]: value of a field on the AdS boundary corresponds to the source for a CFT operator,

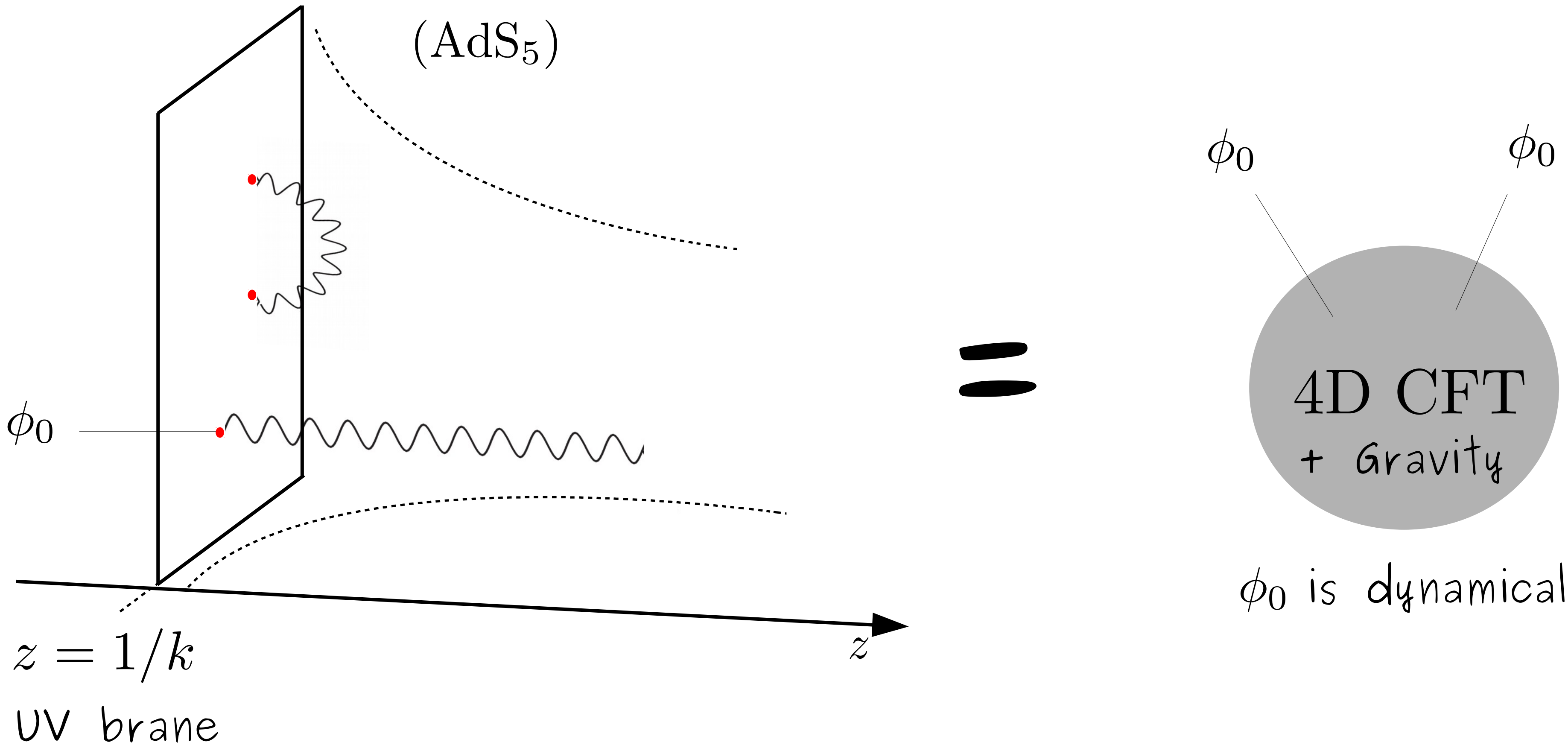
$$\int_{\phi_0} \mathcal{D}\phi e^{iS_{\mathrm{AdS}}[\phi]} \equiv \int \mathcal{D}\phi_{\mathrm{CFT}} e^{i\left(S_{\mathrm{CFT}}+\int d^4x \phi_0 \mathcal{O}_{\mathrm{CFT}}\right)}$$

AdS/CFT and Holography



$$\int_{\phi_0} \mathcal{D}\phi e^{iS_{\mathrm{AdS}}[\phi]} \equiv \int \mathcal{D}\phi_{\mathrm{CFT}} e^{i\left(S_{\mathrm{CFT}} + \int d^4x \phi_0 \mathcal{O}_{\mathrm{CFT}}\right)}$$

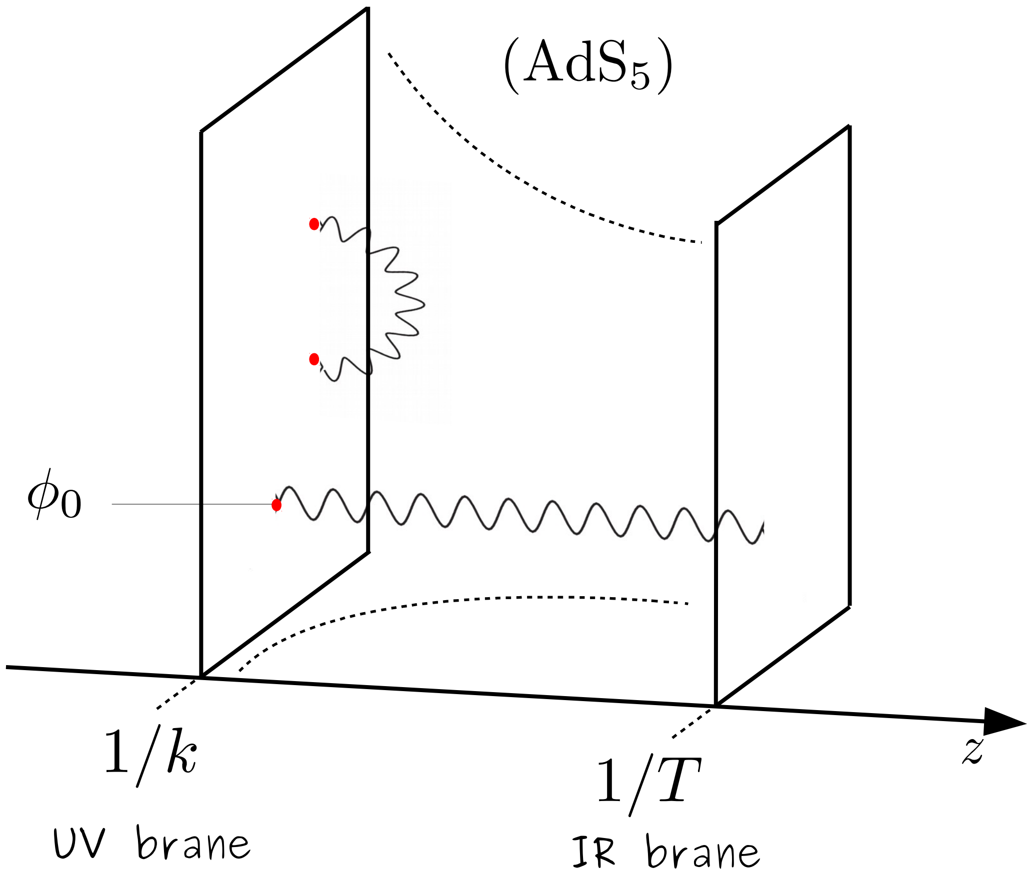
Holography with a UV brane



[Arkani-Hamed/Porrati/Randall '00]

[Rattazzi/Zaffaroni '01]

Holography in a slice of AdS



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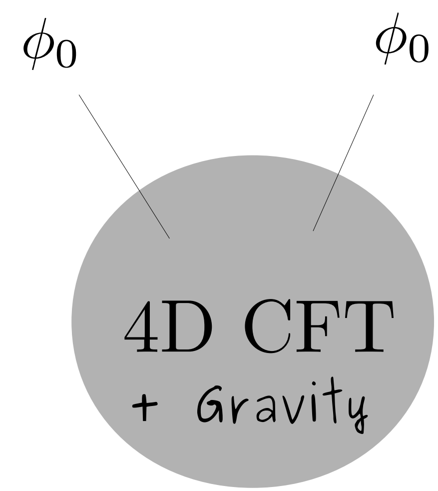


ϕ_0 is dynamical

Scale invariance spontaneously broken in the IR at $\mu \sim T$.

Resonances with $m_n \sim nT$

[Arkani-Hamed/Porrati/Randall '00]

[Rattazzi/Zaffaroni '01]

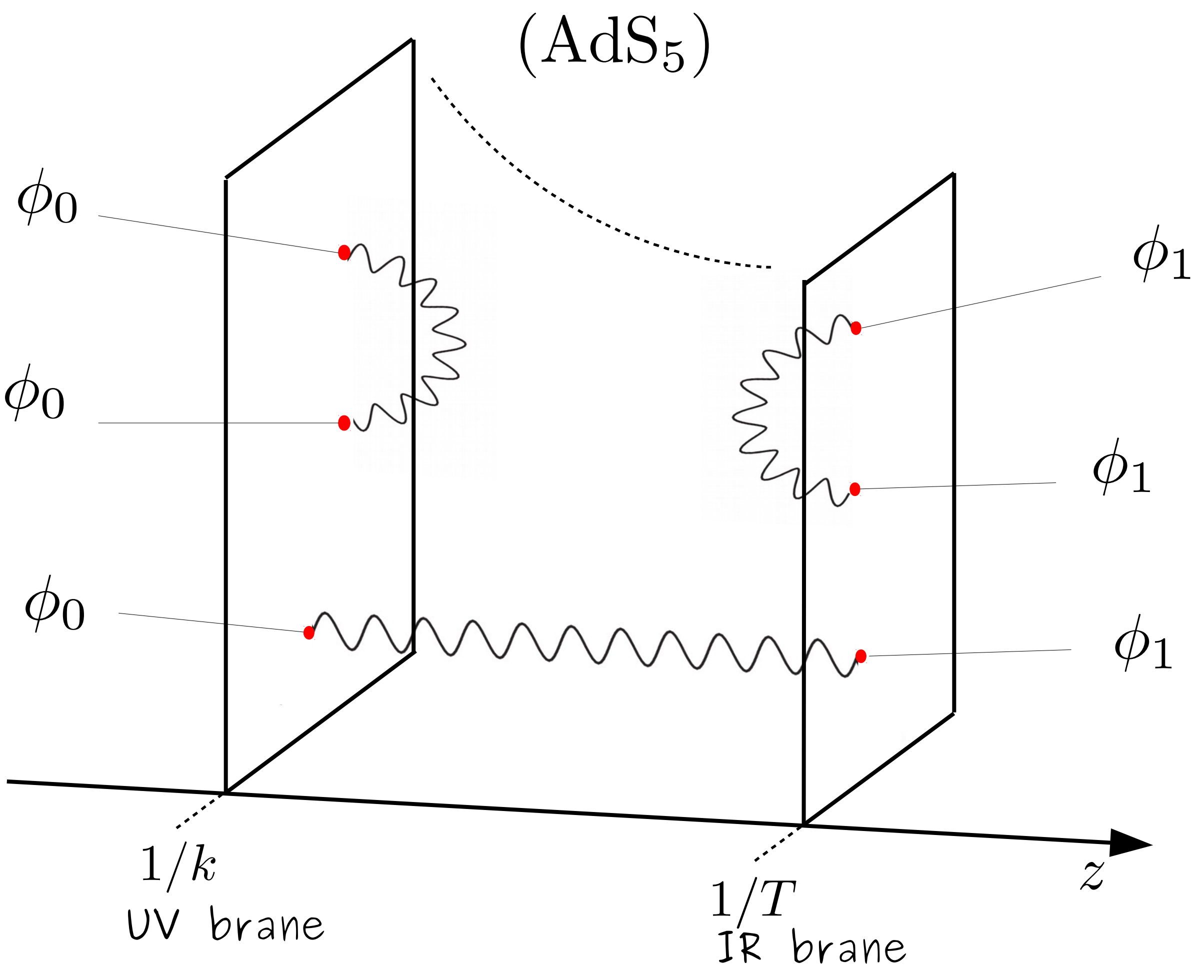
Proposal

The spontaneous breaking of conformal symmetry, the emergence of resonances, all of this is related to the presence of the IR brane. But we do holography on the UV brane. Are we missing some information?

Proposal :

- Treat the IR brane holographically
- Figure out the dual CFT theory
- Hopefully, get new insights/model about how the CFT breaks, how resonances appear, ...

Double Holography



The holographic action:

Hol. self-energies

$$S[\phi_0, \phi_1] = \int \frac{d^4p}{(2\pi)^4} \frac{1}{2} \left(\phi_0 \Pi_0 \phi_0 + 2\phi_0 \Pi_{01} \phi_1 + \phi_1 \Pi_1 \phi_1\right)$$

Double Holography

Given an interval $y \in [y_0, y_1]$, the bulk EOM gives

$$\Phi(y) = Af(y) + Bg(y) \tag{3.1}$$

where A, B are independent of y and the solution $f(y)$, $g(y)$ are independent solutions so that the Wronskian in non zero,

$$W = f'(y)g(y) - f(y)g'(y) \neq 0\,. \tag{3.2}$$

The values of the field on the boundaries are named

$$\Phi(y_0) = \phi_0\,, \quad \Phi(y_1) = \phi_1\,, \tag{3.3}$$

and will be the variables of the holographic action. We also define

$$f(y_0) = f_0\,, \quad f(y_1) = f_1\,, \quad g(y_0) = g_0\,, \quad g(y_1) = g_1\,, \tag{3.4}$$

The A, B constants can be translated into the holographic variables,

$$A = \frac{\phi_0 g_1 - \phi_1 g_0}{f_0 g_1 - f_1 g_0}\,, \quad B = -\frac{\phi_0 f_1 - \phi_1 f_0}{f_0 g_1 - f_1 g_0}\,. \tag{3.5}$$

Double Holography

The holographic action is given by

$$S[\phi_0, \phi_1] = \int \frac{d^4p}{(2\pi)^4} \frac{1}{2} \left(\phi_0 (\partial_y - kb_0) \Phi|_{y_0} - \phi_1 (\partial_y - kb_1) \Phi|_{y_1} \right) . \quad (3.6)$$

One introduces

$$\tilde{f}_i = f'(y_i) - kb_i f(y_i) \quad (3.7)$$

so that

$$(\partial_y - kb_i)\Phi|_{y_i} = A\tilde{f}_i + B\tilde{g}_i . \quad (3.8)$$

Substituting with the holographic variables we get the holographic self energies

$$S[\phi_0, \phi_1] = \int \frac{d^4p}{(2\pi)^4} \frac{1}{2} \left(\phi_0 \Pi_0 \phi_0 + 2\phi_0 \Pi_{01} \phi_0 + \phi_1 \Pi_1 \phi_1 \right) \quad (3.9)$$

with

$$\Pi_0 = \frac{\tilde{f}_0 g_1 - \tilde{g}_0 f_1}{f_0 g_1 - g_0 f_1} , \quad \Pi_1 = \frac{\tilde{f}_1 g_0 - \tilde{g}_1 f_0}{f_0 g_1 - g_0 f_1} , \quad \Pi_{01} = \frac{W}{f_0 g_1 - g_0 f_1} . \quad (3.10)$$

Double Holography

The Green functions of Φ with Neumann boundary conditions takes the form

$$G^{++}(y,y') = -\frac{1}{W}\frac{(\tilde{f}_0 g_1(y_<) - \tilde{g}_0 f_1(y_<))(\tilde{f}_1 g_0(y_>) - \tilde{g}_1 f_0(y_>))}{\tilde{f}_0\tilde{g}_1 - \tilde{g}_0\tilde{f}_1} \tag{3.11}$$

It turns out that the Π_0 (Π_1) self-energy corresponds to the inverse brane-to-brane propagator with Neumann BC on the brane and Dirichlet BC on the opposite brane, *i.e.*

$$\Pi_0 = \frac{1}{G_p^{+-}(y_0, y_0)}, \quad \Pi_1 = \frac{1}{G_p^{-+}(y_1, y_1)}. \tag{3.12}$$

The Π_{01} term can be expressed as

$$\Pi_{01} = \frac{W}{f_0 g_1 - g_0 f_1} = W\frac{1}{\tilde{f}_0\tilde{g}_1 - \tilde{g}_0\tilde{f}_1}\frac{\tilde{f}_0\tilde{g}_1 - \tilde{g}_0\tilde{f}_1}{\tilde{f}_0 g_1 - \tilde{g}_0 f_1}\frac{\tilde{f}_0 g_1 - \tilde{g}_0 f_1}{f_0 g_1 - g_0 f_1} \tag{3.13}$$

$$= -\frac{G_p^{++}(y_0, y_1)}{G_p^{++}(y_1, y_1)\, G_p^{+-}(y_0, y_0)} \tag{3.14}$$

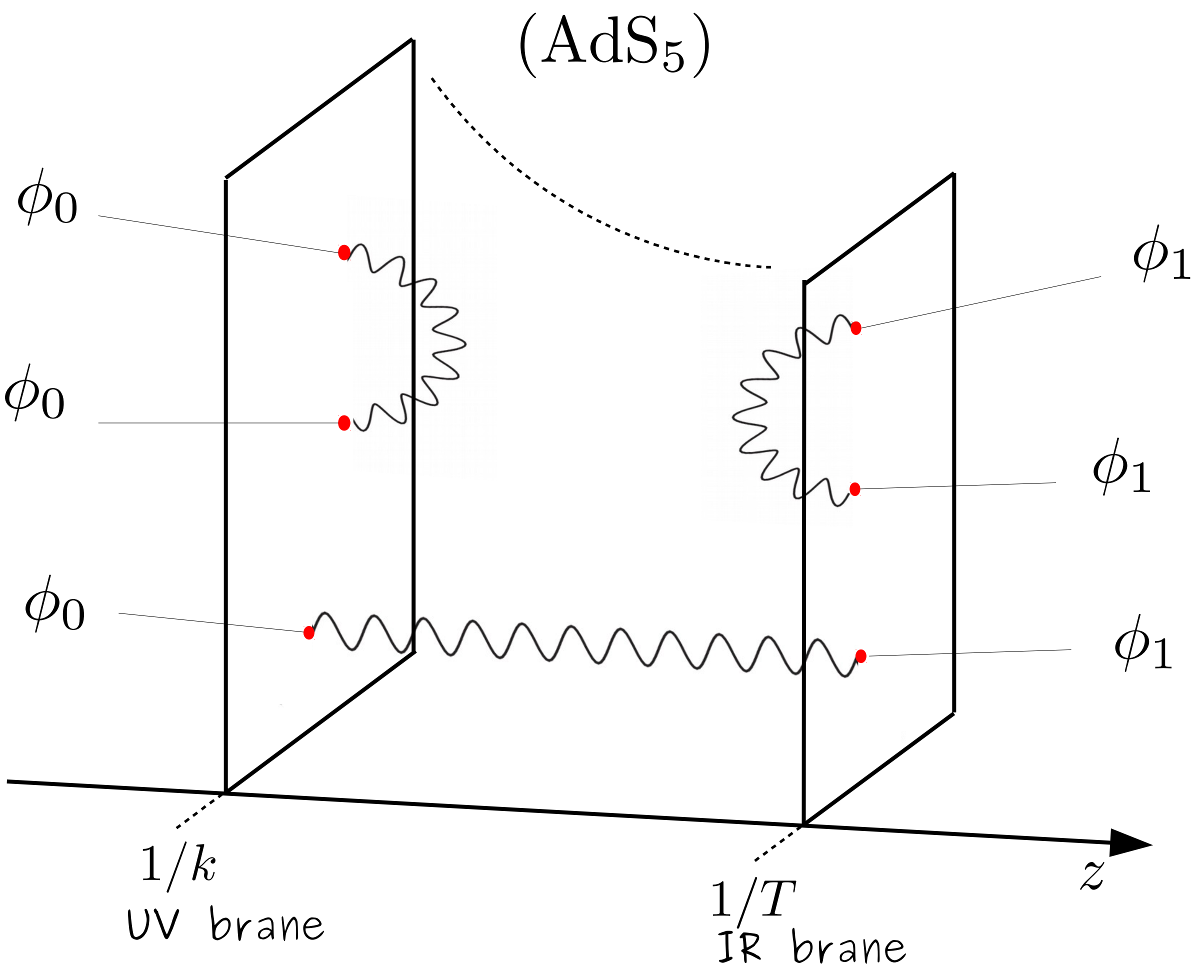
$$= -\frac{G_p^{++}(y_0, y_1)}{G_p^{++}(y_0, y_0)\, G_p^{-+}(y_1, y_1)}. \tag{3.15}$$

One recognizes amputated brane-to-bulk propagators in these expressions. In particular we have

Usual holographic profile

$$\Pi_{01} = -K(y_1)\Pi_1 \tag{3.16}$$

Double Holography



The holographic action:

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Double Holography in a slice of AdS

We apply the formalism to a slice of AdS. Focus on the $T < p < k$ momentum region. We use massive sources, i.e. generic brane mass terms $b_i k$.

UV self-energy:

CFT correlator

$$\Pi_0 = b_{\rm UV} k + (b_{\rm UV} + 2\alpha)k \left(\frac{p}{2k}\right)^{2\alpha} \frac{\Gamma(-\alpha)}{\Gamma(\alpha)} S_\alpha \qquad S_\alpha = \frac{\sin\left(\frac{p}{T} - \frac{\pi}{4}(1-2\alpha)\right)}{\sin\left(\frac{p}{T} - \frac{\pi}{4}(1+2\alpha)\right)}$$

ϕ_0 = massive source

- This is the same result as for UV holography with massive source.
- $S_\alpha \sim 1$ whenever a small imaginary part is included near the poles. Hence the poles tend to vanish and one recovers the unbroken CFT. The IR brane becomes irrelevant.

Double Holography in a slice of AdS

We apply the formalism to a slice of AdS. Focus on the $T < p < k$ momentum region. We use massive sources, i.e. generic brane mass terms $b_i k$.

IR self-energy:

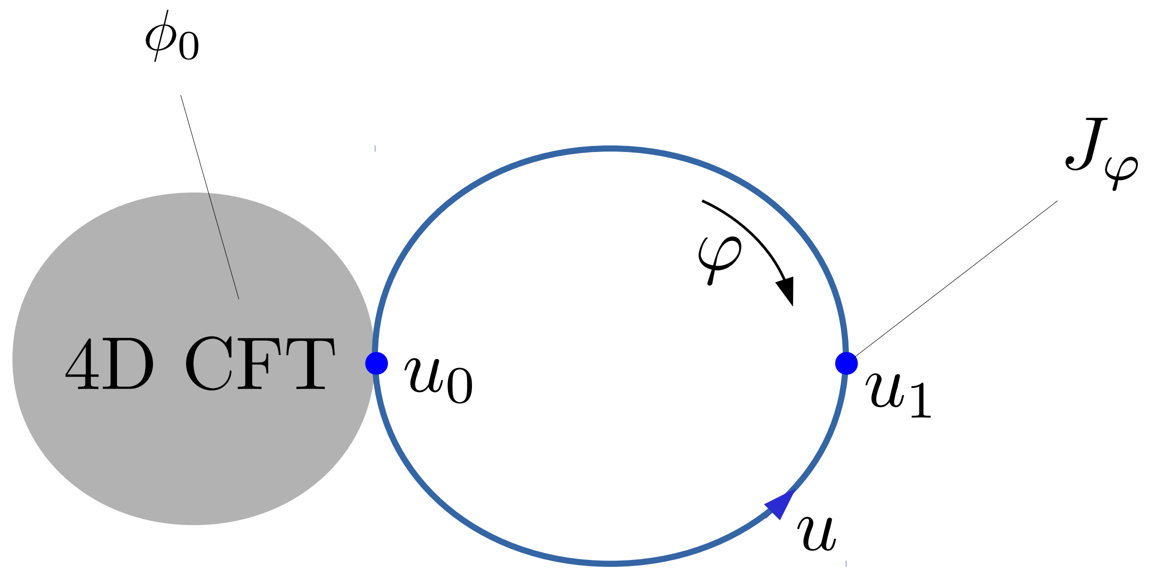
Evenly-spaced zeros and poles

$$\Pi_1 = k\frac{p}{T}\cot\left(\frac{p}{T} + \frac{\pi}{4}(1-2\alpha)\right)$$

- This is the inverse propagator of a free scalar field on circle or on an interval.
- The poles do not tend to vanish.
- This self-energy seems to describe the bound states arising from the CFT. Hence we call $\phi_1 \equiv \varphi$ the meson field.

With that we can start to figure out the CFT dual

Proposed CFT dual (naive version)



$\varphi \equiv \phi_1$

$u \in [0, 2/T]$

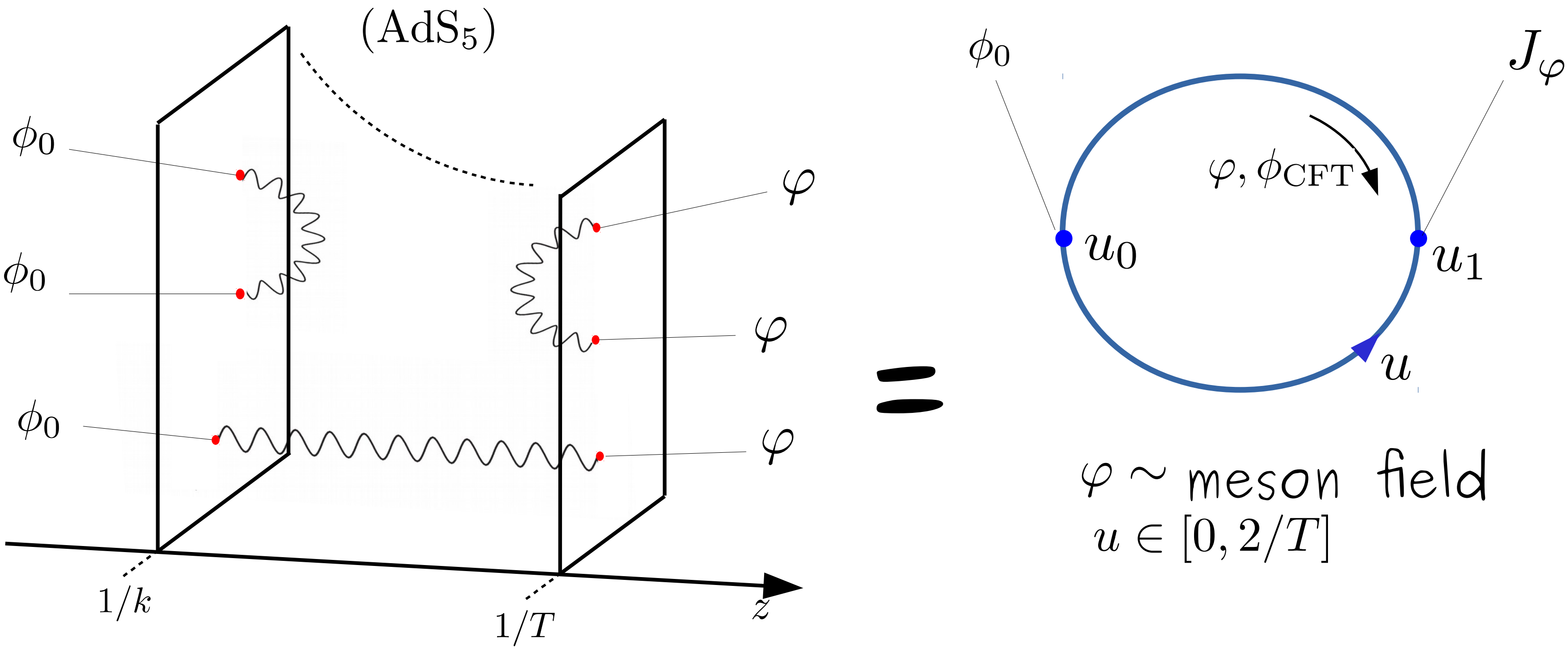
Holographic functional:

$$W[\phi_0, \varphi] = i \log \left[\int \mathcal{D}\phi_{\rm CFT} \mathcal{D}\varphi e^{i\left(S_{\rm CFT} + S[\varphi] + \int d^4x\, \phi_0 \mathcal{O}|_{u=u_0} + \int d^4x\, J_\varphi \varphi|_{u=u_1}\right)} \right]$$

$$- \int d^4x\; J_\varphi \varphi|_{u=u_1} \qquad \text{with} \;\; S[\varphi] = \int d^4x \int_0^{2/T} du (\partial_M \varphi)^2 \,.$$

W is a free energy in ϕ_0 and an effective action in φ

Conclusion



- A two-sources holographic formalism is proposed
- Hopefully it could lead to new insights about CFT breaking
- It's an ongoing work, comments are very welcome

THANKS

Double Holography

Finally, let us check that we recover the standard holographic action when integrating over the ϕ_1 variable. For a Dirichlet boundary condition on y_1, we have $\phi_1 = 0$. It follows trivially that

$$S[\phi_0] = \int \frac{d^4p}{(2\pi)^4} \frac{\phi_0^2}{2\, G_p^{+-}(y_0, y_0)} \quad ✓ \tag{3.17}$$

which is the expected result. For a Neumann boundary condition on y_1, we have $(\partial_y - kb_1)\Phi|_{y_1} = 0$, which implies

$$\phi_1 = \phi_0 \frac{W}{g_0 \tilde{f}_1 - f_0 \tilde{g}_1} \, . \tag{3.18}$$

Replacing in the action one obtains

$$S[\phi_0] = \int \frac{d^4p}{(2\pi)^4} \frac{\phi_0^2}{2\, G_p^{++}(y_0, y_0)} \quad ✓ \tag{3.19}$$

which is again the expected result.

Double Holography in a slice of AdS

We turn to the mixed UV/IR self-energy Π_{01} .

- The general double holography formalism gives $\Pi_{01} = -K(y_1)\Pi_1$ where $K(y_1)$ is the ``holographic profile'', i.e. the Green function giving $\phi(y,p) = \phi_0(p)K(y,p)$, i.e. the amputated UV-brane-to-bulk propagator

$$K_p(z) = \frac{G_p^{++}(z_0, z)}{G_p^{++}(z_0, z_0)}$$

- In a slice of AdS:

$$K(y) = 2\sqrt{\frac{\pi T}{p}} \left(\frac{p}{2k}\right)^{\alpha} \frac{1}{\Gamma(\alpha)\cos\left(\frac{p}{T} + \frac{\pi}{4}(1-2\alpha)\right)} \qquad T < p < k$$

- The other self-energies are about nearly exact CFT and meson states respectively. Hence this one must contain information about the link between both.

Proposed CFT dual

Using the CFT functional we already set, we have

$$\Pi_{01} \equiv \frac{\delta^2 W[\phi_0, \varphi]}{\delta\phi_0(p)\delta\varphi(-p)}$$

Using chain rule and definition of Legendre transform,

$$\frac{\delta^2 W[\phi_0, \varphi]}{\delta\phi_0\delta\varphi} = \frac{\delta^2 W[\phi_0, \varphi]}{\delta\phi_0\delta J_\varphi}\frac{\delta J_\varphi}{\delta\varphi} = -\frac{\delta^2 W[\phi_0, \varphi]}{\delta\phi_0\delta J_\varphi}\Pi_1 \,.$$

Therefore $$\boxed{K(y_1) = \frac{\delta^2 W[\phi_0, \varphi]}{\delta\phi_0\delta J_\varphi} = -i\,\langle \mathcal{O}(p, u_0)\varphi(-p, u_1)\rangle_{\rm conn}\,.}$$

→ The holographic profile corresponds to the connected correlator between the nearly exact operator and the meson field.

A simple model of CFT breaking (ongoing work)

This is nice but up to now $\langle \mathcal{O}(p, u_0) \varphi(-p, u_1) \rangle_{\text{conn}}$ is only predicted by the AdS side.

- Let us try to figure out a model on the CFT side that reproduces $\langle \mathcal{O}(p, u_0) \varphi(-p, u_1) \rangle_{\text{conn}}$ and remains consistent with the CFT picture already established.

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- We have already introduced a compact extradimension. So let's assume that the CFT also lives in the extradimension. We use 4d momentum/position space and large momentum

$$\langle \mathcal{O}(p,0)\mathcal{O}(-p,u)\rangle = iG_{CFT}(p,u) = \frac{i\pi}{4\Gamma(\Delta)}\left(\frac{p}{2u}\right)^{\Delta-2} H^{(2)}_{2-\Delta}(pu)$$

$$\approx i\sqrt{\frac{2\pi}{pu}}\frac{1}{4\Gamma(2+\alpha)}\left(\frac{p}{2u}\right)^{\alpha} e^{-i\left(pu+\frac{\pi}{4}(2\alpha-1)\right)}$$

if $p > 1/u$

$\Delta = 2+\alpha$

A simple model of CFT breaking (ongoing work)

- Now let's apply the compactification but only to the phase. A prescription which does it is

$$G^{\text{comp}}_{CFT}(p,u) = u^{-\Delta+3/2} \sum_{n=-\infty}^{\infty} (u+n2L)^{\Delta-3/2}\, G_{CFT}(p,u+n2L) + Z_2, Z_2' \text{ reflections.}$$

- Neglecting the phases for simplicity, we get

$$G^{\text{comp}}_{CFT}(p,L) = \sqrt{\frac{2\pi}{pL}}\frac{1}{4\Gamma(2+\alpha)}\left(\frac{p}{2L}\right)^{\alpha}\frac{1}{2p\sin(pL)} \qquad (T<p<k)$$

  This is pretty close from $K(y_1)$

- Morever Π_0 remains unchanged, i.e. CFT still nearly exact from the viewpoint of ϕ_0 because for $u=0$ the $n=0$ winding mode dominates.

- In summary the model is

$$\langle \mathcal{O}(p,0)\varphi(-p,L)\rangle_{\text{conn}} \equiv \langle \mathcal{O}(p,0)\mathcal{O}(-p,L)\rangle^{\text{comp}}_{\text{conn}}\ .$$