Possible LQCD projects in Brazil?

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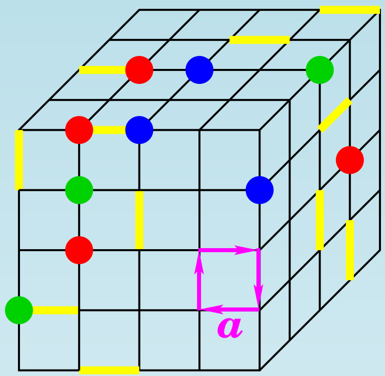
Lattice QCD



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⇒ Different approach to QFT, direct access to (representative) gauge-field configurations

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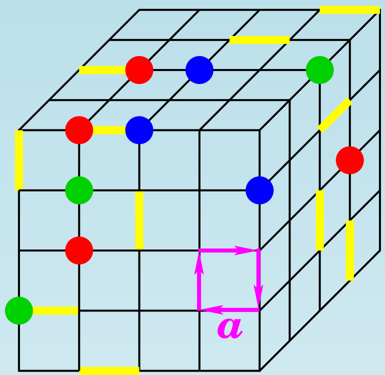

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Current importance for predictions and data analyses in Particle Physics and Nuclear Physics ⇒ `hep-lat` section, 36th Annual Lattice Symposium, ...)

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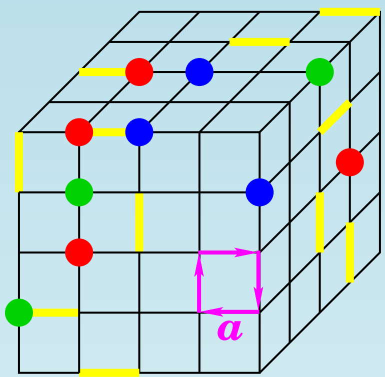

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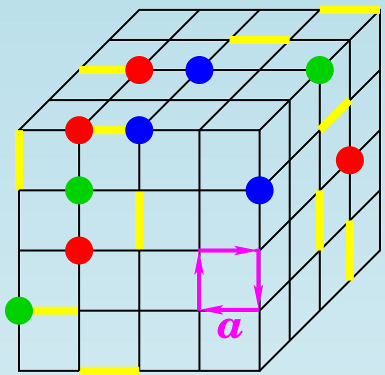

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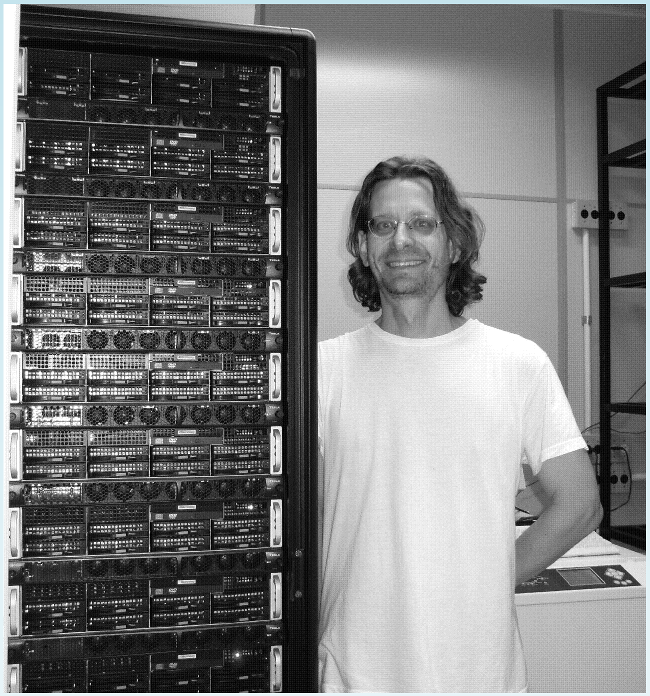
Possibility to understand qualitative features of QCD, e.g. color confinement nonperturbatively

One of the challenges in the simulations is to consider large volumes to reach the IR limit

Our Group at IFSC–USP



To learn more: http://lattice.ifsc.usp.br/

The Lattice Action

The Wilson action (1974)

$$S \; = \; -\frac{\beta}{3} \sum_{\Box} \mathrm{Re\,Tr}\, U_{\Box} \,, \quad U_{x,\mu} \; \equiv \; e^{i g_0 a A^b_\mu(x) T_b} \,, \quad \beta \; = \; 6/{g_0}^2$$

- written in terms of oriented plaquettes formed by the link variables $U_{x,\mu}$, which are group elements
- under gauge transformations: $U_{x,\mu} \to g(x)\, U_{x,\mu}\, g^\dagger(x+\mu)$, where $g \in SU(3)$ $\Rightarrow$ closed loops are gauge-invariant quantities
- integration volume is finite: no need for gauge-fixing

At small β (i.e. strong coupling) we can perform an expansion analogous to the high-temperature expansion in statistical mechanics. At lowest order, the only surviving terms are represented by diagrams with “double” or “partner” links, i.e. the same link should appear in both orientations, since $\int dU\, U_{x,\mu} = 0$

(Numerical) Lattice QCD

Classical Statistical-Mechanics model with the partition function

$$Z \;=\; \int \mathcal{D}U\, e^{-S_g} \int \mathcal{D}\psi\, \mathcal{D}\overline{\psi}\;\; e^{-\int d^4x\; \overline{\psi}(x)\, K\, \psi(x)} \;=\; \int \mathcal{D}U\, e^{-S_g}\; \det K(U)$$

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Evaluate expectation values

$$\langle \mathcal{O} \rangle = \int \mathcal{D}U \, \mathcal{O}(U) \, P(U) = \frac{1}{N} \sum_i \mathcal{O}(U_i)$$

with the weight

$$P(U) = \frac{e^{-S_g(U)} \, \det K(U)}{Z}$$

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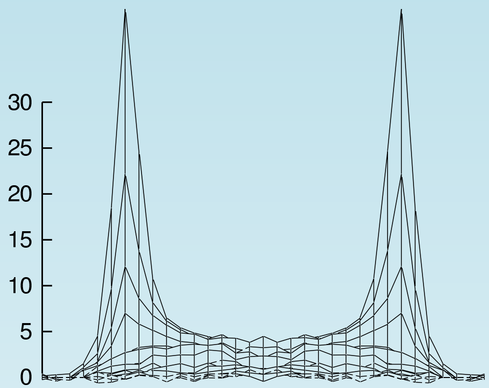
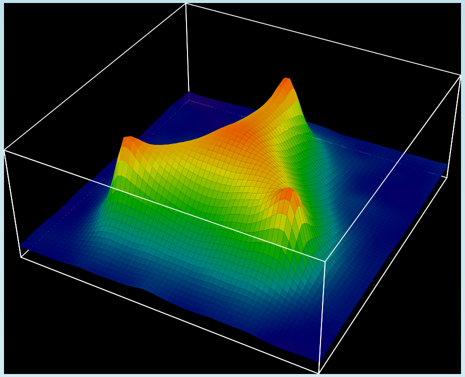
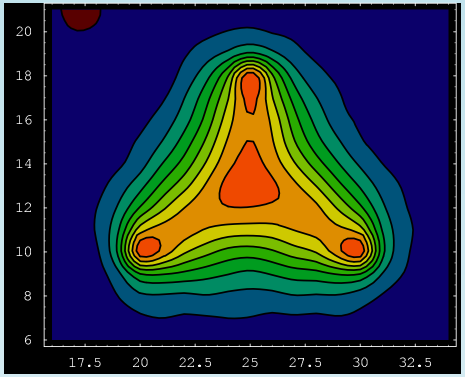
$\Rightarrow$ Monte Carlo simulations: sample representative gauge configurations, then compute O and take average

Confinement from Simulations

May observe formation of flux tubes







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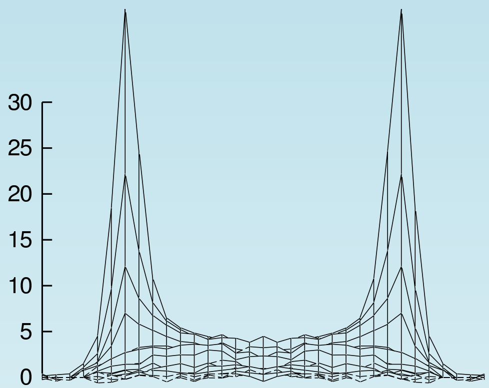
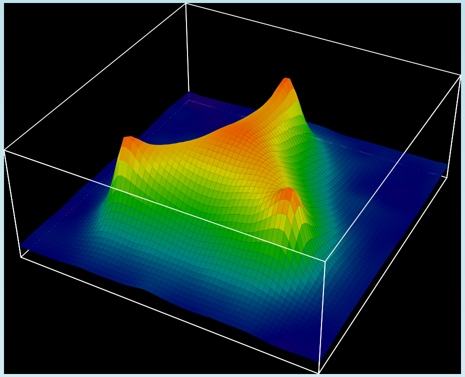
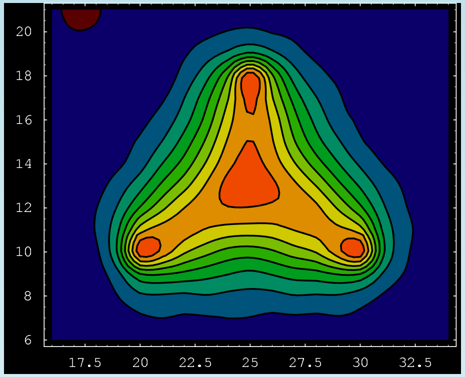
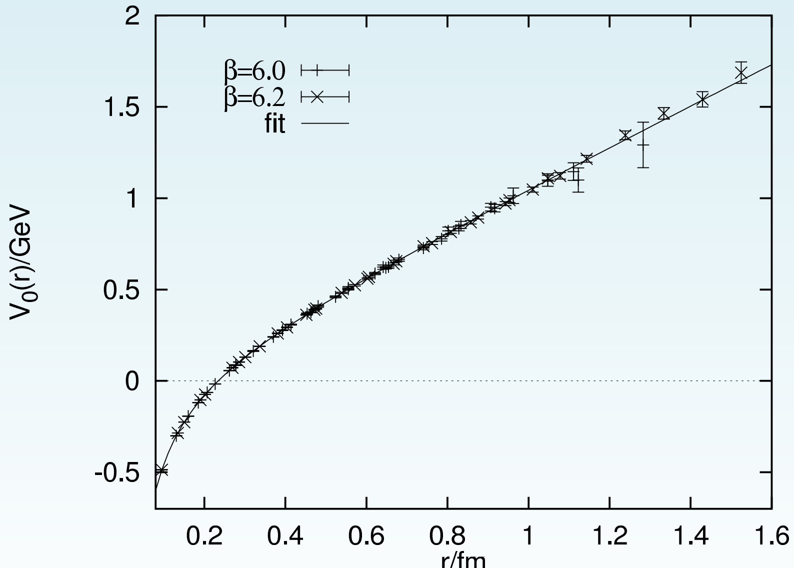
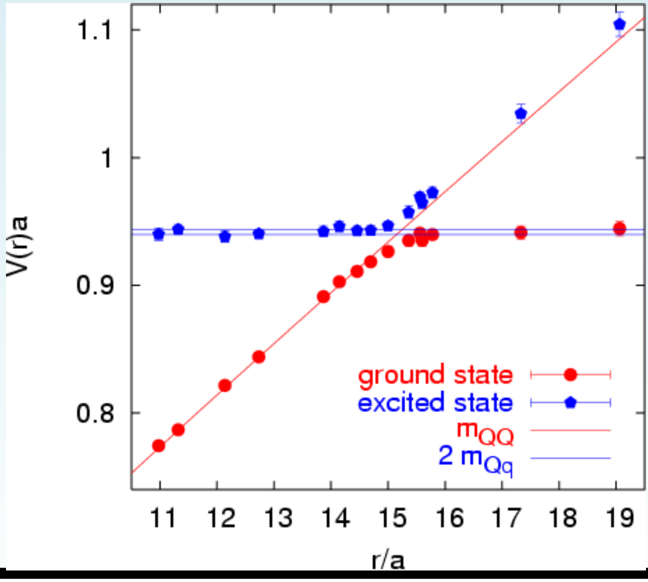






Linear Growth of potential between quarks, string breaking





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- Gribov-Zwanziger confinement scenario based on suppressed gluon propagator and enhanced ghost propagator in the infrared

IR gluon propagator and confinement

- Green's functions carry all information of a QFT's physical and mathematical structure.
- Gluon propagator (two-point function) as the most basic quantity of QCD.
- Confinement given by behavior at large distances (small momenta) $\Rightarrow$ nonperturbative study of IR gluon propagator.

Landau gluon propagator

$$
\begin{aligned}
D^{ab}_{\mu\nu}(p) &= \sum_x e^{-2i\pi k\cdot x} \langle A^a_\mu(x)\, A^b_\nu(0)\rangle \\
&= \delta^{ab} \left(g_{\mu\nu} - \frac{p_\mu\, p_\nu}{p^2} \right) D(p^2)
\end{aligned}
$$

NOT IR enhanced...

Quantization and Gribov Copies

The invariance of the Lagrangian under local gauge transformations implies that, given a configuration $\{A(x), \psi_f(x)\}$, there are infinitely many gauge-equivalent configurations $\{A^g(x), \psi_f^g(x)\}$ (gauge orbits). In the path integral approach we integrate over all possible configurations

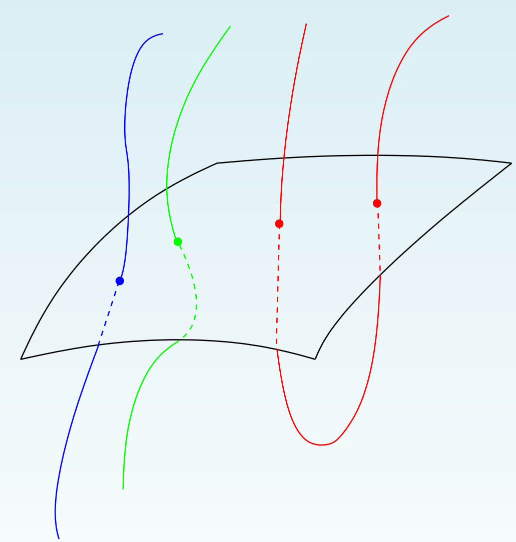
$$Z = \int DA \, \exp\left[-\int d^4x \, \mathcal{L}(x)\right] \, .$$

There is an infinite factor coming from gauge invariance: $\int DA = \int D\overline{A}^g J Dg$ and $\int Dg = \infty$.

To solve this problem we can choose a representative $\overline{A}$ on each gauge orbit (gauge fixing) using a gauge-fixing condition $f(\overline{A}) = 0$. The change of variable $A \to \overline{A}$ introduces a Jacobian in the measure.

Question: does the gauge-fixing condition select one and only one representative on each gauge orbit?

Answer: in general this is not true (Gribov copies).



Lattice Landau Gauge (I)

In the continuum: $\partial_\mu A_\mu(x) = 0$. On the lattice the Landau gauge is imposed by minimizing the functional

$$\mathcal{E}[U;g] = -\sum_{x,\mu} Tr\, U_\mu^{(g)}(x) \,,$$

where $g(x) \in SU(N_c)$ and $U_\mu^{(g)}(x) = g(x)\, U_\mu(x)\, g^\dagger(x + \hat{e}_\mu)$ is the lattice gauge transformation.

By considering the relations $U_\mu(x) = e^{i\, A_\mu(x)}$ and $g(x) = e^{i\,\tau\,\gamma(x)}$, we can expand $\mathcal{E}[U;g]$ (for small τ):

$$\mathcal{E}[U;g] = \mathcal{E}[U;\mathbb{1}] + \tau\, \mathcal{E}^{'}[U;\mathbb{1}](b,x)\, \gamma^b(x)$$

$$+ \frac{\tau^2}{2}\, \gamma^b(x)\, \mathcal{E}^{''}[U;\mathbb{1}](b,x;c,y)\, \gamma^c(y) + \ldots \,,$$

where $\mathcal{E}^{''}[U;\mathbb{1}](b,x;c,y) = \mathcal{M}(b,x;c,y)[A]$ is a lattice discretization of the Faddeev-Popov operator $-D \cdot \partial$

Lattice Landau Gauge (II)

At any local minimum (stationary solution)

$$\mathcal{E}'(0) = 0 \quad \forall \ \{\gamma^b(x)\} \quad \Rightarrow \quad [(\nabla \cdot A)(x)]^b = 0 \quad \forall \ x, b \,,$$

where

$$A_\mu(\vec{x}) = \frac{1}{2\,i} \left[U_\mu(\vec{x}) - U_\mu^\dagger(\vec{x}) \right]_{\text{traceless}}$$

is the gauge field and

$$\left(\nabla \cdot A^b\right)(\vec{x}) = \sum_{\mu=1}^{d} A_\mu^b(\vec{x}) - A_\mu^b(\vec{x} - \hat{e}_\mu)$$

is the (minimal) Landau gauge condition on the lattice

Ghost Propagator

Ghost fields are introduced as one evaluates functional integrals by the Faddeev-Popov method, which restricts the space of configurations through a gauge-fixing condition. The ghosts are unphysical particles, since they correspond to anti-commuting fields with spin zero.

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On the lattice, the (minimal) Landau gauge is imposed as a minimization problem and the ghost propagator is given by

$$G(p) \,=\, \frac{1}{N_c^2 - 1} \sum_{x,\,y,\,a} \frac{e^{-2\pi i\, k\cdot(x-y)}}{V} \,\langle\, \mathcal{M}^{-1}(a,x;a,y) \,\rangle \,,$$

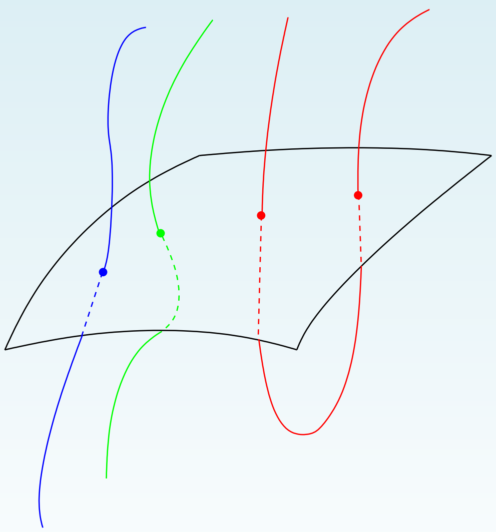
where the Faddeev-Popov (FP) matrix $\mathcal{M}$ is obtained from the second variation of the minimizing functional.

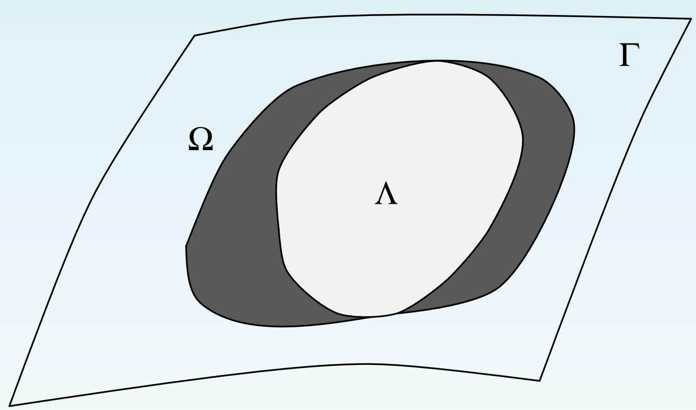
Early simulations: Suman & Schilling, PLB 1996; Cucchieri, NPB 1997

Ghost Enhancement

Gribov's restriction beyond quantization using Faddeev-Popov (FP) method implies taking a minimal gauge, defined by a minimizing functional in terms of gauge fields and gauge transformation

$\Rightarrow$ FP operator (second variation of functional) has non-negative eigenvalues. First Gribov horizon $\partial\Omega$ approached in infinite-volume limit, inducing ghost enhancement





GZ Scenario: Confinement by Ghost

Formulated for Landau gauge, predicts gluon propagator

$$D^{ab}_{\mu\nu}(p) = \sum_x e^{-2i\pi k\cdot x} \langle A^a_\mu(x)\, A^b_\nu(0)\rangle = \delta^{ab} \left(g_{\mu\nu} - \frac{p_\mu\, p_\nu}{p^2} \right) D(p^2)$$

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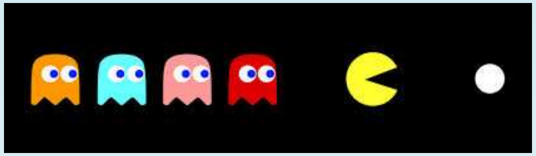

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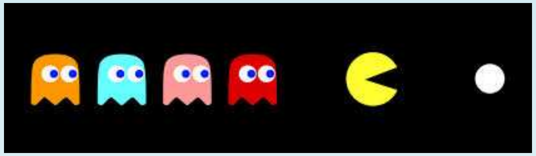

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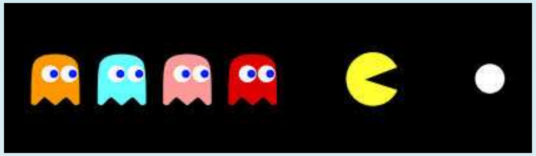

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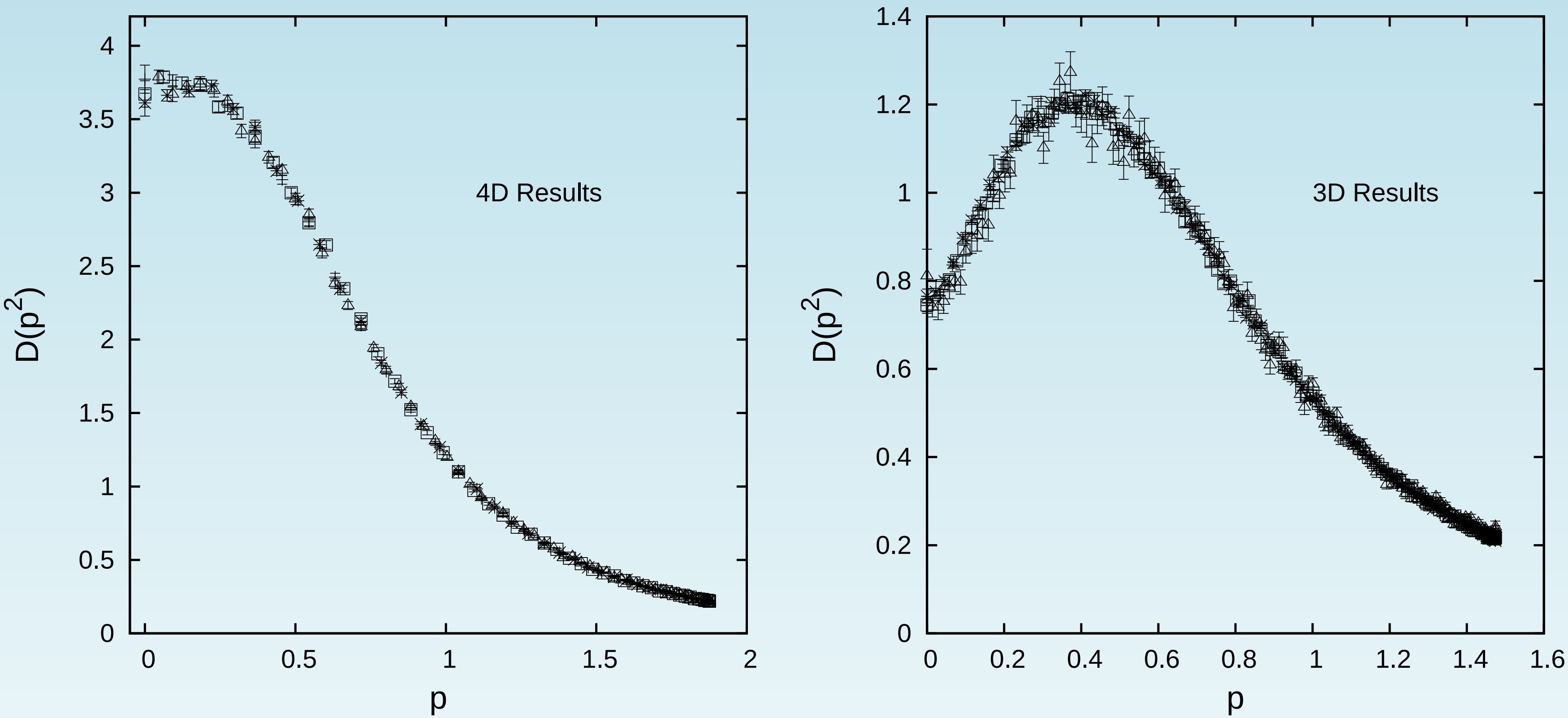
- Infinite volume favors configurations on the first Gribov horizon, where minimum nonzero eigenvalue λ_{min} of Faddeev-Popov operator $\mathcal{M}$ goes to zero
- In turn, $G(p)$ should be IR enhanced, introducing long-range effects, which are related to the color-confinement mechanism

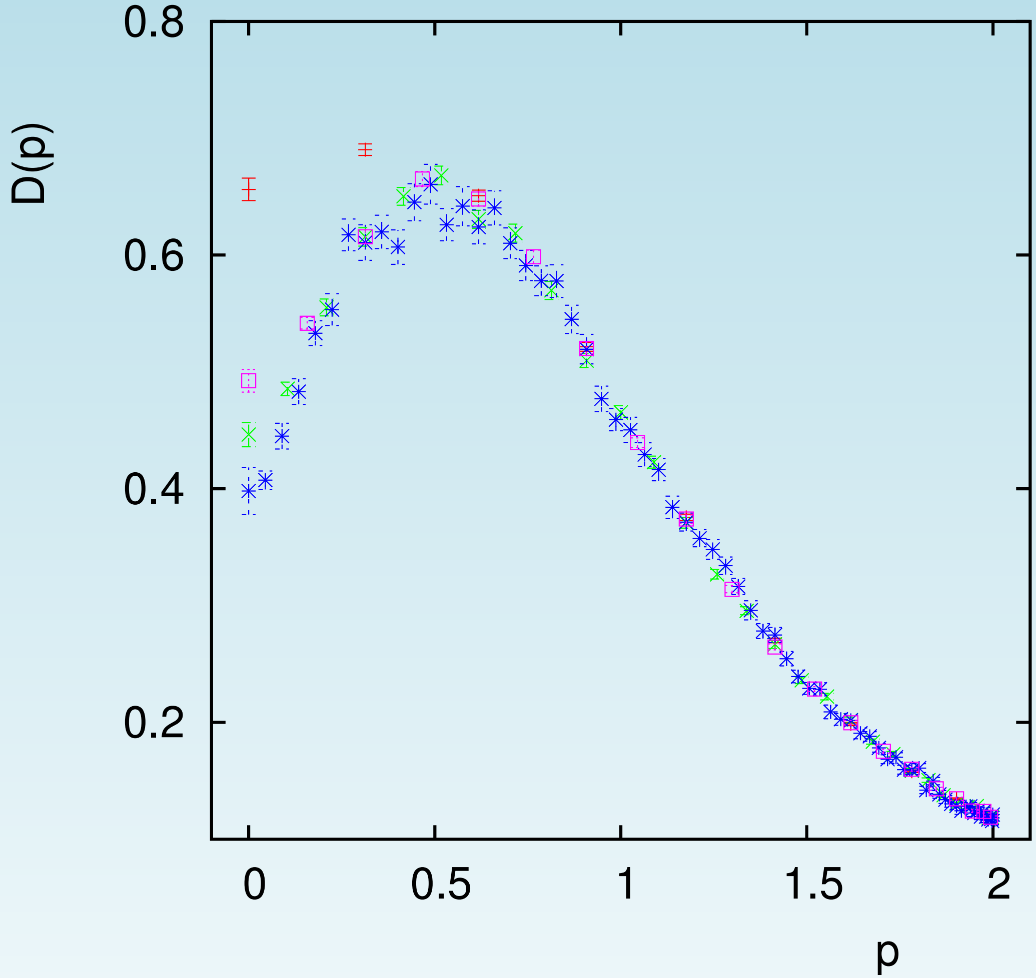
Gluon Propagator at "Infinite" Volume

Attilio Cucchieri & T.M. (2008)



Gluon propagator $D(k)$ as a function of the lattice momenta k (both in physical units) for the pure-$SU(2)$ case in $d = 4$ (left), considering volumes of up to 128^4 (lattice extent ~ 27 fm) and $d = 3$ (right), considering volumes of up to 320^3 (lattice extent ~ 85 fm)

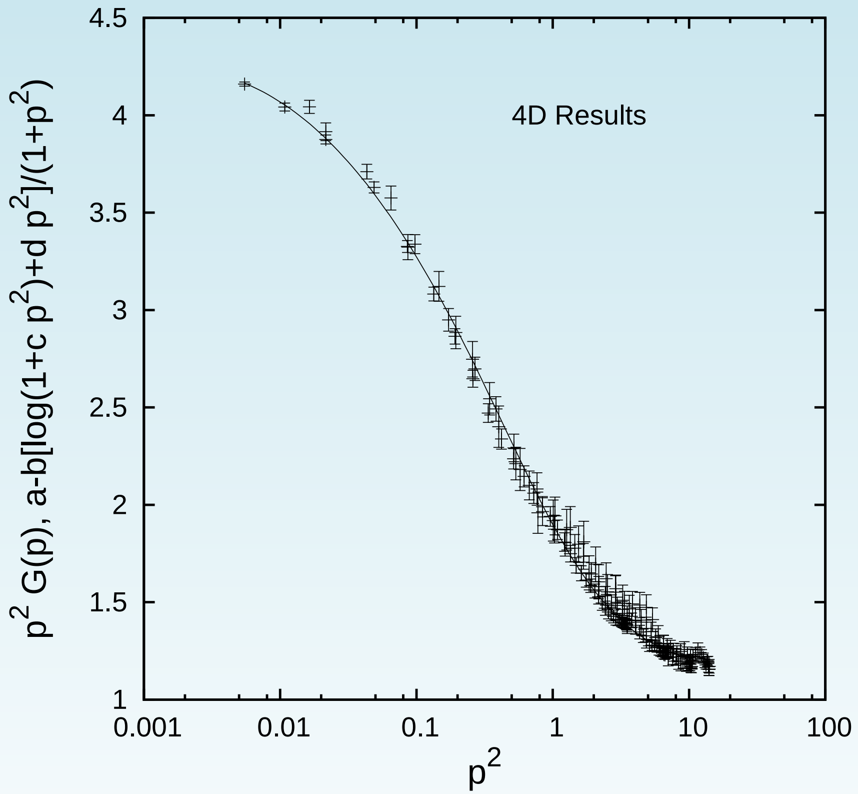
Gluon Propagator: Volume Effects



Gluon propagator as a function of the lattice momentum p for lattice volumes $V = 20^3$, 40^3, 60^3 and 140^3 at $\beta = 3.0$. About 100 days using a 13 Gflops PC cluster (2003)

Ghost Propagator Results

Fit of the ghost dressing function $p^2G(p^2)$ as a function of p^2 (in GeV) for the 4d case ($\beta = 2.2$ with volume 80^4). We find that $p^2G(p^2)$ is best fitted by the form $p^2G(p^2) = a - b[\log(1 + cp^2) + dp^2]/(1 + p^2)$, with



$a = 4.32(2)$,

$b = 0.38(1)\, GeV^2$,

$c = 80(10)\, GeV^{-2}$,

$d = 8.2(3)\, GeV^{-2}$.

In IR limit $p^2G(p^2) \sim a$.

Attilio Cucchieri & T.M. (2008)

Relating λ_{min} and Geometry

Using properties of Ω and the concavity of the minimum function, one can show (A. Cucchieri, TM, PRD 2013)

$$\lambda_{min}\left[\,\mathcal{M}[A]\,\right] \;\geq\; [1-\rho(A)]\, p^2_{min}$$

Here $1-\rho(A) \leq 1$ measures the distance of a configuration $A \in \Omega$ from the boundary $\partial\Omega$ (in such a way that $\rho^{-1} A \equiv A' \in \partial\Omega$). This result applies to any Gribov copy belonging to Ω

Recall that $A' \in \partial\Omega \Longrightarrow$ the smallest non-trivial eigenvalue of the FP matrix $\mathcal{M}[A']$ is null, and that the smallest non-trivial eigenvalue of (minus) the Laplacian $-\partial^2$ is p^2_{min}

In the Abelian case one has $\mathcal{M} = -\partial^2$ and $\lambda_{min} = p^2_{min}$
$\Longrightarrow$ non-Abelian effects are included in the $(1-\rho)$ factor

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- get FP matrix without considering ghost fields explicitly
- Lattice momenta given by $\hat{p}_\mu = 2\sin\left(\pi\, n_\mu/N\right)$ with $n_\mu = 0, 1, \ldots, N/2$ $\Leftrightarrow$ $p_{min} \sim 2\pi/(a\,N) = 2\pi/L$, $p_{max} = 4/a$ in physical units

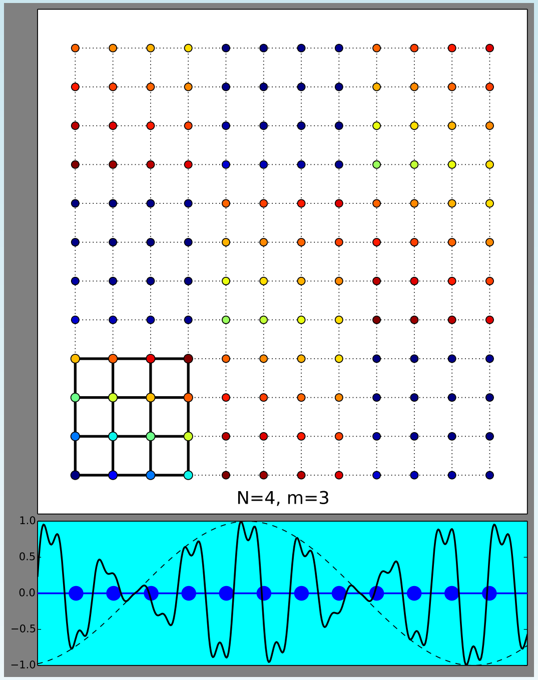
3-Step Code

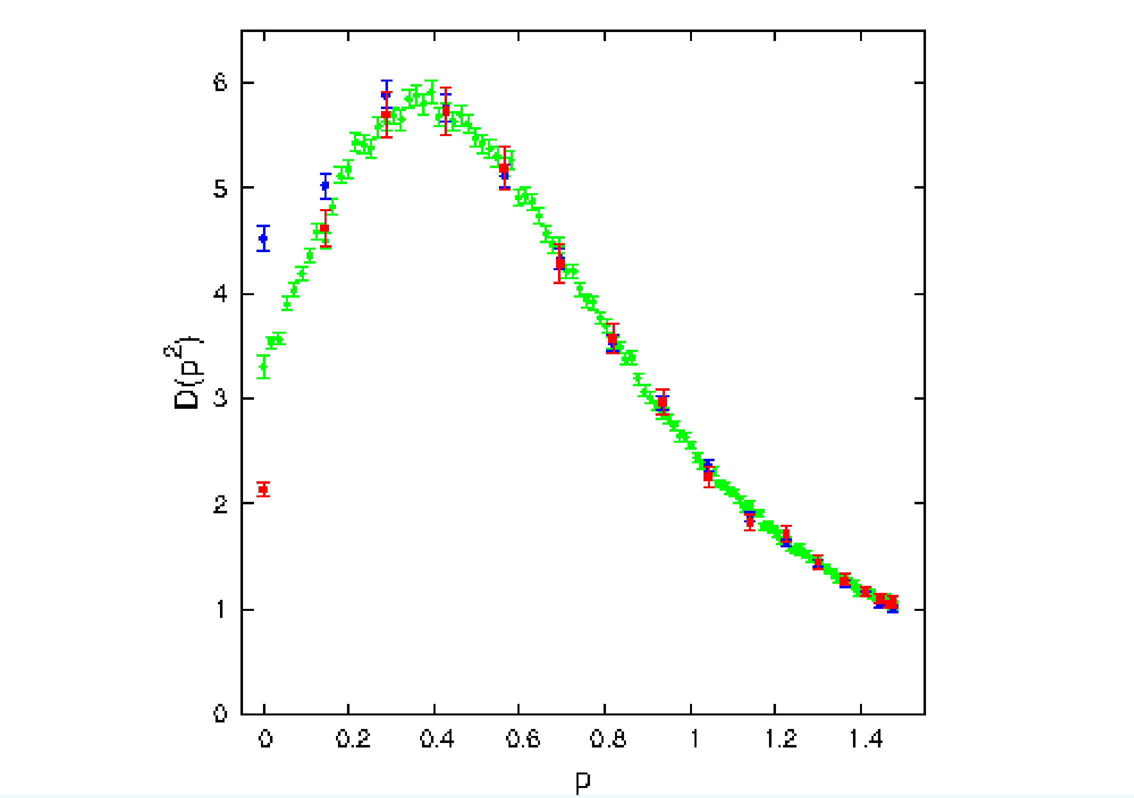
```
main() {
/* set parameters: beta, number of configurations NC,
                         number of thermalization sweeps NT */
     read_parameters();
/* {U} is the link configuration */
     set_initial_configuration(U);
/* cycle over NC configurations */
     for (int c=0; c < NC; c++) {
          thermalize(U,beta,NT);
          gauge_fix(U,g);
          evaluate_propagators(U[g]);
     }
}
```

Algorithms: Heat-Bath and Micro-canonical (thermalization), overrelaxation and simulated annealing (gauge fixing), conjugate gradient and Fourier transform (propagators, etc.).

Large Lattices via Bloch's Theorem

Perform thermalization step on small lattice, then replicate it and use Bloch's theorem from condensed-matter physics to obtain gauge-fixing step for much larger lattice (A. Cucchieri, TM, PRL 2017)





Applications

IR propagators may be used to extract the strong coupling constant α_s

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IR propagators may be used to extract the strong coupling constant α_s

Propagators and running coupling from SU(2) lattice gauge theory, J. C. R. Bloch, A. Cucchieri, K. Langfeld e T. Mendes, Nucl. Phys. B **687**, 76–100 (2004).

Gluon propagator provides NP input in calculations of heavy quark-antiquark potential

Lattice Gluon Propagator and One-Gluon-Exchange Potential, A. Cucchieri, T. Mendes and W. M. Serenone, arXiv:1704.08288 [hep-lat].

Collaborations

Present Collaborations

- Univ. Paris-Sud, Laboratoire de Physique Theorique d'Orsay (support by CNRS–COFECUB)
- Univ. Paris Diderot & UPMC Sorbonne
- Univ. KU Leuven & Univ. of Coimbra
- Visitor in August 2018: A. Vladikas from INFN-Rome

Previously

- Hadron Spectroscopy (Univ. Roma "Tor Vergata")
- Finite Temperature QCD (Univ. Bielefeld)
- Heavy-quark physics (DESY–Zeuthen, Humboldt Fellowship)

Conclusions

It hasn't been easy to set up a LQCD collaboration in Brazil...

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Local group with expertise in computations of infrared propagators, contact to international LQCD collaborations, students

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Let's talk