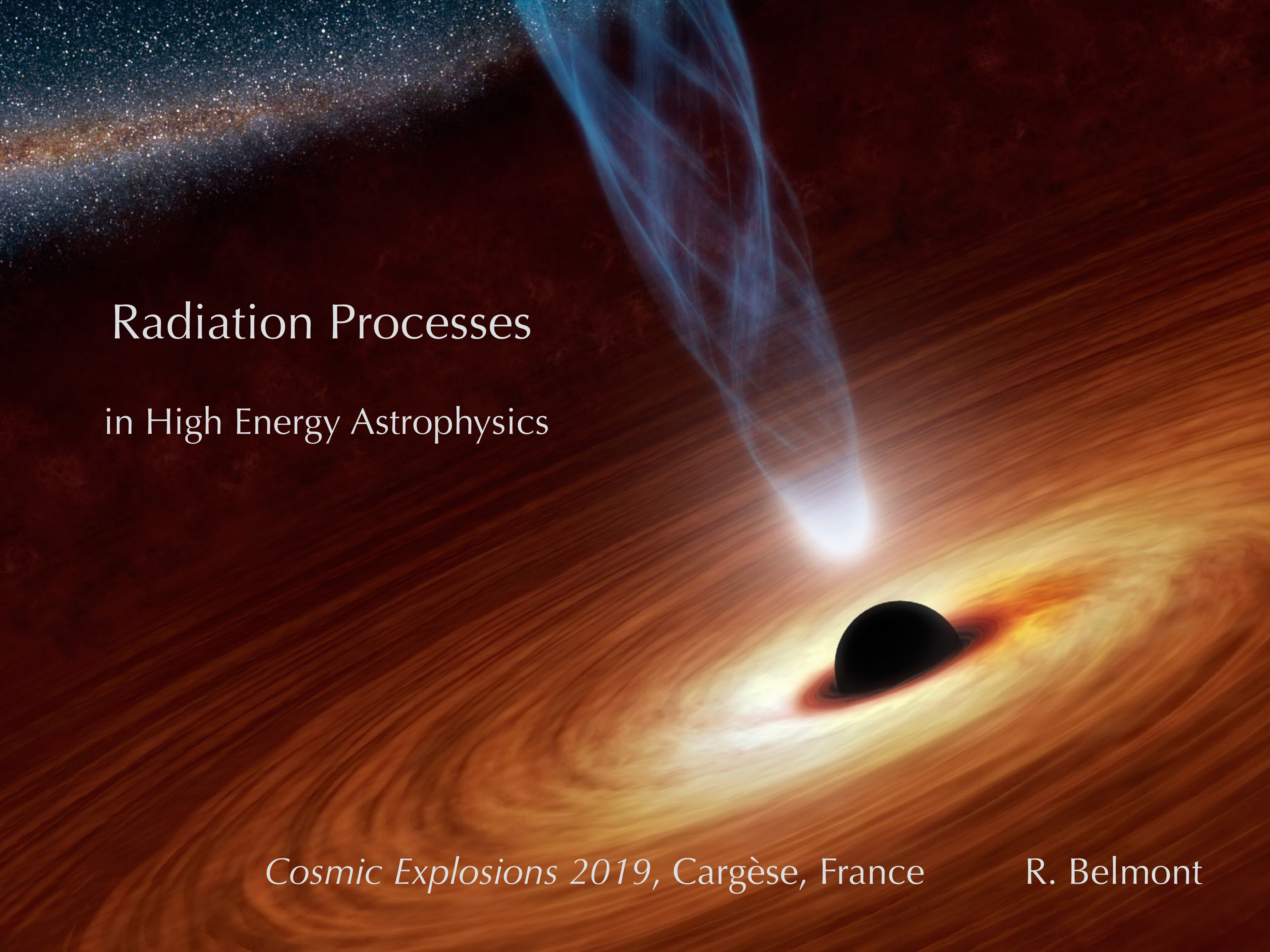


Radiation Processes

in High Energy Astrophysics

Cosmic Explosions 2019, Cargèse, France

R. Belmont

Outline

I. Back to Basics

- Roles of radiation
- Energy distributions
- Transfer
- Special Relativity
- Emission in CED

II. Bremsstrahlung

III. Magneto-bremsstrahlung

- Synchrotron
- Curvature
- Diffuse synchrotron

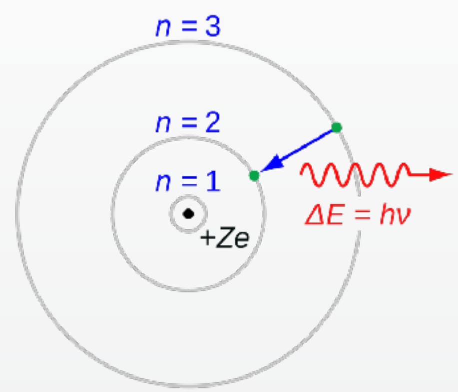
IV. Compton scattering

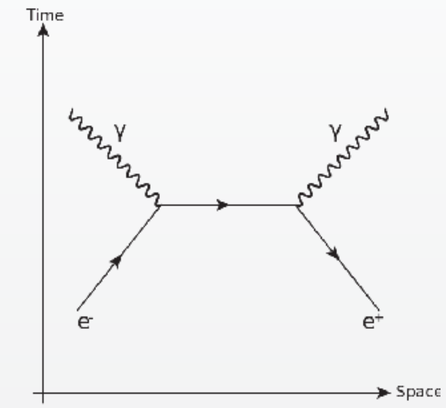
V. Photon annihilation

I. Back to Basics

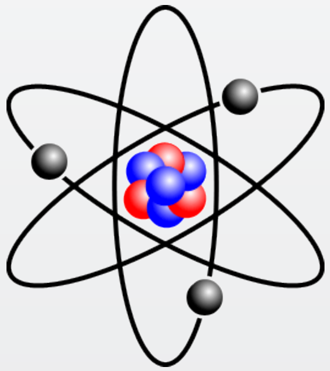
Interpreting observations

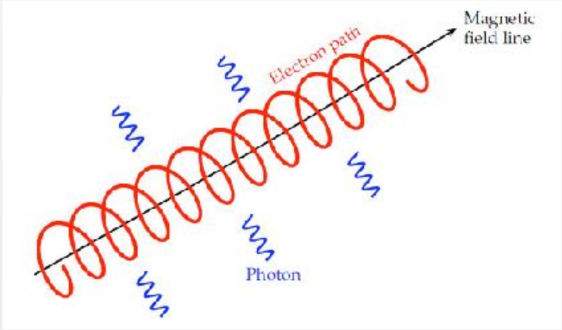
- **Many processes**
  - Atomic/nuclear lines, bremsstrahlung, synchrotron, Compton, pair production/ annihilation, particle physics…

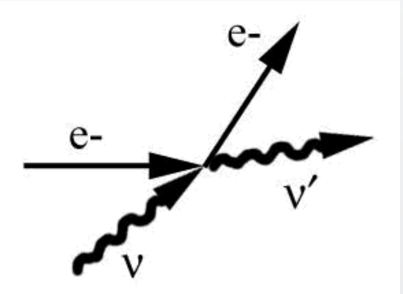




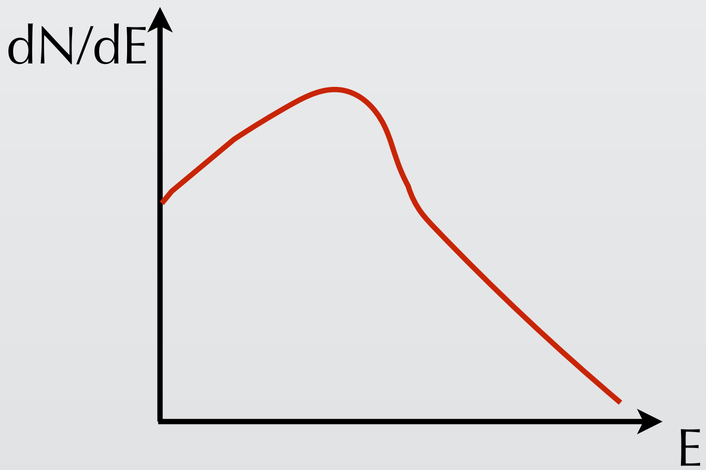
- **Particle nature**
  - leptons, protons, ions

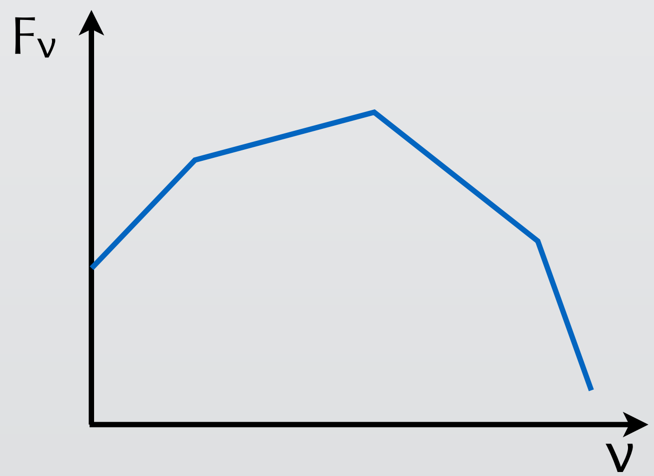






- **Transfer**
  - Thin/thick
- **Particle distributions**
  - Thermal, non-thermal, hybrid etc..
- **Homogeneous/homogeneous**
- **Isotropic/anisotropic medium**
- **Challenges:**
  - Identifying all these key aspects
  - Derive simple ways to model the emission



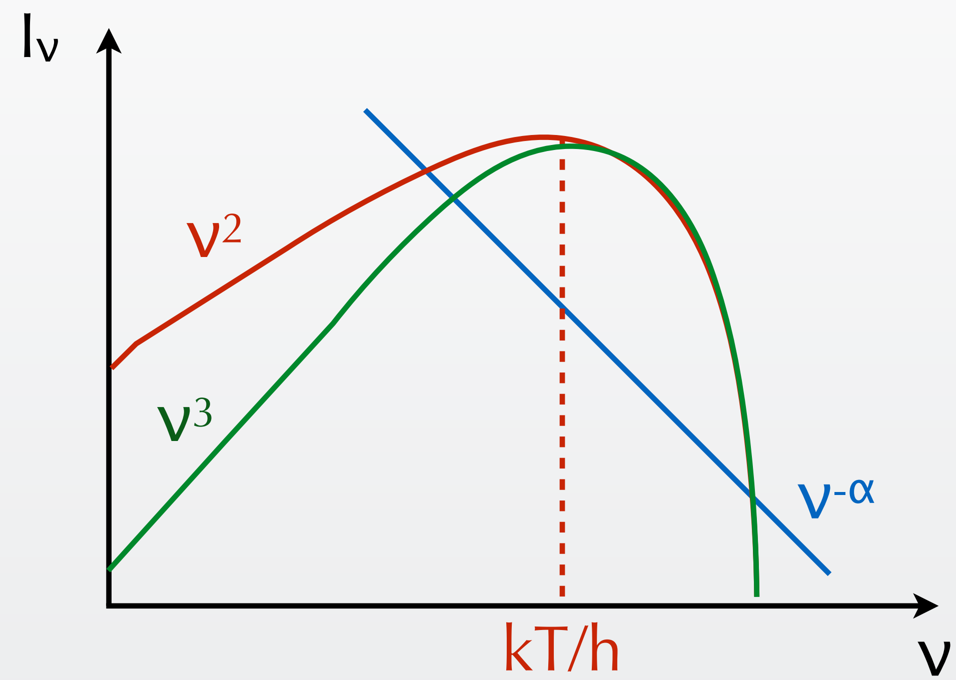


Impacting Matter

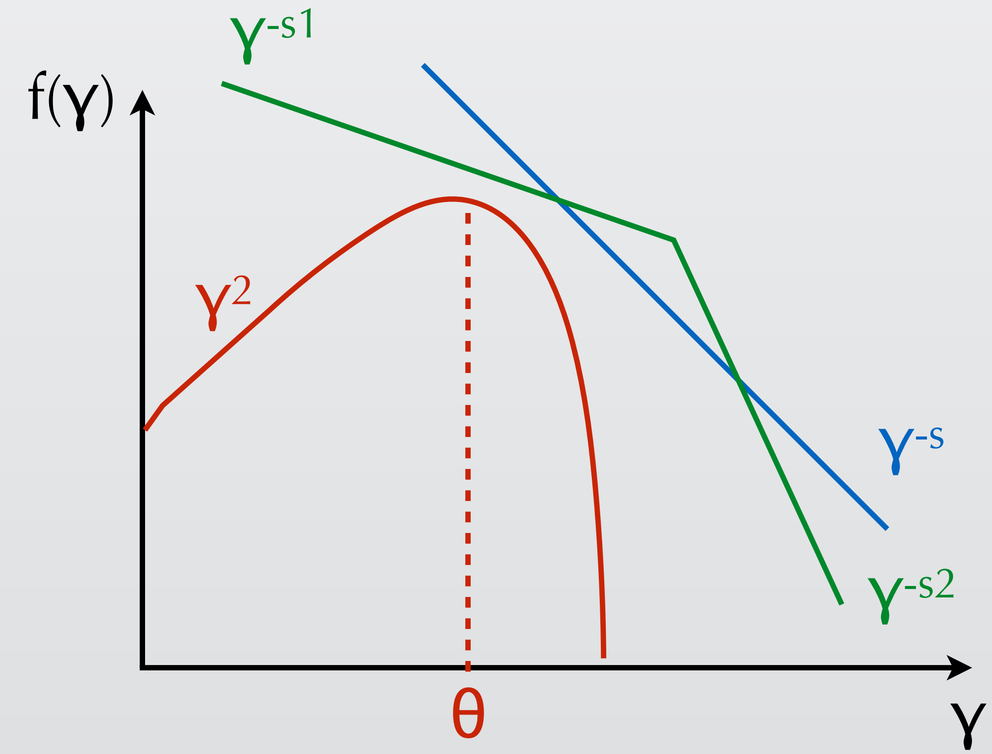
- **Defining the properties of matter**
- **Dynamics**
- **Energetics:**
  - Cooling
  - heating
  - Thermalisation
- **Competition with**
  - acceleration processes
  - escape
  - adiabatic cooling

Particles and Photons

- **Photon properties**
  - Energy: $E=h\nu$
  - Isotropic energy distributions:
    - Power-law $I_\nu \propto \nu^{-\alpha_i}$
    - Planck $I_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$
    - Wien $I_\nu = \frac{2h\nu^3}{c^2}e^{-h\nu/kT}$

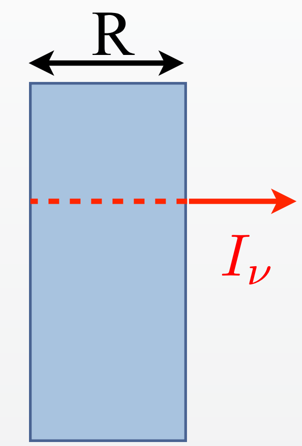


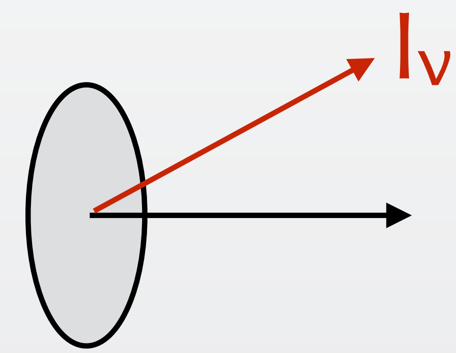
- **Particle properties**
  - Velocity: $\beta=v/c$
  - Energy: $\gamma = E/mc^2$
  - Kinetic energy: $\gamma-1 = (E-mc^2)/mc^2$
  - Momentum: $p = \beta\gamma = P/mc$
  - Isotropic energy distributions:
    - Maxwell-Juttner (θ=kT/mc2): $f(\gamma) = \frac{\gamma p\, e^{-\gamma/\theta}}{\theta K_2(1/\theta)}$
    - Power-law: $f(\gamma) \propto \gamma^{-s}$
    - Hybrid
    - Broken power-laws



Radiation Transfer

- **Transfer equation:** $\left(\frac{1}{c}\frac{\partial}{\partial t} + \vec{n}.\vec{\nabla}\right) I_\nu = j_\nu - \alpha_\nu I_\nu + \alpha_s(\langle I_\nu \rangle - I_\nu)$
  - I_ν (erg/s/Hz/cm/str) = Specific intensity
  - j_ν (erg/s/Hz/cm3/str) = Specific emissivity
  - $\alpha_{a,\nu}$ (1/cm) = Absorption coefficient $\tau_{a,\nu} \sim \alpha_{a,\nu} R$
  - $\alpha_{s,\nu}$ (1/cm) = Scattering coefficient $\tau_s \sim \alpha_s R$
  - Effective optical depth: $\tau^* \sim \sqrt{\tau_a(\tau_a + \tau_s)}$
  - Optically thin/thick: τ=1





- **Processes for isotropic distributions:**
  - Power emitted per particle $P_\nu(\gamma)$: $j_\nu = \frac{1}{4\pi}\int P_\nu(\gamma) f(\gamma) d\gamma$
  - Absorption/scattering cross section $\sigma_\nu(\gamma)$: $\alpha_\nu = \frac{1}{4\pi}\int \sigma_\nu(\gamma) f(\gamma) d\gamma$
  - Emission and absorption are linked: $\sigma_\nu(\gamma) = \frac{c^2}{2h\nu^3}\frac{1}{p\gamma}\left[(p\gamma P_\nu)_{\gamma+h\nu/mc2} - (p\gamma P_\nu)_\gamma\right]$ (True absorption; Stimulated emission)
  - At large optical depth:
    - Thermal distribution: $\frac{j_\nu}{\alpha_\nu} = B_\nu \sim \nu^2$
    - Power-law distribution: $\frac{j_\nu}{\alpha_\nu} \sim \nu^{5/2}$

Special Relativity

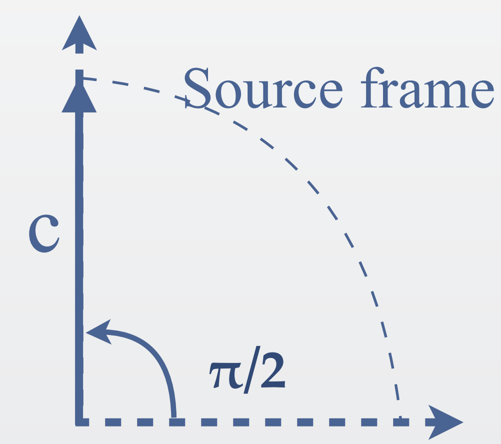
- **Two frames: source(‘)/observer**

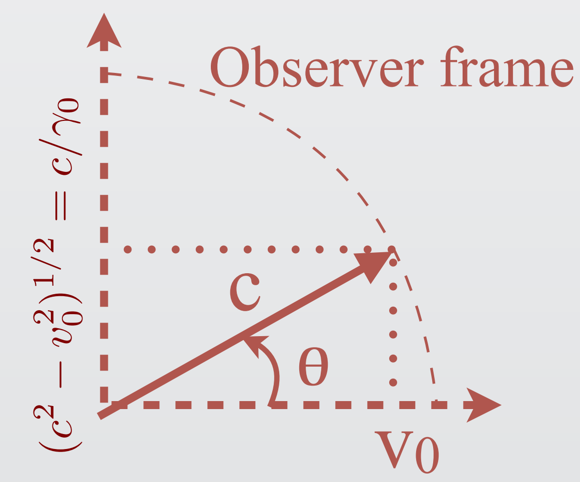
- **Relativistic Doppler:**
  - Doppler factor: $\nu = \nu' \delta$

$$\frac{1}{2\gamma_0} \leq \delta = \frac{1}{\gamma_0(1-\mu\beta_0)} \leq 2\gamma_0$$

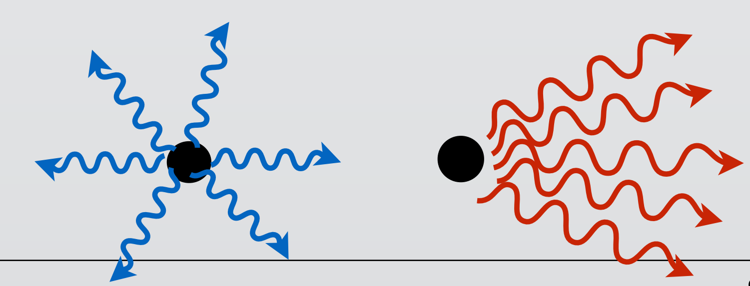
Time dilation   classical Doppler (propagation)

  - Photon energy of approaching sources is increased
  - e.g. spectral features in Blazars, GRBs



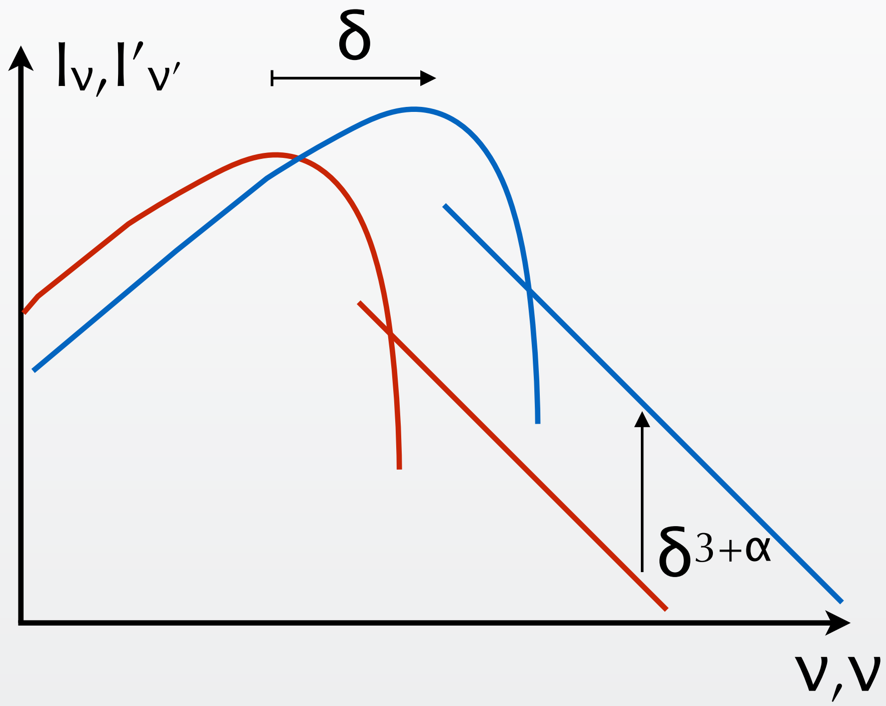


- **Relativistic Aberration:**
  - Ex: Perpendicular photon emission
    - Observed with an angle: $\cos\theta = v_0/c$
    - For relativistic velocities: $\beta_0 \sim 1 - 1/(2\gamma_0^2)$ => $\theta \sim 1/\gamma_0$
  - For any smooth angular distribution: $\theta \leq \dfrac{1}{\gamma_0}$

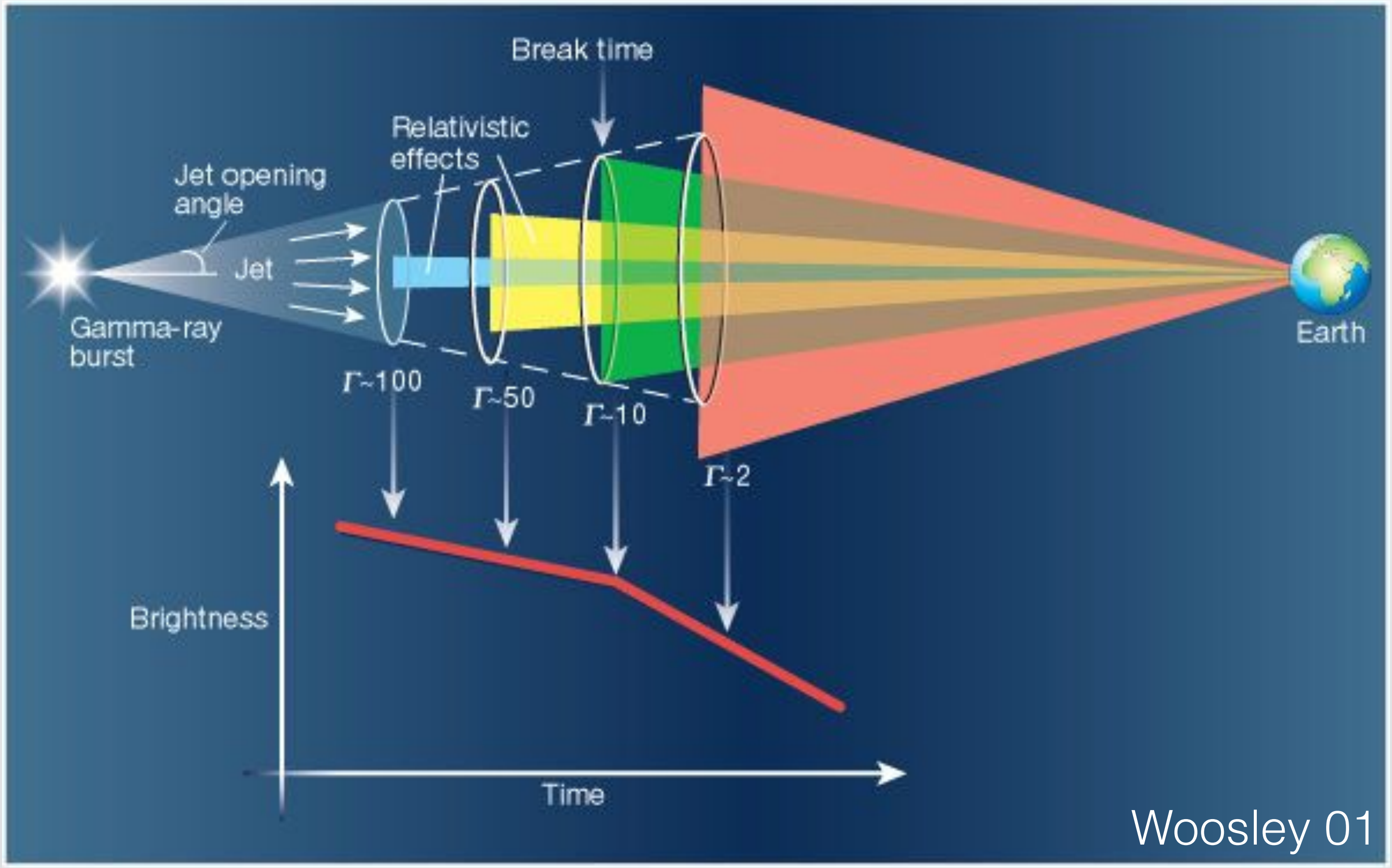


Special Relativity

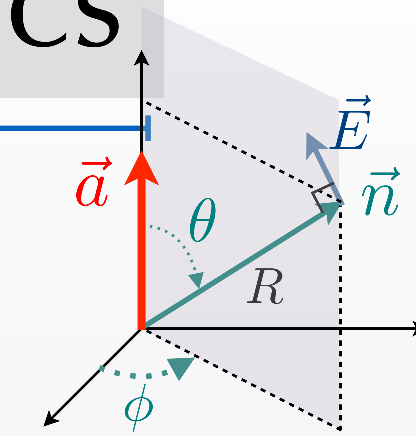
- **Beaming:** $d\Omega = d\Omega'/\delta^2$
- **Boosting:** $I_\nu = \delta^3 I'_{\nu'}$
- **Spectrum:**
  - Temperature (effective or brightness): $T = \delta\, T'$
  - Power-law: $I'_{\nu'} \propto (\nu')^{-\alpha} \;\Rightarrow\; I_\nu \propto \delta^{3+\alpha} \nu^{-\alpha}$



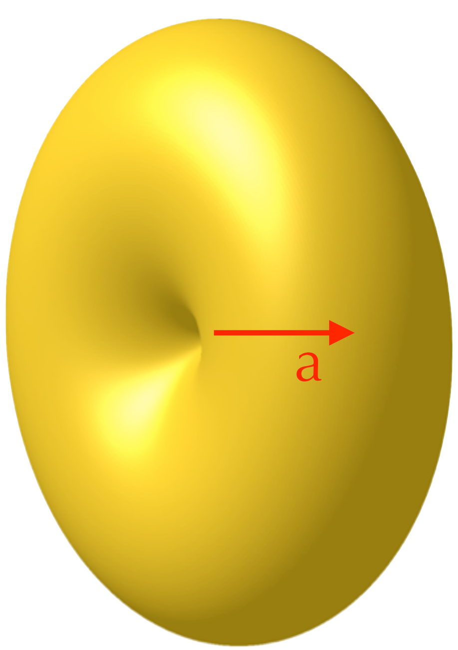
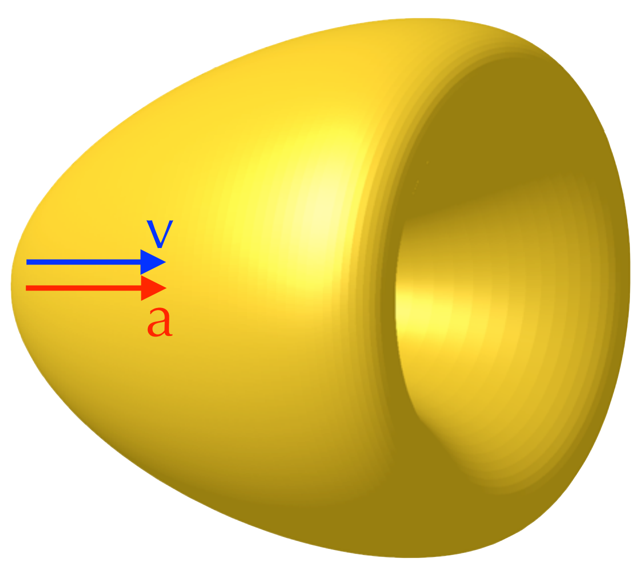
- **Jet break in GRBs**
  - Constant θ_{jet}
    - Expansion
    - Flux decrease
  - $\Gamma_{jet} >> 1/\theta_{jet} >> 1$
    - Beaming
    - small visible area
  - Deceleration
    - Increased visible area
    - Flux decrease partly compensated
  - When $\Gamma_{jet} < 1/\theta_{jet}$
    - no more compensation
    - Steepening of the flux decrease



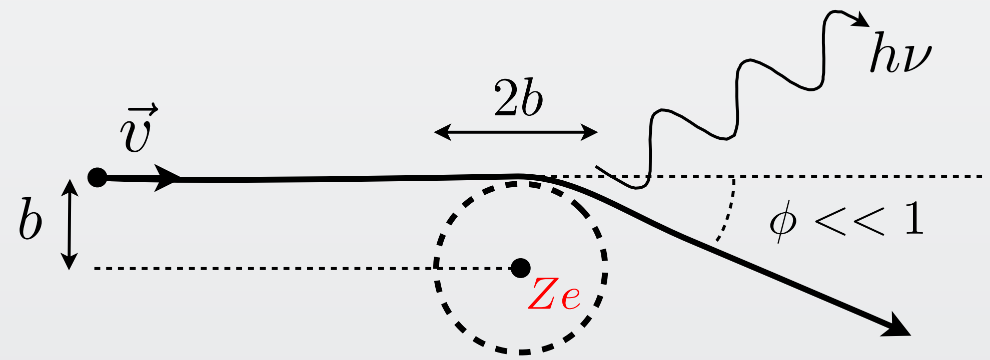
Emission in classical electrodynamics



- **Emission due to accelerated charges**
- **In the particle rest frame:** $\vec{E} = \frac{q}{Rc^2}\,\vec{n} \times (\vec{n} \times \vec{a})$

	Non Relativistic	Any Regime	
		Parallel	Perpendicular
Total Power	$P = \frac{2q^2a^2}{3c^3}$	$P = \frac{2q^2a^2}{3c^3}\gamma^6$	$P = \frac{2q^2a^2}{3c^3}\gamma^4$
Angular distribution	$\frac{dP}{d\Omega} = \frac{q^2a^2}{4\pi c^3}\sin^2\theta$	$\frac{dP}{d\Omega}(\theta, \phi)$	
	Dipolar emission	Beamed emission	
			

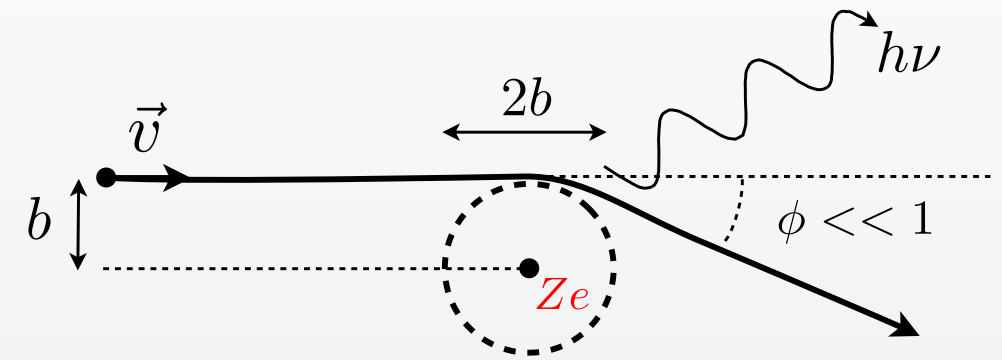
II. Bremsstrahlung



- **Radiation of charges accelerated by the Coulomb field of other particles**

II. Bremsstrahlung

- **Non-relativistic + small deflexion angle**

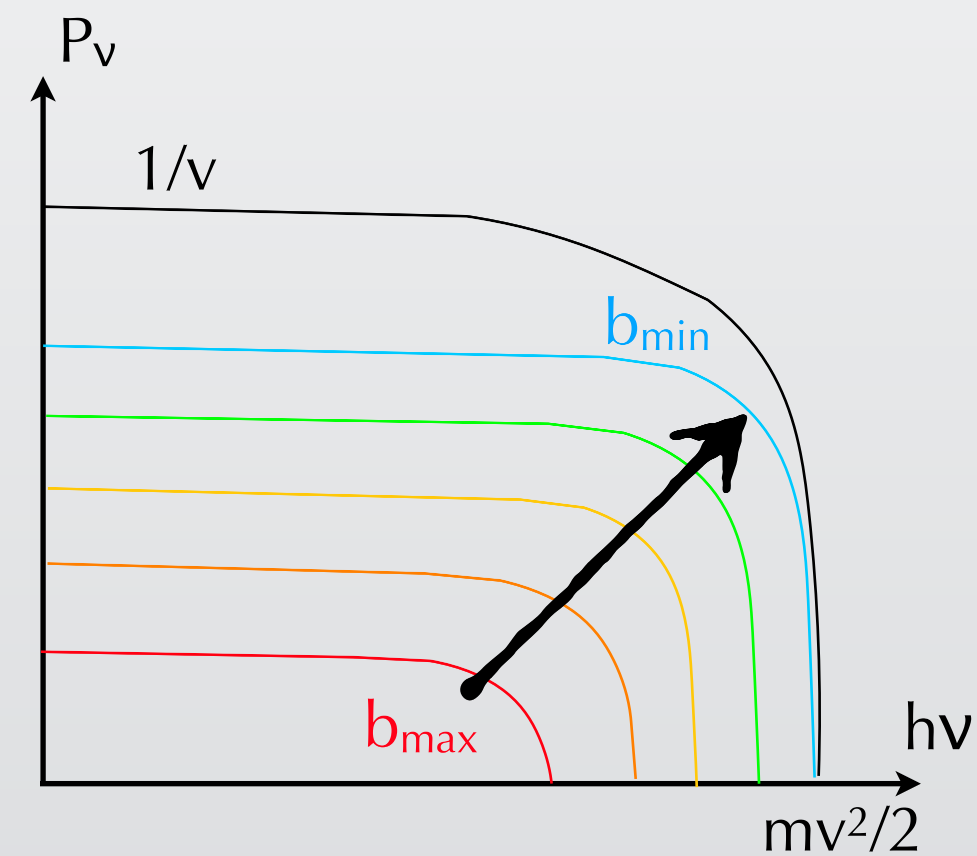


- **Simple derivation for a single interaction event:**
  - Acceleration: $a = \frac{Ze^2}{mb^2}$
  - Interaction duration: $\tau = \frac{b}{v}$
  - Energy radiated: $E = \tau P_{int} = \tau \frac{e^2 a^2}{c^3} = \frac{Z^2 e^6}{m^2 c^3 v b^3}$
  - Cutoff frequency: $\nu_c = \frac{1}{\tau}$
  - Spectral energy radiated up to ν_c: $E_\nu = \frac{E}{\nu_c} = \frac{Z^2 e^6}{m^2 c^3 v^2 b^2}$

- **Emission from many interactions**
  - Power radiated for many interactions:

$$P_\nu = n_e n_i v \int E_\nu 2\pi b db = n_e n_i \frac{Z^2 e^6}{m^2 c^3 v} g_{\rm ff}(v, \nu)$$

  - Single Gaunt factor: $g_{\rm ff}(v, \nu) = \ln \left(\frac{b_{\rm max}}{b_{\rm min}} \right)$
  - Cutoff frequency: $h\nu_c < mv^2/2$
  - Total Power: $P = n_e n_i \frac{Z^2 e^6}{mc^3 h} v$



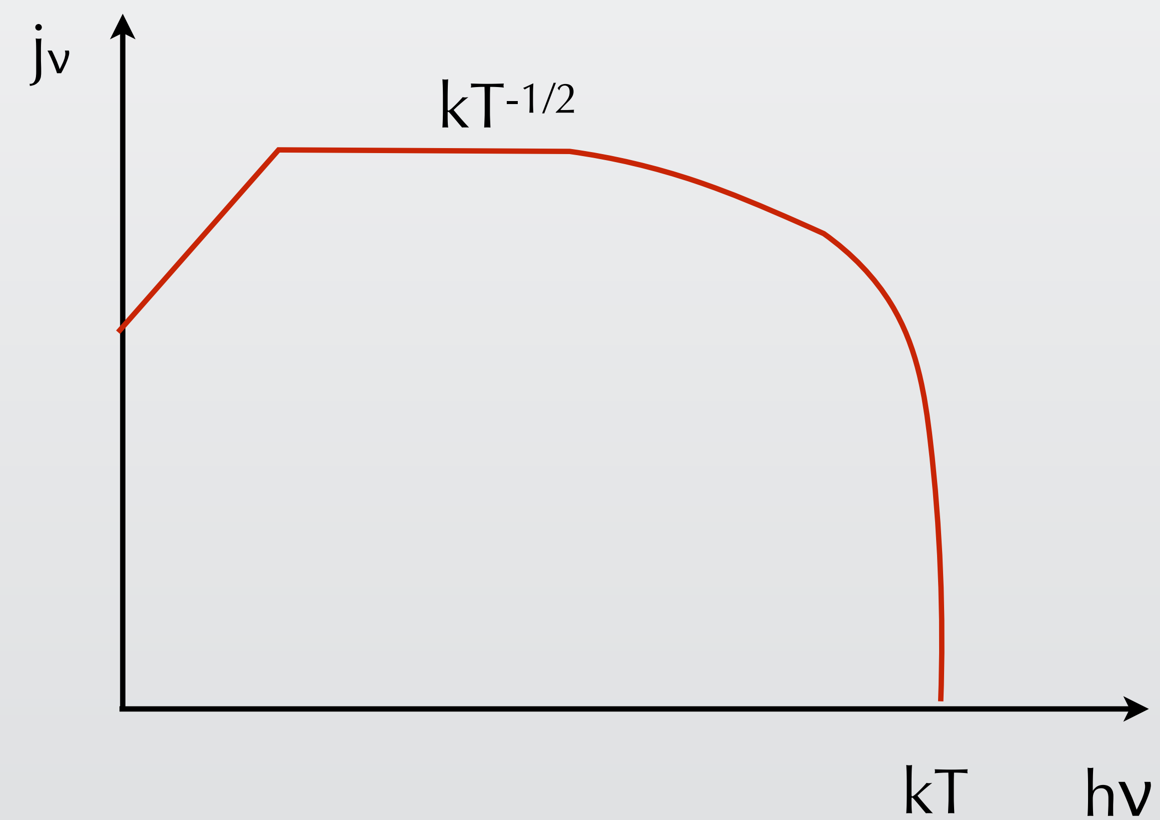
Bremsstrahlung

- **For isotropic particle distributions:** $j_\nu^{\rm ff} = \frac{1}{4\pi}\int P_\nu f(v) dv$

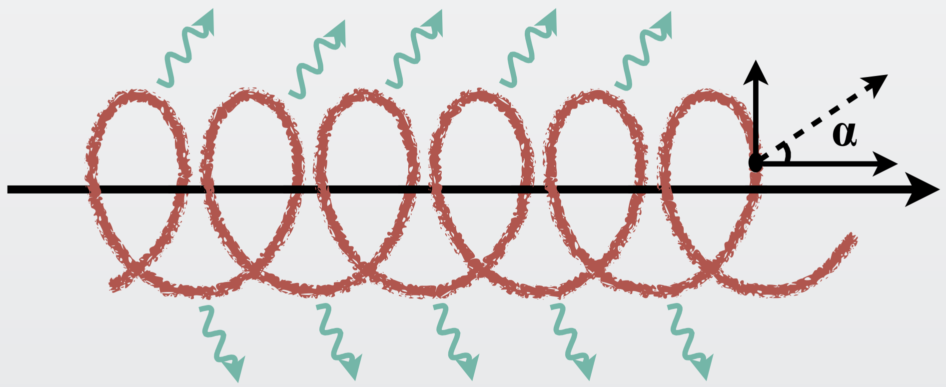
- **For thermal distributions:**
  - Flat spectrum: $j_\nu^{\rm ff} \propto n_i n_e Z^2 T^{-1/2} e^{-\frac{h\nu}{k_B T}}$
  - Cutoff frequency $h\nu_c = k_B T$
  - Total Power: $j^{\rm ff} \propto n_i n_e Z^2 T^{1/2}$

- **Exact calculations imply corrections in T and ν (Gaunt factors)**

- **Absorption for thermal distributions:**
  - coefficient: $\alpha_\nu^{\rm ff} \propto n_i n_e Z^2 T^{-3/2} \nu^{-2}$
  - Low energy self-absorption



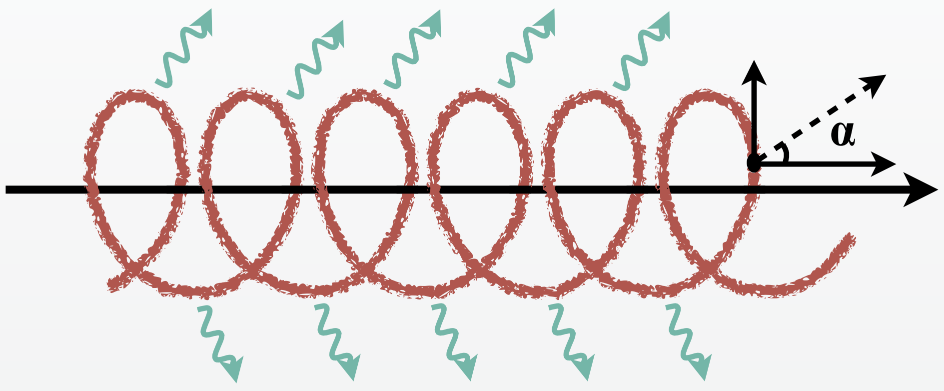
III. Magneto-Bremsstrahlung



- **Emission of charges defected by a magnetic field:**
  - Uniform field: cyclo-synchrotron radiation
  - Curved field: curvature radiation
  - Turbulent field: diffusive synchrotron radiation

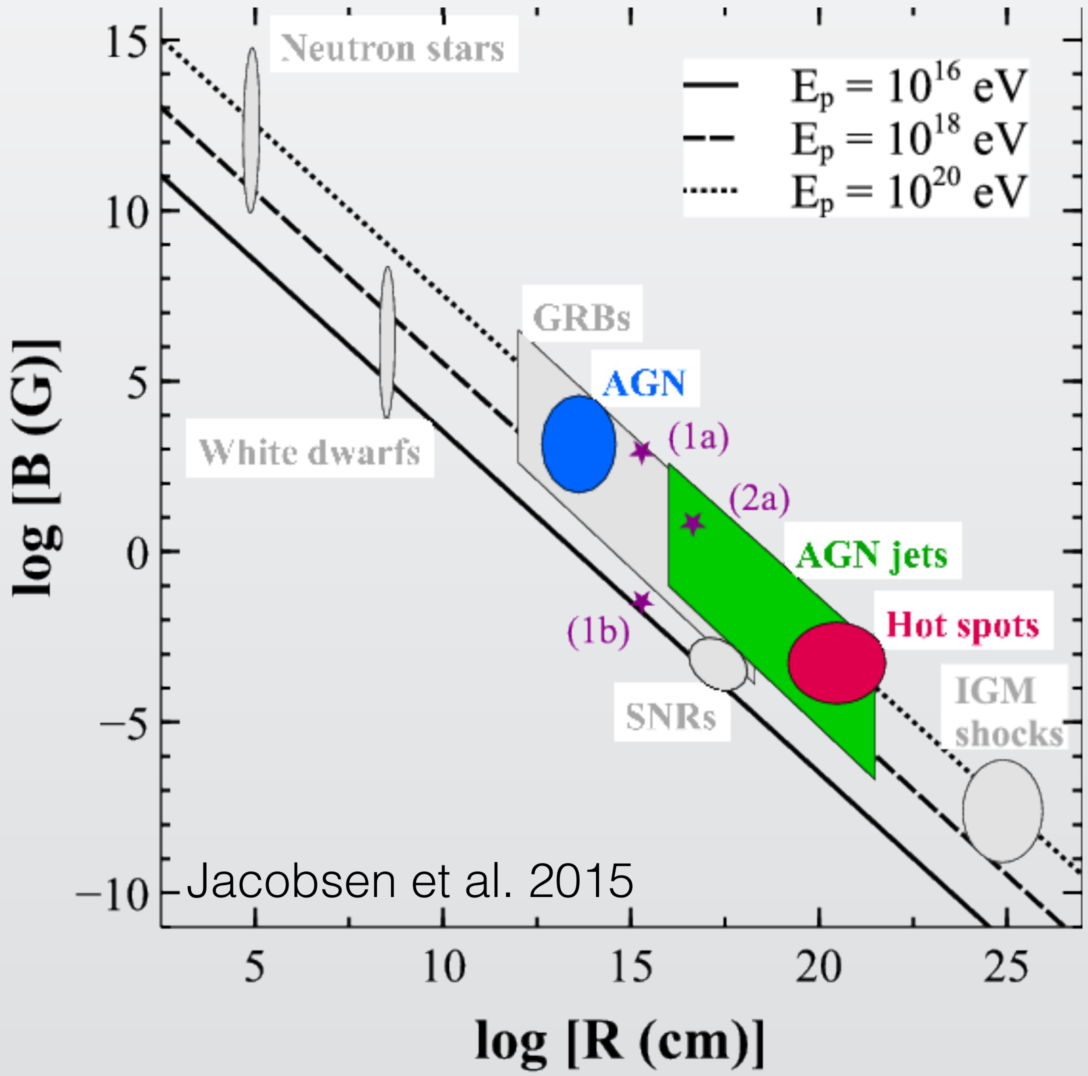
Orbital motion

- **Orbital motion:**
  - Pitch angle α
  - Larmor frequency: $\nu_B = \frac{\nu_L}{\gamma} = \frac{1}{\gamma}\frac{qB}{2\pi mc} \propto \frac{B}{\gamma}$
  - Larmor radius: $r_B = \frac{mc}{qB}p_\perp \propto \frac{\gamma}{B}$



- **Hillas criterium for particle acceleration:**

$$r_B < R_{\text{source}}$$
$$E_{\max} = qBR_{\text{source}}$$



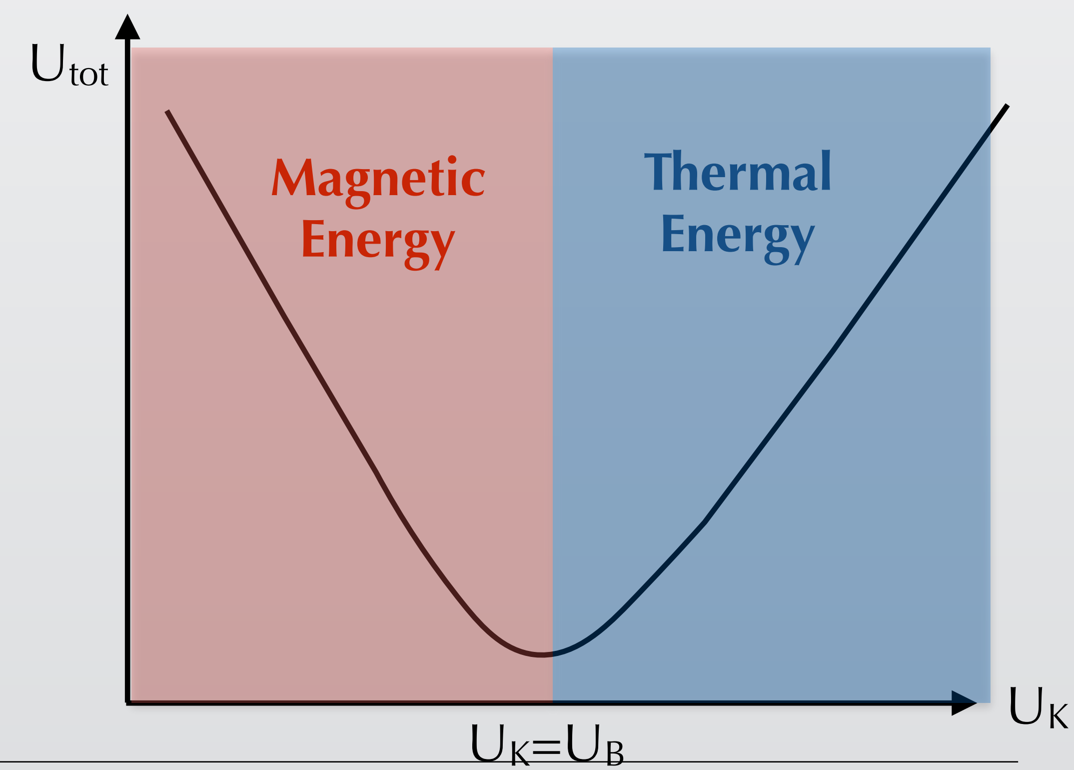
Jacobsen et al. 2015

Cyclo-Synchrotron Radiation

- **Assumptions:**
  - Uniform field + small energy losses + Classical mechanics
- **Power radiated per particle:**
  - Perpendicular acceleration: $a_\perp = \frac{qB}{mc}\frac{v_\perp}{\gamma}$
  - Pitch-angle dependent power: $P = 2c\sigma_T U_B p_\perp^2$
  - Average over isotropic distributions: $P = \frac{4}{3}c\sigma_T U_B p^2 \propto B^2\gamma^2$
- **Cooling time:**

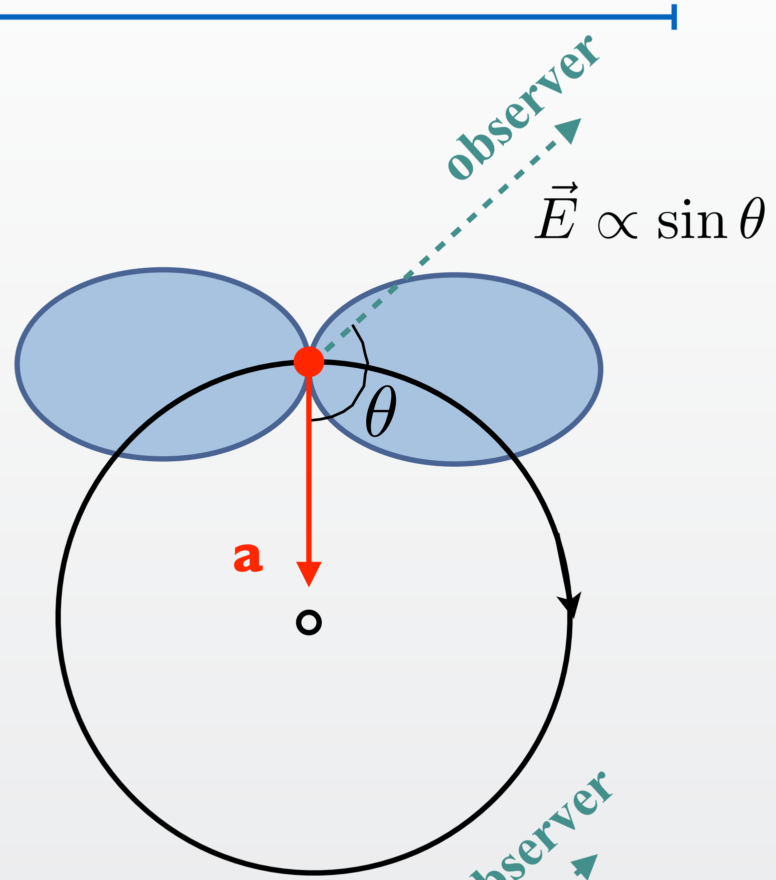
$$t_s = \frac{(\gamma-1)mc^2}{P} = \frac{3mc^2}{4c\sigma_T U_B(\gamma+1)} \propto B^{-2}\gamma^{-1}$$

- **Minimal energy theorem:**
  - $U_{tot} = U_K + U_B$
  - $L = N\ U_B\ U_k^q$
  - $U_{tot} = U_K + L/(NU_k^q)$
  - $U_K = q\ U_B$ corresponds to the minimal energy



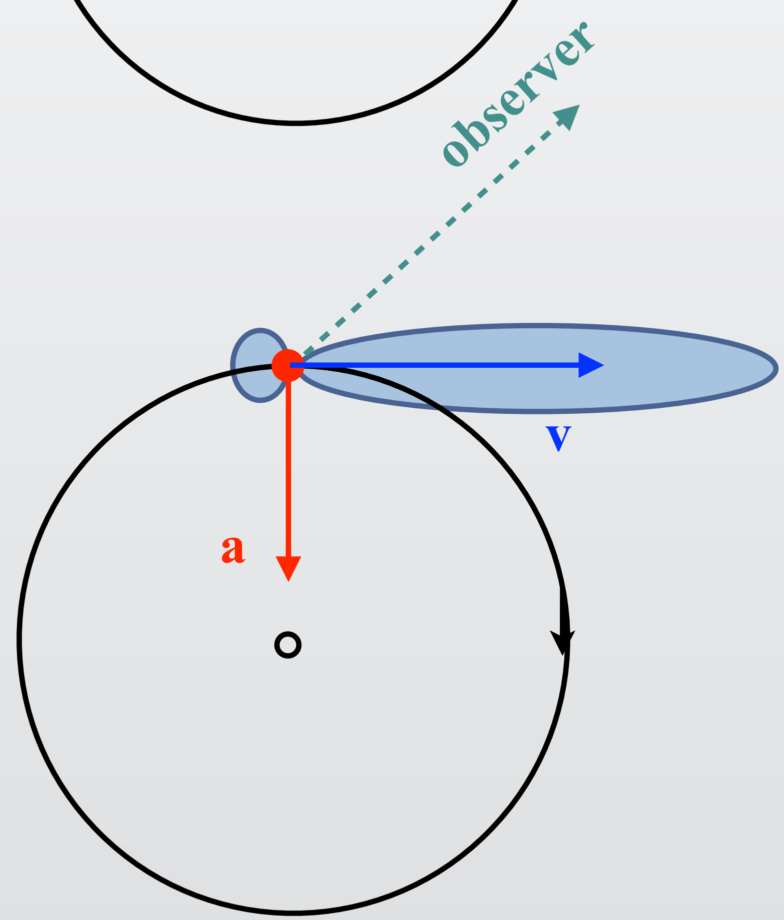
Cyclo-Synchrotron Spectrum

- **Assuming:**
  - Pure perpendicular motion
  - Observer in the orbital plane



- **Cyclotron line:**
  - Sinus electric field
  - Spectrum = single line at ν_L

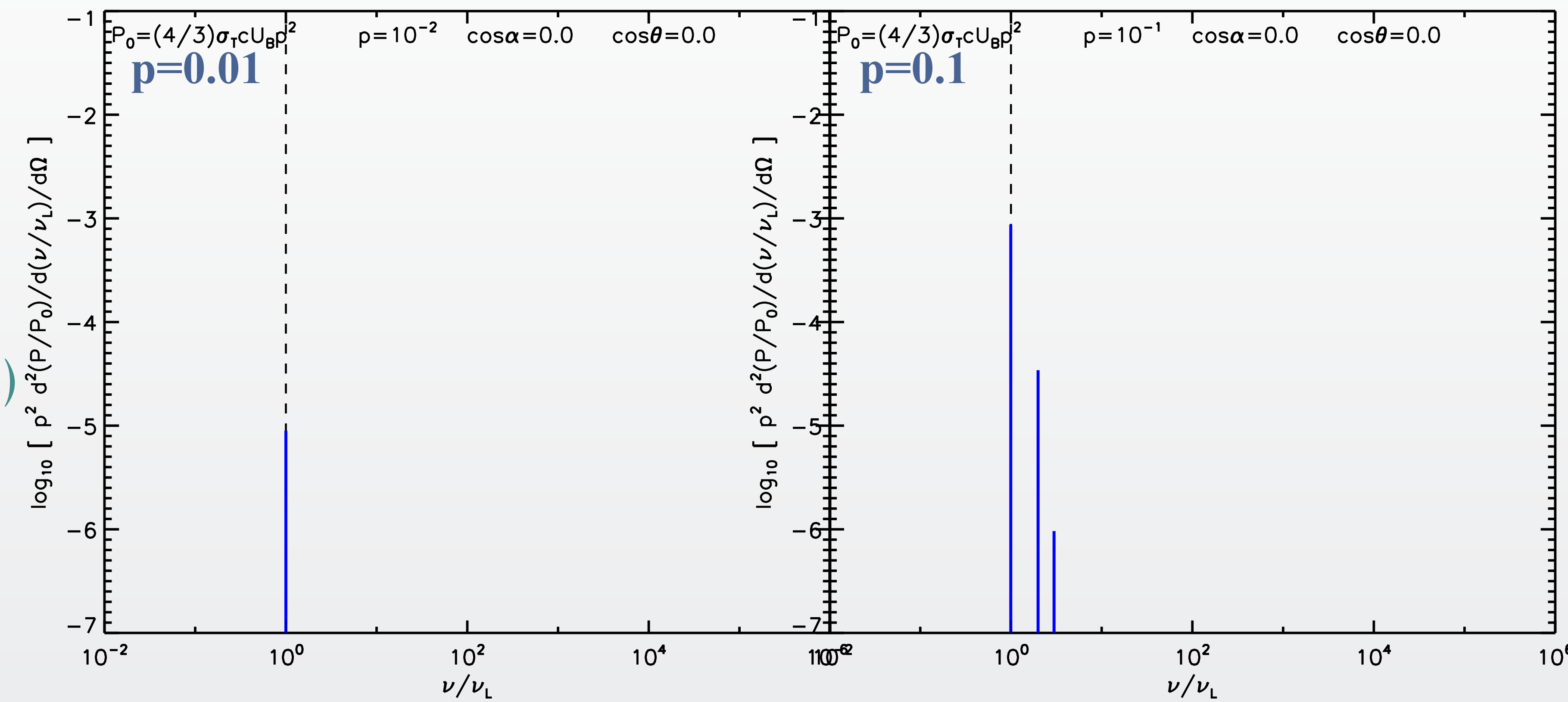
- **Cyclo-synchrotron spectrum:**
  - Pulsed electric field
    - Pulse separation: $\Delta t = 1/\nu_B$
    - Pulse duration $\delta t \sim (1-\beta)\frac{\delta\theta}{\nu_B} \sim (\gamma^3 \nu_B)^{-1}$
  - Spectrum:
    - many harmonic lines at $k\nu_B$
    - Enveloppe peaking at $\nu_c \sim \gamma^2 \nu_L \propto \gamma^2 B$

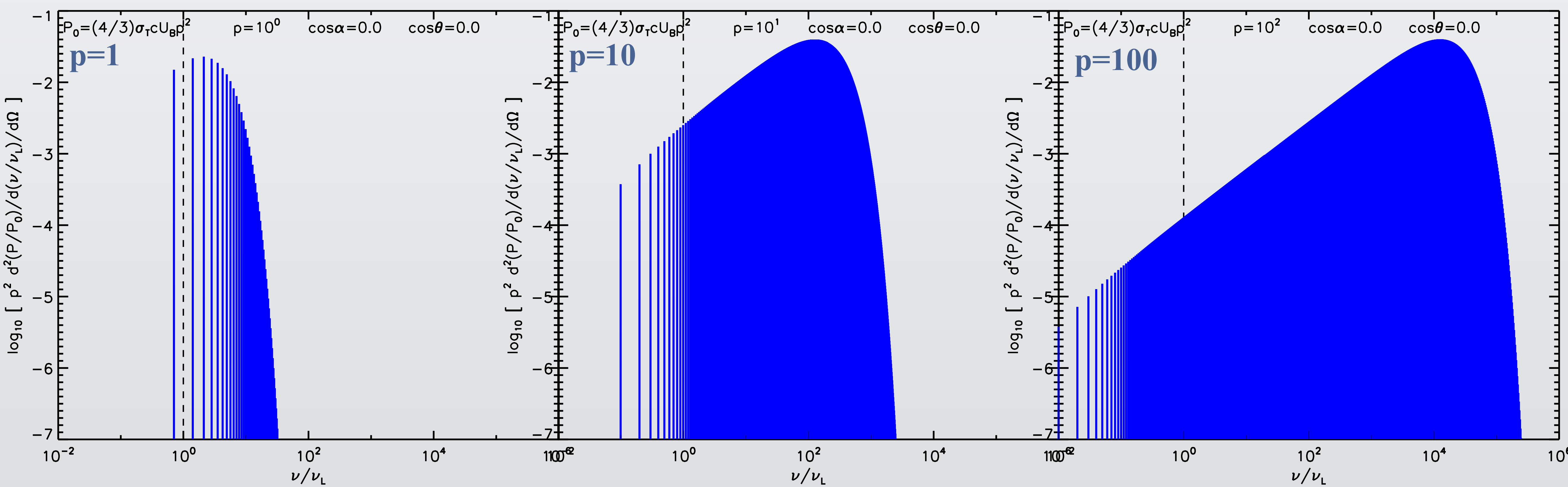


Cyclo-Synchrotron Spectrum

α=π/2 (pure perpendicular motion)

θ=π/2 (observer in the orbital plane)

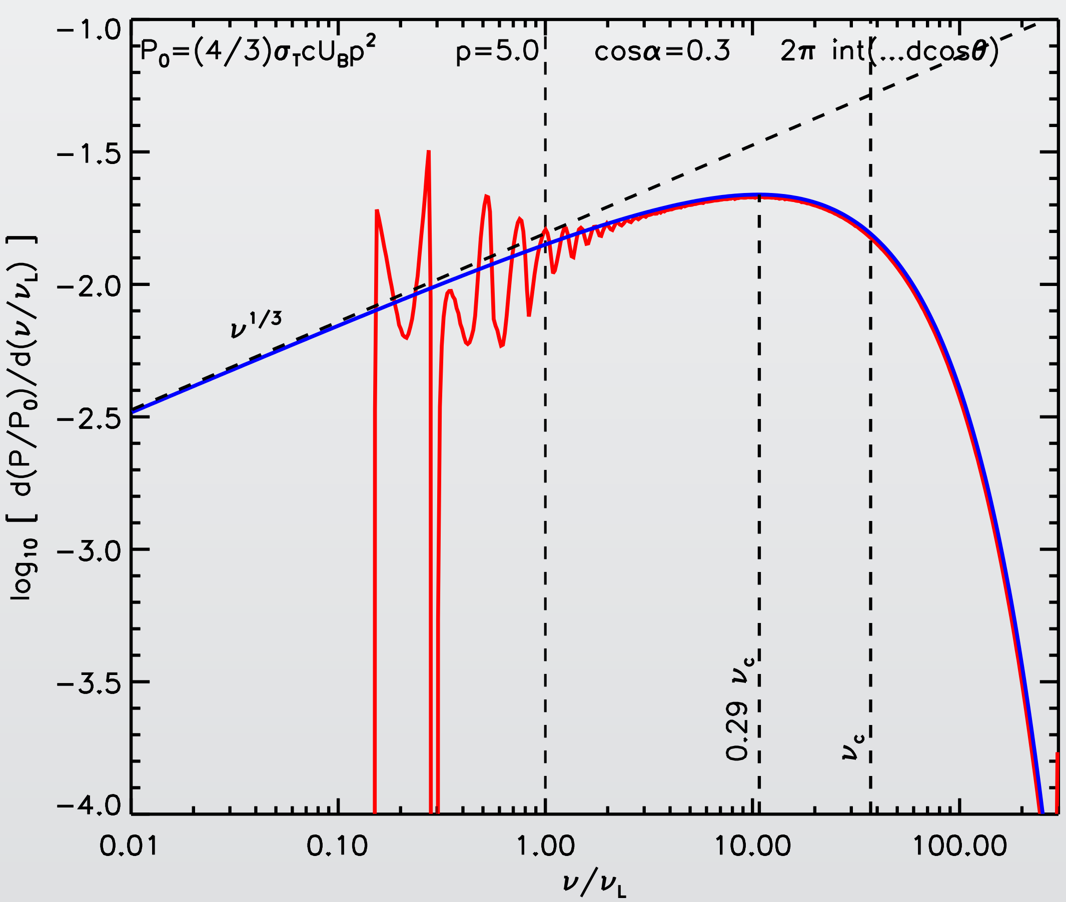


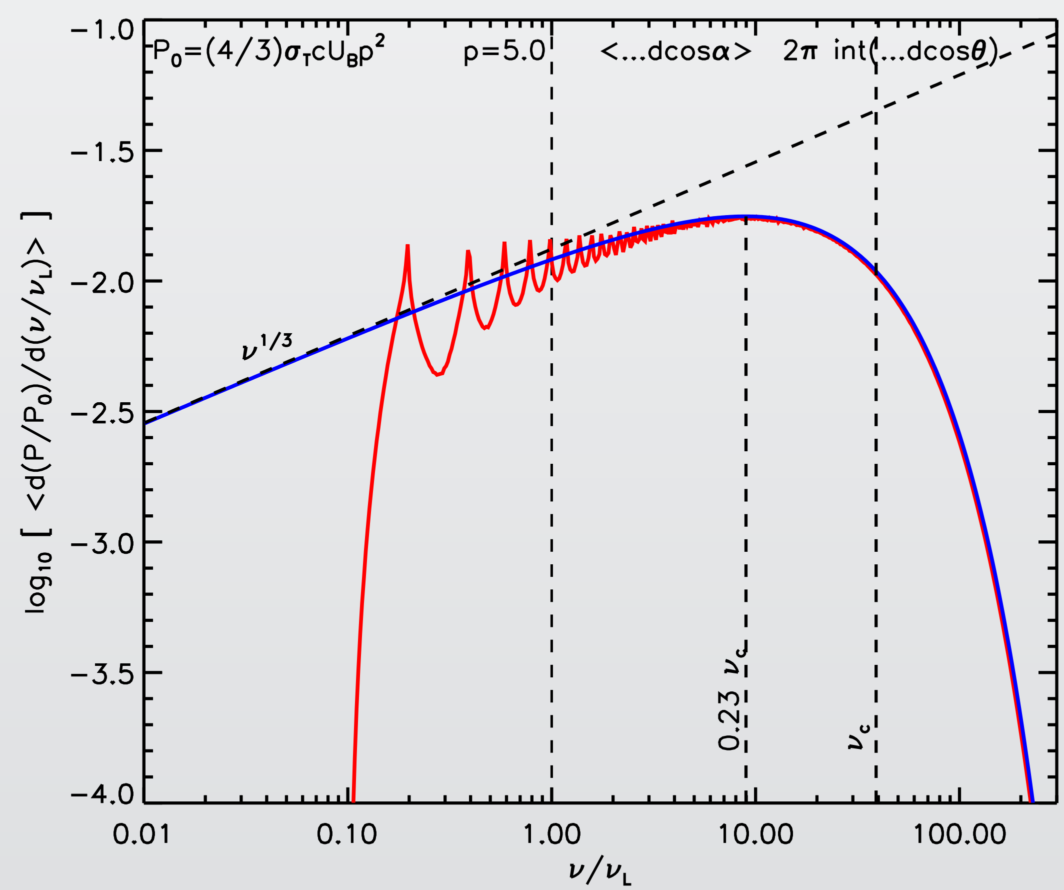


Synchrotron Spectrum

- **For ultra-relativistic particles (γ>>1), emission in a narrow shell**

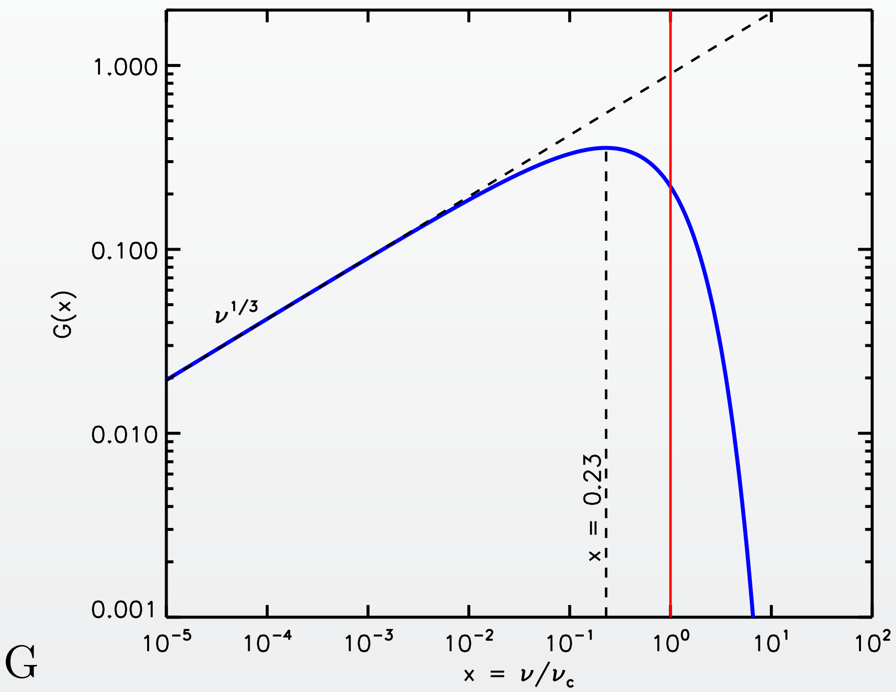
Pitch Angle	Anisotropic (dα>1/γ, aligned with los)	Isotropic (average)
Spectrum	$\frac{dP}{d\nu} = \frac{P_0}{\nu_c}\frac{9\sqrt{3}}{8\pi}F\left(\frac{\nu}{\nu_c}\right)$	$\frac{dP}{d\nu} = \frac{P_0}{\nu_c}\frac{27\sqrt{3}}{8\pi}H\left(\frac{\nu}{\nu_c}\right)$
Total Power	$P_0 = 2c\sigma_T U_B \gamma^2 \sin^2\alpha$	$P_0 = \frac{4}{3}c\sigma_T U_B \gamma^2$
Cutoff Frequency	$\nu_c = \frac{3}{2}\gamma^2\nu_L \sin\alpha$	$\nu_c = \frac{3}{2}\gamma^2\nu_L$
Functional	$F(x) = x\int_x^\infty K_{5/3}(x')dx'$	$H(x) = \left(\frac{x}{2}\right)^2\left[K_{4/3}(x/2)K_{1/3}(x/2) - \frac{3x}{10}\left(K_{4/3}^2(x/2) - K_{1/3}^2(x/2)\right)\right]$





Synchrotron Spectrum

- **Summary:**
  - Total power (erg/s): $P \sim c\sigma_T U_B \gamma^2 \propto B^2\gamma^2$
  - Cutoff frequency: $\nu_c \sim \gamma^2 \nu_L \propto \gamma^2 B$

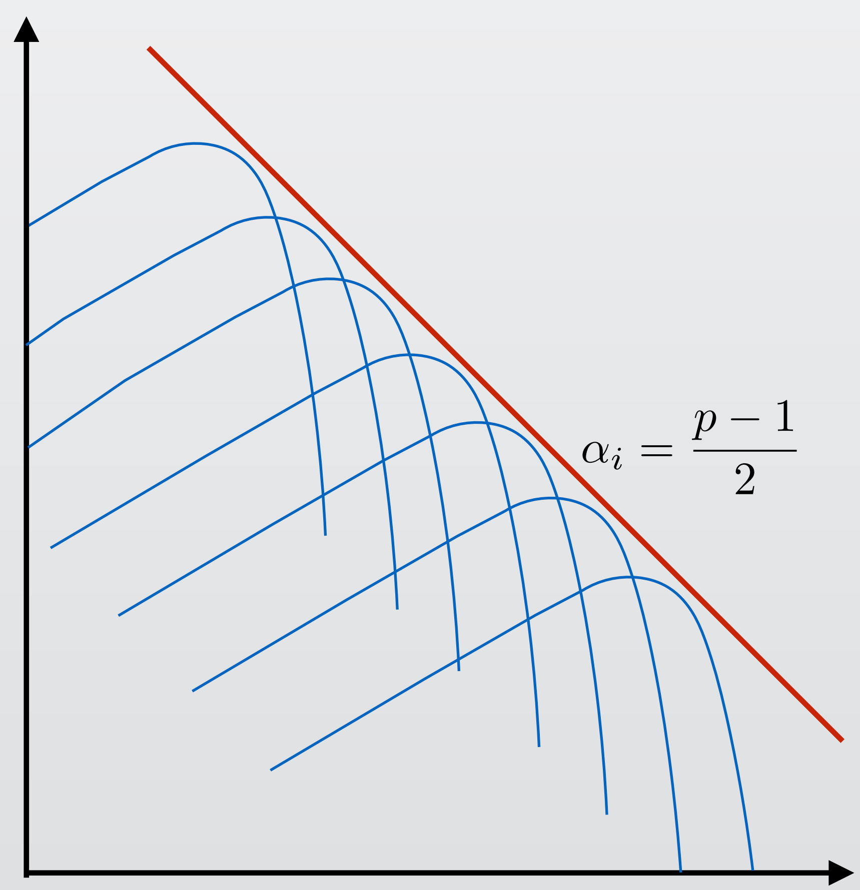


- **Maximal energy of synchrotron photons:**
  - Small loss approximation: $\nu_B t_s > 1$

$$B\gamma(\gamma+1) < \frac{3q}{\sigma_T} = 2.17 \times 10^{15}\mathrm{G}$$

$$h\nu_c < \frac{27}{16\pi}\frac{mc^2}{\alpha_f}$$

  - For electrons: $E_{e,\max} \sim 50$ MeV
  - For protons: $E_{p,\max} \sim 100$ GeV
  - Can be overcome by continuous injection of HE particles (production, reconnection)

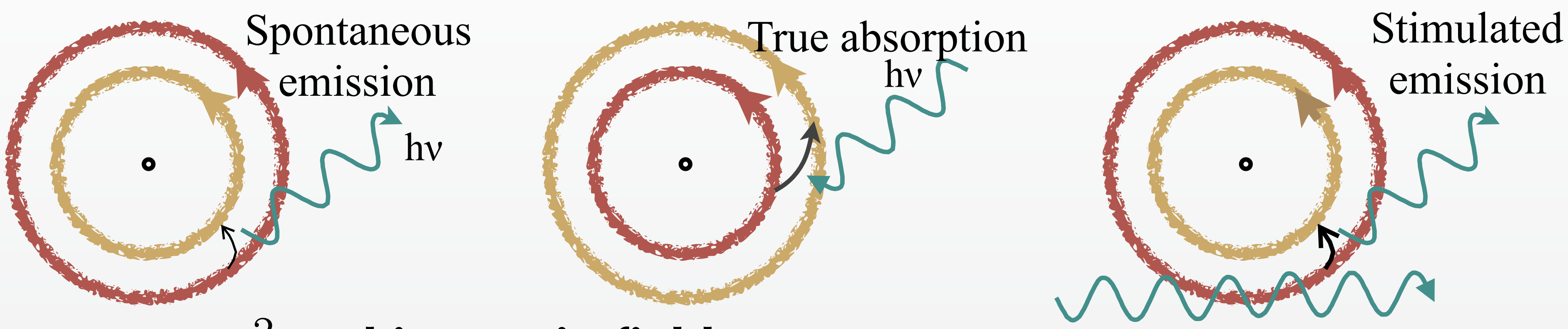


- **Particle distributions:** $j_\nu = \frac{1}{4\pi}\int P_\nu(\gamma) f(\gamma) d\gamma$
  - Power-law distribution of particles (index p)
  - => Power-law spectrum $j(\nu) \propto B^{\frac{p+1}{2}} \nu^{-\alpha_i}$ $\quad \alpha_i = \frac{p-1}{2}$

Synchrotron Polarisation

- **Ordered magnetic field:**
  - Linearly polarised perp to the projected field: $0 < \frac{P_\parallel}{P_\perp} < 1/3$
  - Mono-energetic distributions: $\Pi(\nu, \gamma, \alpha) = \frac{P_\perp - P_\parallel}{P_\perp + P_\parallel} = \frac{x K_{2/3}(x)}{F(x)} \geq 50\%$
  - Power-law particle distribution: $\Pi(\alpha, \nu) = \frac{p+1}{p+7/3} \geq 70\%$
- **Tangled magnetic field:**
  - Net polarisation cancels out!
- **Polarisation helps constraining the emission mechanism**
- **Ex: prompt GRB emission**
  - Many models (photospheric, synchrotron, Compton…)
  - Π~10% (Troja et al. 2015)
  - Spatial mixing
  - Time mixing (Zhang et al. 2019)
  - Energy dependent polarisation

Synchrotron Self-Absorption



- For $h\nu << \gamma mc^2$ and isotropic fields:

$$\sigma_\nu = \frac{1}{2m\nu^2}\frac{1}{\gamma p}\frac{d(\gamma p P_\nu)}{d\gamma} \qquad \alpha_\nu = \frac{1}{4\pi}\int \sigma_\nu(\gamma)f(\gamma)d\gamma$$

- Strong absorption at low photon energy!

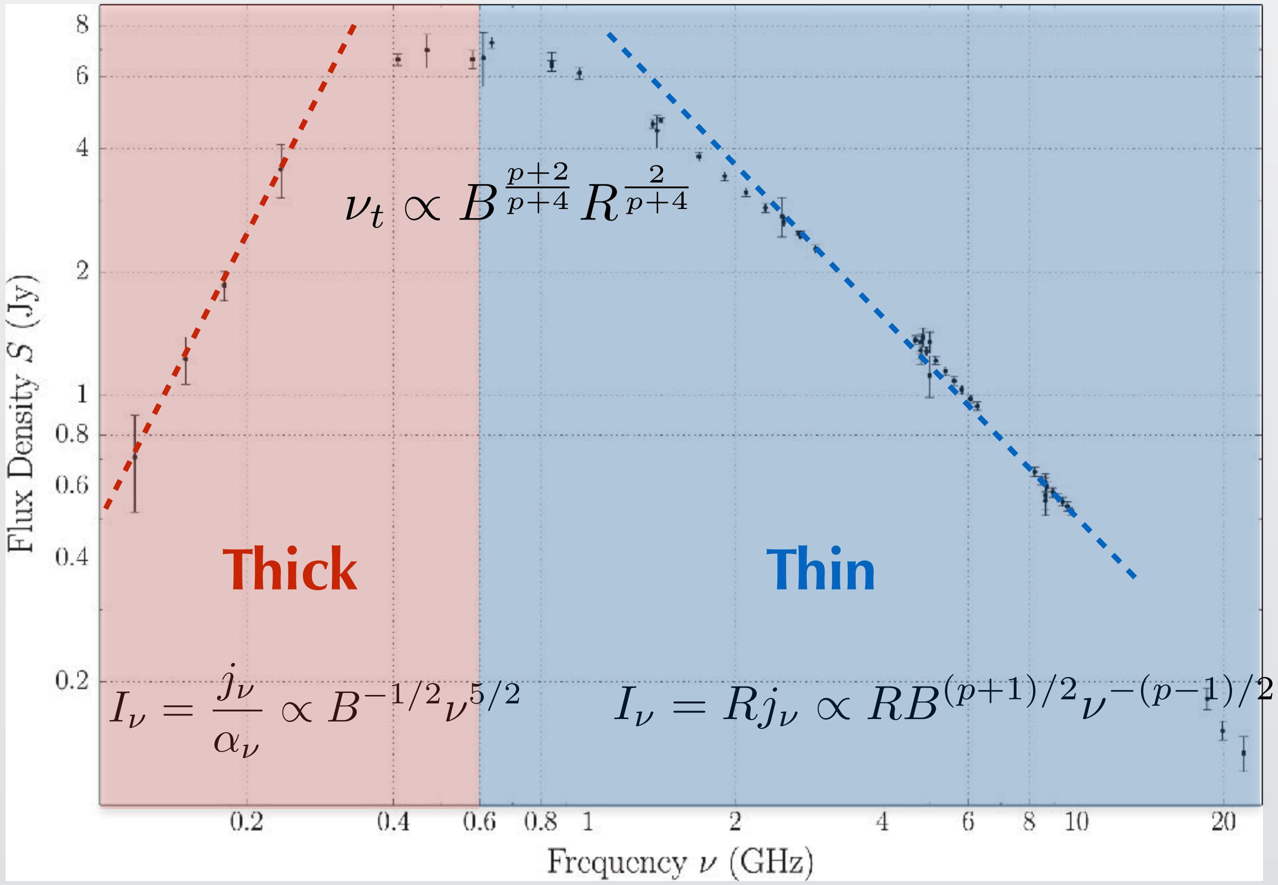
- Ex: for isotropic power-law distribution of particles:

$$j(\nu) \propto B^{\frac{1}{2}+\frac{p}{2}}\nu^{\frac{1}{2}-\frac{p}{2}}$$

$$\alpha_\nu \propto B^{1+\frac{p}{2}}\nu^{2-\frac{p}{2}}$$

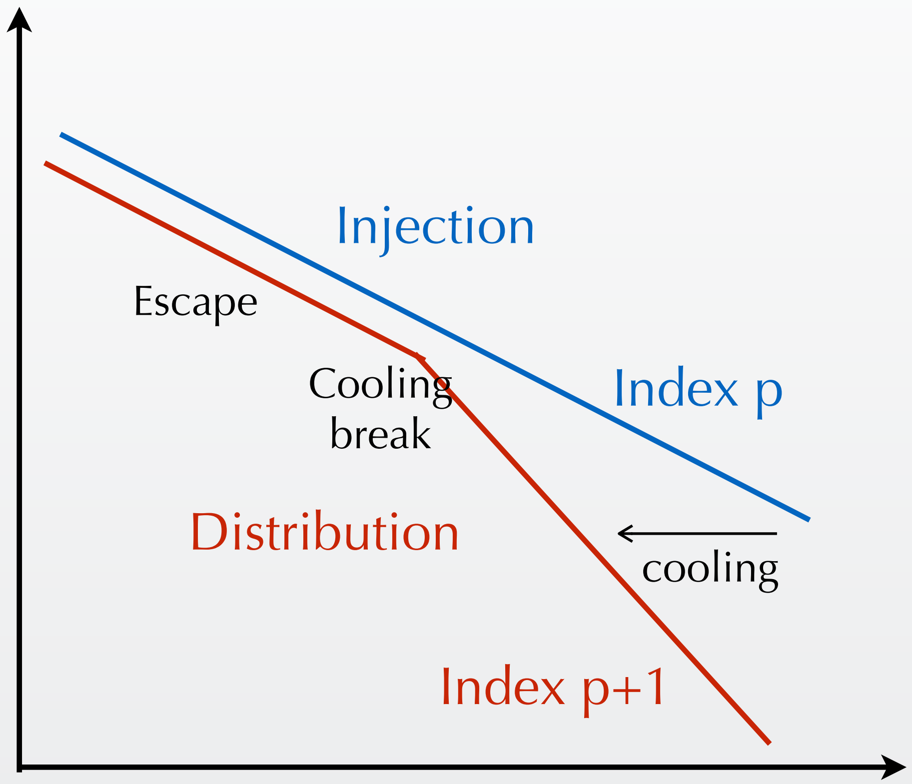
  - Turn-over frequency => B
  - Issues with inhomogeneous media
  - Other competing processes (free-free absorption, induced Compton)

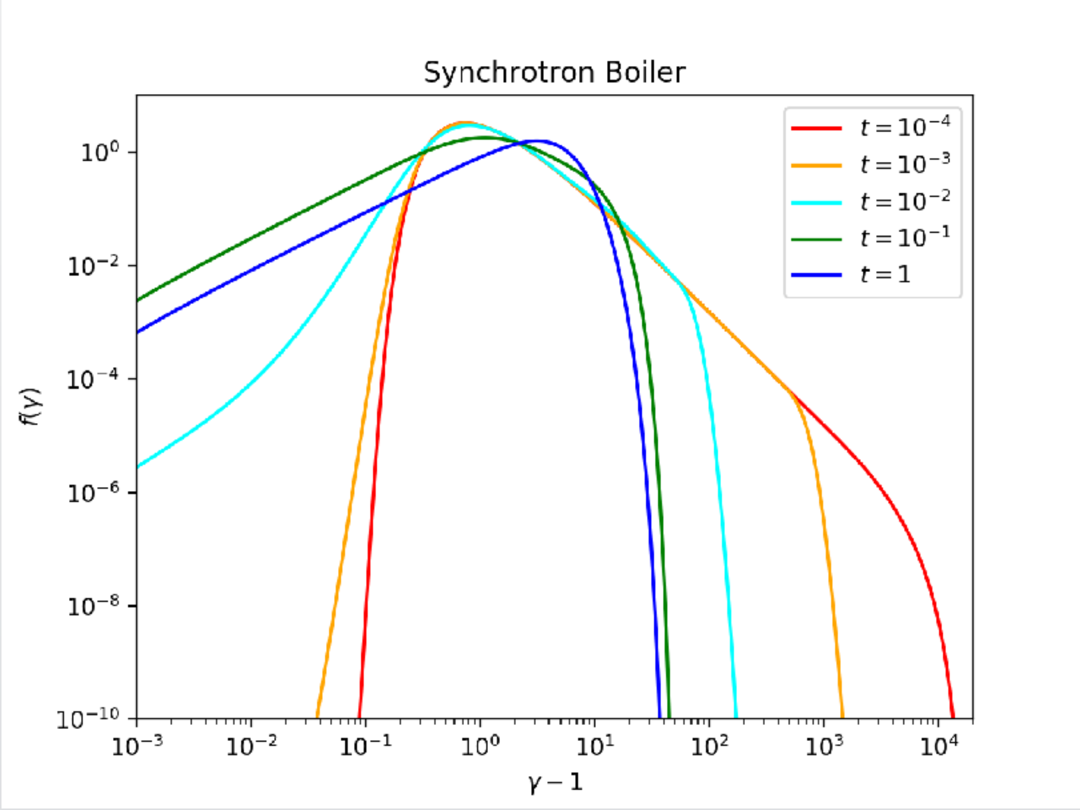
PKS B0008-421, Callingham+15



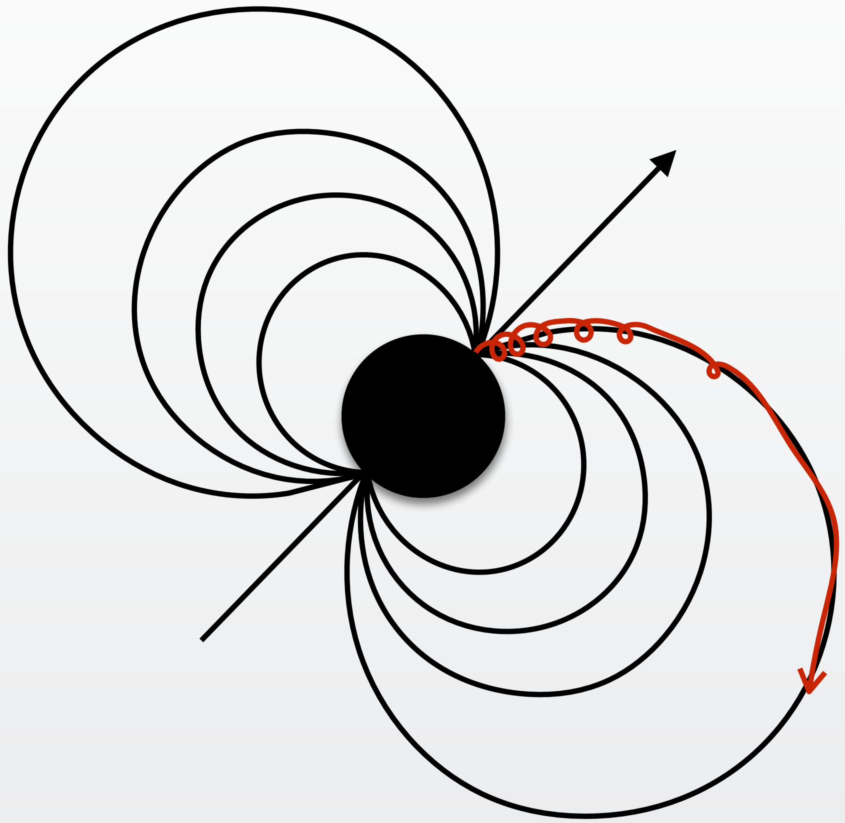
Synchrotron Interactions

- **Particle Cooling rate:** $\frac{dE}{dt} = \frac{4}{3} c \sigma_T U_B p^2$
- **Maximal energy:**
  - Local competition with acceleration
  - Ex: DSA in SNR: $\left.\frac{dE}{dt}\right|_{\text{acc}} = \frac{r-1}{r(r+1)} ZeBv_{\text{shock}}^2$
  - Maximal particle energy:
    $$E_{\max} = 240 \left(\frac{v_{\text{shock}}}{10^4 \text{km/s}} \right) \left(\frac{B}{1mT} \right)^{-1/2} \text{TeV}$$
- **Distribution slope:**
  - Cooling after acceleration
  - Evolution equation: $\frac{dN_\gamma}{dt} = \frac{d}{d\gamma}(-\dot{\gamma} N_\gamma) + Q_{\text{inj}} - \frac{N_\gamma}{t_{\text{esc}}}$
  - Non thermal injection: $Q_{\text{inj}}(\gamma) \propto \gamma^{-p}$
  - => cooling break and steepening $N_\gamma \propto \gamma^{-(p+1)}$
- **Thermalisation:**
  - Emission + Absorption => synchrotron boiler



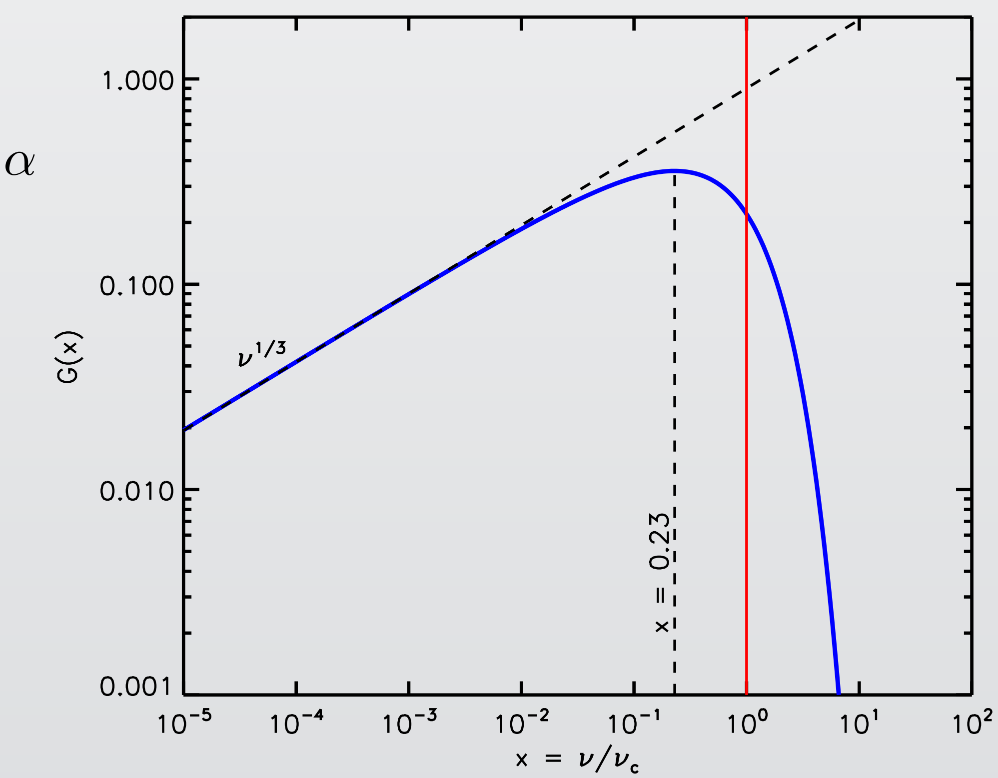


Curvature Radiation



- **Emission of particles guided by circular magnetic field lines**
- **e.g Pulsar magnetosphere**
  - Strong electric acceleration // field lines
  - $\begin{cases} v_{\parallel} \to c \\ v_{\perp} \to 0 \end{cases}$ => weak synchrotron
- **Circular motion => results for synchrotron apply**
  - Total power: $P = \frac{2}{3}\frac{qc\gamma^4}{R^2}$ $\qquad P_{\rm synch} \propto \gamma^2 B^2$
  - Spectrum
  - Cutoff frequency: $\nu_c = \frac{3}{4\pi}\frac{c\gamma^3}{R}$ $\qquad \nu_{\rm synch} \propto \gamma^2 B \sin\alpha$
  - Can produce higher energy photon than pure synchrotron in the outer gaps

$$h\nu_c \approx 3\left(\frac{\gamma}{10^7}\right)^3 \left(\frac{R}{10^6\rm cm}\right)^{-1} \rm GeV$$

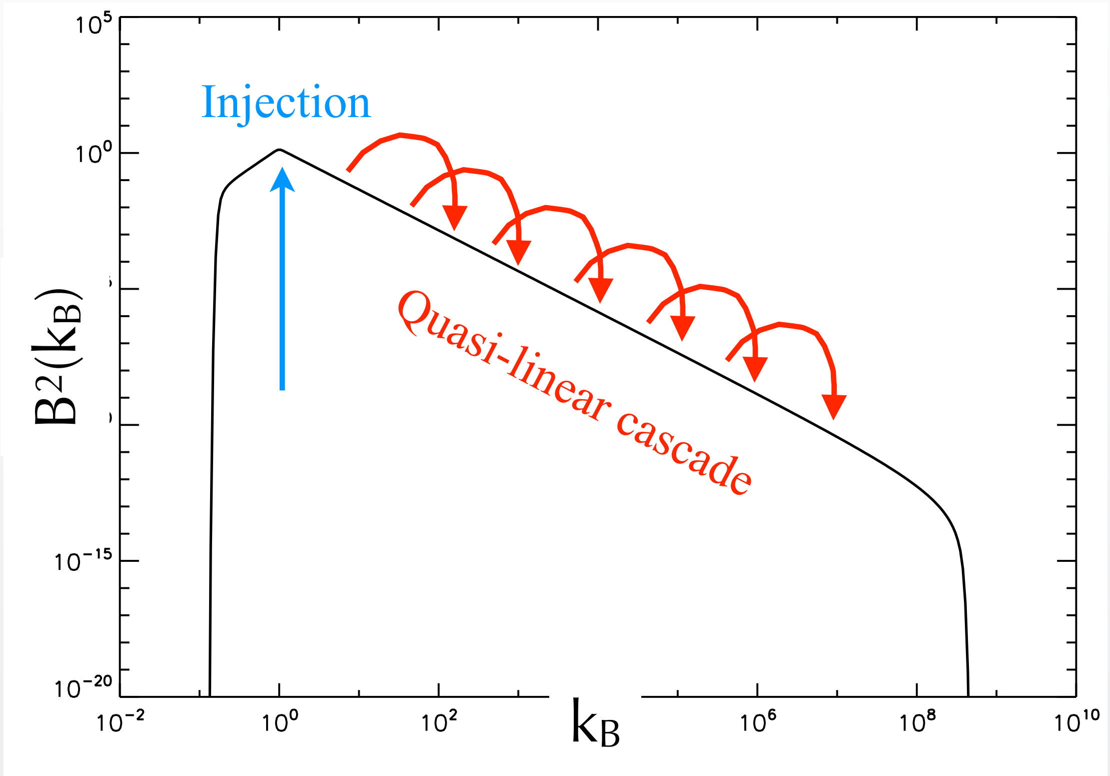


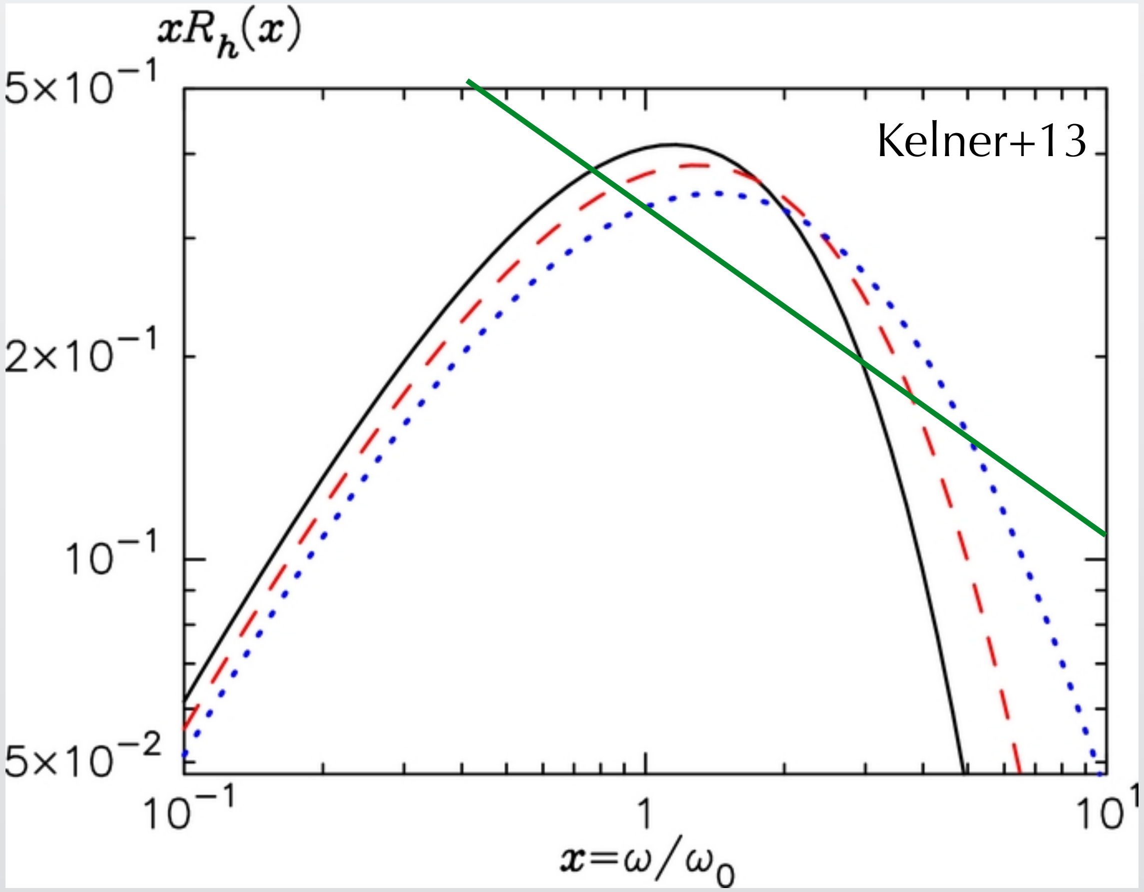
- **Most generally: Synchro-curvature radiation**

Diffuse Synchrotron Radiation

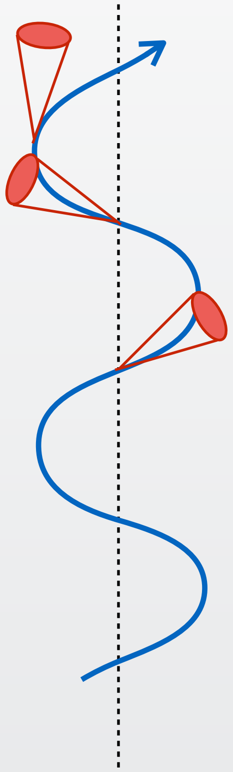
- **Turbulent magnetic field with many-scale perturbations**

- **Large scale fluctuations (λ>>R_L):**
  - => Inhomogeneous synchrotron radiation
  - Scale is irrelevant => turbulence described by amplitude distribution
  - Simple convolution of the synchrotron spectrum
  - For peaked distributions, shifted and broaden emission
  - For power-law distribution, power-law spectrum $P(B) \propto B^{-s}$ $F_\nu \propto \nu^{-s+2}$





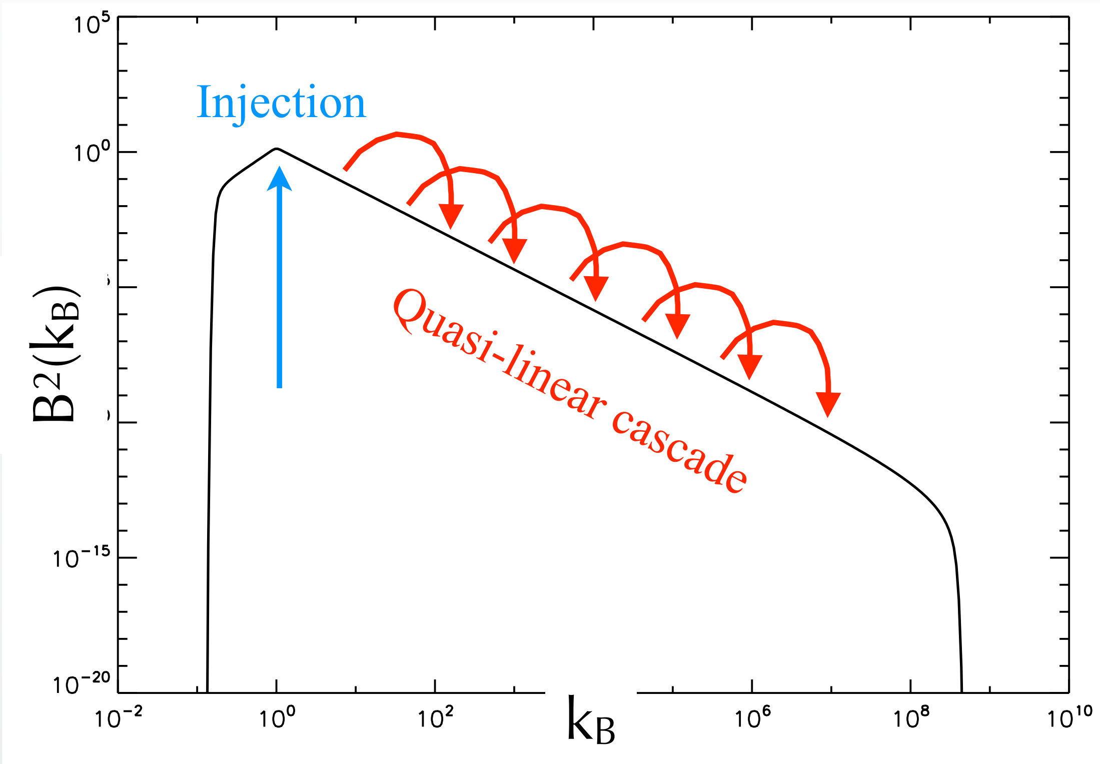
Diffuse Synchrotron Radiation

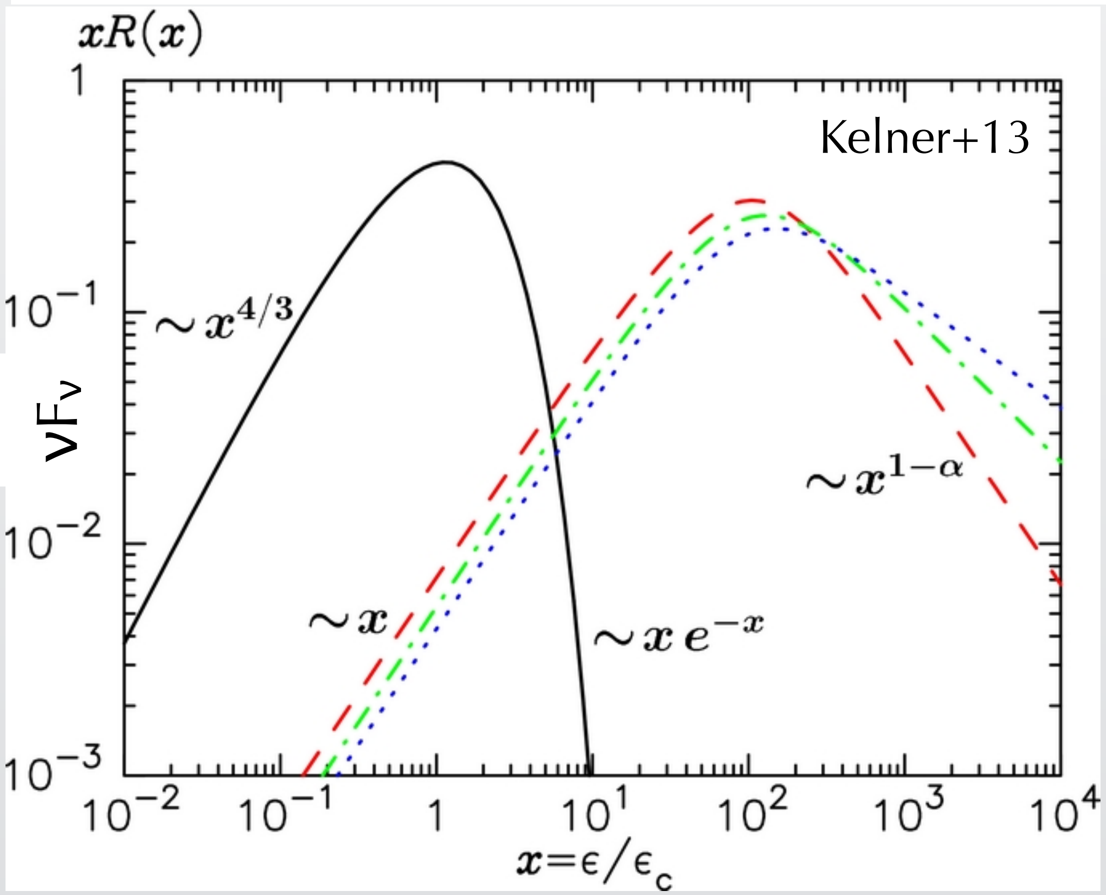


- **Small scale fluctuations (λ<<R_L):**
  - => Jitter radiation
  - Mono-chromatic wave $k_B=2\pi/\lambda_B$ => line

$$\nu_j \sim \gamma^2 k_B c \sim \frac{R_L}{\lambda_B}\,\nu_{c,\mathrm{synch}} > \nu_{c,\mathrm{synch}}$$

  - Small scale turbulence => large photon energy
  - Convolution with the field turbulent spectrum: $B^2(k_B) \propto k_B^{-\alpha}$
    - Slower rise
    - Higher energy cutoff
    - Power-law tail
  - Applications to
    - GRBs
    - Blazars
    - Crab flares





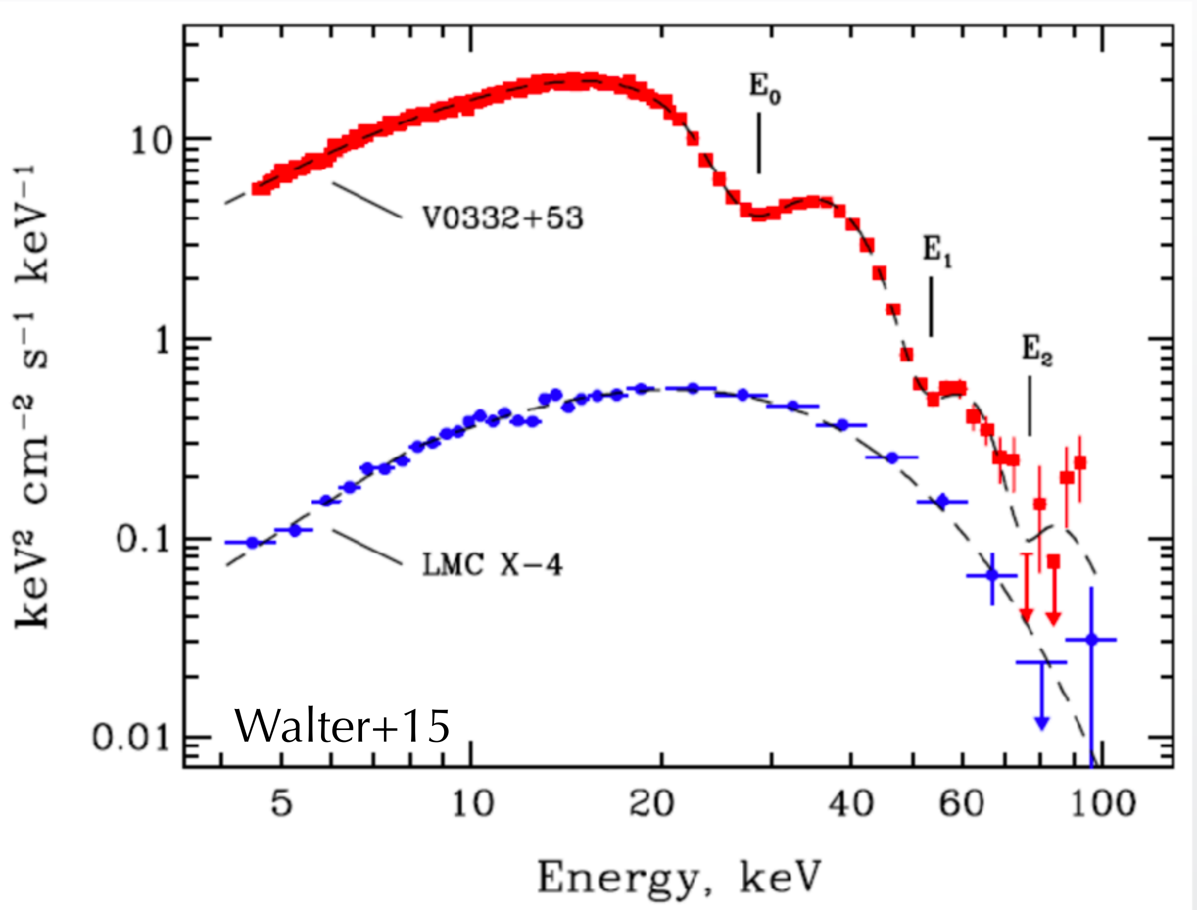
Strong Fields

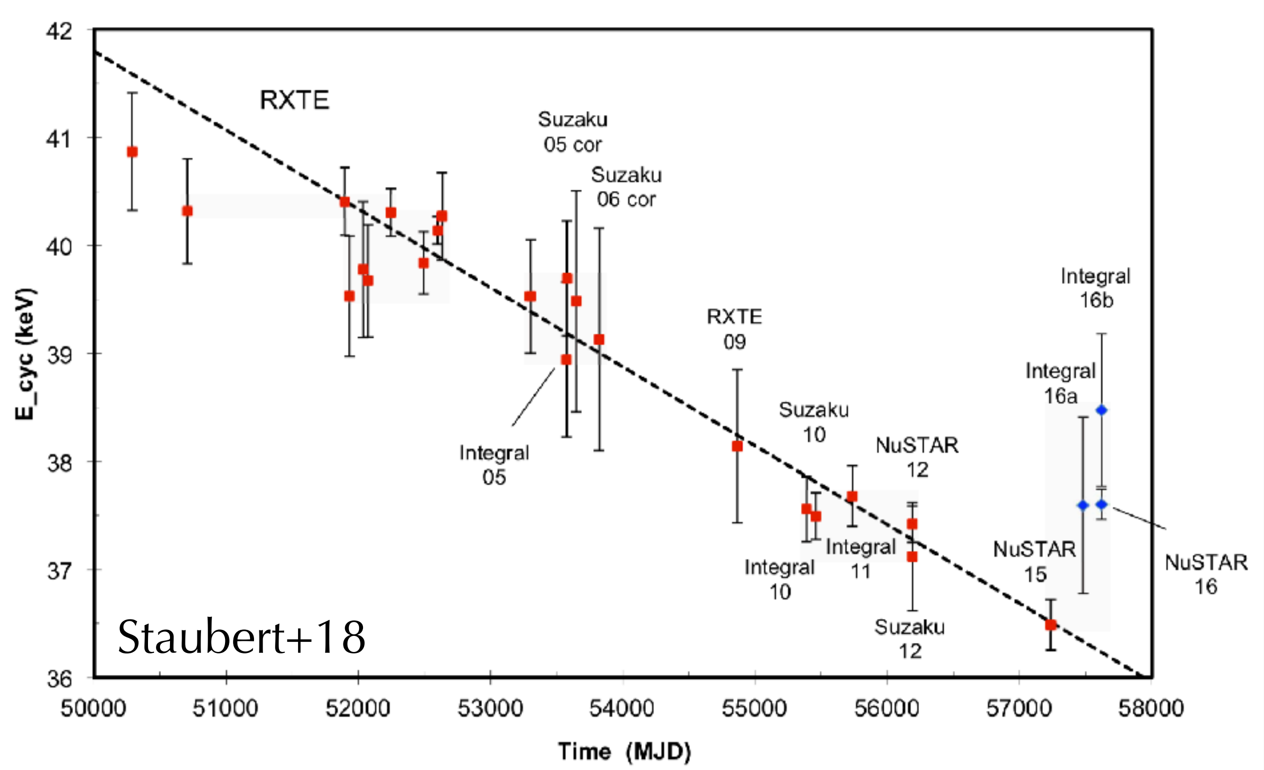
- **Classical limit:** $h\nu_B << mc^2$ **i.e.** $B << \gamma B_c$

- **Critical field:** $B_c = \dfrac{2\pi m^2 c^3}{qh}$
  - For leptons: $B_c \approx 4.14 \times 10^{13}$ G

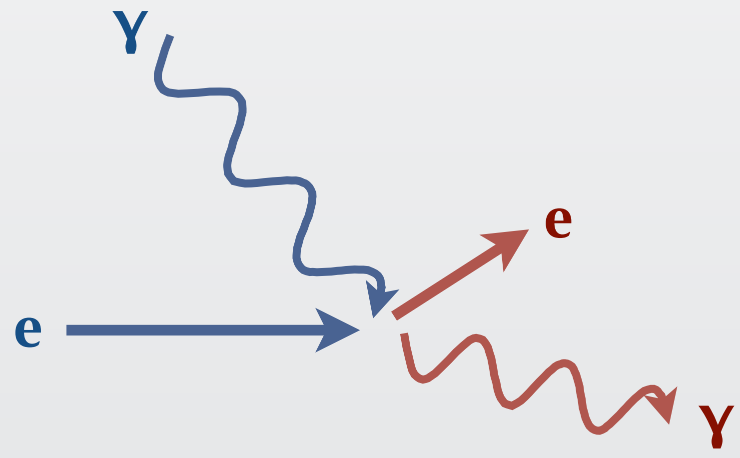
- **Quantification:** $p_n^2 = 2n\dfrac{B}{B_c} = 2n\dfrac{h\nu_L}{mc^2}$
  - Large energy and/or weak field: n >>1
  - Low energy and/or large field: n~1

- **In strongly magnetised neutron stars:**
  - Clear quantification (Trumper+78): n~1
    - kT_e ~ 10 keV => p^2 ~ 0.06
    - B ~ 10^{12} G => 2B/Bc ~ 0.05
  - Sub-relativistic => cyclotron line(s)
  - However combined with resonant Compton interaction => absorption features





IV. Compton Scattering

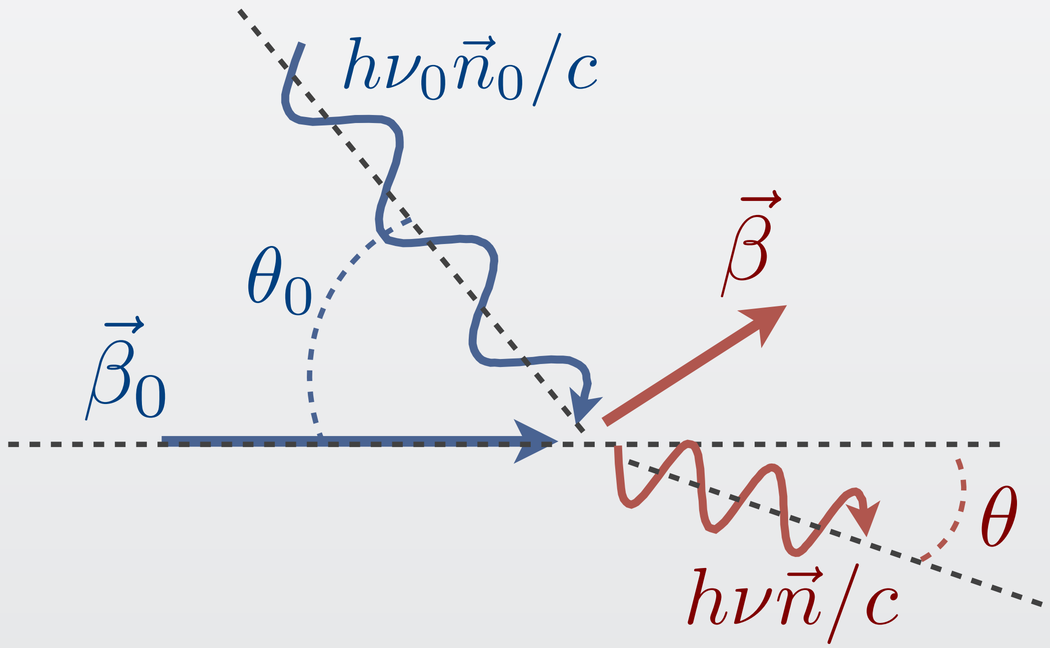


- **Scattering of photons by charged particles**

Compton Scattering

- **Echange of energy and momentum**

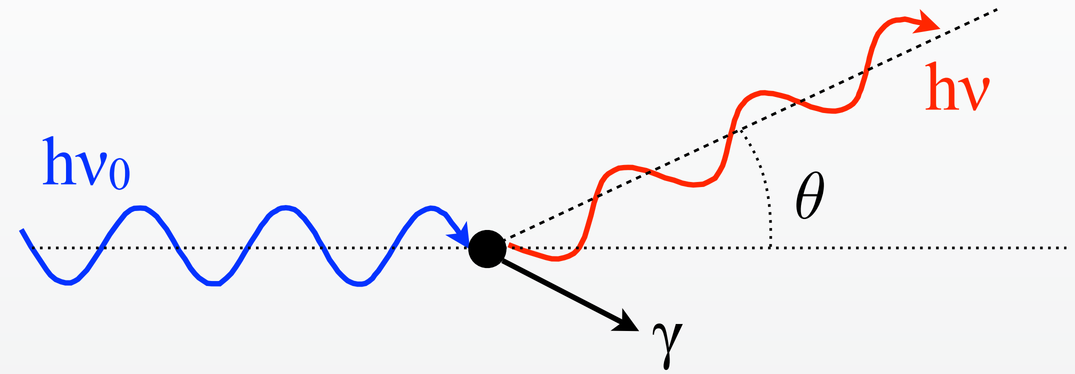
- **The outcome**
  - Depends on energies of incoming photons/particles
  - Depends on interaction angle
  - Is described by distributions (angle, energy)



- **In most cases:**
  - If $E_{part} < E_{phot}$: Compton down-scattering
  - If $E_{part} > E_{phot}$: Compton up-scattering

- **Two main regimes:**
  - Thomson: classical mechanics, no particle recoil
  - Klein-Nishina: quantum mechanics, particle recoil

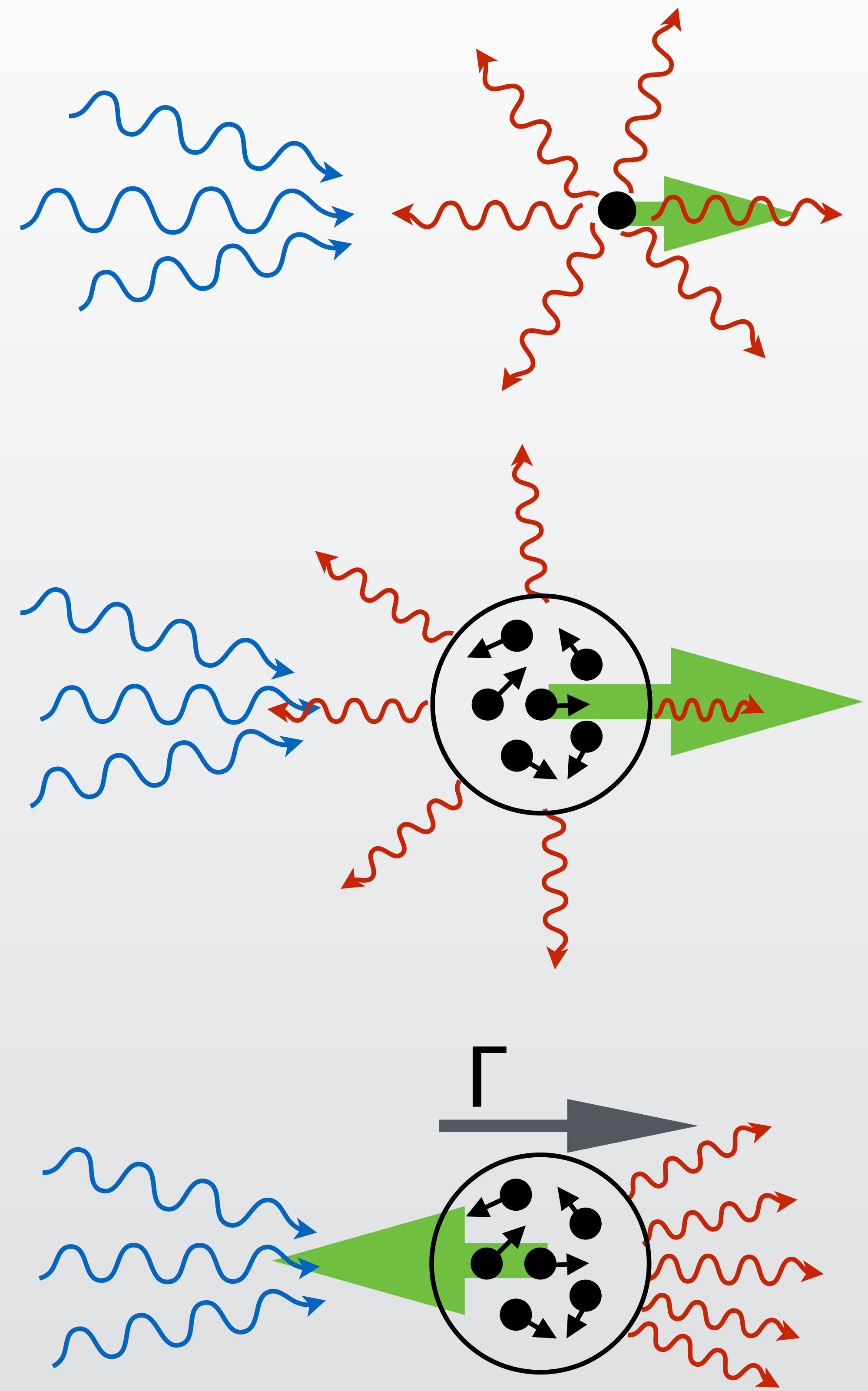
In the particle rest frame



- **Depends only on the photon energy E_{phot}**
- **The Thomson regime ($E_{phot} < mc^2$):**
  - Particle recoil can be neglected
  - Particle oscillates as a response to a linearly polarised wave => radiation
  - Monochromatic emission: coherent scattering
    - Mean acceleration: $< a^2 >= \frac{1}{2}\left(\frac{eE}{m}\right)^2$
    - Emitted Power $P = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 \frac{cE^2}{8\pi}$
    - Thomson cross section: $\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 \approx 6.65 \times 10^{-25}\mathrm{cm}^2$
    - Dipolar emission
  - For unpolarised radiation:
    - quasi-Isotropic emission
- **The Klein-Nishina regime ($E_{phot} > mc^2$):**
  - Anisotropic radiation
  - Energy transfer photon->particle => scattered spectrum
  - Drop of the cross section => barely relevant to physical cases

In the source frame

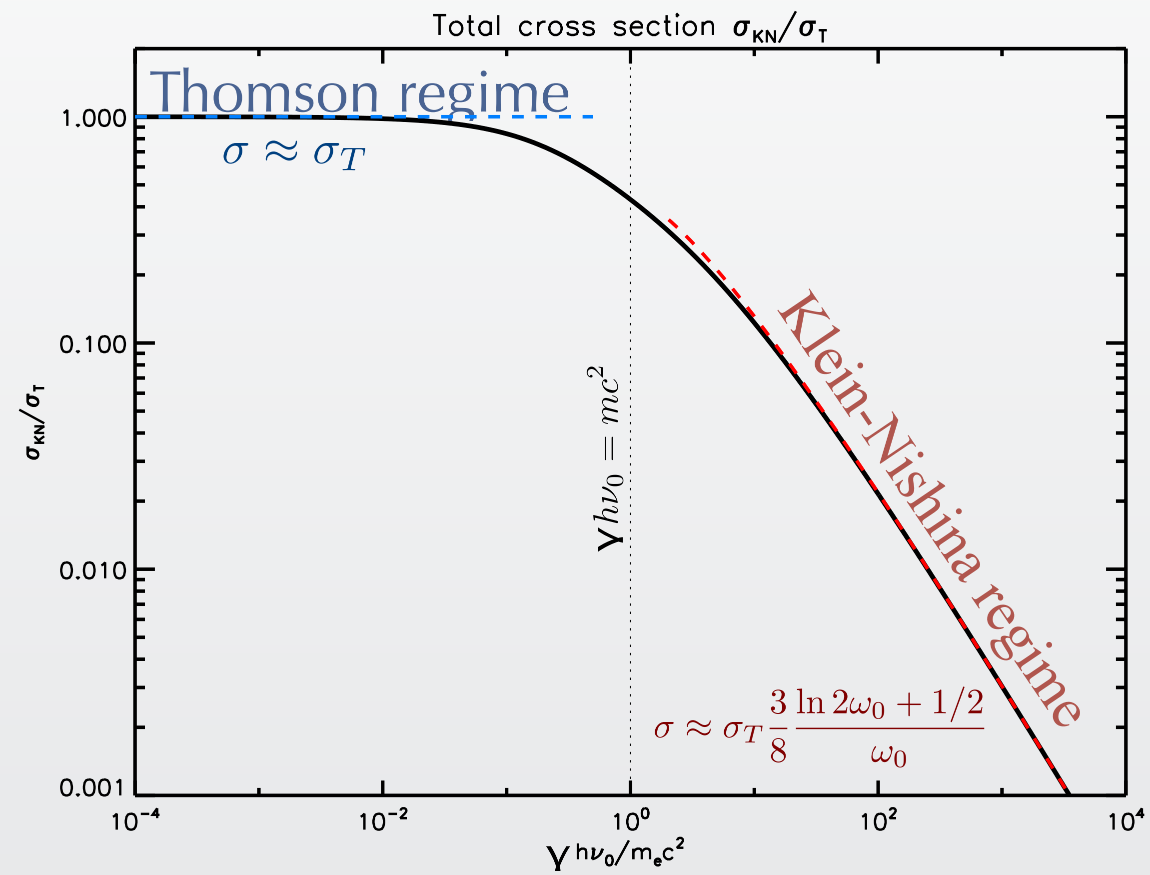
- **Also depends on the interaction angle and the particle energy**
- **Example of a single scattering event:**
  - Head-on collision: $\nu_0' = 2\gamma\, \nu_0$
  - Coherent Thomson scattering: $\nu' = \nu_0'$
  - Backward scattering: $\nu = 2\gamma\nu'$
  - Up-scattering by a factor $A = 4\gamma^2$
- **Example of anisotropic interaction: Compton on bulk motion (AGN, GRBs)**
  - Radiation pressure from an anisotropic field on cold matter: $f = (\sigma_T/c)S$
  - Hot plasma: more efficient by about γ_{th}^2 (Compton rocket)
  - Relativistic bulk motion: emission beamed in forward direction => recoil force (Compton drag) => Γ saturates.



Isotropic distributions

- **Average over interaction and scattering angles**
- **Total cross section drops at:** $\gamma_0 \frac{h\nu_0}{mc^2} = 1$

	$h\nu_0$	$E_{max} = \gamma_{max} mc^2 = (mc^2)^2/h\nu_0$
CMB	1 K	PeV
Radio jet of blazars	10^{13} Hz	10 TeV
Star	10 000 K	100 GeV
AGN accretion disk	10 eV	10 GeV
NS/SMBH accretion disk	1 keV	100 MeV



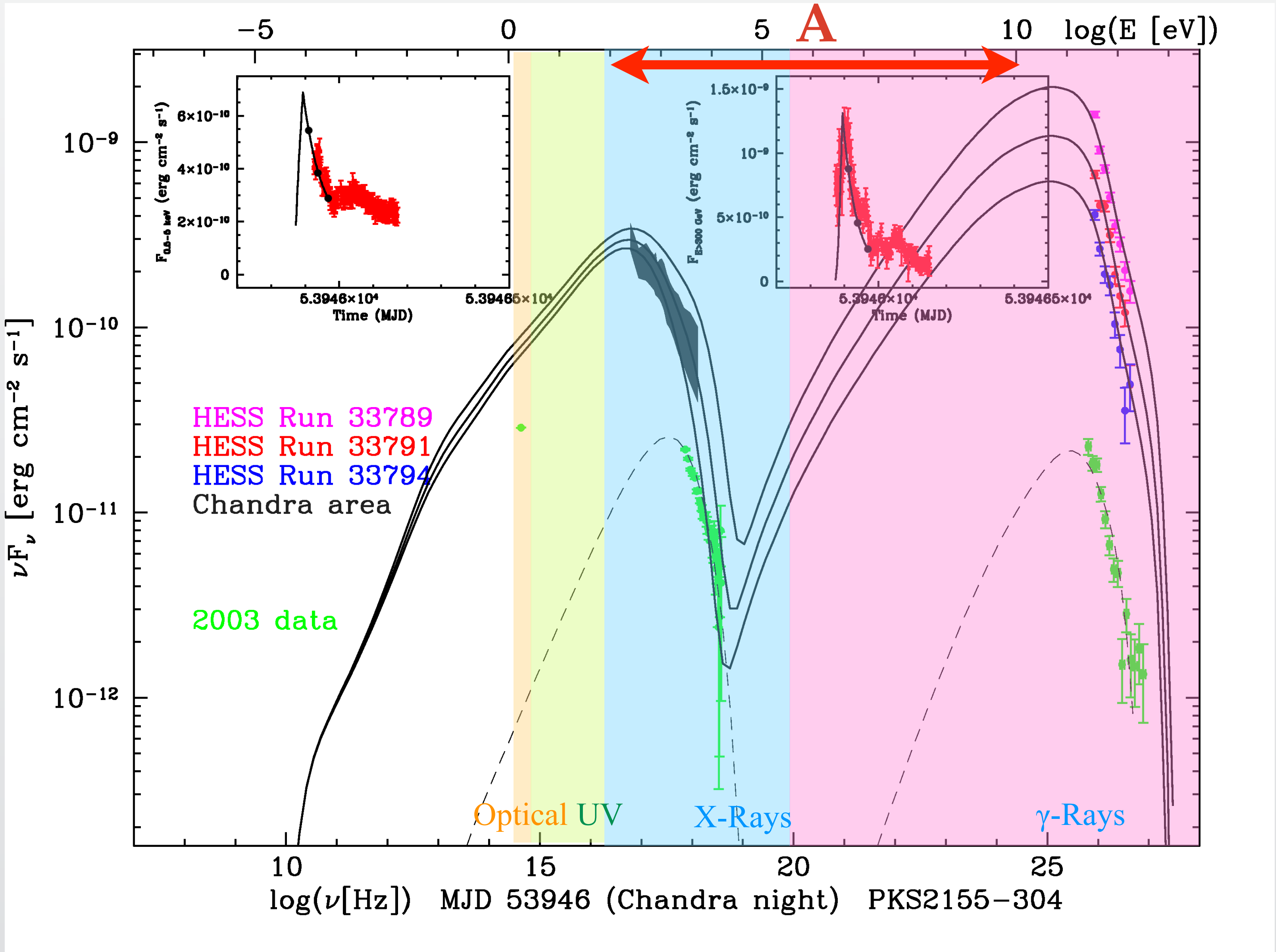
  - KN rarely relevant…

- **Scattered spectrum in the Thomson regime:** $\frac{d\sigma}{d\nu'}(\nu, \gamma \to \nu')$
  - No simple expression
  - Average amplification factor in Thomson regime, for up-scattering : $\langle A \rangle = 1 + \frac{4}{3} p^2$

$$\langle A \rangle \sim \gamma^2$$

Isotropic distributions

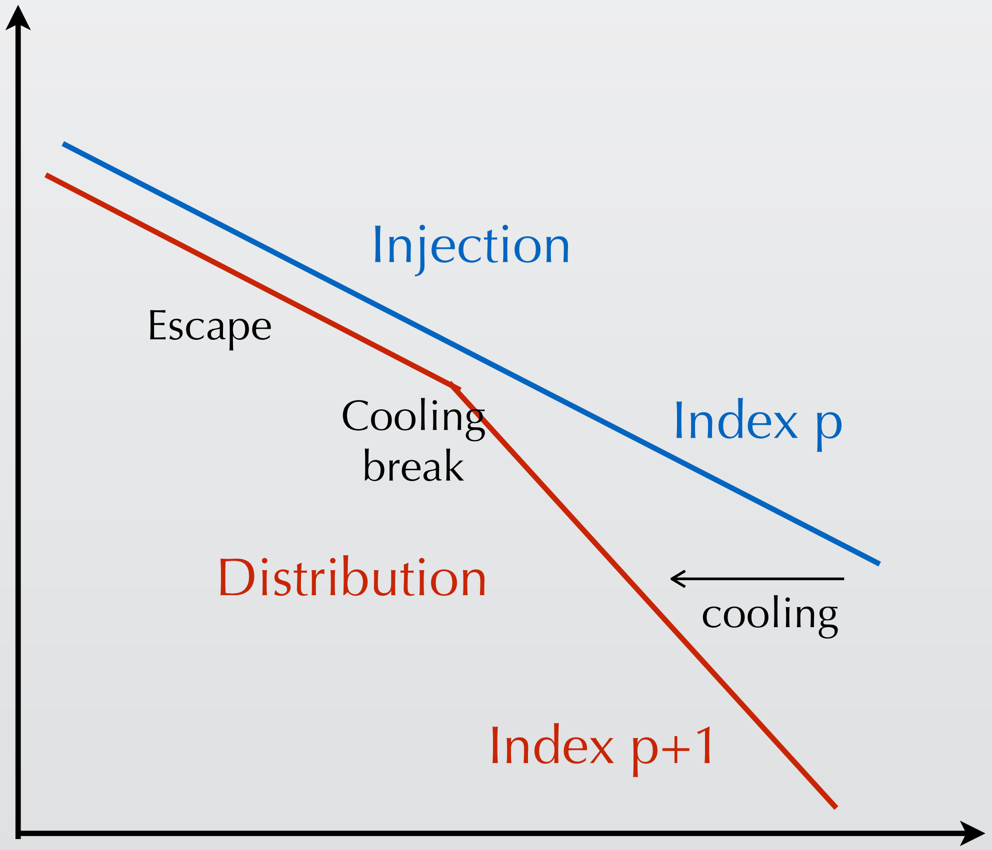
- **Convolved with the seed photon spectrum**
- **Convolution with the particle energy distribution:**
  - Thermal: substitution for $\langle p^2 \rangle \approx 3\theta(1+4\theta)$
  - Power law of index s => power-law spectrum with index $\alpha_i = \dfrac{p-1}{2}$
- **Ex: Blazar spectrum**



  - Synchrotron self-Compton
  - A ~10^8
  - If no Doppler: $\gamma \sim 10^4$
  - $h\nu_0$=100 eV => $\gamma h\nu_0/mc^2>1$
  - KN regime

Effect on particles

- **Particle Energy variation in the Thomson regime:** $\frac{\langle \Delta E \rangle}{mc^2} = (\langle A \rangle - 1)\frac{h\nu_0}{mc^2} = \frac{4}{3}\frac{h\nu_0}{mc^2}p^2$
- **With soft photon density U$_{\mathrm{ph}}$:** $\dot{\gamma} = c\sigma_T \int \frac{\langle \Delta E \rangle}{mc^2} dN_\nu = \frac{4}{3}\frac{c\sigma_T U_{\mathrm{ph}}}{mc^2}p^2$
  - Compare to synchrotron cooling rate $\frac{\dot{\gamma}_{\mathrm{ic}}}{\dot{\gamma}_{\mathrm{synch}}} = \frac{U_{\mathrm{ph}}}{U_B}$
  - ex: Compton catastrophe in high brightness temperature radio sources
    - Synchrotron self-absorbed radio sources (AGN)
    - Compton cooling dominates for $T_B \gtrsim 10^{12}$ K
    - Need for coherent emission in high brightness temperature sources (e.g. FRBs)



- **Cooling time:** $t_c = \frac{3}{4}\frac{mc^2}{c\sigma_T U_{\mathrm{ph}}}\frac{1}{\gamma + 1}$
- **Example of kinetic equation for particles:**

$$\frac{dN_\gamma}{dt} = \frac{d}{d\gamma}(\dot{\gamma}N_\gamma) + S(\gamma) - p_{esc}N_\gamma$$

  - For Power-law injection:
    - Cooling break
    - Steepening $S(\gamma) \propto \gamma^{-p} \;\Rightarrow\; N_\gamma \propto \gamma^{-(p+1)}$

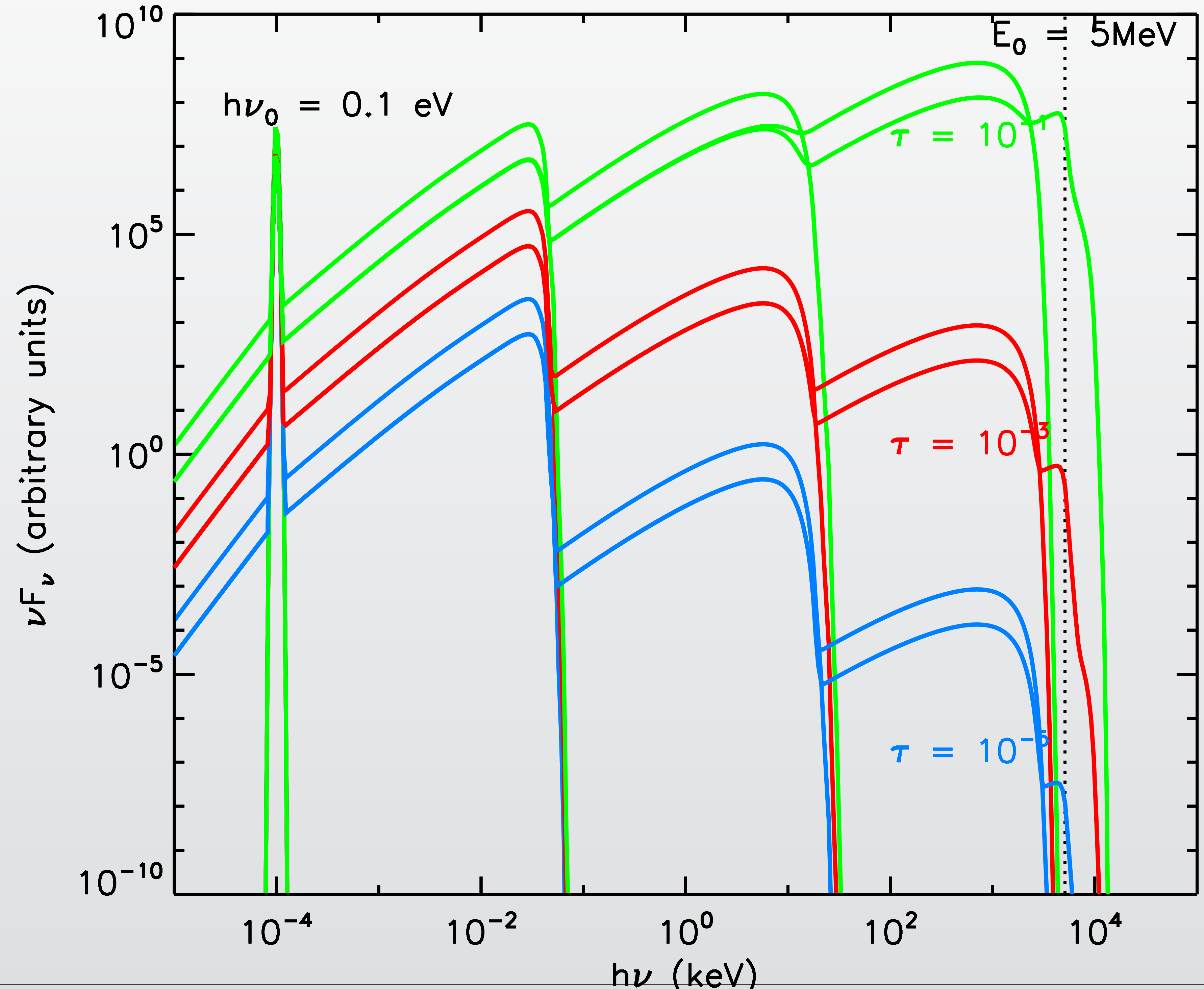
Multiple Scattering

- **Radiation transfer problem for a source of given size R**
- **Characterised by 2 quantities:**
  - Optical depth: $\tau_T = \sigma_T n_e R$
  - The y-parameter: $y = N_{\rm scat} \frac{\Delta E}{E} \sim \frac{4}{3} p^2 N_{\rm scat}$

	Thomson Thin $\tau_T < 1$	Thomson thick $\tau_T > 1$
Photon scattering rate per unit volume	$\dot{N}_{\rm scat} \sim c\sigma_T n_e$	$\dot{N}_{\rm scat} \sim c\sigma_T n_e$
Escape time	R/c	$N_{\rm scat}^{1/2}\ R/c$
Mean number of scatterings	τ_T	τ_T^2
y	$4\theta(1+4\theta)\tau$	$4\theta(1+4\theta)\tau^2$

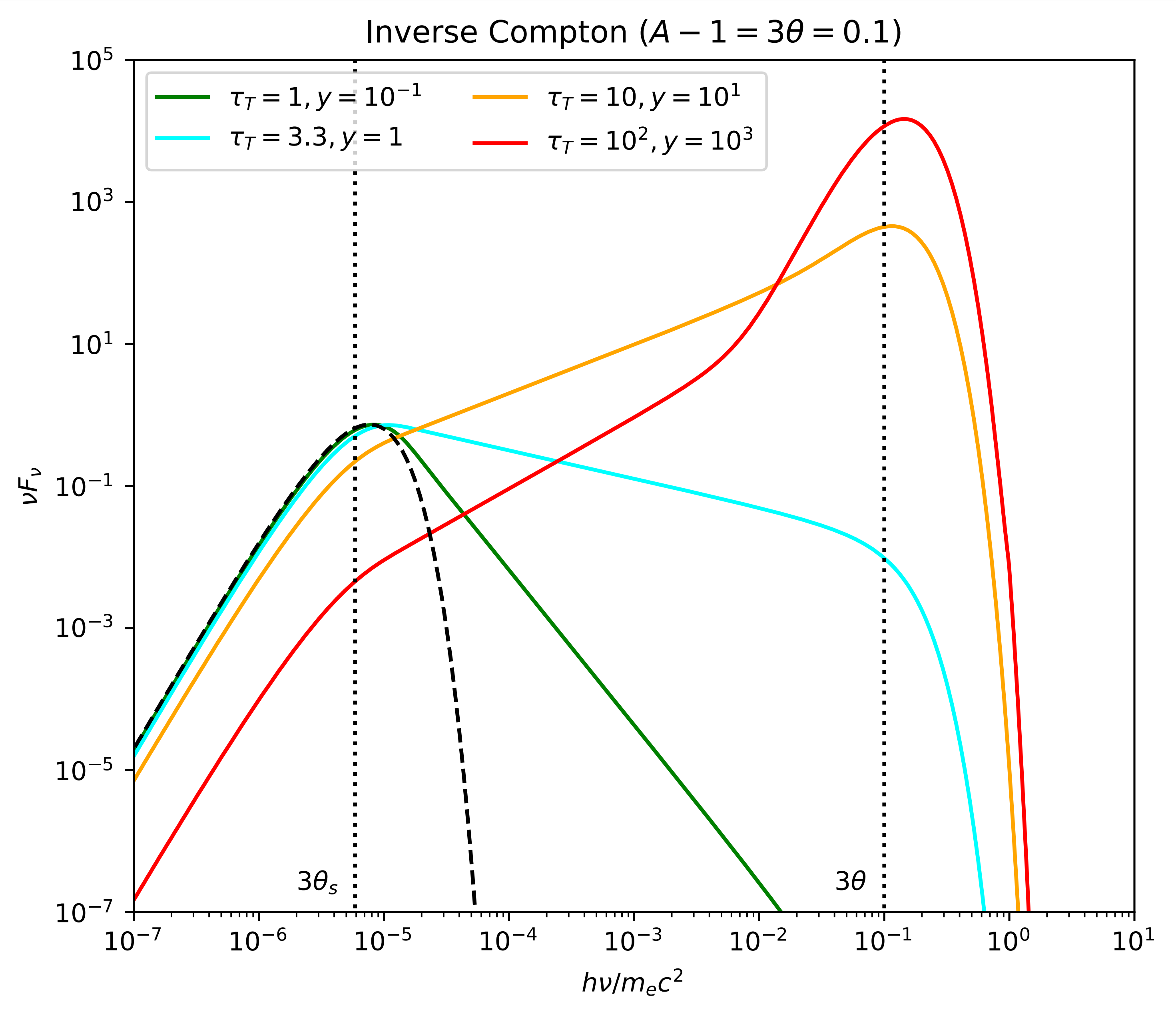
Example of Multiple Scattering

- γ=10, hν_0=0.1eV, Thomson



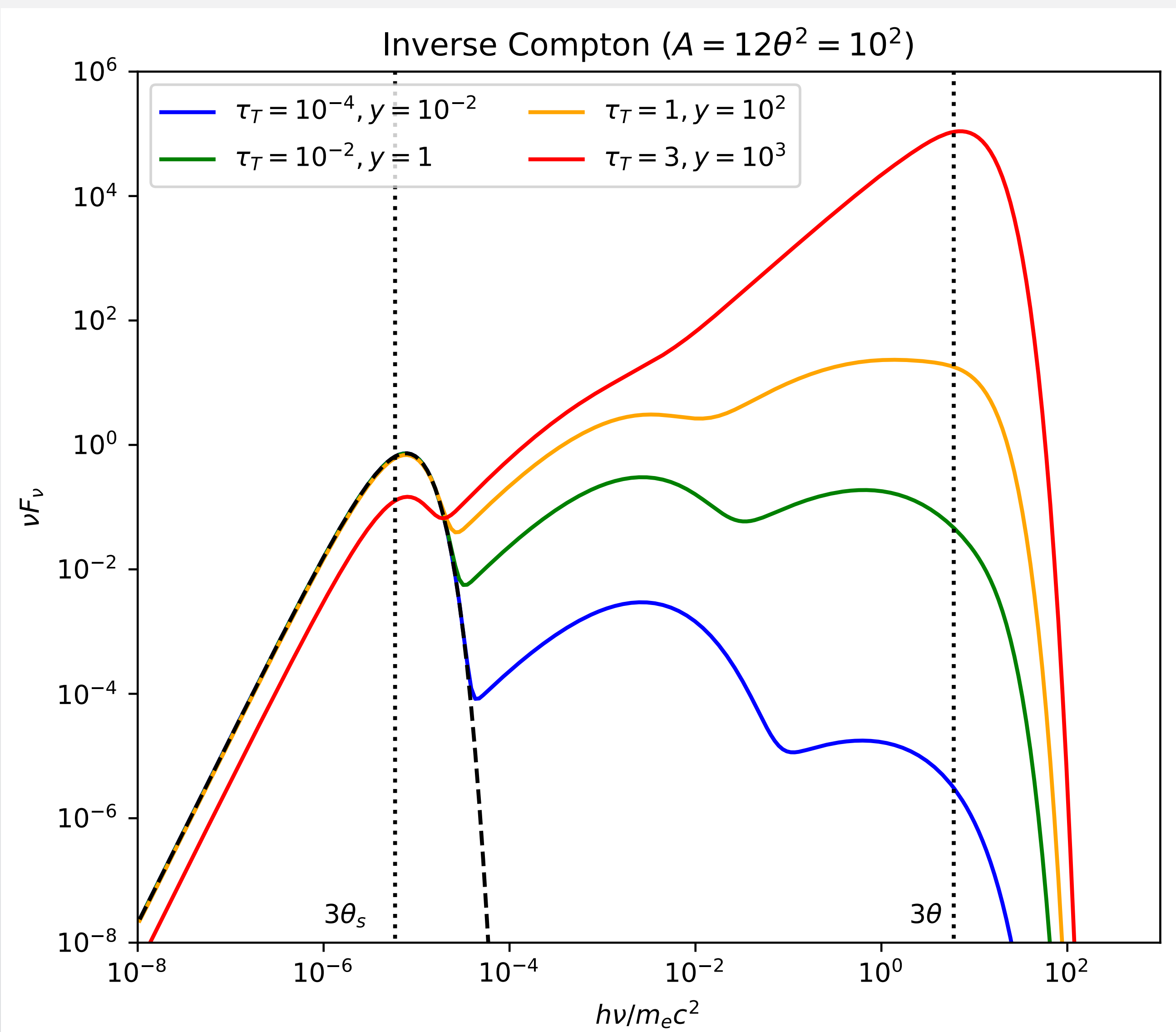
Regimes of Multiple Scattering

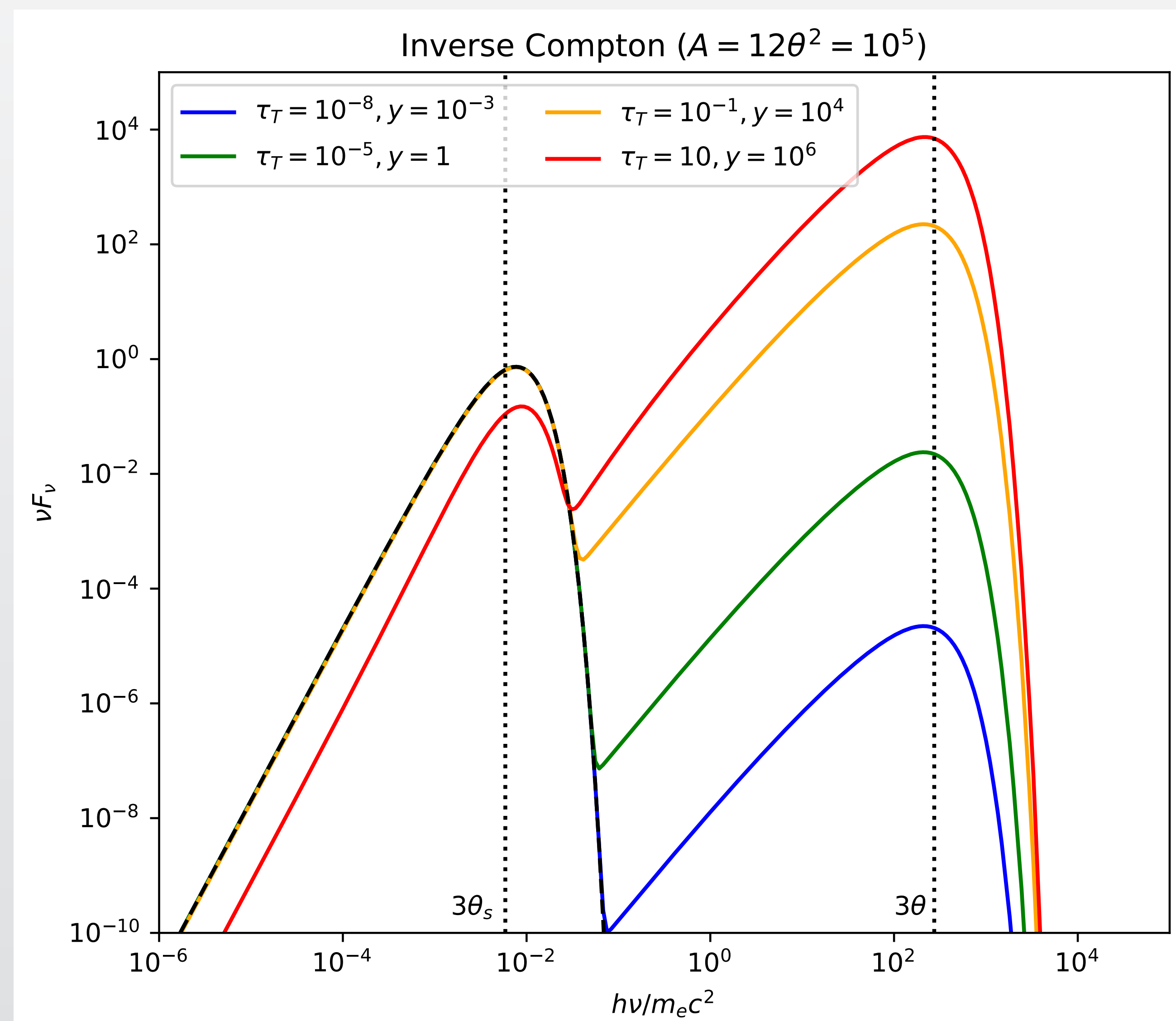
- **$\gamma\, h\nu_0/mc^2 < \gamma(\gamma-1) < 1$:**
  - Thomson
  - Small A => Power-law
  - Small tau (y<<1) => cutoff at kT
  - Large tau (y>>1) => saturation at kT



Regimes of Multiple Scattering

- $\gamma\, h\nu_0/mc^2 < 1 < \gamma(\gamma\text{-}1)$:
  - Thomson
  - large A => Several Compton orders
- $1 < \gamma\, h\nu_0/mc^2 < \gamma(\gamma\text{-}1)$:
  - KN, large A => Only one Compton order





Kinetic Equation

- **General kinetic equation for photons:**

$$\frac{dN_\nu}{dt} = c \iint \left[\frac{d\sigma}{d\nu}(\nu', \gamma \to \nu) - \frac{d\sigma}{d\nu'}(\nu, \gamma \to \nu') \right] dN_{\nu'} dN_\gamma$$

  - Steady state solution for thermal distribution: Wien spectrum $N_\nu \propto \nu^2 e^{-\frac{h\nu}{k_B T}}$

- **In the sub-relativistic regime: The Kompaneets equation**

$$\frac{dN_\omega}{dt} = n_e c \sigma_T \left[\frac{d}{d\omega} \left(\omega(\omega - 4\theta) N_\omega \right) + \theta \frac{d^2(\omega^2 N_\omega)}{d\omega^2} \right] \qquad \omega = \frac{h\nu}{mc^2}$$

- **Must be included in some sort of transfert equation**
  - Source and escape terms
  - Other processes

Induced Compton Scattering

- **Total scattering rate:** $(1+n(\nu'))\ \frac{d\sigma}{d\nu'}(\nu,\gamma\to\nu')$
  - With occupation number: $n(\nu') = \frac{c^3}{8\pi}\frac{N'_\nu}{\nu'^2}$
- **n is a strongly decreasing function of ν**
  - ex: Planck spectrum: $n \propto T/\nu$
  - => favours down-scattering
- **Steady state solution for thermal particle distributions:**
  - Bose-Einstein distribution $N_\nu = \frac{8\pi}{c^3}\frac{\nu^2}{\lambda_N e^{\frac{h\nu}{kT}} - 1}\ +\ C_N\delta(\nu)$
  - Fugacity and chemical potential: $\lambda_N = e^{\mu_N} > 1$
  - Harder than pure Compton scattering at low frequency ($F_\nu \propto \nu^3$ instead of $F_\nu \propto \nu^2$)
- **Implied and high brightness temperature radio sources…**

Two-Photon Scattering

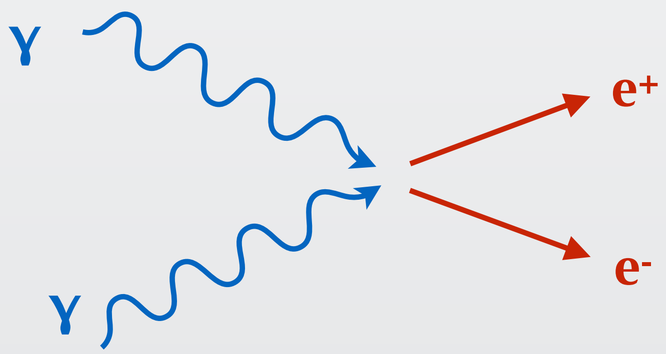
- **Particle physics also predicts multiple-photon scattering** $\gamma + e \;\rightarrow\; \gamma + \gamma + e$
  $\gamma + e \;\rightarrow\; \gamma + \gamma + \gamma + e$

- **Cross-section decreases as** α_f^n

- **However, it becomes a sources of photons that can compete with other emission mechanisms**

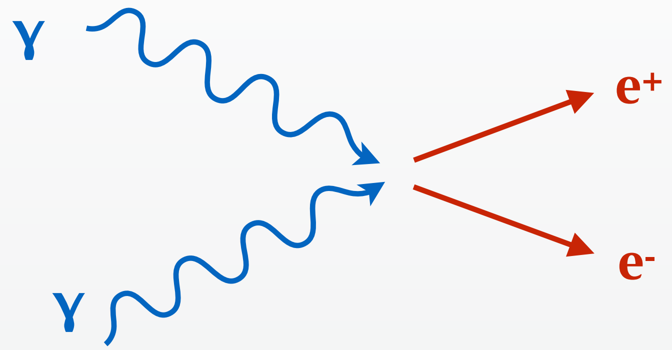
- **Ex:**
  - Pulsar magnetosphere:
    - Need for photons to produce pairs and fill the magnetosphere
    - In competition with photon splitting $\gamma + B \rightarrow \gamma + \gamma$
  - GRB photosphere:
    - Photon production rates decreases as the jet expands until it freezes
    - Double Compton might play a role in transition thermal->non-thermal
    - In competition with bremsstrahlung

V. Pair production



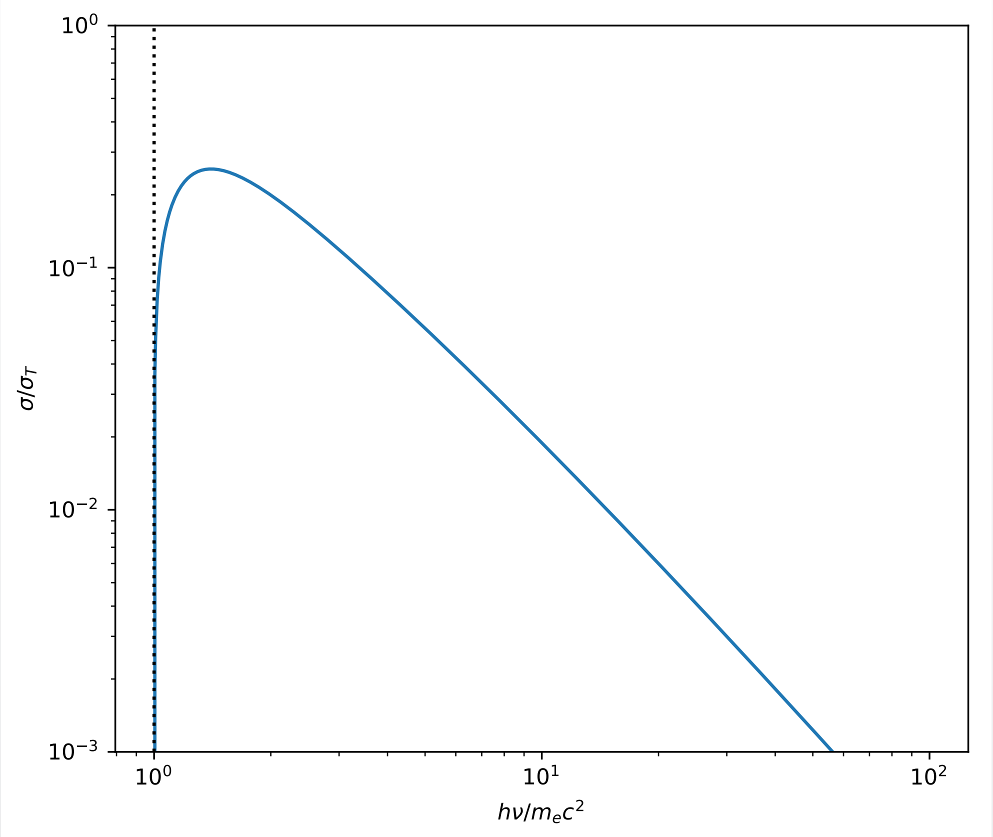
- **Photon-photon annihilation**

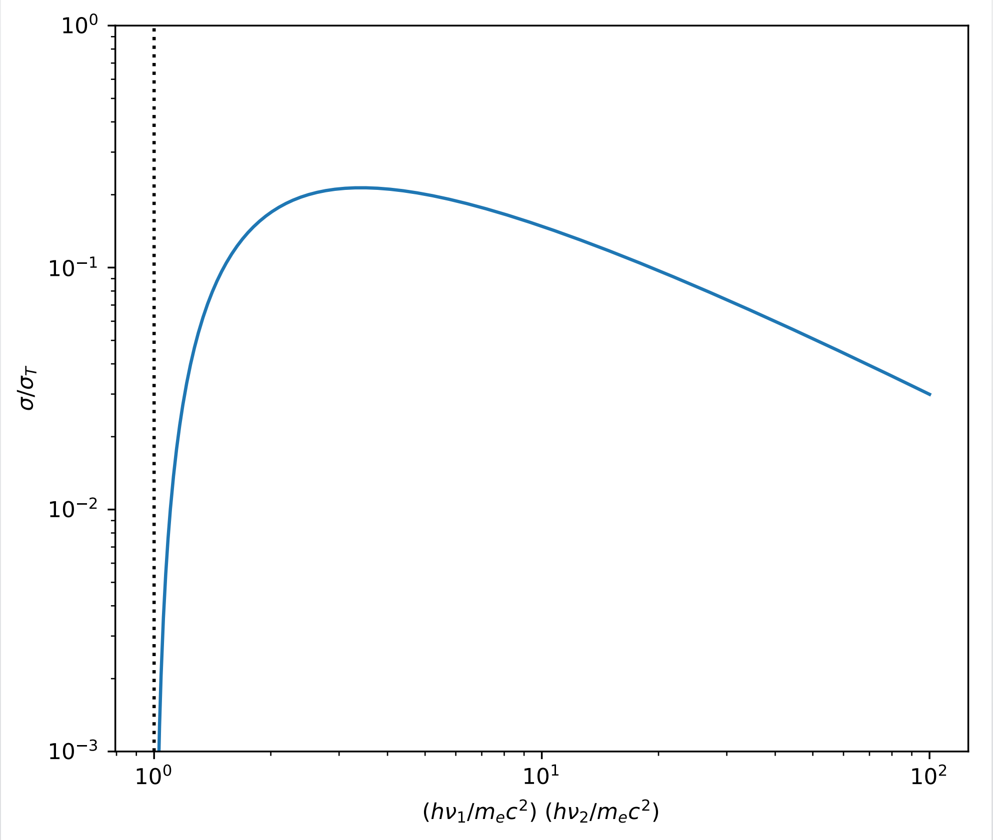
Cross Section



- **In the center of momentum frame:**
  - 2 photons of same energy hν′
  - 2 leptons of same energy γ′
  - Energy threshold: $h\nu' > m_e c^2$
  - Energy conservation: $h\nu' = \gamma' m_e c^2$
  - Annihilation requires MeV photons

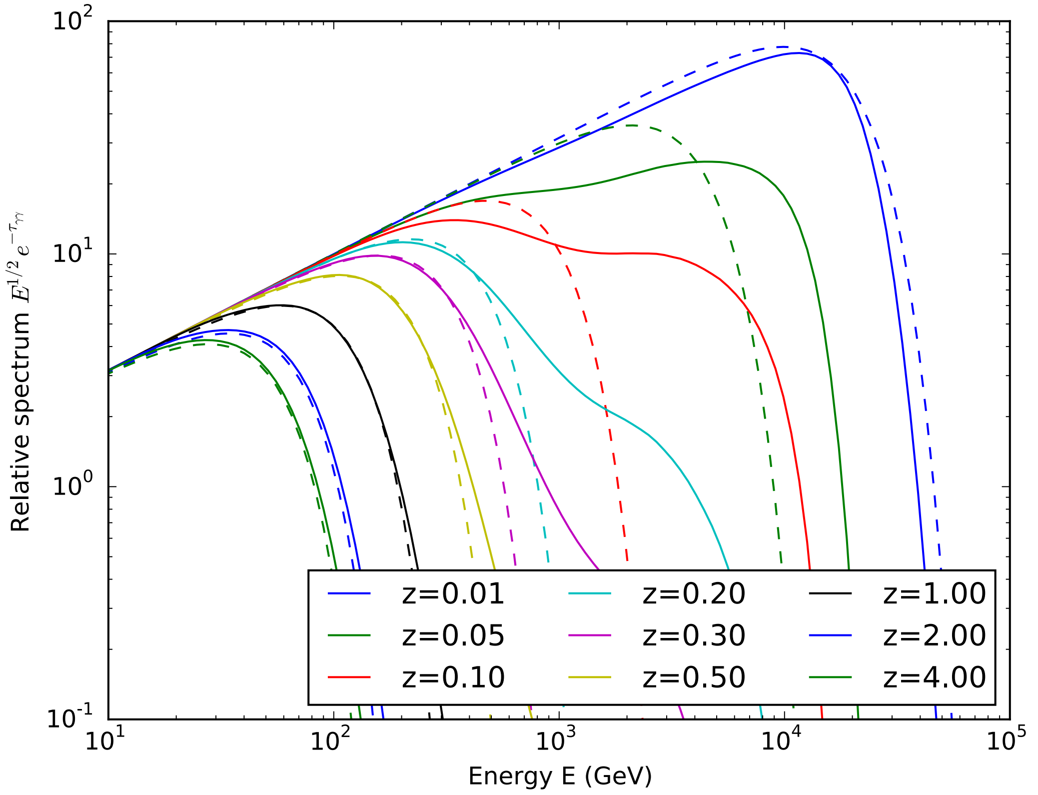
- **In the source frame:**
  - Results depends on the angles
  - Threshold: $\frac{h\nu_1}{m_e c^2} \frac{h\nu_2}{m_e c^2} \frac{1-\cos\theta}{2} \geq 1$
    - Head-on collisions: softer threshold
    - Tail-on collisions: harder threshold
  - Average for isotropic photons fields:
    - Efficiency peaks at $\frac{h\nu_1}{m_e c^2} \frac{h\nu_2}{m_e c^2} \sim 1$





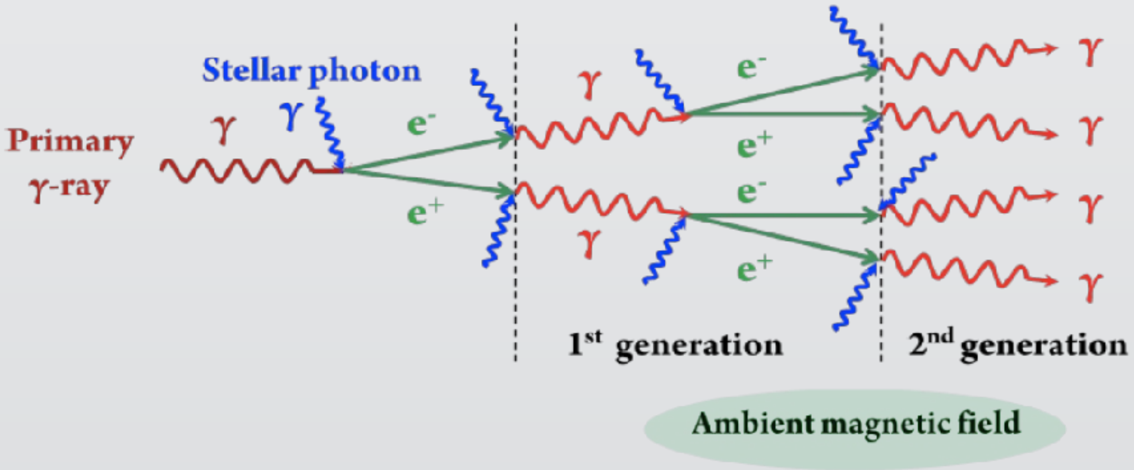
Photon absorption

- **Photon-photon optical depth:** $\tau_{\gamma\gamma}(\nu_1) \sim \frac{\sigma_T}{5} n_\gamma(\nu_2) R$ $\quad h\nu_2 = \frac{(m_e c^2)^2}{h\nu_1}$
- **Absorption:** $I_\nu = I_\nu^0 e^{-\tau_{\gamma\gamma}}$
  - Optically thick/thin: $\tau_{\gamma\gamma} = 1$



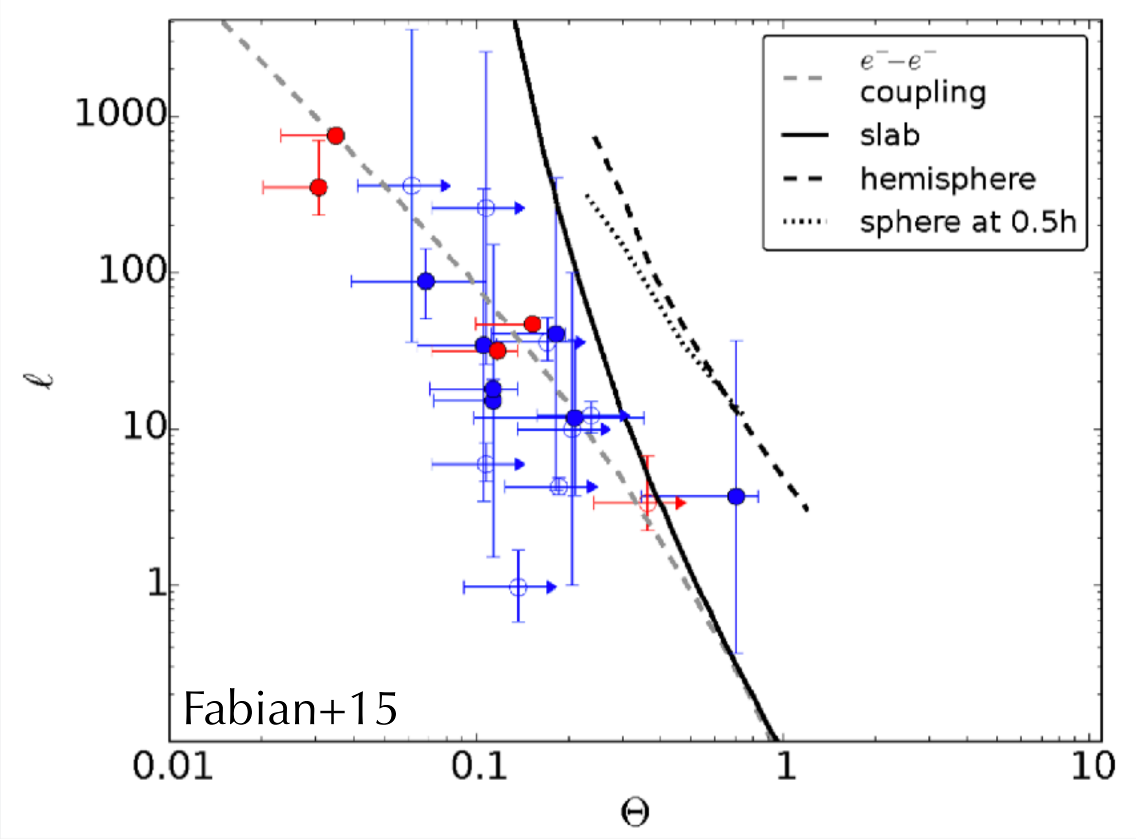
- **Absorption within the source: estimating Γ in Fermi GRBs:**
  - Emission medium is not opaque to gamma-rays at 100 MeV ($\tau_{\gamma\gamma}$< 1)
  - + Variability (0.1s) => size: $R = \Gamma^2 c\delta t$
  - + Observed flux at (1MeV)^2/100 MeV = 10 keV
  - = Constraint on Γ>100-1000

- **Absorption along the line of sight: pair cascades**
  - Extragalactic TeV photon are absorbed by EBL photons
  - Produced pairs up-scatter CMB photons
  - etc… => shower

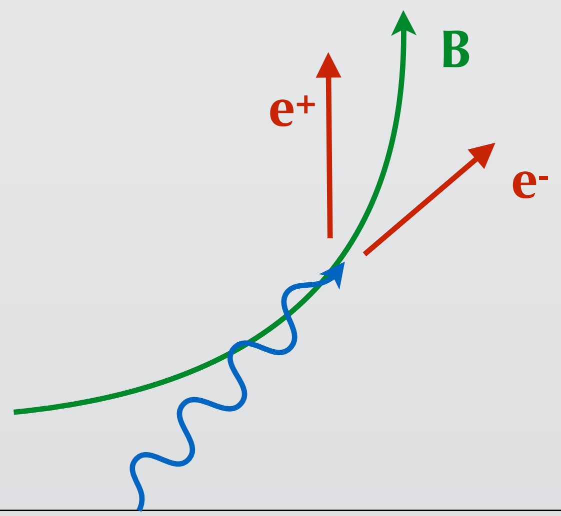


Pair production

- **Matter production and temperature decrease**
- **Pair thermostat in AGN (e.g. Svensson+84):**
  - For $kT > m_e c^2$: MeV photons (e.g. Compton)
  - For high photon density sources ($l = \frac{\sigma_T L}{R m_e c^3} > 1$): strong pair production
  - => Temperature saturation



- **Pair instability supernovae (e.g. Gilmer+17)**
  - For M~100 M_o
  - Pair production => temperature and pressure decrease
  - Collapse trigger
- **Pulsar magnetosphere**
  - Pair plasma
  - Production by
    - double photon annihilation
    - Single photon annihilation in strong field
    - Curvature radiation



References

- **Main text books:**

  - Rybicki G.B. & Lightman A.P., Radiative processes in astrophysics (1979)
  - Aharonian F. A., Very High energy cosmic gamma radiation (2004)
  - Ghisellini, Radiative processes in high energy astrophysics (2012)
  - Shu F.H., The physics of astrophysics, vol.II: radiation (1992)
  - Jackson J.D., Classical Electrodynamics (1998)

- **Detailed cross sections**

  - Jauch J.M. & Rohrlich F., The theory of photons and electrons (1980)
  - Heitler W., Quantum theory of radiation (1954)
  - Blumenthal G.R. & Gould R.J., Reviews of Modern Physics (1970)