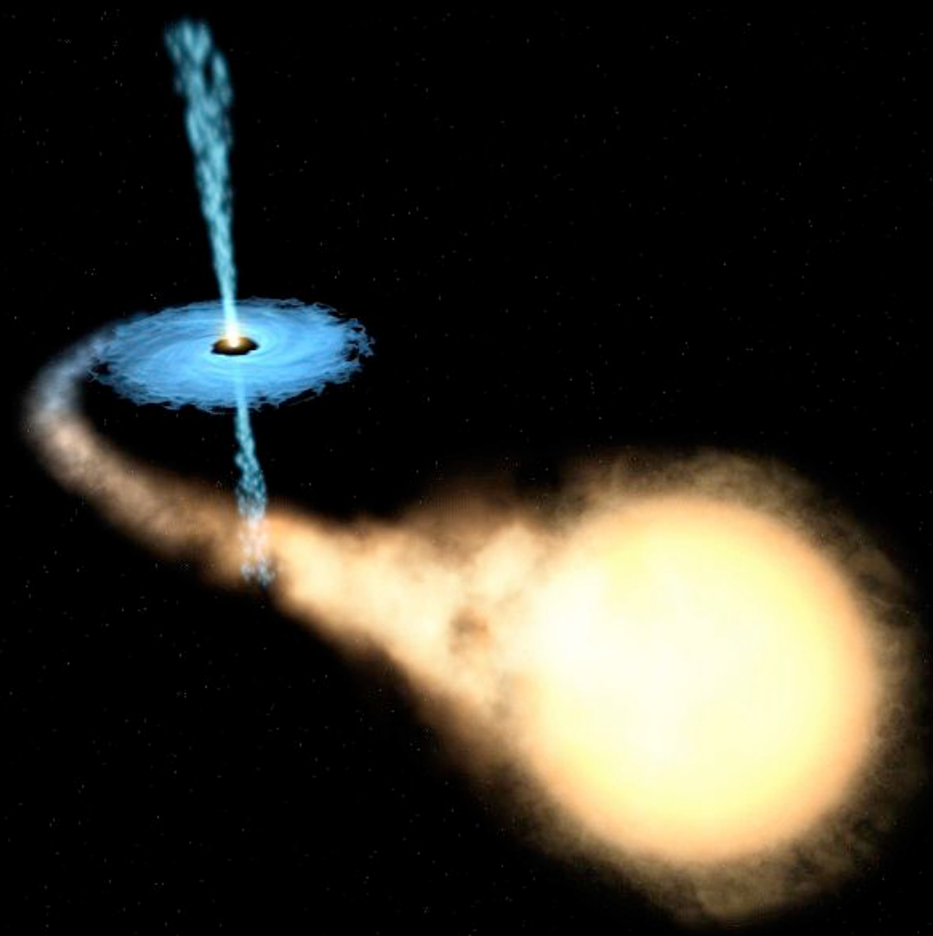
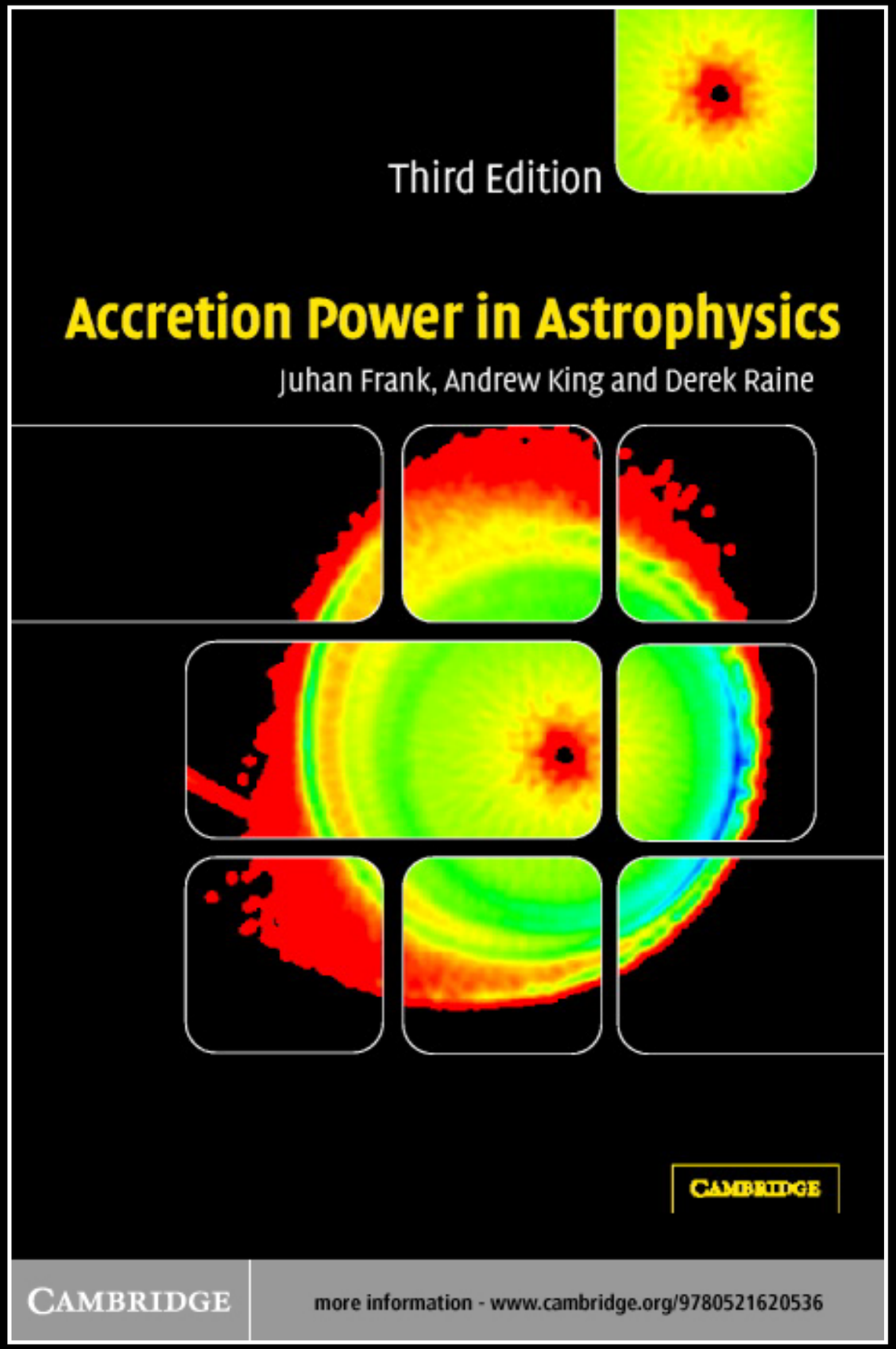
Accretion

Frédéric Daigne (Institut d'Astrophysique de Paris - Sorbonne University)



Reference:


Third Edition
Accretion Power in Astrophysics
Juhan Frank, Andrew King and Derek Raine
CAMBRIDGE
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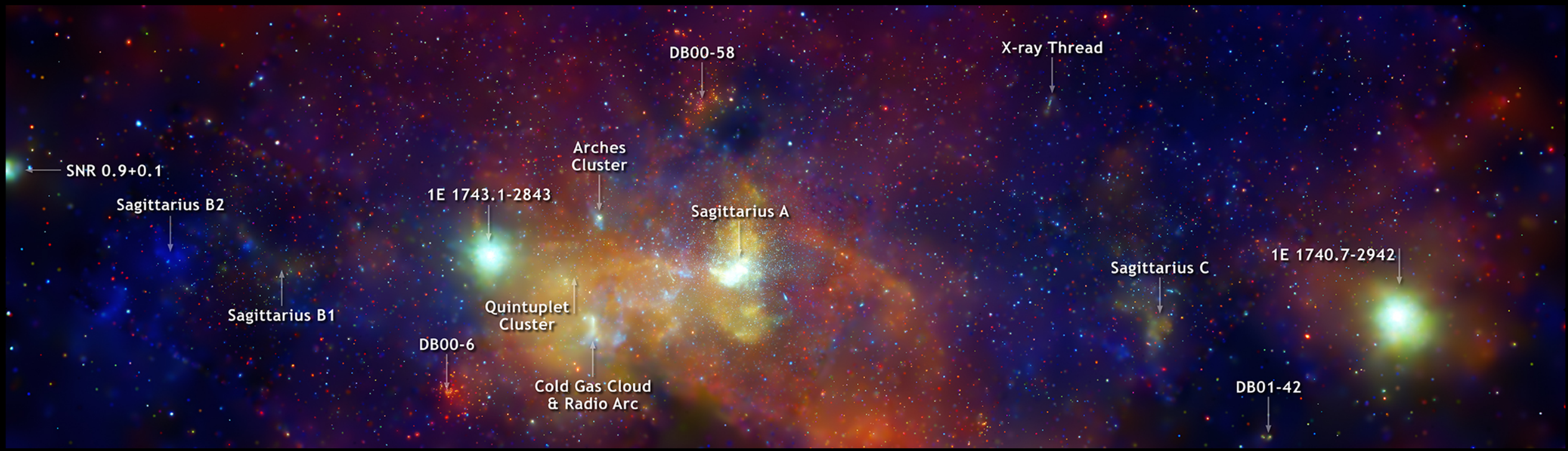
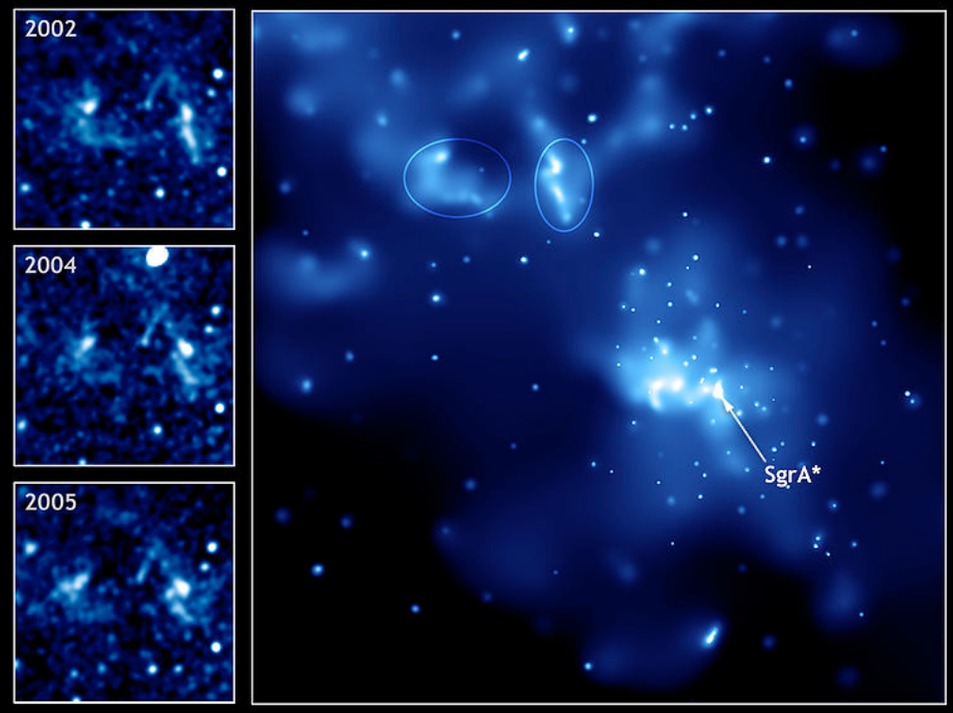
more information - www.cambridge.org/9780521620536

Accreting compact objects in the HE sky

The region near the Galactic center in X-rays

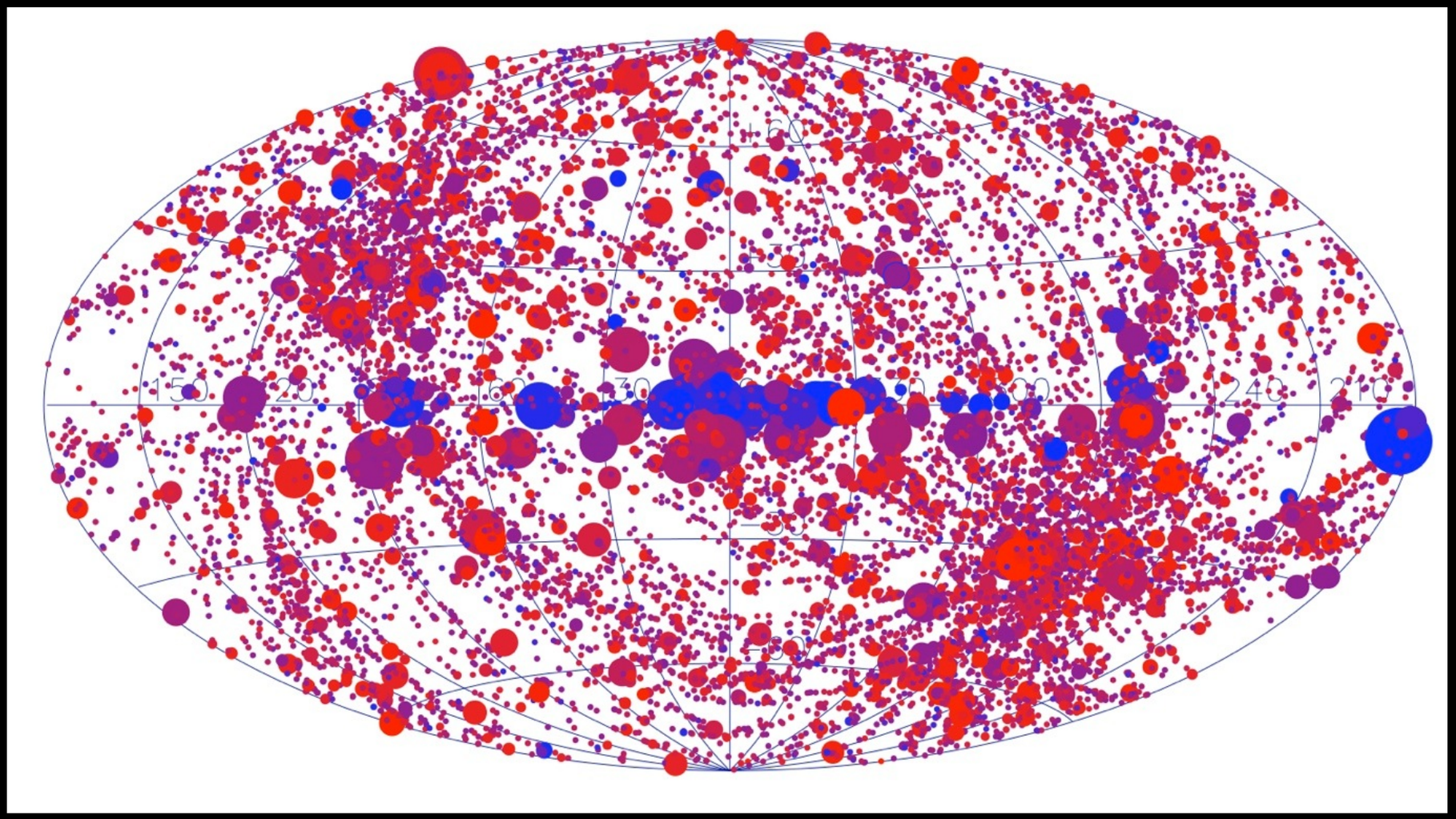


Sgr A* **= Galactic supermassive BH**
SNR 0.9+0.1 = supernova remnant
Sgr B2 = molecular cloud emitting X-rays
(interaction with gamma-ray radiation from Sgr A* during an outburst 350 years ago ?)
1E 1743.1-2843 = BH candidate (X-ray binary)
1E 1740.7-2942 = BH candidate (X-ray binary, microquasar)



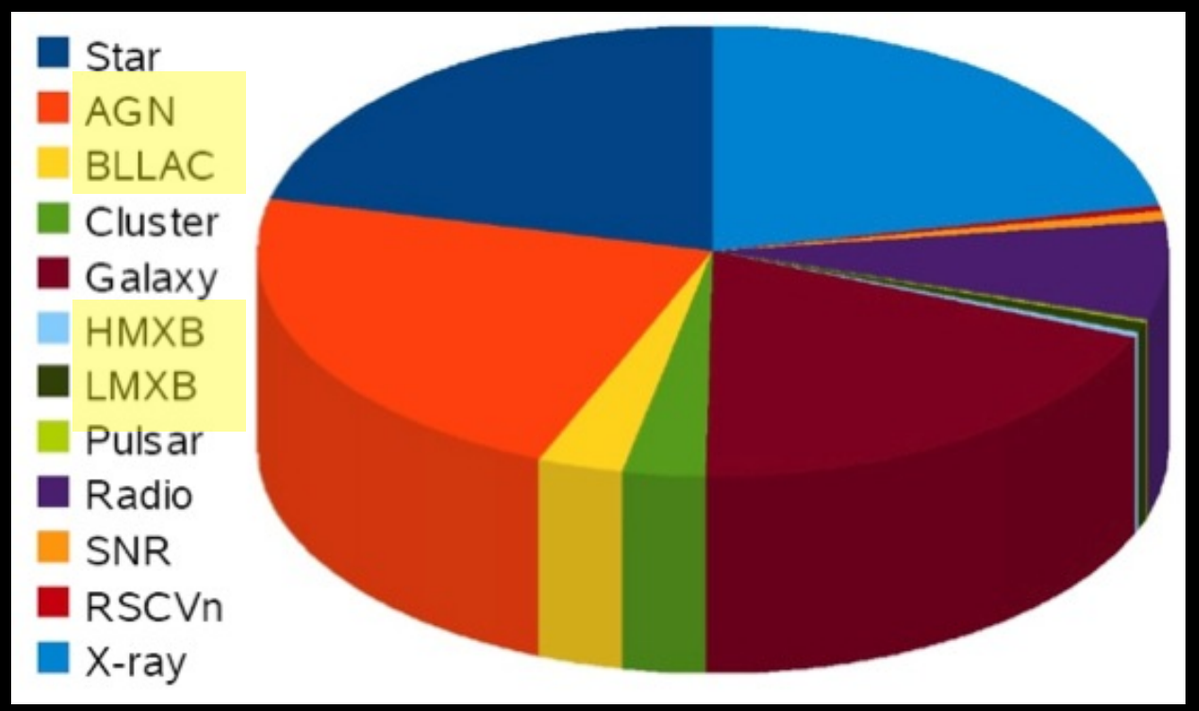
Chandra

The sky in soft X-rays (0.2-12 keV : XMM-Newton)

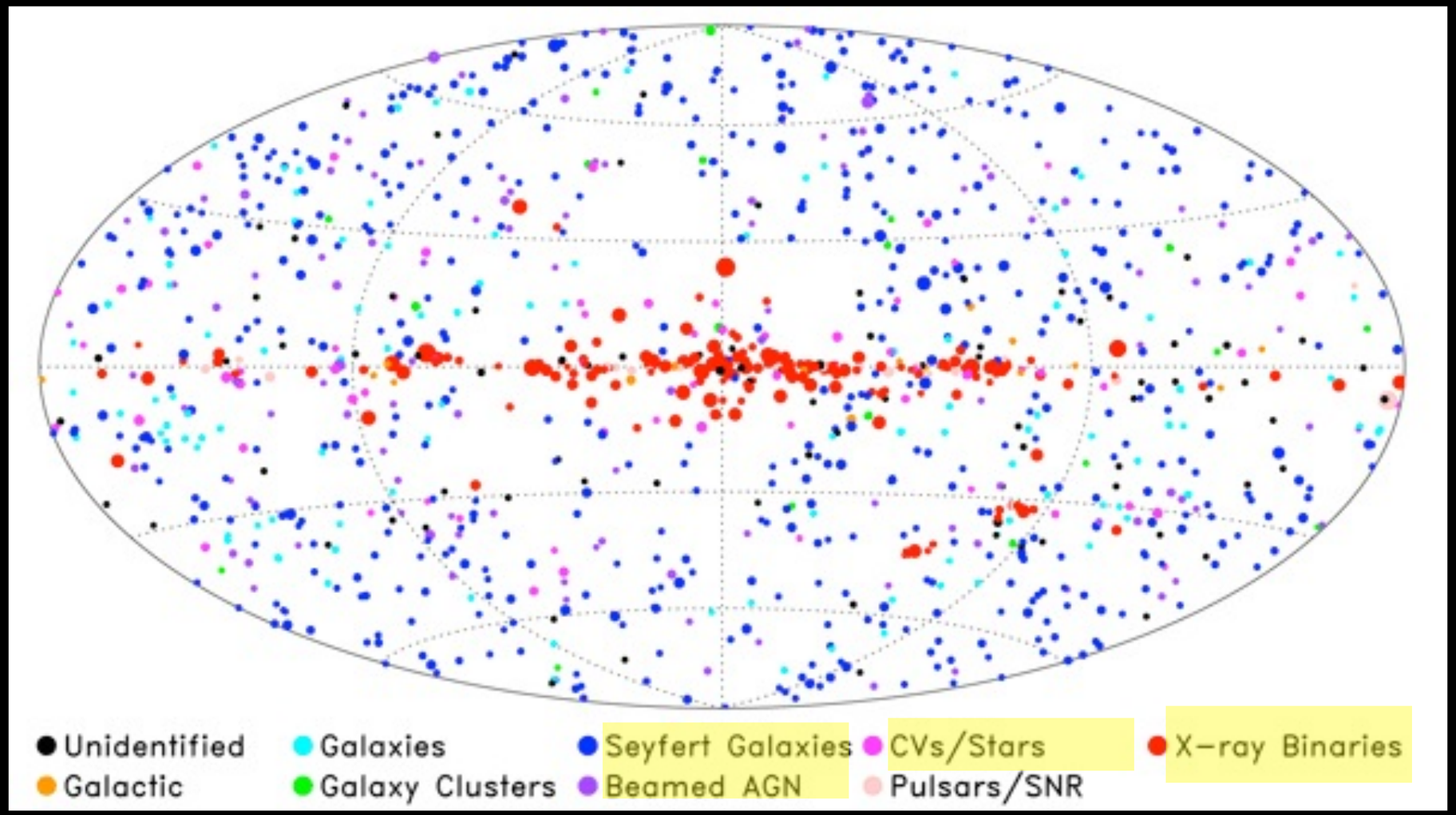


Red : 0.2-2 keV
Blue : 2-12 keV

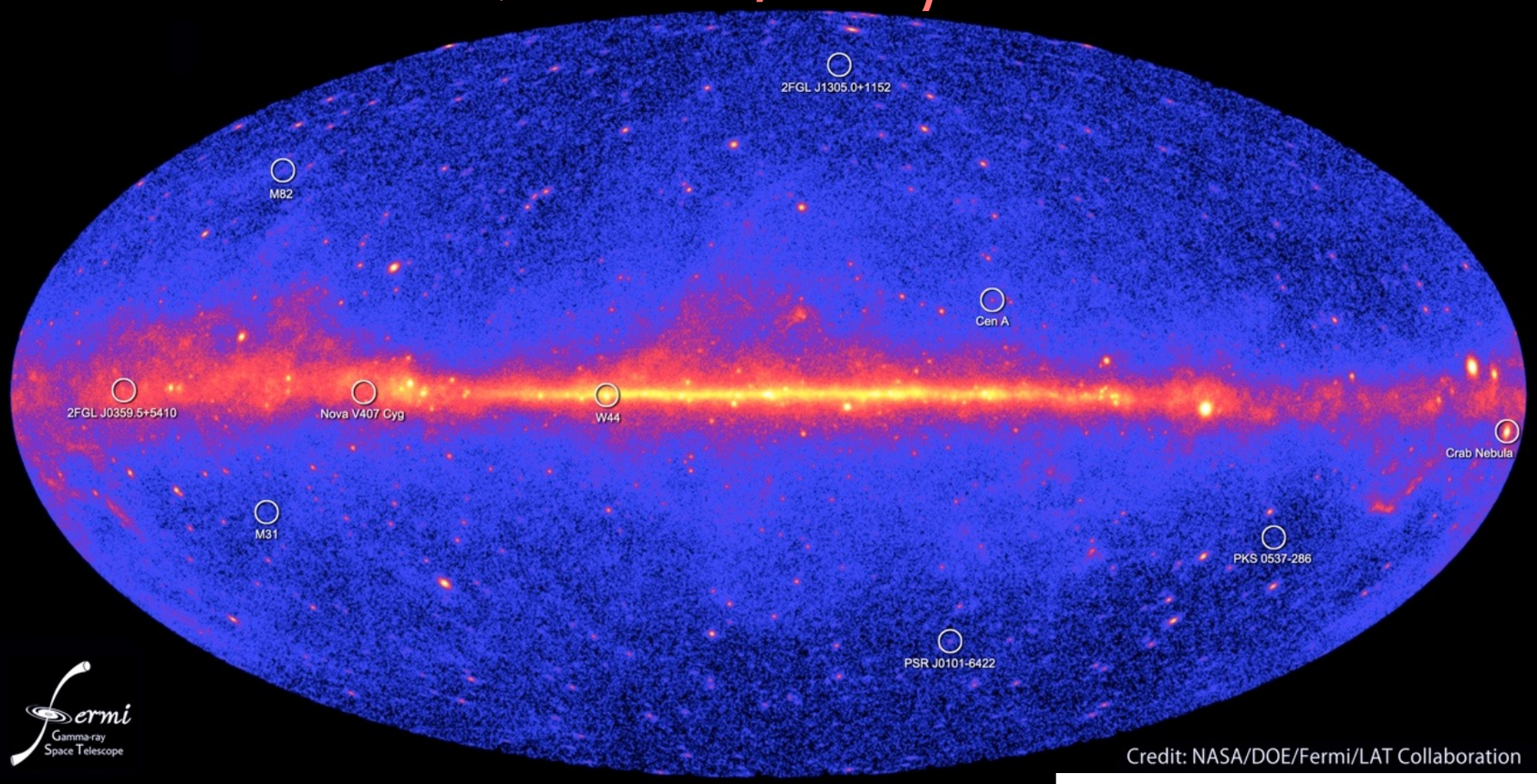
Size ∝ flux



The sky in hard X-rays (14-195 keV : Swift/BAT)



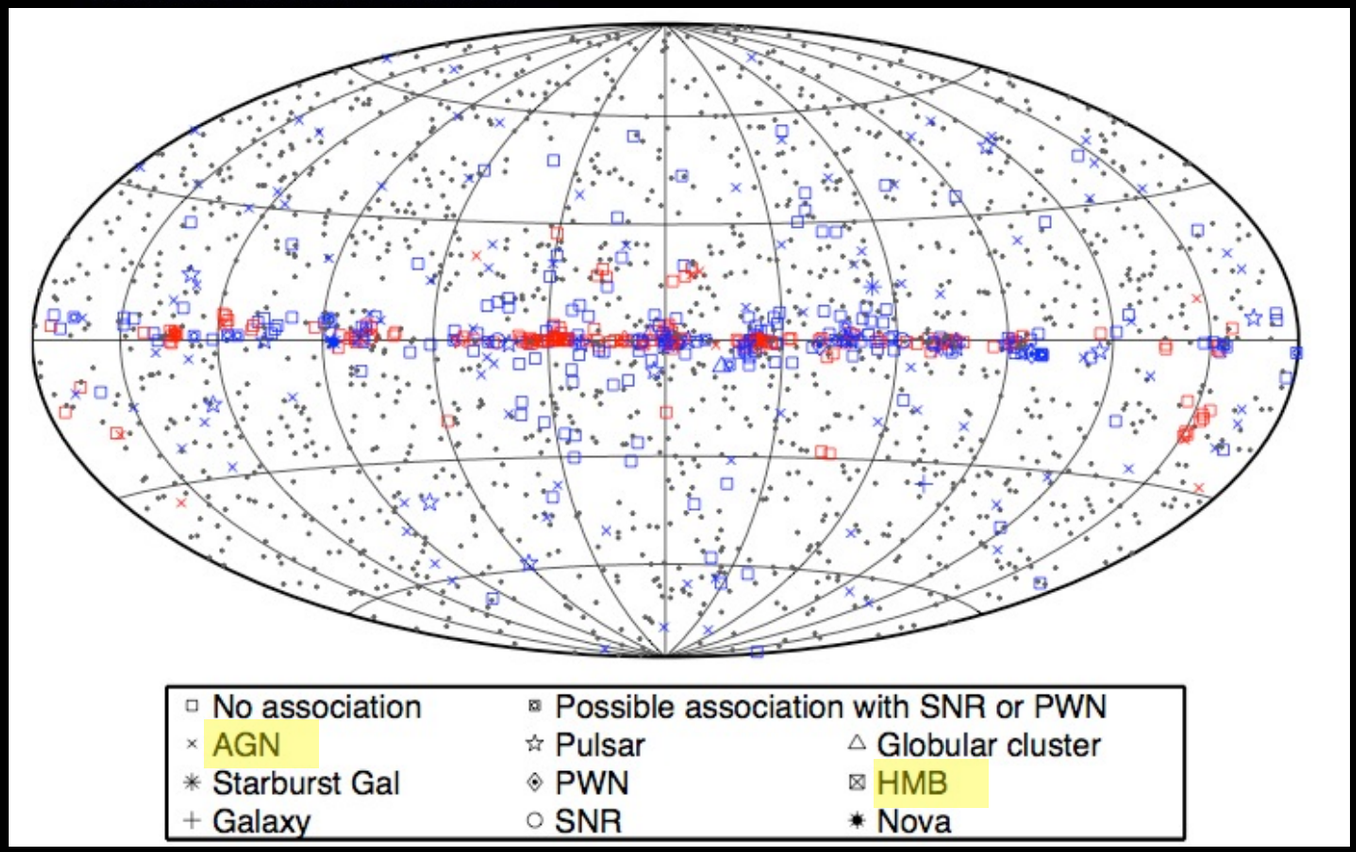
The sky in high energy gamma-rays (0.1-100 GeV ; Fermi/LAT)



Diffuse emission:
interaction between cosmic rays and the interstellar medium

Sources :

Credit: NASA/DOE/Fermi/LAT Collaboration



Main classes of accreting compact objects

Accretion in compact binary systems

Cataclysmic variables (CV) and X-ray binaries (XB) [XB: L_X=250 to 25000 $L_\odot$]

1 = compact object: WD in a CV - NS or BH in a XB

2 = companion

Classification: based on the mass ratio companion / compact object q = M2/M1

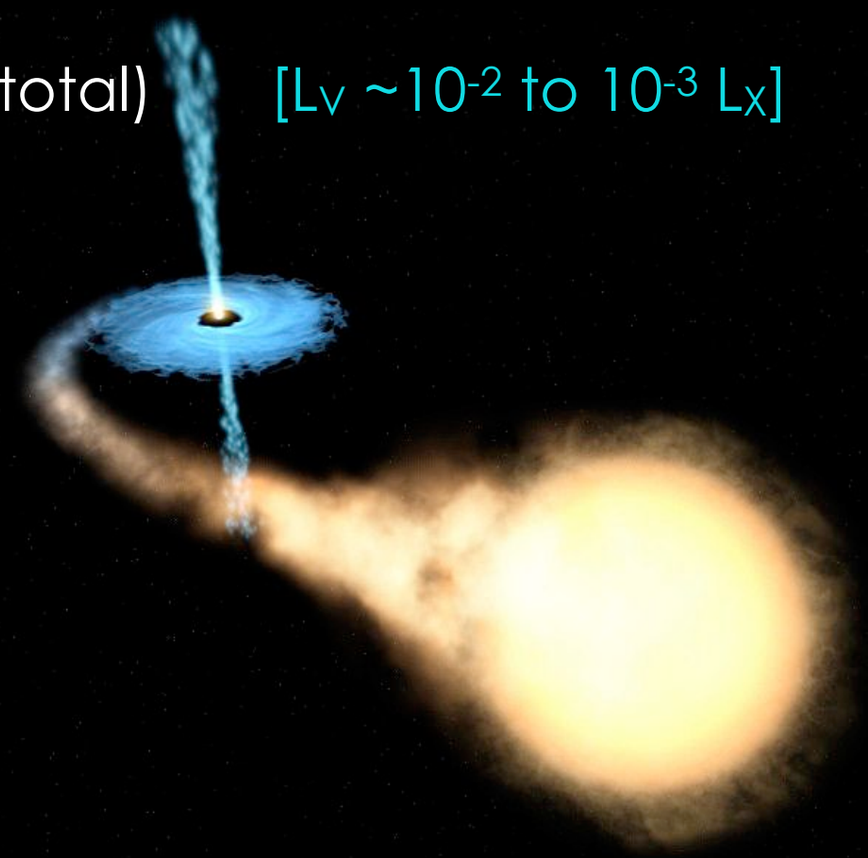
HMXB : 2 = O, B star or blue supergiant: M2/M1>1 [T= day to year]

- X-rays: bright source(accretion)
- Visible: emission is dominated by 2 [L_V ~10^{-1} to 10^3 L_X]
- examples: Cyg X1, Vela X1, …

LMXB : 2 = main sequence star or WD : M2/M1 < 1 [T= 10 min to 100 days]

- X-rays: bright source (accretion)
- Visible: emission is fainter than in a LMXB (visible < 1% total) [L_V ~10^{-2} to 10^{-3} L_X]
- LMXB are more frequent than HMXB

~ 100 LMXB in the Galaxy, 30 % in globular clusters



Additional classification: based on other phenomena

* **Novae, X-ray bursts**: CV or XB with thermonuclear explosions on the surface
* **Microquasar**: XB with relativistic ejections

examples: GRS 1915+105, Cyg X1

Active Galactic Nuclei (AGN)

AGN: supermassive black hole + accretion disk (+ outflow/jet)

Classification:

- **Radio-quiet AGN: emission from jets/lobes is faint**

Seyfert galaxies: - narrow emission lines in the nucleus
- strong X-ray emission
- possible radio jet
- host galaxy: usually spiral/irregular
- sub-classes: Seyfert 1 = strong broad emission lines
Seyfert 2 = no such lines

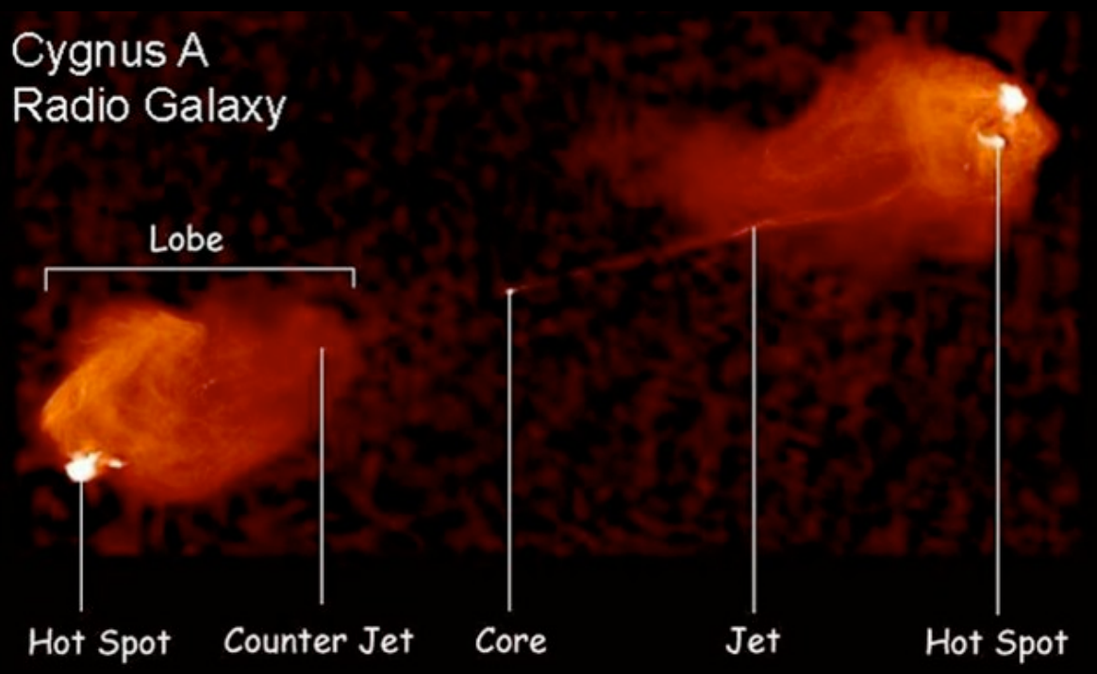
Radio-quiet quasars: same as Seyfert galaxies, but brighter

- **Radio-loud AGN: emission from jets/lobes is bright (strong interaction with the external medium)**

Starburst galaxies:
strong star formation, very bright IR emission

Radiogalaxies:
usually elliptical,
no emission lines like in a Seyfert galaxy

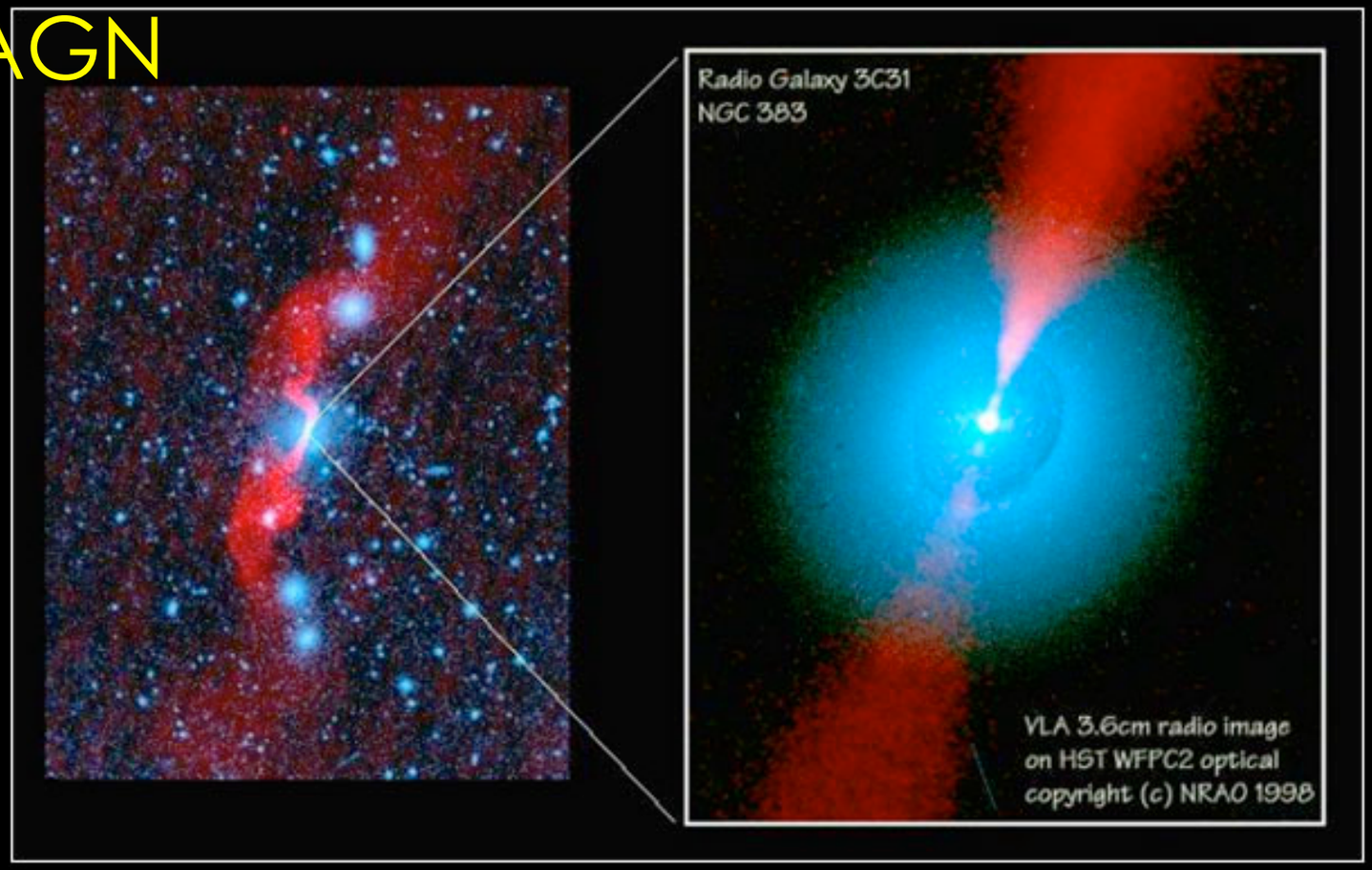
Radio-loud quasars: like a radio-quiet QSO, with a jet in addition



The central region of an AGN

Two radio loud AGN

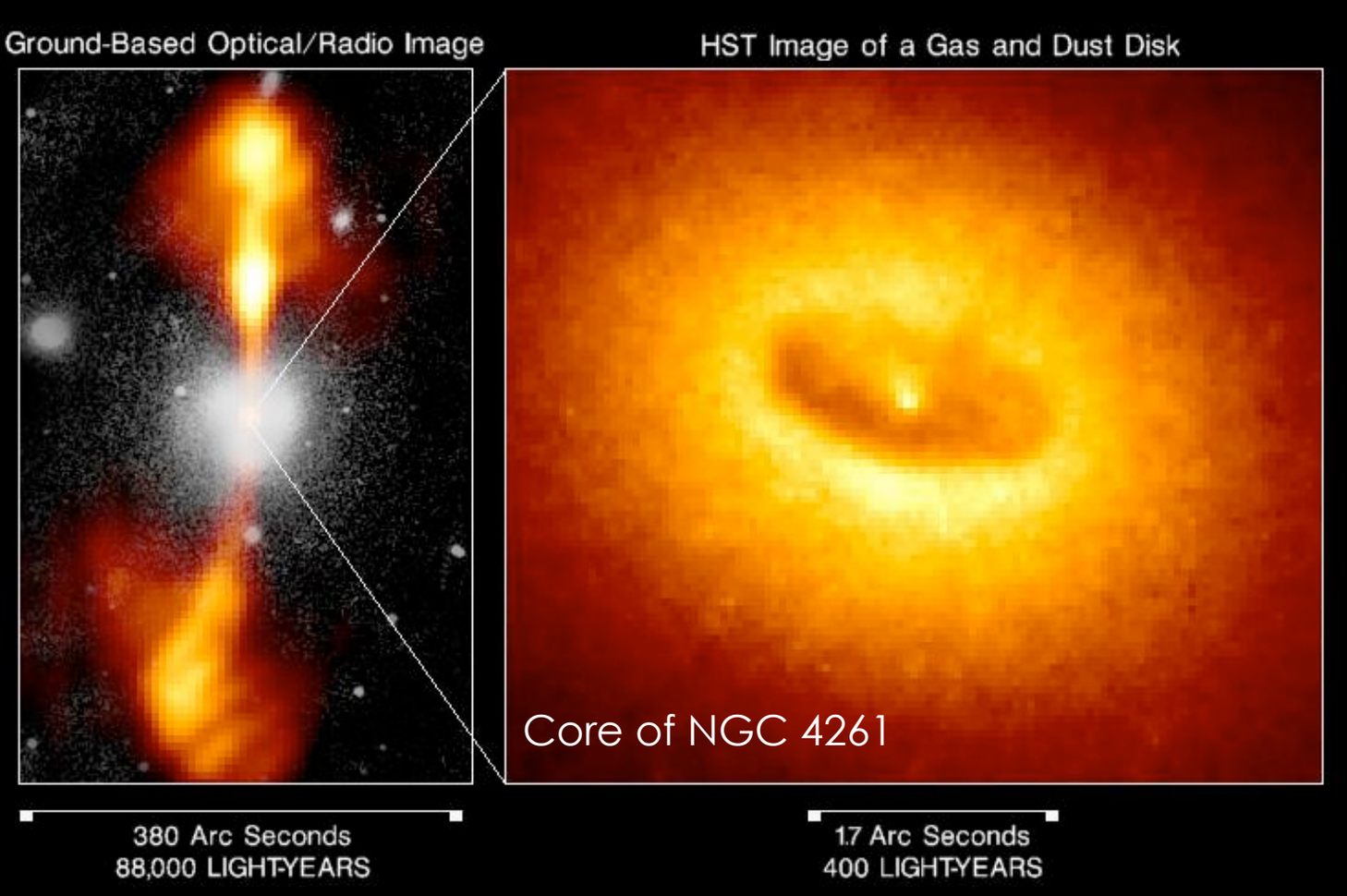
(radio-galaxies)



NGC 4261:

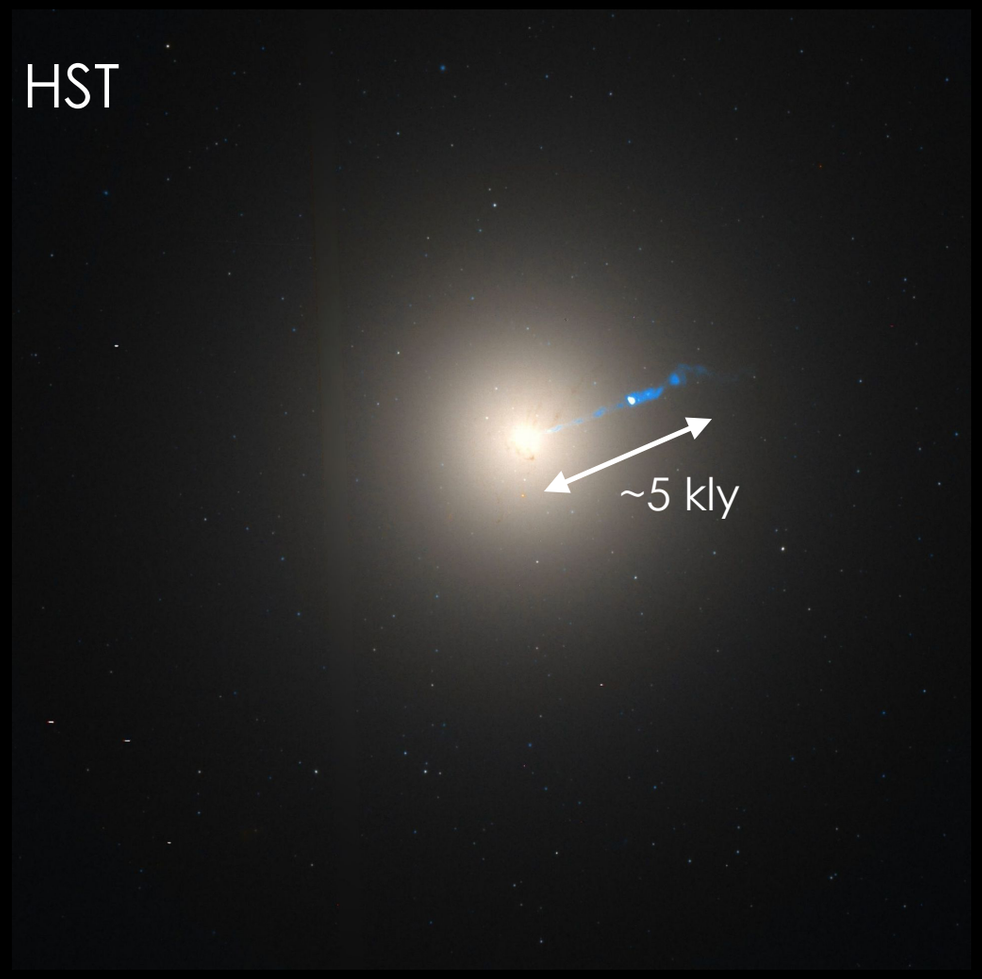
$M_{SMBH} \sim 4\ 10^8\ M_\odot$

$R_S \sim 8$ AU << 400 ly

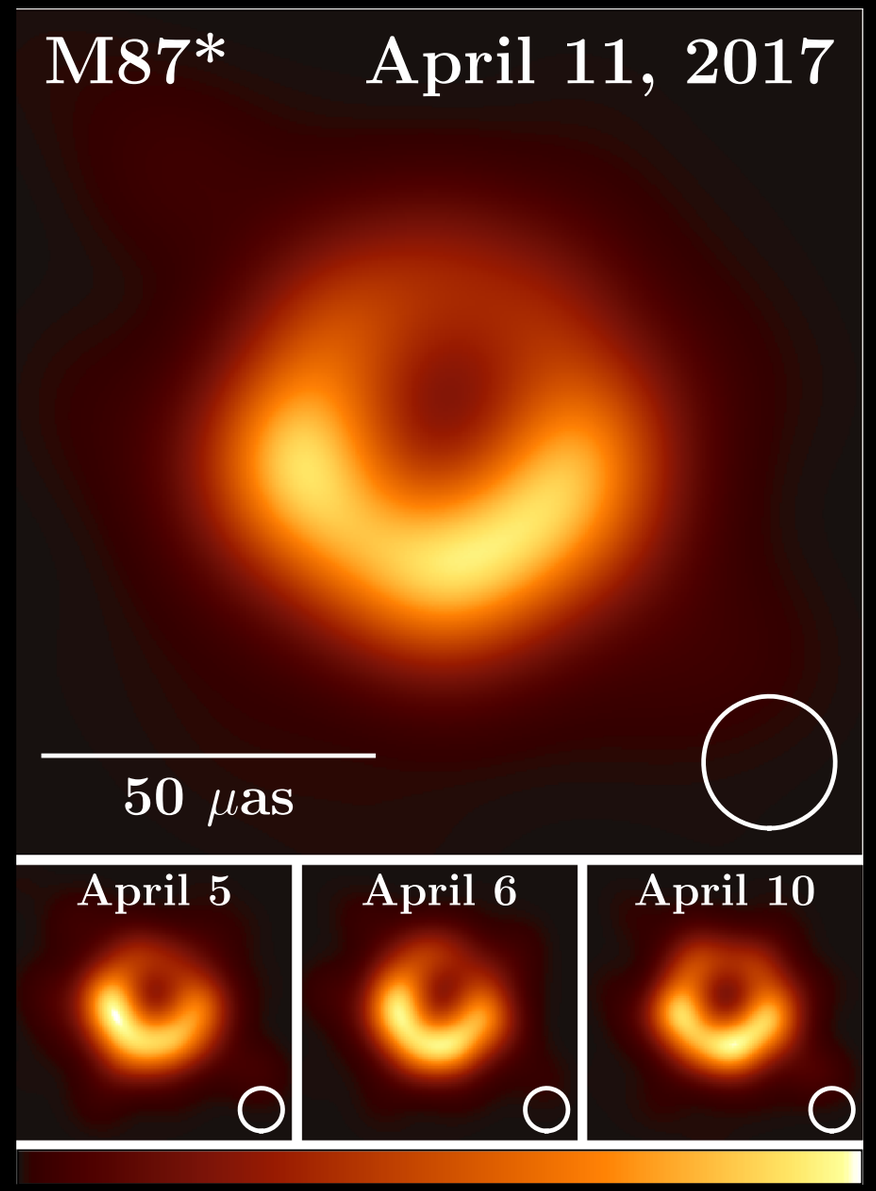


M87

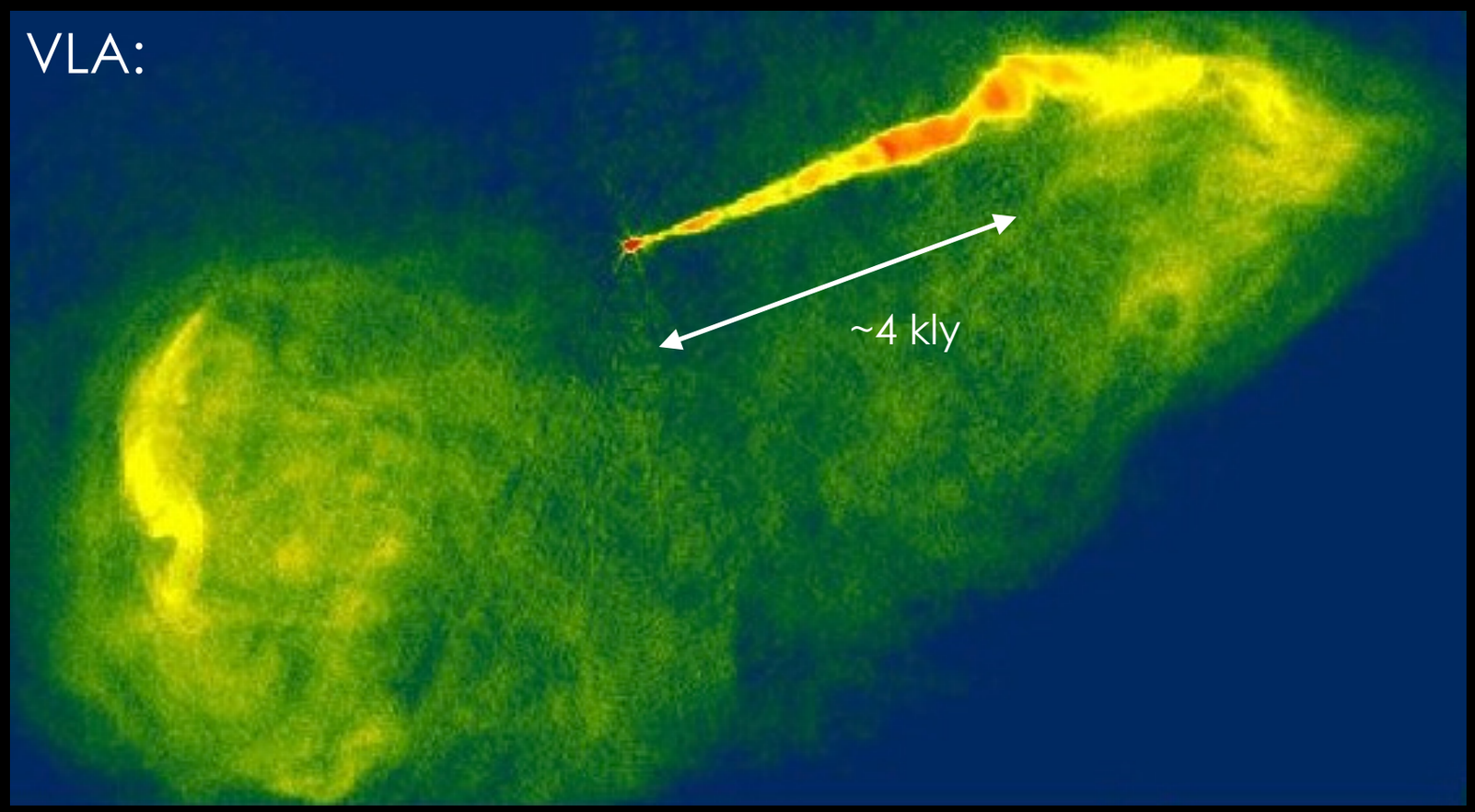
HST



EHT



VLA:

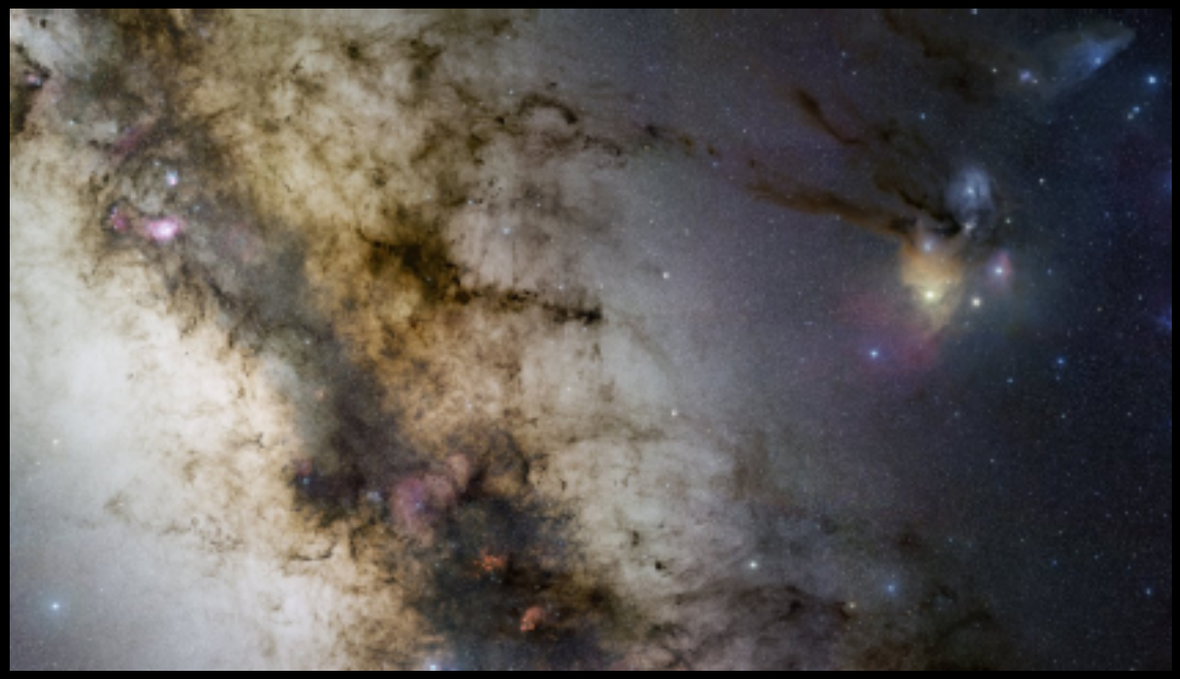


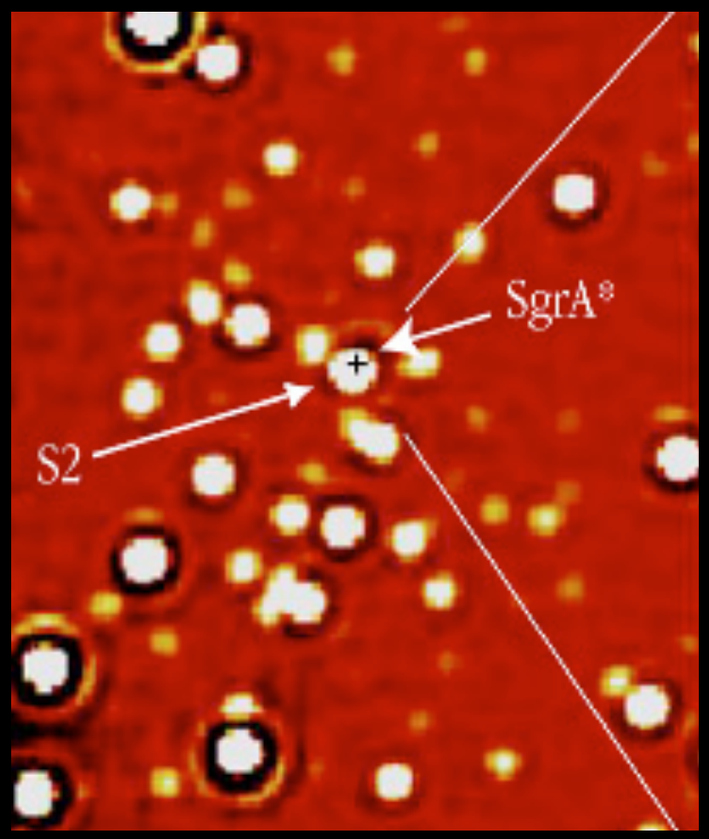
$M_{SMBH} \sim 6.5\ 10^9\ M_\odot$

$R_S \sim 130$ AU (~8 μas)

Supermassive black holes with a weak activity

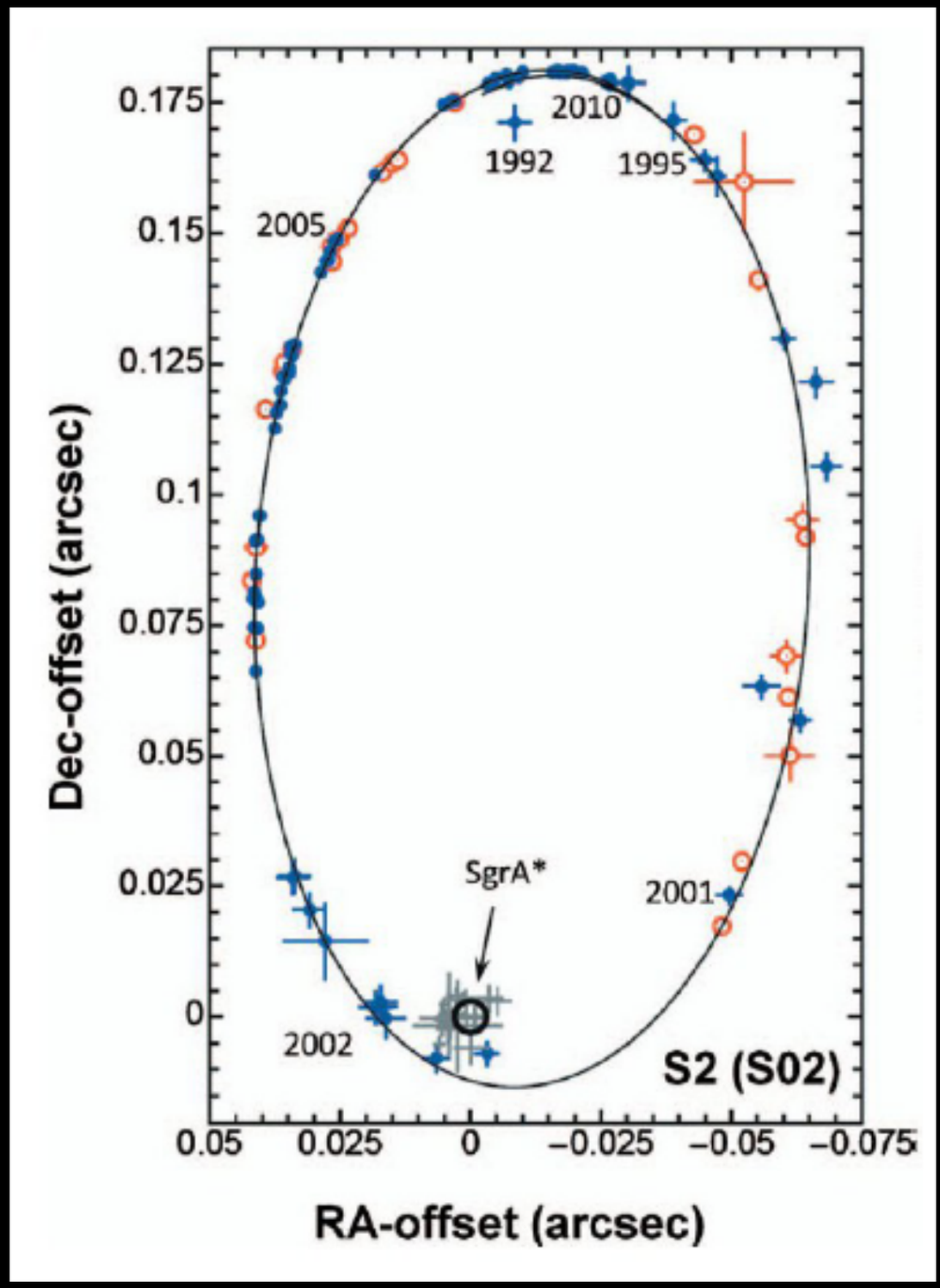
Milky Way: supermassive black hole at the center (SgrA*) - M ~ 4 10^6 $M_\odot$





Orbit S2:
Period: T= 16 yr

Periastron:
r=120 UA = 1400 R_S
v=0.02 c



Genzel et al. 2010

Accretion: orders of magnitude

Accretion: luminosity

Maximum luminosity that can be extracted via the accretion process:

Accretion luminosity (not necessarily radiated):

$$L_{\rm acc} = \Xi \dot{m} c^2 = \begin{cases} 0.32 \left(\frac{M}{M_\odot}\right) \left(\frac{R}{10^4\,{\rm km}}\right)^{-1} \left(\frac{\dot{m}}{10^{-10}\,M_\odot.{\rm yr}^{-1}}\right) L_\odot \ ({\rm WD}) \\ 320 \left(\frac{M}{M_\odot}\right) \left(\frac{R}{10\,{\rm km}}\right)^{-1} \left(\frac{\dot{m}}{10^{-10}\,M_\odot.{\rm yr}^{-1}}\right) L_\odot \ ({\rm NS}) \\ 250 \left(\frac{\dot{m}}{10^{-10}\,M_\odot.{\rm yr}^{-1}}\right) L_\odot \ ({\rm BH}) \\ 2.5 \times 10^{11} \left(\frac{\dot{m}}{0.1\,M_\odot.{\rm yr}^{-1}}\right) L_\odot \ ({\rm SMBH}) \end{cases}$$

Compact object:

$$\Xi = \frac{GM}{Rc^2}$$

WD: $\Xi_{\rm WD} \simeq 10^{-4} - 10^{-3}$

NS: $\Xi_{\rm NS} \simeq 0.2$

BH: $\Xi_{\rm BH} = \frac{GM}{R_{\rm ISCO} c^2} \simeq \frac{1}{6}$

Maximum luminosity that can be radiated from a static atmosphere:

Eddington luminosity: ➟ **corresponding limit on mass flux**

$$L_{\rm Edd} = \frac{4\pi G M m_{\rm p} c}{\sigma_{\rm T}} \simeq 3.4 \times 10^4 \left(\frac{M}{M_\odot}\right) L_\odot$$

Accretion: luminosity

Maximum luminosity that can be extracted via the accretion process:

Accretion luminosity (not necessarily radiated):

$$L_{\rm acc} = \Xi \dot{m} c^2 = \begin{cases} 0.32 \left(\frac{M}{M_\odot}\right) \left(\frac{R}{10^4\,\mathrm{km}}\right)^{-1} \left(\frac{\dot{m}}{10^{-10}\,M_\odot.\mathrm{yr}^{-1}}\right) L_\odot\,(\mathrm{WD}) \\ 320 \left(\frac{M}{M_\odot}\right) \left(\frac{R}{10\,\mathrm{km}}\right)^{-1} \left(\frac{\dot{m}}{10^{-10}\,M_\odot.\mathrm{yr}^{-1}}\right) L_\odot\,(\mathrm{NS}) \\ 250 \left(\frac{\dot{m}}{10^{-10}\,M_\odot.\mathrm{yr}^{-1}}\right) L_\odot\,(\mathrm{BH}) \\ 2.5 \times 10^{11} \left(\frac{\dot{m}}{0.1\,M_\odot.\mathrm{yr}^{-1}}\right) L_\odot\,(\mathrm{SMBH}) \end{cases}$$

CV

XB

XB

AGN

quiescent SMBH: ~ 10^{-5}-10^{-3} $M_\odot$/yr

Stellar mass objects:

~ 0.1 $M_\odot$ in 1 Gyr

SMBH:

~ 10^{-5} – 1 $M_\odot$ in 1 yr

Accretion: luminosity

Maximum luminosity that can be extracted via the accretion process:

Accretion luminosity (not necessarily radiated):

$$L_{\rm acc} = \Xi \dot{m} c^2 = \begin{cases} 0.32 \left(\frac{M}{M_\odot}\right) \left(\frac{R}{10^4\,{\rm km}}\right)^{-1} \left(\frac{\dot{m}}{10^{-10}\,M_\odot.{\rm yr}^{-1}}\right) L_\odot \ ({\rm WD}) \\ 320 \left(\frac{M}{M_\odot}\right) \left(\frac{R}{10\,{\rm km}}\right)^{-1} \left(\frac{\dot{m}}{10^{-10}\,M_\odot.{\rm yr}^{-1}}\right) L_\odot \ ({\rm NS}) \\ 250 \left(\frac{\dot{m}}{10^{-10}\,M_\odot.{\rm yr}^{-1}}\right) L_\odot \ ({\rm BH}) \\ 2.5 \times 10^{11} \left(\frac{\dot{m}}{0.1\,M_\odot.{\rm yr}^{-1}}\right) L_\odot \ ({\rm SMBH}) \end{cases}$$

CV

XB

XB

AGN

quiescent SMBH: ~ 10^{-5}-10^{-3} $M_\odot$/yr

Stellar mass objects:

~ 0.1 $M_\odot$ in 1 Gyr

SMBH:

~ 10^{-5} – 1 $M_\odot$ in 1 yr

Maximum luminosity that can be radiated from a static atmosphere:

Eddington luminosity: usual sources are well below this limit

$$L_{\rm Edd} = \frac{4\pi G M m_{\rm p} c}{\sigma_{\rm T}} \simeq 3.4 \times 10^4 \left(\frac{M}{M_\odot}\right) L_\odot$$

Accretion: spectral range

Accretion: spectral range of the emission (optically thick regime)

$$kT_{\rm thick} \simeq k\left(\frac{L_{\rm acc}}{4\pi R^2\sigma}\right)^{1/4} \simeq \begin{cases} 3\,{\rm eV}\,\left(\frac{M}{M_\odot}\right)^{\frac{1}{4}}\left(\frac{R}{10^4\,{\rm km}}\right)^{-\frac{3}{4}}\left(\frac{\dot m}{10^{-10}\,M_\odot.{\rm yr}^{-1}}\right)^{\frac{1}{4}} & ({\rm WD}) \\ 0.6\,{\rm keV}\,\left(\frac{M}{M_\odot}\right)^{\frac{1}{4}}\left(\frac{R}{10\,{\rm km}}\right)^{-\frac{3}{4}}\left(\frac{\dot m}{10^{-10}\,M_\odot.{\rm yr}^{-1}}\right)^{\frac{1}{4}} & ({\rm NS}) \\ 0.2\,{\rm keV}\,\left(\frac{M}{10\,M_\odot}\right)^{-\frac{1}{2}}\left(\frac{\dot m}{10^{-10}\,M_\odot.{\rm yr}^{-1}}\right)^{\frac{1}{4}} & ({\rm BH}) \\ 2\,{\rm eV}\,\left(\frac{M}{10^9\,M_\odot}\right)^{-\frac{1}{2}}\left(\frac{\dot m}{0.1\,M_\odot.{\rm yr}^{-1}}\right)^{\frac{1}{4}} & ({\rm SMBH}) \end{cases}$$

Accretion: spectral range of the emission (optically thin regime)

$$kT_{\rm thin} \simeq \frac{2}{3}\Xi\mu m_{\rm p}c^2 \simeq \begin{cases} 30\,{\rm keV}\,\left(\frac{\Xi}{10^{-4}}\right)\left(\frac{\mu}{0.5}\right) & ({\rm WD}) \\ 60\,{\rm MeV}\,\left(\frac{\Xi}{0.2}\right)\left(\frac{\mu}{0.5}\right) & ({\rm NS}) \\ 50\,{\rm MeV}\,\left(\frac{\mu}{0.5}\right) & ({\rm BH,\ SMBH}) \end{cases}$$

Accretion: spectral range

$$kT_{\rm thick} \simeq k\left(\frac{L_{\rm acc}}{4\pi R^2\sigma}\right)^{1/4} \simeq \qquad kT_{\rm thin} \simeq \frac{2}{3}\Xi\mu m_{\rm p}c^2$$

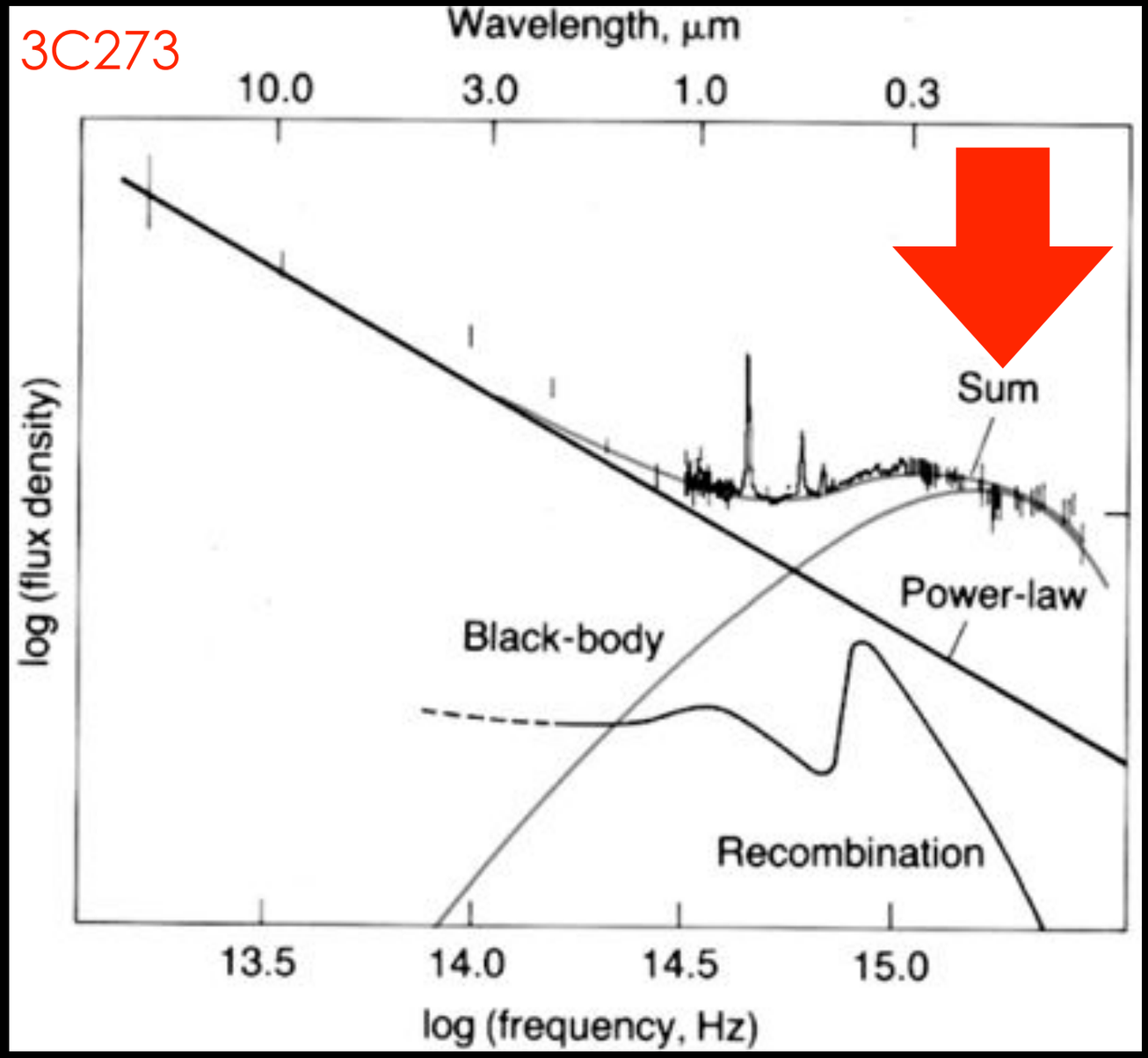
Expected spectral range for usual sources:

Accreting white dwarfs (CV) : 1 eV → 10 keV
visible/UV to X-rays

Accreting neutron stars/stellar mass BH (XB) : 0.1 keV → 100 MeV
soft X-rays to gamma-rays

Accreting supermassive black holes (AGN) : 1 eV → 100 MeV
visible/UV to gamma-rays

AGN: « blue/UV bump »



Direct accretion: Bondi-Hoyle

Direct accretion: Bondi-Hoyle-Littleton

external medium at rest

mass density ρ_∞

sound speed $c_{\mathrm{S},\infty}$

Compact object

spherical symmetry
+ steady state

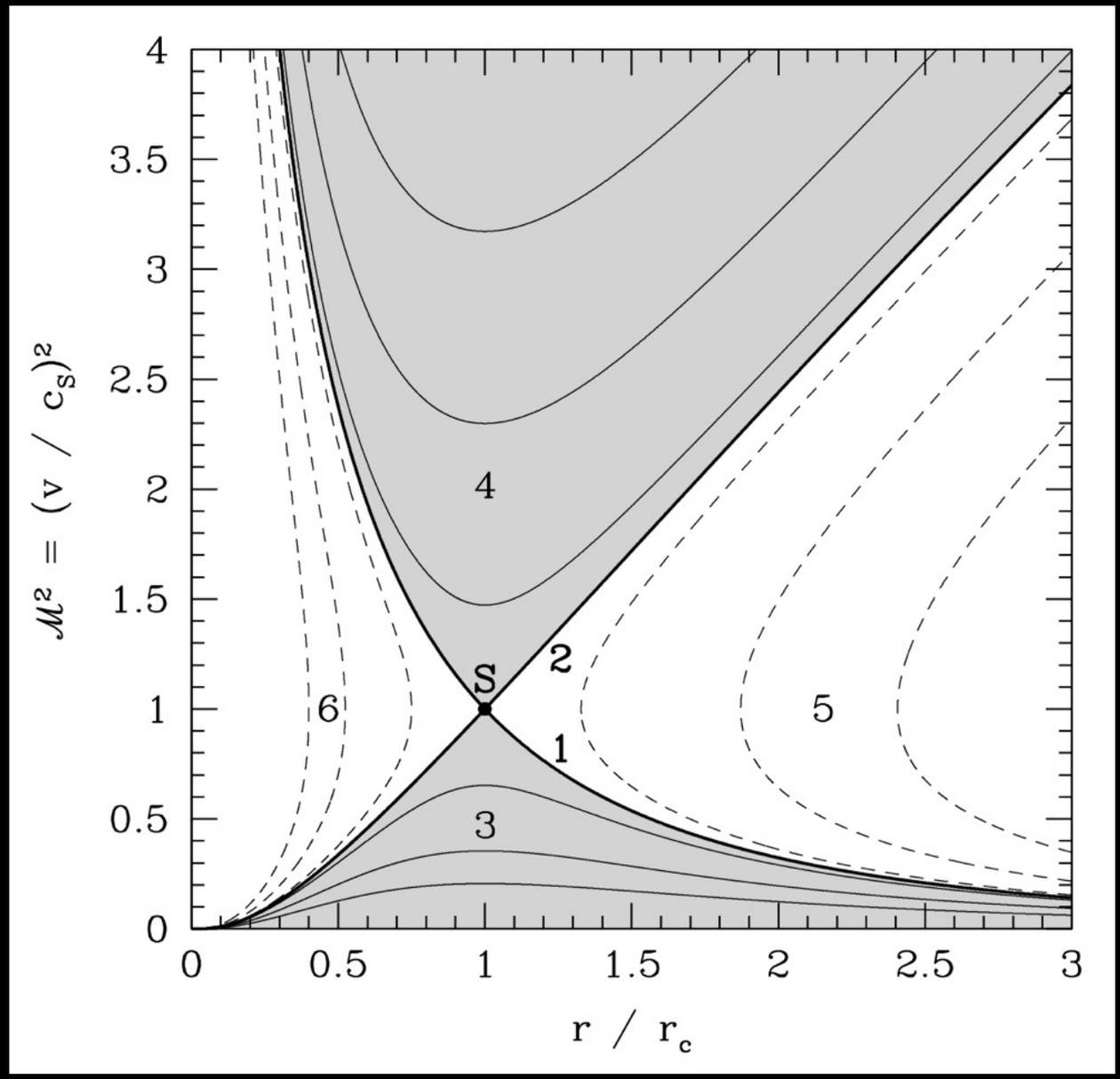
$$\Xi = \frac{GM}{Rc^2}$$

What is the mass flux?

Direct accretion: Bondi-Hoyle-Littleton

$\dot{m}$, $h_{\rm tot}$ and γ : fixed.

Each line: a different value of κ .



Direct accretion: Bondi-Hoyle-Littleton

Bondi-Hoyle accretion: mass flux

$$\dot{m} \simeq 2.2 \times 10^{-15}\,\mathrm{M_\odot.yr^{-1}} \left(\frac{M}{M_\odot}\right)^2 \left(\frac{\rho(\infty)/m_\mathrm{p}}{1\,\mathrm{cm^{-3}}}\right) \left(\frac{c_\mathrm{S}(\infty)}{10\,\mathrm{km.s^{-1}}}\right)^{-3}$$

Direct accretion is not efficient and cannot reproduce the observed luminosities from CV, XB or AGN.

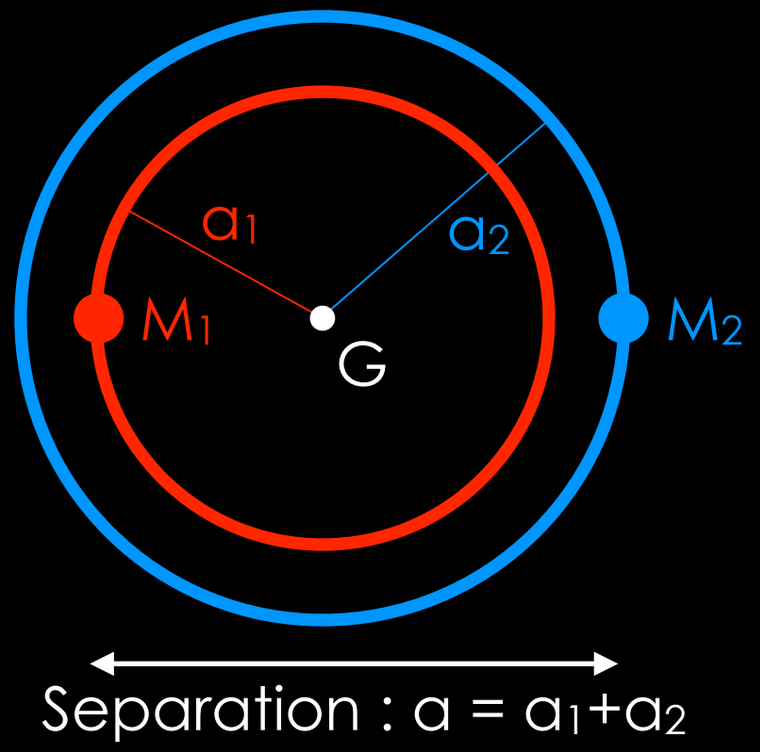
Bondi-Hoyle accretion gives the right order of magnitude for quiescent black holes such as SgrA* (assuming a low radiative efficiency).

Accretion in a binary system: from the companion to the compact object

Accretion in a binary system: geometry

- Close binary: circular orbits (tidal effects)

$$q = \frac{M_2}{M_1}$$

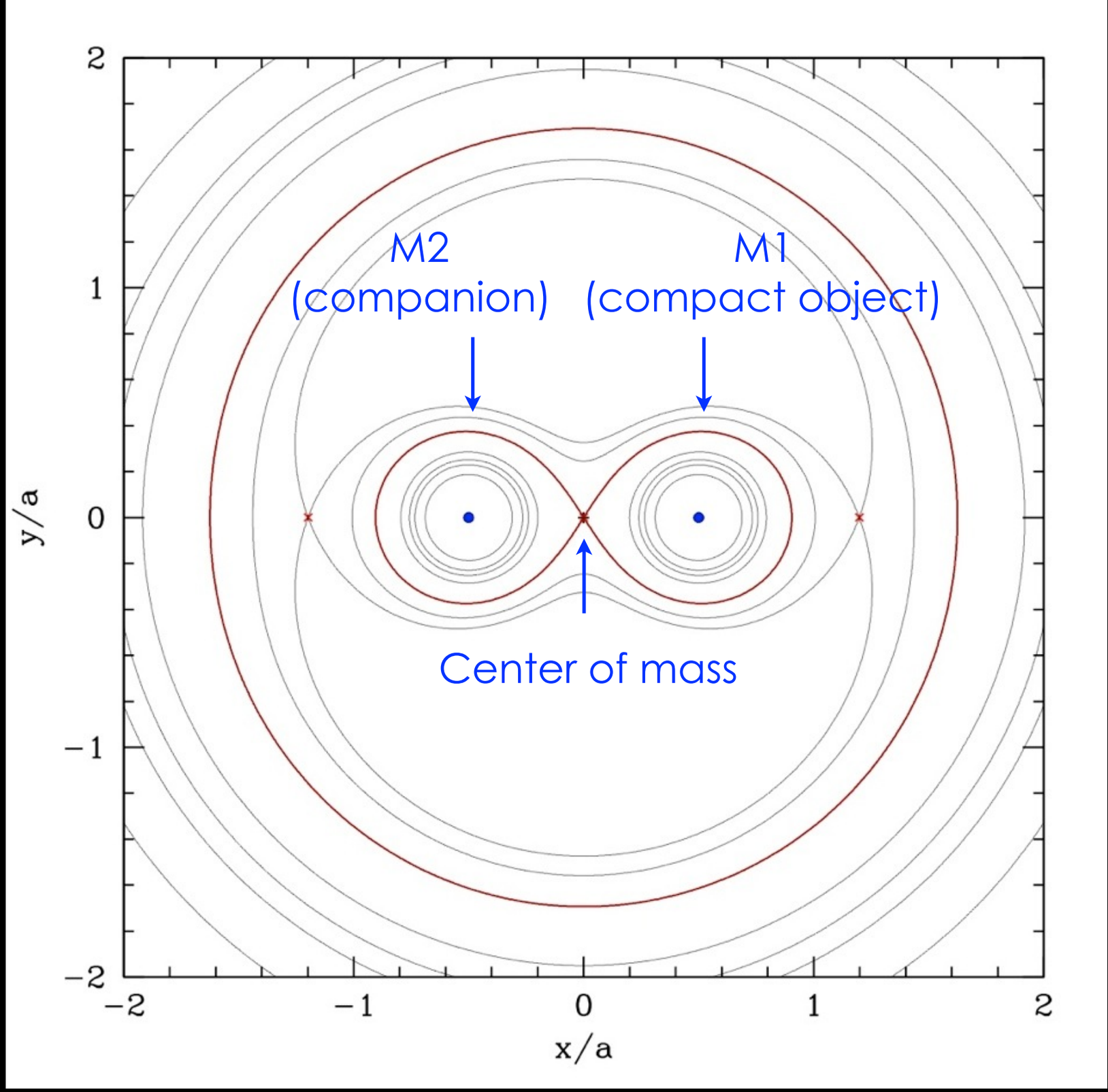


- Third law of Kepler:

$$a = 1\,\mathrm{A.U.}\,(1+q)^{1/3}\left(\frac{M_1}{1\,\mathrm{M}_\odot}\right)^{1/3}\left(\frac{T}{1\,\mathrm{yr}}\right)^{2/3}$$

$$\simeq \begin{cases} 3.5\times10^5\,\mathrm{km}\,(1+q)^{1/3}\left(\frac{M_1}{1\,\mathrm{M}_\odot}\right)^{1/3}\left(\frac{T}{1\,\mathrm{h}}\right)^{2/3} & (\mathrm{CV},\ \mathrm{LMXB}) \\ 2.9\times10^6\,\mathrm{km}\,(1+q)^{1/3}\left(\frac{M_1}{1\,\mathrm{M}_\odot}\right)^{1/3}\left(\frac{T}{1\,\mathrm{d}}\right)^{2/3} & (\mathrm{HMXB}) \end{cases}$$

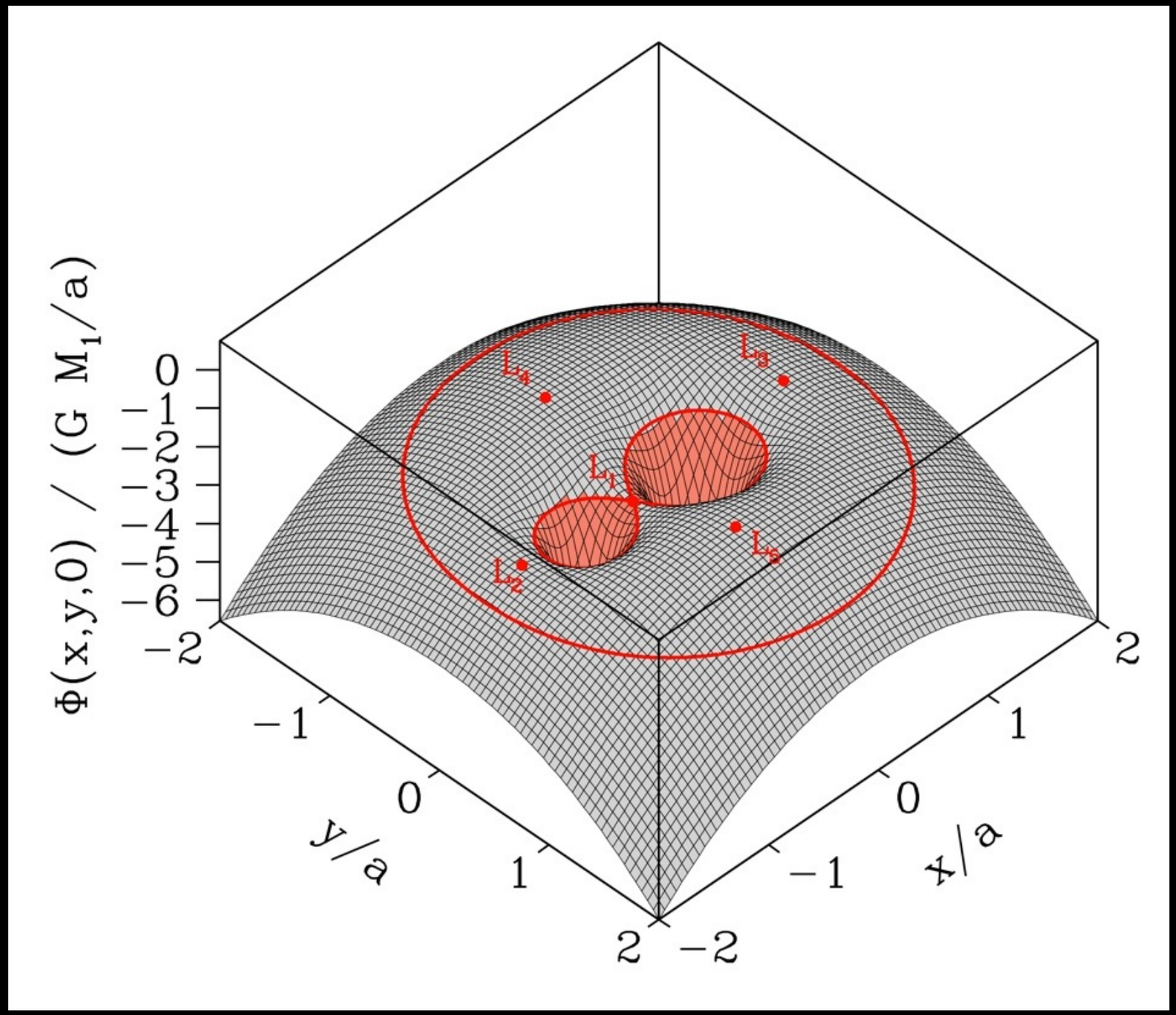
Accretion in a binary system: Roche lobes



Corotating frame

q = M2/M1 = 1,0

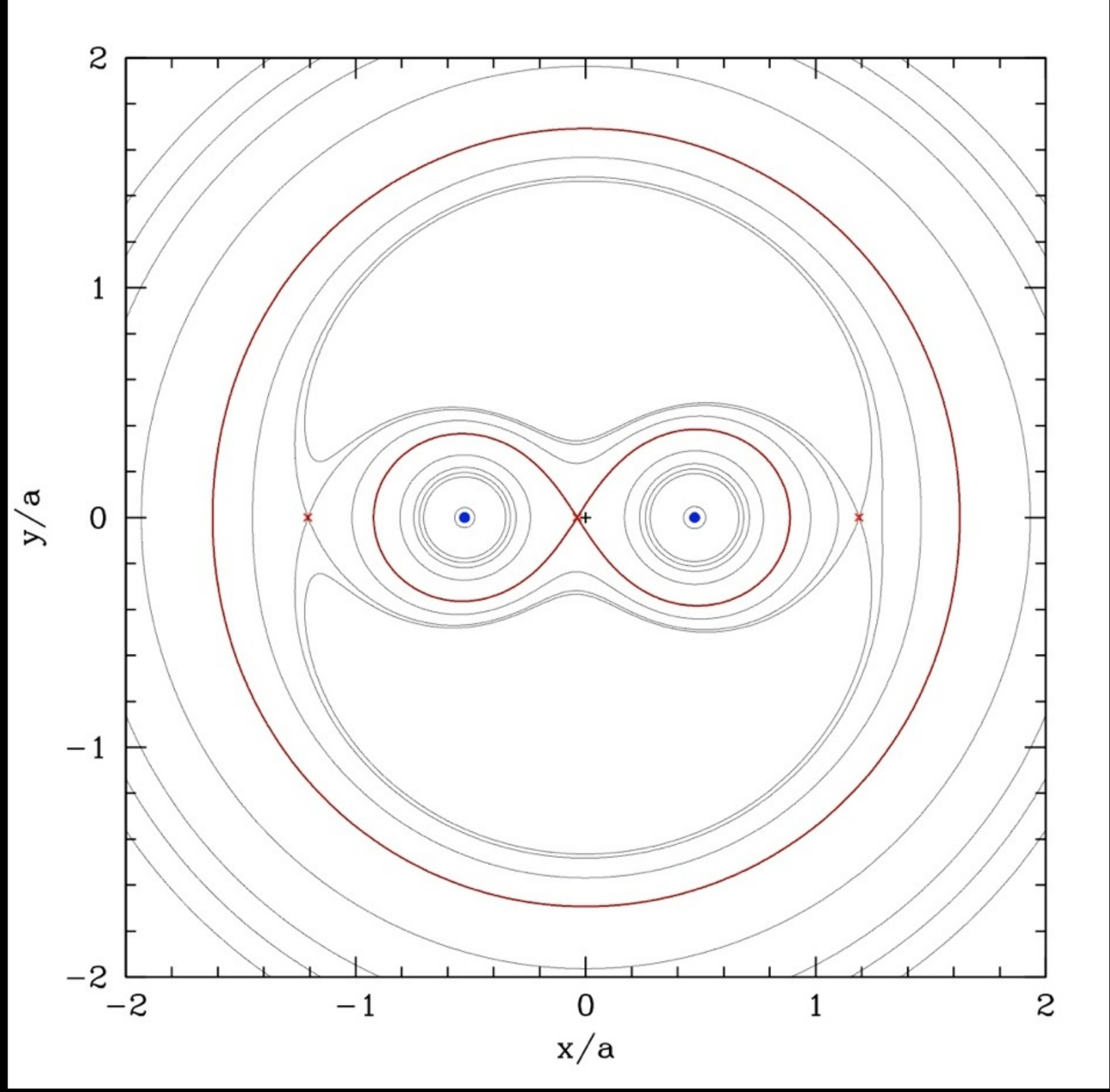
Accretion in a binary system: Roche lobes



Corotating frame

q = M2/M1 = 0,5

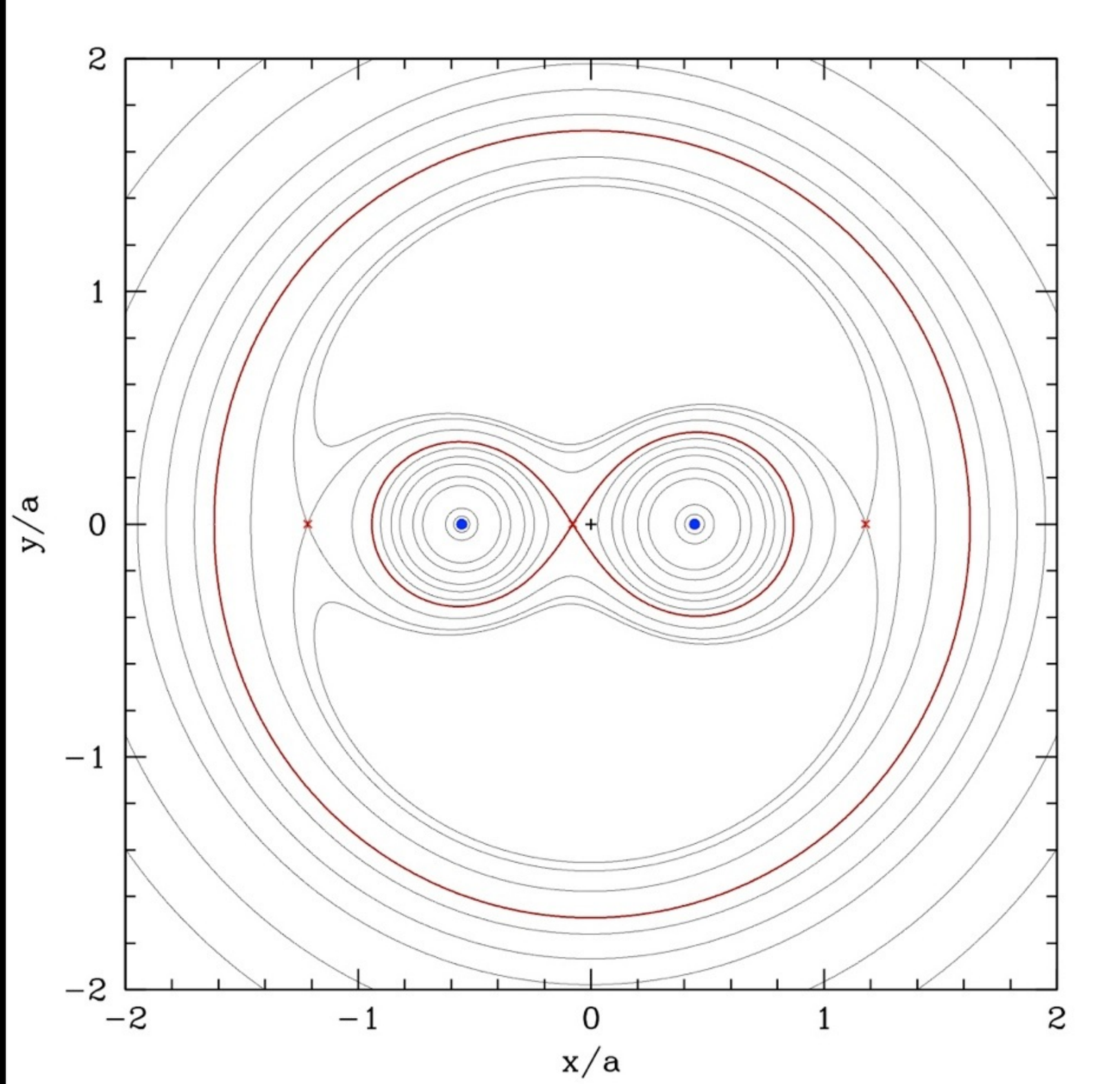
Accretion in a binary system: Roche lobes



Corotating frame

q = M2/M1 = 0,9

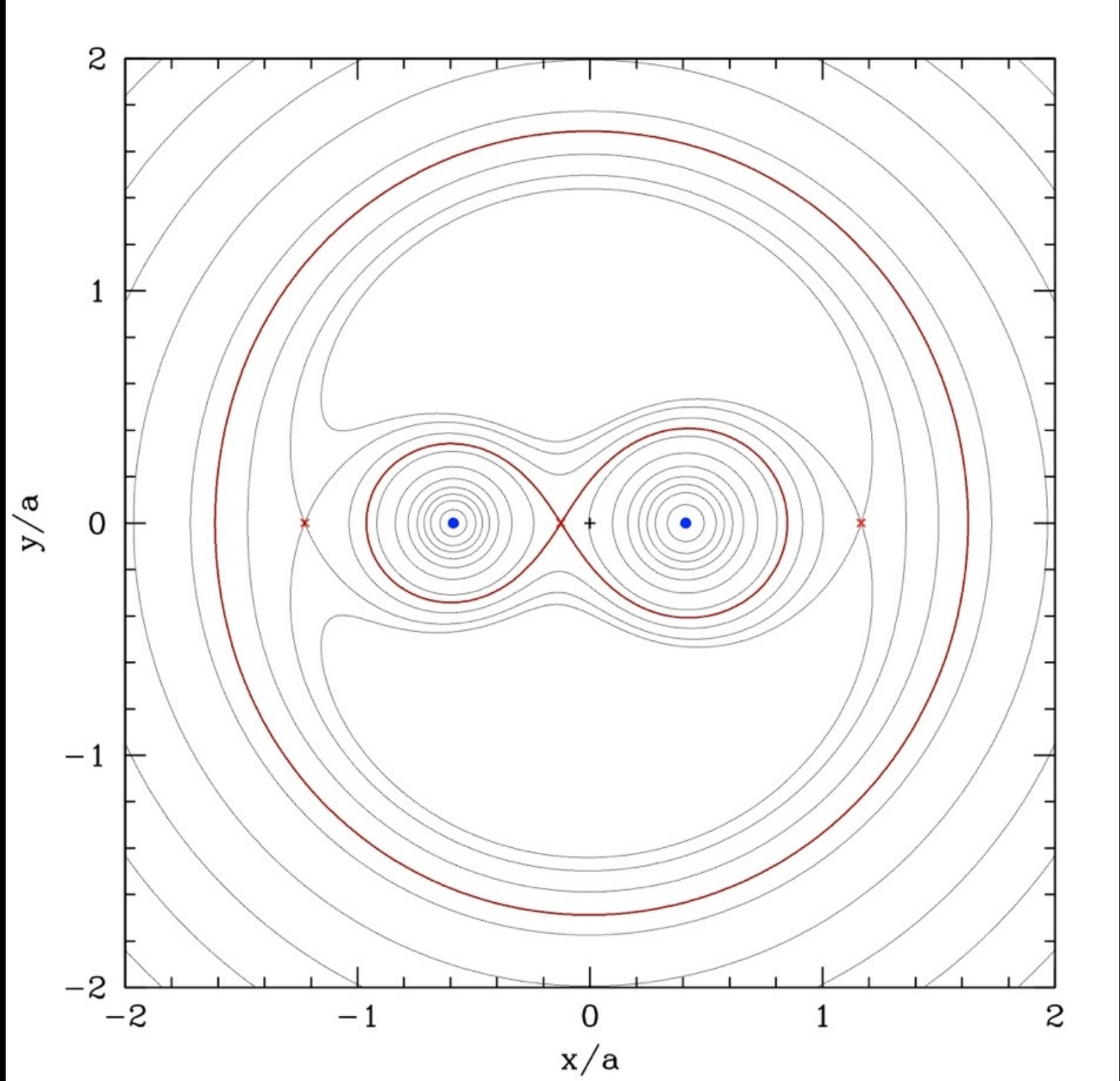
Accretion in a binary system: Roche lobes



Corotating frame

q = M2/M1 = 0,8

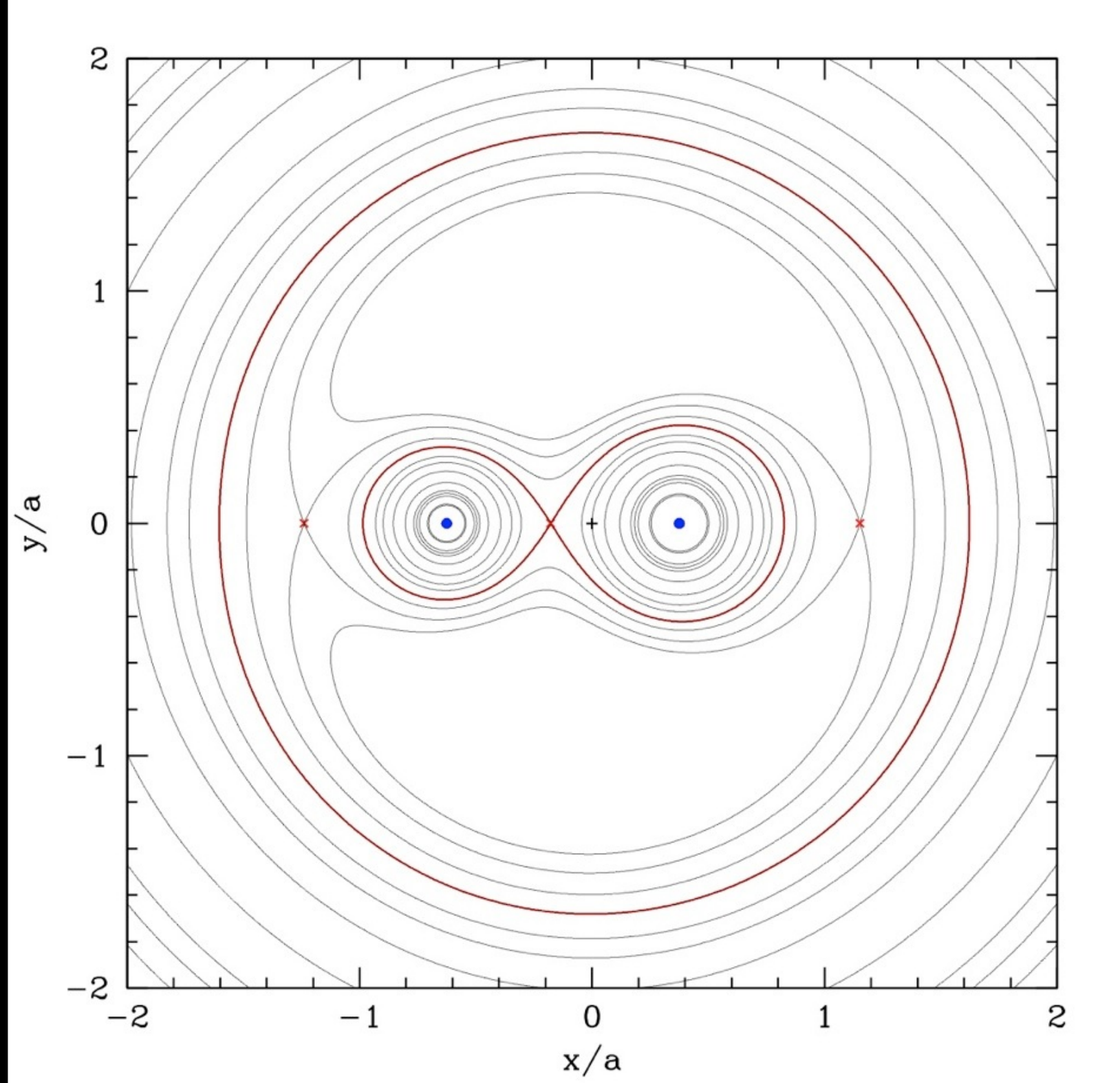
Accretion in a binary system: Roche lobes



Corotating frame

q = M2/M1 = 0,7

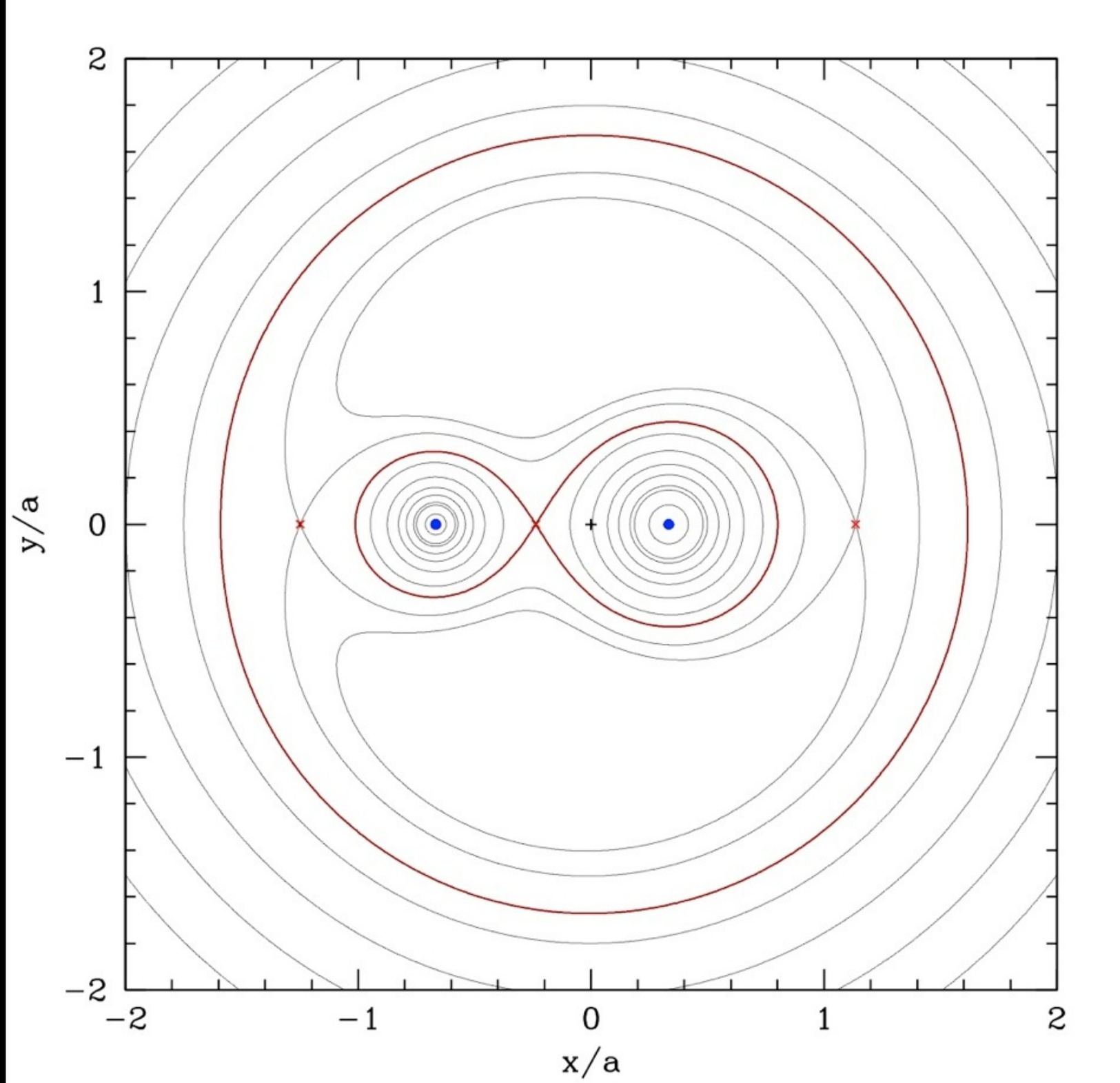
Accretion in a binary system: Roche lobes



Corotating frame

q = M2/M1 = 0,6

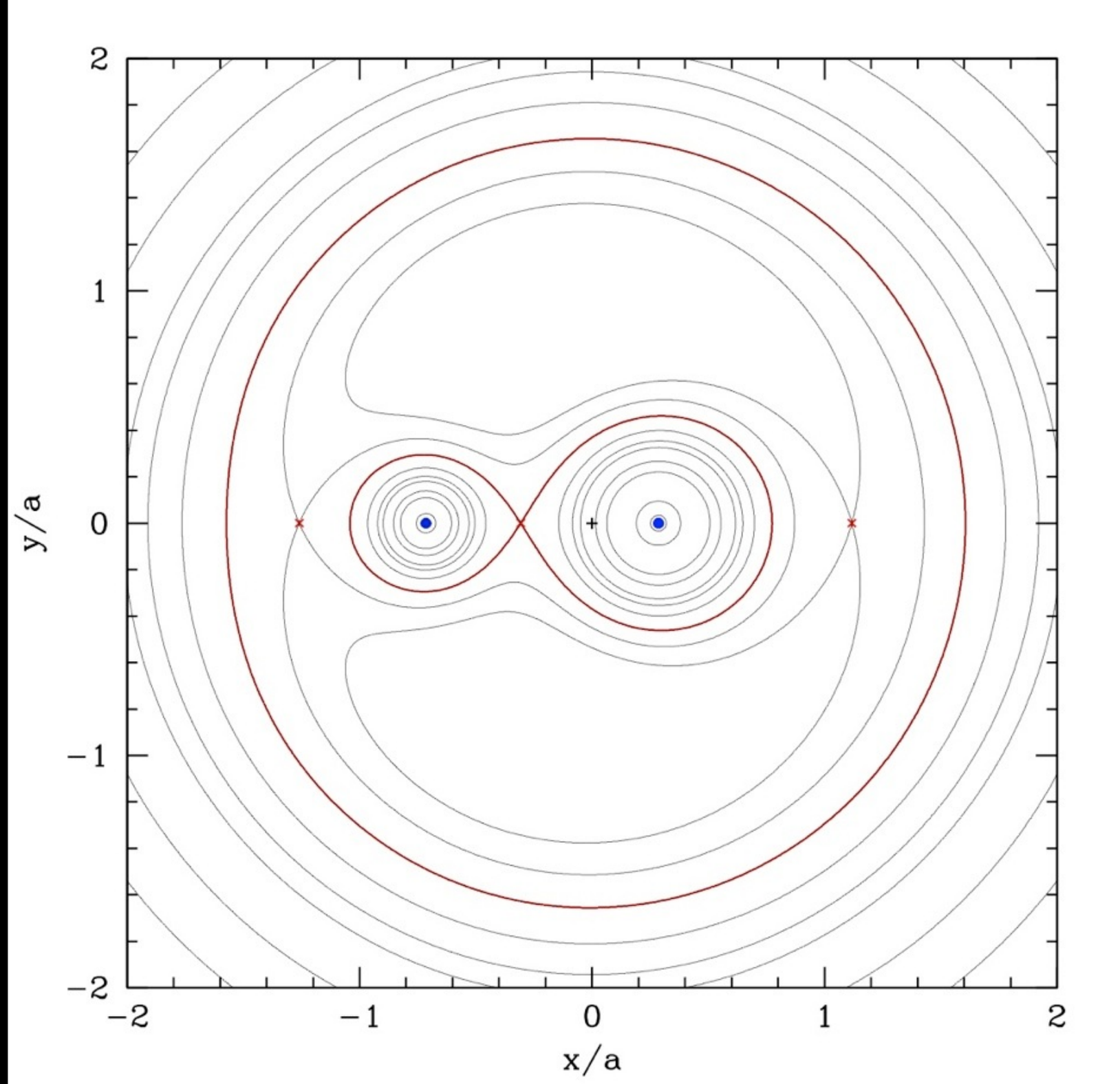
Accretion in a binary system: Roche lobes



Corotating frame

q = M2/M1 = 0,5

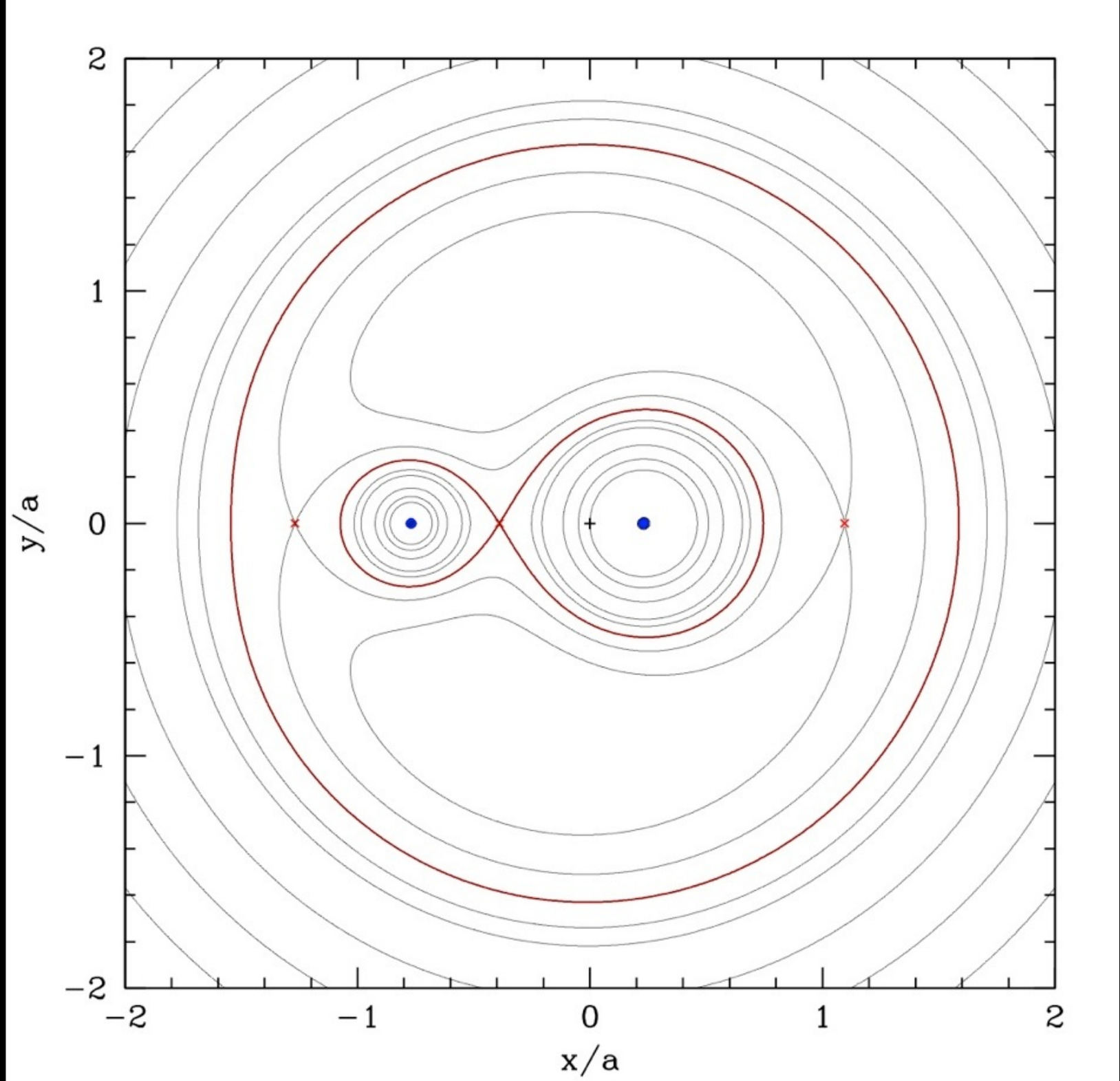
Accretion in a binary system: Roche lobes



Corotating frame

q = M2/M1 = 0,4

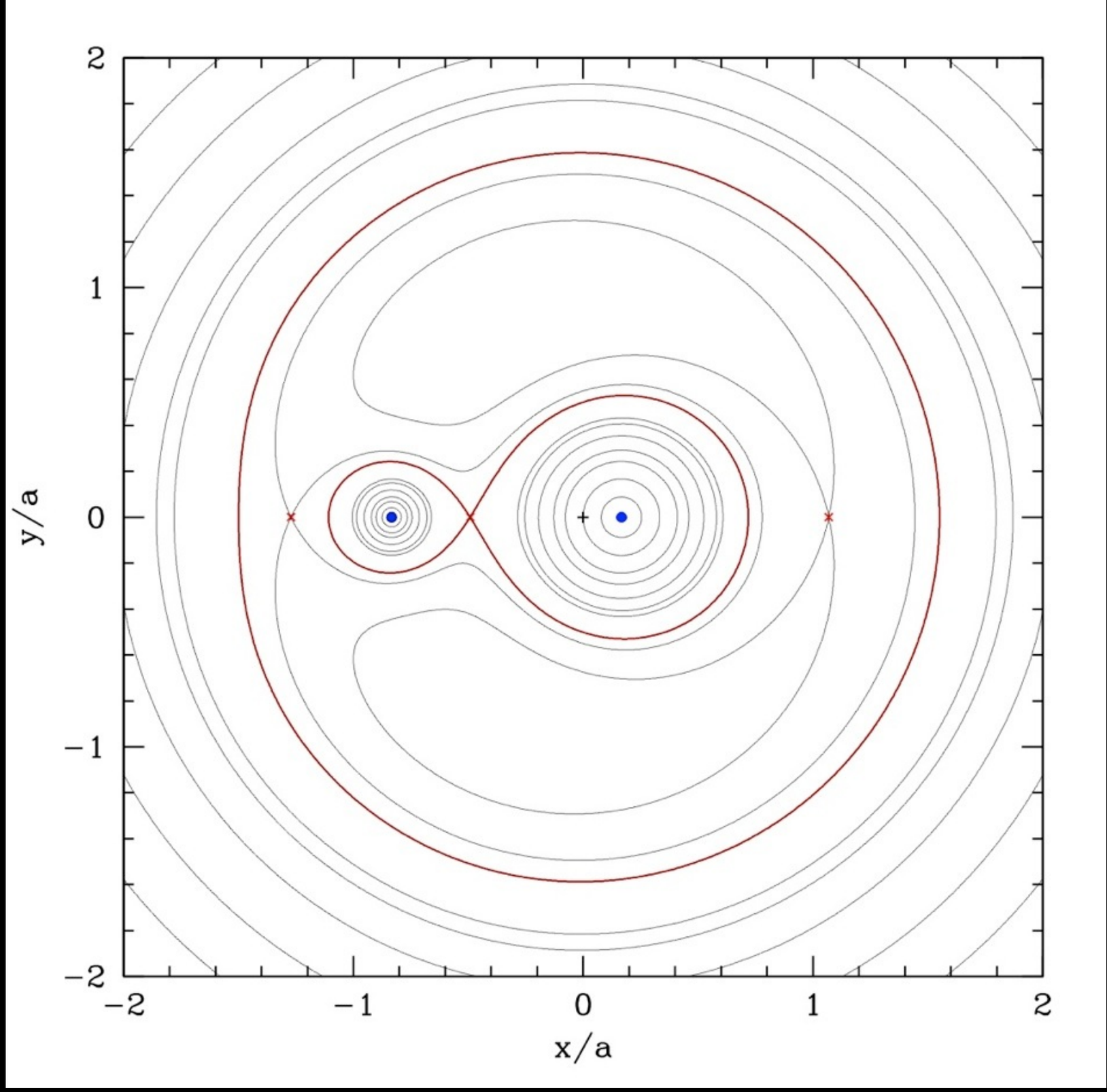
Accretion in a binary system: Roche lobes



Corotating frame

q = M2/M1 = 0,3

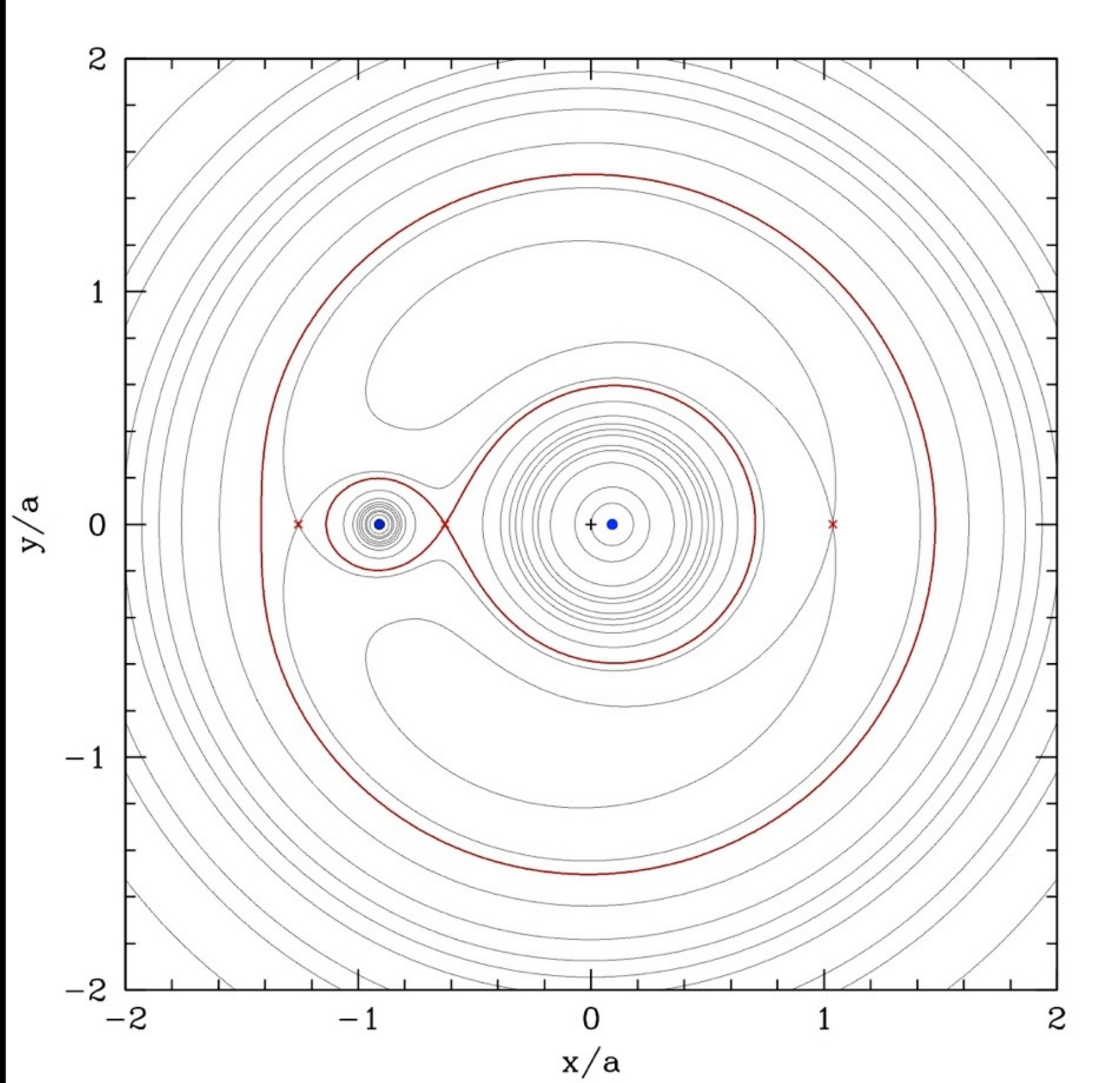
Accretion in a binary system: Roche lobes



Corotating frame

q = M2/M1 = 0,2

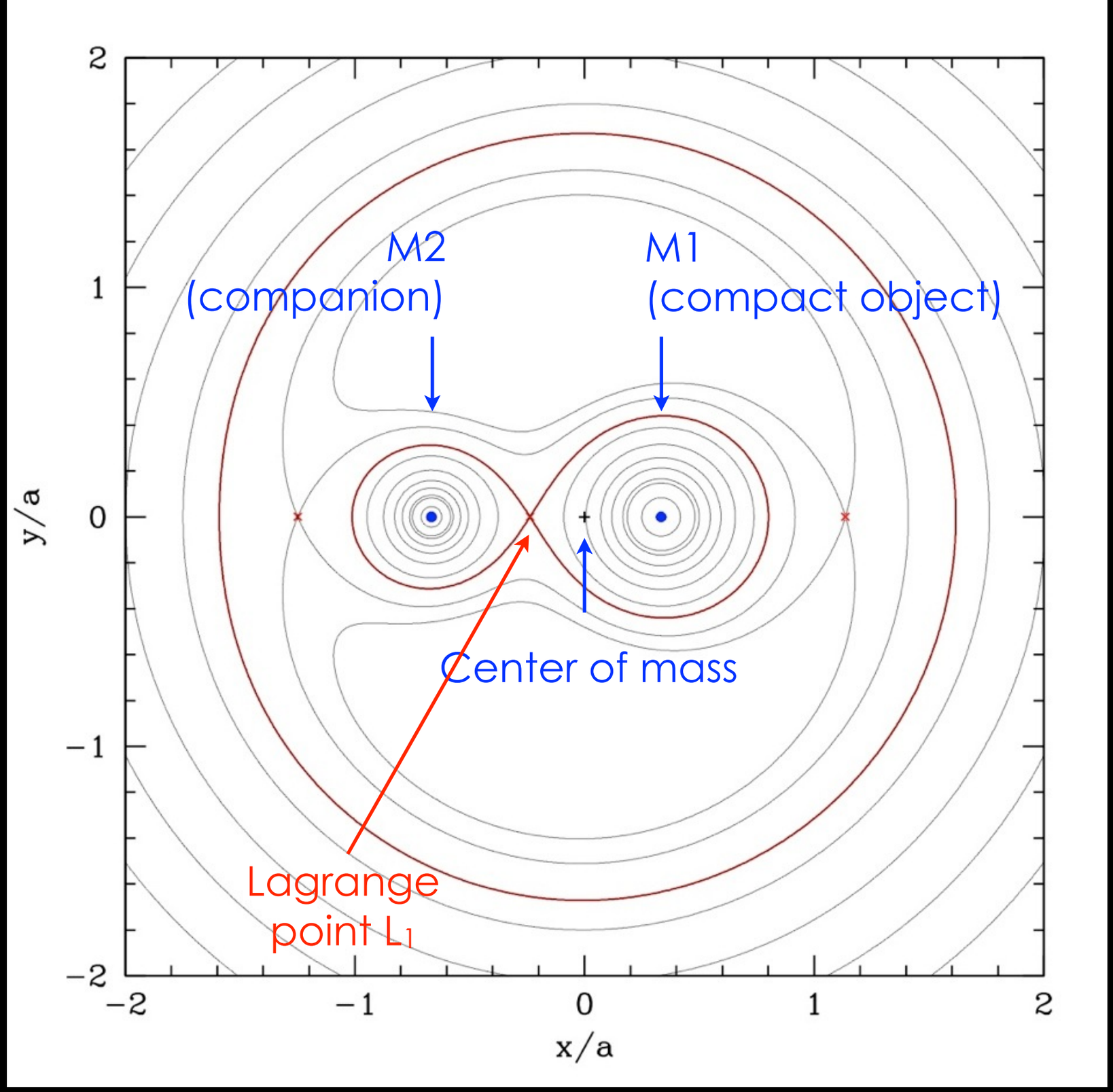
Accretion in a binary system: Roche lobes



Corotating frame

q = M2/M1 = 0,1

Accretion in a binary system: Roche lobes



Size of the Roche lobe of the companion :
(approximate formula given by Paczynski)

$$\bar{R}_2 \simeq \beta a \left(\frac{q}{1+q}\right)^{1/3} \quad \text{pour } 0 \leq q \lesssim 0.8\,, \text{ avec } \beta = 2/3^{4/3} \simeq 0.462$$

Accretion in a binary: companion filling its Roche lobe?

- Approximate size of the Roche lobe of the companion:

$$\bar{R}_2 \simeq \beta a \left(\frac{q}{1+q}\right)^{1/3} \quad \text{pour } 0 \leq q \lesssim 0.8\,, \text{ avec } \beta = 2/3^{4/3} \simeq 0.462$$

- The mean mass density within the companion's Roche lobe depends only on the orbital period:

$$\bar{\rho}_2 = \frac{3M_2}{4\pi \bar{R}_2^3} \simeq \frac{3\pi}{\beta^3}\frac{1}{GT^2} \simeq 0.11 \times 10^6\,\mathrm{kg.m^{-3}} \left(\frac{T}{1\,\mathrm{h}}\right)^{-2}$$

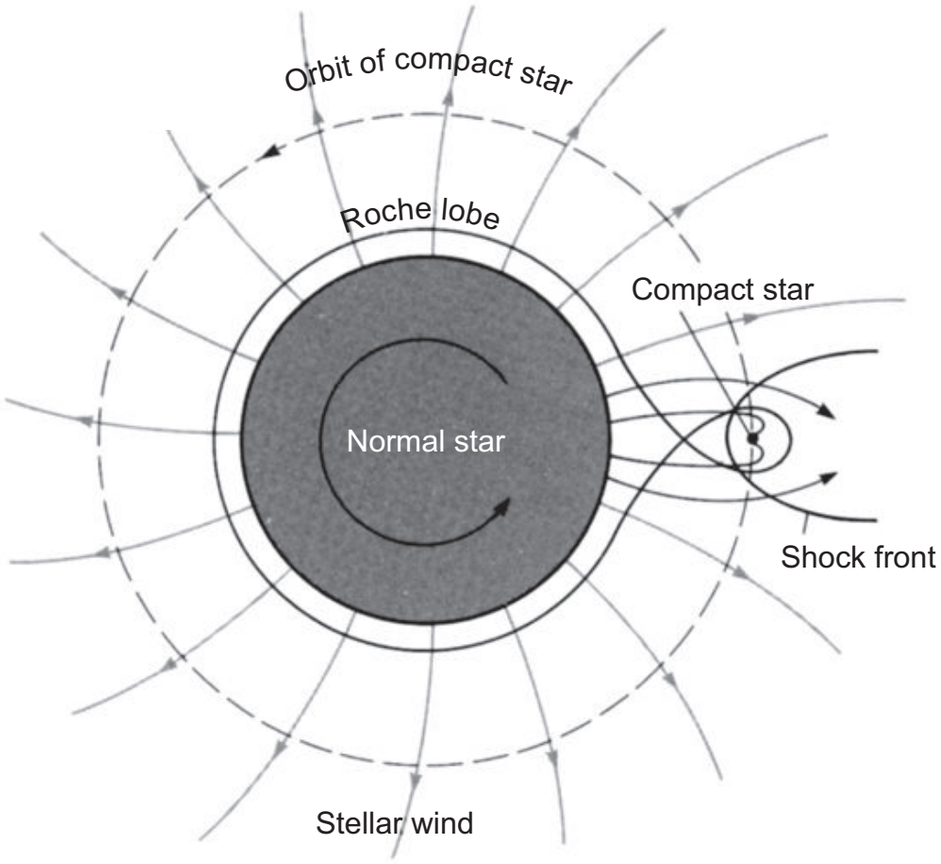
- If the companion reaches the density above, it fills its Roche lobe:

For a main sequence star:

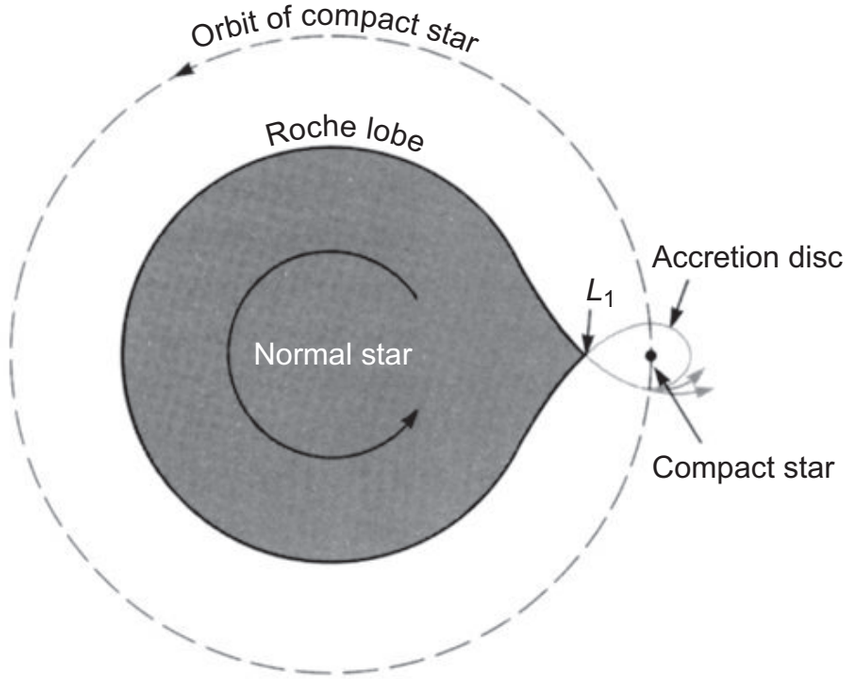
$$\bar{\rho} \simeq 1.4 \times 10^3\,\mathrm{kg.m^{-3}}\ (M/\mathrm{M}_\odot)^{-2}$$

$$M_2 \simeq 0.11\,\mathrm{M}_\odot\ (T/1\,\mathrm{h})$$

Detached and semi-detached systems:



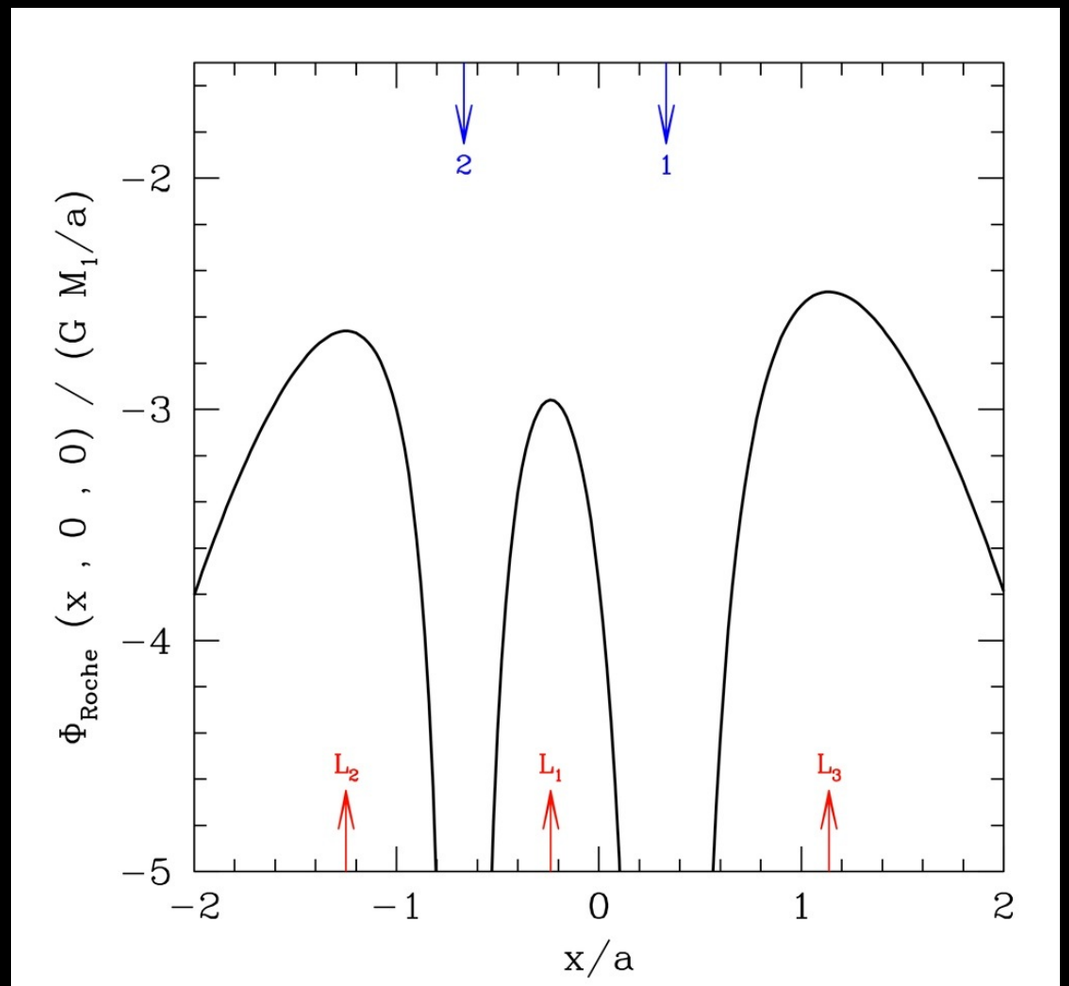
(a)Detached system: stellar wind

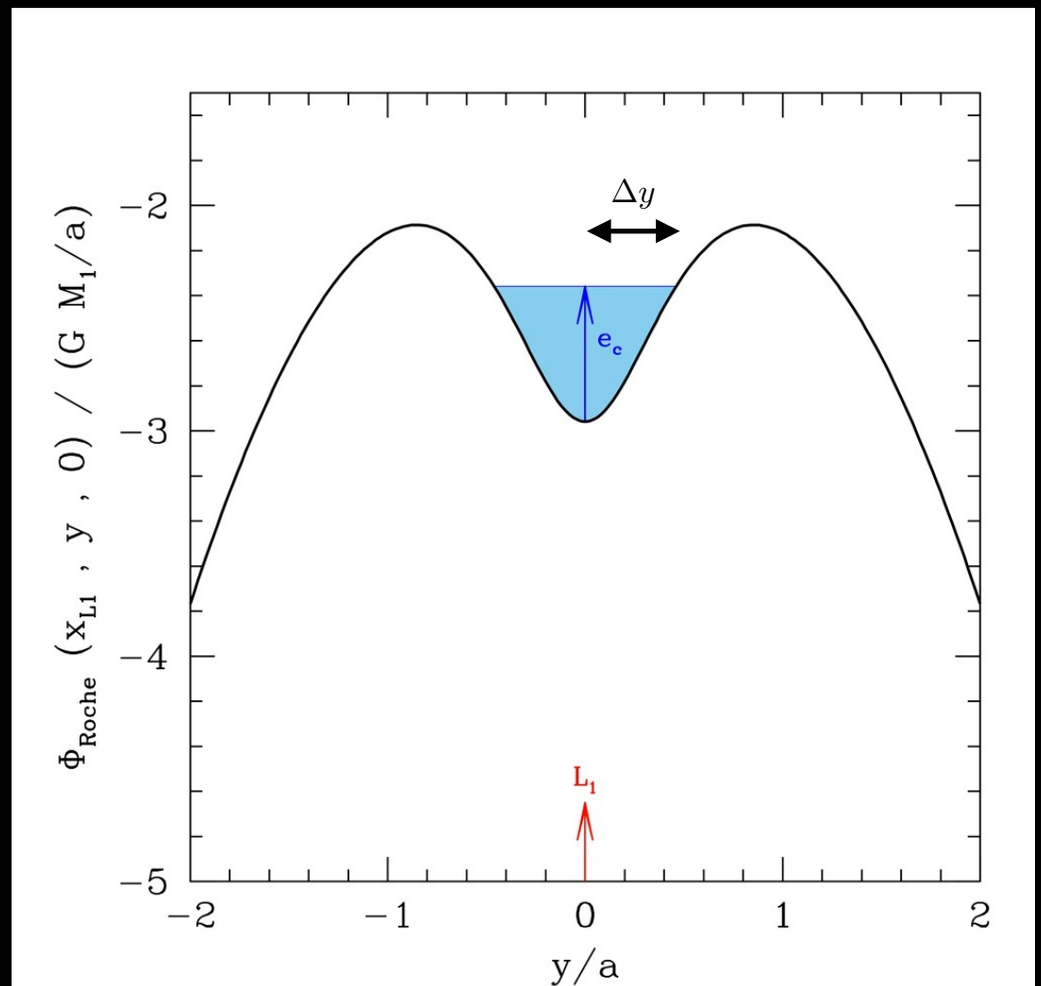


(b) Semi-detached system: Roche lobe overflow

Accretion in a binary system: Roche lobe overflow

Shape of the Roche potential in the vicinity of the Lagrange point L_1





Corotating frame

Accretion in a binary system: Roche lobe overflow

Semi-detached systems: mass flux through the Lagrange point L_1

$$\dot{m} \simeq \frac{\pi}{2K} \frac{\rho_\star \, c_{\mathrm{S}\star}^3}{\Omega^2} \simeq 10^{-8} \, \mathrm{M}_\odot.\mathrm{yr}^{-1} \left(\frac{T}{1\,\mathrm{h}} \right)^2$$

Main sequence star :

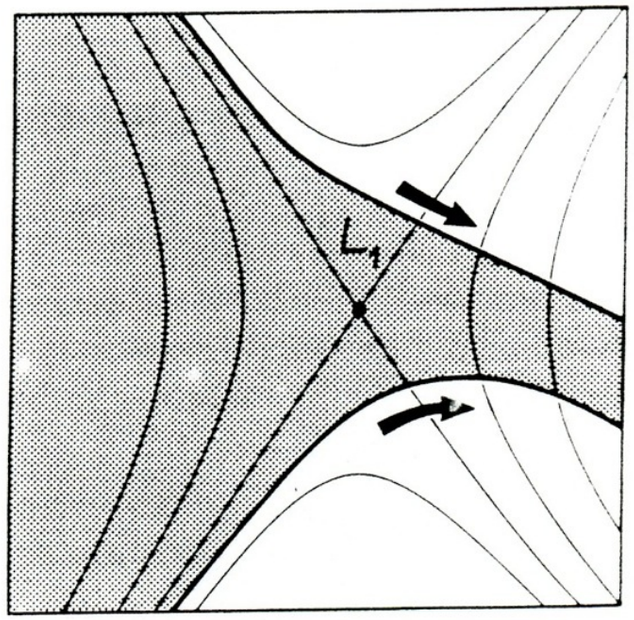
$T_{\mathrm{S}\star} \simeq 3000 - 4000\,\mathrm{K} \quad c_{\mathrm{S}\star} \simeq 5 - 6\,\mathrm{km.s}^{-1}$

$\rho_\star \simeq 10^{-3}\,\mathrm{kg.m}^{-3}$

Accretion in a binary system: flow through L_1

Mass flow through the Lagrange point L_1

Fig. 1.6. A representation of mass flow through the L_1-point. Star 1 on the left overfills its Roche lobe and so its surface potential lies just above the critical potential through L_1. Matter on star 1 near the L_1-point therefore flows down into the potential well of star 2. The flow speed at the L_1-point is roughly sonic and the flow is deflected by Coriolis force.



Corotating frame

Within the Roche lobe of the compact object

Mass flow within the Roche lobe of the compact object

Corotating frame

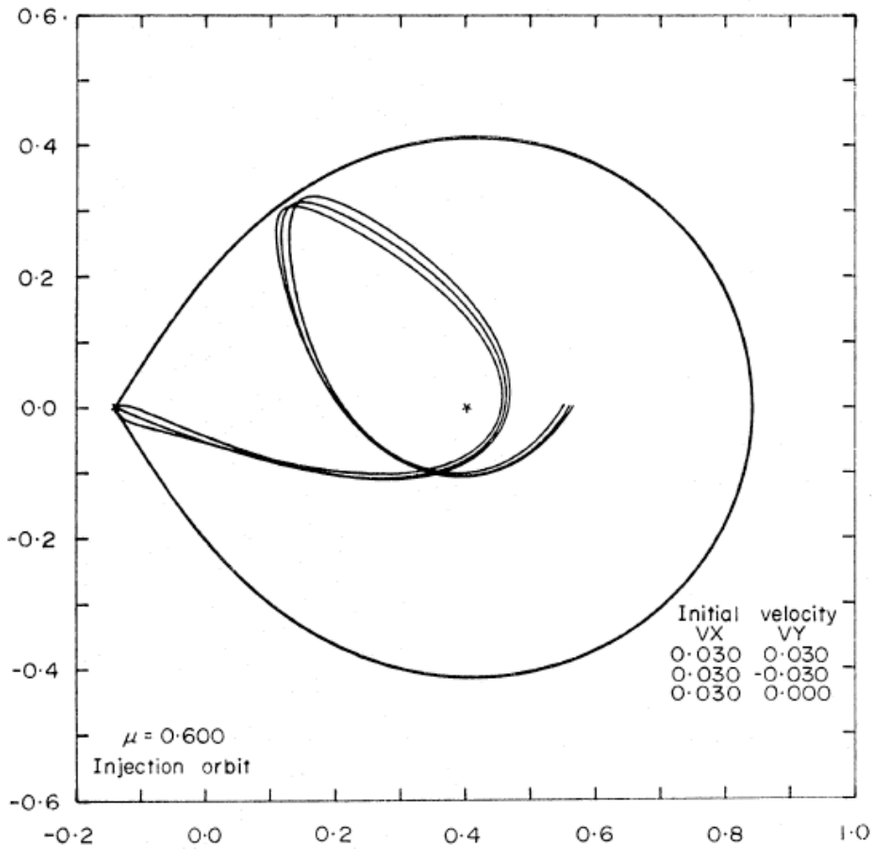


FIG. 4. *Particle trajectories in the orbital plane corresponding to thermal evaporation from the inner Lagrangian point for a mass fraction of* 0·6. *The bounding curve represents the Roche equipotential through* L_1. *Note that the trajectories cross before reaching the region of hot spot formation, which is* $r_d = 0{\cdot}10$ *for this mass fraction.*

(Flannery 1975)

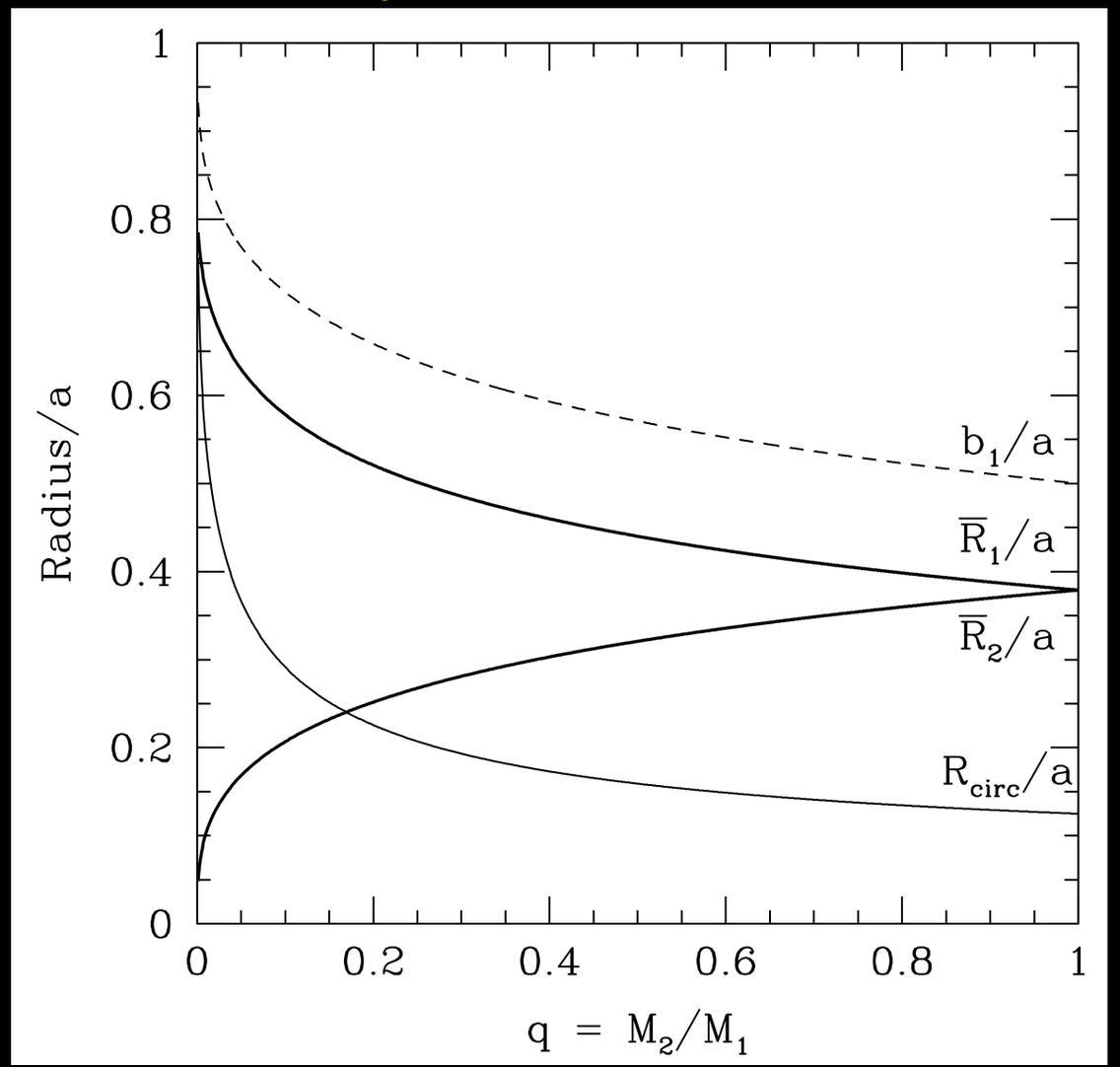
Circularization around the compact object: disk

Characteristic radii:

a= separation
b_1 = distance L_1-compact object

R_1,R_2= radii of Roche lobes

R_{circ}=circularization orbit



State state geometrically thin/optically thick disk theory

Steady state + axisymmetry

+ neglecting the disk self-gravity $\Phi(r,z) = -\dfrac{GM}{\sqrt{r^2+z^2}}$

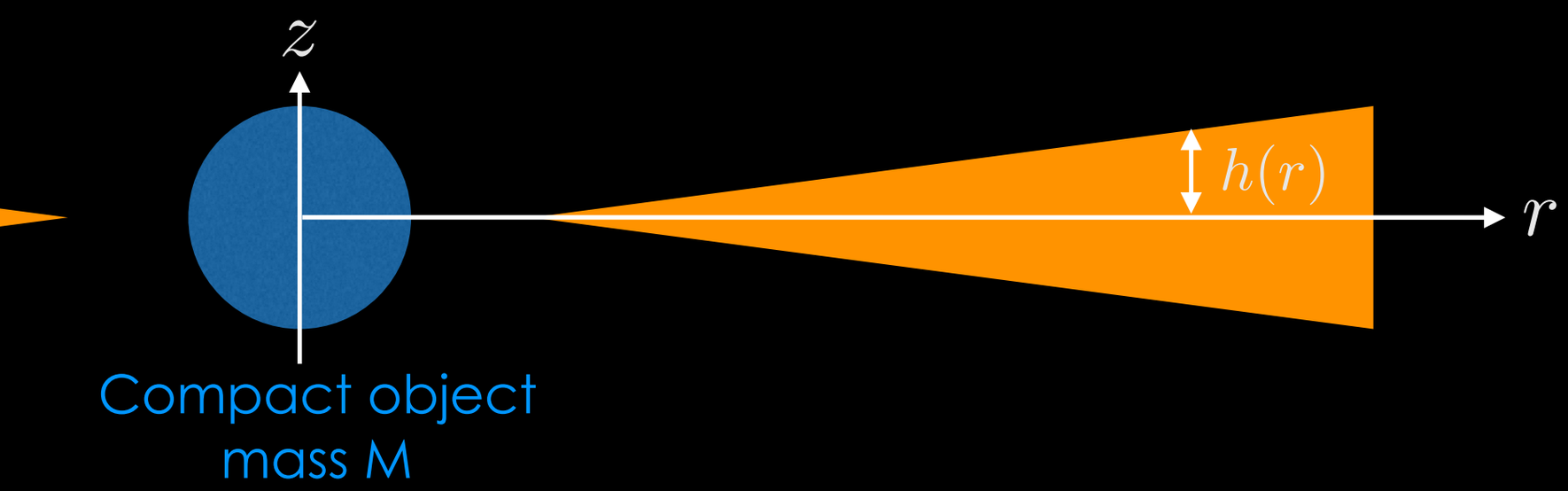
+ geometrically thin assumption $h(r) \ll r$

averaged quantities:

$$\Sigma(r) = \int_{-\infty}^{\infty} \rho(r,z)\mathrm{d}z$$

$$\bar{Q}(r) = \frac{1}{\Sigma(r)} = \int_{-\infty}^{\infty} \rho(r,z)Q(r,z)\mathrm{d}z$$

with $Q = v_r$, P ...



State state geometrically thin/optically thick disk theory

(1) Mass conservation: $\dot{m} = -2\pi r \Sigma \bar{v}_r = \mathrm{cst}$

State state geometrically thin/optically thick disk theory

(2) Leading orders in Navier stokes equation:

Velocity: $|v_z| \ll |v_r| \ll |v_\theta$

(r) $$-\frac{v_\theta^2}{r} \simeq -\frac{\partial \Phi}{\partial r}$$ ➠ quasi-keplerian disk: $v_\theta \simeq \sqrt{\frac{GMr^2}{(r^2+z^2)^{3/2}}}$

$$v_\theta \simeq \sqrt{\frac{GM}{r}}$$ in disk plane (z=0)

$$\Omega = \frac{v_\theta}{r} \simeq \sqrt{\frac{GM}{r^3}}$$

$$l = r v_\theta = r^2 \Omega \simeq \sqrt{GMr}$$

State state geometrically thin/optically thick disk theory

(2) Leading orders in Navier stokes equation:

Velocity: $|v_z| \ll |v_r| \ll |v_\theta$

(r) ➠ quasi-keplerian disk: $v_\theta \simeq \sqrt{\frac{GM}{r}}$

$$\Omega = \frac{v_\theta}{r} \simeq \sqrt{\frac{GM}{r^3}}$$

$$l = r v_\theta = r^2 \Omega \simeq \sqrt{GMr}$$

(θ) $$\rho v_r \frac{\partial l}{\partial r} \simeq \frac{1}{r}\frac{\partial}{\partial r}\left(\rho \nu r^3 \frac{\partial \Omega}{\partial r}\right)$$

➠ angular momentum transport $$\Sigma \bar{v}_r \frac{\partial l}{\partial r} \simeq \frac{1}{r}\frac{\partial}{\partial r}\left(\Sigma \bar{\nu} r^3 \frac{\partial \Omega}{\partial r}\right)$$

$$\bar{v}_r \simeq -\frac{3}{\Sigma l}\frac{\partial}{\partial r}(\Sigma \bar{\nu} l)$$

State state geometrically thin/optically thick disk theory

(2) Leading orders in Navier stokes equation:

Velocity: $|v_z| \ll |v_r| \ll |v_\theta$

(r) ➠ quasi-keplerian disk: $v_\theta \simeq \sqrt{\frac{GM}{r}}$

$$\Omega = \frac{v_\theta}{r} \simeq \sqrt{\frac{GM}{r^3}}$$

$$l = r v_\theta = r^2 \Omega \simeq \sqrt{GMr}$$

(θ) ➠ angular momentum transport $\bar{v}_r \simeq -\frac{3}{\Sigma l}\frac{\partial}{\partial r}\left(\Sigma \bar{\nu} l\right)$

(z) $\frac{1}{\rho}\frac{\partial P}{\partial z} \simeq \frac{\partial \Phi}{\partial z}$ ➠ hydrostatic equilibrium in z direction

hydrostaticdisk height scale: $\frac{1}{h}\frac{P}{\rho} \simeq \frac{GM}{r^3}h$

« isothermal sound speed »: $c_{\mathrm{S,T}}^2 = \frac{P}{\rho}$ $h \simeq \frac{c_{\mathrm{S,T}}}{\Omega}$

State state geometrically thin/optically thick disk theory

(2) Leading orders in Navier stokes equation:

(r) ➠ quasi-keplerian disk: $v_\theta \simeq \sqrt{\frac{GM}{r}}$

$$\Omega = \frac{v_\theta}{r} \simeq \sqrt{\frac{GM}{r^3}}$$

$$l = r v_\theta = r^2 \Omega \simeq \sqrt{GMr}$$

(θ) ➠ angular momentum transport $\bar{v}_r \simeq -\frac{3}{\Sigma l}\frac{\partial}{\partial r}\left(\Sigma \bar{\nu} l\right)$

(z) ➠ hydrostatic equilibrium in z direction $h \simeq \frac{c_{\mathrm{S,T}}}{\Omega}$ then: $\bar{\rho} \simeq \frac{\Sigma}{2h}$ etc.

A posteriori check: $|v_z| \ll |v_r| \ll |v_\theta|$

State state geometrically thin/optically thick disk theory

(3) Energy dissipation in the disk (viscosity) $\rho\Psi_{\rm visc} \simeq \rho\nu\left(r\frac{{\rm d}\Omega}{{\rm d}r}\right)^2$ (power/volume)

$$D(r) \simeq \Sigma\bar{\nu}\left(r\frac{{\rm d}\Omega}{{\rm d}r}\right)^2$$ (power/surface)

State state geometrically thin/optically thick disk theory

(3) Energy dissipation in the disk (viscosity) $D(r) \simeq \Sigma\bar{\nu}\left(r\frac{\mathrm{d}\Omega}{\mathrm{d}r}\right)^2$ (power/surface)

Equation of transport of the angular momentum can be integrated: $\bar{v}_r \simeq -\frac{3}{\Sigma l}\frac{\partial}{\partial r}\left(\Sigma\bar{\nu}l\right)$

$$K - \frac{\dot{m}}{2\pi}l(r) \simeq \Sigma\bar{\nu}r^3\frac{\mathrm{d}\Omega}{\mathrm{d}r}$$

Integration constant?

We assume that the rotation of the compact object is well below the Keplerian limit

$$\Omega_* < \sqrt{\frac{GM}{R^3}}$$

Then: boundary layer between the disk and the compact object

Ω decreases from the Keplerian value to Ω_* from R_{min} to R

Inner radius of the disk defined by $\frac{\mathrm{d}\Omega}{\mathrm{d}r}\left(R_{\mathrm{min}}\right) = 0$

Constant: $K = \frac{\dot{m}}{2\pi}\,l\left(R_{\mathrm{min}}\right)$

State state geometrically thin/optically thick disk theory

(3) Energy dissipation in the disk (viscosity) $D(r) \simeq \Sigma\bar{\nu}\left(r\frac{\mathrm{d}\Omega}{\mathrm{d}r}\right)^2$ (power/surface)

Equation of transport of the angular momentum can be integrated:

$$K - \frac{\dot{m}}{2\pi}l(r) \simeq \Sigma\bar{\nu}r^3\frac{\mathrm{d}\Omega}{\mathrm{d}r} \qquad K = \frac{\dot{m}}{2\pi}\,l\left(R_{\min}\right)$$

$$\Sigma\bar{\nu} \simeq \frac{\dot{m}}{3\pi}\left[1 - \frac{l(R_{\min})}{l(r)}\right] \simeq \frac{\dot{m}}{3\pi}\left[1 - \left(\frac{r}{R_{\min}}\right)^{-1/2}\right]$$

State state geometrically thin/optically thick disk theory

(3) Energy dissipation in the disk (viscosity) $D(r) \simeq \Sigma\bar{\nu}\left(r\frac{\mathrm{d}\Omega}{\mathrm{d}r}\right)^2$ (power/surface)

Equation of transport of the angular momentum can be integrated:

$$\Sigma\bar{\nu} \simeq \frac{\dot{m}}{3\pi}\left[1-\frac{l(R_{\rm min})}{l(r)}\right] \simeq \frac{\dot{m}}{3\pi}\left[1-\left(\frac{r}{R_{\rm min}}\right)^{-1/2}\right]$$

This gives the radial velocity:

$$\bar{v}_r \simeq -\frac{3}{2}\frac{\bar{\nu}}{r}\left[1-\left(\frac{r}{R_{\rm min}}\right)^{-1/2}\right]^{-1} \simeq -\frac{3}{2}\frac{r}{t_{\rm visc}}$$

Viscous time: $t_{\rm visc} = \frac{r^2}{\bar{\nu}}$

State state geometrically thin/optically thick disk theory

(3) Energy dissipation in the disk (viscosity) $D(r) \simeq \Sigma\bar{\nu}\left(r\frac{\mathrm{d}\Omega}{\mathrm{d}r}\right)^2$ (power/surface)

Equation of transport of the angular momentum can be integrated:

$$\Sigma\bar{\nu} \simeq \frac{\dot{m}}{3\pi}\left[1-\frac{l(R_{\rm min})}{l(r)}\right] \simeq \frac{\dot{m}}{3\pi}\left[1-\left(\frac{r}{R_{\rm min}}\right)^{-1/2}\right]$$

This also gives the dissipated power: $D(r) \simeq \frac{3GM\dot{m}}{4\pi r^3}\left[1-\left(\frac{r}{R_{\rm min}}\right)^{-1/2}\right]$

After integration over the disk:

$$L_{\rm visc}(r) = \int_{R_{\rm min}}^{R_{\rm max}} \mathrm{d}r\, 2\pi r\, D(r) \simeq \frac{GM\dot{m}}{2R_{\rm min}}\left[1+\left(\sqrt{\frac{R_{\rm min}}{R_{\rm max}}}-3\right)\frac{R_{\rm min}}{R_{\rm max}}\right]$$

State state geometrically thin/optically thick disk theory

(3) Energy dissipation in the disk (viscosity)

$$D(r) \simeq \frac{3GM\dot{m}}{4\pi r^3}\left[1-\left(\frac{r}{R_{\rm min}}\right)^{-1/2}\right]$$

$$L_{\rm visc} \simeq \frac{GM\dot{m}}{2R_{\rm min}} \quad \text{for} \quad R_{\rm max} \gg R_{\rm min}$$

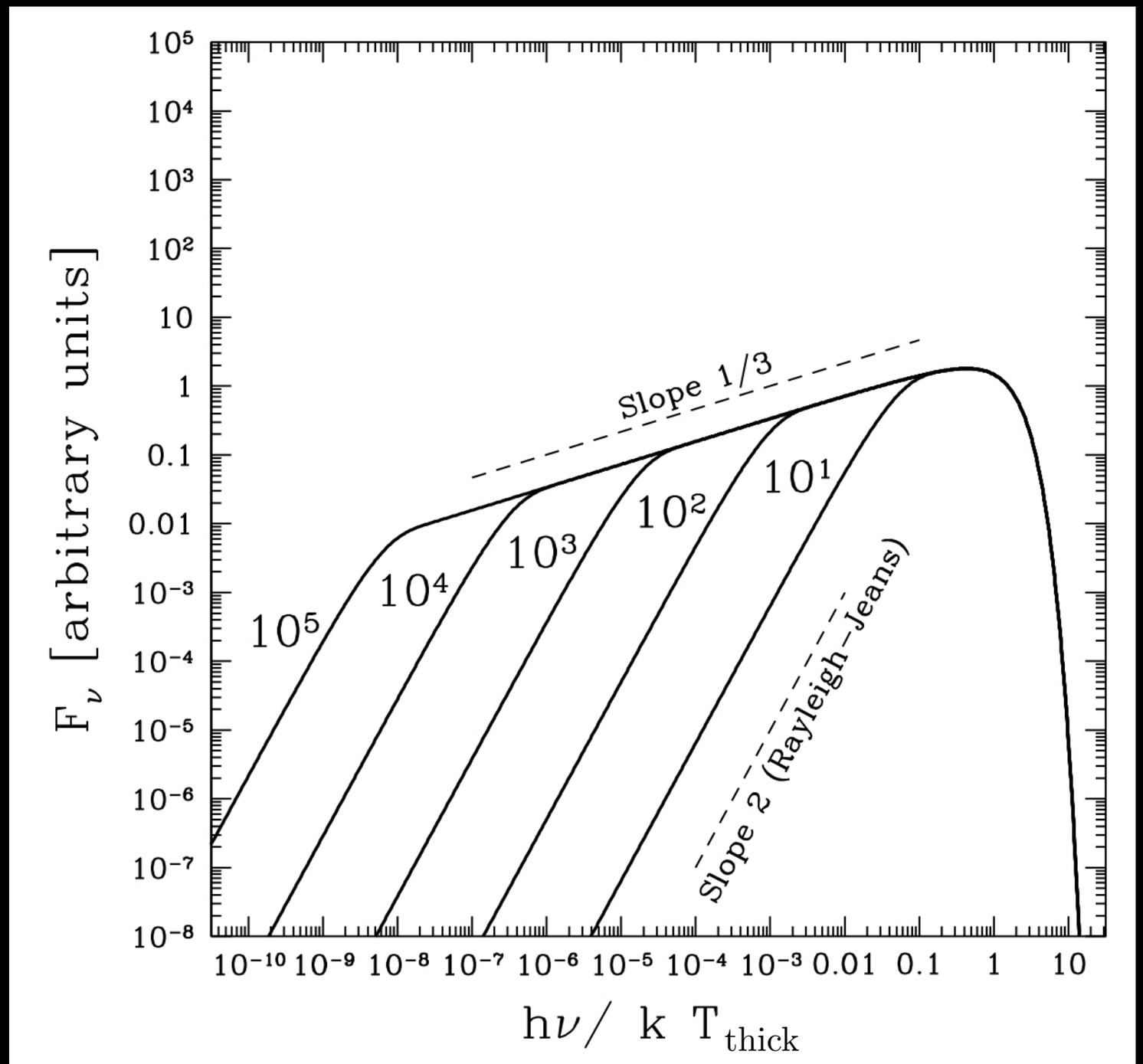
Optically thick disk: thermal radiation $D(r) = 2 \times \sigma T_{\rm S}^4(r)$

Surface temperature: $$T_{\rm S}(r) \simeq \left[\frac{3GM\dot{m}}{8\pi\sigma r^3}\left[1-\left(\frac{r}{R_{\rm min}}\right)^{-1/2}\right]\right]^{1/4}$$

Emission from the accretion disk

$$L_{\rm visc} \simeq \frac{1}{2}\frac{R}{R_{\rm min}} L_{\rm acc} \qquad T_{\rm S}(r) \simeq T_{\rm thick} \times \left(\frac{3}{2}\right)^{1/4} \left(\frac{r}{R}\right)^{-3/4}$$

Multicolor spectrum:



Spectrum of the black hole candidate Cygnus X1

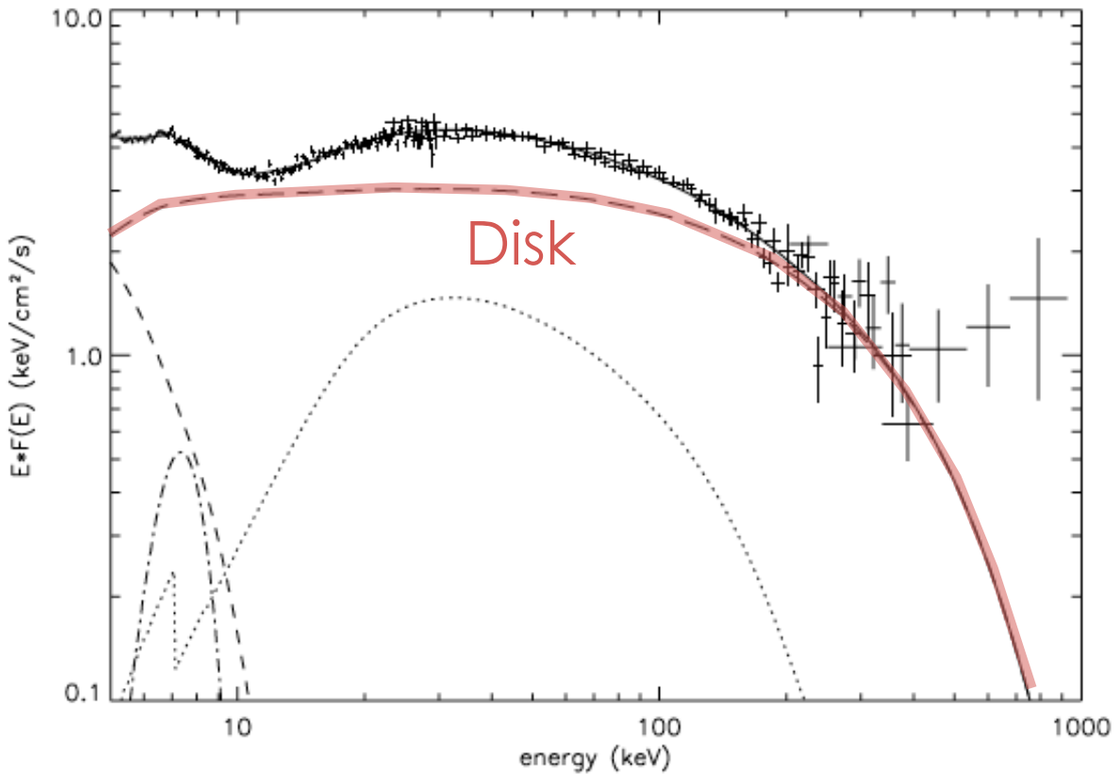


Fig. 7. Epoch 2 unabsorbed $EF(E)$ spectrum of Cygnus X-1 along with the best-fit model described in Table 2 with the JEM-X, SPI and IBIS (ISGRI and PICsIT) data. *Dashed*: Disc. *Dotted-dashed*: Gaussian line. *Dotted*: Reflection. *Long dashes*: Comptonization. *Thick*: Total model.

« Soft » state

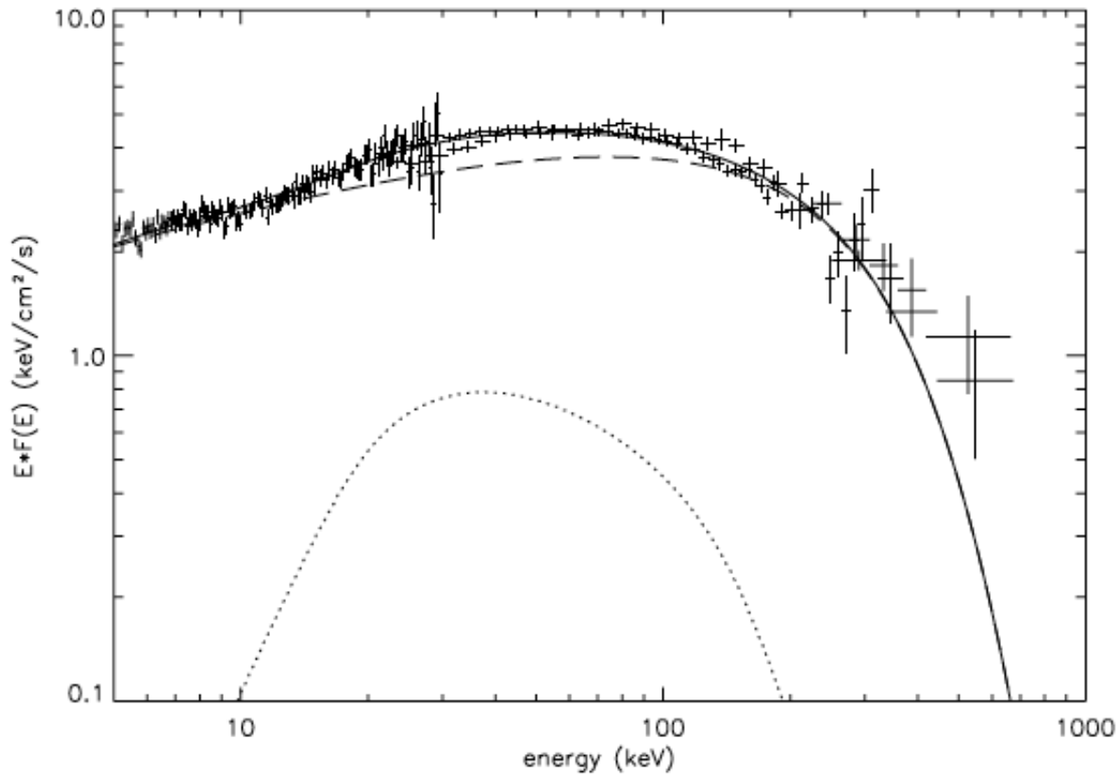


Fig. 5. Epoch 1 unabsorbed $EF(E)$ spectrum of Cygnus X-1 along with the best-fit model described in Table 2 with the JEM-X, SPI and IBIS (ISGRI and PICsIT) data. *Dotted*: reflection. *Long dashes*: comptonization. *Thick*: total model.

« hard » state

Observed spectrum : multi-components
(non-thermal process, corona ? jet ? etc.)

Change of spectral state = change of flow geometry ?

(Cadolle Bel et al. 2006)

Is it independent of the viscosity?

$$L_{\rm visc} \simeq \frac{1}{2}\frac{R}{R_{\rm min}} L_{\rm acc} \qquad T_{\rm S}(r) \simeq T_{\rm thick} \times \left(\frac{3}{2}\right)^{1/4} \left(\frac{r}{R}\right)^{-3/4}$$

No: the viscosity is hidden in the mass flux... $\dot{m} = -2\pi r \Sigma \bar{v}_r = \mathrm{cst}$

$$\bar{v}_r \simeq -\frac{3}{2}\frac{r}{t_{\rm visc}}$$

Problem: what is the viscosity?

α prescription: $\bar{\nu}(r) = \alpha\, h(r)\, \bar{c}_{\rm S,T}(r)$
(Shakura & Sunyaev 1973)

Physical prescription? turbulence, MRI, ...

Full structure of the disk

Add: EOS + opacity

Limit of the thin disk solution:

$$\dot{m} \ll \dot{m}_{\rm Edd}$$

$$
\begin{aligned}
\dot{m} &= -2\pi r \Sigma \bar{v} = \mathrm{cst}\,, \\
\Sigma\bar{v} &\simeq \frac{\dot{m}}{3\pi}\left[1-\left(\frac{r}{R_{\rm min}}\right)^{-1/2}\right], \\
\bar{\rho} &\simeq \frac{\Sigma}{2h}\,, \\
h &\simeq \frac{\bar{c}_{\rm S,T}}{\Omega}\,, \\
\bar{c}^2_{\rm S,T} &\simeq \frac{\bar{P}}{\bar{\rho}}\,, \\
T_{\rm S}^4 &\simeq \frac{D}{2\sigma}\,, \\
\left(\frac{\bar{T}}{T_{\rm S}}\right)^4 &\simeq \frac{3}{4}\bar{\kappa}\Sigma + 1\,, \\
D &\simeq \frac{3GM\dot{m}}{4\pi r^3}\left[1-\left(\frac{r}{R_{\rm min}}\right)^{-1/2}\right], \\
&\simeq \frac{3\dot{m}}{4\pi}\Omega^2\left[1-\left(\frac{r}{R_{\rm min}}\right)^{-1/2}\right], \\
v_\theta &= \frac{l}{r} = r\Omega \simeq \sqrt{\frac{GM}{r}}\,, \\
\bar{v}_r &\simeq -\frac{3}{2}\frac{\bar{\nu}}{r}\left[1-\left(\frac{r}{R_{\rm min}}\right)^{-1/2}\right]^{-1}, \\
\bar{v}_z &\simeq \frac{h}{r}\bar{v}_r\,.
\end{aligned}
$$

Full structure of the disk

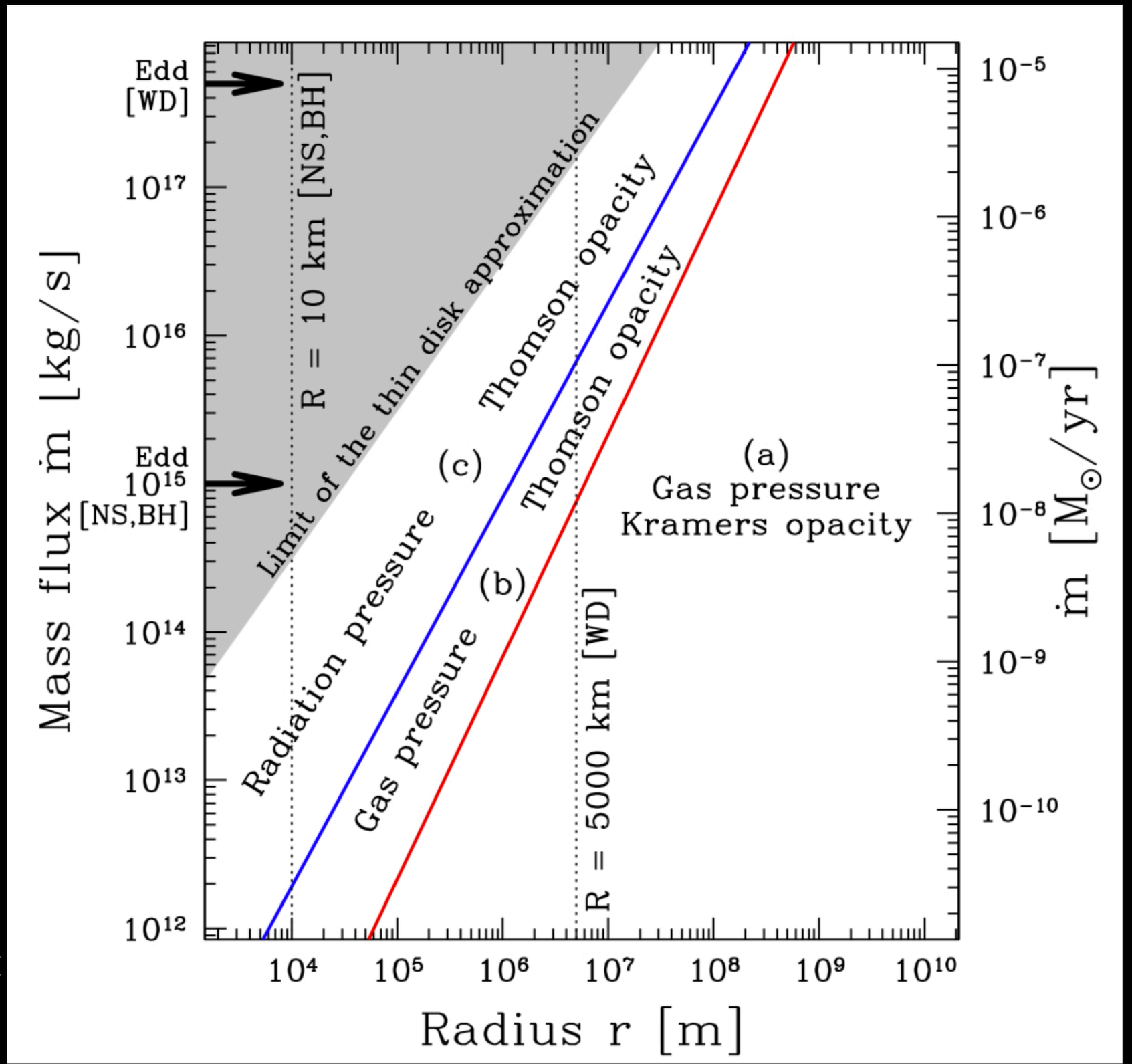
Dominant term:
EOS & Opacity

EOS: $P = P_{th}+P_{rad}$

Opacity:
e- scatterings (Thomson)
+
photo-ionization
/inverse Bremmstrahlung
(Kramers)

Limit of the thin disk solution:

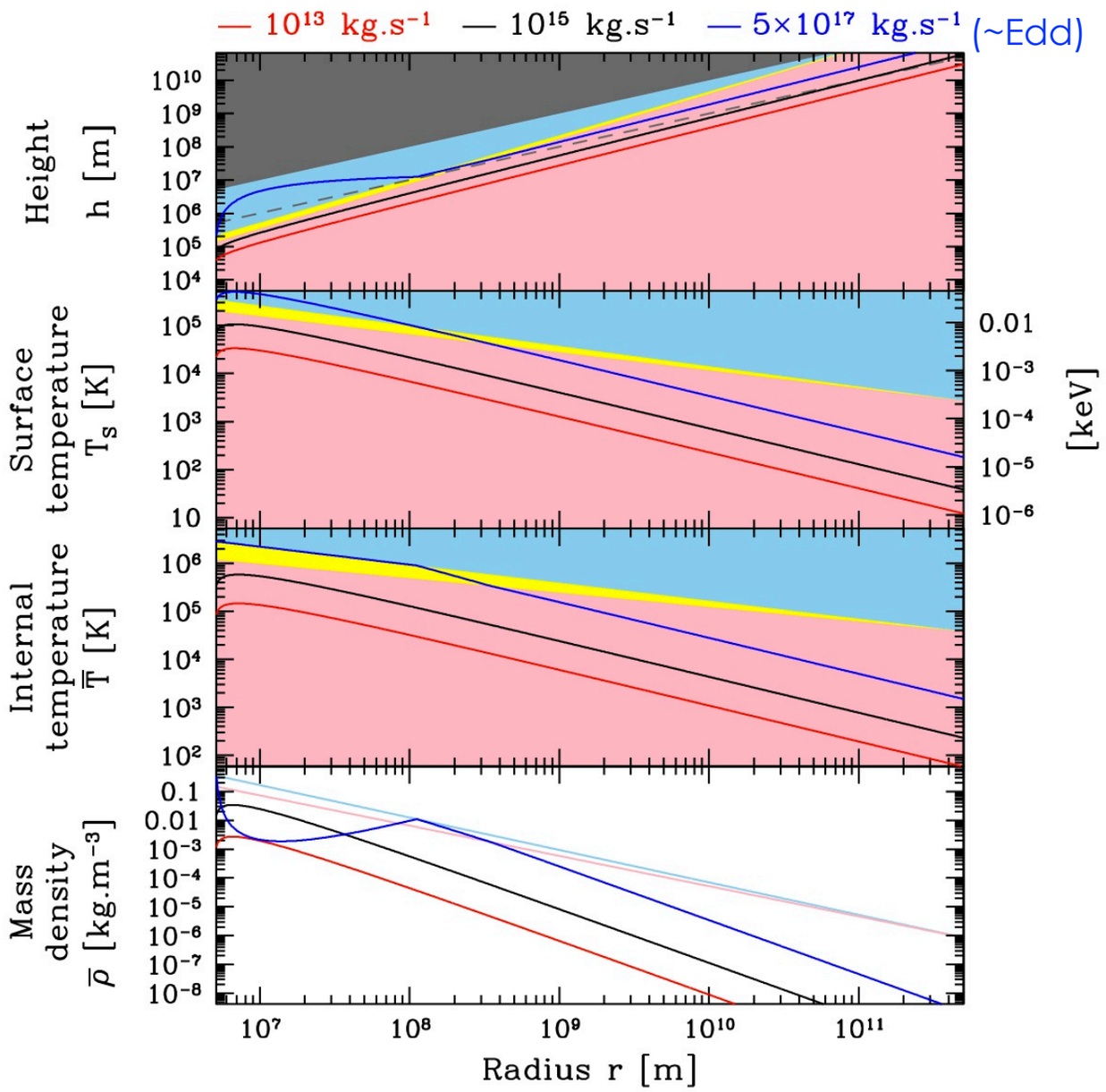
$$\dot{m} \ll \dot{m}_{\rm Edd}$$



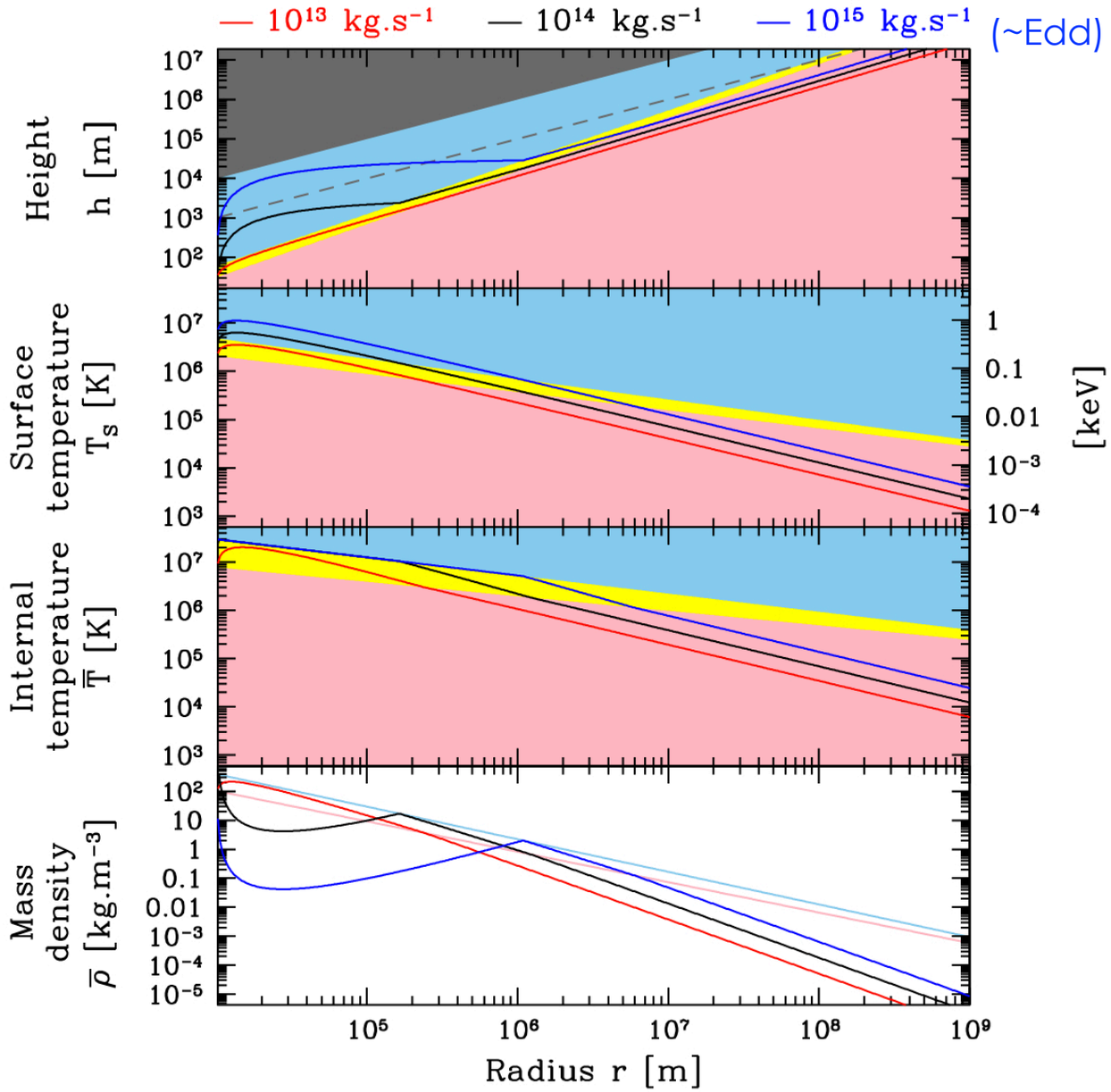
M = 1 $M_\odot$; μ = 0.5 ; μ_e = 1 ; α = 0.1

Full structure of the disk

R_{min} = 5000 km (WD)



R_{min} = 10 km (NS)



Dominant terms:

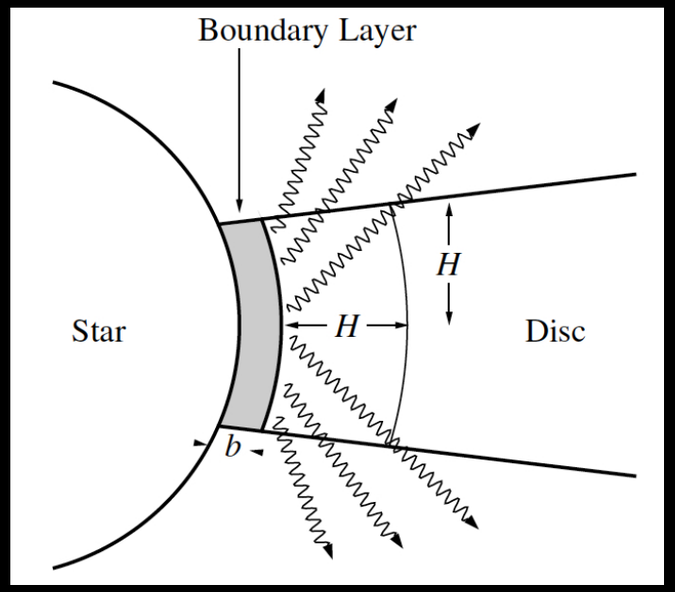
- P_{th} + Kramers (« low T »)
- P_{th} + Thomson
- P_{rad} + Thomson (« high T »)

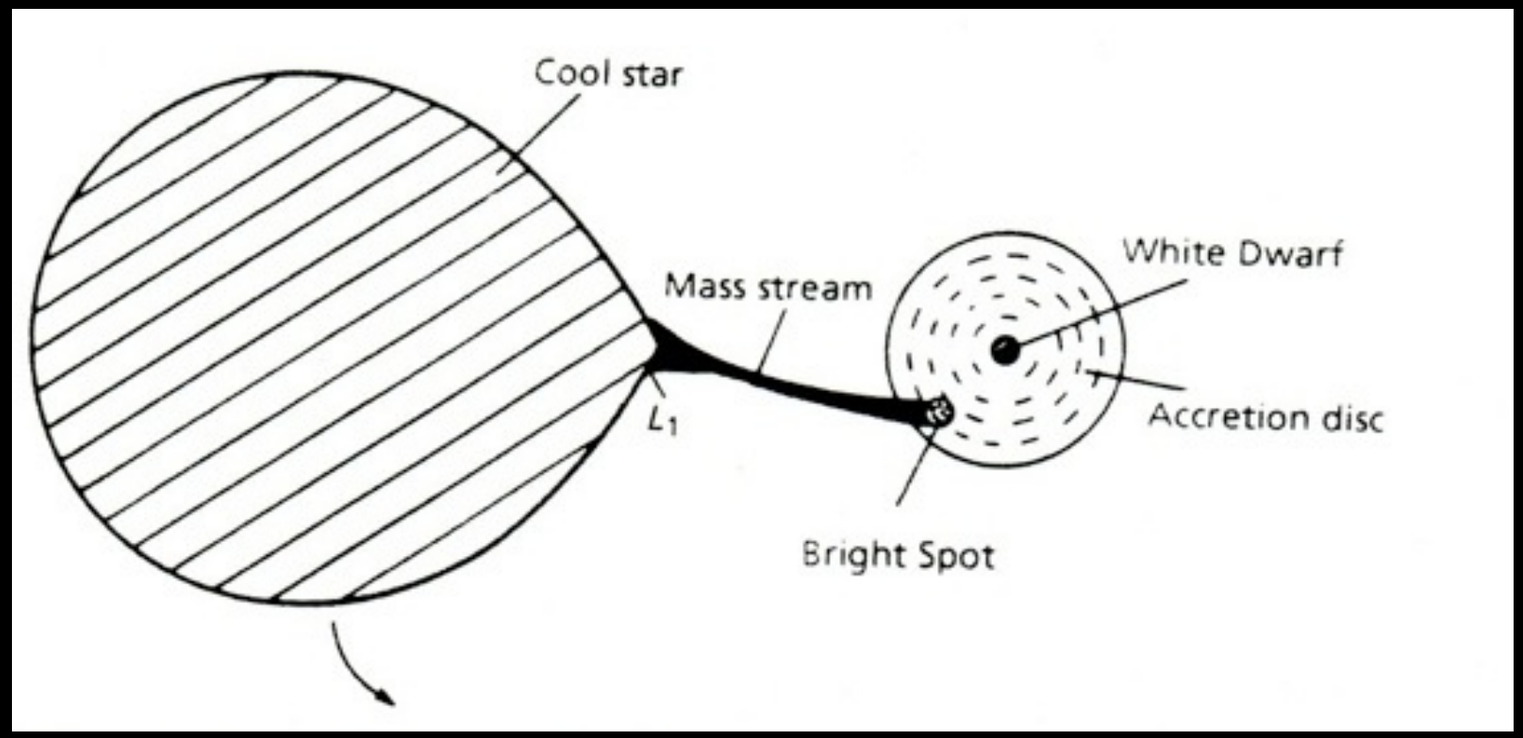
Limit of the thin disk solution: $\dot{m} \ll \dot{m}_{\mathrm{Edd}}$

M = 1 $M_\odot$; μ = 0.5 ; μ_e = 1 ; α = 0.1

From the disk to the compact object

unmagnetized CV





$$L_{\rm c.l.} \simeq \frac{L_{\rm acc}}{2}\left(1-\frac{\Omega}{\Omega_{\rm K}}\right)^2 . \qquad T_{\rm c.l.} \simeq T_{\rm thick}\left(\frac{R}{2h}\right)^{1/4}$$

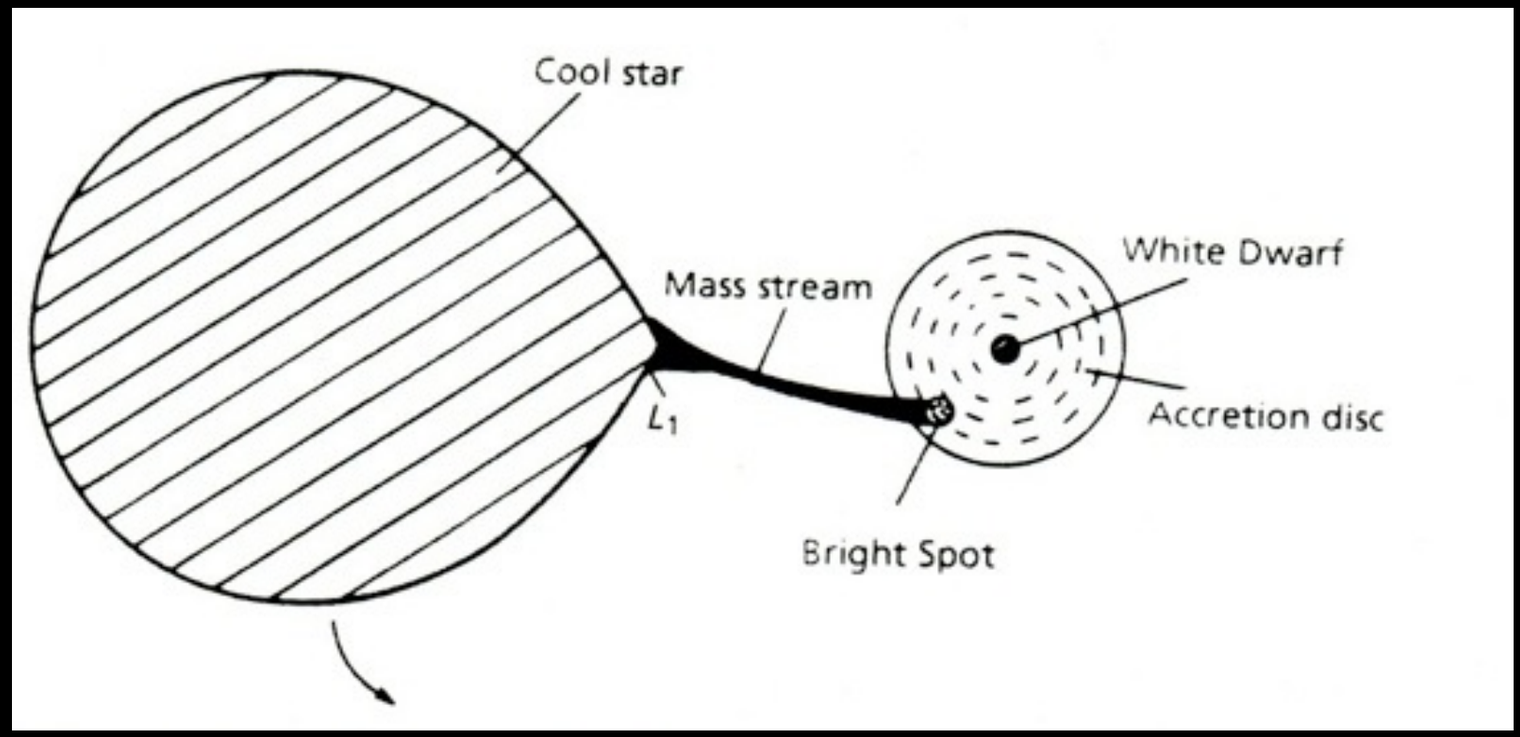
Unmagnetized WD,NS (=surface) : boundary layer

Similar geometry for a NS

From the disk to the compact object

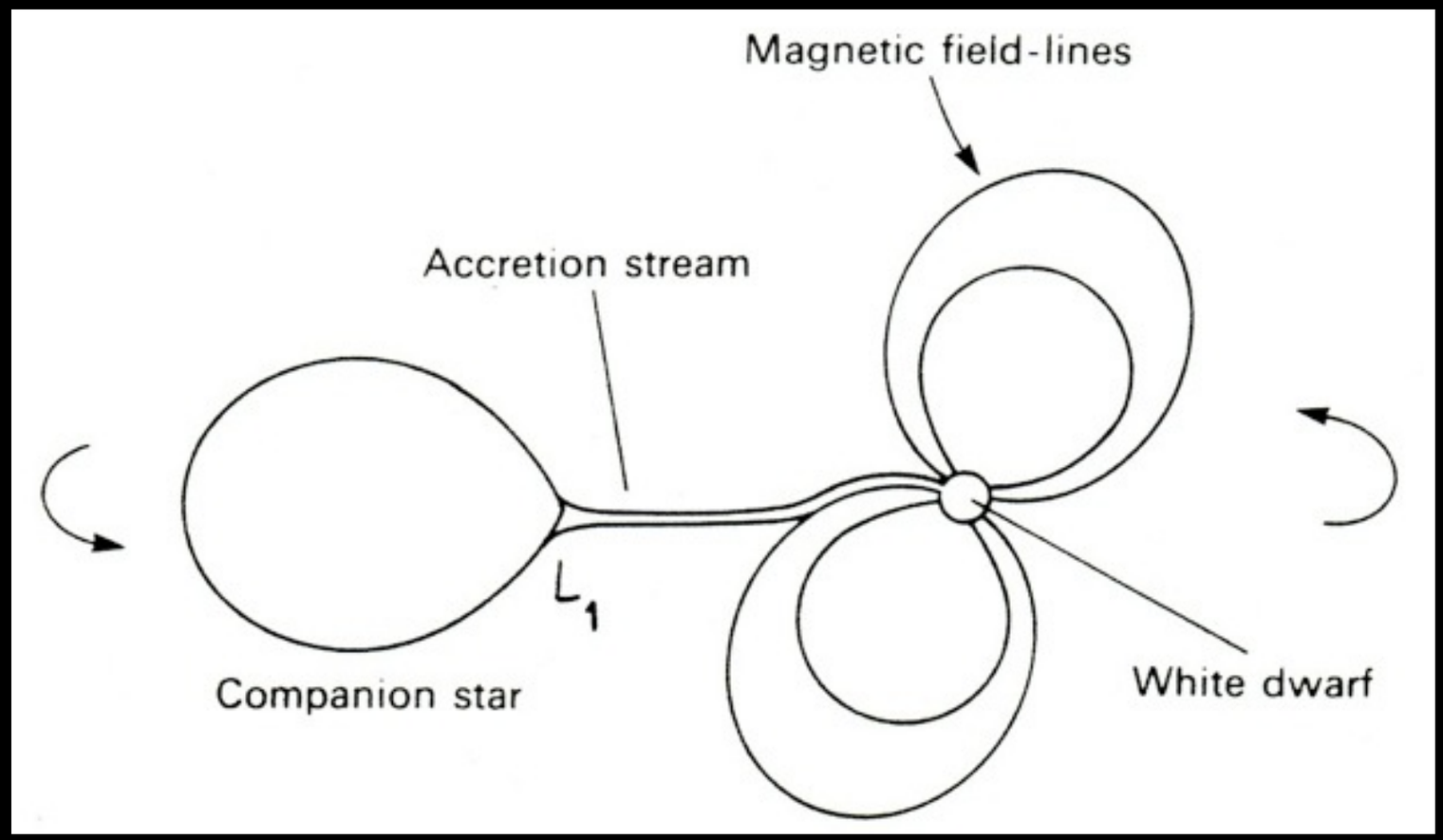
unmagnetized CV

Unmagnetized WD,NS: boundary layer



magnetized CV (polar)

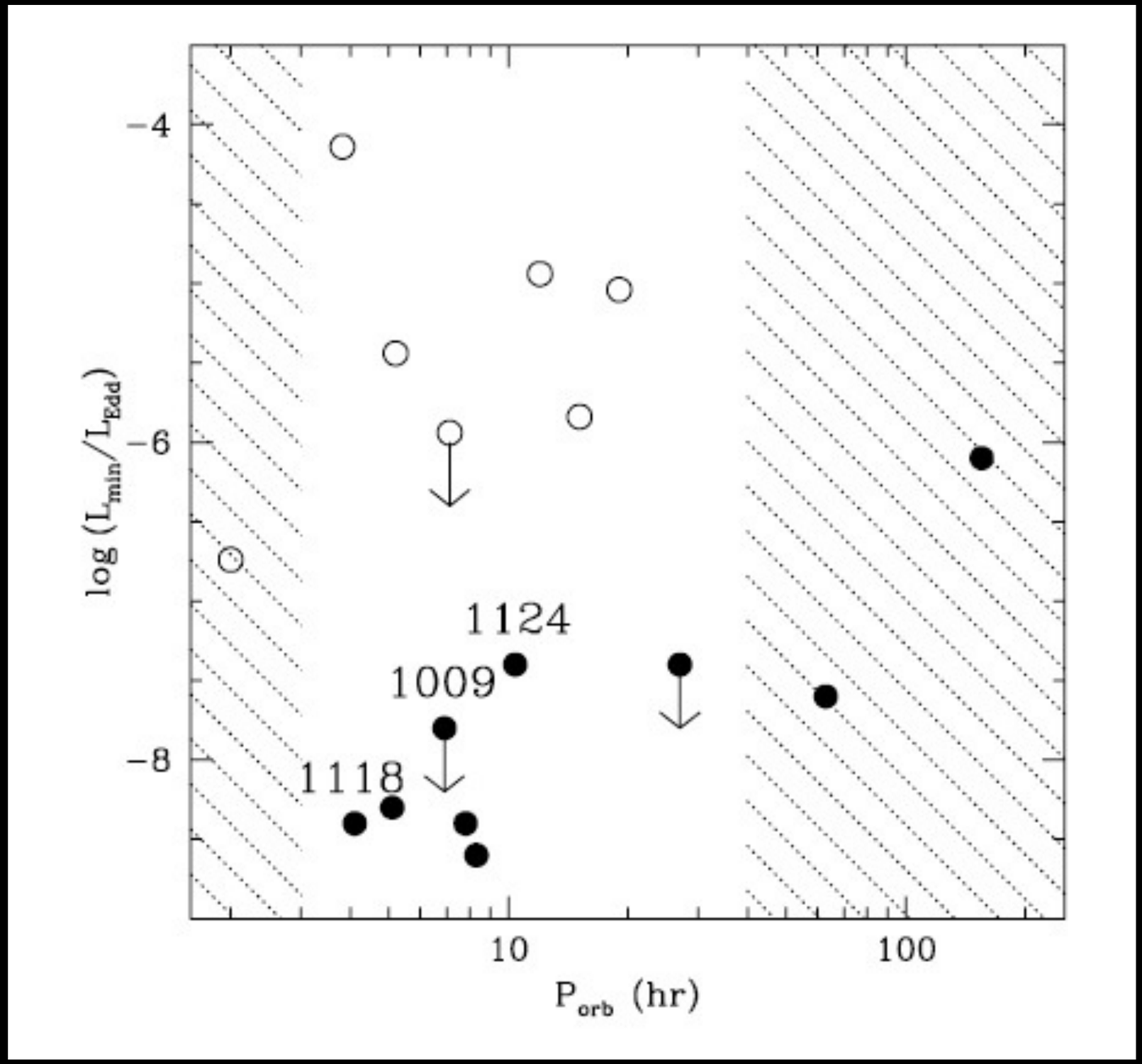
Magnetized WD,NS: truncated disk, polar accretion (accretion column)



Similar geometry for a NS

Luminosity of a quiescent X-ray binary: NS vs BH

BH: no surface (horizon) → no boundary layer → less bright



Neutron stars

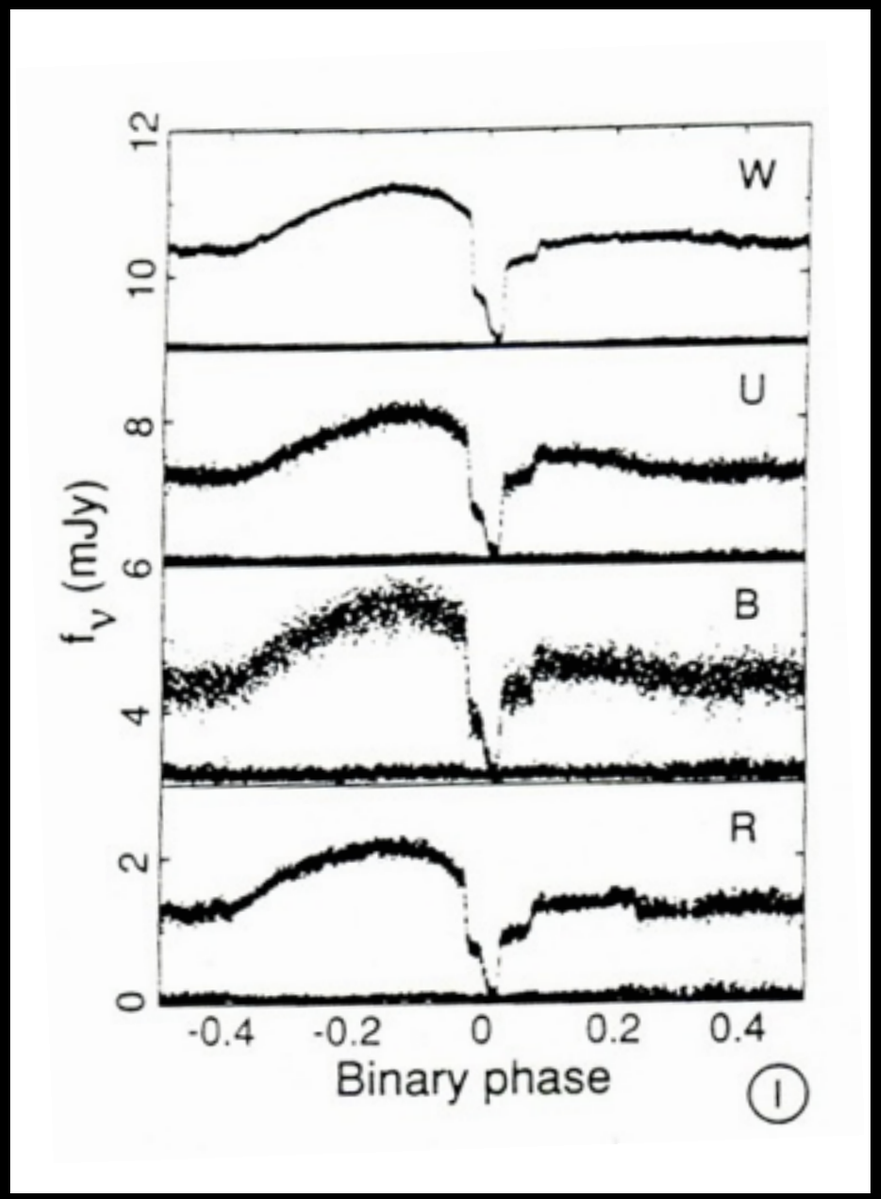
Black hole candidates

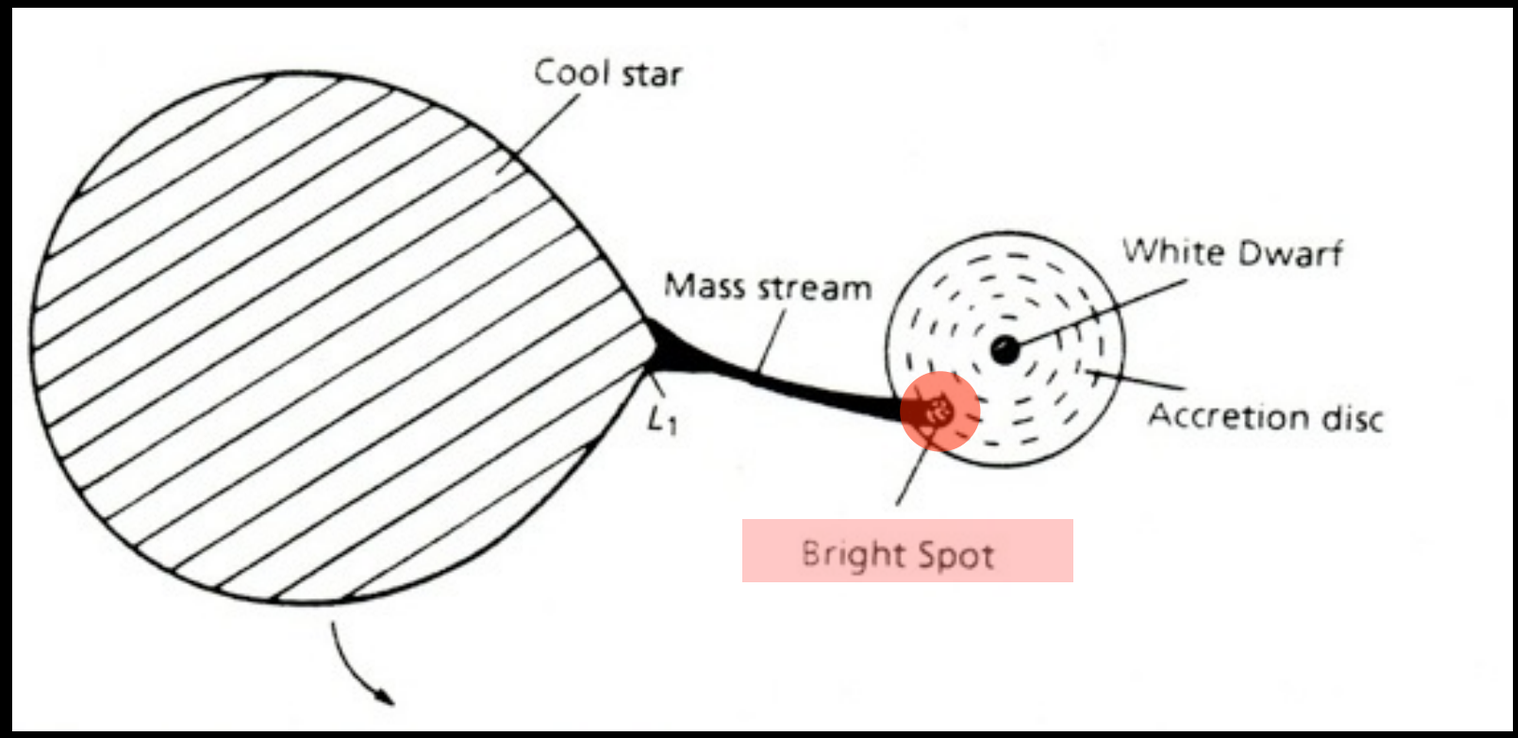
« mass flux »

Bright spot

Accreting WD:
OY Carinae
(Wood et al. 1989)

Eclipsing system:
visible lightcurve (1 orbit)

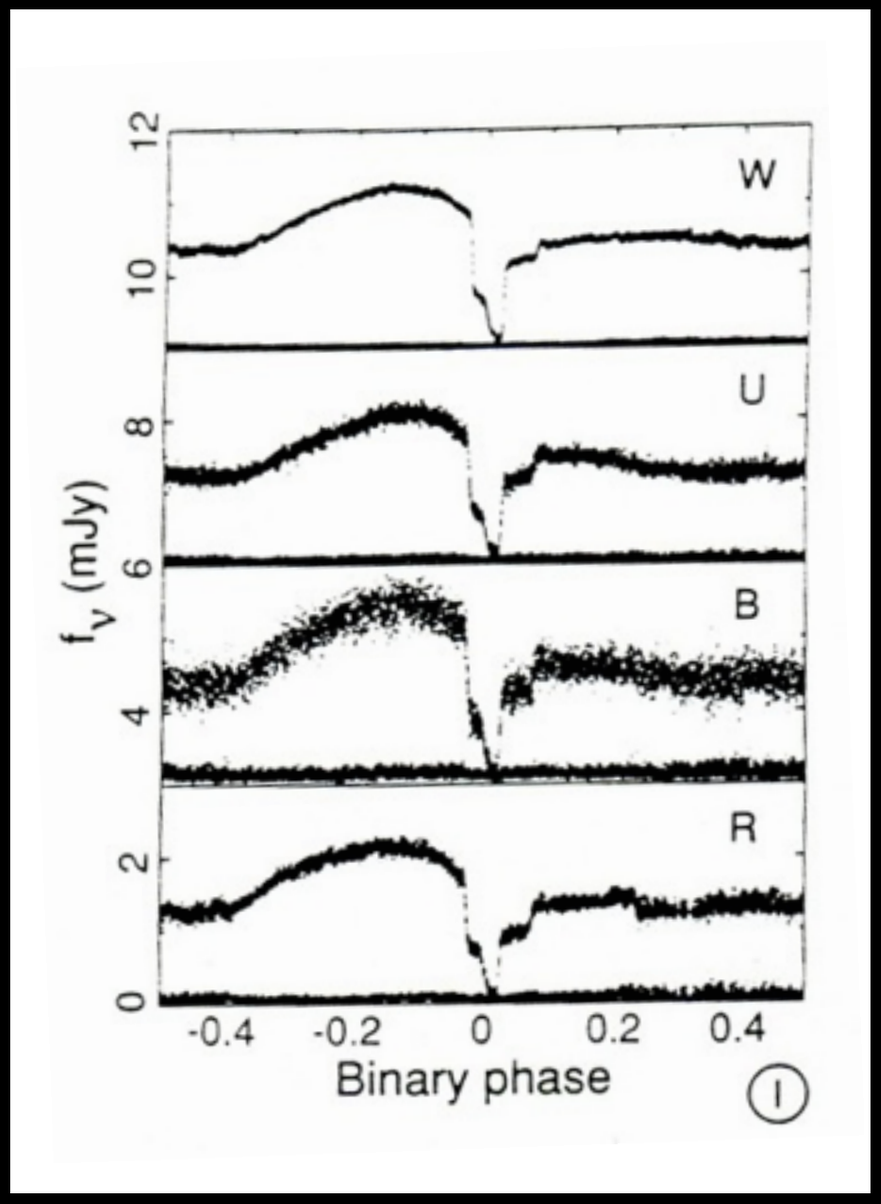




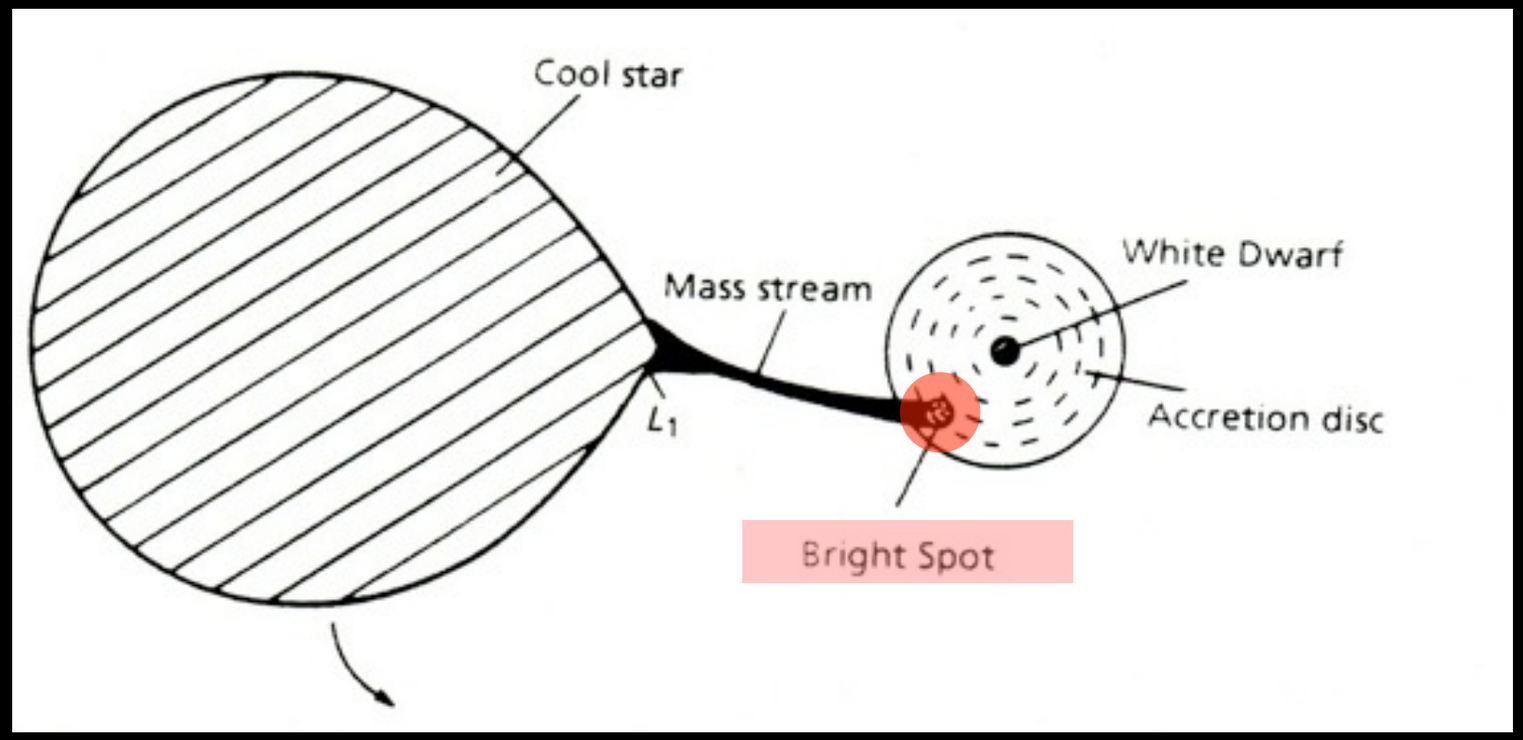
Bright spot

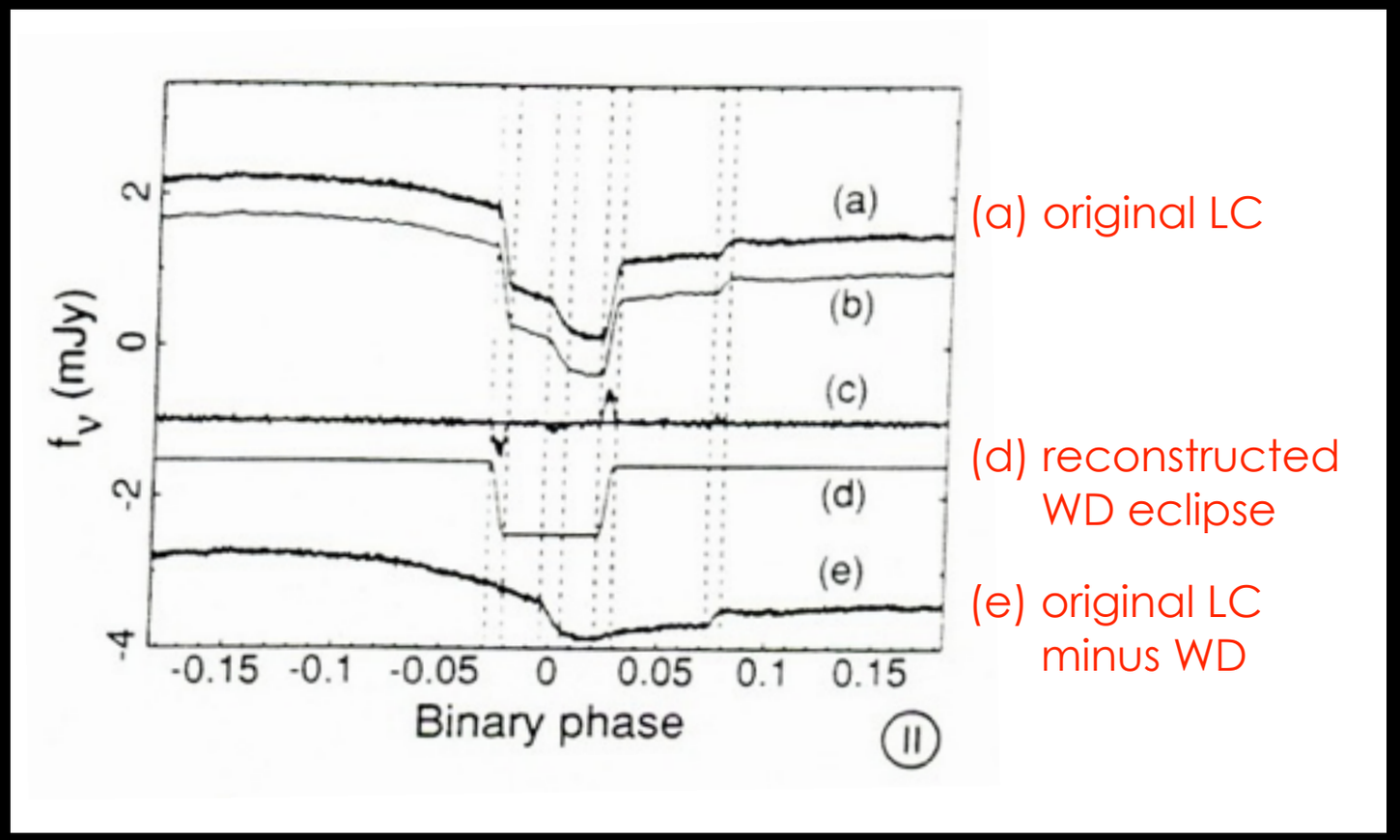
Accreting WD:
OY Carinae
(Wood et al. 1989)

Eclipsing system:
visible lightcurve (1 orbit)



Extract eclipse





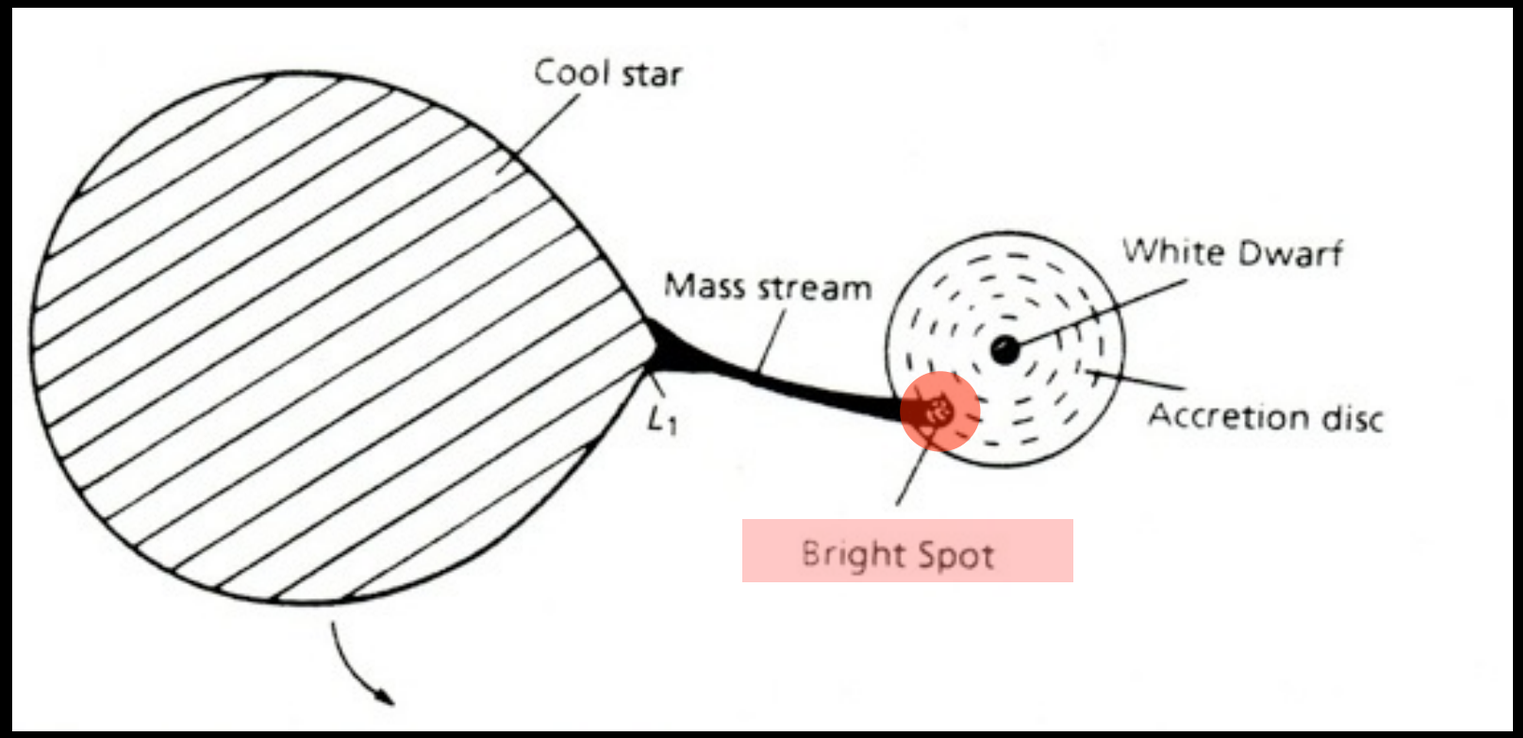
(a) original LC

(d) reconstructed WD eclipse

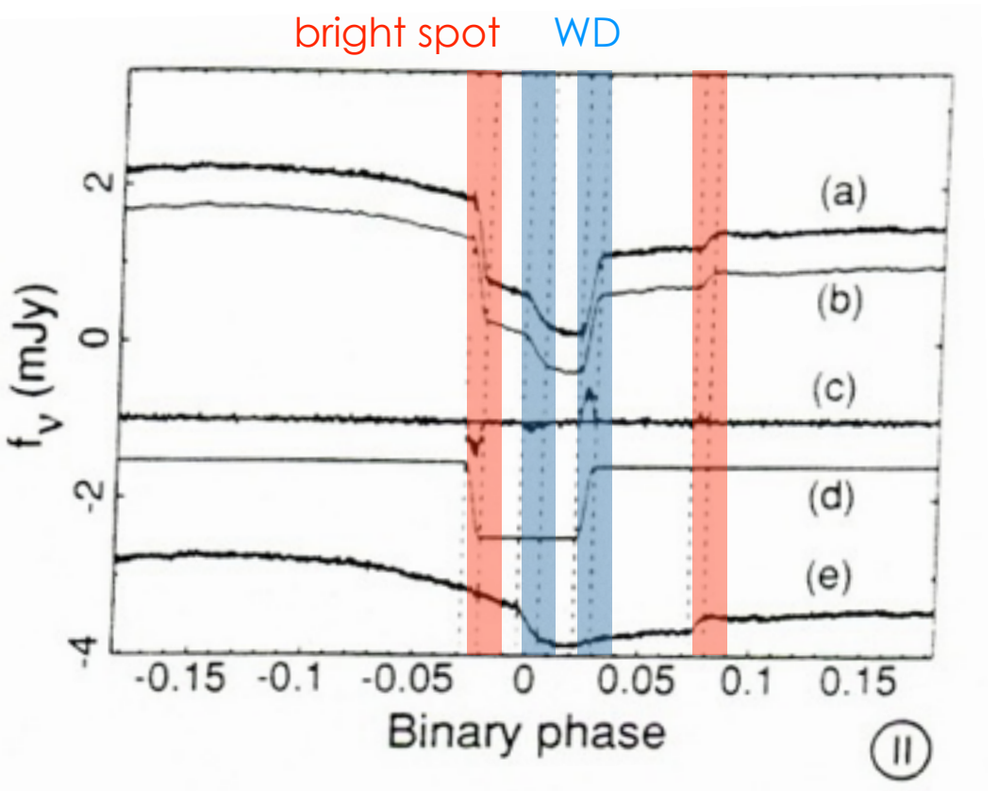
(e) original LC minus WD

Bright spot

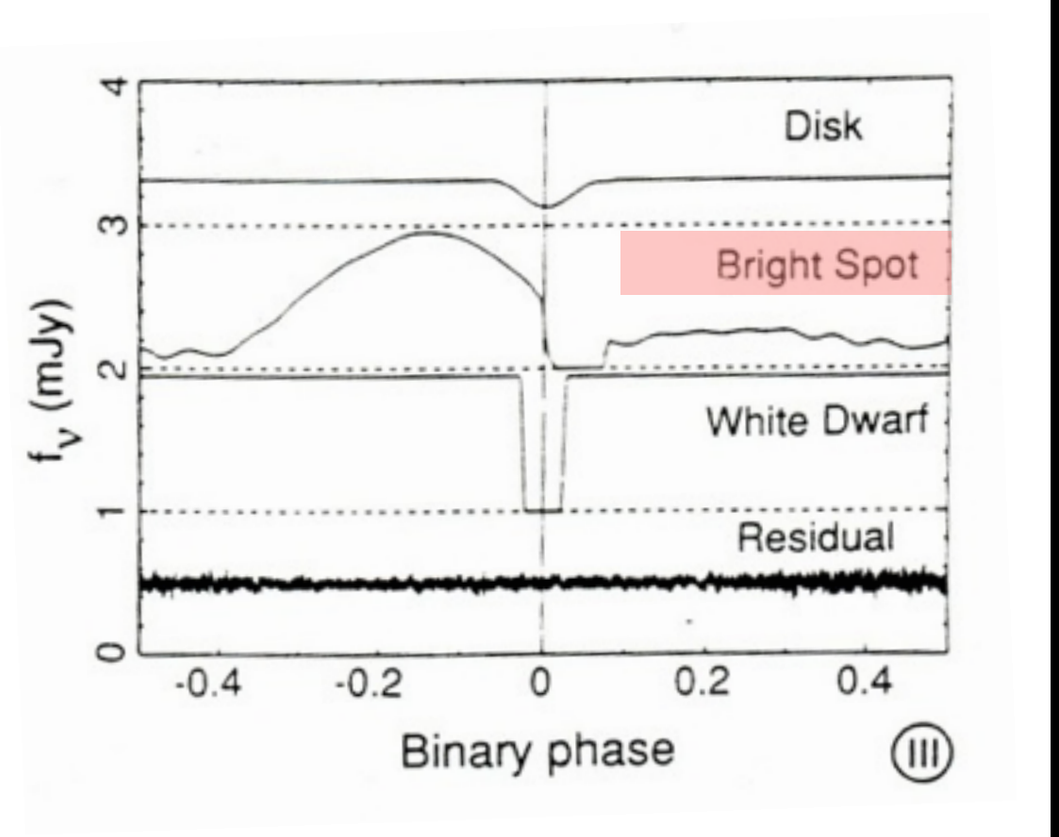
Accreting WD:
OY Carinae
(Wood et al. 1989)



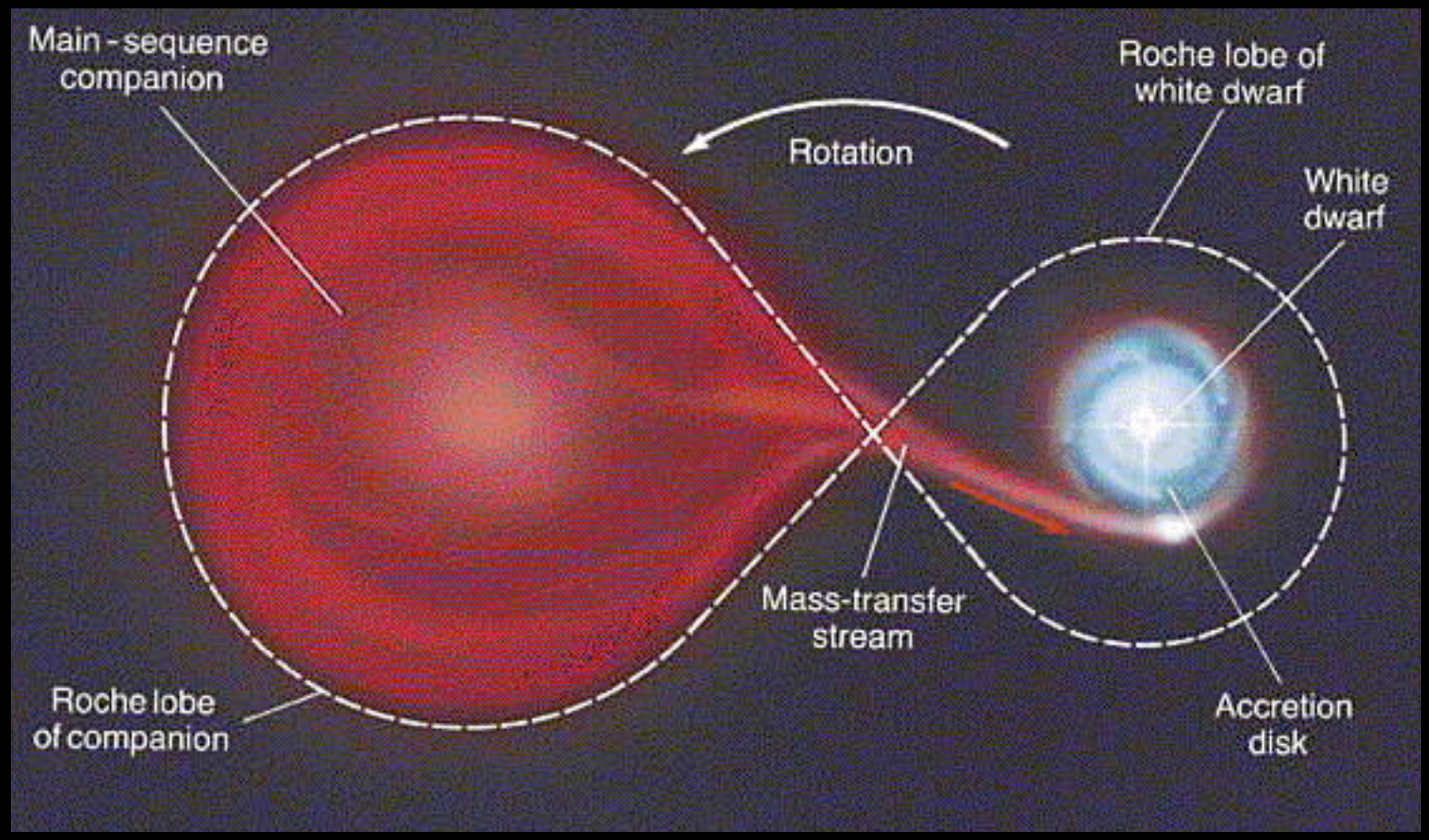
Contact: bright spot Contact: WD



Model of the mean white dwarf LC:



Nova: a thermonuclear explosion on a white dwarf



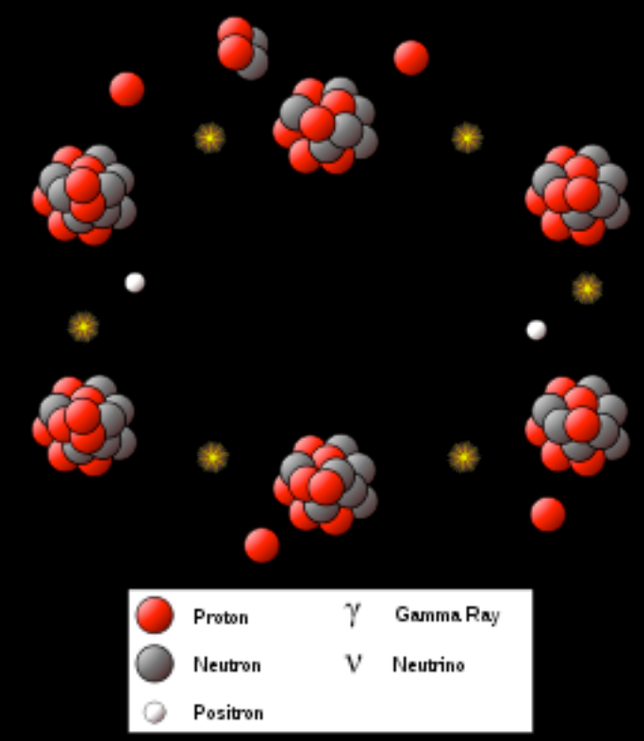
Matter is accumulated on the surface (accretion)

It is compressed and heated (gravity)

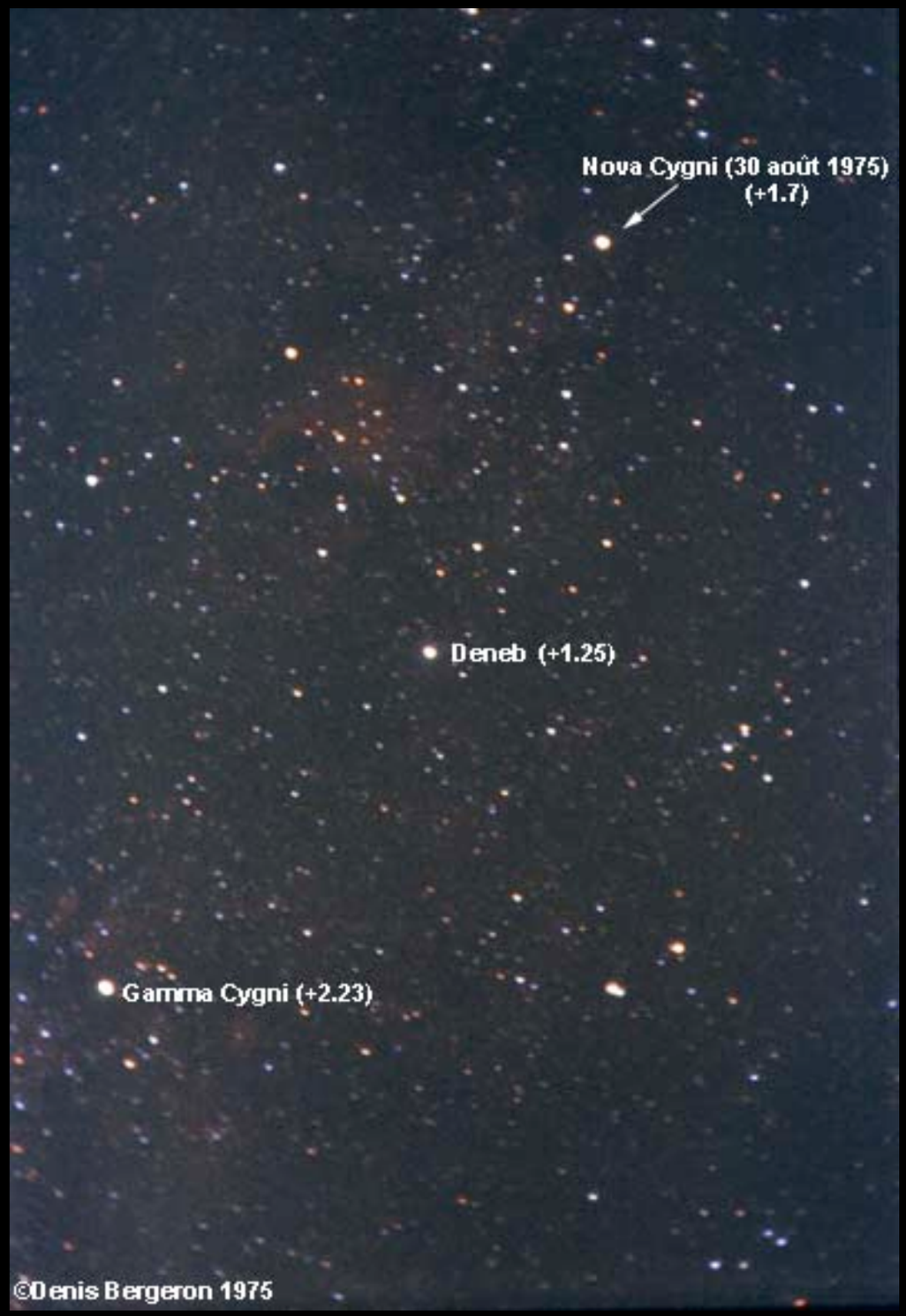
When the temperature threshold is reached:

Hydrogen nuclear fusion (CNO cycle)

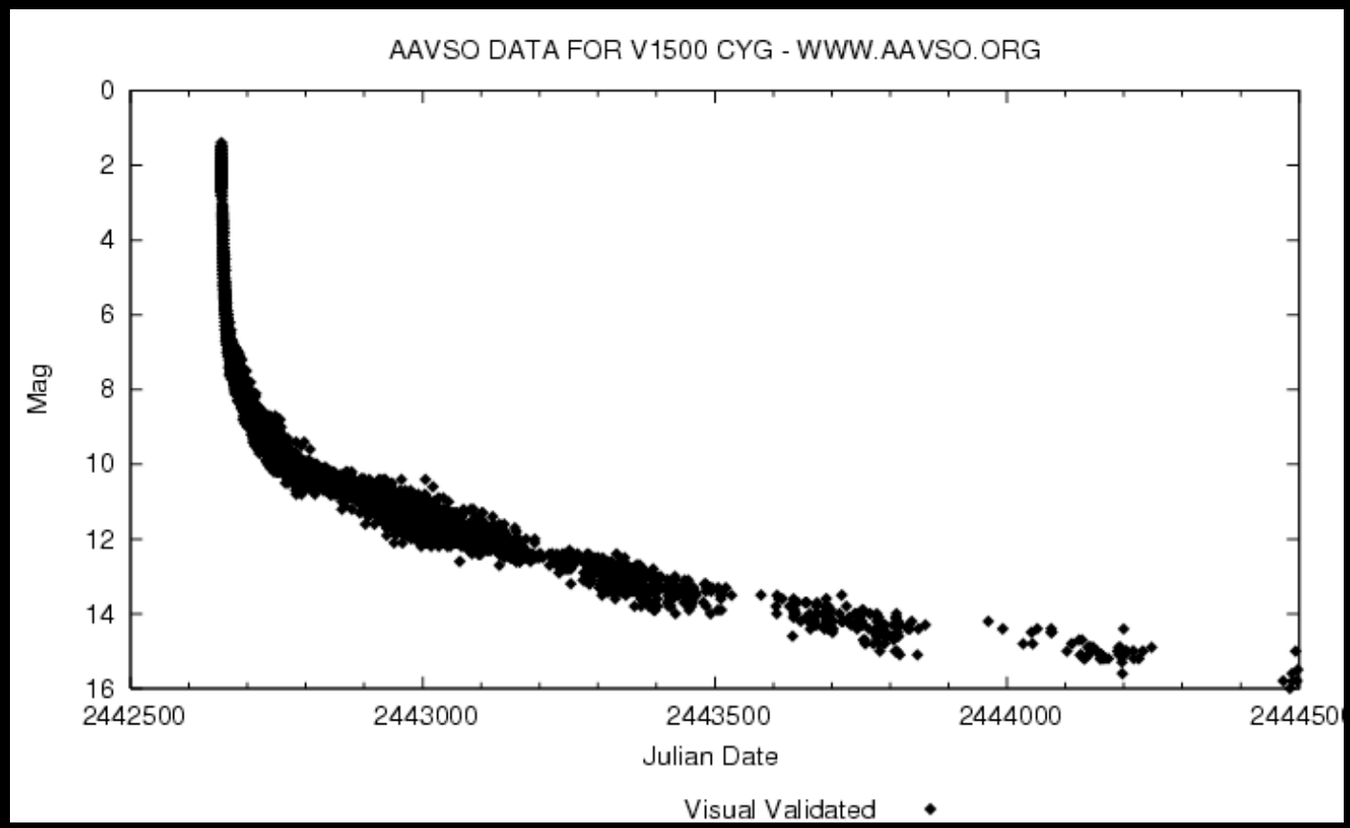
Degenerate matter: no regulation of temperature



Novae



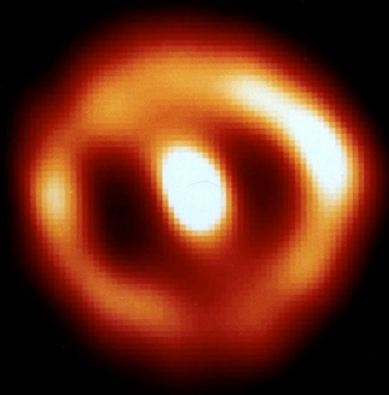
Nova Cyg 1975



Nova Cyg 1975

Amateur astronomer

Novae: matter ejection



HST (27 december 1995) :
Ejection of a shell of matter

v ~ 800-1000 km/s

R ~ 600 UA

D ~ 2 kpc

Type I X-ray burst: thermonuclear explosion on a NS

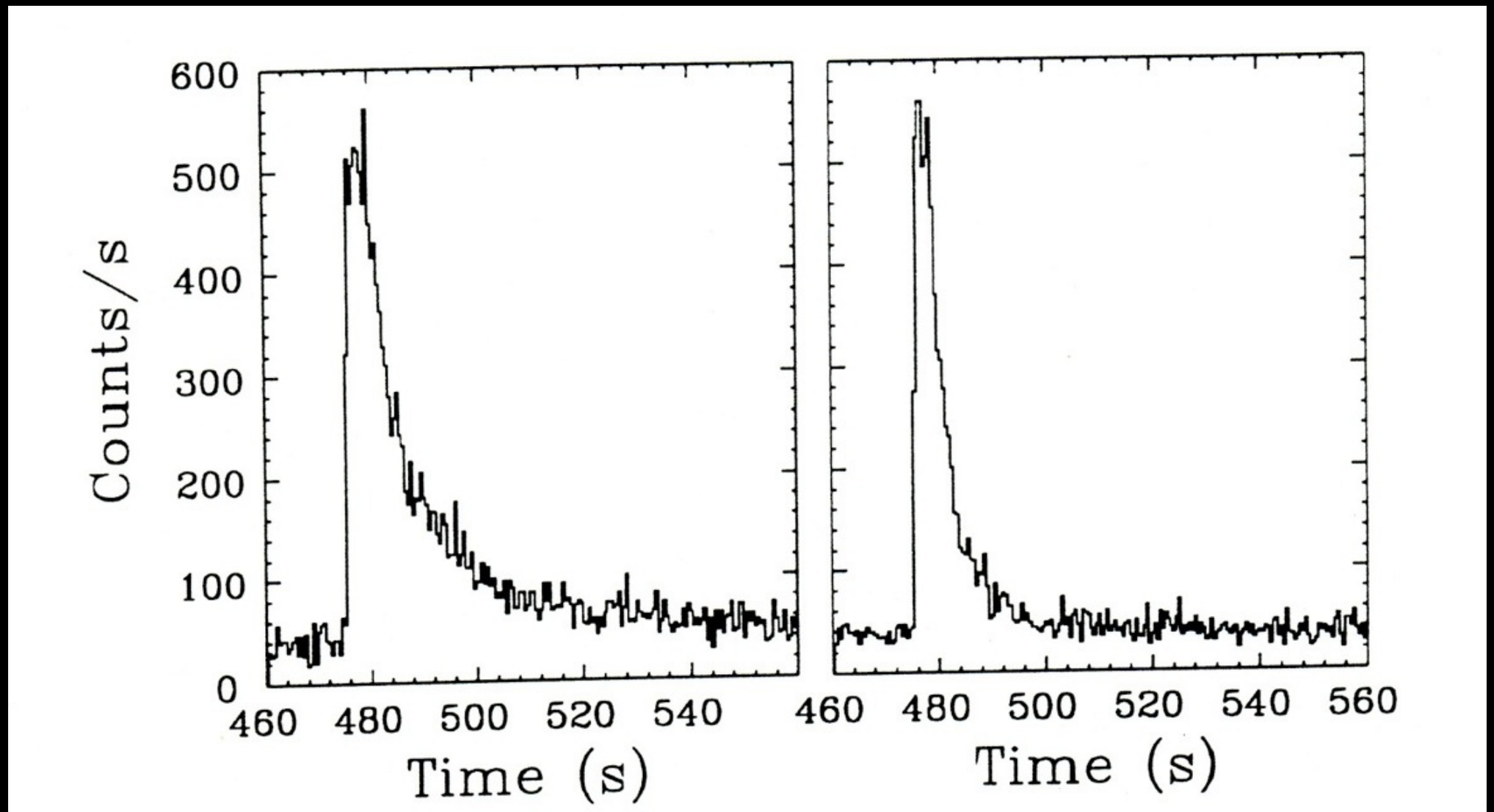


Fig. 7. Type I X-ray burst from 1702-42 as observed with Exosat in the 1.2 - 5.3 keV band (top) and the 5.3 - 19.0 keV band (bottom); the softening of the X-ray burst spectrum is apparent as a longer tail in the low-energy burst profile (courtesy T. Oosterbroek).

Survey of 33 X-ray binaries by RXTE during 8 years (Remillard et al. 2006):

-13 accreting NS: 135 type I X-ray bursts

-18 accreting BH candidates: 0 bursts = no surface but horizon

Accretion by Roche lobe overflow: stability

Evolution of the binary system: (semi-detached system)

Mass of the system:

$$M = M_1 + M_2$$

Conservative system:

$$M = \mathrm{cst}$$

$$\dot{M}_2 = -\dot{M}_1 < 0$$

Angular momentum of the system:

$$\vec{J} = \left(M_1 a_1^2 + M_2 a_2^2\right) \Omega \vec{u}_z = \sqrt{\frac{Ga}{M}} \left(M_1 M_2\right) \vec{u}_z$$

Roche lobe of the companion:

$$\bar{R}_2 \simeq \beta a \left(\frac{M_2}{M}\right)^{1/3}$$

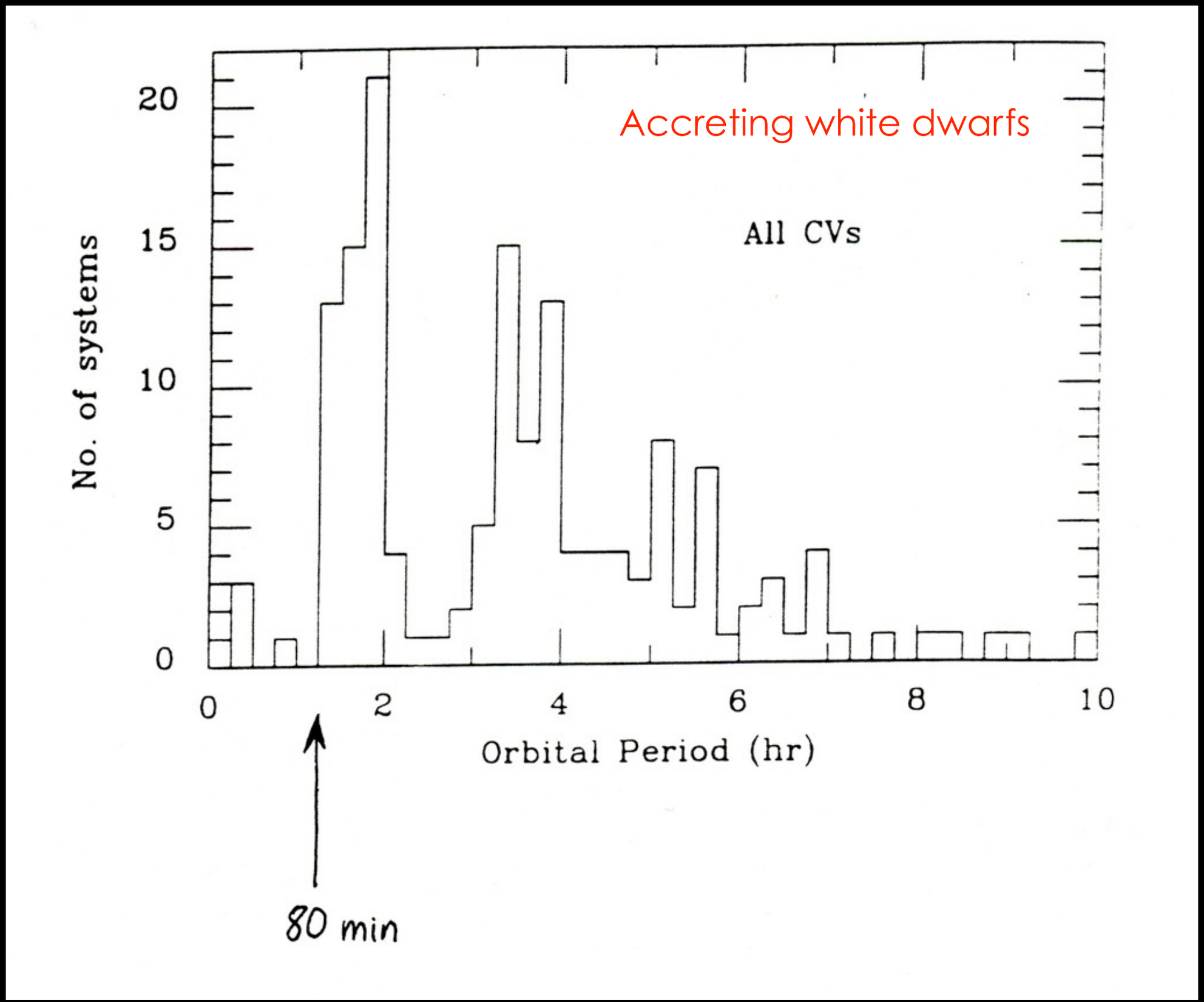
Size of the companion:

$$R_2 \simeq \alpha M_2^x$$

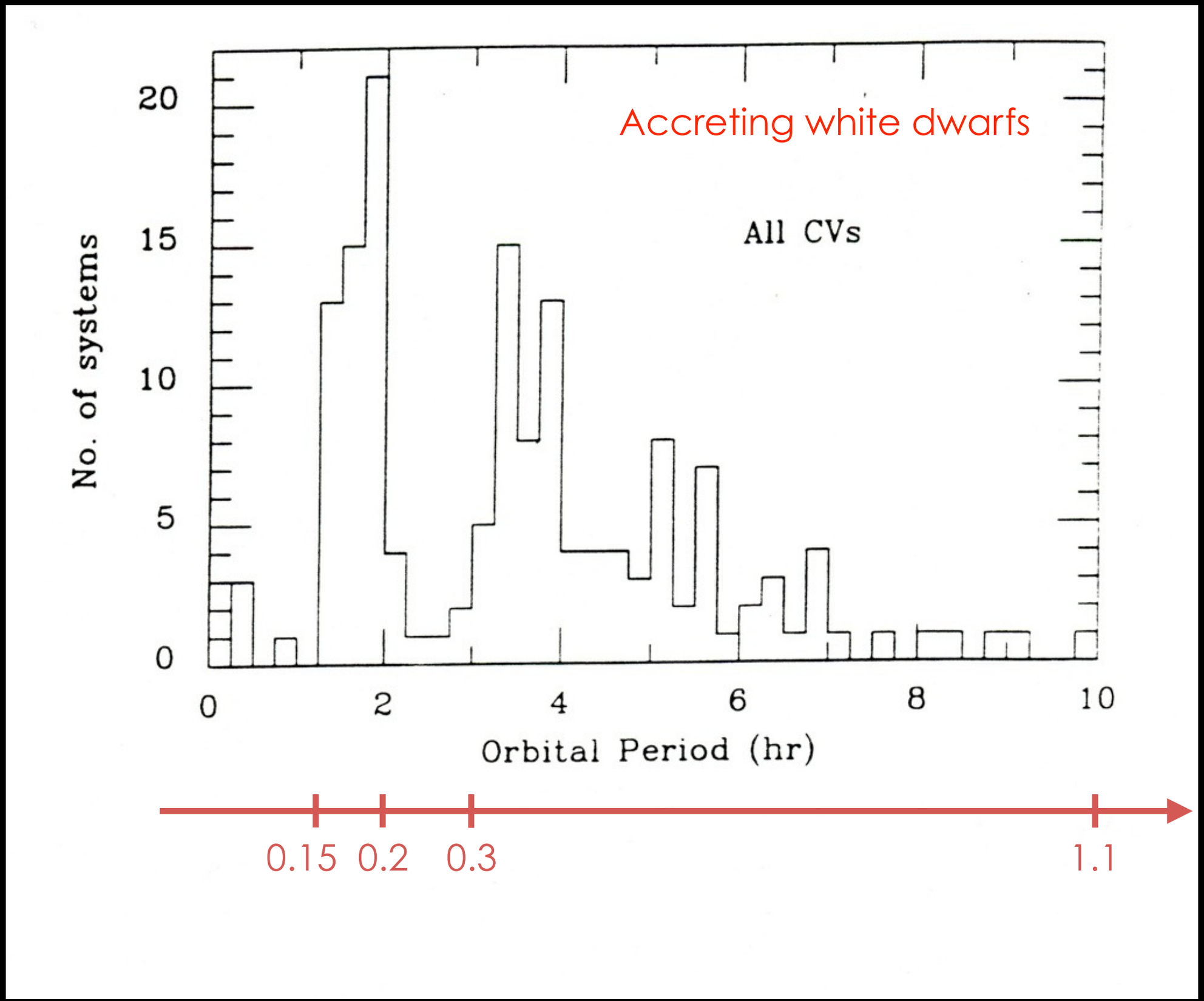
MS : $x \simeq 1$

WD : $x \simeq -1/3$

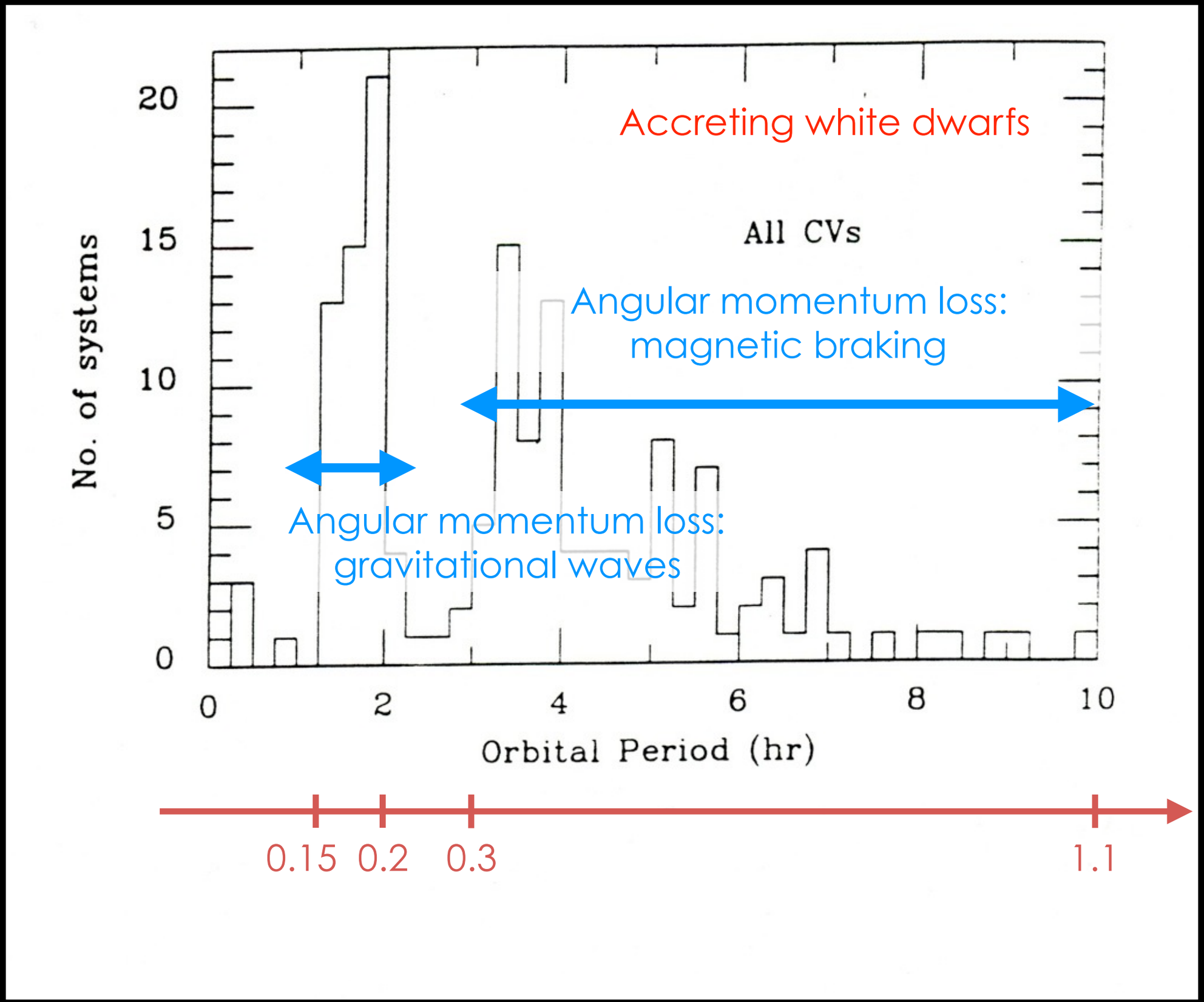
Stability of the mass transfer:
need for a loss of angular momentum



Stability of the mass transfer:
need for a loss of angular momentum



Stability of the mass transfer: need for a loss of angular momentum



Stability of the mass transfer: need for a loss of angular momentum

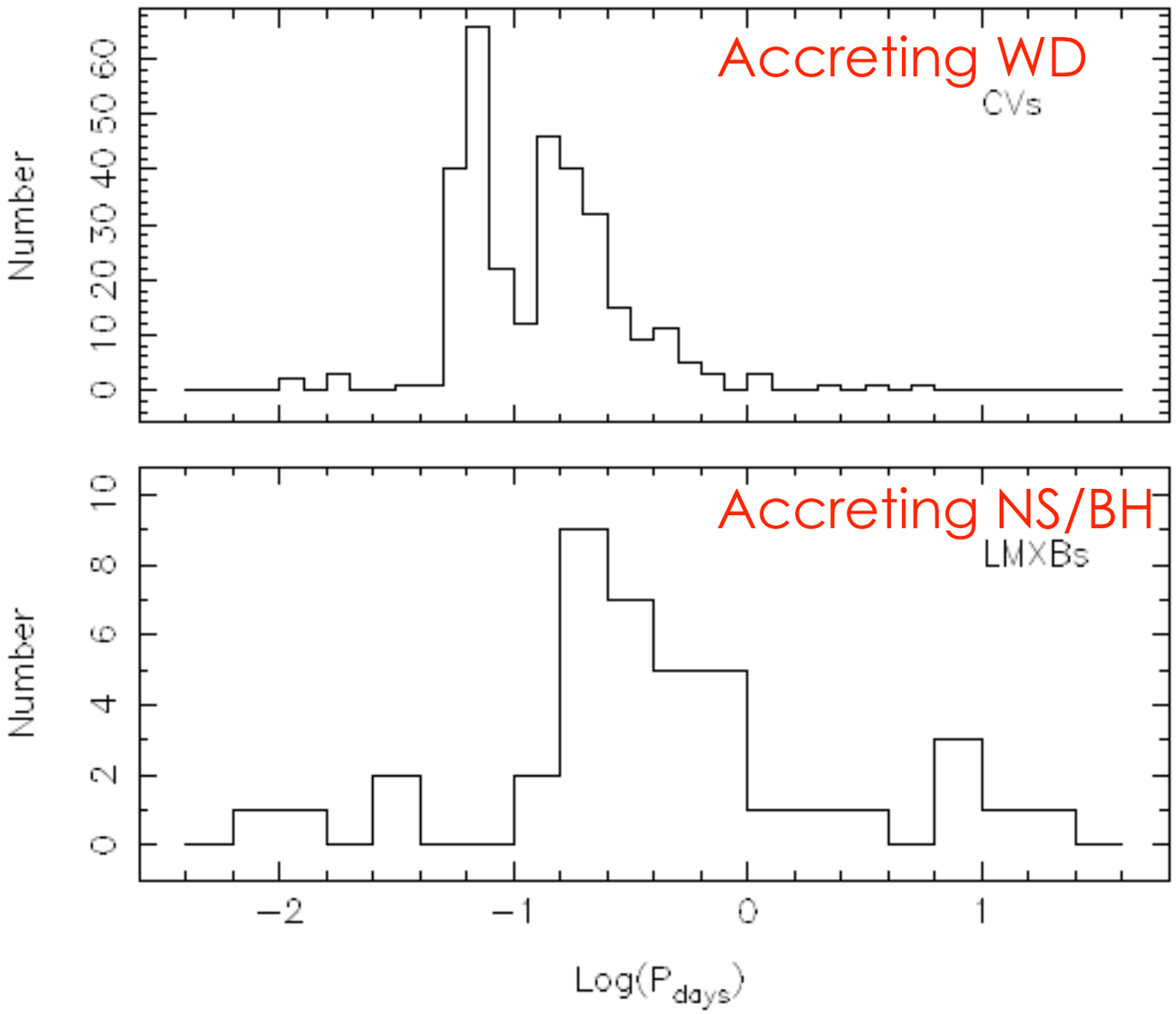


Figure 6. Distributions of orbital periods for low-mass X-ray binaries and cataclysmic variables. Data have been taken from Van Paradijs (1995), Ritter & Kolb (1997), and the references given in Table 1.

(van Paradijs 1998)

Accretion: beyond standard theory

Accretion: beyond standard theory

- **Beyond the alpha prescription:**
  physical mechanism for the transport of the angular momentum (MRI …)

- **Beyond the geometrically thin assumption:**
  thick disks/tori (especially important for super-Eddington systems)

- **Beyond the radiative energy transport/viscous transport of angular momentum:**
  convection, advection (ADAF, …)

- **Beyond the steady state regime:**
  instabilities (e.g. dwarf novae), evolution governed by mass flux, QPOs, etc.

- **Beyond the multicolor spectrum:**
  non-thermal components (corona, shocks)

- **Role of magnetic field**

- **Accretion-ejection connection**

- **etc.**

The case of binary systems composed of two compact objects

Compact object binaries

- **WD+WD:** ~10 systems are known: T = 10 min à 1 h
  Mass transfer from 2 to 1
  (prototype: AM Canum Venaticorum: 0.68+0.13 M⊙, T=17 min !)

  Merger: SN Ia progenitor?
- **WD+NS:** a few cases among LMXB

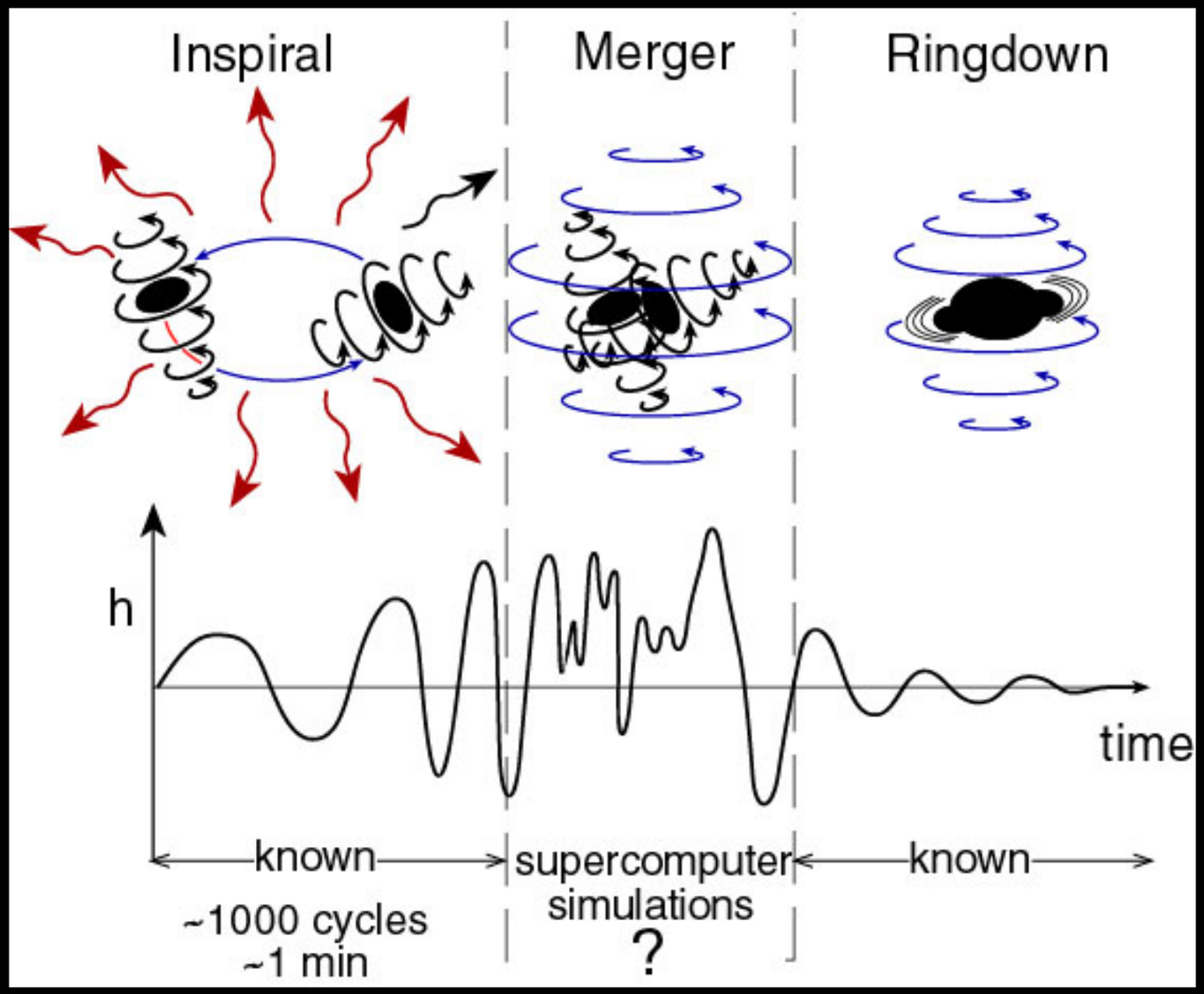
  Merger: NS+disk? + planets?
- **NS+NS:** ~10 systems are known

  Merger: short GRB + kilonova?

Nom	Période du pulsar P (ms)	Période orbitale T (jour)	Excentricité orbitale e	Âge du pulsar t (Myr)	Temps de coalescence τ_c (Myr)	Masses M_1, M_2 ($M_\odot$)
J0737-3039	22.7/2770	0.102	0.088	200/50	79	1.35/1.24
J1906+0746	144.1	0.17	0.085	0.13	320	1.29/1.32
B1913+16	59.0	0.3	0.62	100	320	1.441/1.387
B2127+11C	30.5	0.3	0.68	100	200	1.36/1.35
J1756-2251	28.5	0.32	0.18	400	$1.6\,10^4$	1.34 / 1.23
B1534+12	37.9	0.4	0.27	250	$2.5\,10^3$	1.333 / 1.345
B1829-2456	41.0	1.18	0.14	$1.3\,10^4$	$6.3\,10^4$	?/?
J1518-4904	40.9	8.6	0.25	$2.0\,10^4$	$2.5\,10^6$	< 1.17 / > 1.55
J1811-1736	104.2	18.8	0.83	$1.0\,10^3$	$1.0\,10^7$	$M_1 + M_2 = 2.57$, $M_2 > 0.93$
B1820-11	279.8	357.8	0.79	3.2	$6.3\,10^9$	?/?

- **NS+BH:** no system known, should exist (GW: already 1 candidate in O3?)
- **BH+BH:** first detection = GW150914 (GW only), 10 in 01-O2, many more in O3

Gravitational waves



GW emission: three steps

- inspiral down to the last stable orbit: very well understood
  → allows to measure the properties of the initial binary

- merger: highly uncertain

- ringdown: well understood (for BH)
  → allows to characterise the final post-merger object

Gravitational waves: (circularized) binary system

Quadrupole formula:

$$L_{\mathrm{GW}} = \frac{32 G^4 M^3 \mu^2}{5 a^5 c^5}$$

$$\mu = \frac{M_1 M_2}{M_1 + M_2}$$

$$M = M_1 + M_2$$

Gravitational signal (on-axis):

$$h_{ij}(t) = -h \begin{pmatrix} \cos\left(2\omega\left(t - \frac{D}{c}\right)\right) & \sin\left(2\omega\left(t - \frac{D}{c}\right)\right) & 0 \\ \sin\left(2\omega\left(t - \frac{D}{c}\right)\right) & -\cos\left(2\omega\left(t - \frac{D}{c}\right)\right) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

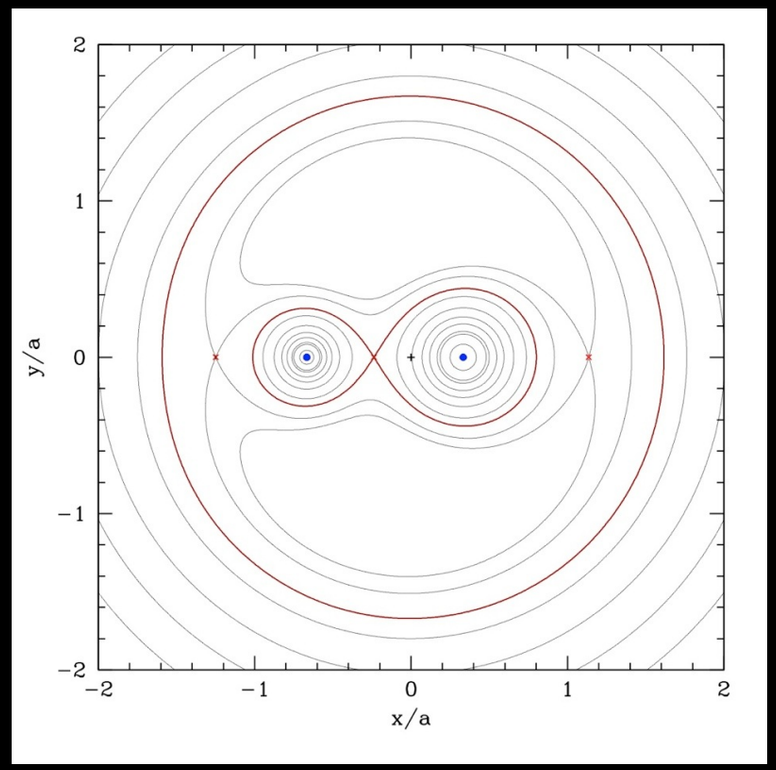
$$h = \frac{4 G \mu a^2 \omega^2}{D c^4}$$

Gravitational waves: (circularized) binary system

Merger time of a circularised binary:

$$\begin{aligned}
\tau_c &= \frac{5}{256}\frac{c^5 a_0^4}{G^3 M^2 \mu}\,, \\
&= \frac{5}{32}\frac{a_0^4}{c\left(2GM/c^2\right)^2\left(2G\mu/c^2\right)} \\
&= 5\times 10^4\,\mathrm{yr}\left(\frac{M}{3\,M_\odot}\right)^{-2}\left(\frac{\mu}{0.75\,M_\odot}\right)^{-1}\left(\frac{a_0}{10^{-3}\,\mathrm{AU}}\right)^4
\end{aligned}$$

Mergers: general discussion



Condition 1: merger time $\tau_c \lesssim 10\,\mathrm{Gyr}$

Condition 2: type of merger?

- can 2 fill its Roche lobe?
  - no: « direct collision »
  - yes: tidal disruption of 2 starts, mass transfer=same formalism as for standard accretion
- if 2 fills its Roche lobe: stable mass transfert?
  - no: merger by unstable mass transfer from 2 to 1 on a dynamical timescale (~orbital period)
  - yes: merger by stable mass transfer, regulated by the loss of angular momentum via GW emission: timescale?

    peculiar case: BH, check where is R_{ISCO}...

Evolution of a binary system with mass transfer: (semi-detached system)

Total mass:

$$M = M_1 + M_2$$

Conservative case:

$$M = \mathrm{cst}$$

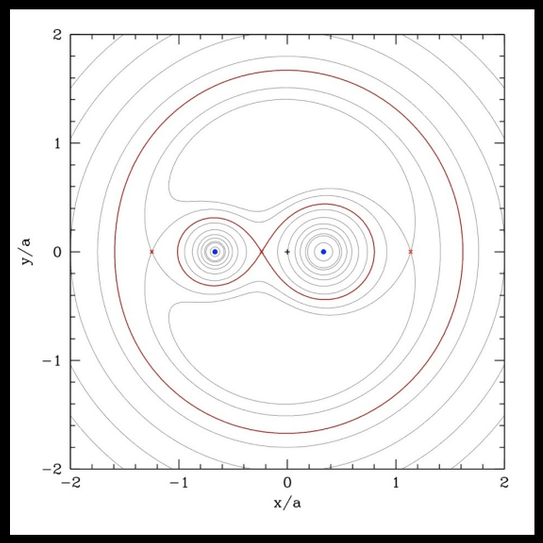
$$\dot{M}_2 = -\dot{M}_1 < 0$$

Total angular momentum:

$$\vec{J} = \left(M_1 a_1^2 + M_2 a_2^2\right) \Omega \vec{u}_z = \sqrt{\frac{Ga}{M}} \left(M_1 M_2\right) \vec{u}_z$$

Roche lobe of 2:

$$\bar{R}_2 \simeq \beta a \left(\frac{M_2}{M}\right)^{1/3}$$



Size of 2:

$$R_2 \simeq \alpha M_2^x$$

Mergers: general discussion

Condition 1: merger time $\tau_c \lesssim 10\,\mathrm{Gyr}$

> **Condition 1 pour la coalescence : temps de coalescence inférieur à l'âge de l'Univers (système circularisé)**
>
> $$a_0 \lesssim 4.5\,\mathrm{R}_\odot \left(\frac{M}{3\,M_\odot}\right)^{1/2} \left(\frac{\mu}{0.75\,M_\odot}\right)^{1/4}$$

Condition 2: type of merger?

Critical separation where 2 fills its Roche lobe (tidal disruption):

$$a_{c,WD} \simeq \frac{\alpha}{\beta}\frac{M^{1/3}}{M_2^{2/3}} \simeq \begin{cases} 2.7\,10^4\,\mathrm{km}\left(\frac{M}{1.4\,\mathrm{M}_\odot}\right)^{1/3}\left(\frac{M_2}{0.7\,\mathrm{M}_\odot}\right)^{-2/3} & (\mathrm{WD}+\mathrm{WD}) \\ 3.1\,10^4\,\mathrm{km}\left(\frac{M}{2.1\,\mathrm{M}_\odot}\right)^{1/3}\left(\frac{M_2}{0.7\,\mathrm{M}_\odot}\right)^{-2/3} & (\mathrm{NS}+\mathrm{WD}) \\ 5.4\,10^4\,\mathrm{km}\left(\frac{M}{10.7\,\mathrm{M}_\odot}\right)^{1/3}\left(\frac{M_2}{0.7\,\mathrm{M}_\odot}\right)^{-2/3} & (\mathrm{BH}+\mathrm{WD}) \\ 2.4\,10^6\,\mathrm{km}\left(\frac{M}{10^6\,\mathrm{M}_\odot}\right)^{1/3}\left(\frac{M_2}{0.7\,\mathrm{M}_\odot}\right)^{-2/3} & (\mathrm{SMBH}+\mathrm{WD}) \end{cases}.$$

$$a_{c,NS} \simeq \frac{R_2}{\beta}\left(\frac{M}{M_2}\right)^{1/3} \simeq \begin{cases} 27\,\mathrm{km}\left(\frac{M}{2.8\,\mathrm{M}_\odot}\right)^{1/3}\left(\frac{M_2}{1.4\,\mathrm{M}_\odot}\right)^{-1/3} & (\mathrm{NS}+\mathrm{NS}) \\ 44\,\mathrm{km}\left(\frac{M}{11.4\,\mathrm{M}_\odot}\right)^{1/3}\left(\frac{M_2}{1.4\,\mathrm{M}_\odot}\right)^{-1/3} & (\mathrm{BH}+\mathrm{NS}) \\ 1900\,\mathrm{km}\left(\frac{M}{10^6\,\mathrm{M}_\odot}\right)^{1/3}\left(\frac{M_2}{1.4\,\mathrm{M}_\odot}\right)^{-1/3} & (\mathrm{SMBH}+\mathrm{NS}) \end{cases}.$$

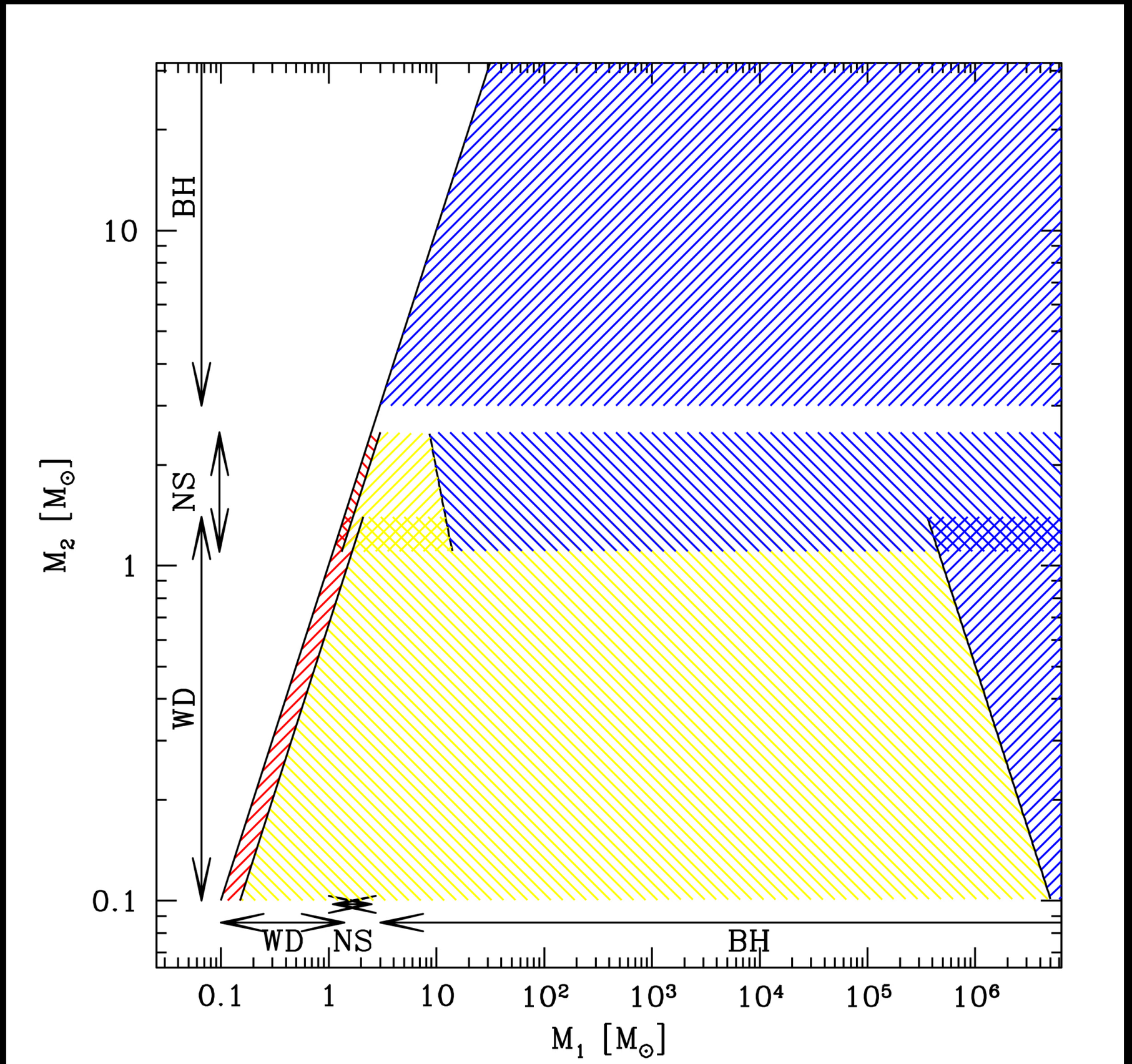
« Direct collision/plunge »:

$$a_c < R_1 + R_2$$

Unstable mass transfer:

$$\frac{M_2}{M_1} > \frac{5}{6} + \frac{x}{2}$$

Type of merger?

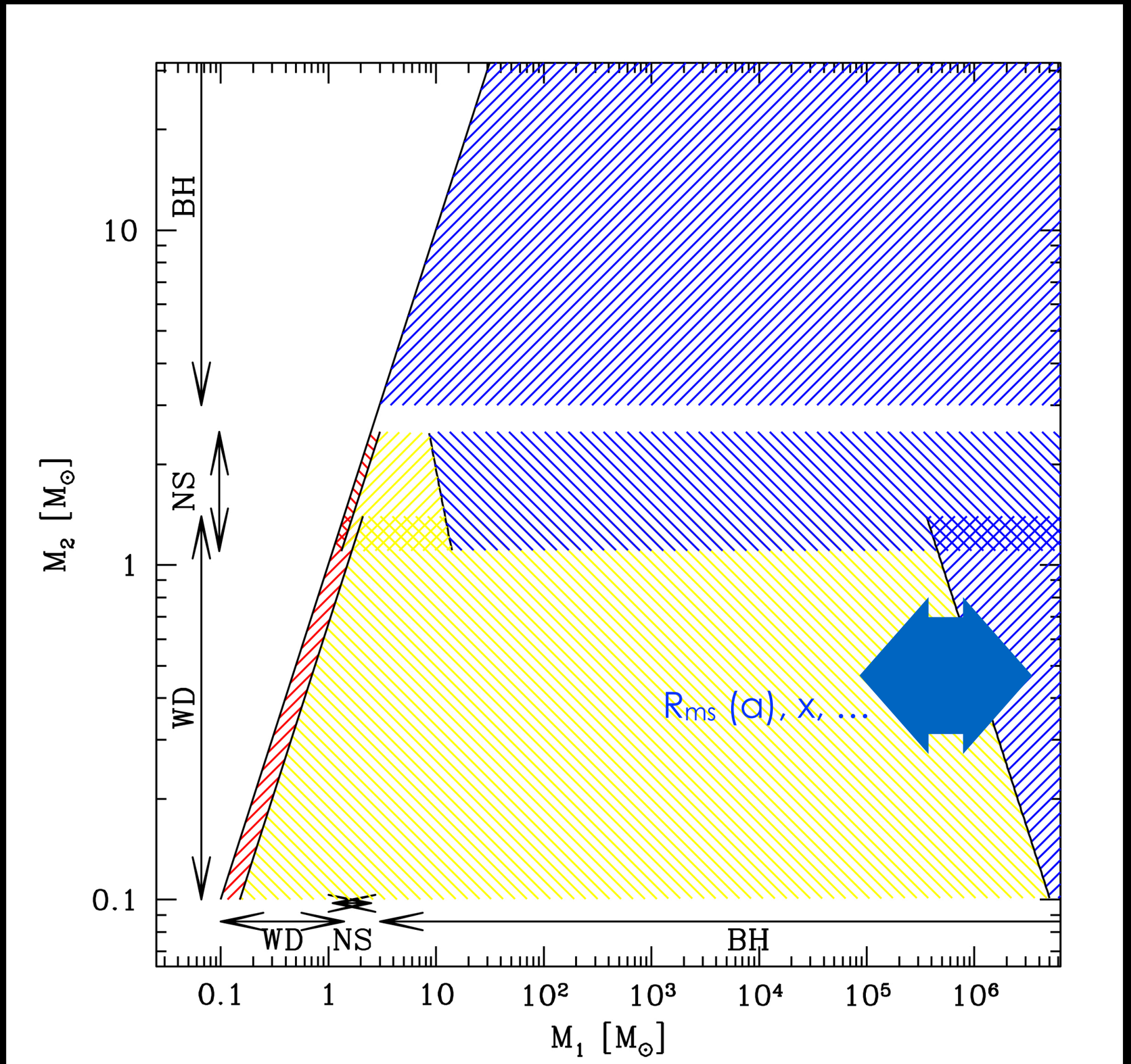


Direct collision/plunge

Stable mass transfer

Unstable mass transfer

Type of merger?



- Direct plunge
- Stable mass transfer
- Unstable mass transfer

Evolution of a binary system with stable mass transfer: (semi-detached system)

$$\frac{(-\dot{M}_2)}{M_2} \simeq \frac{1}{\frac{5}{6}+\frac{x}{2}-q}\left.\frac{(-\dot{J})}{J}\right|_{\rm GW}$$

$$\frac{\dot{a}}{a} = \frac{2}{3}\frac{\dot{T}}{T} \simeq \frac{\frac{1}{3}-x}{\frac{5}{6}+\frac{x}{2}-q}\left.\frac{(-\dot{J})}{J}\right|_{\rm GW}.$$

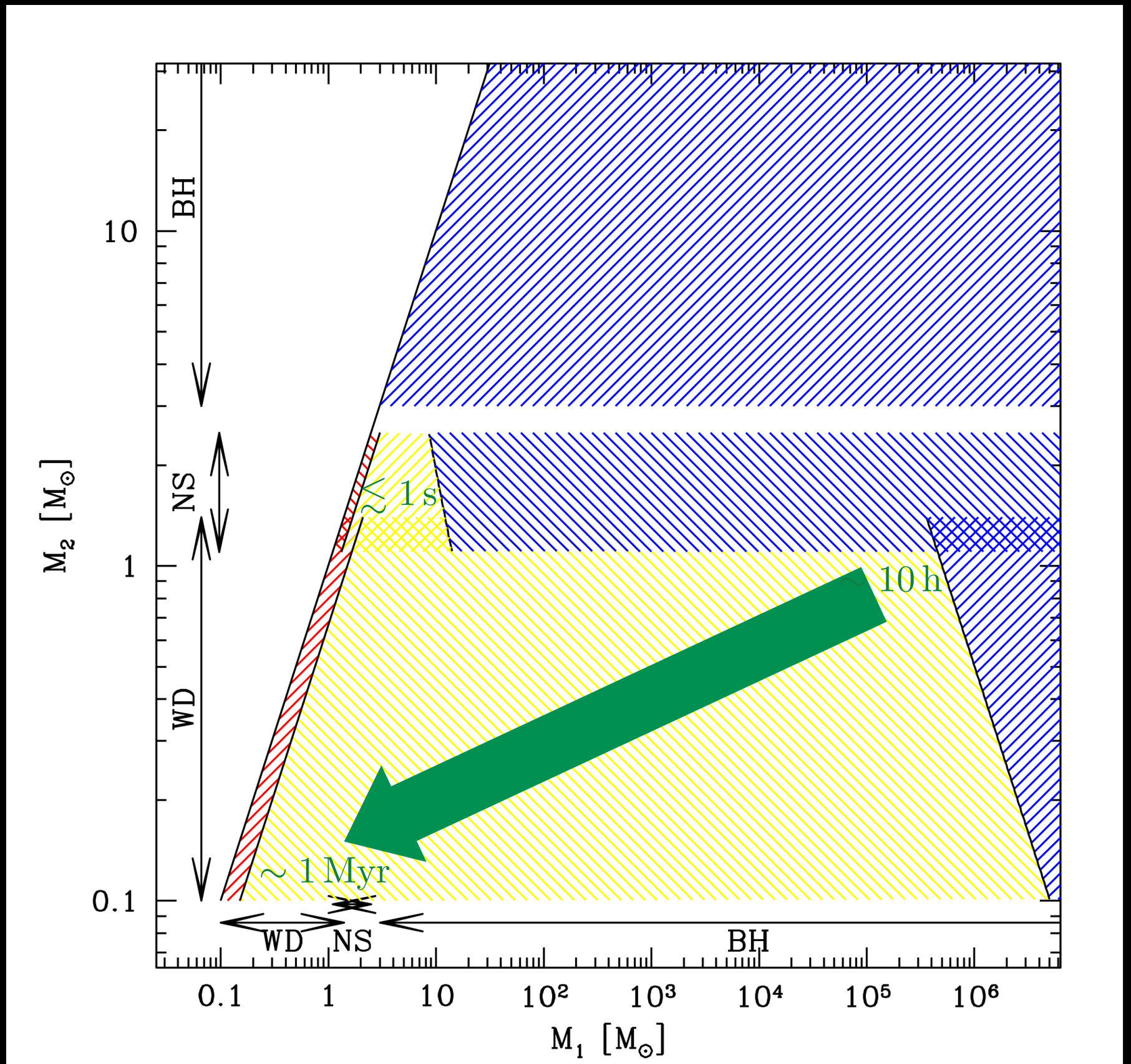
q = M_2/M_1

Loss of angular momentum (GW emission):

$$\left.\frac{(-\dot{J})}{J}\right|_{\rm GW} = \frac{32}{5}\frac{G^3}{c^5}\frac{M_1 M_2 (M_1+M_2)}{a^4}.$$

The loss of angular momentum by GW emission regulates the mass tranfer: the Roche lobe is adjusted to have always the size of 2.

Type of merger?

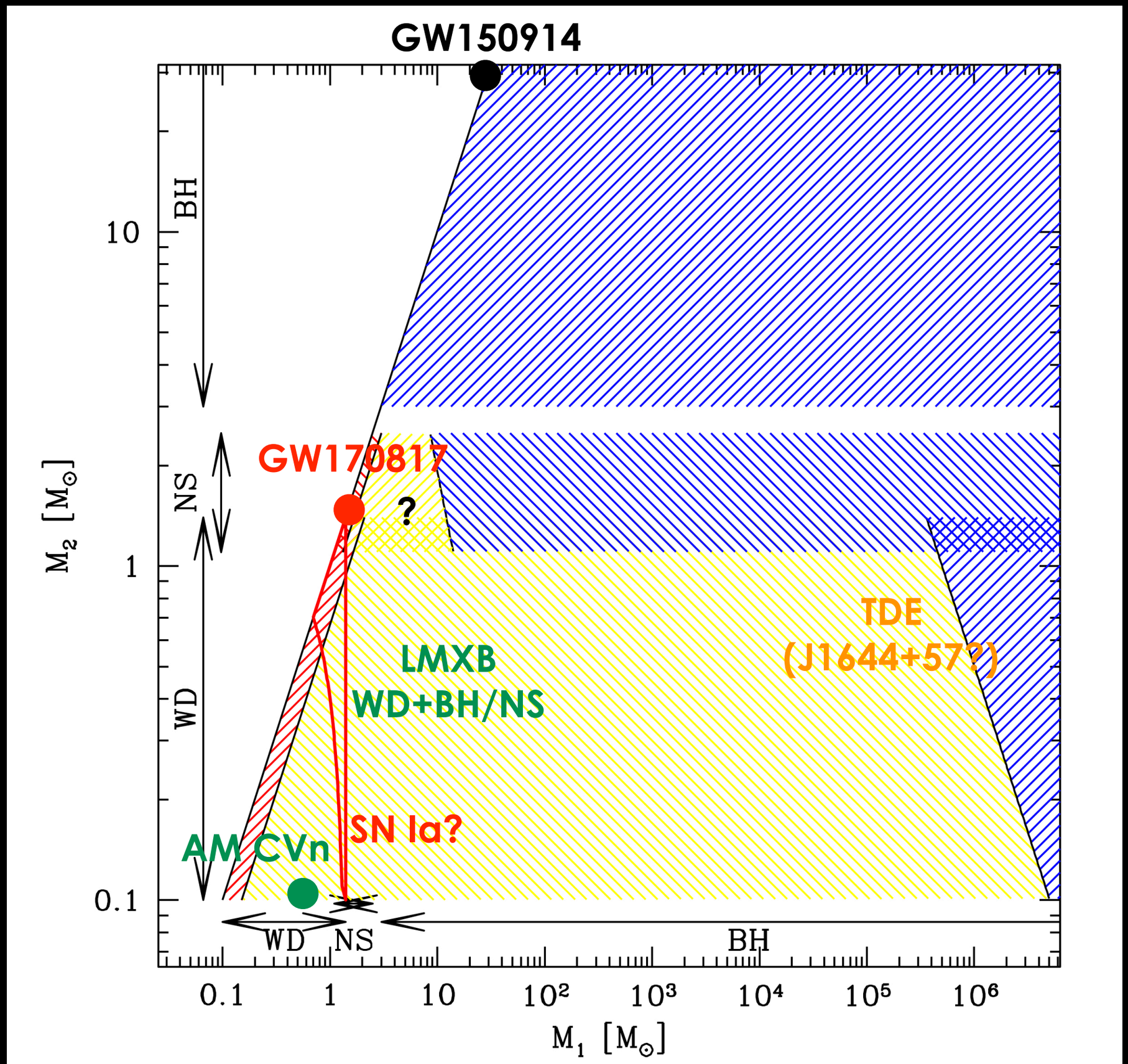


Direct plunge

Stable mass transfer

Unstable mass transfer

Type of merger?



- Direct plunge
- Stable mass transfer
- Unstable mass transfer

Conclusion

Conclusion

- Accretion is the energy extraction mechanism at work in the brightest long-lived sources in the Universe
  (transients: more complicated but accretion usually plays a role)
- It is especially important for compact objects (WD,NS,BH): efficiency (~G M/R c^2) exceeds the efficiency of nuclear fusion in stars (MS: 0.7%/10)

- Steady-state radial accretion (Bondi): very inefficient

- Accretion in a binary system via L1: efficient
  - detached system: wind
  - semi-detached: Roche lobe overflow
- Mass transfer is regulated by the loss of angular momentum

- A standard model is well established (geom. thin/opt. thick accretion disk)
- Some differences among accreting compact objects:
  - surface (WD,NS) or horizon (BH) - surface: boundary layer, thermonuclear explosions, …
  - weak/strong magnetic field - strong B: truncated disk

- Beyond the standard model: many open questions
  - origin of the transport of angular momentum, role of B, …
  - accretion-ejection connection?