COSMIC RAYS
and
PARTICLE ACCELERATION

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COSMIC RAYS:
A BRIEF INTRODUCTION

What are Cosmic Rays?

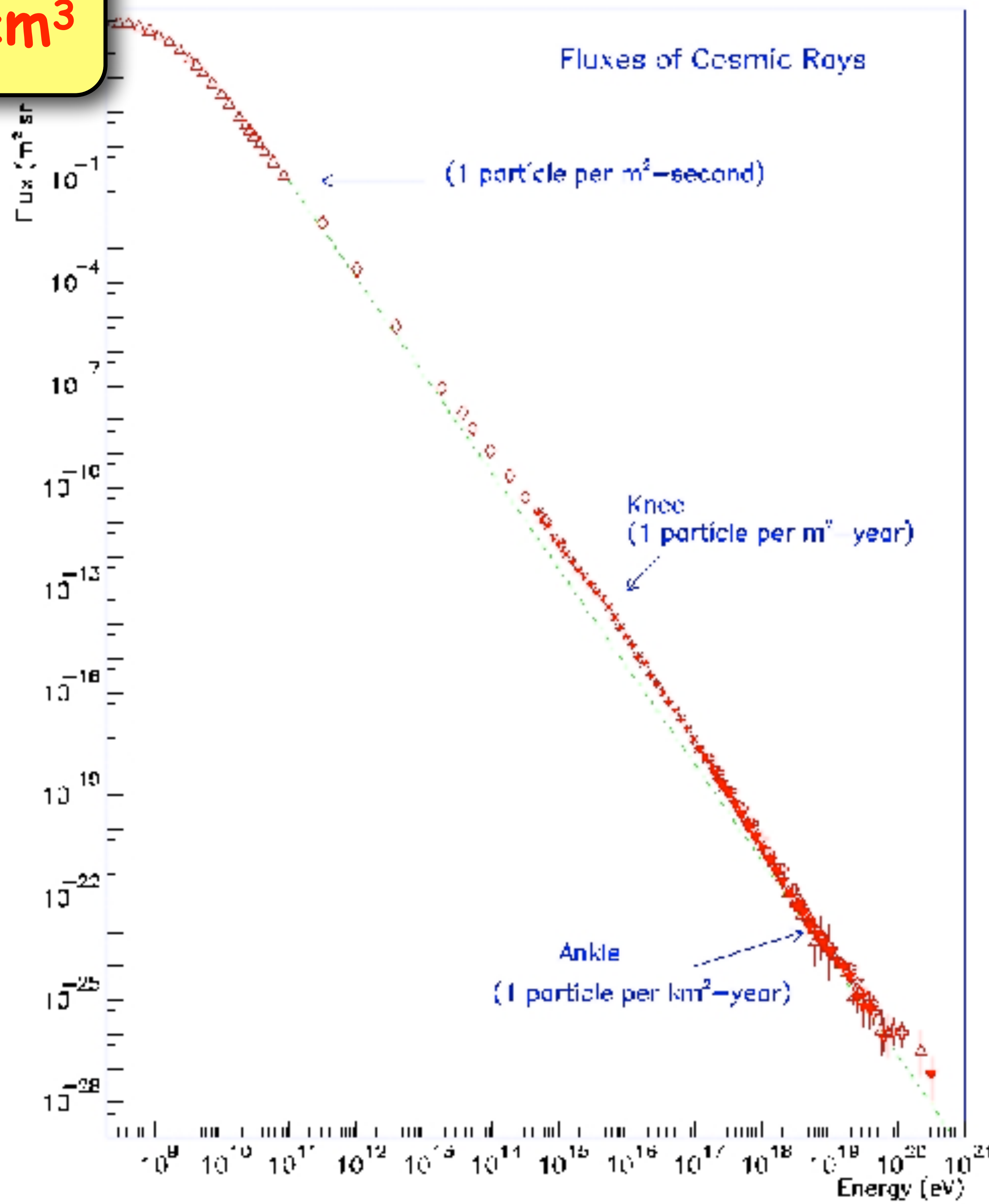
Cosmic rays particles hit the Earth's atmosphere at the rate of about **1000 per square meter per second**. They are ionized nuclei - about **90% protons**, 9% alpha particles and the rest heavy nuclei - and they are distinguished by their high energies. Most cosmic rays are **relativistic**, having energies comparable or somewhat greater than their masses. A very few of them have ultrarelativistic energies extending up to 10^{20} eV (about 20 Joules), eleven order of magnitudes greater than the equivalent rest mass energy of a proton. The fundamental question of cosmic ray physics is, **"Where do they come from?"** and in particular, **"How are they accelerated to such high energies?"**.

T. Gaisser "Cosmic Rays and Particle Physics"

Also **electrons** are present in the cosmic radiation -> **~ 1%**

The (local) Cosmic Ray spectrum

$w_{CR} \sim 1$ eV/cm^3



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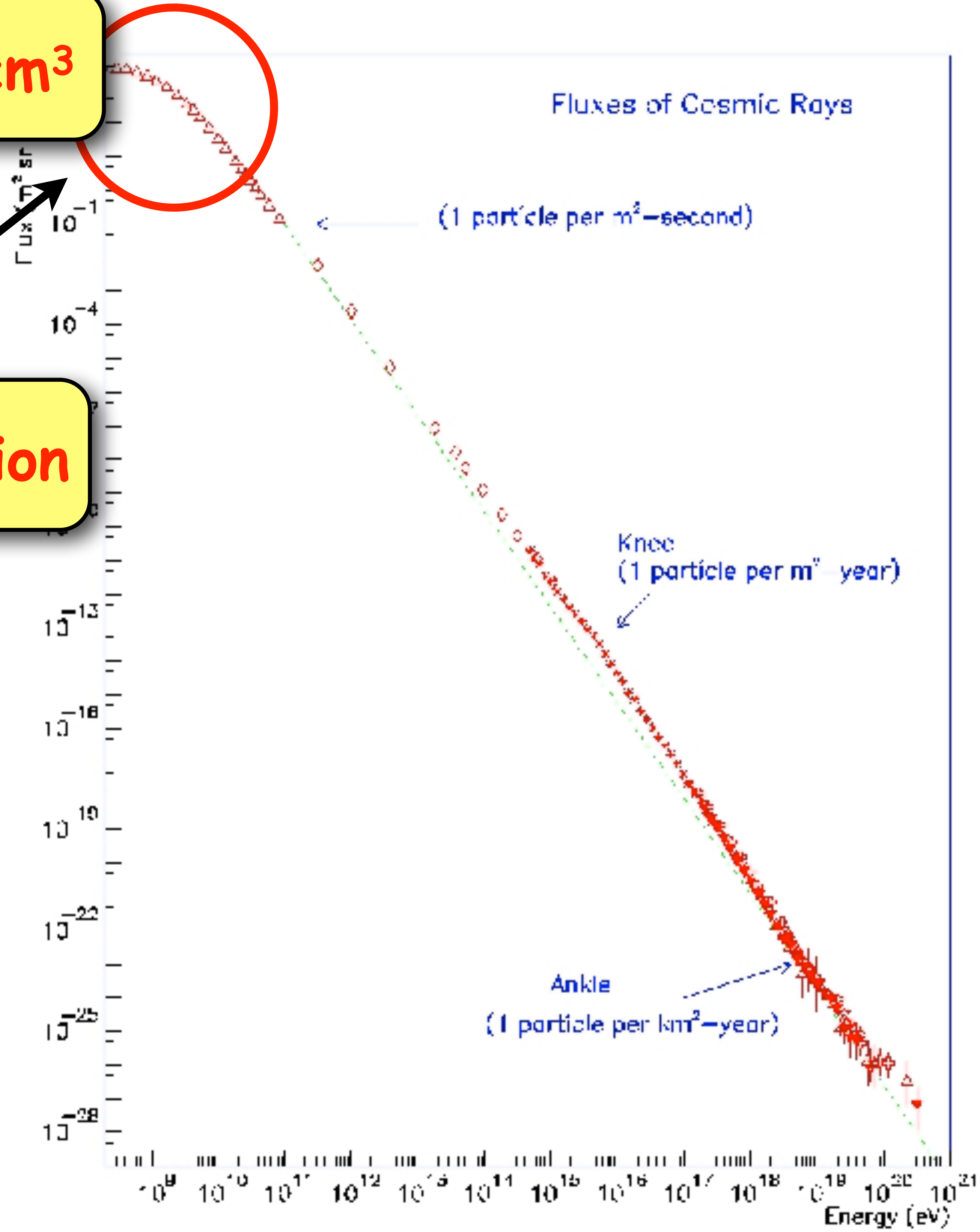
solar modulation



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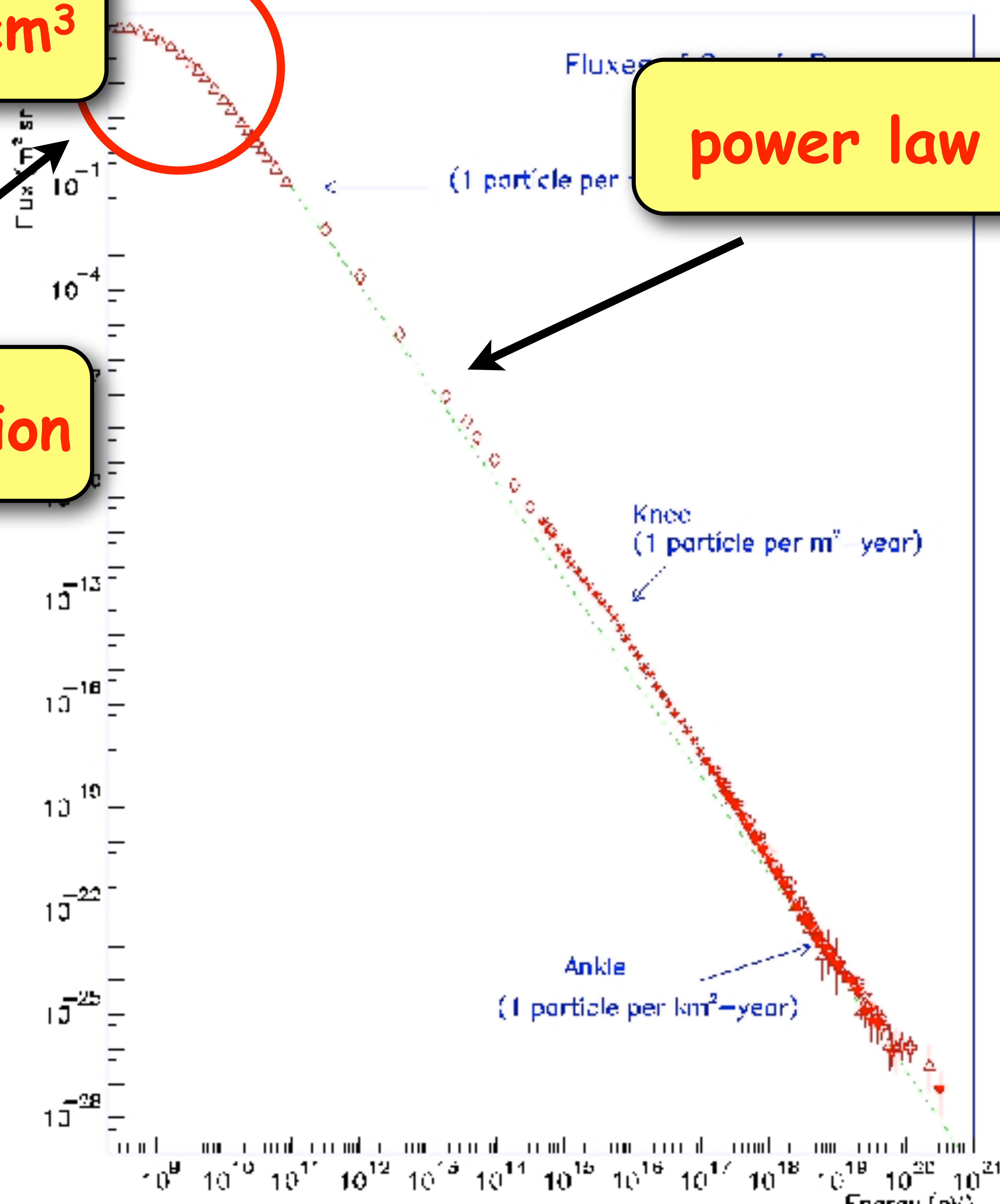
$w_{CR} \sim 1$ eV/cm^3

power law $E^{-2.7}$

solar modulation



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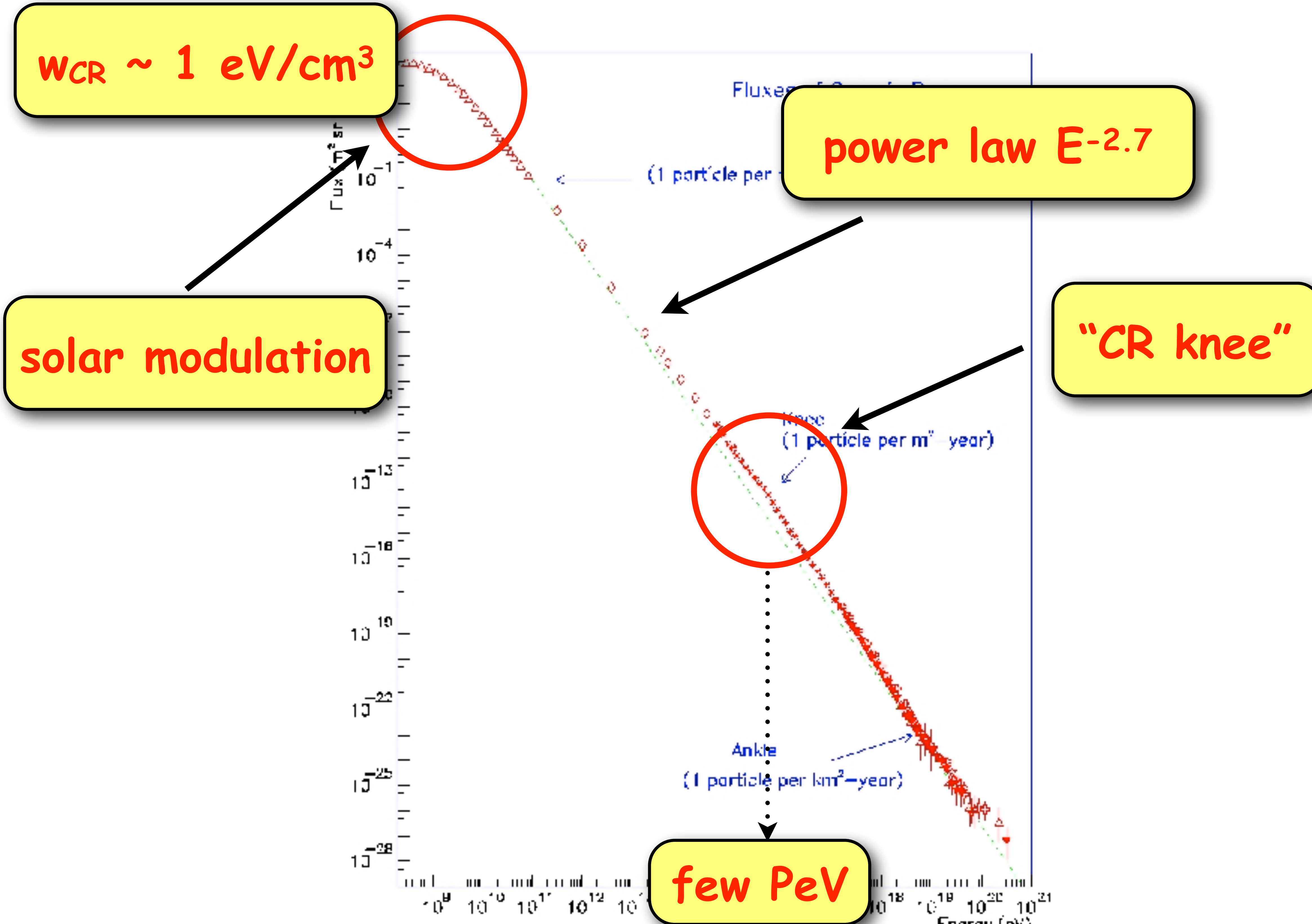

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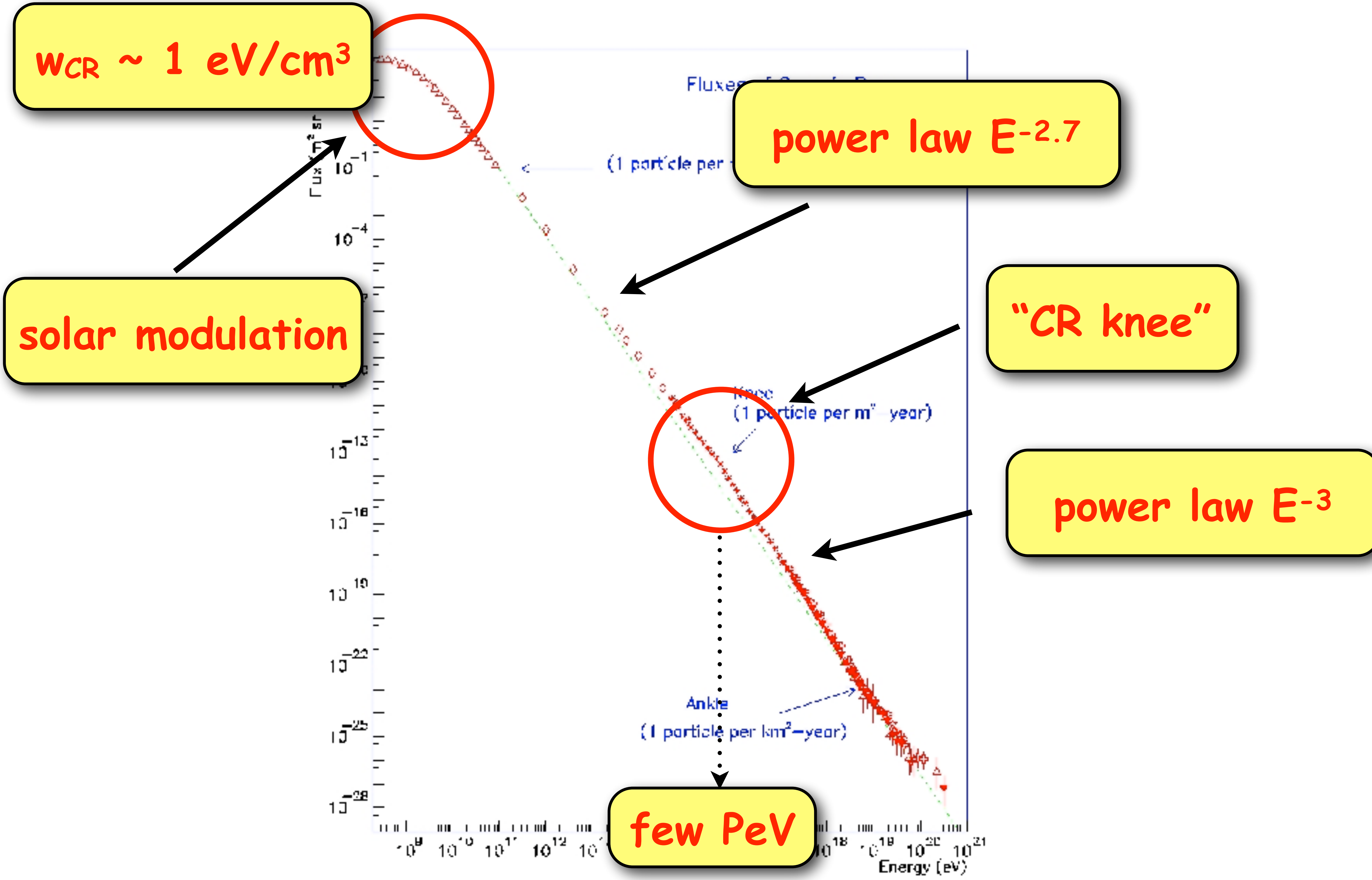

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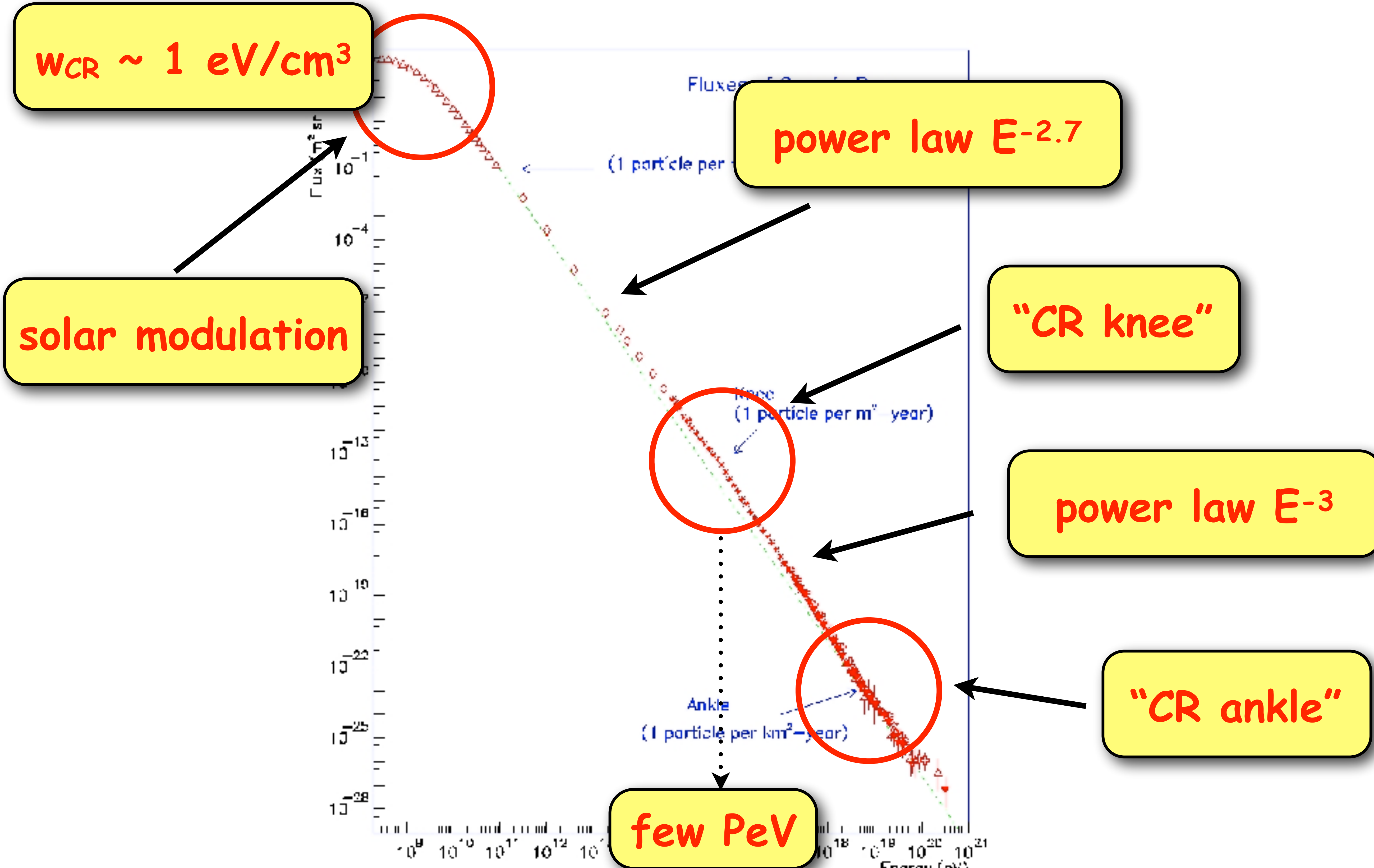

Energy density

Cosmic Ray energy density: $w_{CR} \sim 1 \text{ eV cm}^{-3}$

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Cosmic Ray energy density: $$w_{CR} \sim \boxed{1 \text{ eV cm}^{-3}}$$

Magnetic field energy density: $$w_B = \frac{B^2}{8\pi} \sim \boxed{1 \text{ eV cm}^{-3}}$$

Thermal gas energy density: $$w_{gas}^{turb} = \varrho_{gas} v_{turb}^2 \sim \boxed{1 \text{ eV cm}^{-3}}$$

Variations in time and space

- CR flux at Earth **constant during the last 10^9 yr** (from radiation damages in geological and biological samples, meteorites, and lunar rocks)
- thus the CR flux must be **constant along the orbit of the Sun** around the galactic centre (many revolutions in a Gyr)

Stability in time and (hints for) spatial homogeneity

Cosmic Rays are isotropic

Cosmic Ray anisotropy: $$\delta = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$ (I -> CR intensity)

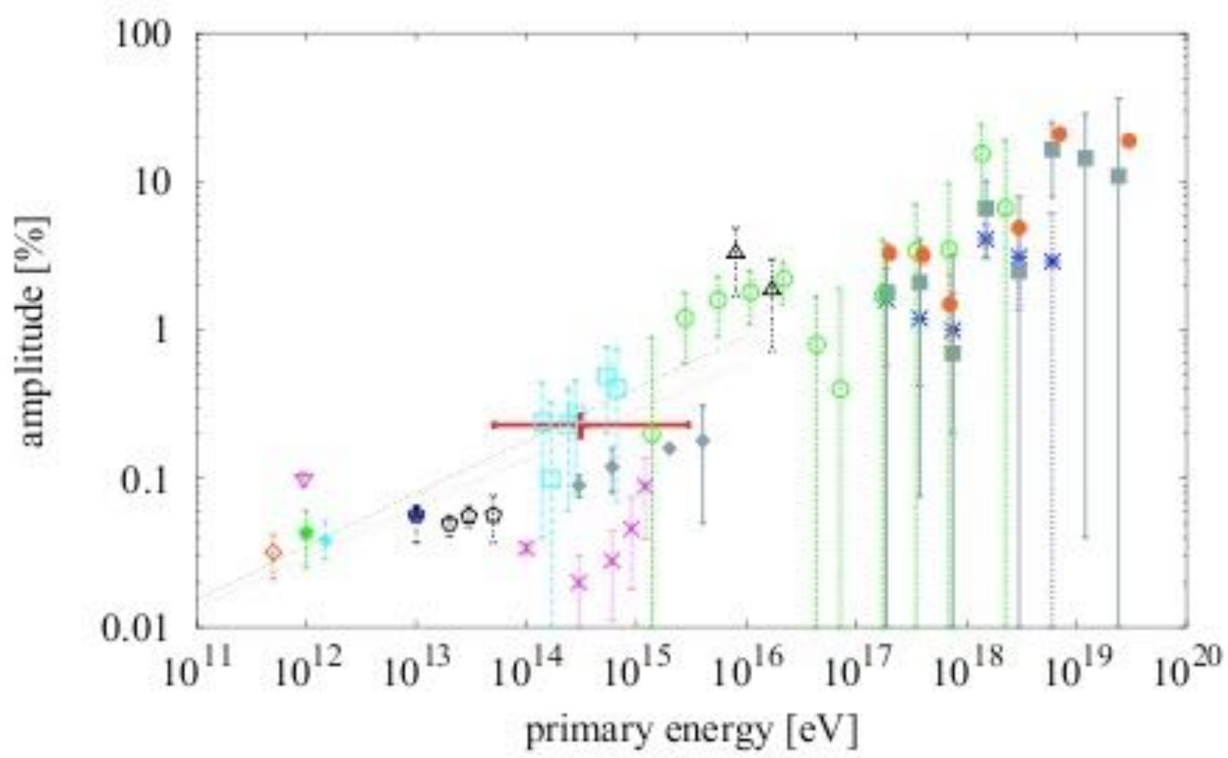

figure from
Iyono et al, 2005

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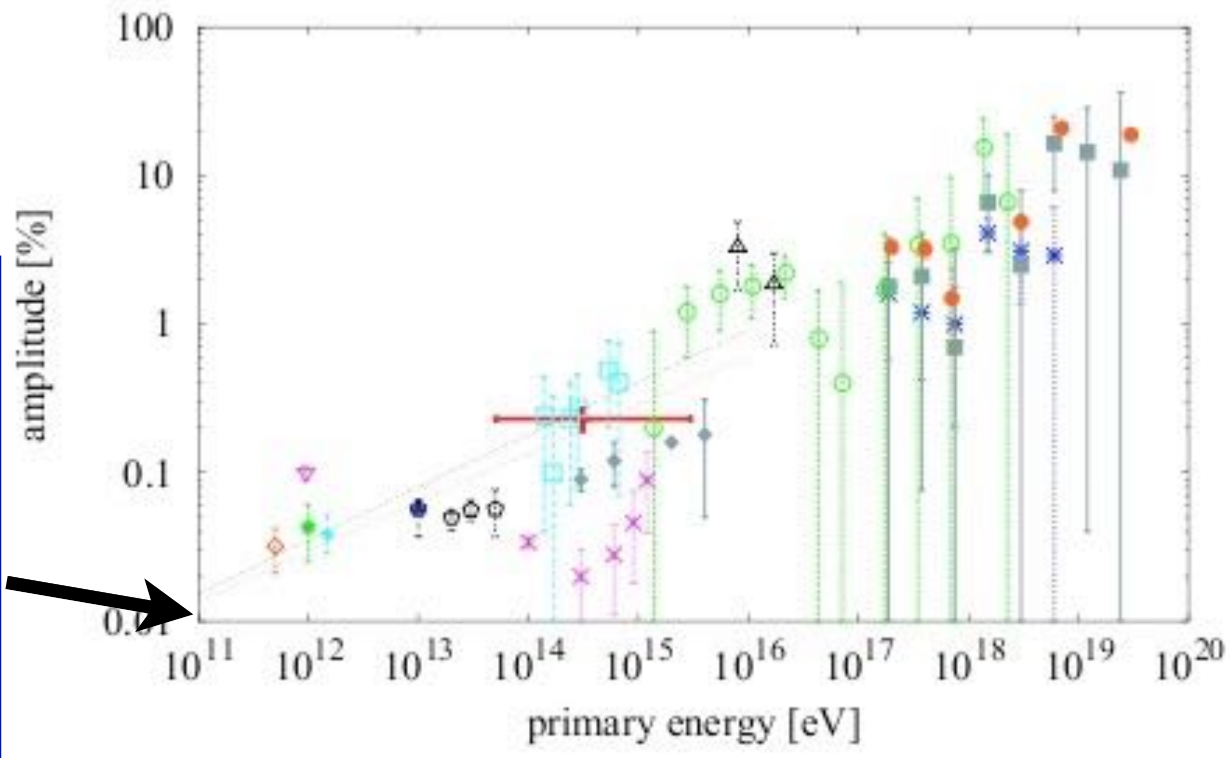
measures available only above ~500 GeV -> magnetic field of the solar system has no effect



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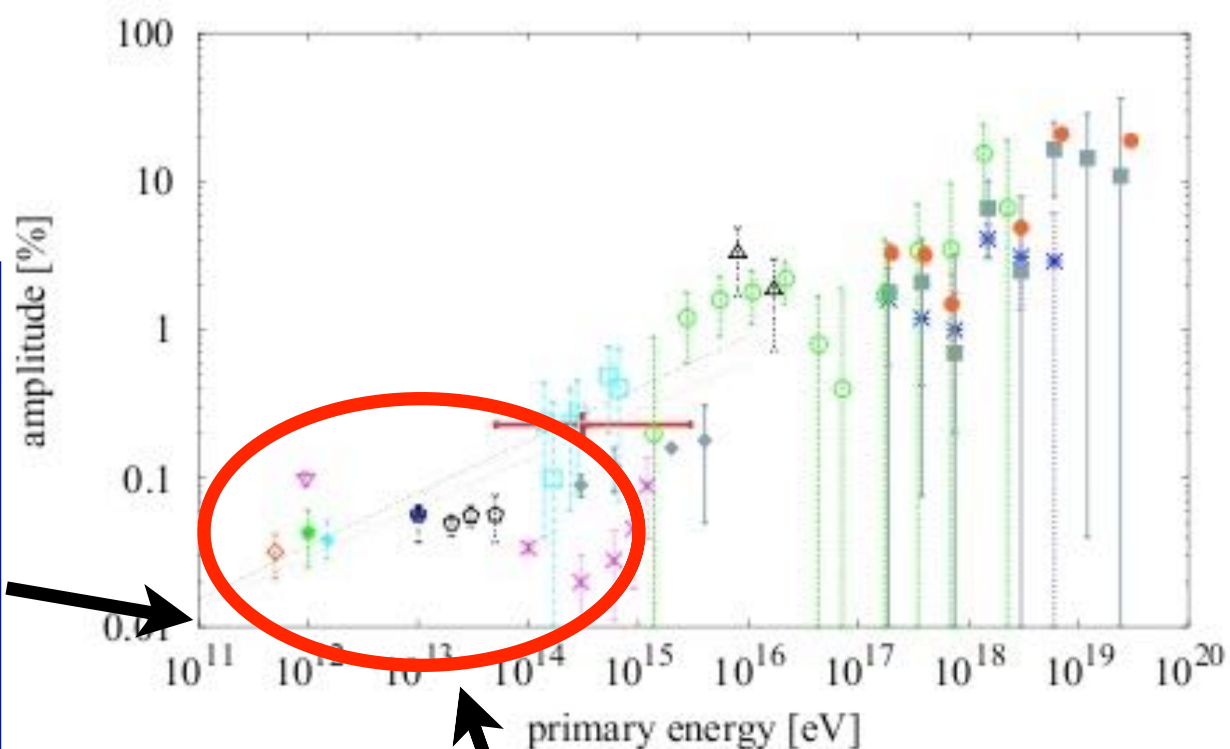

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$\delta \sim 10^{-3}$

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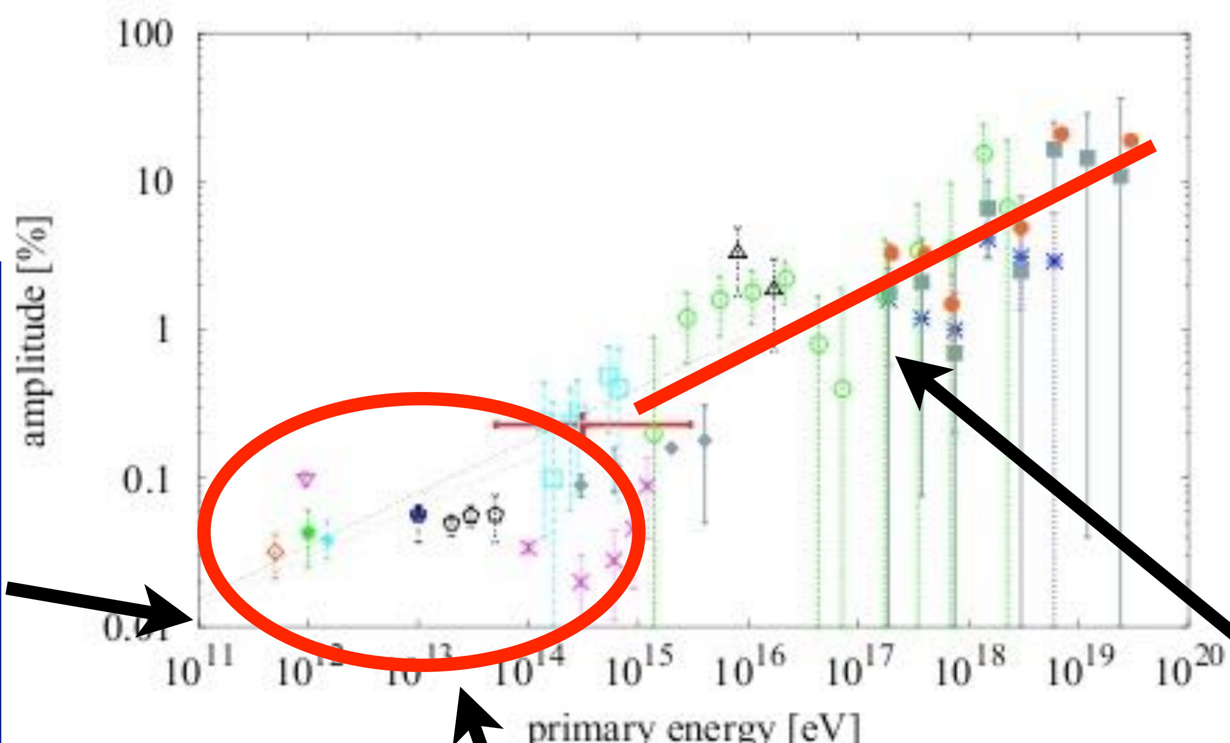

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the anisotropy increases with particle energy

figure from Iyono et al, 2005

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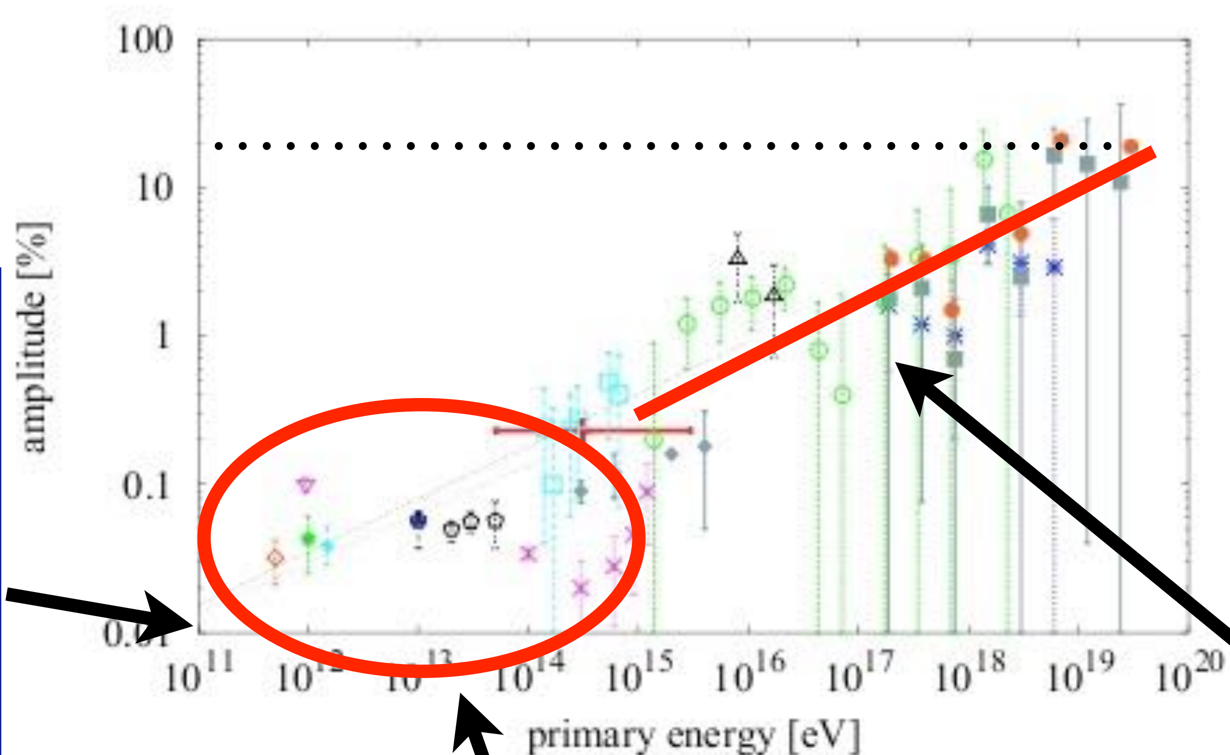


CRs are very isotropic in the sky

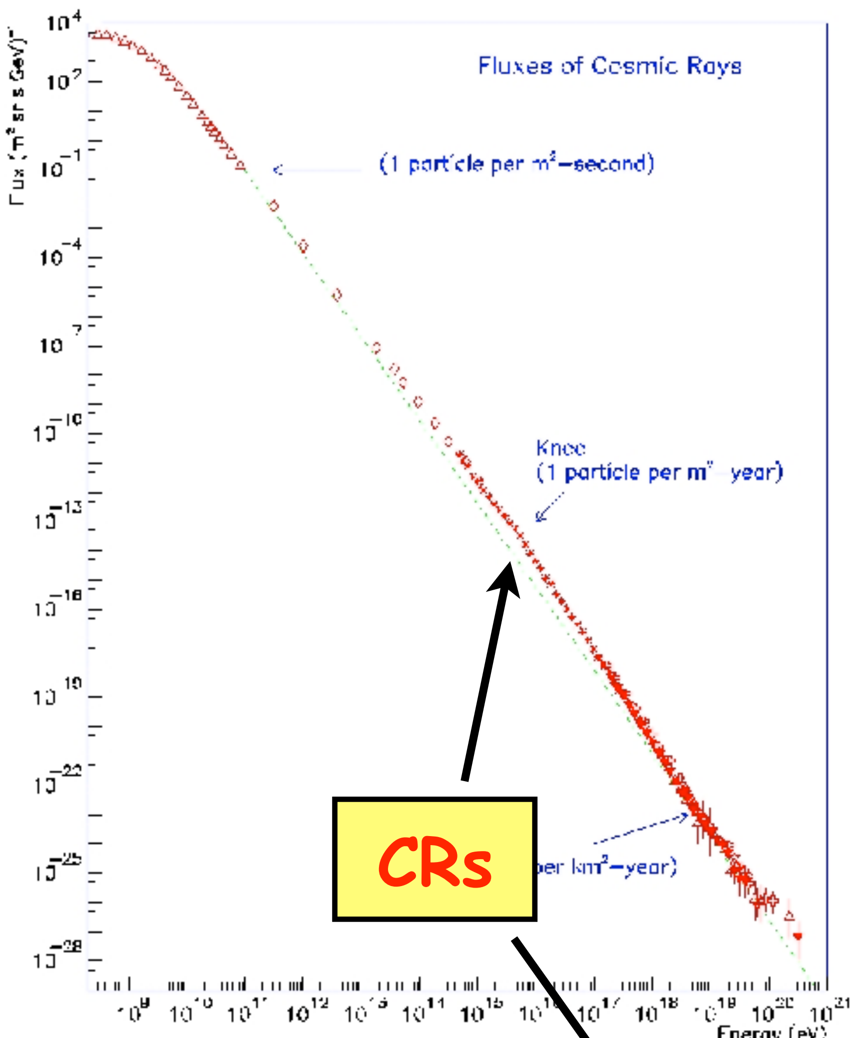
measures available only above ~500 GeV -> magnetic field of the solar system has no effect

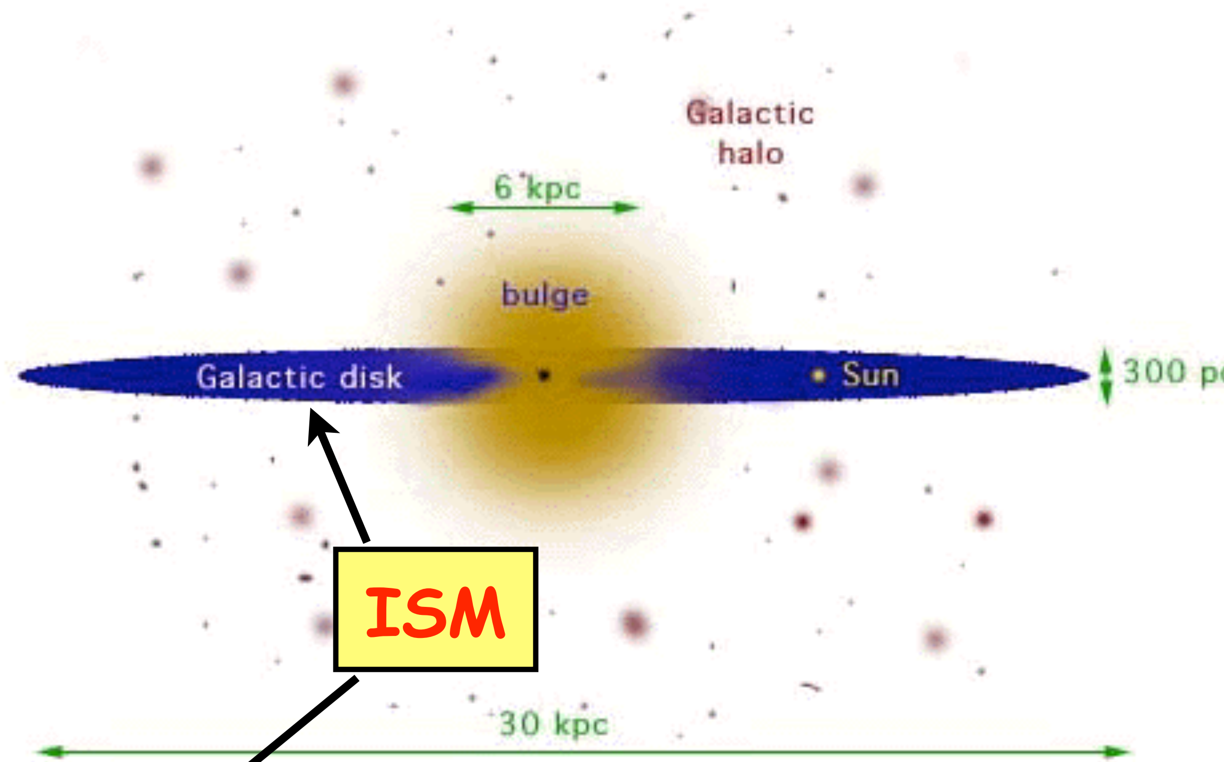
the anisotropy increases with particle energy

$\delta \sim 10^{-3}$

figure from Iyono et al, 2005

Hayakawa's conjecture (1952)





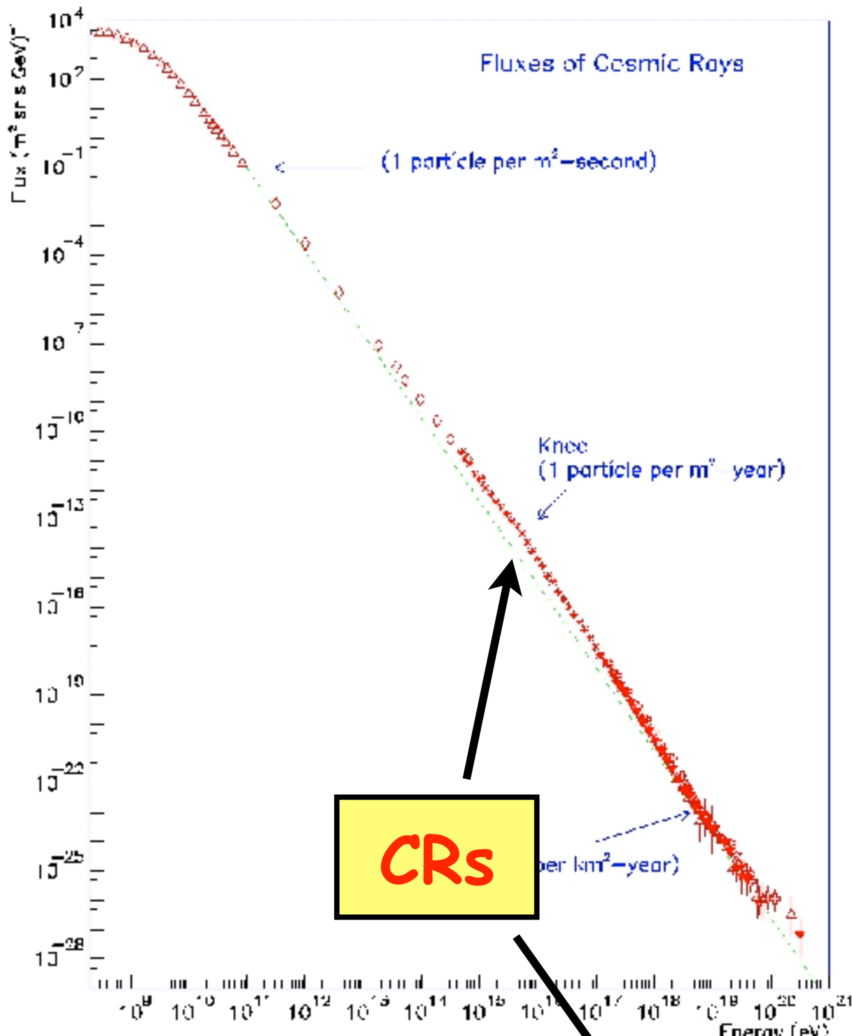
$$p + p \to p + p + \pi^0$$

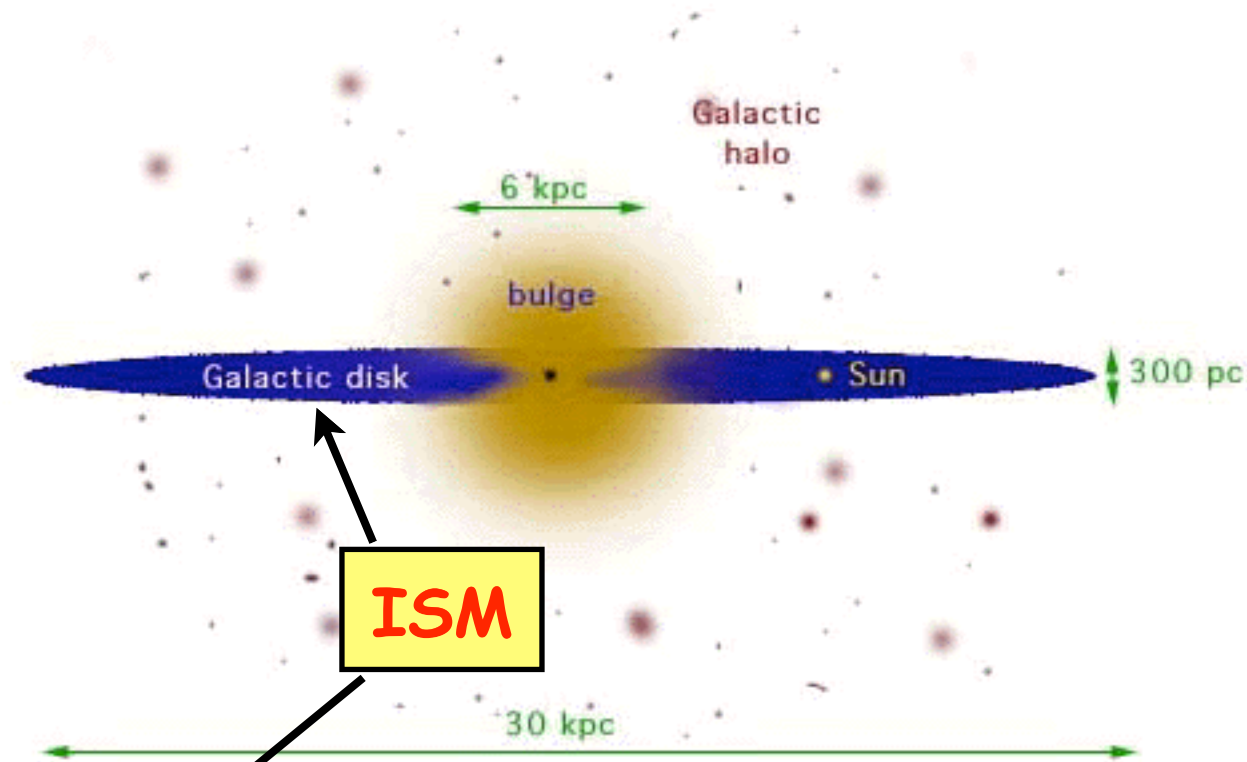
$$\pi^0 \to \gamma + \gamma$$

Cosmic Rays undergo hadronic interactions in the InterStellar Medium

-> the Galaxy should shine in gamma rays!

Hayakawa's conjecture (1952)





CRs

ISM

$$E_{th} \sim 280 \text{ MeV}$$

$$p + p \rightarrow p + p + \pi^0$$

$$\pi^0 \rightarrow \gamma + \gamma$$

$$E_\gamma \sim 0.1 \; E_p$$

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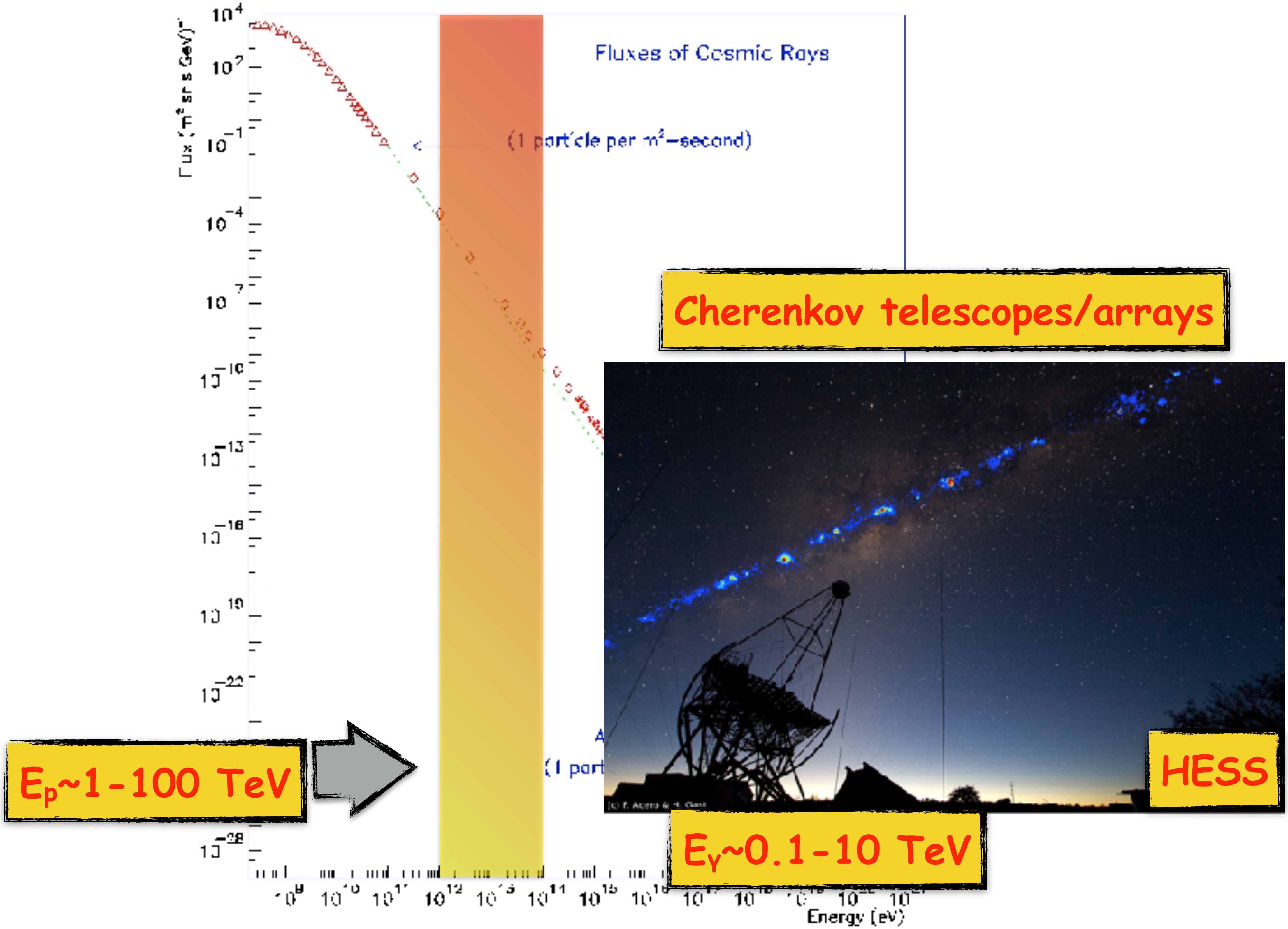
γ-ray observations from ground…

Cherenkov telescopes/arrays

E_γ~0.1-10 TeV

γ-ray observations from ground…



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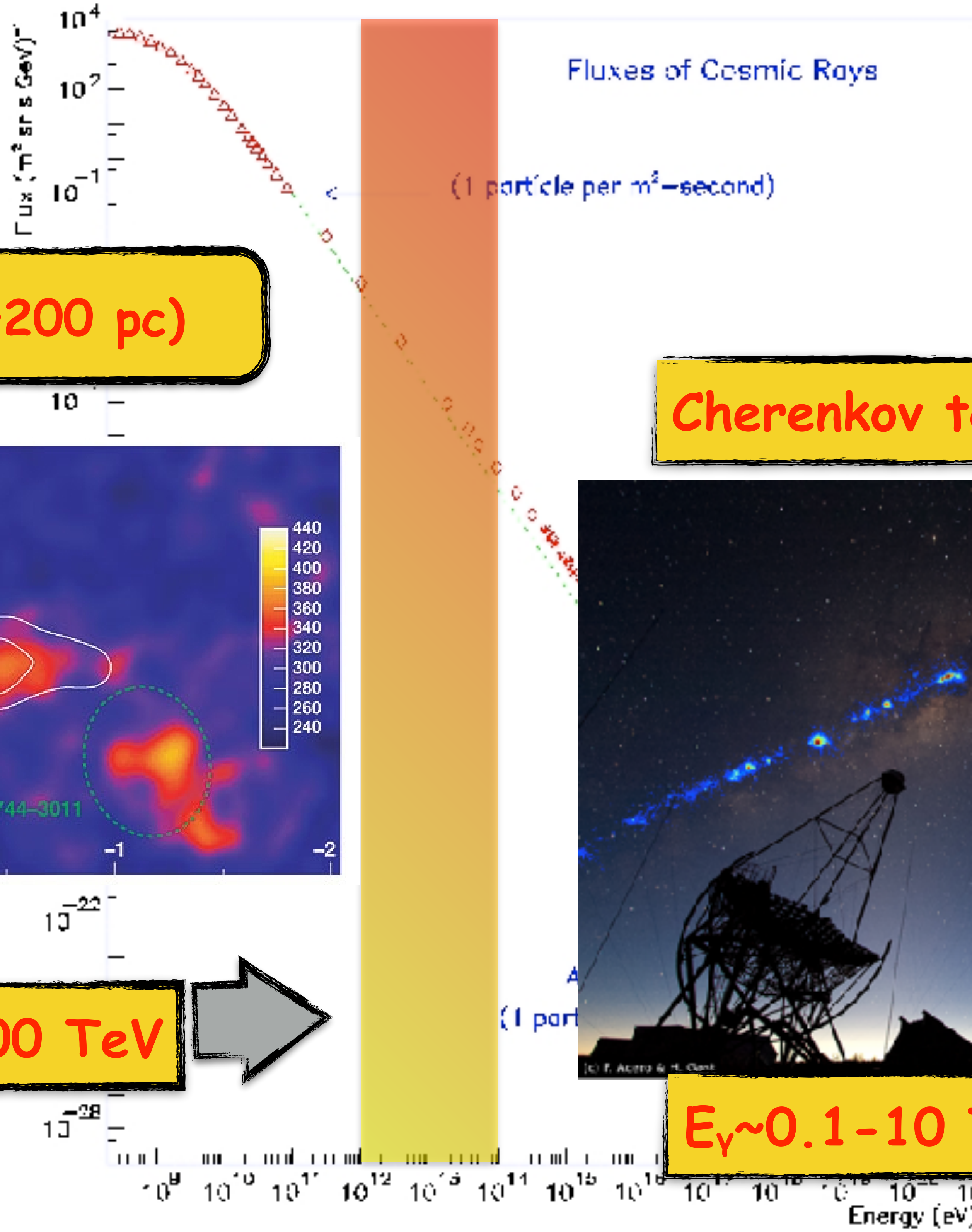
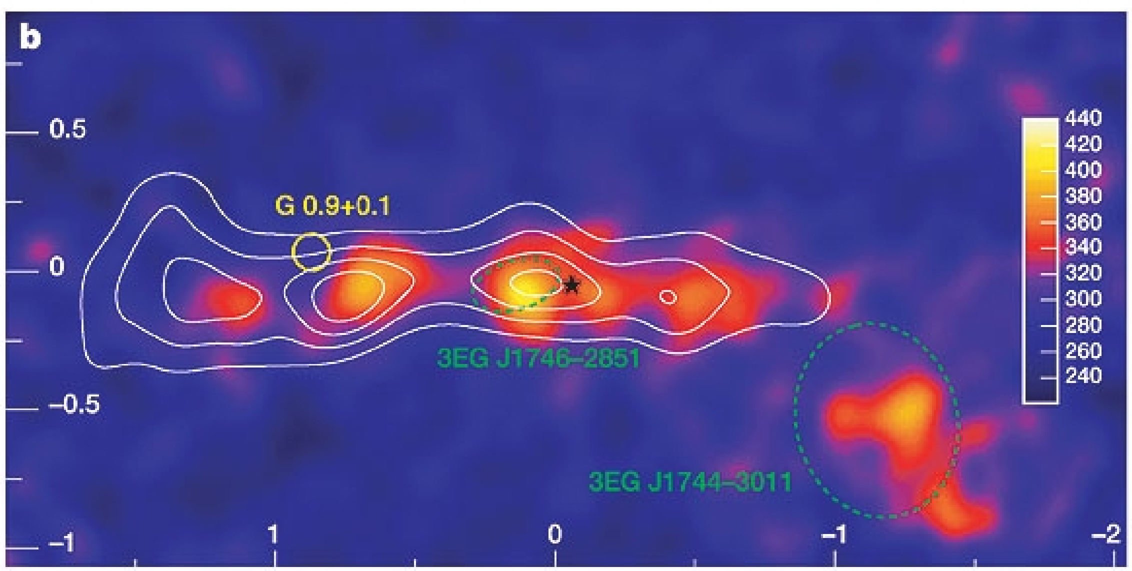


Inner galaxy (~200 pc)



E_p~1-100 TeV

Cherenkov telescopes/arrays

HESS

E_γ~0.1-10 TeV

…and from space

FERMI

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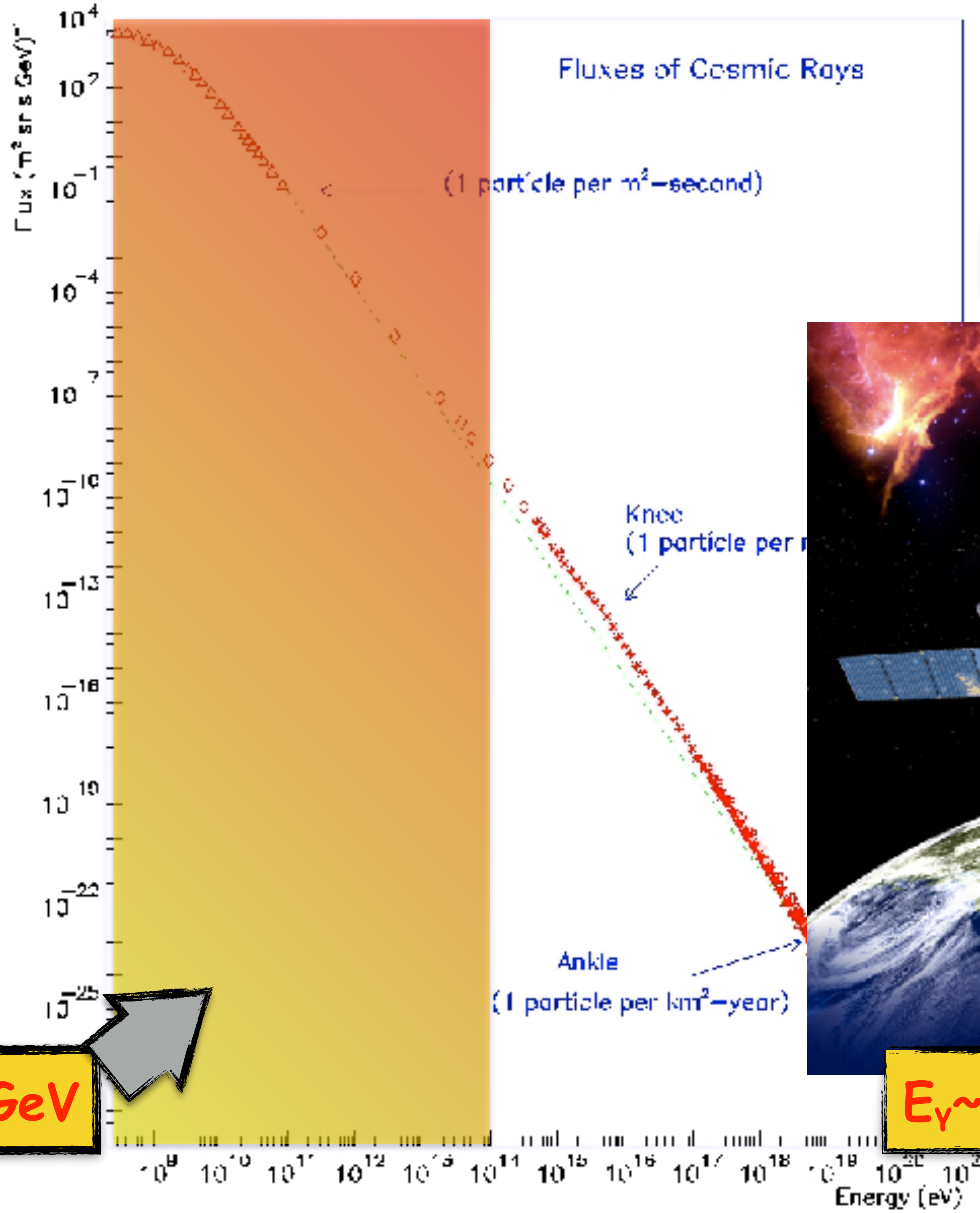


FERMI

E_p~280 MeV-100 GeV

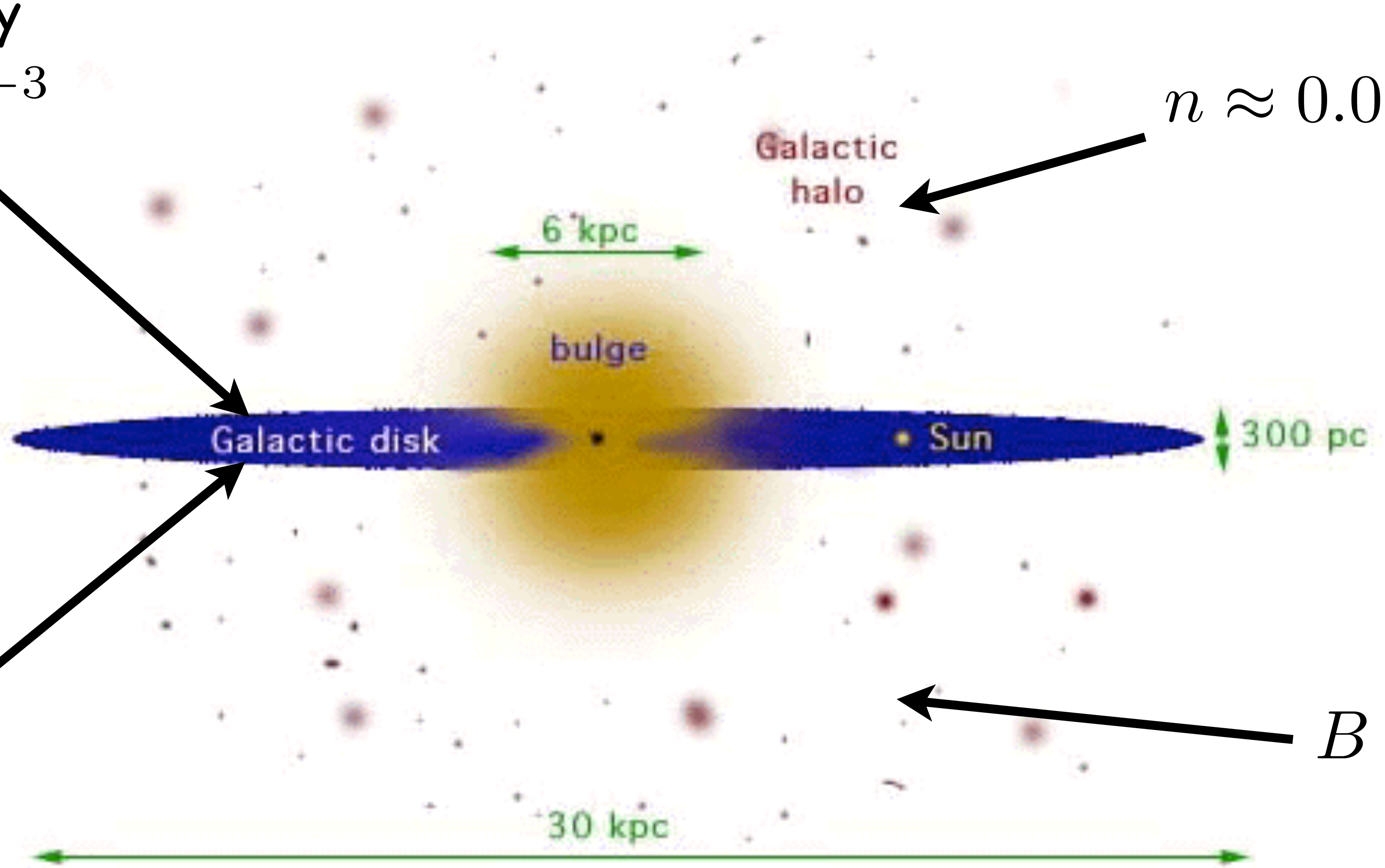
E_γ~0.1-10 GeV

Gamma rays from the Milky Way

gas density

$n \approx 1\ \mathrm{cm}^{-3}$

$n \approx 0.01\ \mathrm{cm}^{-3}$



magnetic field

$B \approx 3\ \mu\mathrm{G}$

$B < 10\ \mu\mathrm{G}$

Schematic view of the Milky Way

Cosmic rays fill the whole galactic disk!

Gamma rays from the Milky Way

gas c

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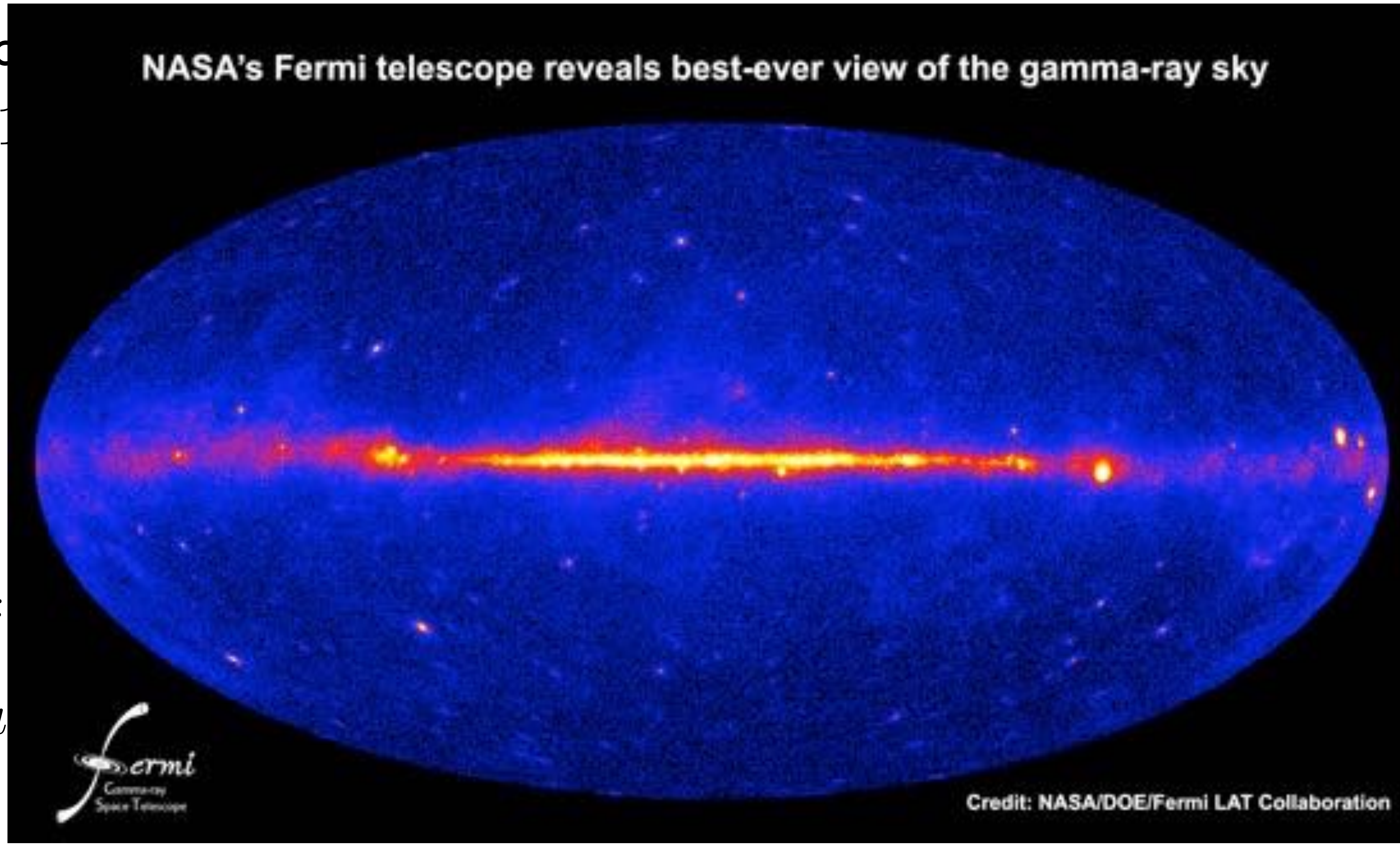

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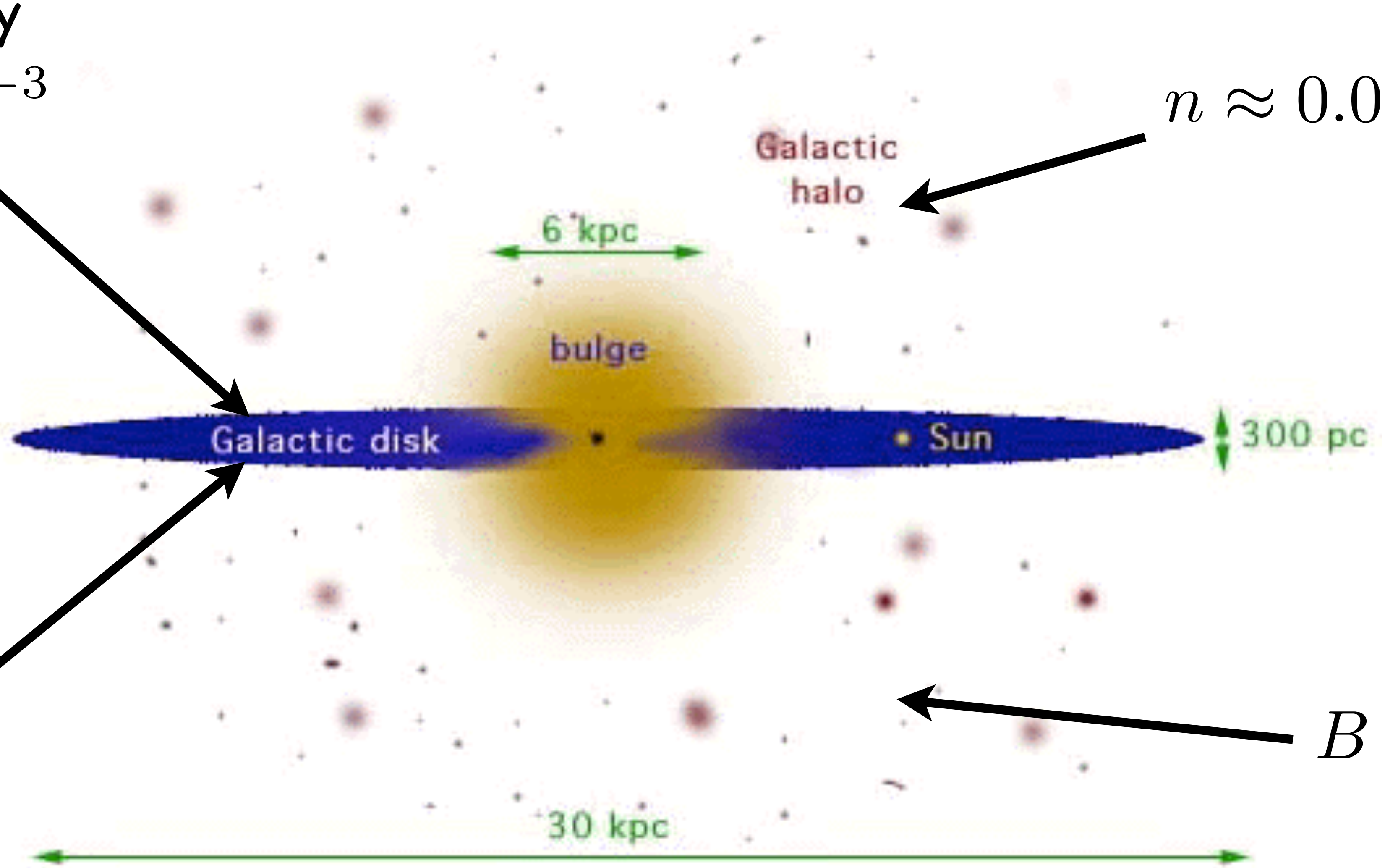

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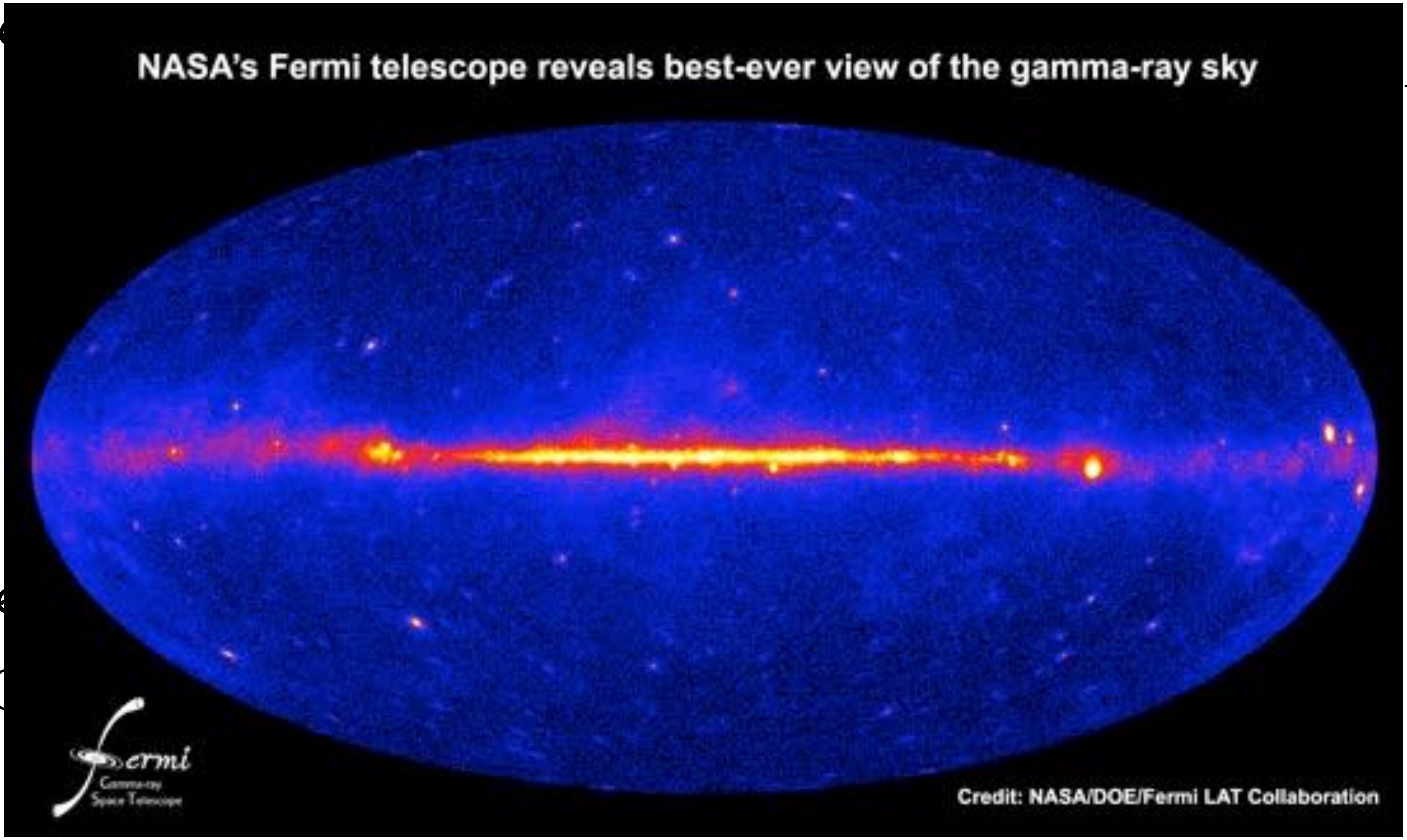

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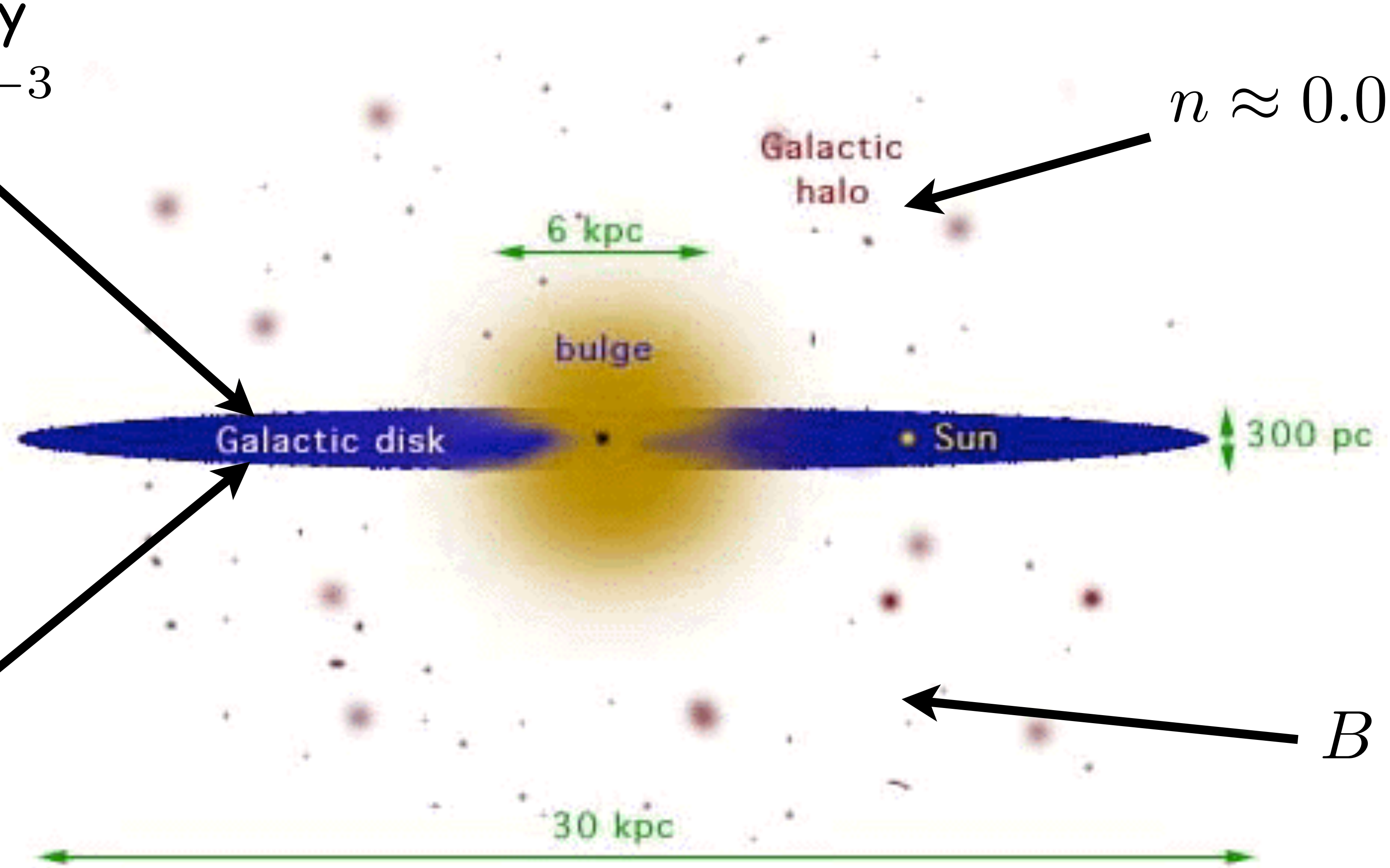

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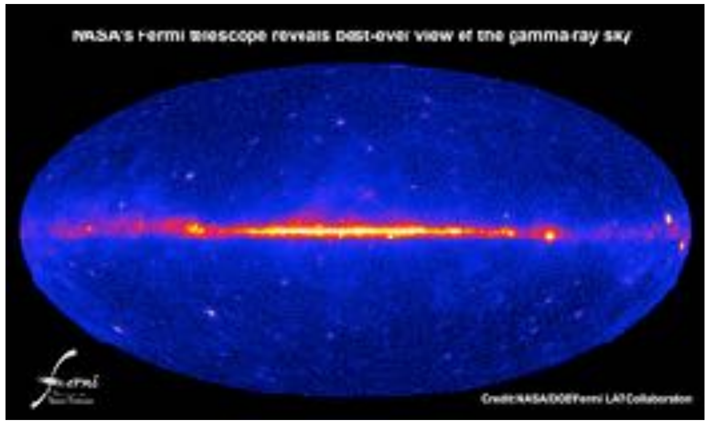
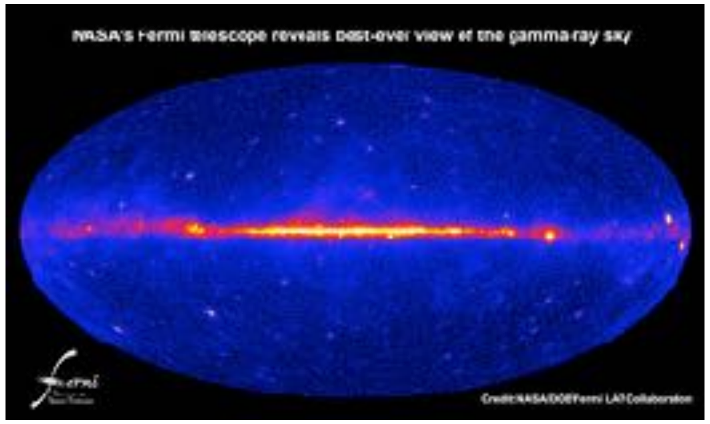
Are CRs universal?



Cosmic rays are homogeneously distributed in the galactic disk.

Hypothesis: are they homogeneously distributed in the whole Universe?

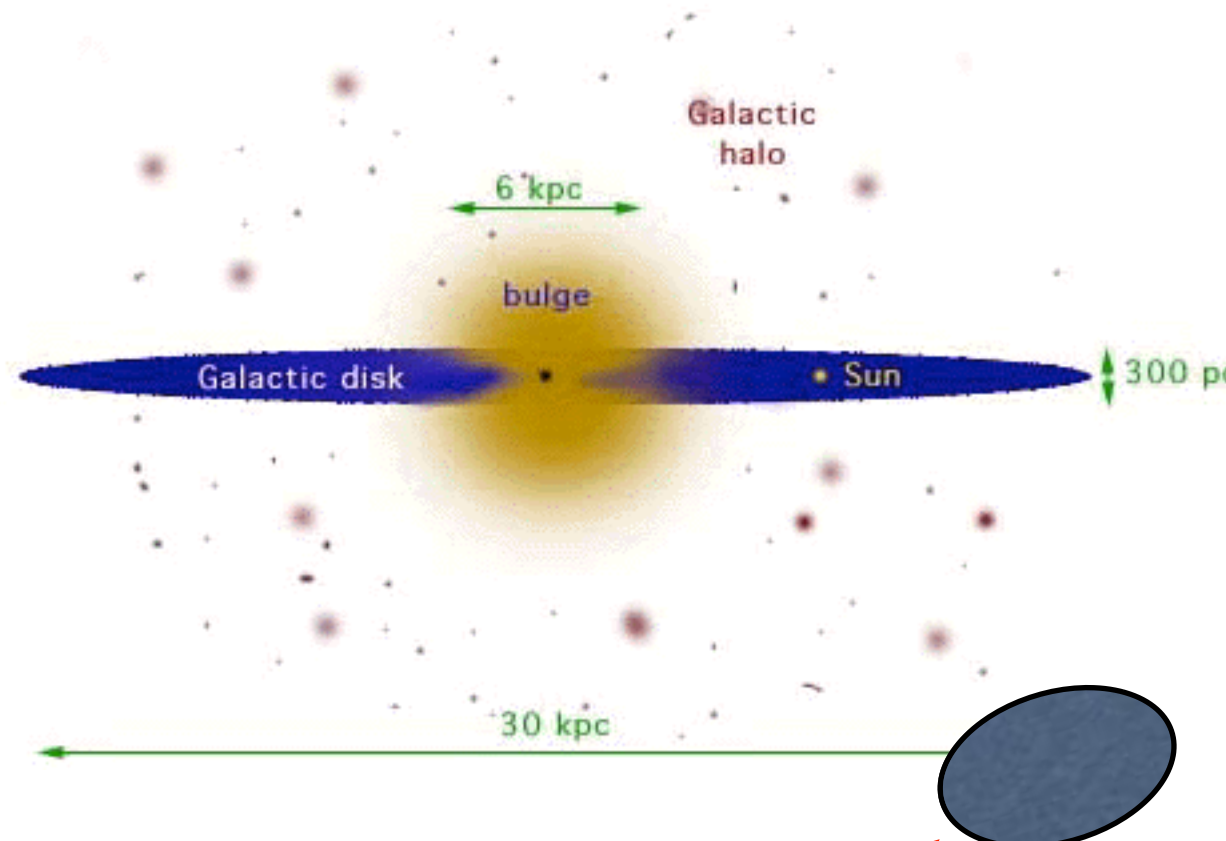
Are CRs universal?



Cosmic rays are homogeneously distributed in the galactic disk.
Hypothesis: are they homogeneously distributed in the whole Universe?

We play the same game with the Small Magellanic Cloud.
Total gas mass -> expected gamma rays

We observe less gammas than expected!

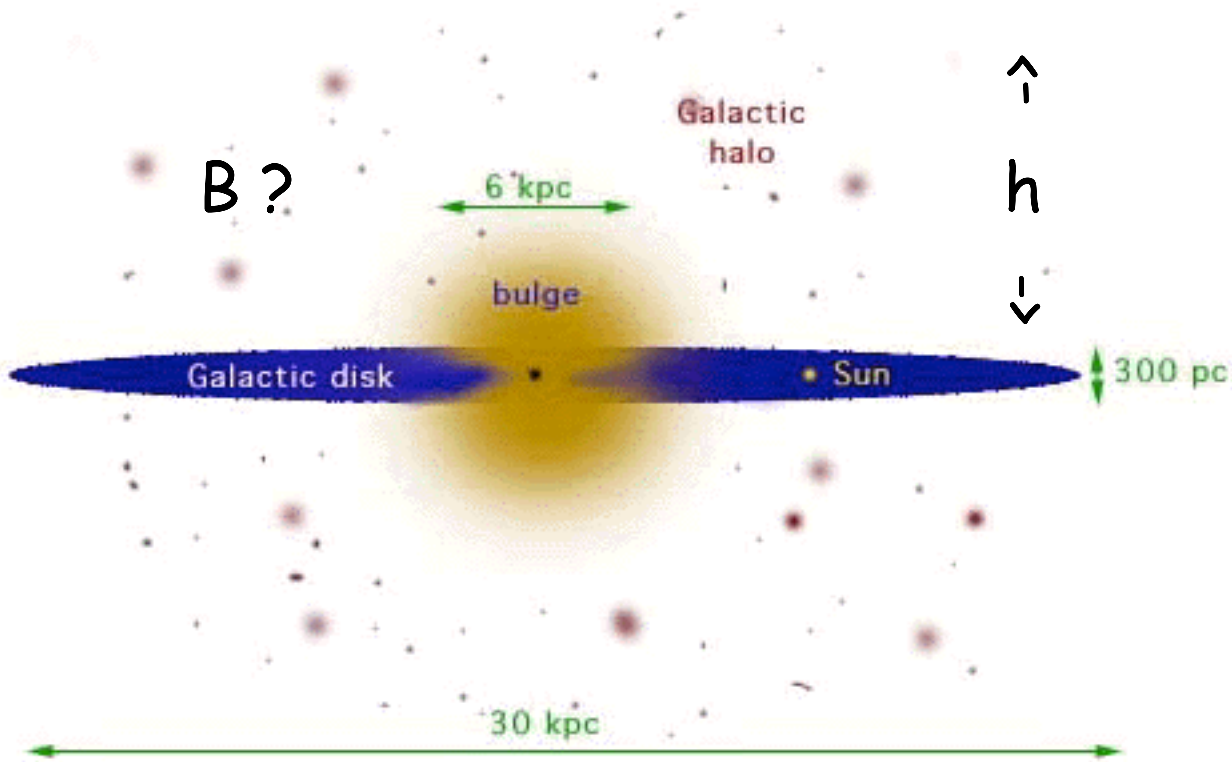


Small Magellanic Cloud

CRs come from the Galaxy

Galactic or extra-galactic?

Which CRs are confined in the Galaxy?



It depends on the values of the magnetic field and thickness of the halo (both poorly constrained...)

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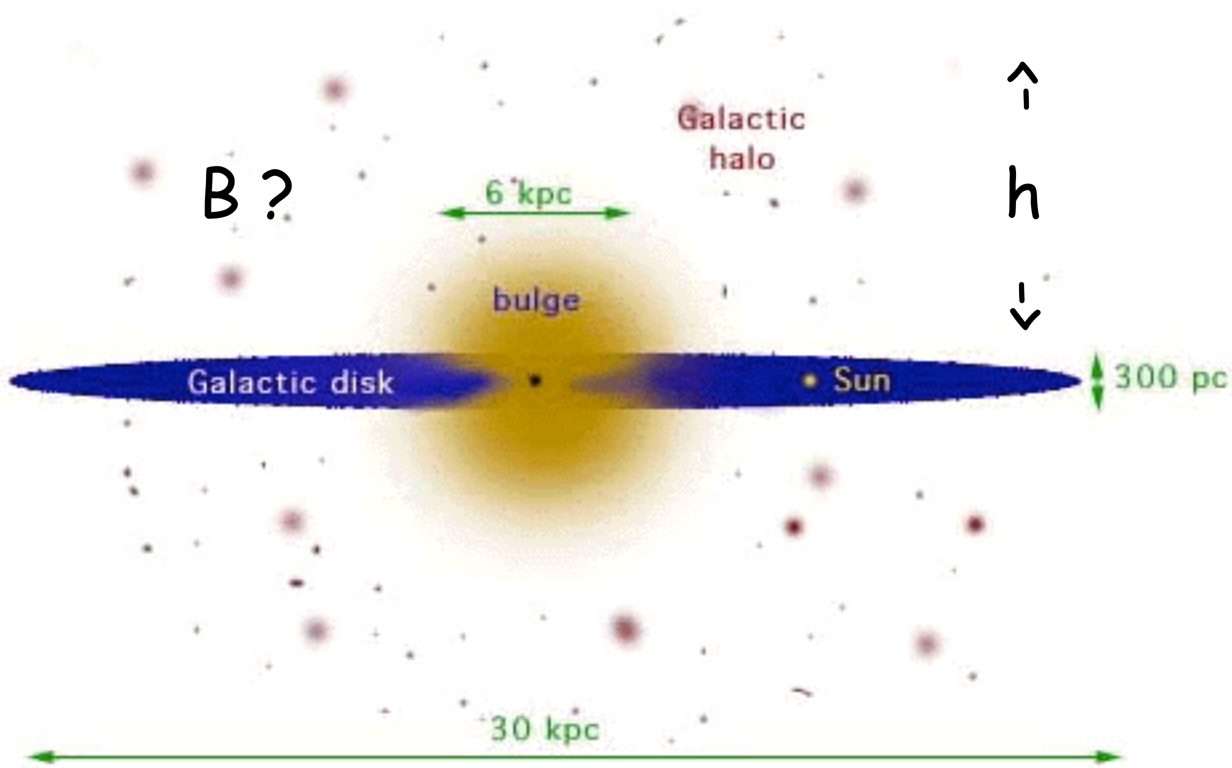

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Confinement condition:

$$R_L < h$$

Larmor radius

halo size

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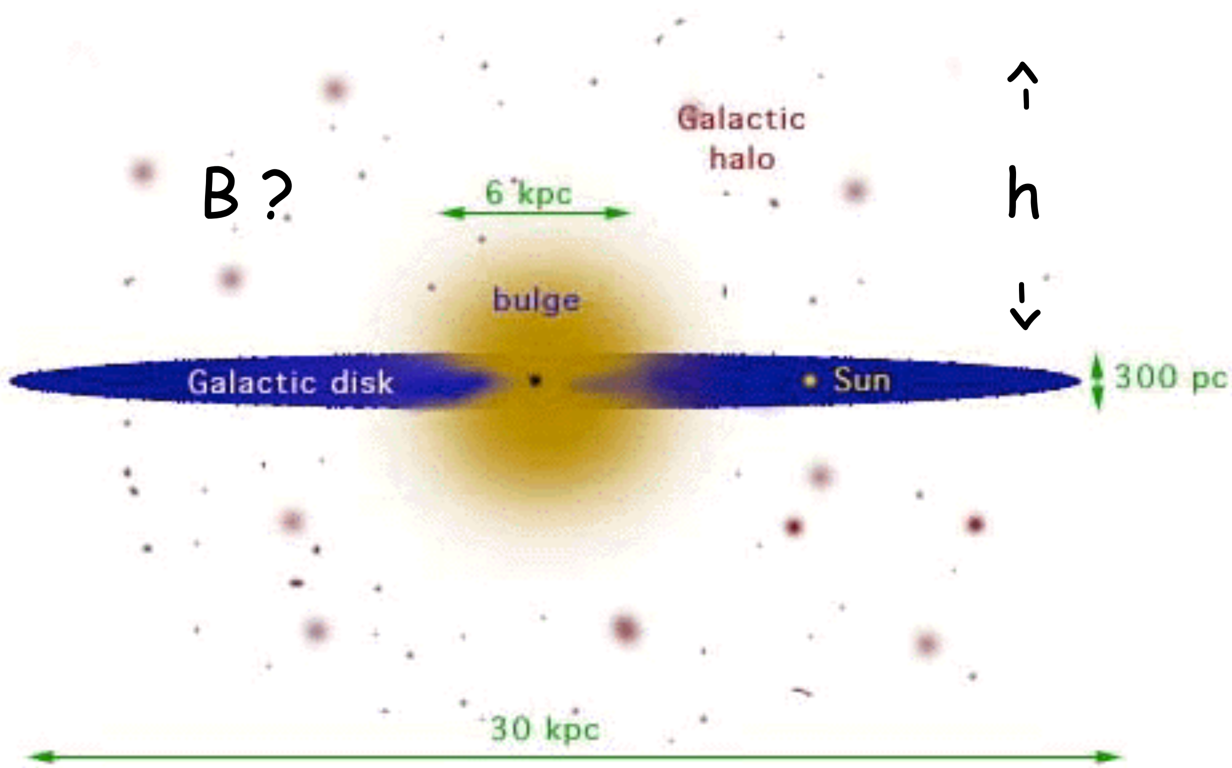

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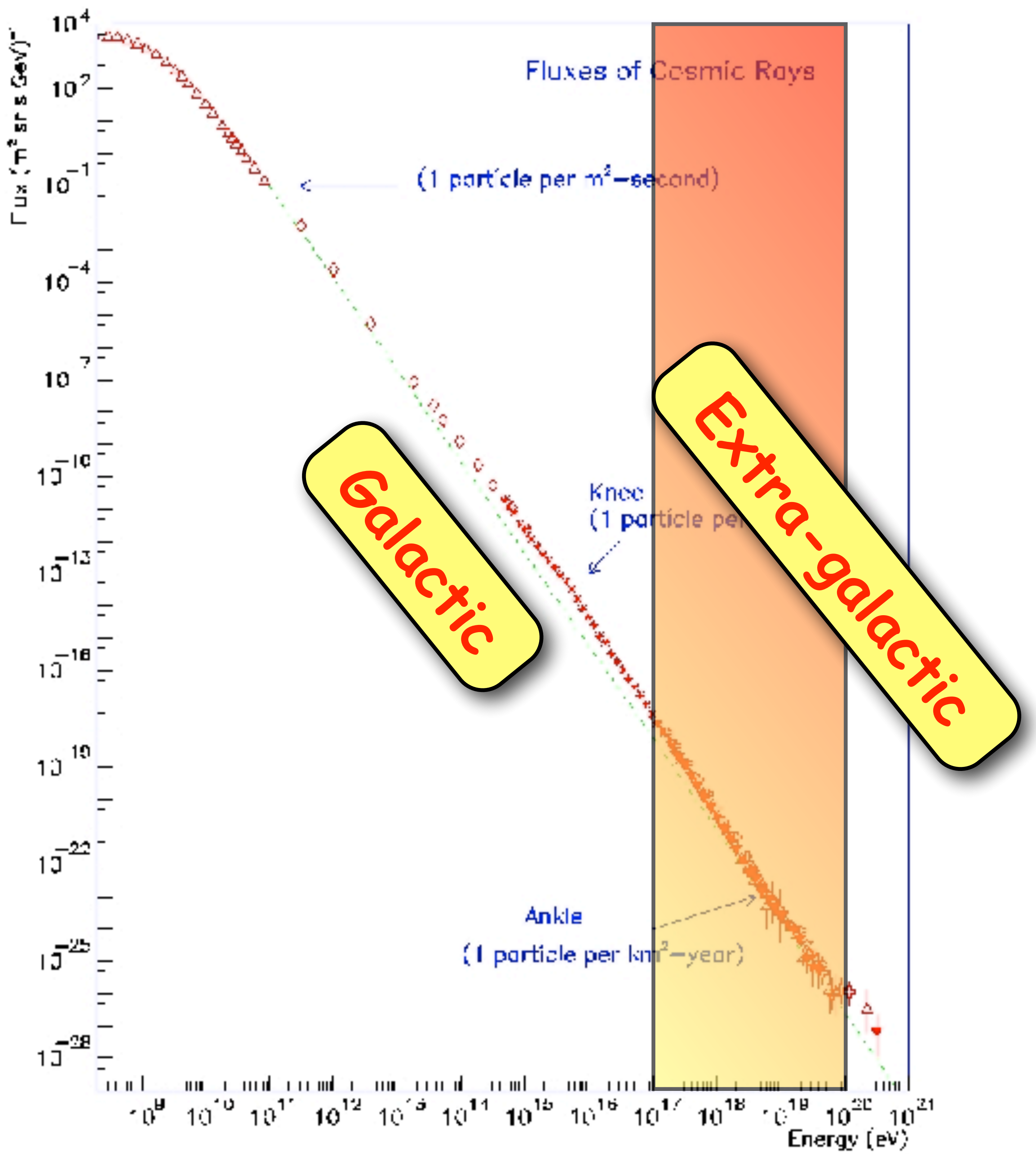
Confinement condition:

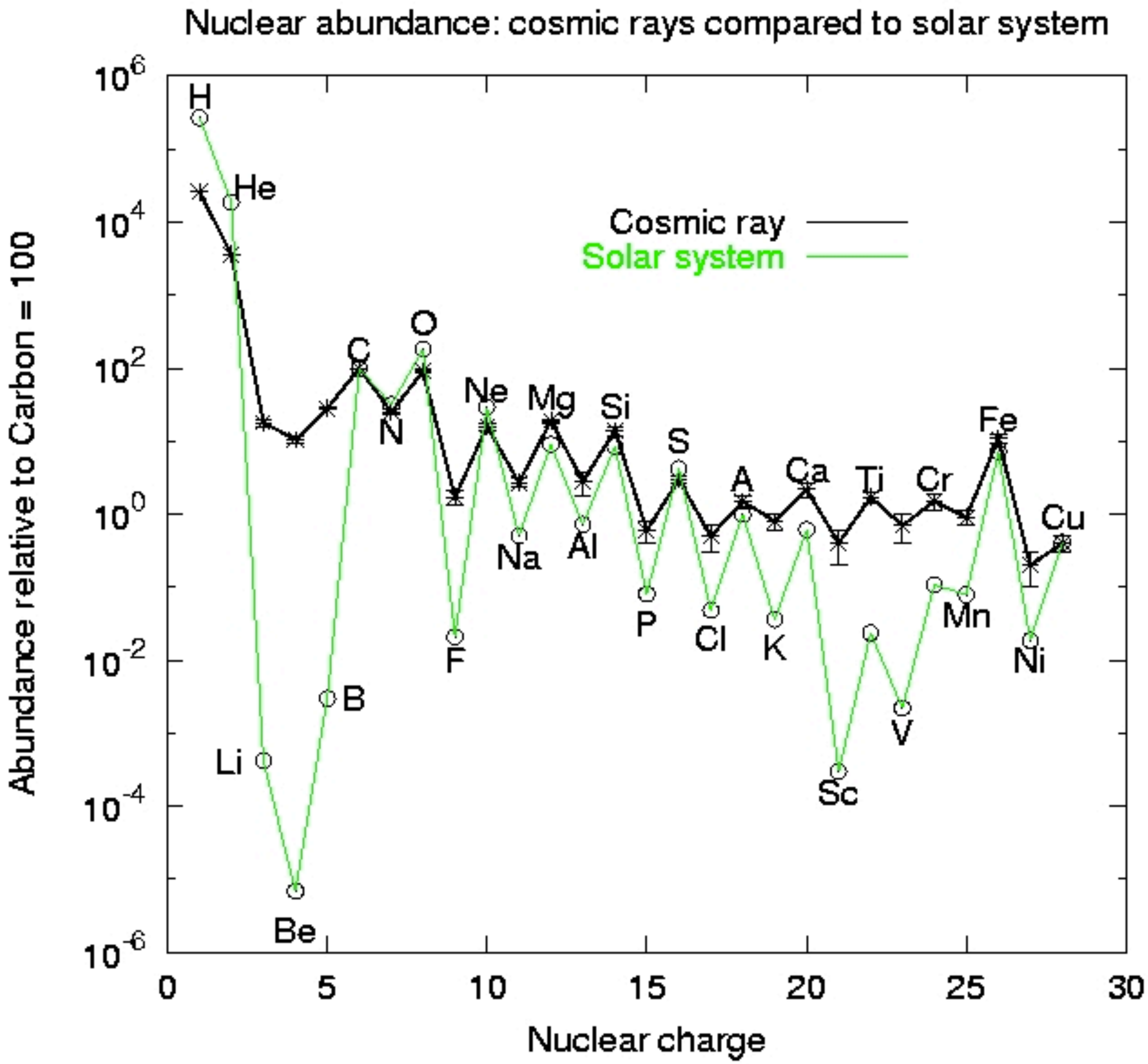
$$\frac{E(\mathrm{eV})}{300\ B(\mathrm{G})} = R_L < h \Rightarrow E < 10^{18} \left(\frac{h}{\mathrm{kpc}}\right) \left(\frac{B}{\mu\mathrm{G}}\right) \mathrm{eV} = 10^{17} \div 10^{20}\ \mathrm{eV}$$

(cm) Larmor radius halo size 1 - 10 0.1 - 10

Galactic or extra-galactic?



Cosmic Ray composition



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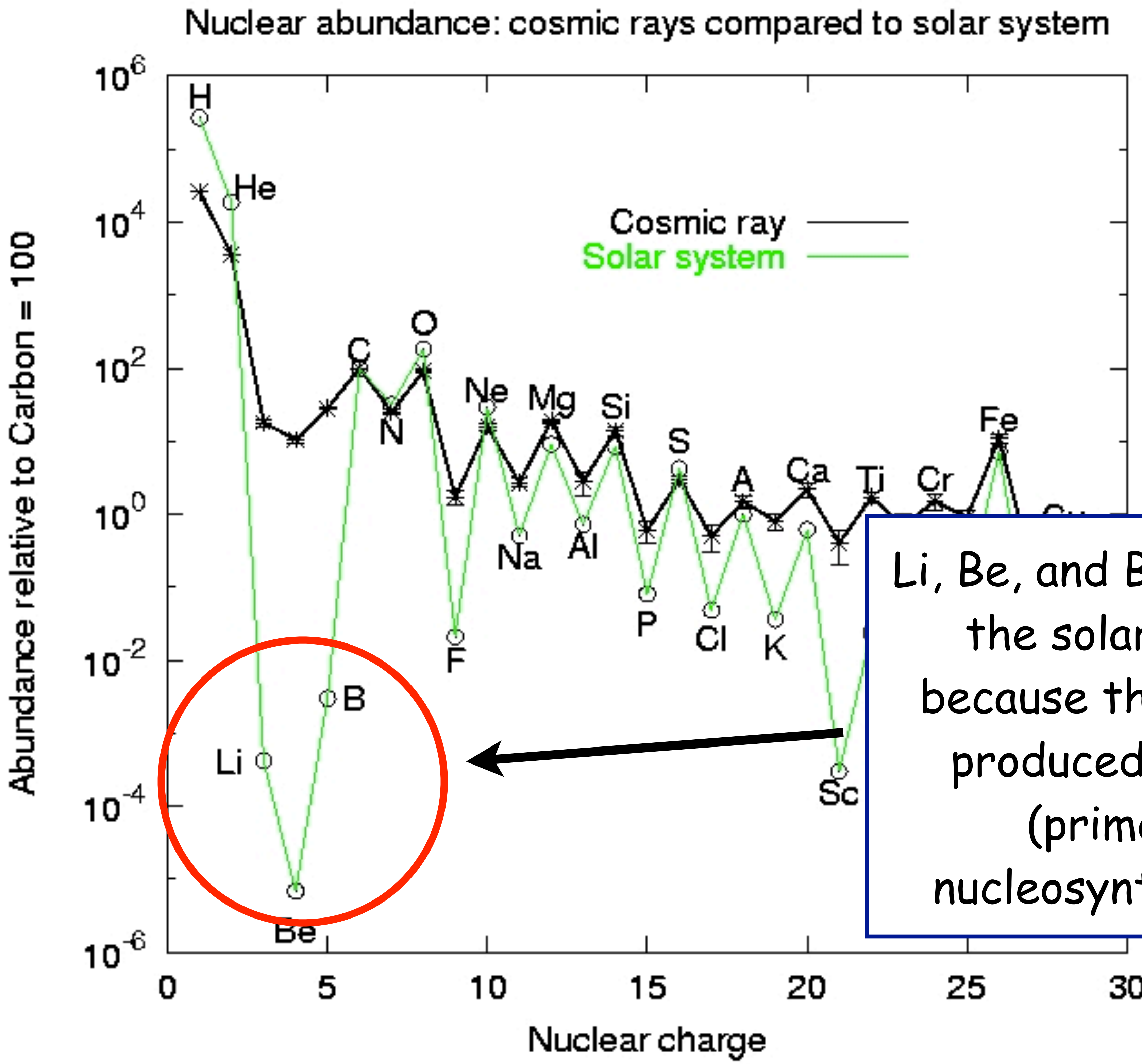

Li, Be, and B are rare in the solar system because they are not produced by stars (primordial nucleosynthesis + ?)

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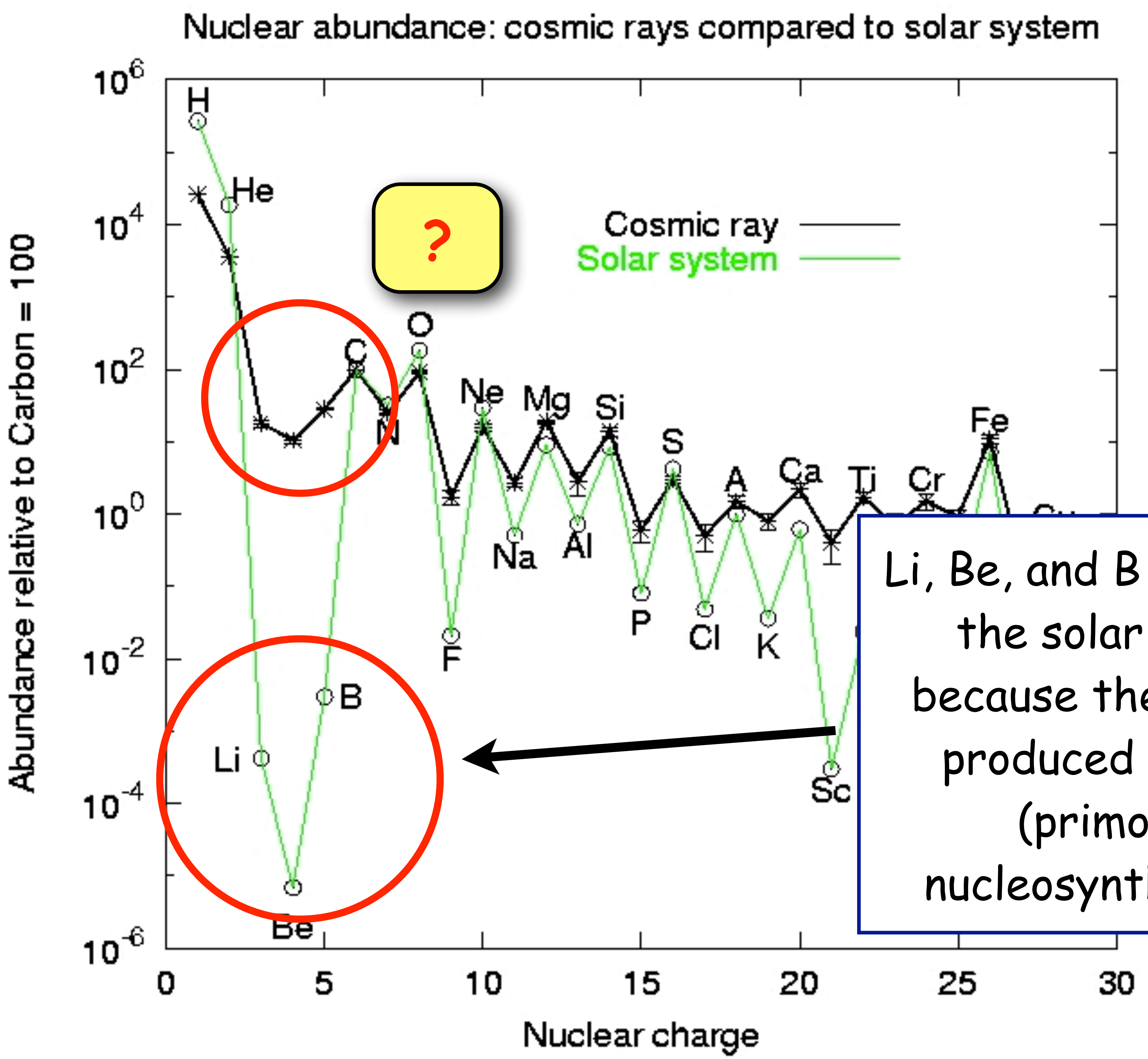

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Cosmic Ray composition: spallation

Spallation: production of light elements as fragmentation products of the interaction of high energy particles with cold matter.

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The anomaly is explained if (~ GeV) CRs transverse $\lambda \approx 5 \ \mathrm{g/cm^2}$

Assuming propagation in the galactic disk: $l_s = \dfrac{\lambda}{\varrho_{ISM}} \approx 1 \ \mathrm{Mpc}$

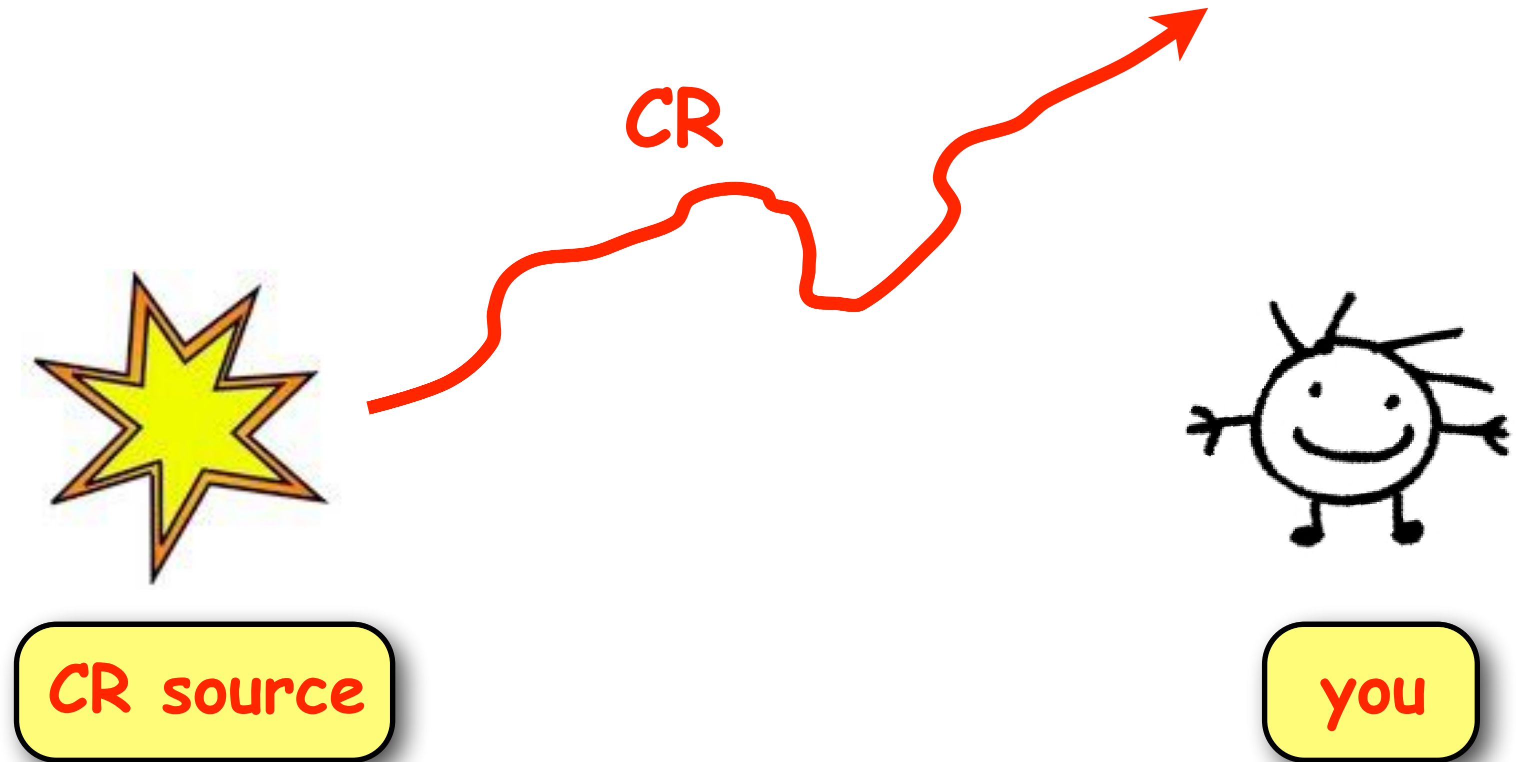
much larger than the size of the disk!!!

CRs don't go straight but are confined in the disk -> diffusive behavior -> isotropy!

CRs don't go straight: consequences

(1) We cannot do CR astronomy

-> difficult to identify sources



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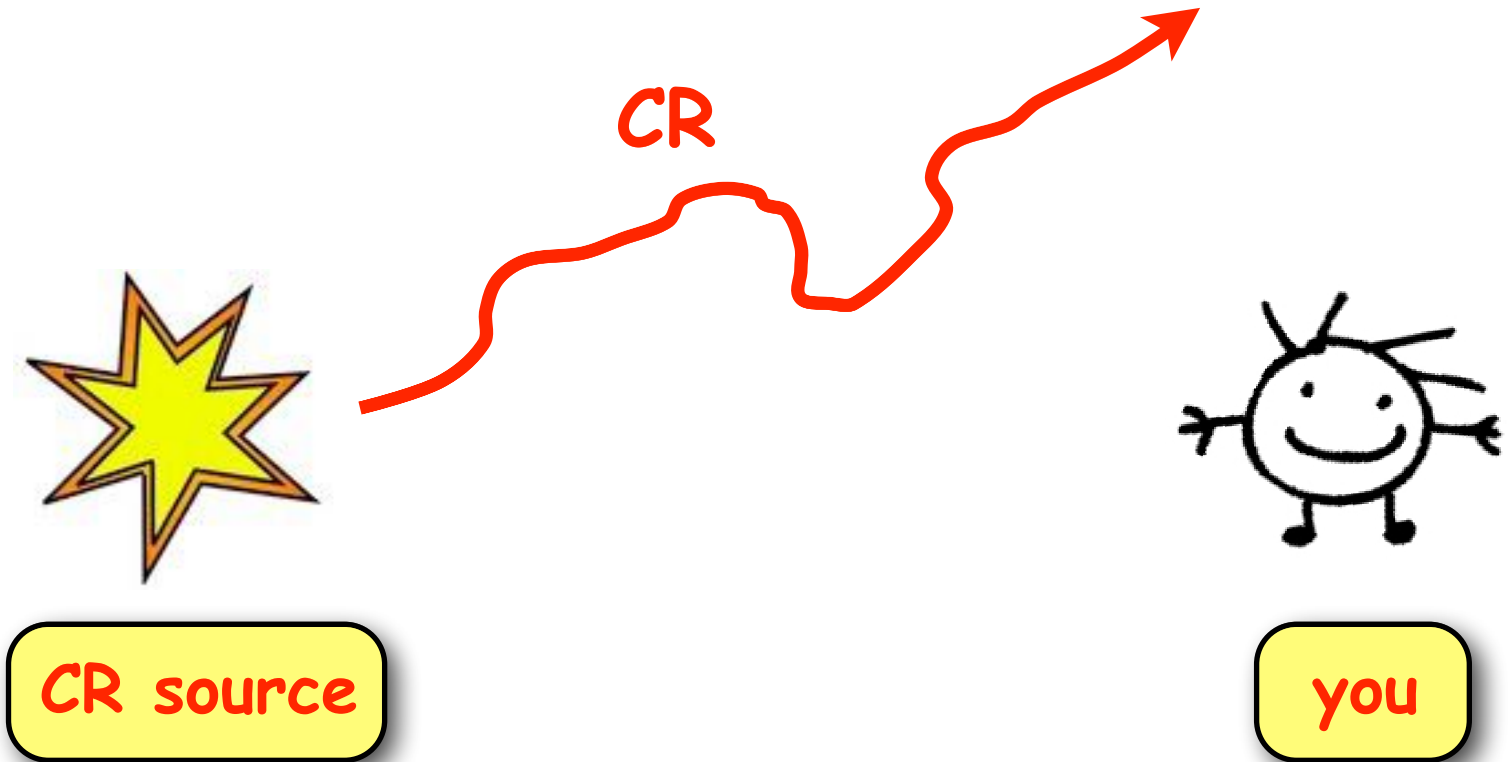
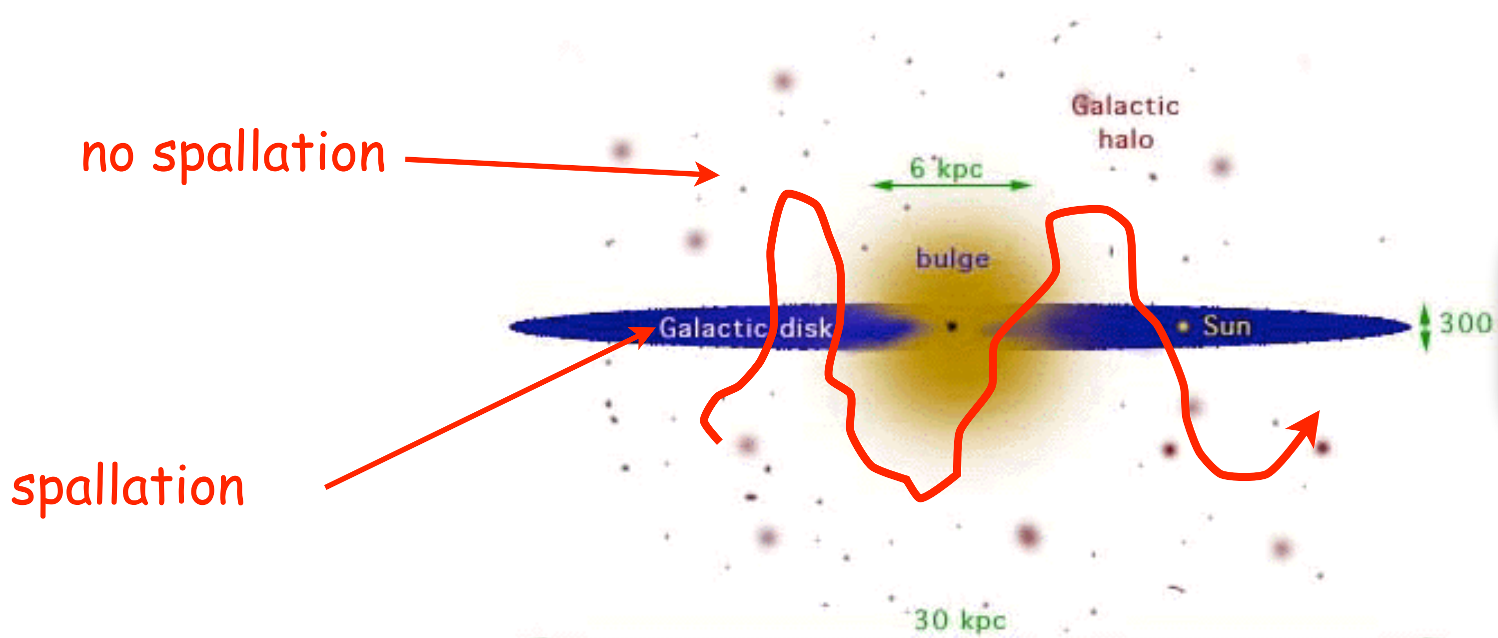


(2) CRs are confined in the Galactic disk

$$t_{disk} = \frac{l_s}{c} \approx 3 \times 10^6 \text{ yr}$$



$$\boxed{t_{halo} \approx 10 \times 10^6 \text{ yr}}$$

Cosmic Ray power in the Galaxy

CR energy density

total CR energy in the disk

MW disk volume

$$w_{CR} \sim 1 \text{ eV/cm}^3 \xrightarrow[\text{spatial homogeneity}]{} \mathcal{E}_{CR} = w_{CR} V_{disk}$$

<u>spatial homogeneity</u>

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$$w_{CR} \sim 1\ \mathrm{eV/cm^3} \longrightarrow \mathcal{E}_{CR} = w_{CR}\, V_{disk}$$

<u>spatial homogeneity</u>

$$\frac{\mathrm{d}\mathcal{E}_{CR}}{\mathrm{d}t} = P_{CR} - \frac{\mathcal{E}_{CR}}{t_{disk}}$$

CR power from CR sources in the disk

Cosmic Ray power in the Galaxy

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stability in time

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CR power from CR sources in the disk

$$P_{CR} = \frac{\mathcal{E}_{CR}}{t_{disk}} = 10^{41}\mathrm{erg/s}$$

Is this correct?

CRs interact with the gas -> $p + p \rightarrow p + p + \pi^0$

Should we use this equation instead?

$$\frac{\mathrm{d}\mathcal{E}_{CR}}{\mathrm{d}t} = P_{CR} - \frac{\mathcal{E}_{CR}}{t_{disk}} - \dot{\mathcal{E}}_{pp}$$

energy loss term due to p-p interactions

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energy loss term due to p-p interactions

Energy loss rate:

$$t_{pp} = (n_{gas}\ \sigma_{pp}\ c\ k)^{-1}$$

4×10^{-26} cm^2 0.45

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energy loss term due to p-p interactions

Energy loss rate:

$$t_{pp} = (n_{gas}\ \underbrace{\sigma_{pp}}_{4 \times 10^{-26}\ \mathrm{cm}^2}\ c\ \underbrace{k}_{0.45})^{-1} \approx 60 \left(\frac{n_{gas}}{\mathrm{cm}^{-3}}\right)^{-1} \mathrm{Myr} >> t_{disk} = 3\ \mathrm{Myr}$$

We can safely neglect CR energy losses

The diffusion of CRs

Spallation measurements tell us that cosmic rays follow tortuous paths before escaping the Galaxy. **Why?**

The galactic magnetic field or, better, **irregularities in the Galactic magnetic field** are responsible for the diffusive propagation of cosmic rays.

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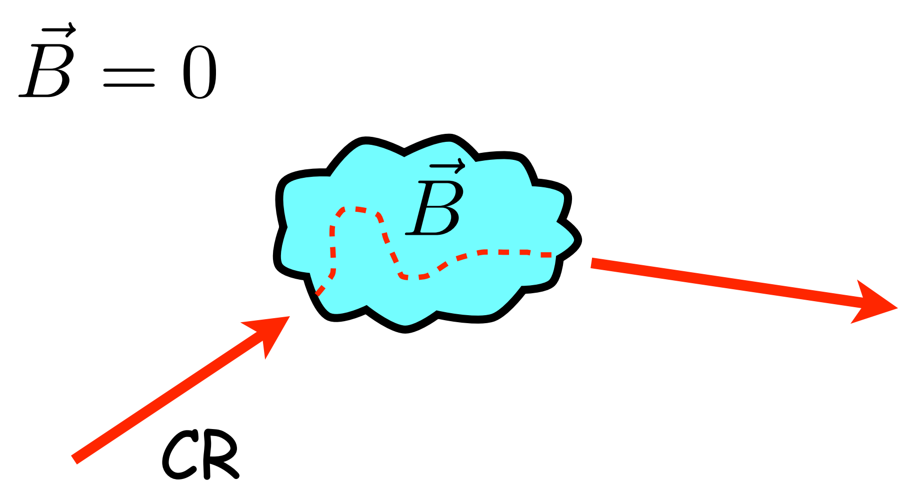
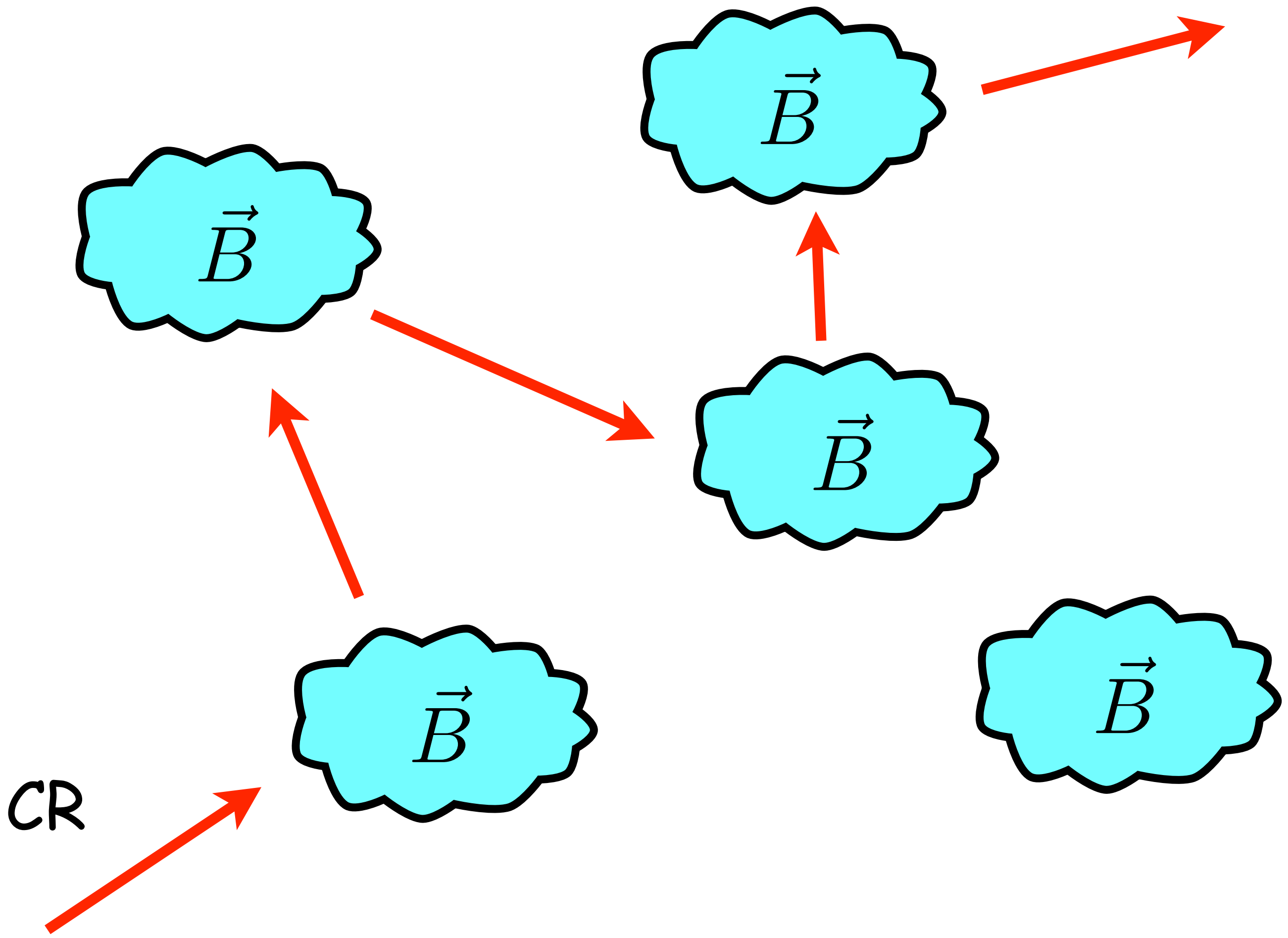
(Oversimplified picture)

magnetized cloudlets in an unmagnetized background



the particle energy is unchanged (Lorentz force)

The diffusion of CRs



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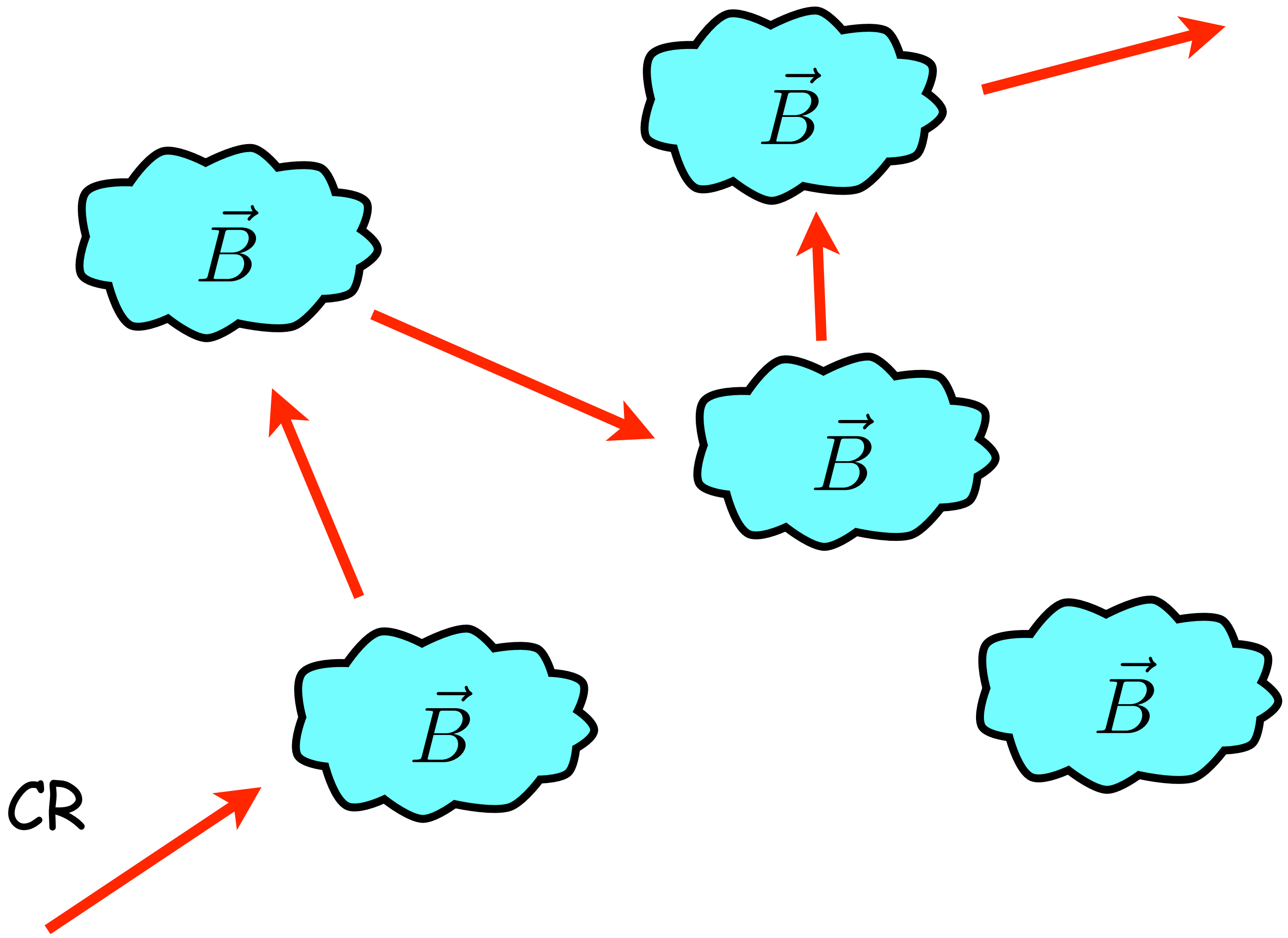
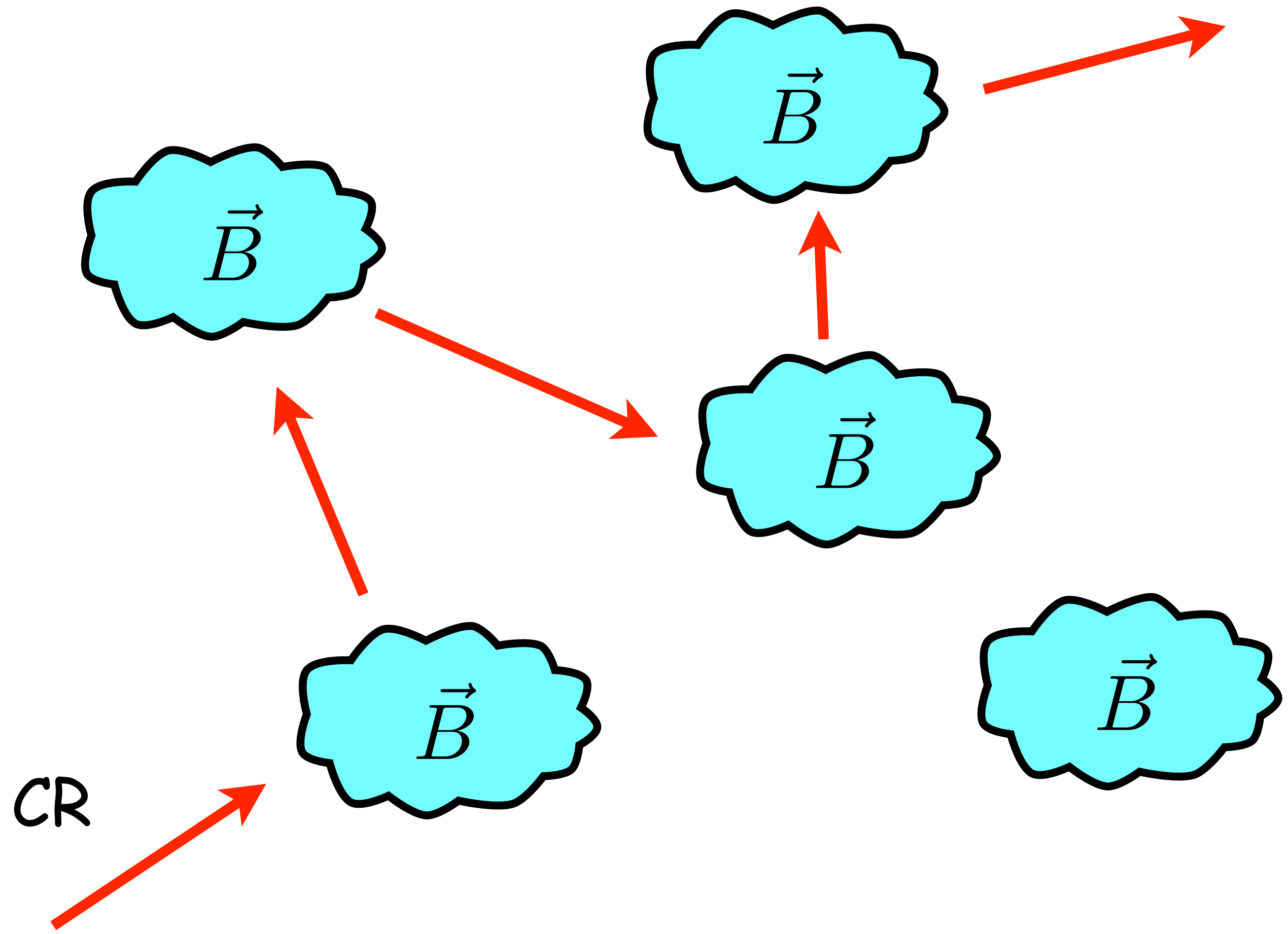


λ -> mean free path

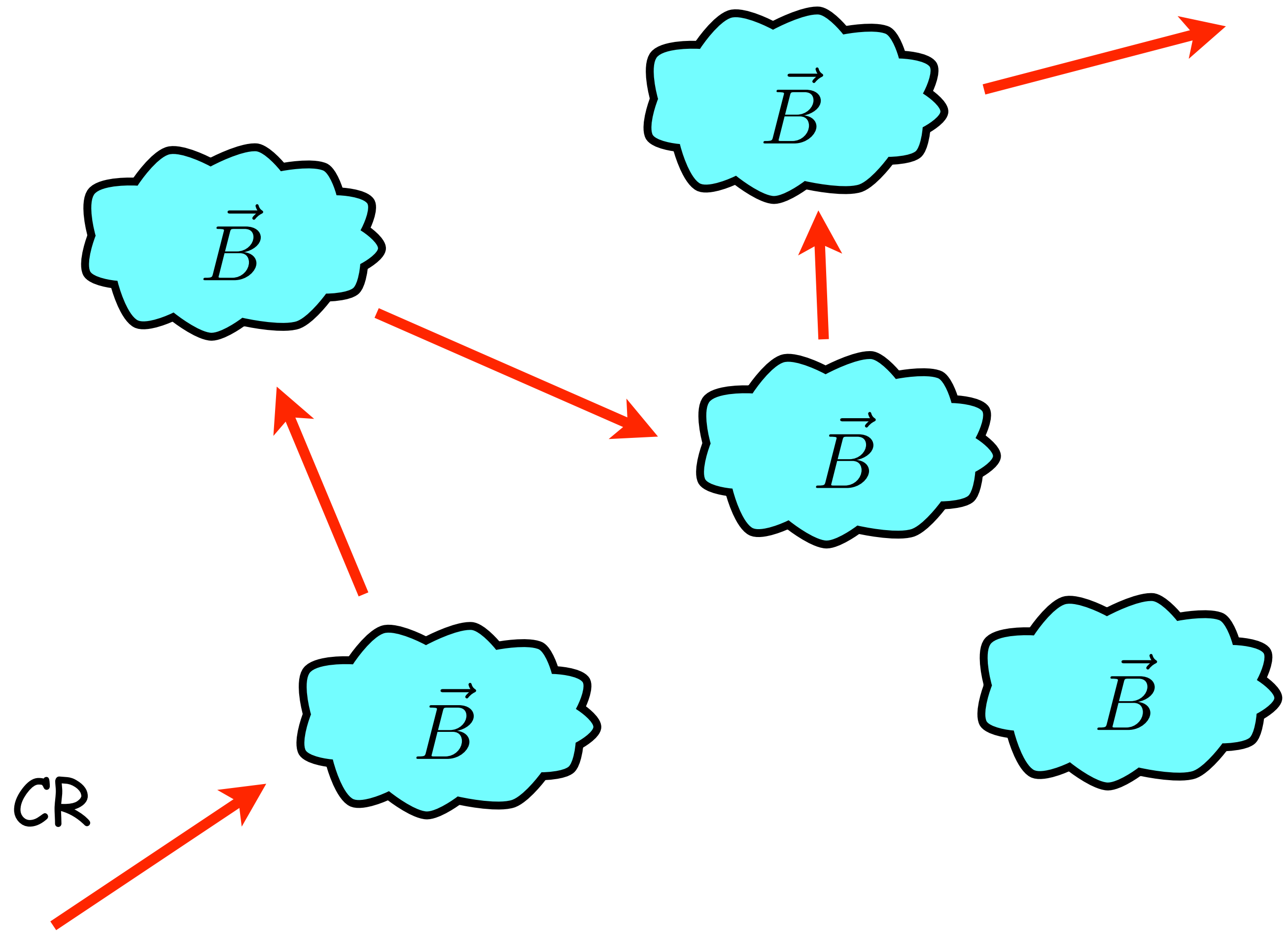
The diffusion of CRs



λ -> mean free path

$\tau_c = \dfrac{\lambda}{c}$ -> collision time

The diffusion of CRs

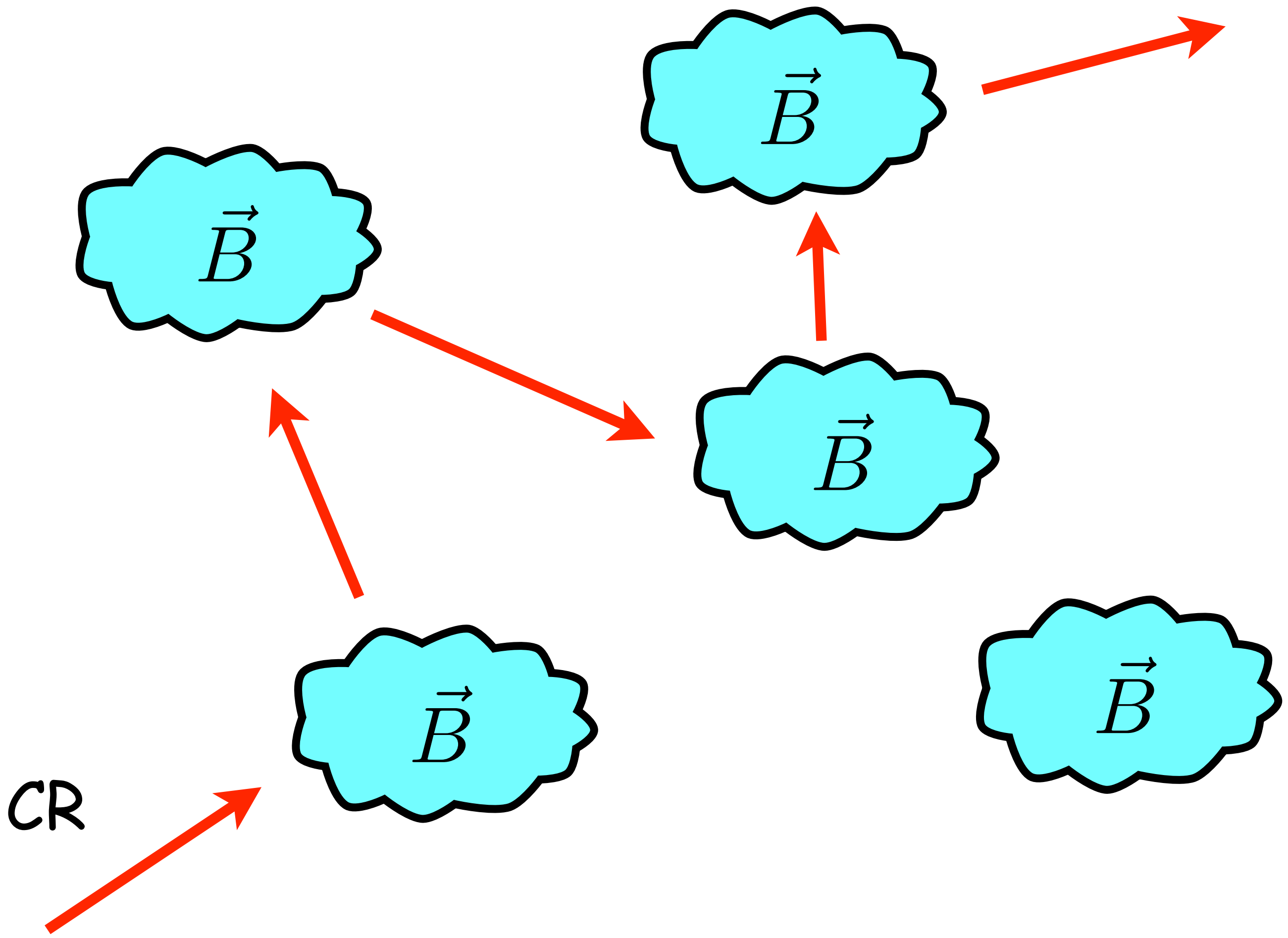


λ -> mean free path

$$\tau_c = \frac{\lambda}{c}$$ -> collision time

$$N = \frac{t}{\tau_c}$$ -> # collisions after time t

The diffusion of CRs



λ -> mean free path

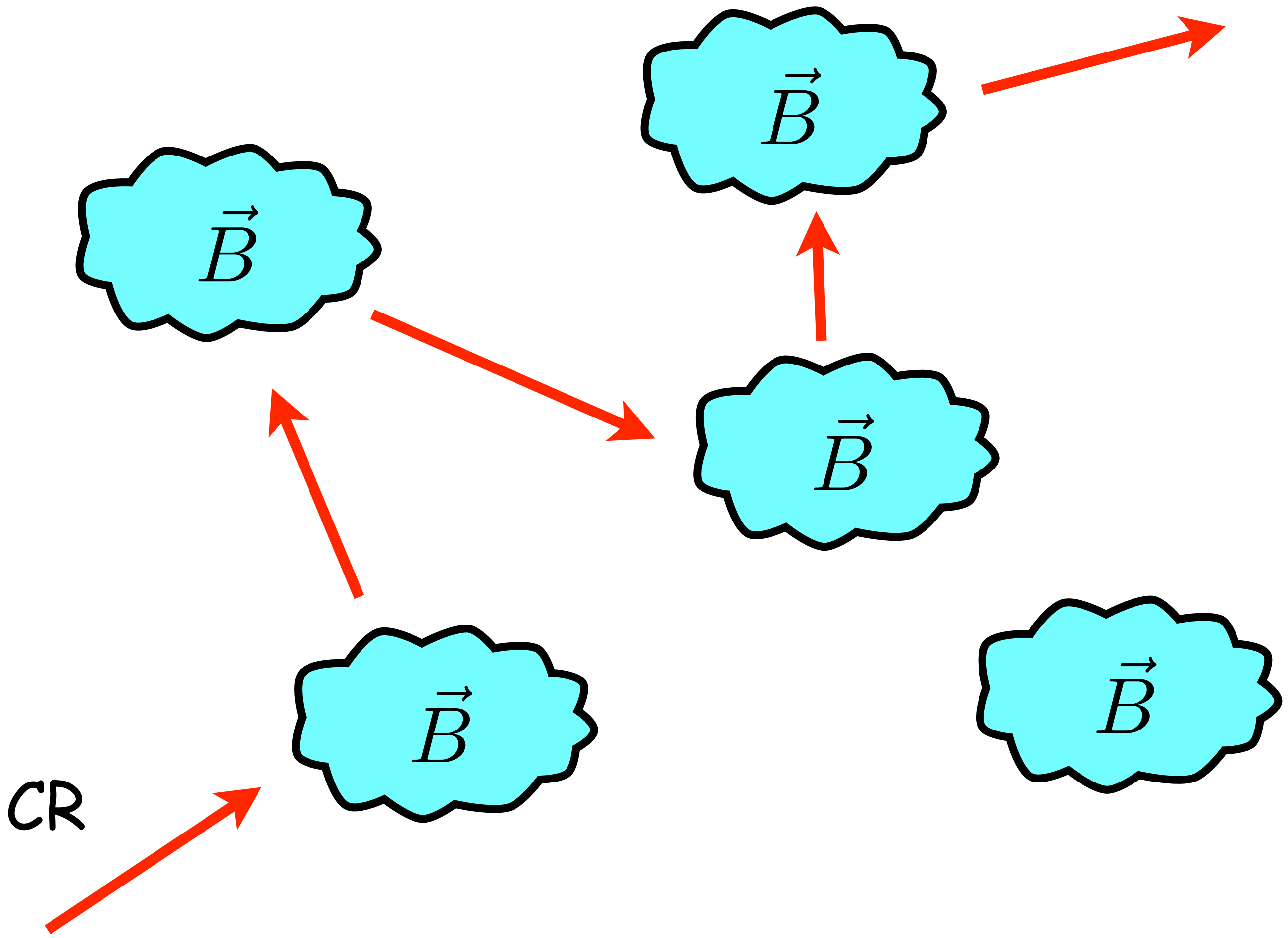
$\tau_c = \dfrac{\lambda}{c}$ -> collision time

$N = \dfrac{t}{\tau_c}$ -> # collisions after time t

diffusion length -> $l_d = \lambda \sqrt{N}$

random walk

The diffusion of CRs



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$$N = \frac{t}{\tau_c}$$ -> # collisions after time t

diffusion length -> $$l_d = \lambda \sqrt{N} = \lambda \sqrt{\frac{t}{\tau_c}} = \lambda \sqrt{\frac{t\ c}{\lambda}} = \sqrt{\lambda\ c\ t}$$

random walk

this product determines the diffusion properties of the particle

The diffusion of CRs

It is convenient to define the quantity $D = \lambda\ c$ called **diffusion coefficient**

diffusive propagation -> $$l_d \ = \ \sqrt{D\ t} \ \propto \sqrt{t}$$

straight line propagation -> $$l_{sl} \ = \ c\ t \ \propto \ t$$

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Spallation measurements allow us to measure the average diffusion coefficient in the Galaxy

$$l_{disk} = \sqrt{D\, t_{disk}} \;\longrightarrow\; D \;=\; \frac{l_{disk}^2}{t_{disk}} \;=\; 10^{28}\ \mathrm{cm}^2/\mathrm{s}$$

~300 pc

3 Myr (from spallation)

@ 10 GeV

CR diffusion is energy dependent

Spallation measurements at different energies -> $t_{disk} \propto E^{-0.3}$

which corresponds to -> $D \propto E^{0.3}$

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$$\frac{\mathrm{d}N_{CR}(E)}{\mathrm{d}t} = Q_{CR}(E) - \frac{N_{CR}(E)}{t_{disk}}$$

CRs injected from sources in the disk

escape rate from the disk

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stability in time

CRs injected from sources in the disk

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$$Q_{CR}(E) = \frac{N_{CR}(E)}{t_{disk}} \propto N_{CR}(E) D(E) \propto E^{-2.4}$$

measured -> $E^{-2.7}$

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*which sources???

A remarkable coincidence

Total CR power in the Galaxy ->

$$P_{CR} = 10^{41} \mathrm{erg/s}$$

A **SuperNova** is the explosion of a massive star that releases ~10^{51} ergs in form of kinetic energy. In the Galaxy the observed supernova rate is of the order of 1/30 yr^{-1}.

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Total CR power in the Galaxy -> $$P_{CR} = 10^{41} \mathrm{erg/s}$$

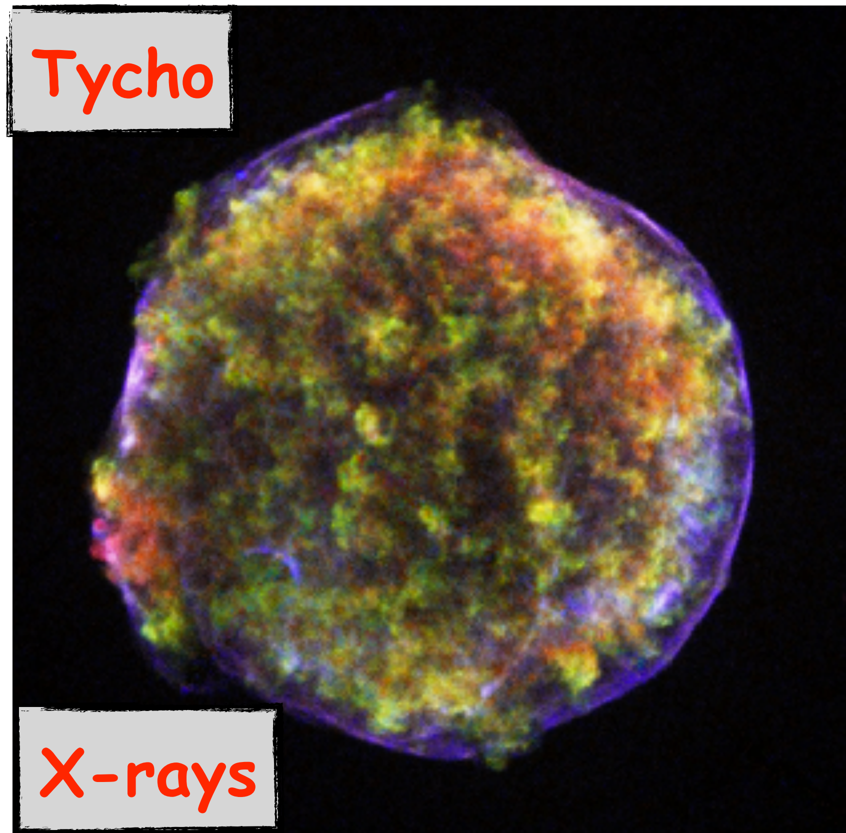
A SuperNova is the explosion of a massive star that releases $\sim 10^{51}$ ergs in form of kinetic energy. In the Galaxy the observed supernova rate is of the order of $1/30\ \mathrm{yr}^{-1}$.

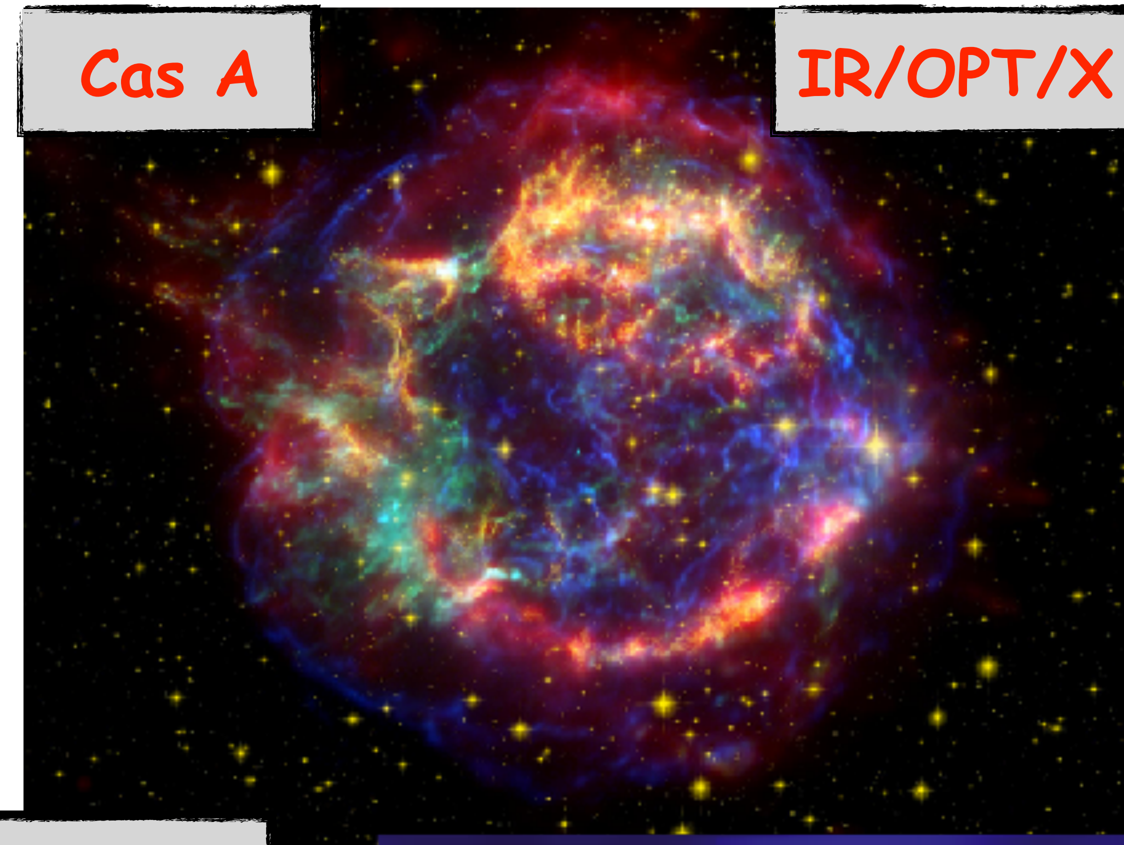
Total SN power in the Galaxy -> $$P_{SN} = 10^{42} \mathrm{erg/s}$$

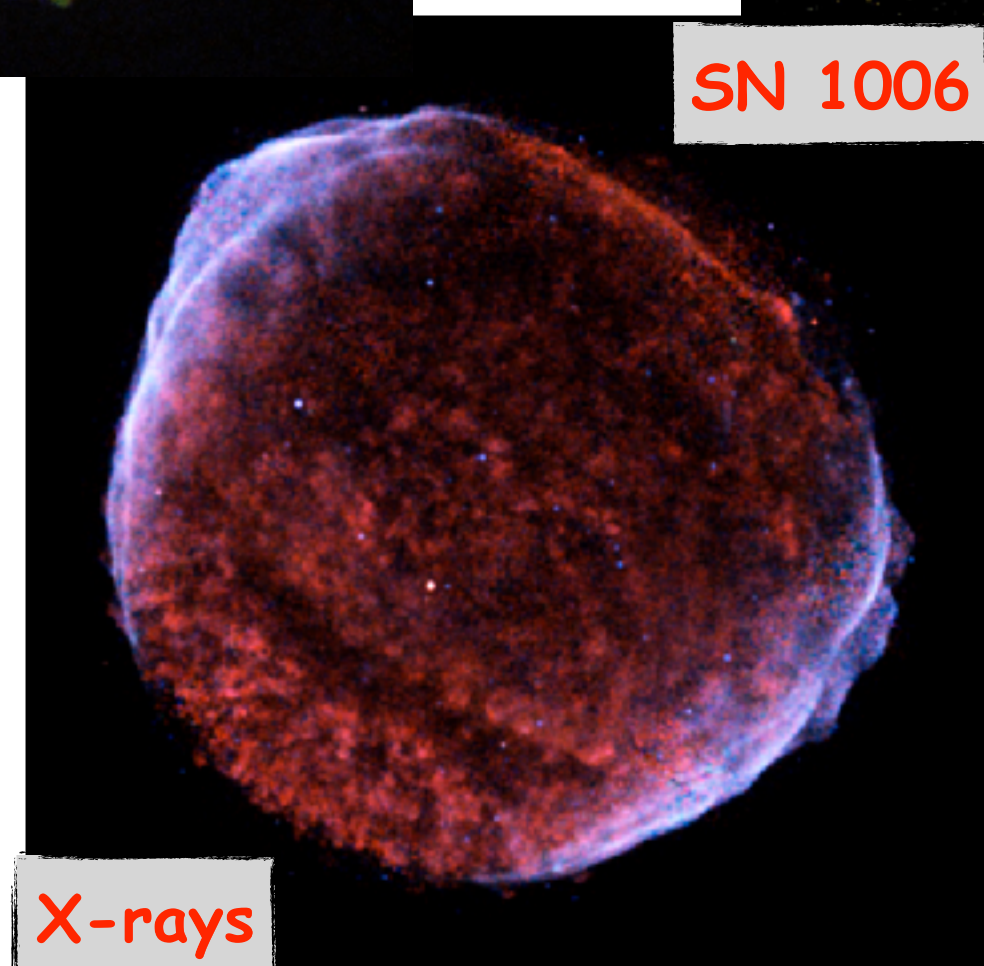
SuperNovae alone could maintain the CR population provided that about 10% of their kinetic energy is somehow converted into CRs

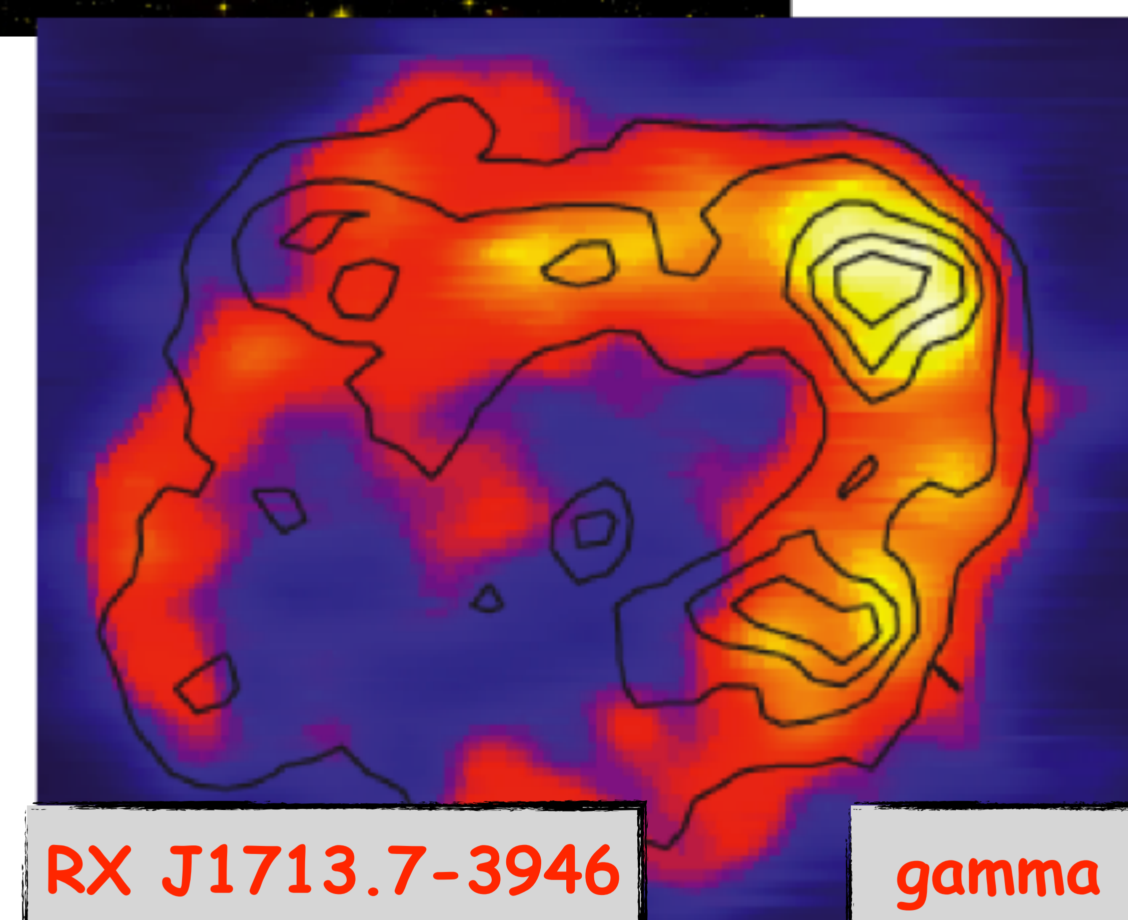
DIFFUSIVE SHOCK ACCELERATION

Supernova remnant shocks









Particle acceleration

Ohm's law

Static electric field -> $$\vec{E} = \frac{\vec{j}}{\sigma} \approx 0$$

electric conductivity -> infinity
in astrophysical plasmas!

Lorentz's force

Static magnetic field -> $$\vec{F} = q\,\vec{v} \times \vec{B}$$

no work done on the particle

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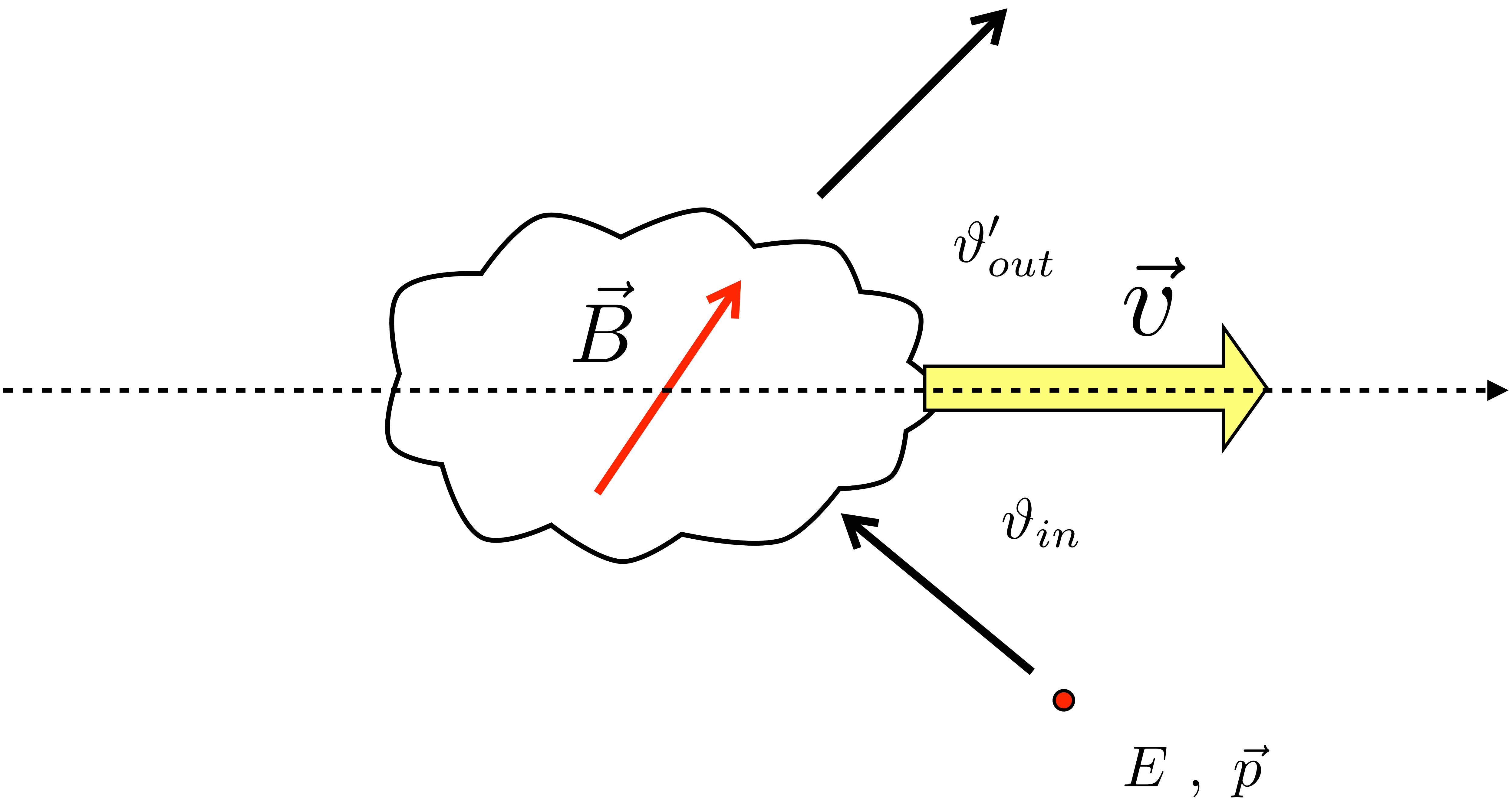
Induced E-field ->

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

Moving B-field ->

$$\vec{E} = -\frac{1}{c}\,\vec{v} \times \vec{B}$$

Fermi's idea (1949, 1954)

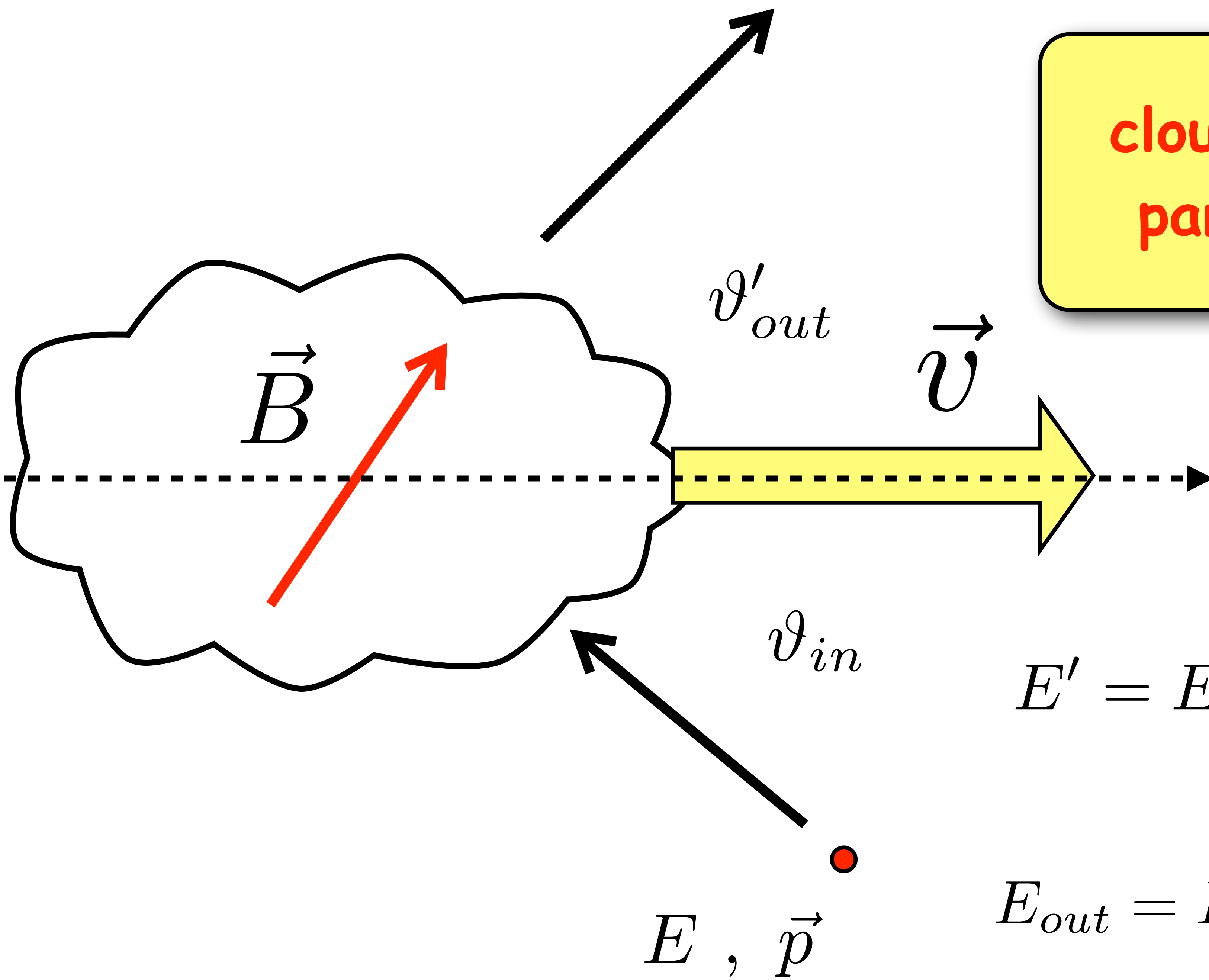


* primed quantities are measured in the rest frame of the cloud

Energy gain (loss) per interaction

$$E' = \gamma_v \left(E - v\, p\, \cos(\vartheta_{in})\right)$$

cloud -> non-relativistic
particle -> relativistic



cloud speed

$$E' = E\left(1 - \beta\cos(\vartheta_{in}) + \frac{\beta^2}{2}\right)$$

$$E_{out} = E'\left(1 + \beta\cos(\vartheta'_{out}) + \frac{\beta^2}{2}\right)$$

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Energy gain (loss) per interaction

$$\frac{\Delta E}{E} = \beta\left[\cos(\vartheta'_{out}) - \cos(\vartheta_{in})\right] + \beta^2\left[1 - \cos(\vartheta_{in})\cos(\vartheta'_{out})\right]$$

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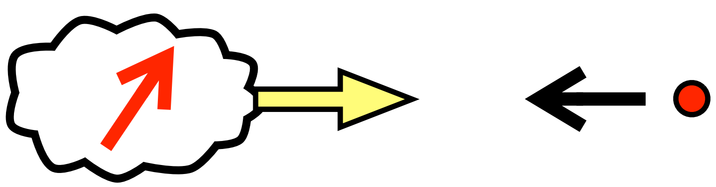
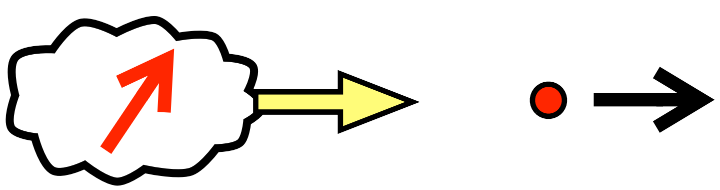
head-on collision

before:



after:



$$\frac{\Delta E}{E} = 2\, \beta\, (1 + \beta)$$

Energy gain (loss) per interaction

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head-on collision

before:

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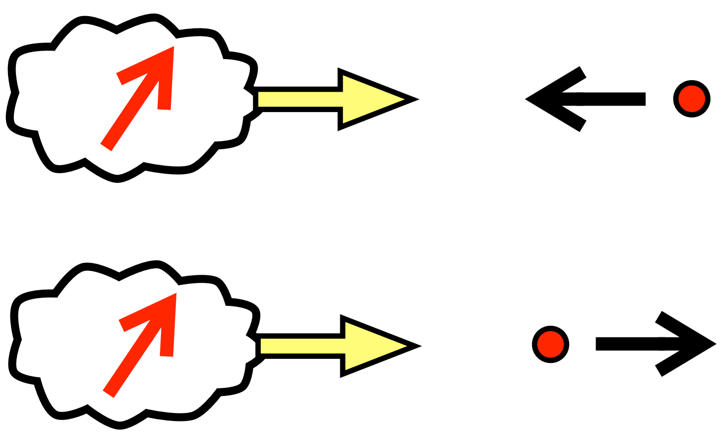
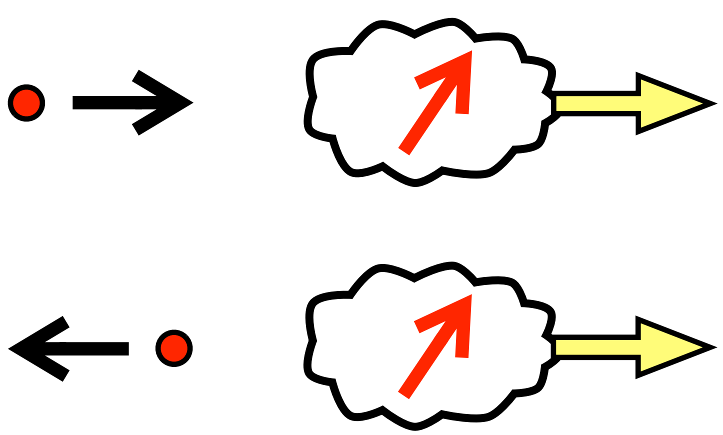


$$\frac{\Delta E}{E} = 2\,\beta\,(1+\beta)$$

tail-on collision

before:

after:



$$\frac{\Delta E}{E} = -2\,\beta\,(1-\beta)$$

Average energy gain

$$\boxed{\frac{\Delta E}{E} = \beta\left[\cos(\vartheta'_{out}) - \cos(\vartheta_{in})\right] + \beta^2\left[1 - \cos(\vartheta_{in})\cos(\vartheta'_{out})\right]}$$

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- isotropy of particles in the cloud frame ----> $\langle \cos(\vartheta'_{out}) \rangle = 0$
- # particles between ϑ_{in} and $\vartheta_{in} + \mathrm{d}\vartheta_{in}$ ----> $\propto \sin(\vartheta_{in})\mathrm{d}\vartheta_{in}$
- rate at which particles enter the cloud prop. to ---> $\propto 1 - \beta \cos(\vartheta_{in})$

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$$\boxed{\langle \cos(\vartheta_{in}) \rangle = -\frac{\beta}{3}}$$

Second order Fermi mechanism

$$\boxed{\left\langle \frac{\Delta E}{E} \right\rangle \approx \frac{4}{3}\,\beta^2}$$

Second order Fermi mechanism

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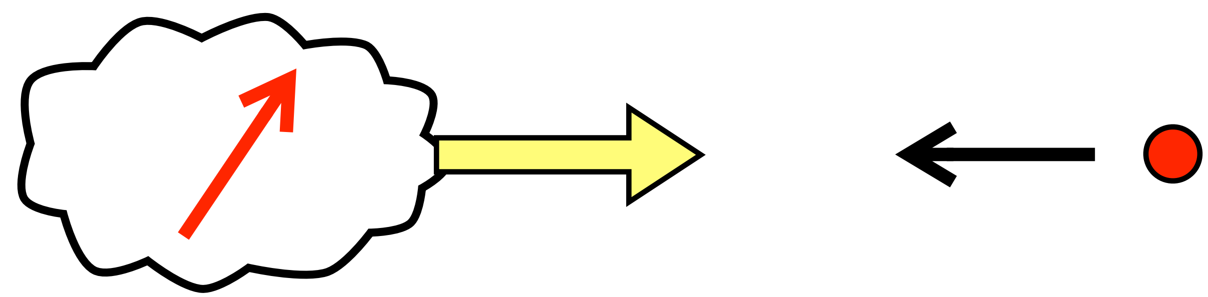
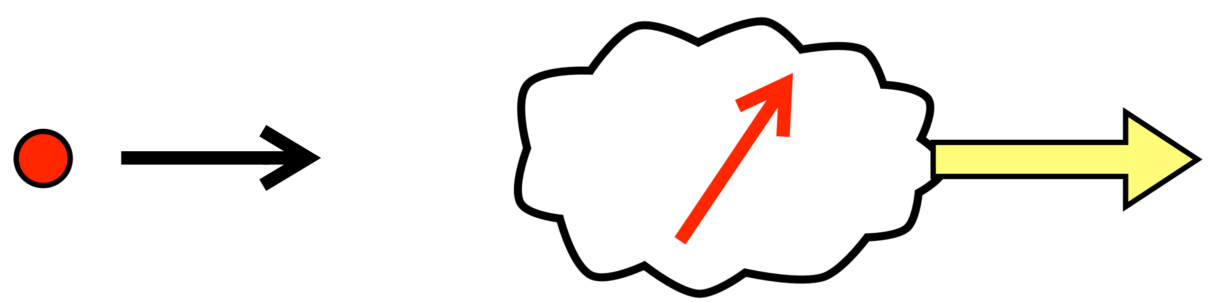
inefficient -> too slow...

A very simple idea

head-on collision



tail-on collision



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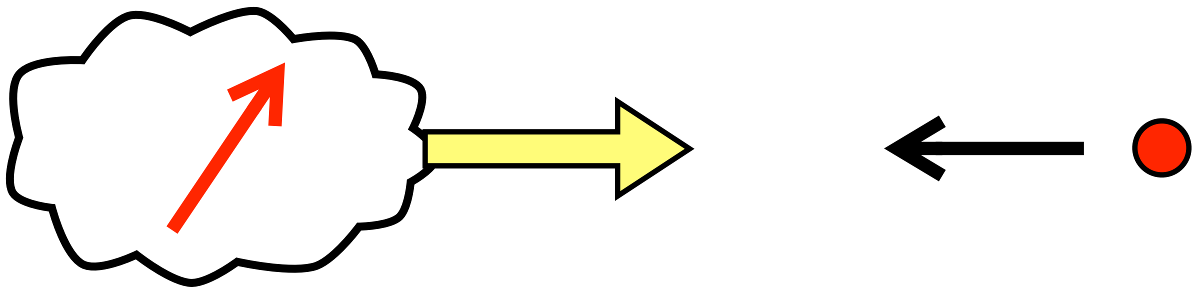

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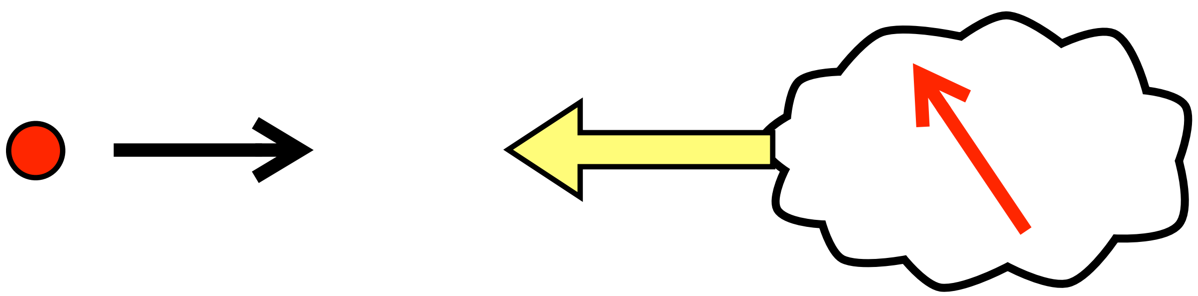

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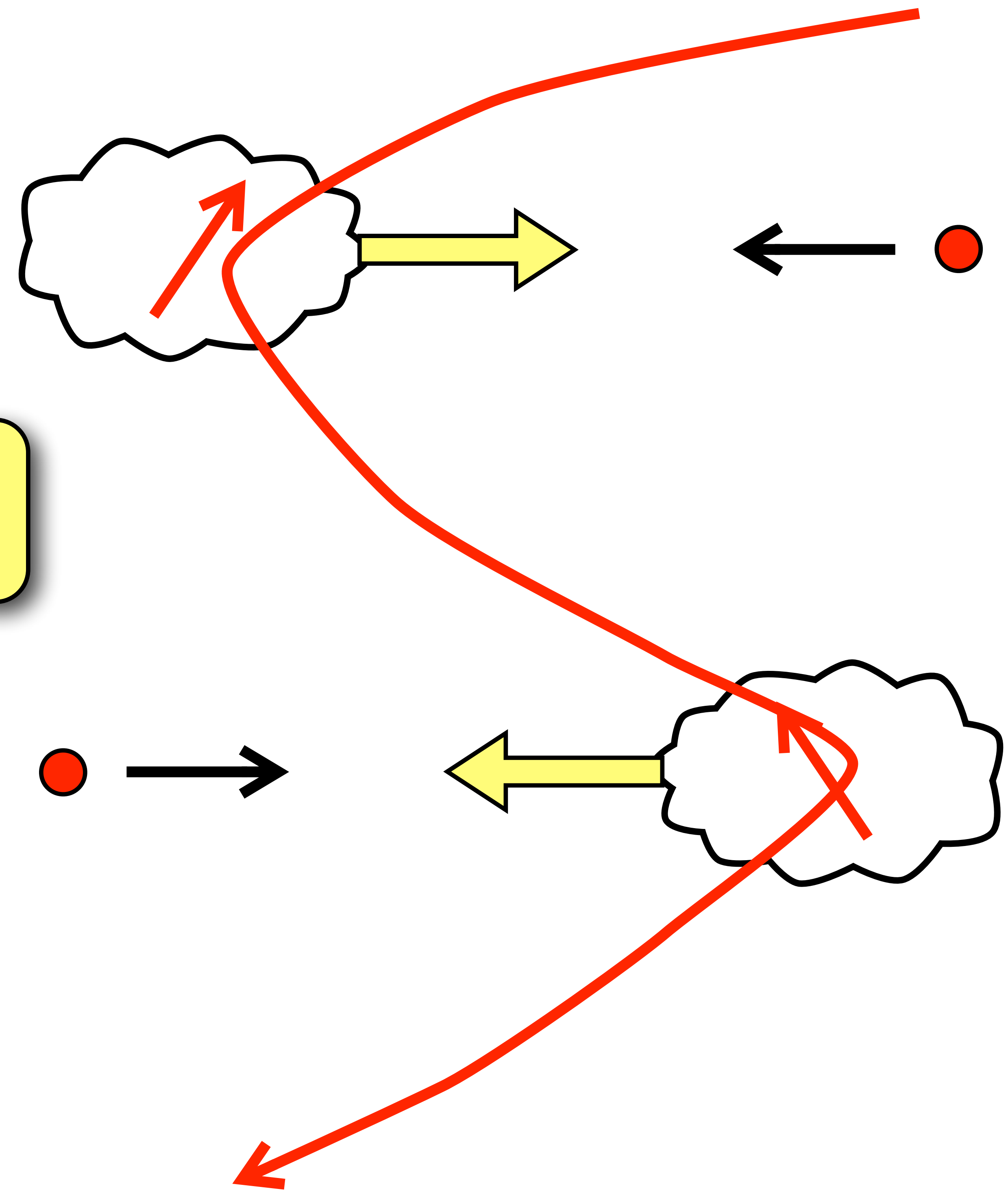
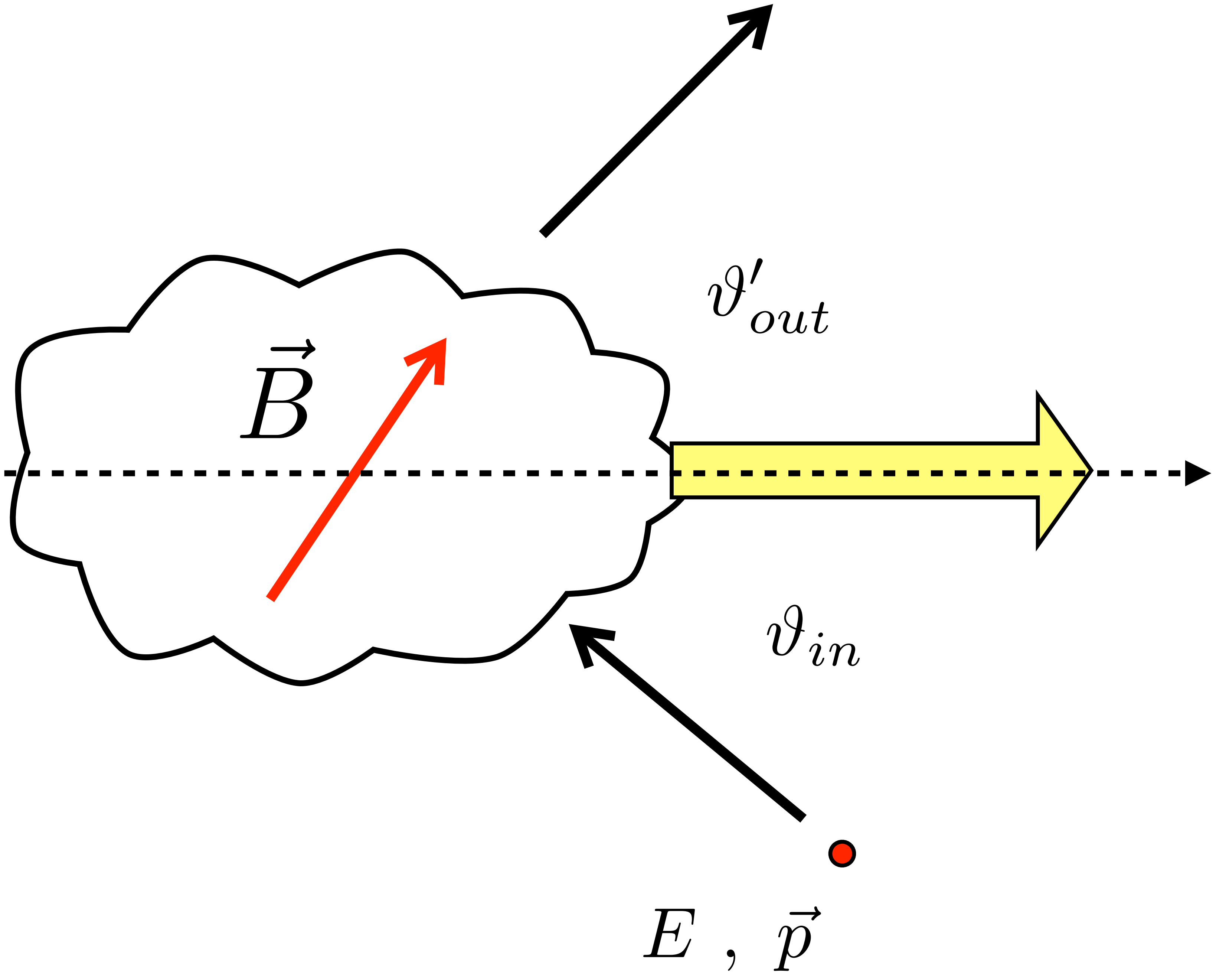


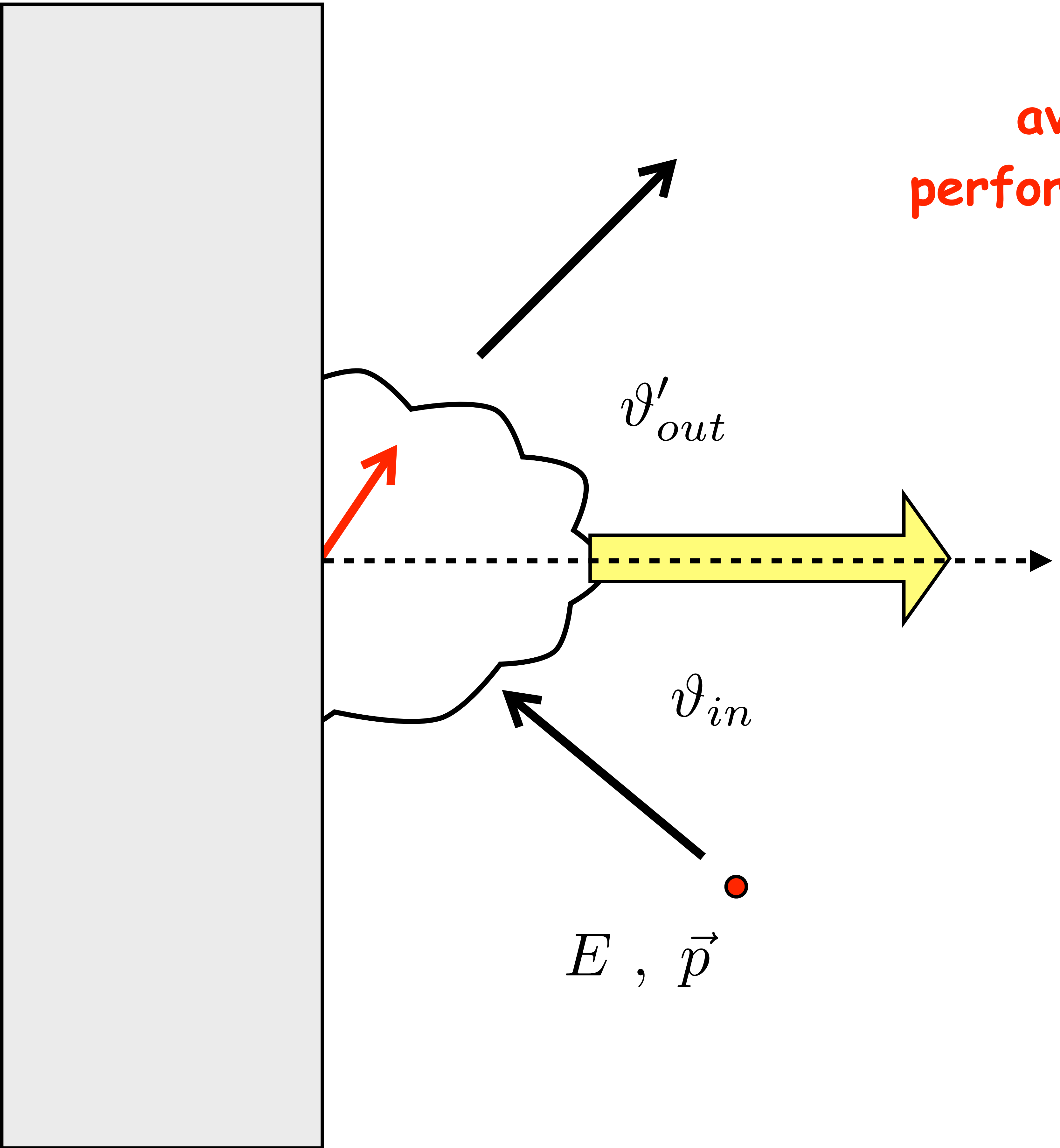
Converging walls

head-on collision

First order Fermi mechanism



First order Fermi mechanism



averages have to be performed over the interval:

$$0 < \vartheta < \frac{\pi}{2}$$

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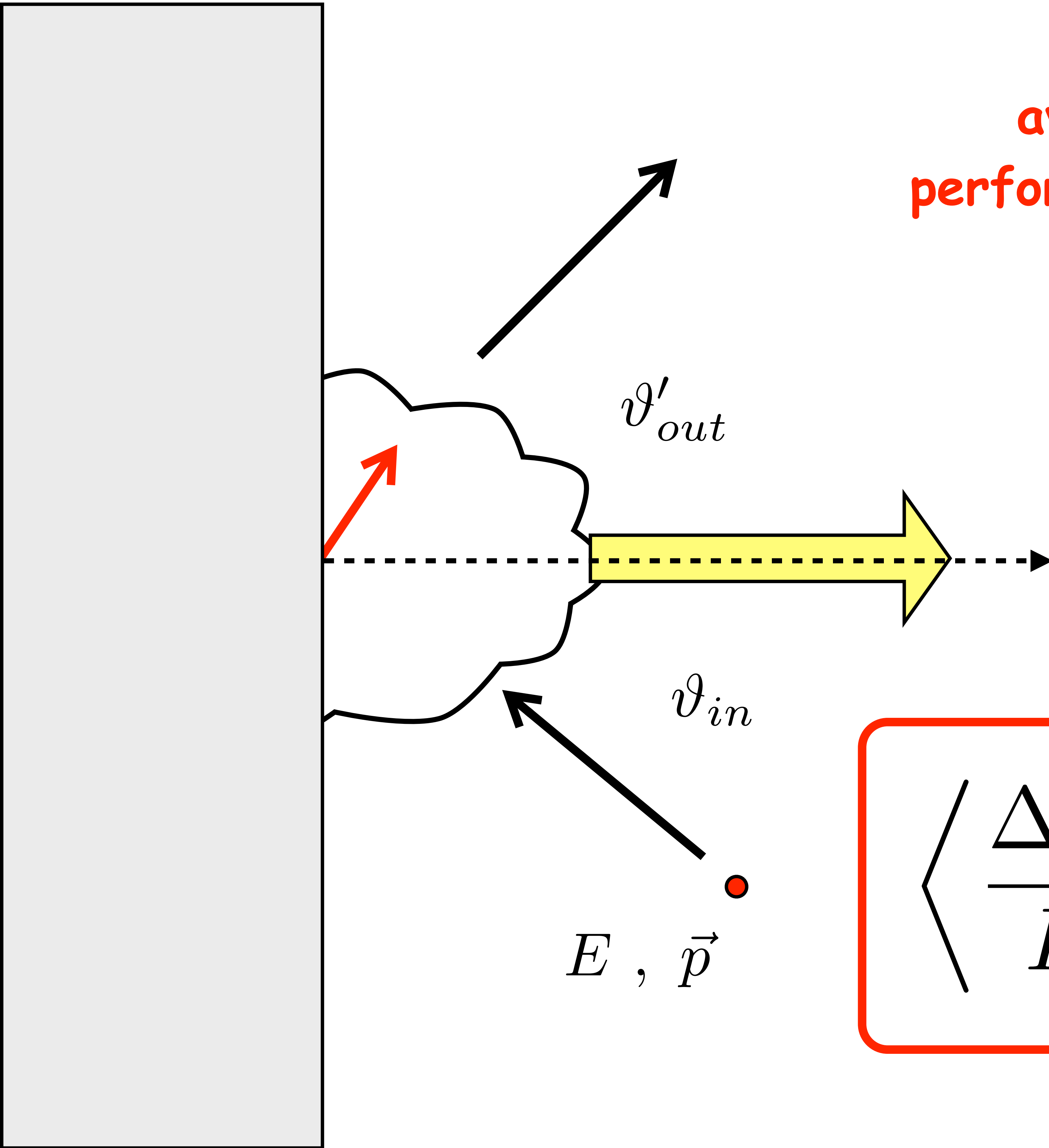

averages have to be performed over the interval:

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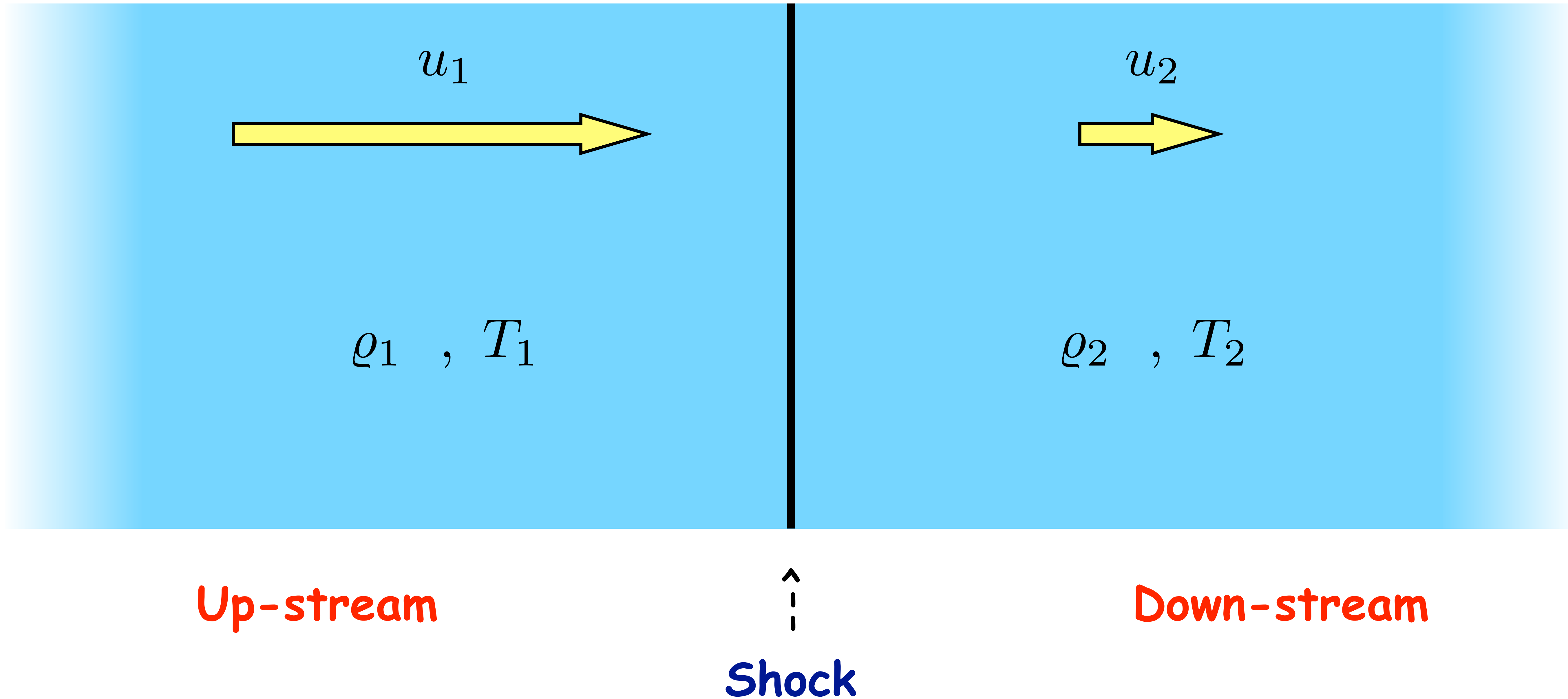
First order!

$$\boxed{\left\langle \frac{\Delta E}{E} \right\rangle \approx \frac{4}{3}\,\beta}$$

Shock waves in one slide

Shock rest frame



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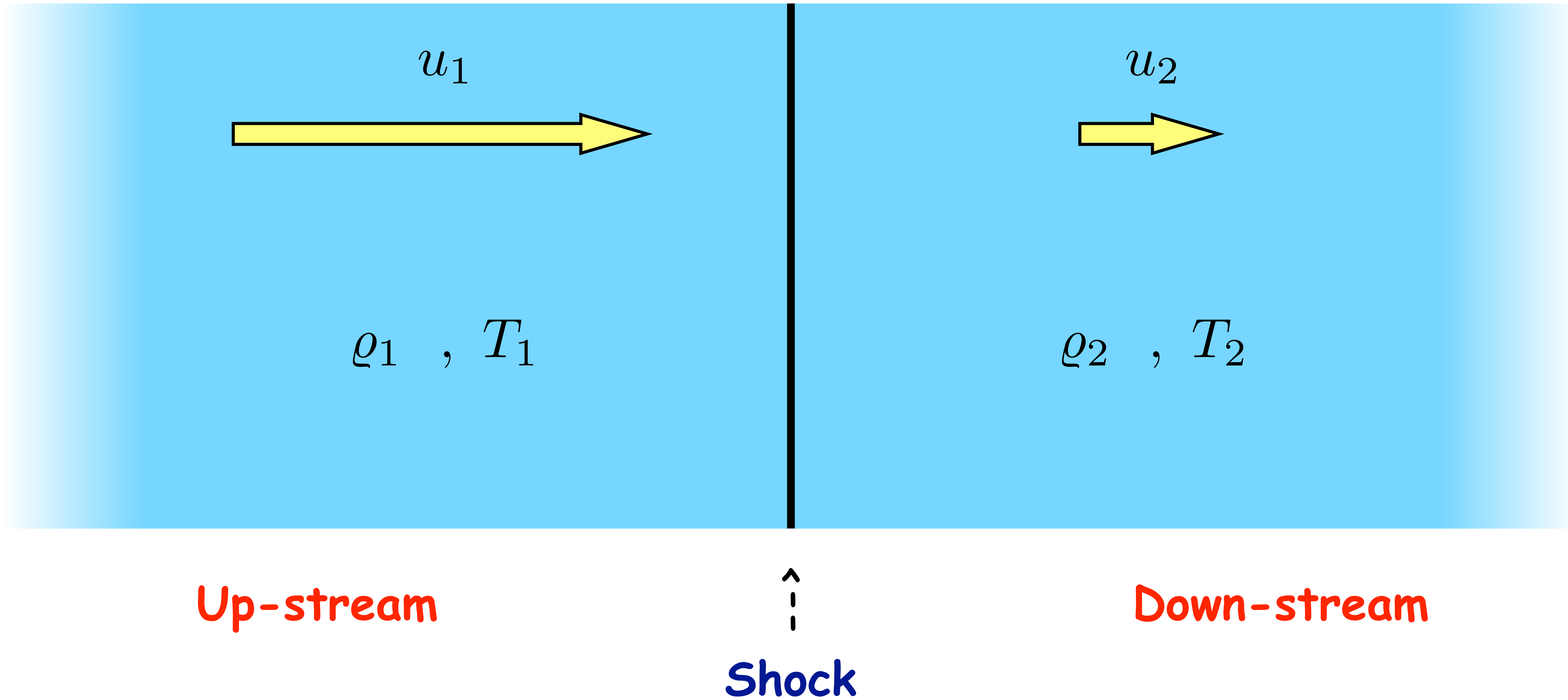
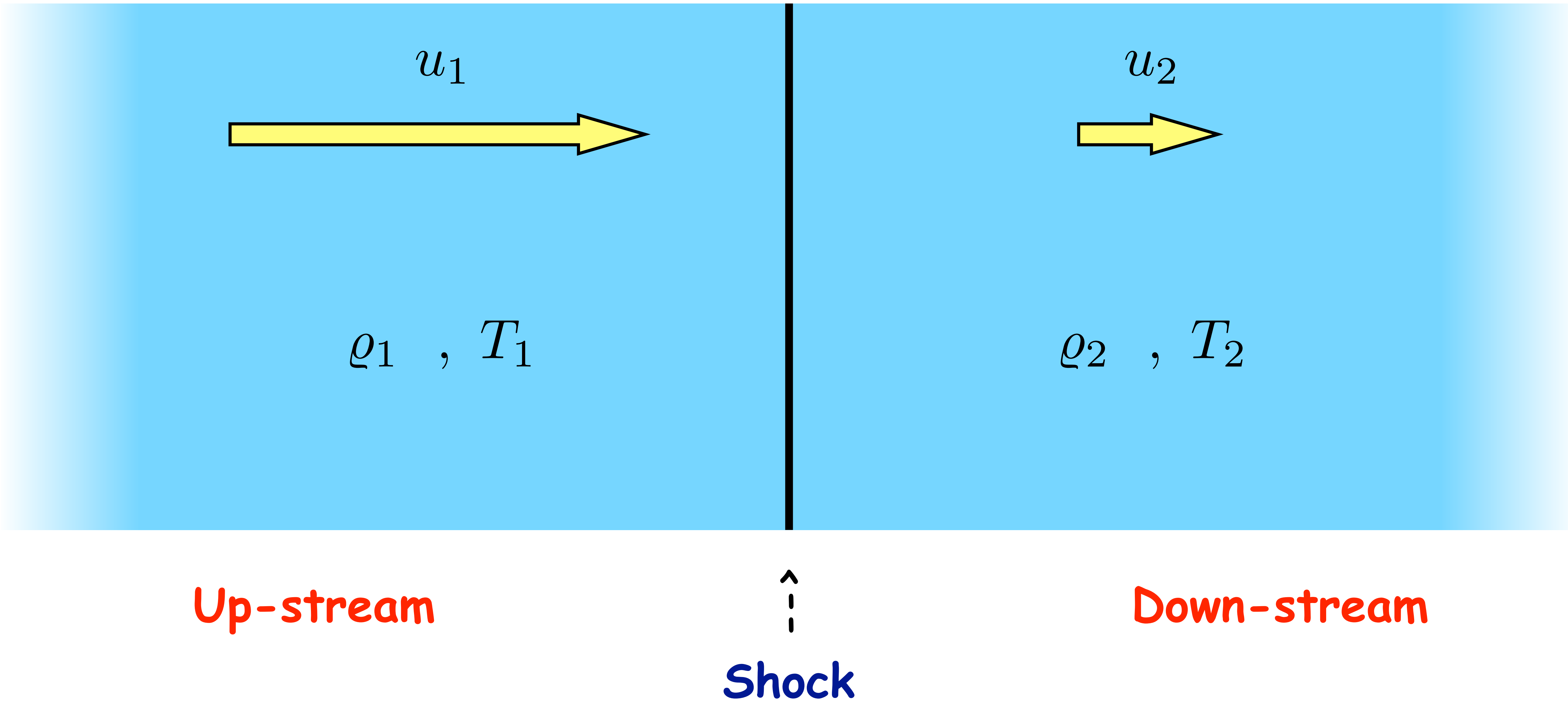


In 1960 (!) F. Hoyle (first?) suggests that shocks accelerate CRs

$$\frac{\varrho_2}{\varrho_1} = \frac{u_1}{u_2} = \frac{\gamma+1}{\gamma-1} = 4 \qquad\qquad p_2 = \frac{2}{\gamma+1}\ \varrho_1 u_1^2$$



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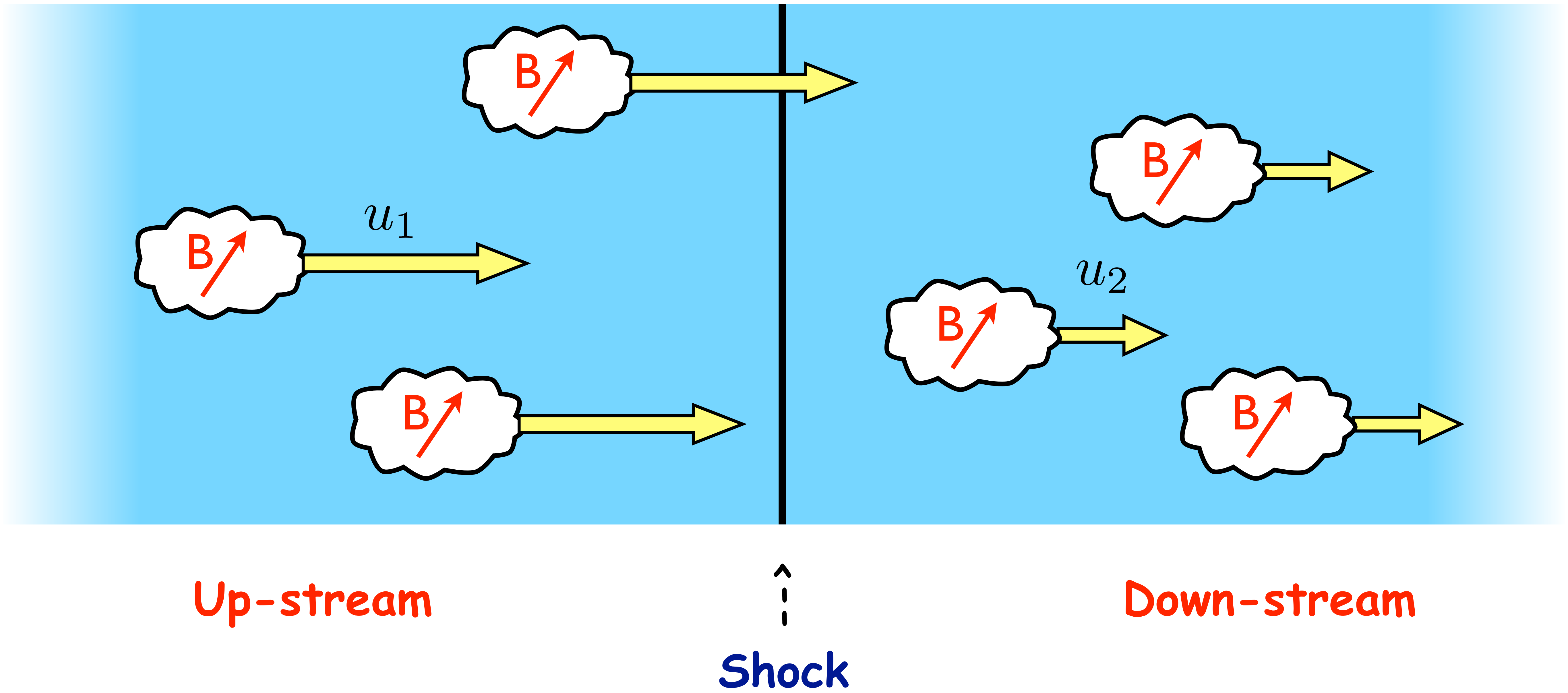
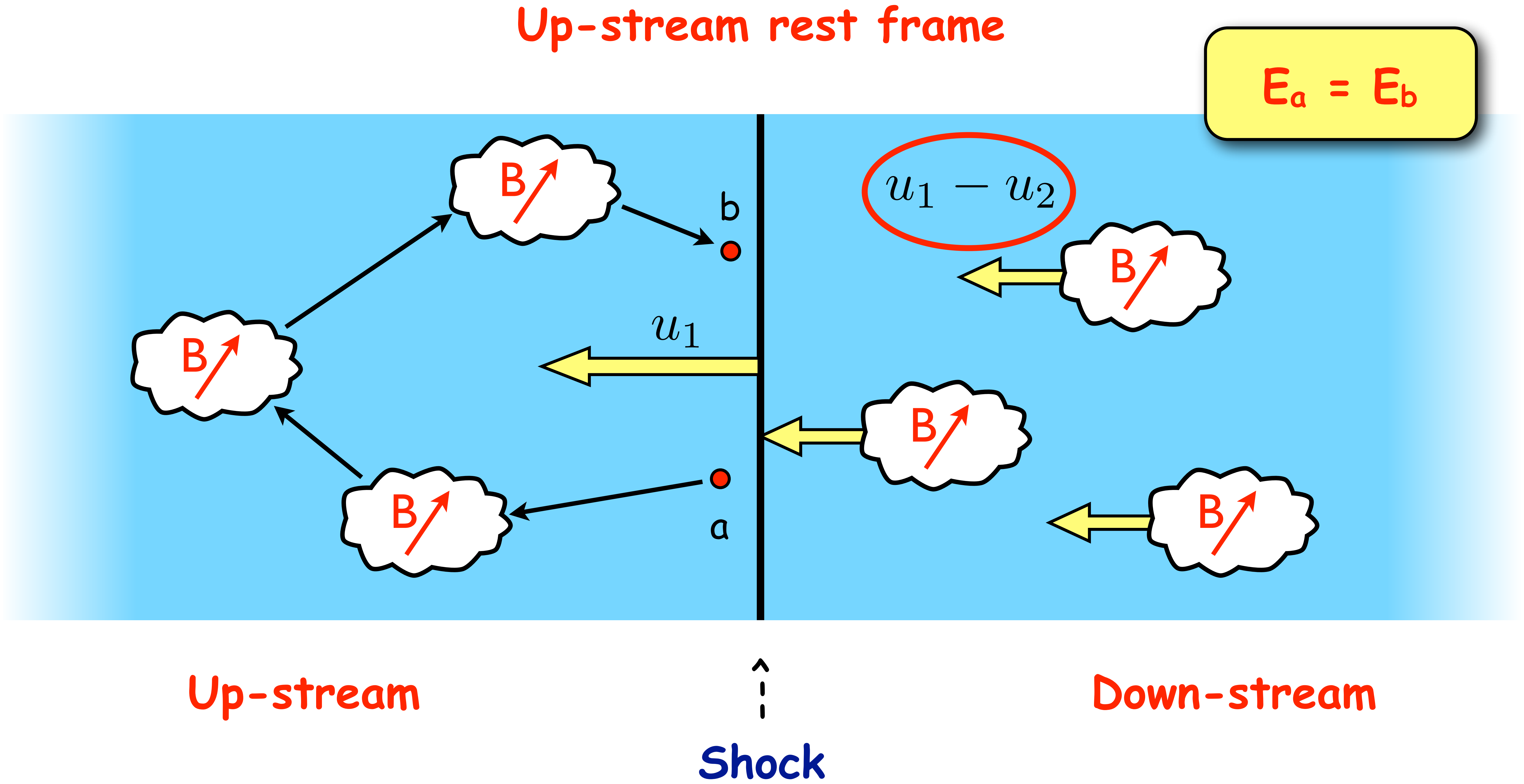
Diffusive Shock Acceleration

Shock rest frame



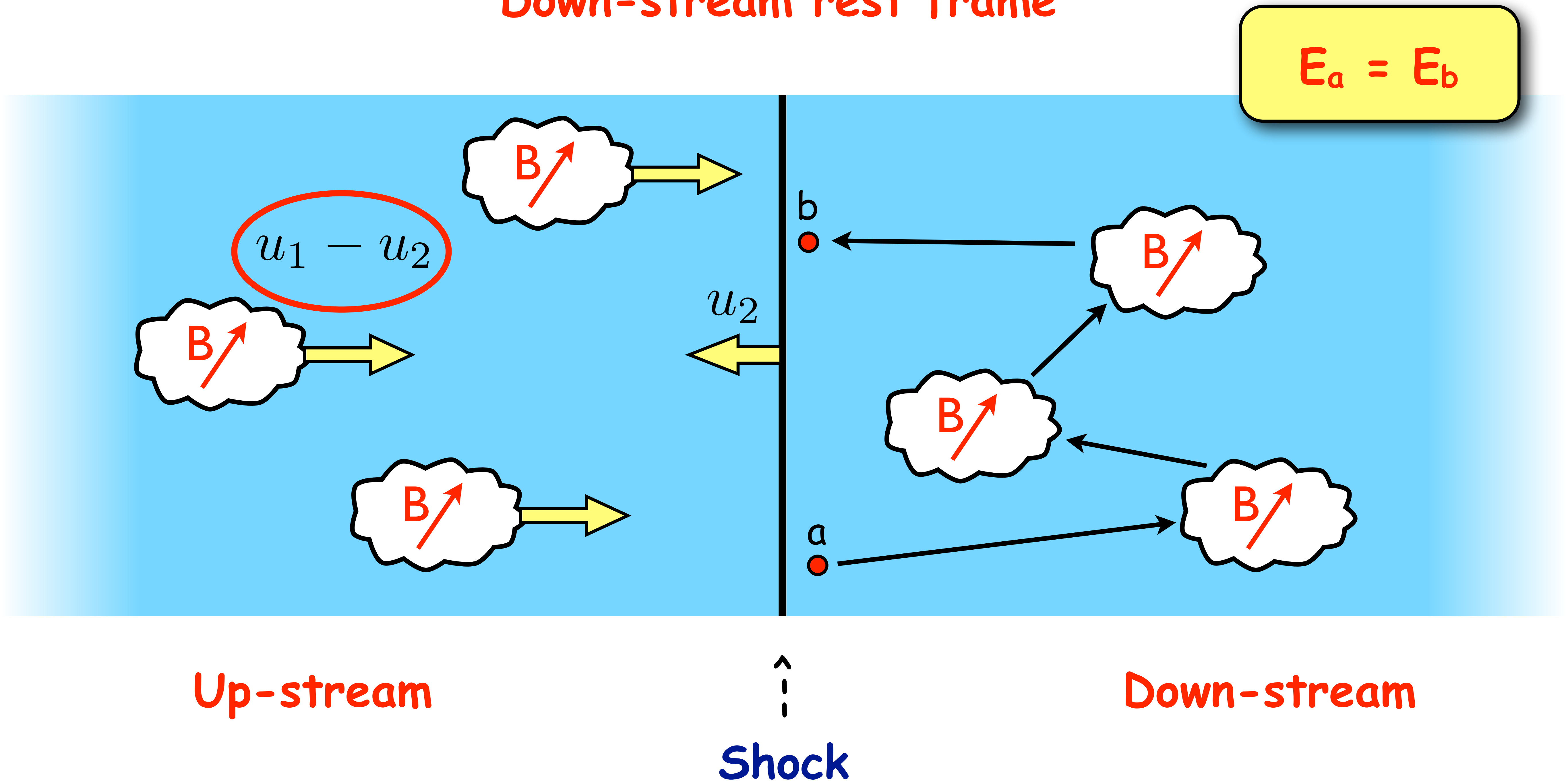
Krymskii 1977, Axford et al. 1977, Blandford & Ostriker 1978, Bell 1978

Diffusive Shock Acceleration



Diffusive Shock Acceleration

Down-stream rest frame



Diffusive Shock Acceleration

Symmetry

Every time the particle crosses the shock (up -> down or down -> up), it undergoes an head-on collision with a plasma moving with velocity u_1-u_2

Diffusive Shock Acceleration

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Asymmetry

(Infinite and plane shock:) Upstream particles always return the shock, while downstream particles may be advected and never come back to the shock

Universality
of diffusive shock acceleration

Let's search for a test-particle solution

Assumption: scattering is so effective at shocks that the distribution of particles is isotropic

-> an universal solution of the problem can be found

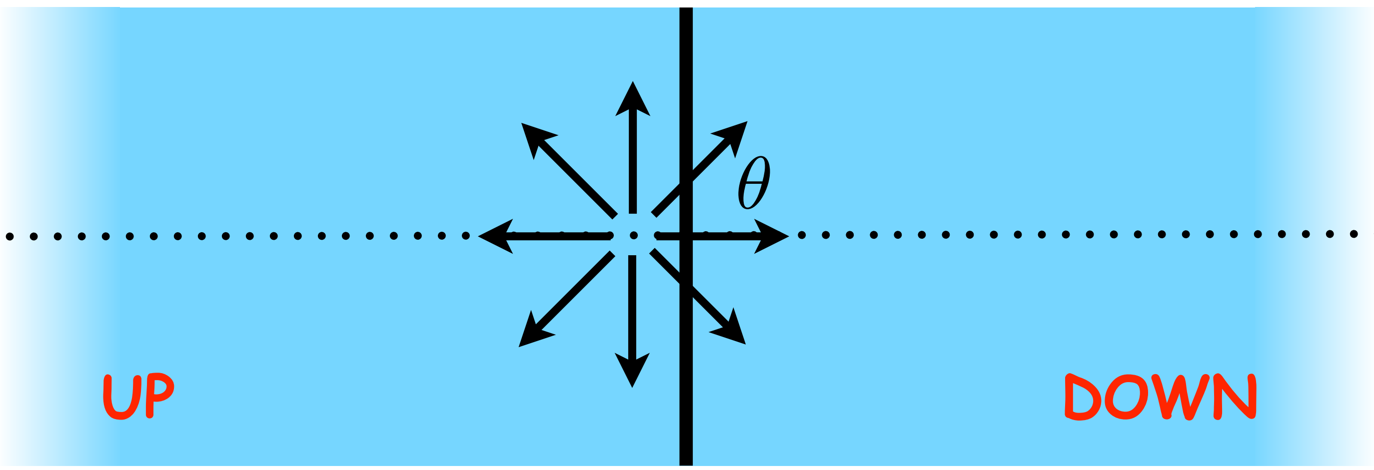
Rate at which particles cross the shock

Let's calculate R_{in}...

n -> density of accelerated particles close to the shock

n is isotropic: $$\mathrm{d}n = \frac{n}{4\pi}\,\mathrm{d}\Omega$$

velocity across the shock: $c\,\cos(\theta)$



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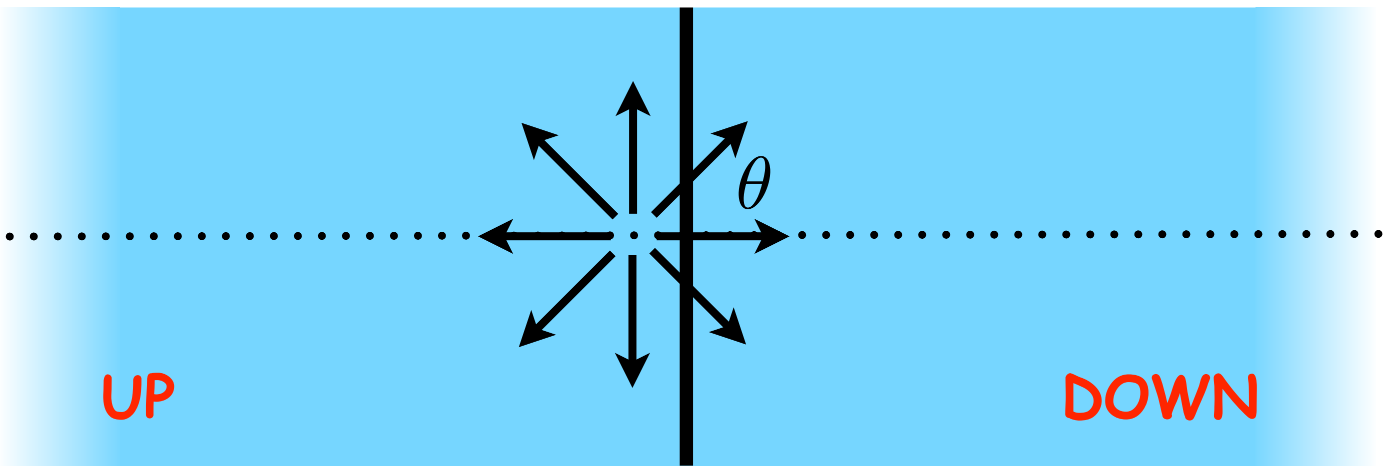

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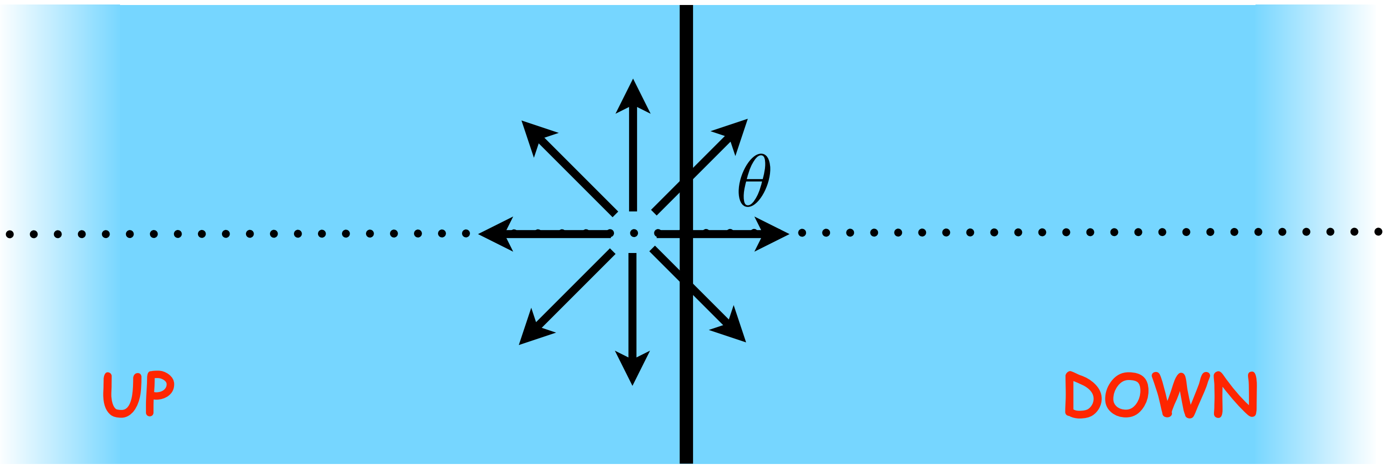

$$R_{in} = \int_{up \rightarrow down} \mathrm{d}n \ c \ \cos(\theta) = \frac{n \ c}{4\pi} \int_0^{\frac{\pi}{2}} \cos(\theta) \sin(\theta) \mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\psi = \frac{1}{4} \ n \ c$$

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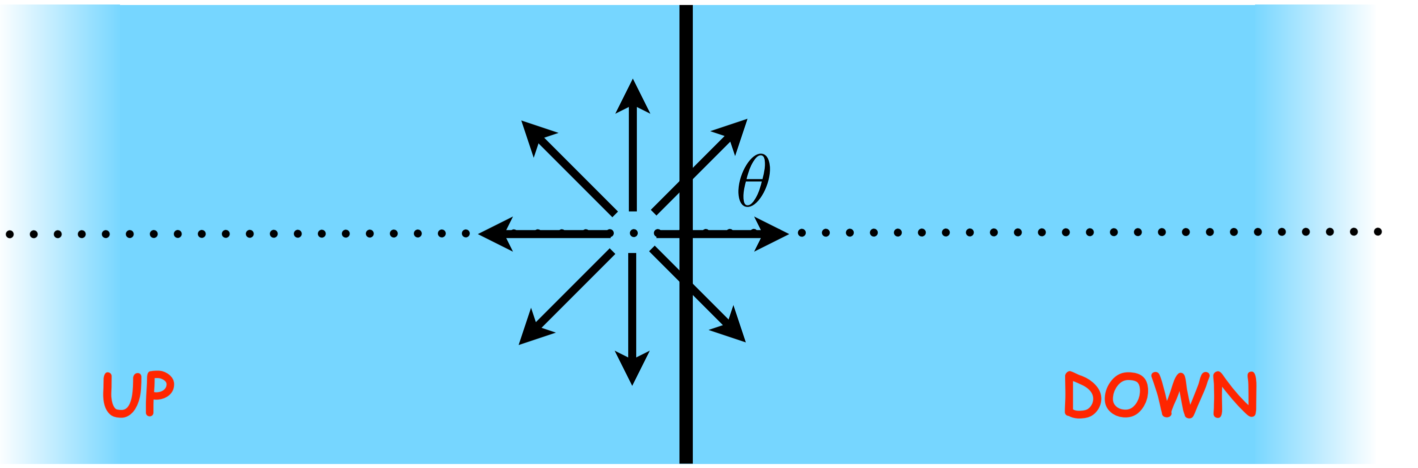

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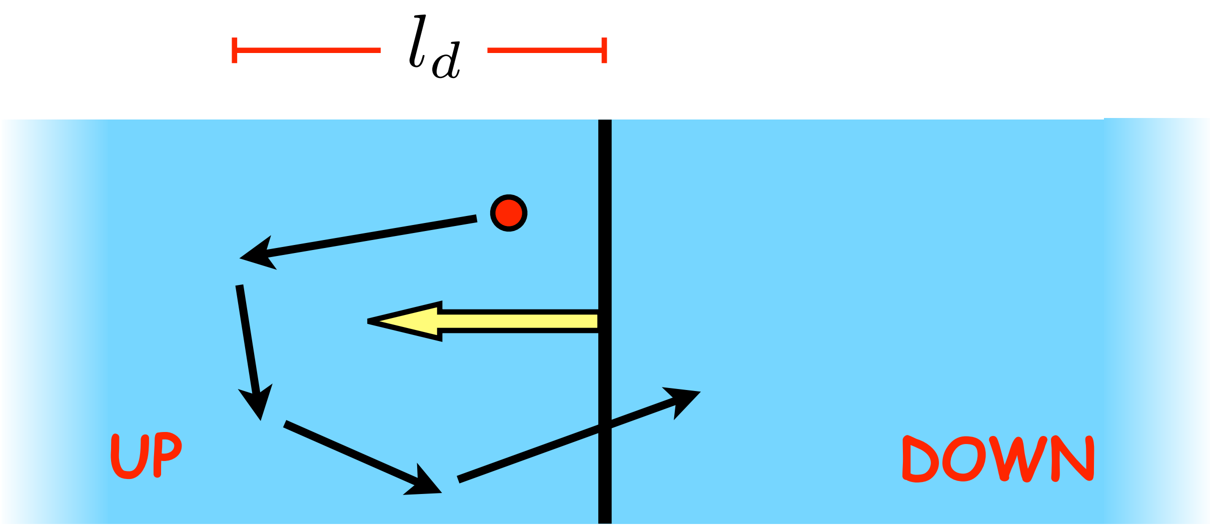
-> the same result is obtained for down -> up

Residence time upstream

-> let's find the STEADY STATE solution upstream of the shock



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D(E) -> diffusion coefficient

very poorly constrained (from both observations and theory)

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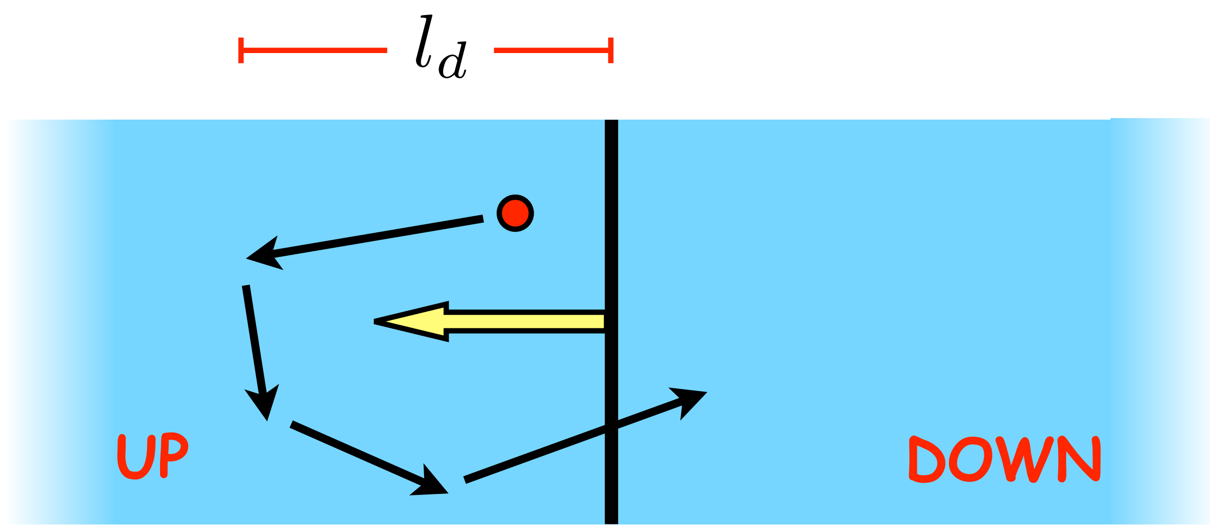

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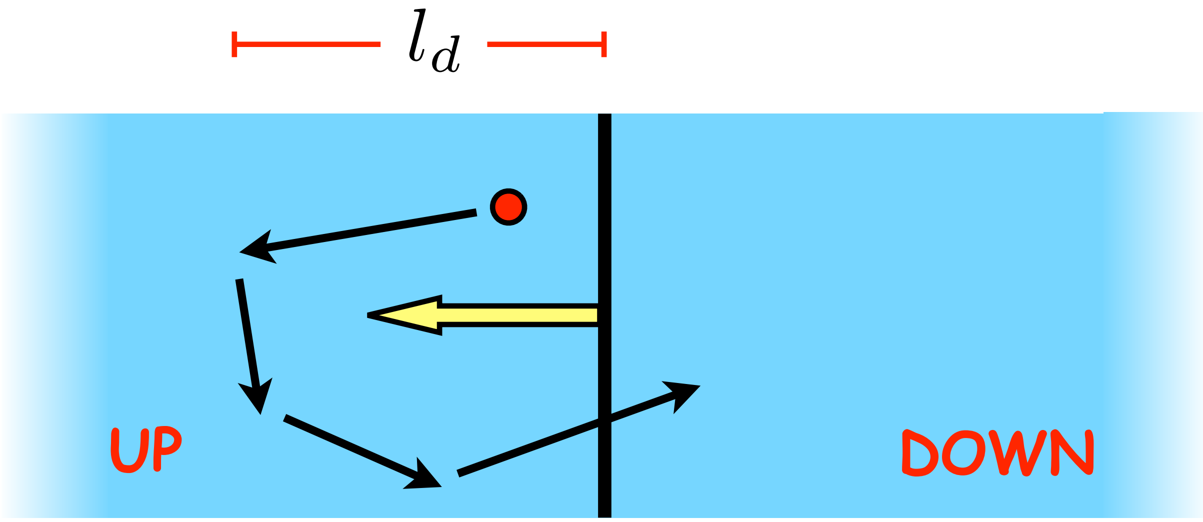

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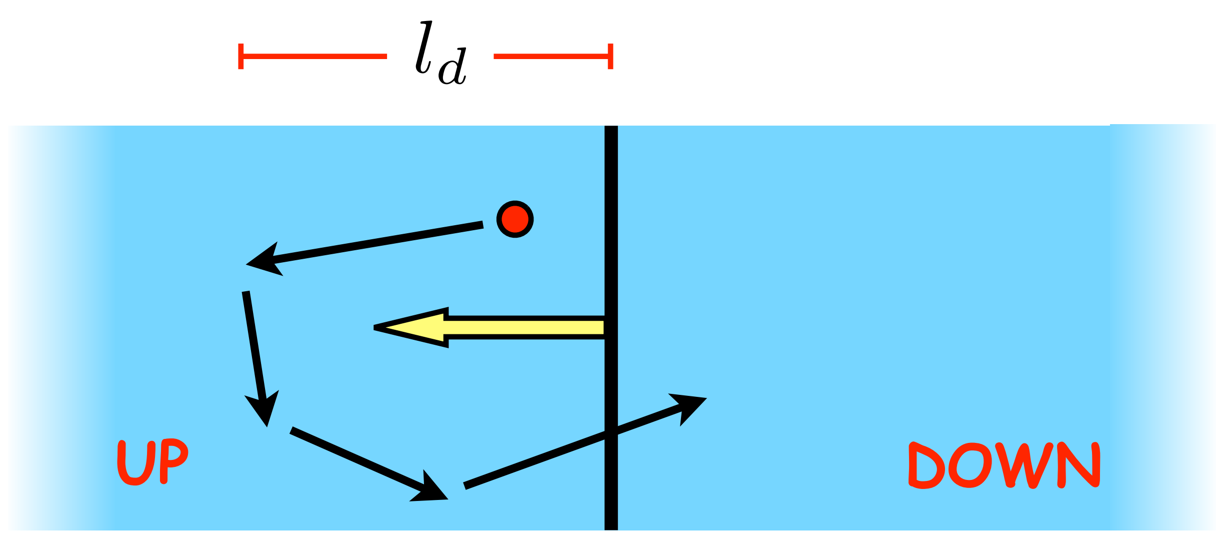

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$$\boxed{l_d \approx \frac{D}{u_1}}$$

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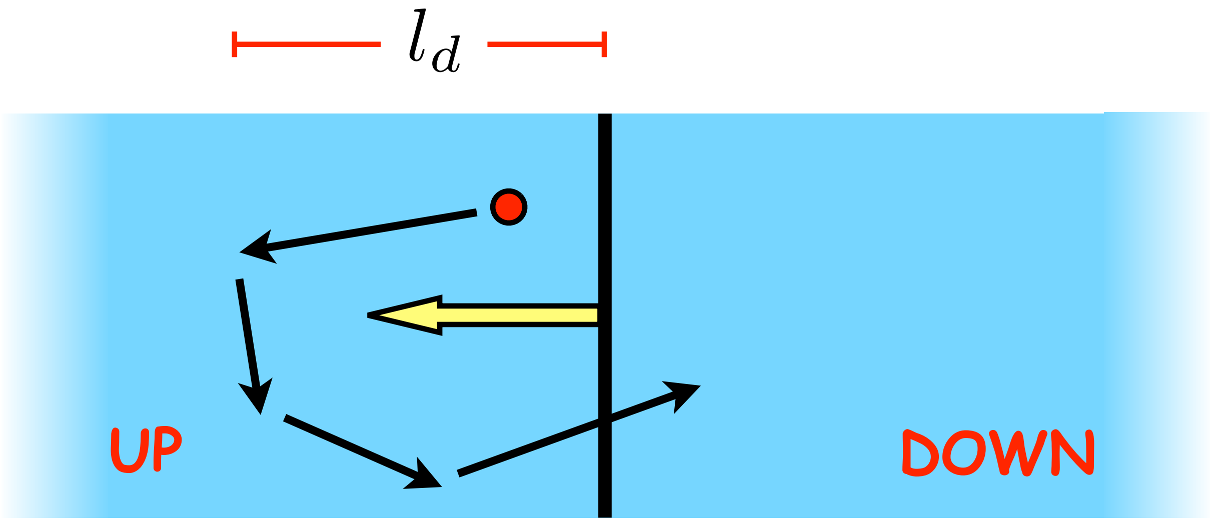

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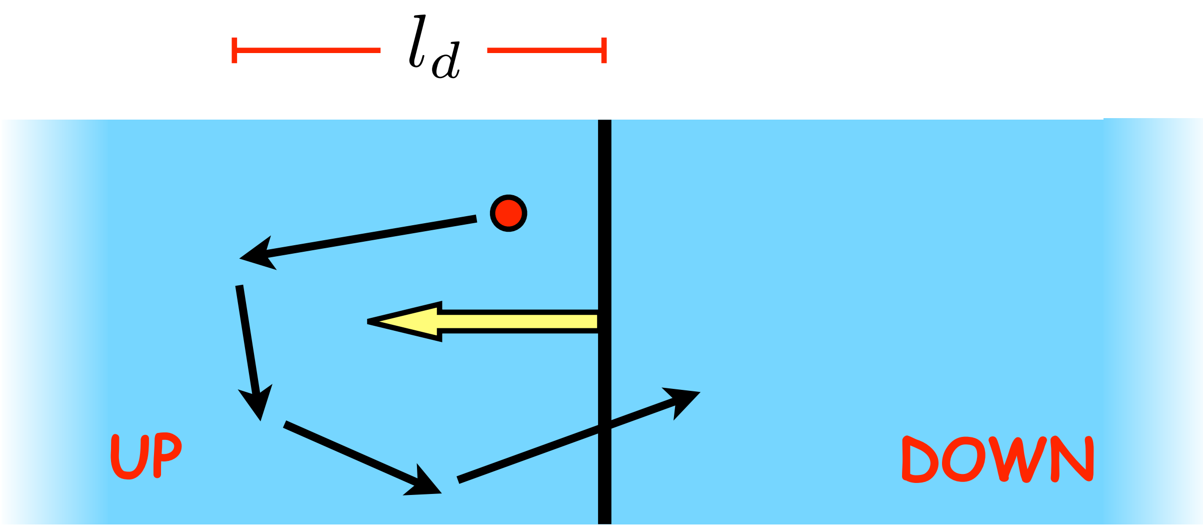

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cosmic ray precursor -> n ~constant up to $l_d \approx \dfrac{D}{u_1}$

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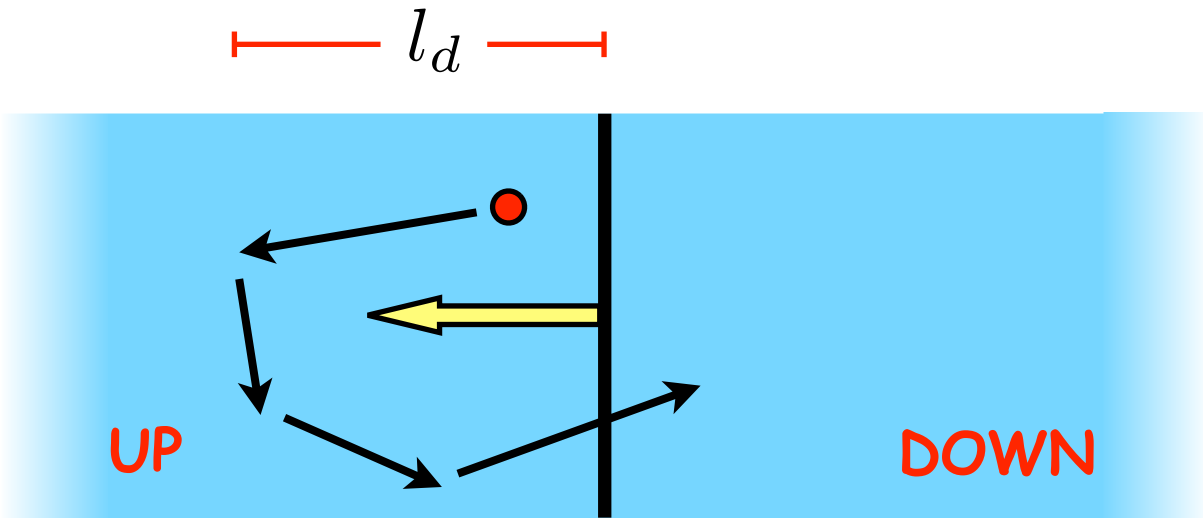

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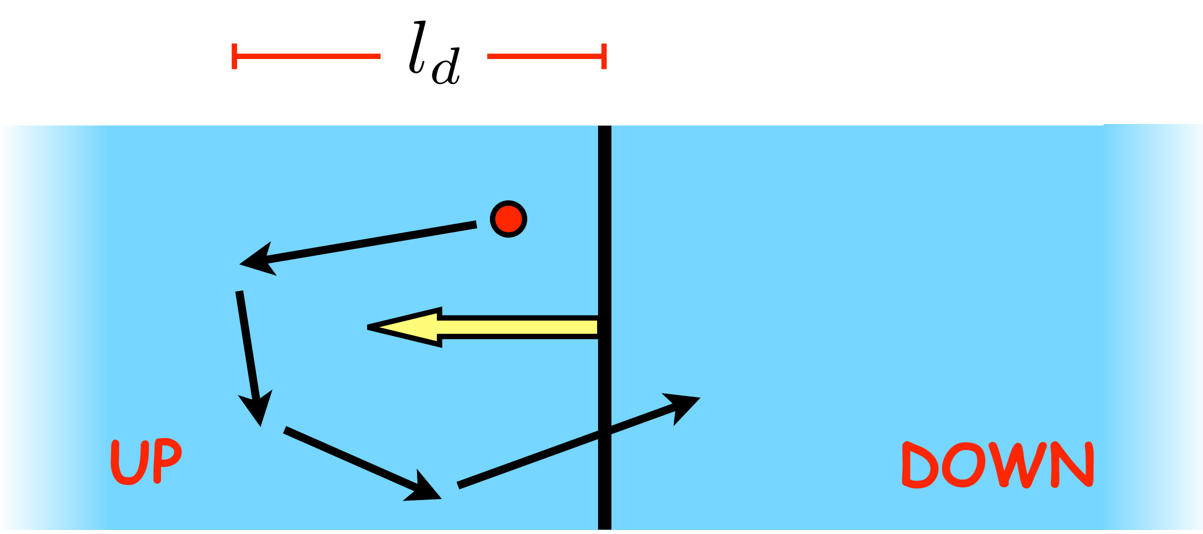
residence time upstream -> $$\tau_{up} = \frac{N_{up}}{R_{in}} = \frac{n\ l_d}{\frac{1}{4}\ n\ c} = \frac{4\ D}{u_1\ c}$$

Residence time upstream

-> let's find the STEADY STATE solution upstream of the shock



behavior of particles is diffusive
D(E) -> diffusion coefficient

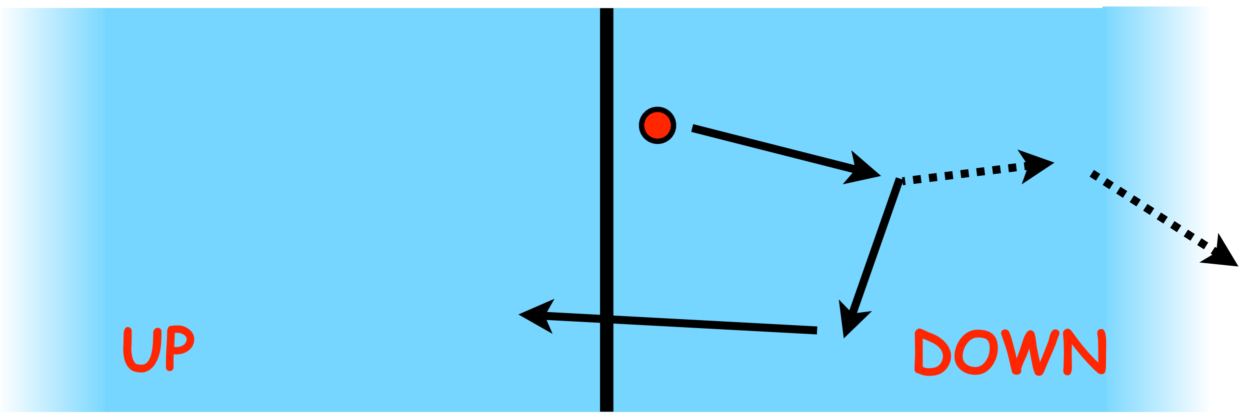
very poorly constrained (from both observations and theory)

cosmic ray precursor -> n ~constant up to $l_d \approx \dfrac{D}{u_1}$

residence time upstream -> $$\tau_{up} = \frac{N_{up}}{R_{in}} = \frac{n\ l_d}{\frac{1}{4}\ n\ c} = \frac{4\ D}{u_1\ c}$$

Residence time downstream

-> a bit more subtle...



n is constant downstream of the shock

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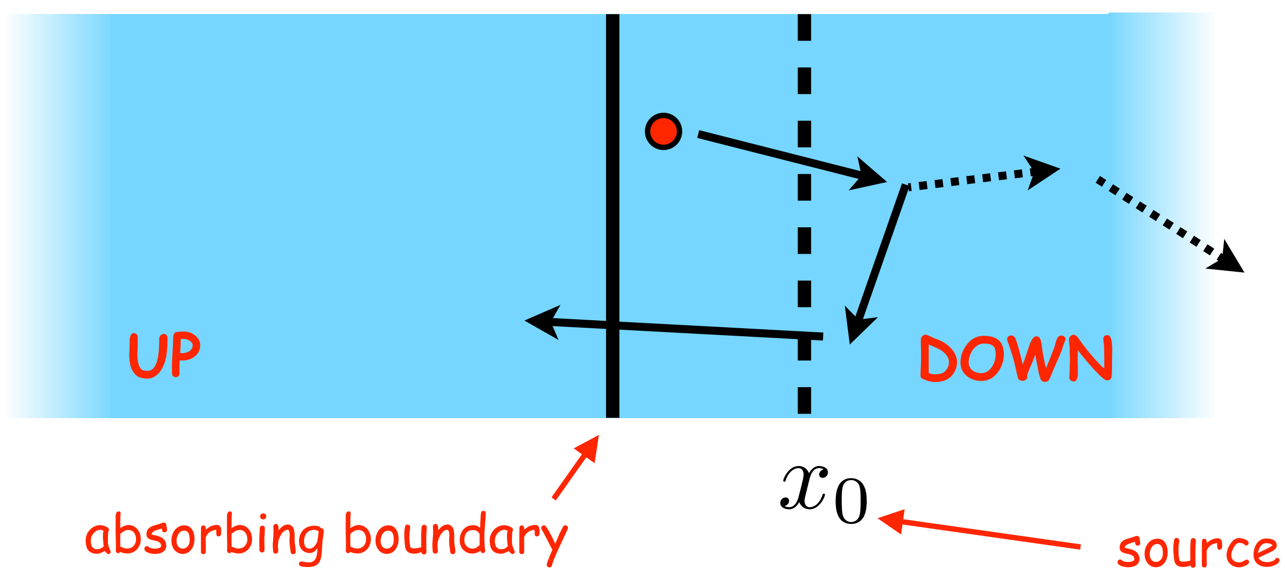

n is constant downstream of the shock

$$u_2 \frac{\partial n}{\partial x} = D \frac{\partial^2 n}{\partial x^2} + Q \ \delta(x - x_0) \qquad n(0) = 0$$

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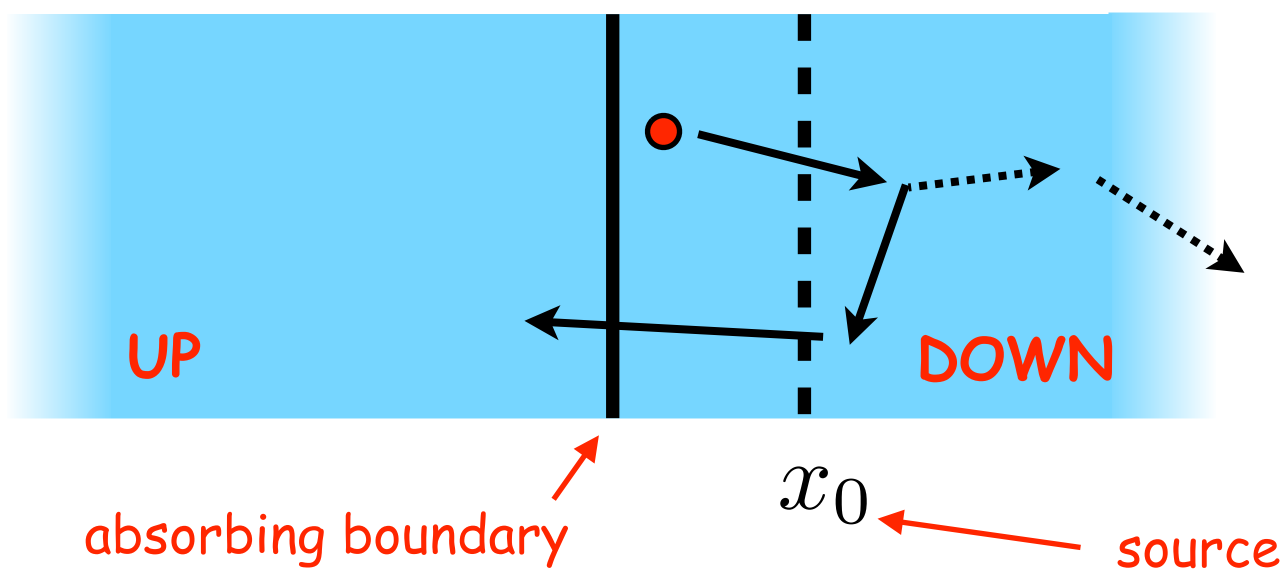

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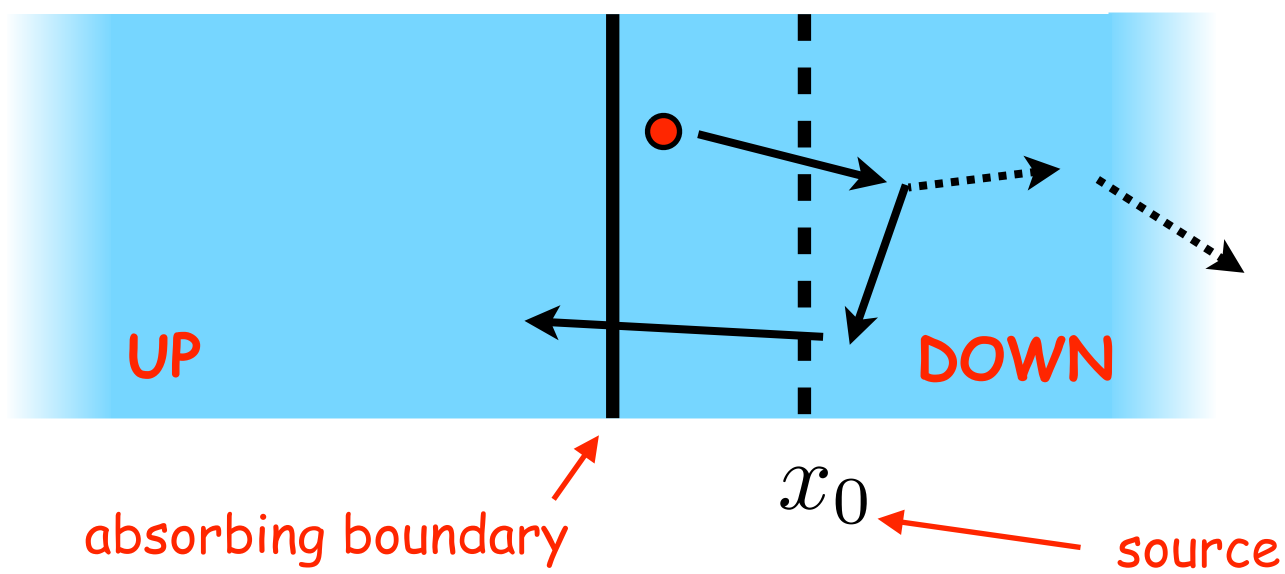

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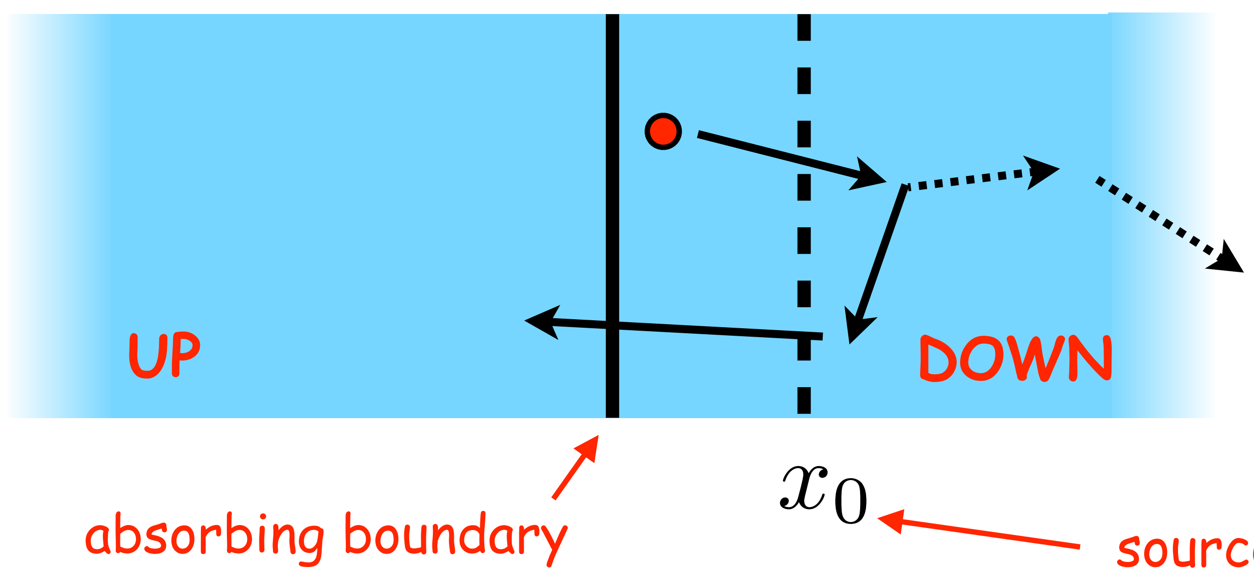

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$$\boxed{P_{ret} = \exp\left(-\frac{x_0 u_2}{D}\right)}$$

Residence time downstream

number of downstream particles that will return to the shock:

$$\int_0^\infty \mathrm{d}x \; P_{ret}(x) \; n \;=\; \frac{D \; n}{u_2}$$

same expression upstream!

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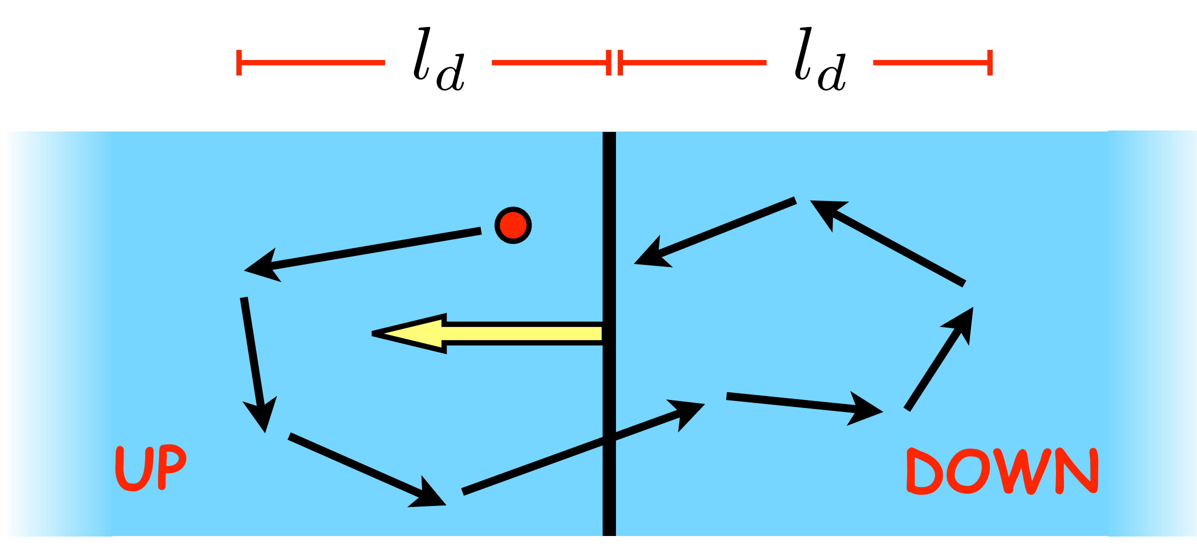


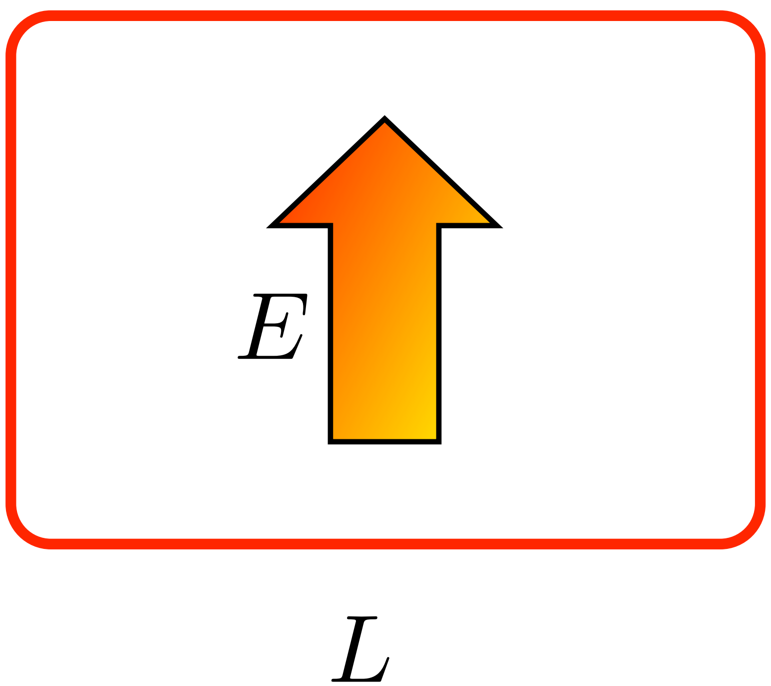
Acceleration rate

L

everything happens within a "box" of size:

$$L = \frac{D}{u_1} + \frac{D}{u_2} \quad = \; c \, \frac{\tau_{up} + \tau_{down}}{4}$$

Acceleration rate



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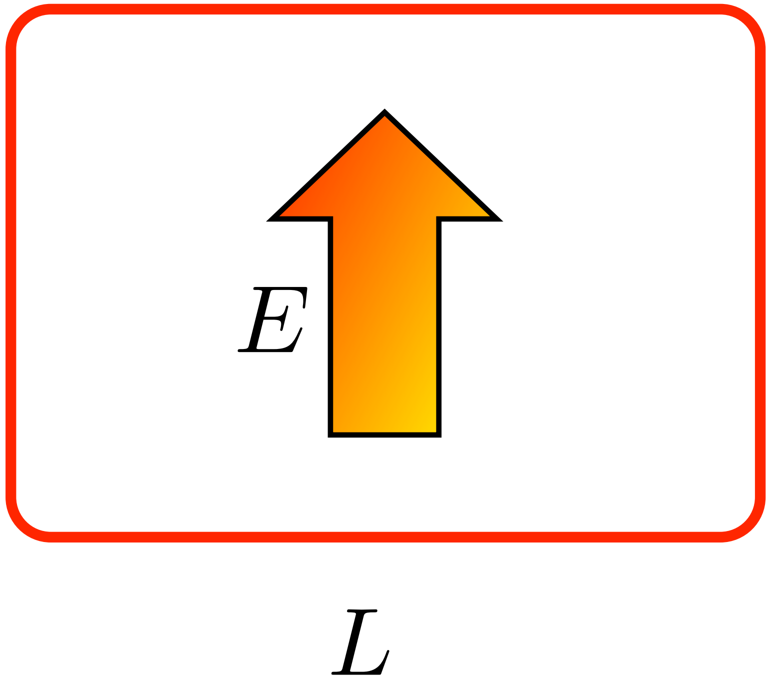

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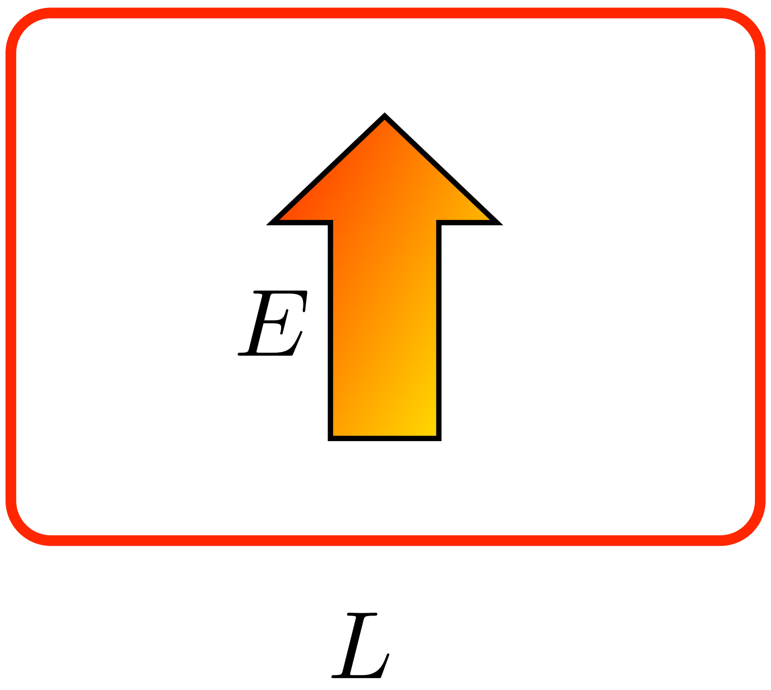
Box model for shock acceleration



particles move up in energy

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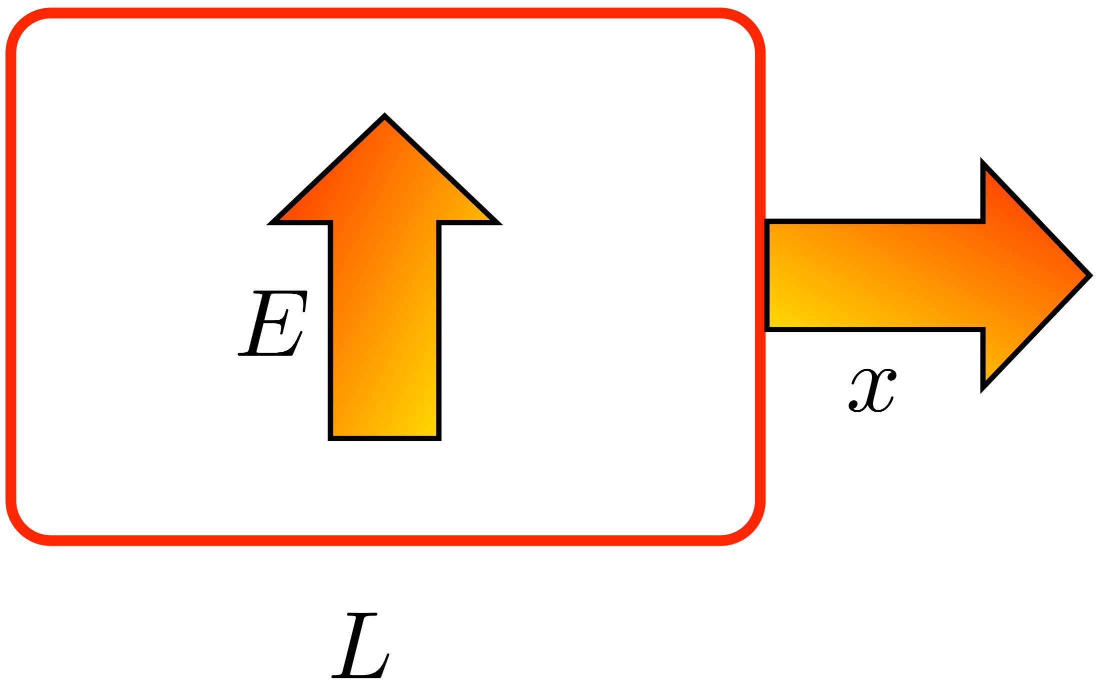

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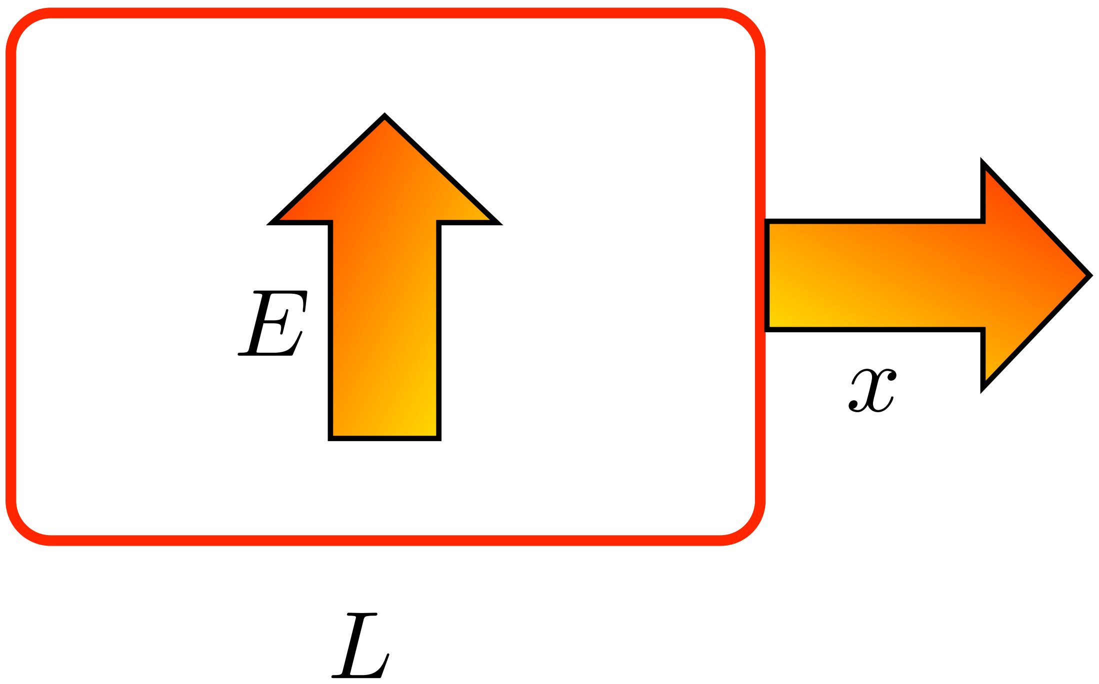

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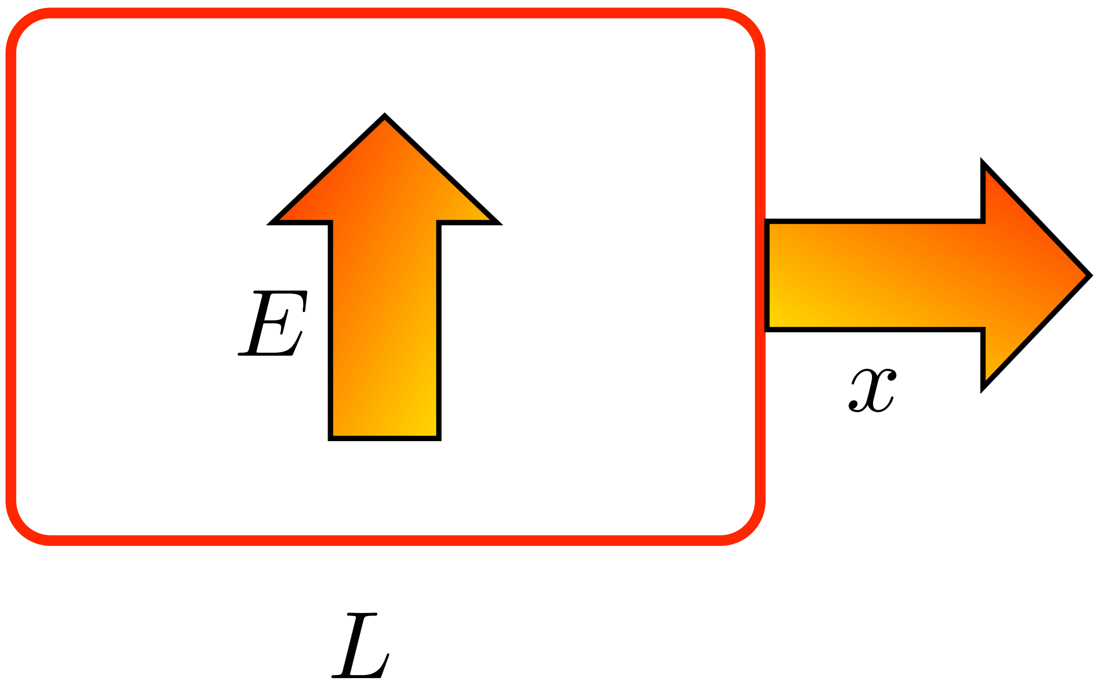

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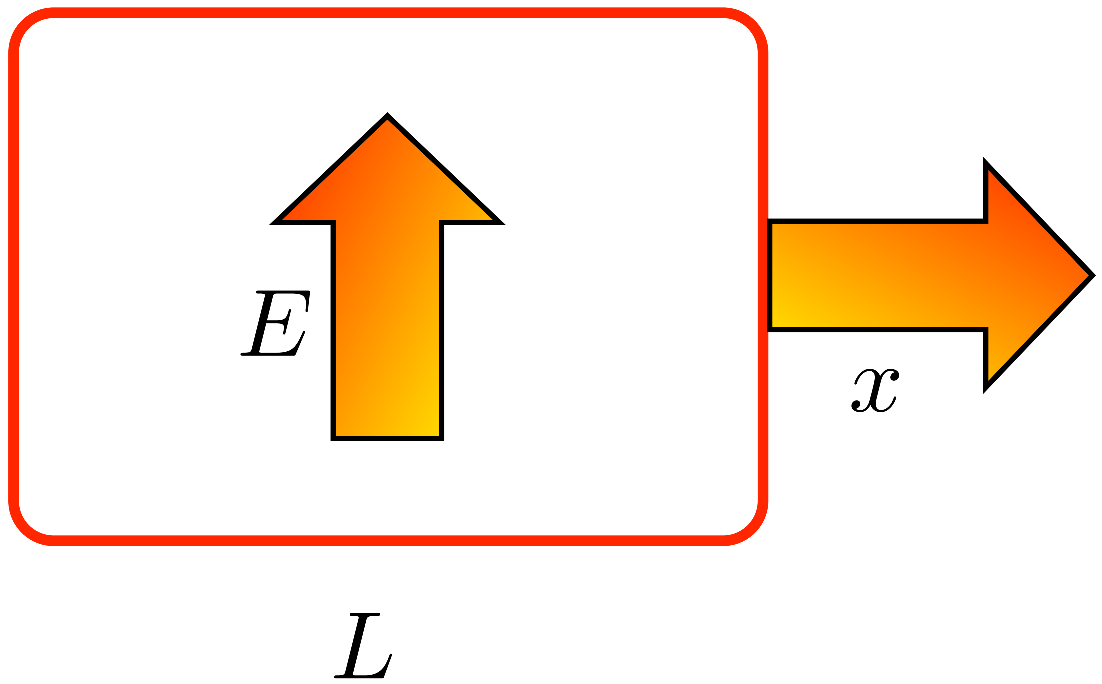

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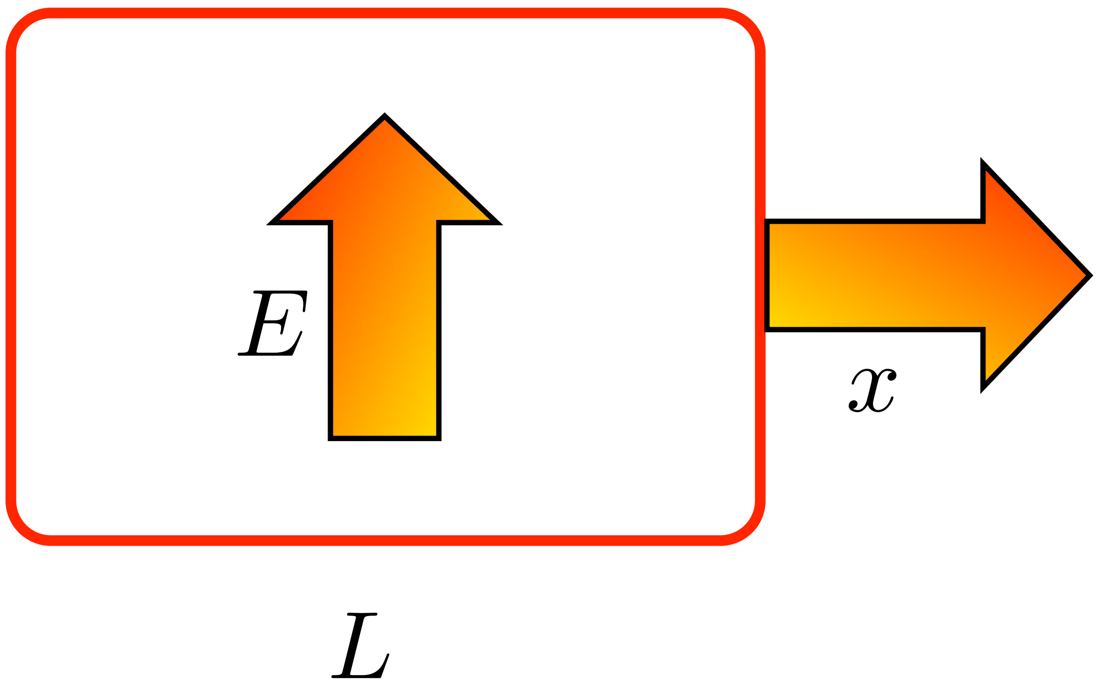

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Universality of diffusive shock acceleration

Let's search for a test-particle solution

Assumption: scattering is so effective at shocks that the distribution of particles is isotropic

-> an universal solution of the problem can be found

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Independent on D !!!

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Bell's approach

Let's start with N_0 particles of energy E_0...

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Universal solution: Bell's approach

$$\log\left(\frac{N}{N_0}\right) = k\log\left(1 - \frac{u_1}{c}\right)$$

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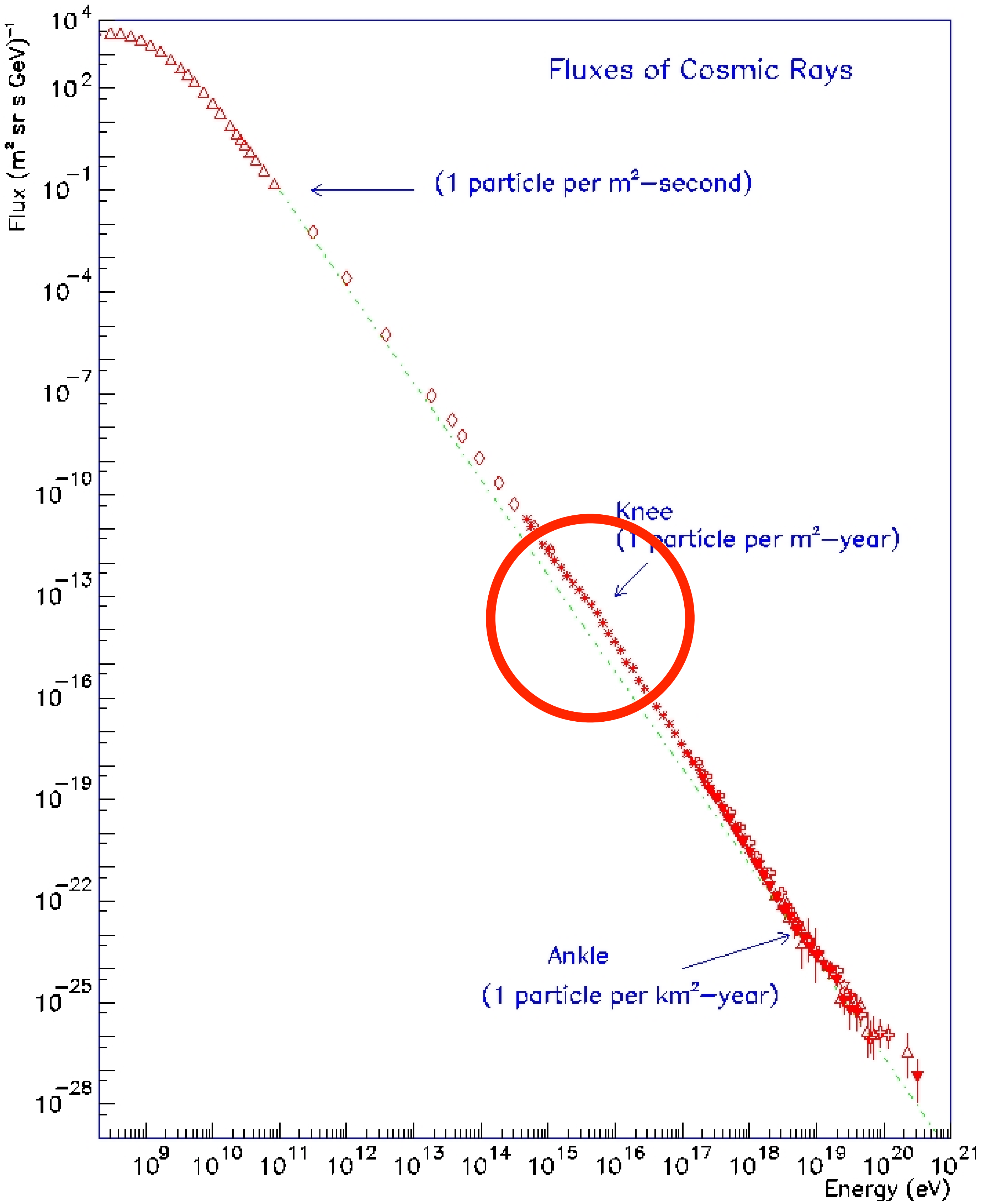
Getting to the knee



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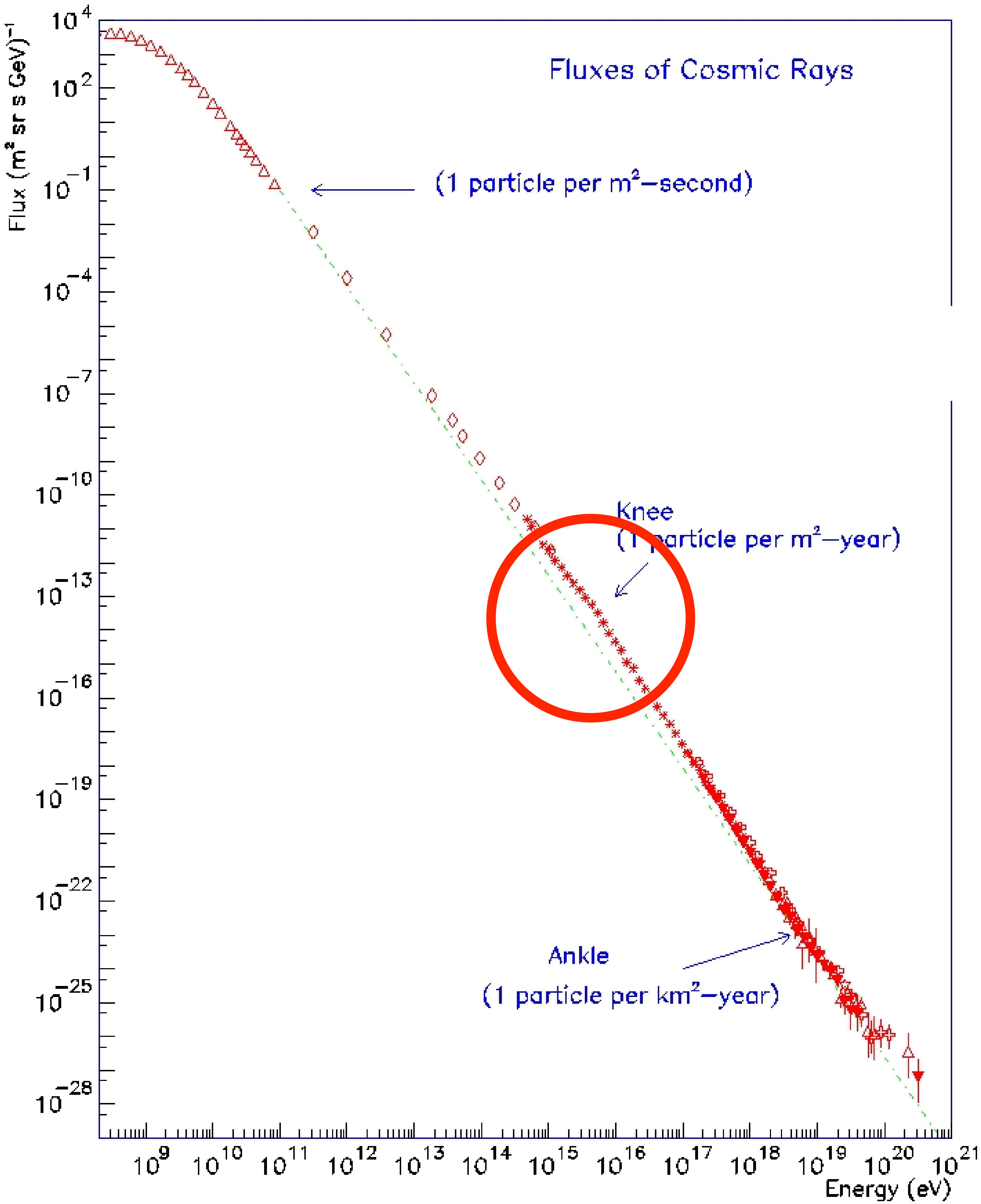
Getting to the knee



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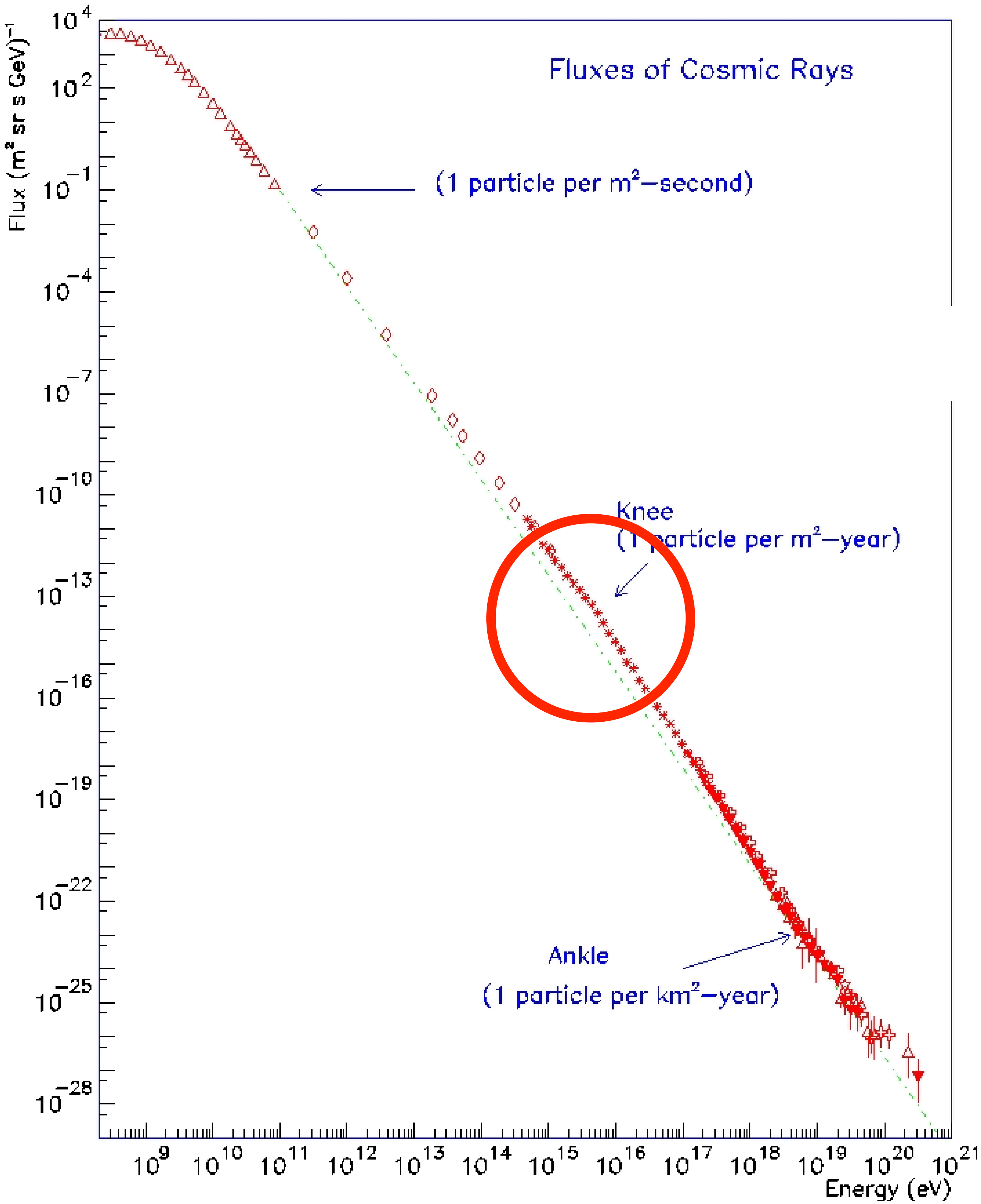
Getting to the knee



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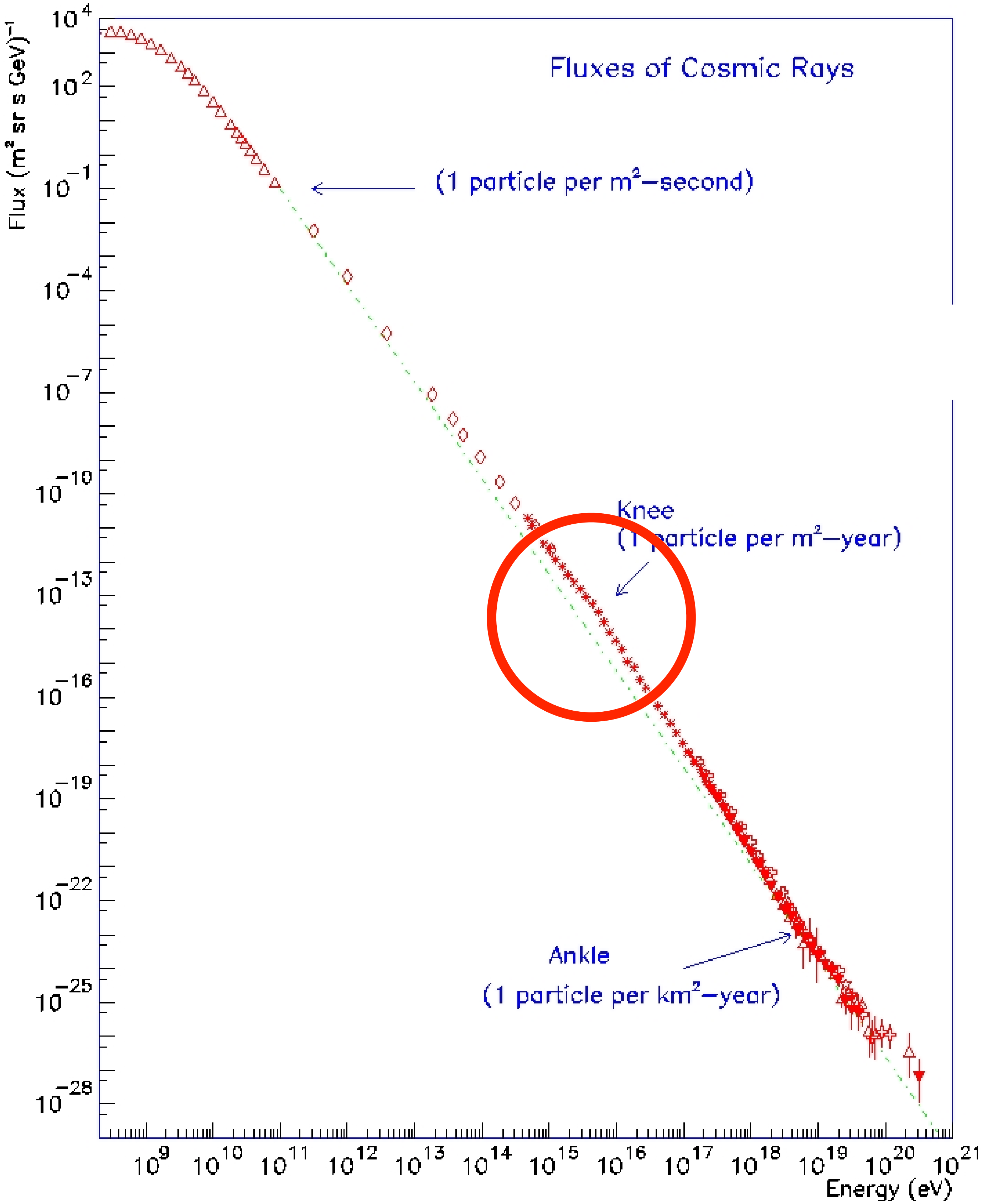

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details matter

Getting to the knee

Lagage & Cesarsky 1983

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- -> CRs are scattered by resonant MHD waves $\lambda \approx R_L$ Larmor radius

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10 times below the knee

How to solve the problem

horribly oversimplified, for a proper treatment see Bell 2004

$$E_{max} \approx B \, u \, R$$

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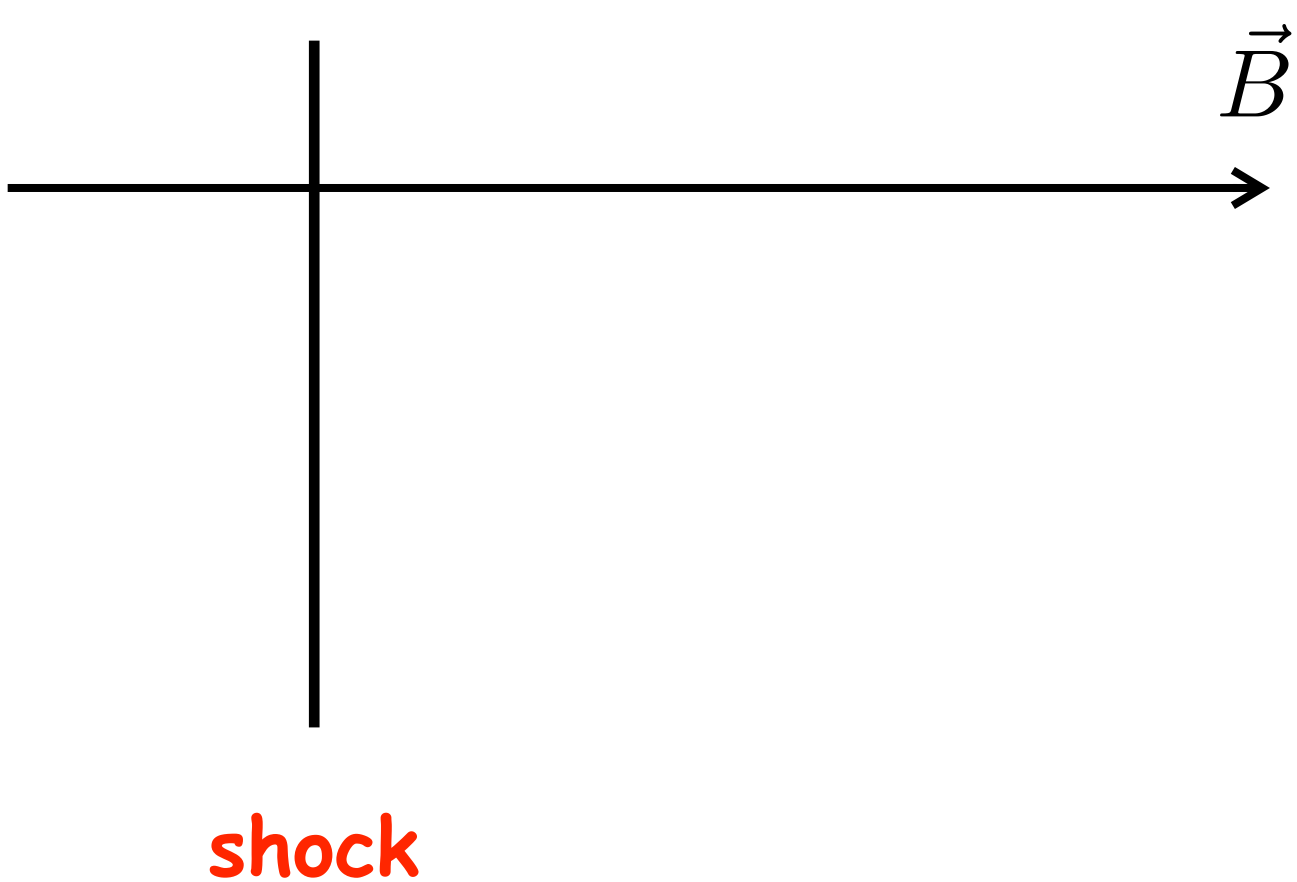

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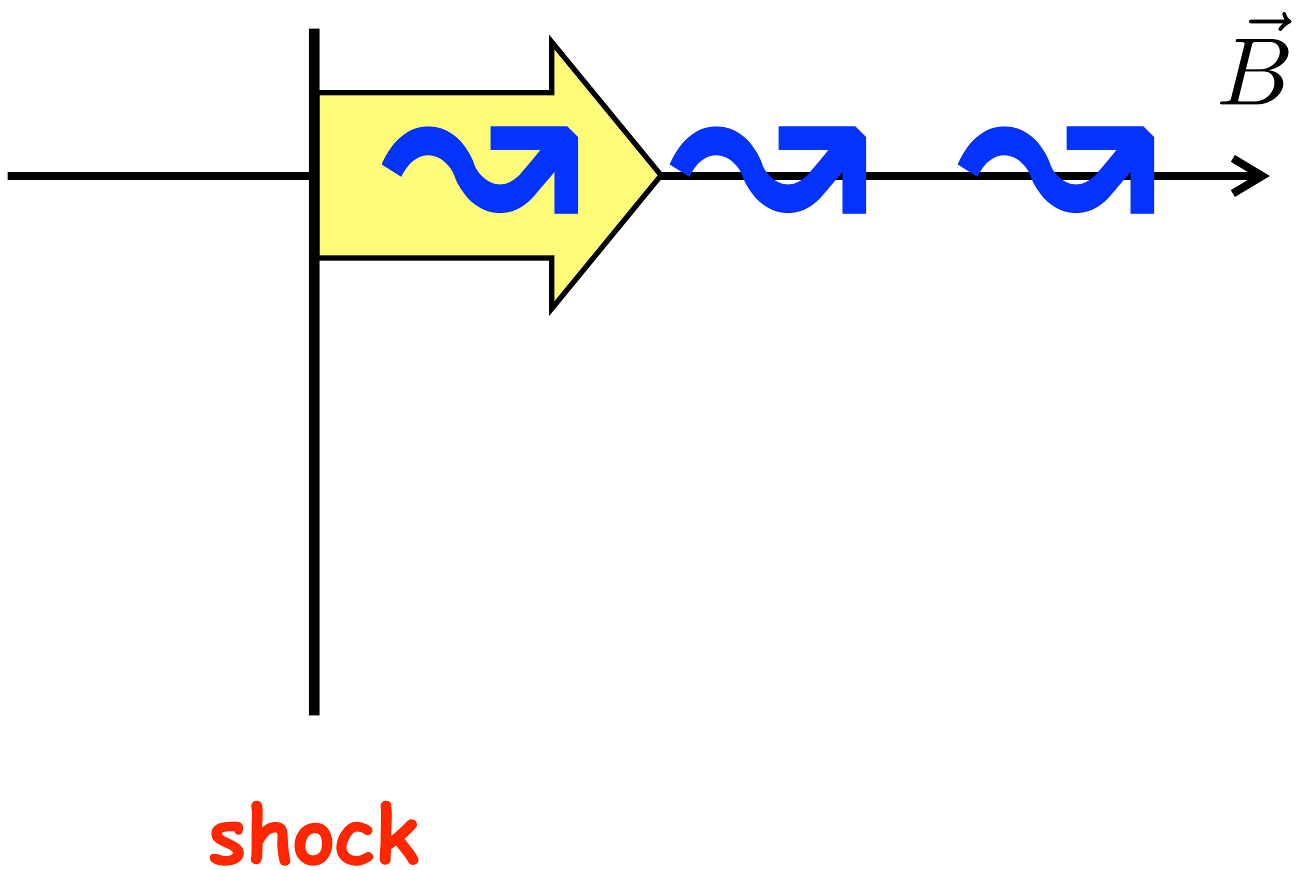

Alfven speed

$$V_A = \frac{B}{\sqrt{4\ \pi\ \varrho}}$$

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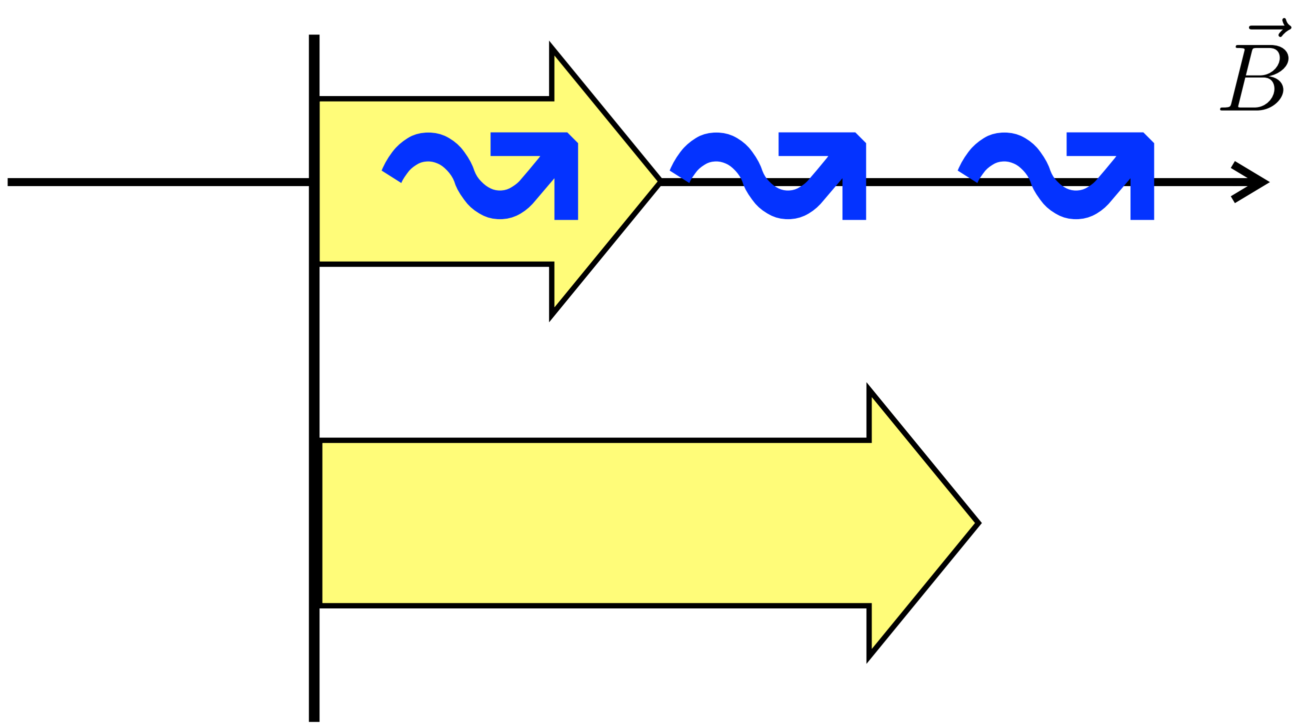

shock

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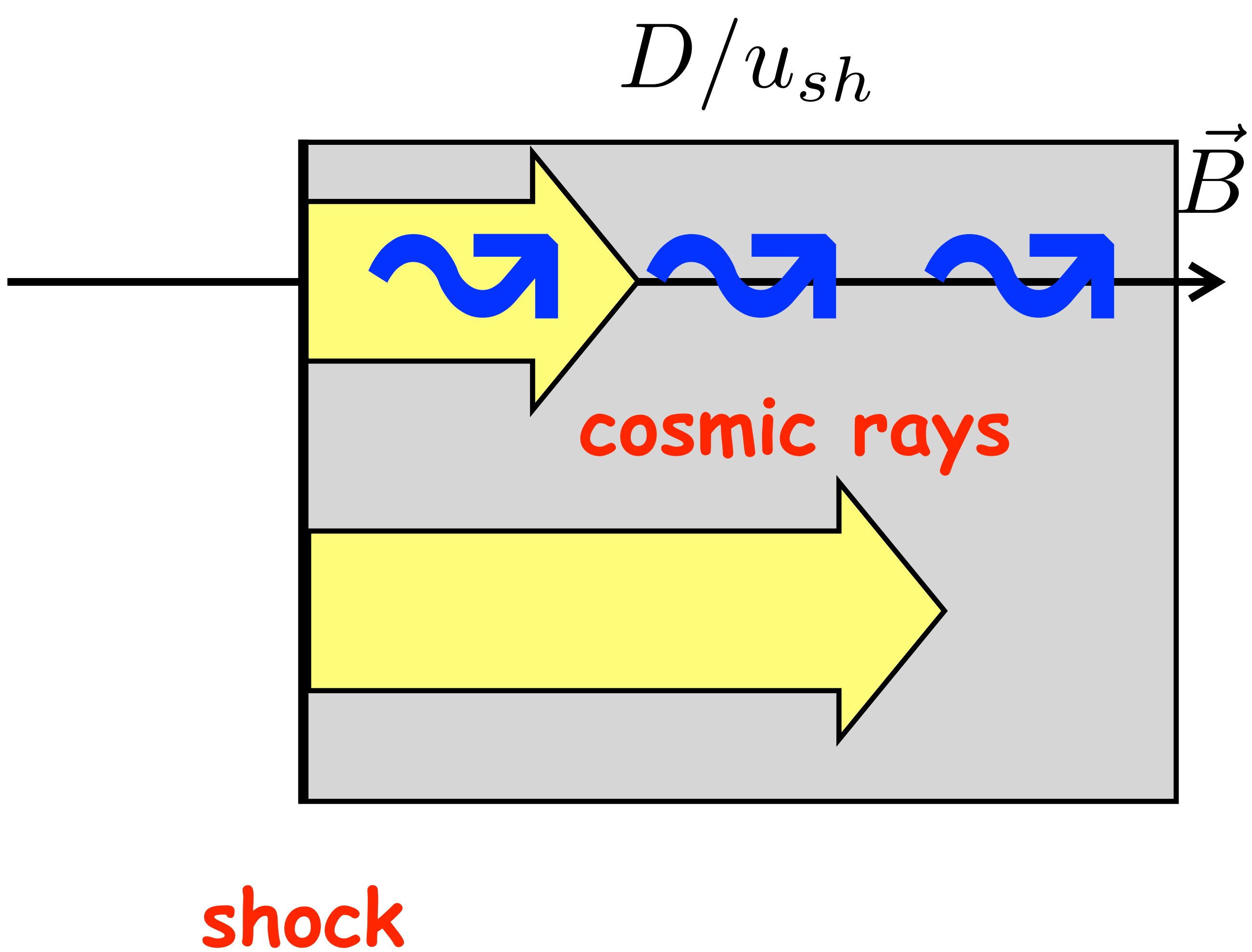

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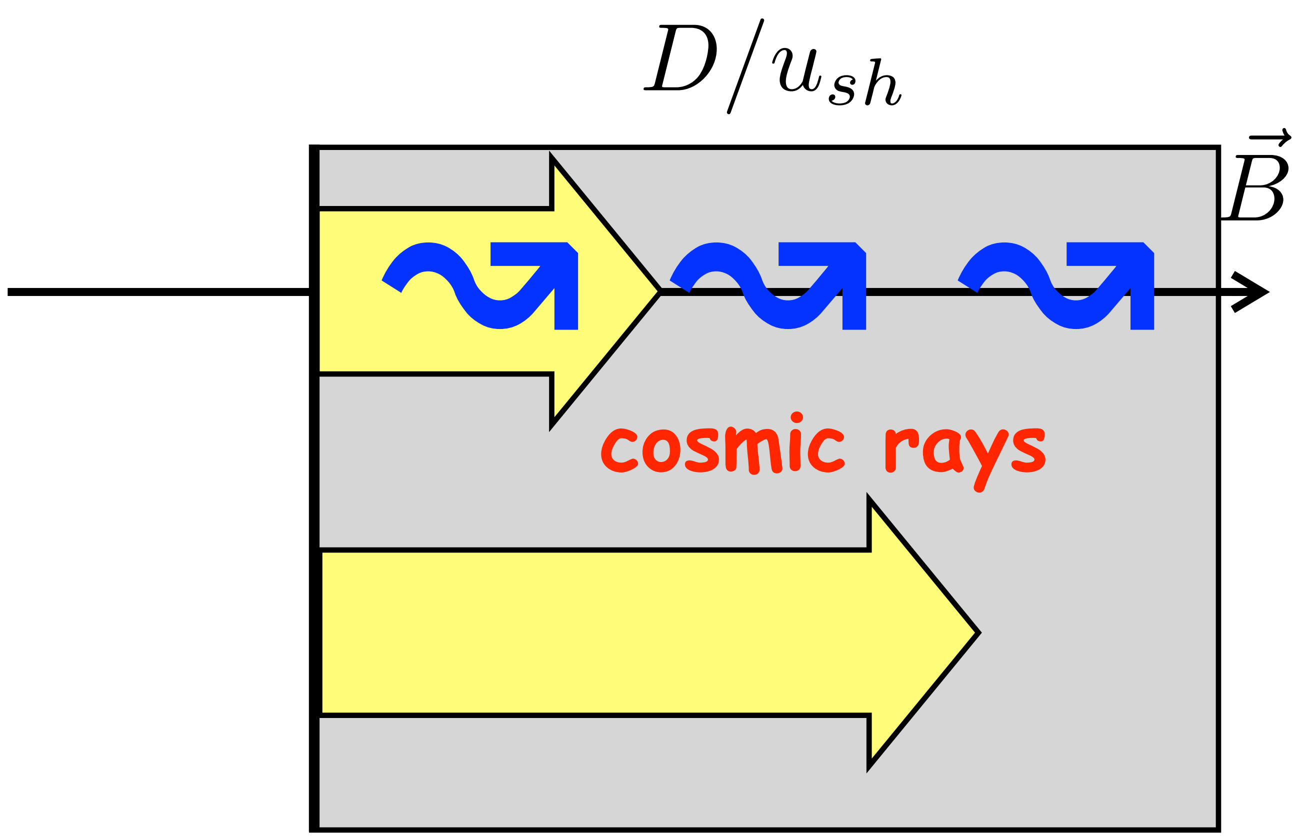

shock

Alfven speed

$$V_A = \frac{B}{\sqrt{4\ \pi\ \varrho}}$$

$$u_{sh} \gg V_A$$

-> CRs move with the shock -> faster than waves -> CR and waves strongly coupled -> V_A increases -> **B increases!**

Observational test: X-ray filaments

electrons $$\tau_{acc}(E) = \tau_{age}$$

Observational test: X-ray filaments

electrons

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Observational test: X-ray filaments

electrons

$$\cancel{\tau_{acc}(E) = \tau_{age}}$$

$$\tau_{acc} \sim \frac{D}{u_s^2} = \tau_{syn}$$

Observational test: X-ray filaments

electrons

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$$\tau_{acc} \sim \frac{D}{u_s^2} = \tau_{syn} \quad \approx E^{-1} B^{-2}$$

Observational test: X-ray filaments

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depends on
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Observational test: X-ray filaments

electrons

$$\cancel{\tau_{acc}(E) = \tau_{age}}$$

$$\tau_{acc} \sim \frac{D}{u_s^2} = \tau_{syn} \approx E^{-1} B^{-2} \longrightarrow E_{max} \sim u_s B^{-1/2}$$

max energy of synchrotron photons ->

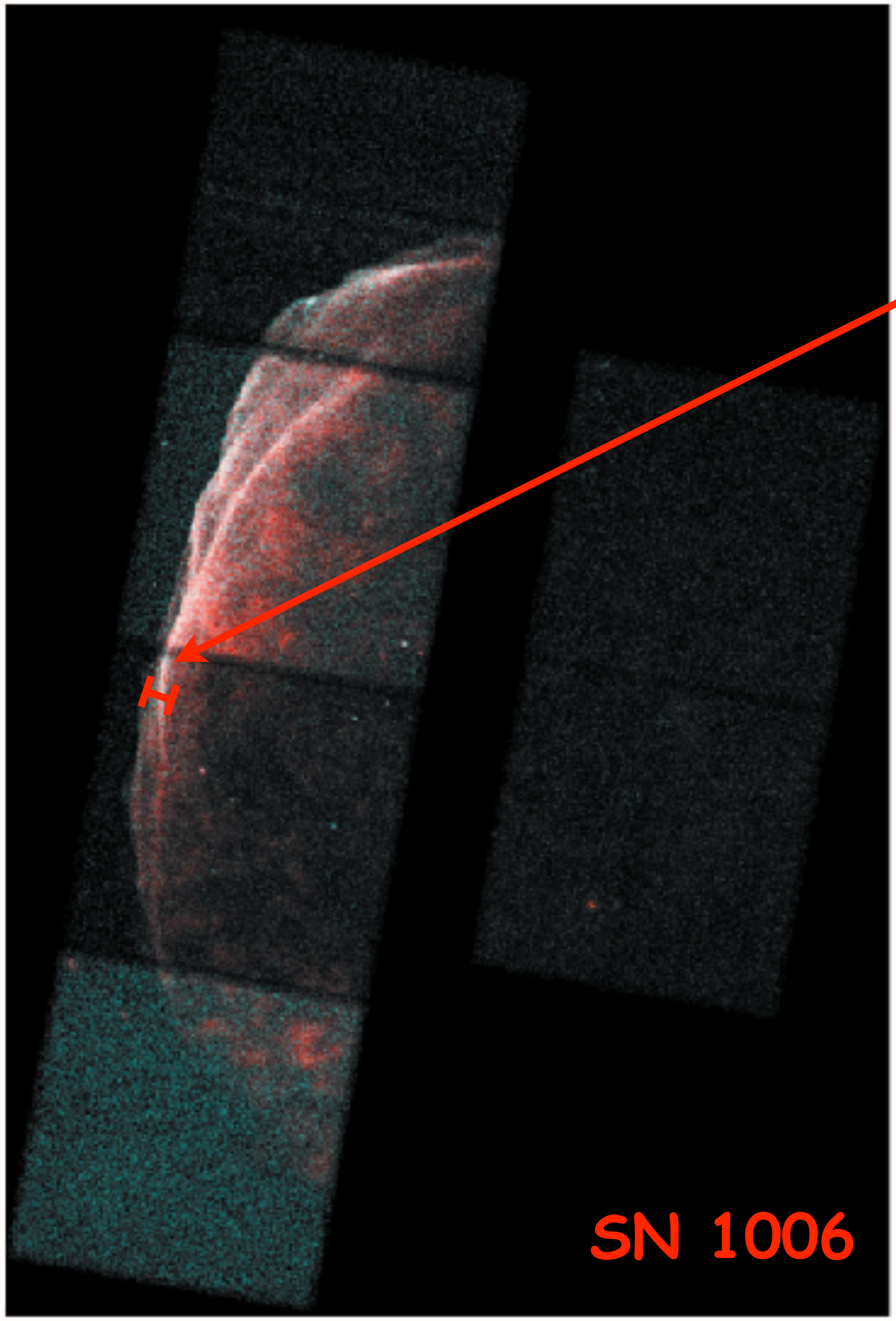
$$E_{syn} \sim E^2 B^2 \sim u_s^2$$

depends on
velocity only!!!

$$u_s \approx 10^3 \mathrm{km/s} \longrightarrow E_{syn}^{max} \approx 1 \mathrm{kev}$$

X-rays!

Observational test: X-ray filaments



$$\Delta l_X \sim \tau_{syn} u_2 \sim B^{-3/2}$$

B ~ hundreds of microGauss !

The SNR paradigm: does it work?

- diffusive transport of cosmic rays in the galaxy —> **ISOTROPY**
  - Why does the dipole anisotropy increases so slowly with E? Small scale anisotropies?
- slope of the spectrum —> **E^{-2} is too hard!**
  - what we see from gamma ray **observations** of SNRs seems to suggest that shock accelerate **steeper spectra**
  - theoreticians proposed tricks (modification of the diffusive shock acceleration theory) to explain this
- if **magnetic field amplification** operates at shocks (???) —> can protons be accelerated **up to the knee (~10^{15} eV)?** Probably yes, but can SNRs accelerate enough protons to the knee?
- things we did not discuss: chemical composition, electrons, …