Introduction to CP violation and the CKM matrix (II)

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Outline

Yesterday:

- A bit of history
- The flavor sector of the Standard Model
- *CP* violation and the CKM matrix
- Theory-free determination of the CKM elements

Today:

- Theoretical methods for heavy flavors
- Examples of predictions
- Beyond the SM and New Physics tests

Hadronic matrix elements

To leading order of the weak interaction, one has

$$\langle f|H_{eff}|i\rangle \sim V_{CKM} \times \langle f|O|i\rangle$$

where the operators O can be further decomposed with the Operator Product Expansion from the weak scale

$$O \sim C_i(\mu)Q_i(\mu)$$

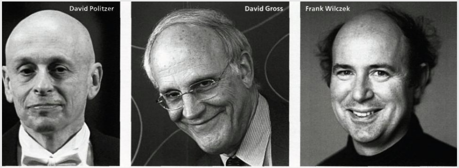
The $C_i(\mu)$ are renormalized Wilson coefficients that can be computed in terms of fundamental couplings in the SM and beyond, and the O_i are (renormalized) quark operators, the matrix elements of them have to be computed in QCD at low energy: they are genuinely non perturbative objects (decay constants, current form factors, non local matrix elements. . .). Alternatively they can be extracted from the data within a phenomenological analysis that relates different observables.

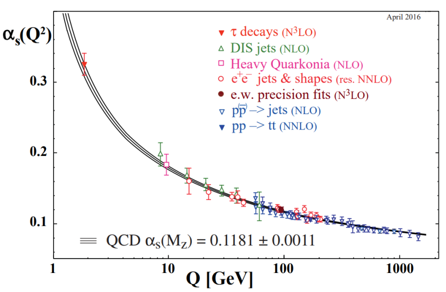
Strong interaction

Strong interaction is strong. . . but only at low energies.

Asymptotic freedom: the renormalized coupling constant tends to zero at large energies.

Typical non perturbative regime: up to 1–2 GeV.





Non perturbative Quantum Field Theory

To make a long story short:

- There is no general exact solution to QFT.
- There is no general analytical and systematic approximation technique to QFT.
- There is a general systematic approximation technique to QFT, which relies on numerical tools: Lattice QFT.

Hence in principle LQCD is the way to go for the strong interaction. This is indeed the privileged choice to compute many hadronic matrix elements relevant to flavor physics.

However there are also many hadronic matrix elements that cannot computed by LQCD for present and near future computer resources: other methods are needed.

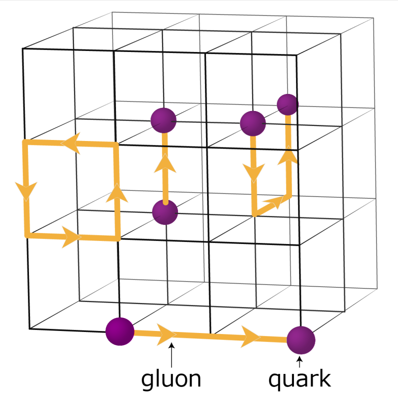
Note also that most often LQCD calculations come together with analytical techniques (power expansions) in order to make the problem more tractable.

Lattice QCD

Any QFT quantity can be represented by a path integral (*i.e.* an integral in which the integration variable is a field, that is a function of spacetime).

The path integral is an ill-defined object in mathematics. However when the infinite continuum spacetime is replaced by a finite lattice, the integral becomes a finite sum.

Discretization parameters: lattice spacing a and lattice size L. Physical spacetime corresponds to the $a \to 0$ and $L \to \infty$ limit.



Applicability of LQCD

A particle can be correctly simulated iff its mass 'fits' on the lattice

$$a \ll \frac{1}{m} \ll L$$

In particular, on present lattices it remains very costly to simulate physical pions, and impossible to simulate b hadrons at their physical mass.

The numerical evaluation of the path integral/sum relies on stochastic methods, on an Euclidean lattice. This is justified as long as the replacement $e^{iS} \to e^{-S}$ can be done in the integrand.

Maiani-Testa 'no-go' theorem (1990): Euclidean correlation functions can only determine multihadrons states in an unphysical (off-shell) configuration. Thus *a priori* at most one hadron in the initial/final state can be simulated.

Example: $B \to \pi\pi$ (for CKM angle α) is a triple nightmare for LQCD: 1) The B is too heavy; 2) The π's are very light; 3) Maiani-Testa theorem states that the continuum limit corresponds to a configuration where the pions are at rest, which is unphysical.

Extending the applicability of LQCD

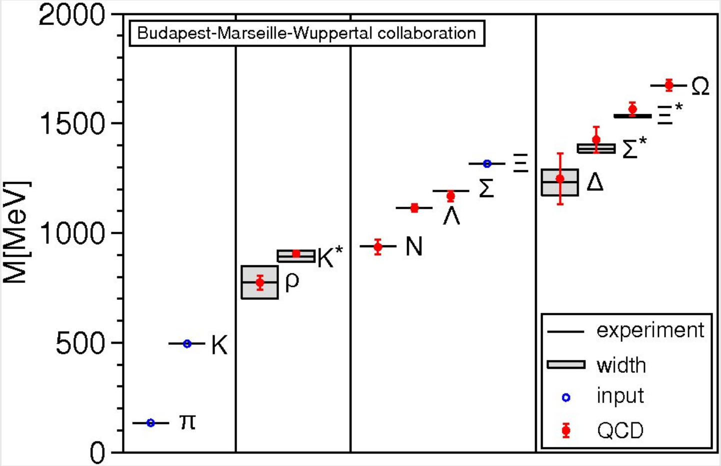
For too light/heavy particles, one can instead perform a simulation at a different mass, and then use a power expansion to extrapolate to the physical region.

To evade Maiani-Testa, one can use a 'small' lattice box, in which multihadron states are discrete and sufficiently well separated. A result by Lellouch & Lüscher relate these states to the physical configurations in the continuum.

Message: LQCD is not a numerical 'moulinette', it is more a different way to use Quantum Field Theory and perform calculations from it.

Spectrum of light hadrons

BMW collaboration 2008



a quantitative success and validation of Lattice QCD

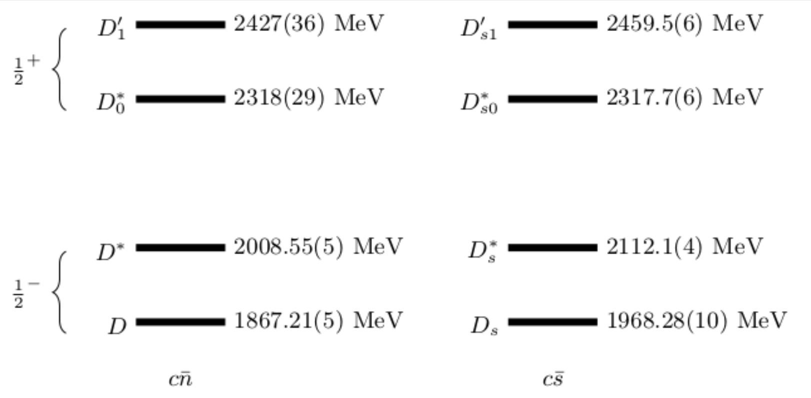
Heavy quark symmetry

Let's consider a heavy meson $(Q\bar{q})$. In the limit $m_Q \gg \Lambda$ where Λ is a typical interaction scale, the heavy quark becomes static: properties of the meson do not depend on m_Q anymore. In addition spin effects are suppressed by $1/m_Q$ (as in the hydrogen atom). [Isgur & Wise 1989]

Heavy quark symmetry = spin-flavor symmetry. Consequence: B, B^*, D and D^* are essentially the same bound state (up to a calculable m_Q scaling).

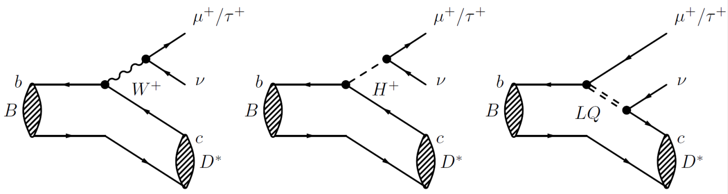
More generally, B and D bound states can be categorized in degenerate spin doublets, and mass differences between the doublets do not depend on the b or c flavor.

D^{**} doublets



$\frac{1}{2}^+$
D_1' 2427(36) MeV
D_0^* 2318(29) MeV
D_{s1}' 2459.5(6) MeV
D_{s0}^* 2317.7(6) MeV
$\frac{1}{2}^-$
D^* 2008.55(5) MeV
D 1867.21(5) MeV
D_s^* 2112.1(4) MeV
D_s 1968.28(10) MeV
$c\bar{n}$
$c\bar{s}$

Heavy-to-heavy form factors



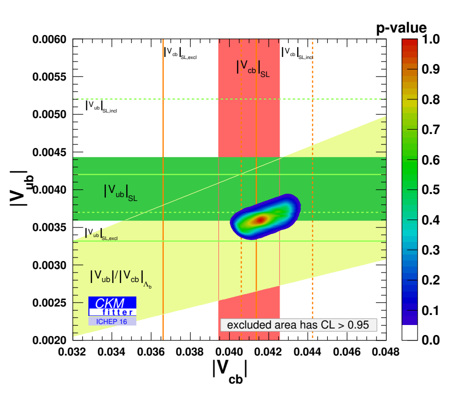
Whatever the mediator, the hadronic part is a matrix element of a two quark current

$$\langle D^{(*)}|\bar{c}\Gamma b|B\rangle$$

Heavy quark symmetry, in the $m_{b,c} \to \infty$ limit, essentially predicts that the initial and final states are the same: hence the matrix element is fully described by a single elastic form factor $\xi(w)$, $w = v_B \cdot v_D$, that is normalized at zero recoil $\xi(1) = 1$ ($w \leftrightarrow q^2$, the lepton invariant mass). Hence in principle in the heavy quark limit $|V_{cb}|$ can be extracted from the measurement of the zero recoil rate without any input from QCD calculations !

The extraction of $|V_{cb}|$

In practice one needs to control the corrections to the heavy mass limit to reach an accuracy of a few %. Form factors are calculated in LQCD, with the help of heavy quark expansion to extrapolate to the physical b mass. $|V_{cb}|$ is the most precisely known short-distance quantity in the B meson sector



Heavy mass expansion

The heavy mass expansion is not simply a symmetry: it is also a rigorous tool to expand many quantities in inverse powers of the heavy mass, leading to the construction of an effective theory (HQET).

The coefficients of these expansions are defined in the heavy mass limit, so they obey HQS relations. This allows to constrain them from data when they are too complicated to be computed theoretically.

Heavy-to-light form factors

Consider a $b \to u$ transition

$$\langle \pi(\rho) | \bar{u} \Gamma b | B \rangle$$

or a $b \to s$ one

$$\langle K^{(*)} | \bar{s} \Gamma b | B \rangle$$

In this case strict HQS is only useful for the heavy initial state, predictions are much looser than in the heavy-to-heavy case. However one can perform a combined $m_b \to \infty$ and $E \to \infty$ expansion, where E is the energy of the final meson in the B rest frame,

$$E = v_b \cdot p_K = (m_B/2)(1 - q^2/mB^2)$$

(E large $\Leftrightarrow q^2 \sim \Lambda^2$ or $q^2 \sim m_B \Lambda$): in this limit the 3 (for $B \to K$) $+7$ (for $B \to K^*$) form factors reduce to three independent ζ, $\zeta_\perp$, $\zeta_\|$ 'soft' form factors, that obey well-defined scaling laws in E. [JC *et al.* '99]

SCET

Effective field theory implementation: Soft-Collinear Effective Theory (SCET). [Bauer *et al.* '00,'01]

The SCET limit is best used in the moderate to large recoil region, $q^2 \sim m_B \Lambda$, where LQCD has no access.

In the low recoil region $q^2 \sim m_B^2$, SCET does not apply. However in this region the form factors can be computed directly on the lattice.

In contrast to most effective theories, SCET is an expansion with respect to kinematical variables (E or q^2), not to constants (masses). For this reason it is significantly more complicated.

Non leptonic decays

Typical matrix elements

$$\langle M_1 M_2 | (\bar{q}_1 \Gamma q_2)(\bar{q}_3 \Gamma' b) | B \rangle$$

One of the most cited work in flavor physics is the series of BBNS papers on QCD factorization: establishes factorization of non leptonic matrix elements into simpler quantities

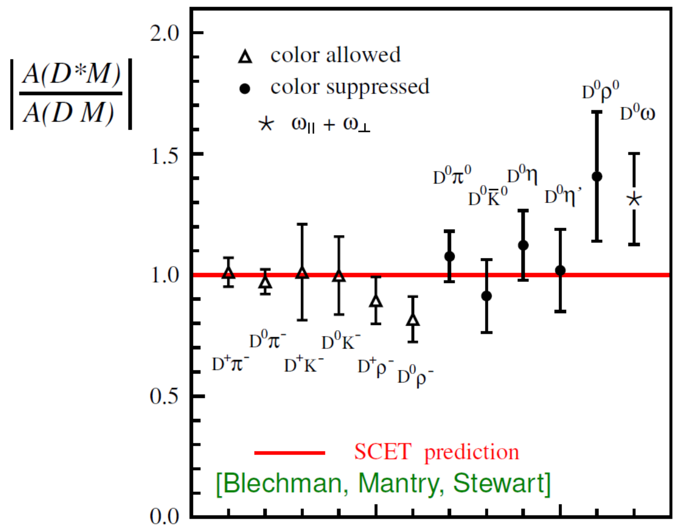
$$\langle M_1 | (\bar{q}_1 \Gamma q_2) | 0 \rangle \times \langle M_2 | (\bar{q}_3 \Gamma' b) | B \rangle$$

(SL form factors, decay constants, distribution amplitudes) at leading order of the heavy mass expansion. QCDF was the long awaited (diagrammatic) proof of Bjorken's color transparency argument, and can be seen as a development of the (also very famous) Brodsky-Lepage approach to hard scattering. Beneke *et al.*, '99-'01

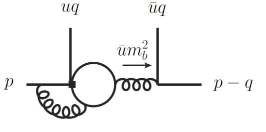
QCDF successfully predicts observables in $B \to D\pi$-like modes and, generally speaking, correct pattern of decay rates, non trivial CP asymmetries etc.

Predictions of QCDF/SCET

B decays to $D^{(*)}+$ light channels



However in charmless decays such as $B \to \pi\pi$ the predictivity of QCDF is spoiled by chirally enhanced power corrections $\sim 2m_\pi^2/((m_u + m_d)m_b) \sim 1$ that are only partly factorizable/computable: it is still not known how to treat these terms.



SCET can also adress non leptonic matrix elements but suffers from the same problem than QCDF. Bauer *et al.* ‘00-‘02

We have introduced the theoretical tools needed to evaluate hadronic matrix elements relevant for flavor physics especially in the B meson sector.

Now we can consider not only the determination of the SM couplings, but also try to test for the presence of New Physics contributions.

Where could be New Physics ?

First answer: *a priori* anywhere; imagine there are right-handed currents, then all the flavor observables are impacted, and we presumably do not have enough theoretical and experimental information to extract both left- and right-handed couplings simultaneously; in other words, the CKM matrix is unknown, and the apparent successes so far are accidental.
Generally speaking, if New Physics is generic and impact many kind of observables, then many things have to be recalculated and a completely global analysis is needed; this is actually very challenging and is not the common practice.
However one may assume that dominant New Physics effects occurs in SM amplitudes that are small because of its specific properties; in particular since Flavor Changing Neutral Currents are suppressed by quantum loops in the SM, this is the first place to look at.

Exercise: the 4 generation CKM matrix

3D CKM matrix:

$$\begin{aligned} s_{12} &= \lambda \\ s_{23} &= A\lambda^2 \\ s_{13}e^{-i\delta} &= A\lambda^3(\rho - i\eta) \\ \rho + i\eta &= \frac{\sqrt{1-A^2\lambda^4}(\bar{\rho}+i\bar{\eta})}{\sqrt{1-\lambda^2}[1-A^2\lambda^4(\bar{\rho}+i\bar{\eta}]} \end{aligned}$$

then expanding in λ one exhibits the hierarchy of flavor transitions.
Taking four generations one gets five more parameters, three mixing angles $\theta_{i4}(i = 1, 2, 3)$ and two CP-phases $\delta^{(\prime)}$
Why not try to generalize the 3×3 λ hierarchy to the fourth generation ?
Then one naturally defines

$$s_{34} \equiv A'\lambda^3\,,\; s_{24}e^{-i\delta'} = A'\lambda^4(\rho' - i\eta')\,,\; s_{14}e^{-i\delta''} = A'\lambda^5(\rho'' - i\eta'')$$

if A‘, ρ', ρ'', η', η'' are of order one, one can expand in λ

$$
\begin{pmatrix}
\frac{1}{8}\left(-\lambda^4-4\lambda^2+8\right) & \lambda & A\lambda^3(i\eta+\rho) & A'\lambda^5(i\eta''+\rho'') \\
\frac{1}{2}A^2\lambda^5(2i\eta-2\rho+1)-\lambda & \frac{1}{8}\left(-4A^2\lambda^4-\lambda^4-4\lambda^2+8\right) & A\lambda^2 & A'\lambda^4(i\eta'+\rho') \\
\frac{1}{2}A\lambda^3\left(-i\eta\left(\lambda^2-2\right)+\rho\left(\lambda^2-2\right)+2\right) & \frac{1}{2}A\lambda^2\left(\lambda^2(2i\eta-2\rho+1)-2\right) & 1-\frac{A^2\lambda^4}{2} & A'\lambda^3 \\
A'\lambda^5(-i\eta'+i\eta''+\rho'-\rho'') & A'\lambda^4(i\eta'+A\lambda-\rho') & -A'\lambda^3 & 1
\end{pmatrix}
$$

In this limit the new couplings that contribute to the neutral meson mixings are strongly suppressed by powers of λ with respect to the SM couplings

$$
\begin{array}{llll}
K\bar{K} \text{ mixing} & \lambda^3 & D\bar{D} \text{ mixing} & \lambda^4 \\
B_d\bar{B}_d \text{ mixing} & \lambda^5 & B_s\bar{B}_s \text{ mixing} & \lambda^5
\end{array}
$$

Thus if the observed geometric hierarchy in the 3×3 Standard Model is valid in the four generation scenario, the impact on flavor observables will be invisible !

Thanksfully, the SM-like 4th generation is now excluded by direct searches. . .

The flavor problem

Recall that the (FCNC) meson mixing operators come with a coupling of the form

$$\frac{g^2}{m_W^2}(V_i V_j^*)^2$$

Similarly, New Physics will contribute to the same operators with couplings

$$\frac{c_{ij}}{\Lambda_{NP}^2}$$

Thus to avoid that NP contributions are larger than SM ones, one needs

$$\frac{\Lambda_{NP}}{\sqrt{c_{ij}}} \gtrsim \frac{4\mathrm{TeV}}{|V_i V_j|}$$

$K\bar{K}$ mixing	10^4 TeV	$D\bar{D}$ mixing	10^4 TeV
$B_d\bar{B}_d$ mixing	10^3 TeV	$B_s\bar{B}_s$ mixing	10^2 TeV

In other words, either New Physics is very far and cannot solve the Higgs mass hierarchy problem, or the new flavor couplings are very small.

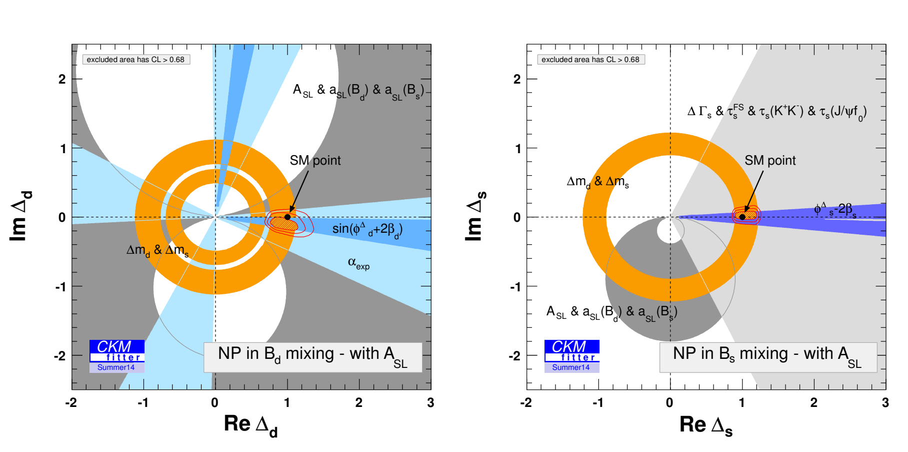
New Physics in B-$\overline{B}$ mixing

Independently of the flavor problem, the natural "to start with" choice is to assume that New Physics only contribute to FCNC.
In the global CKM analysis, only a few new parameters are needed to describe neutral meson mixing, and other FCNC observables can be discarded from the inputs in a first step.
In other words New Physics only enters M_{12} which is the real (dispersive) part of the mixing Hamiltonian.

$$\left\langle B_q \,\middle|\, \mathcal{H}_{\Delta B=2}^{\mathrm{SM+NP}} \,\middle|\, \bar{B}_q \right\rangle \equiv \left\langle B_q \,\middle|\, \mathcal{H}_{\Delta B=2}^{\mathrm{SM}} \,\middle|\, \bar{B}_q \right\rangle \times \left(\mathrm{Re}(\Delta_q) + i\,\mathrm{Im}(\Delta_q)\right)$$

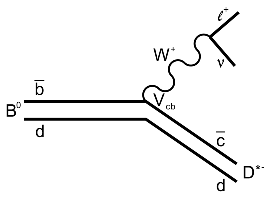
SM is thus located at $\Delta_d = \Delta_s = 1$;
alternative parametrization $\Delta_q = (1 + h_q e^{2i\sigma_q})$.

Constraints on Δ_d and Δ_s



there is still room for New Physics at the 30% level

Lepton universality: $R(D)$ and $R(D^*)$



We have seen that $b \to c$ transitions are much constrained by HQS. Semileptonic decays with a light lepton pair leads to an excellent determination of $|V_{cb}|$.

First measurement of

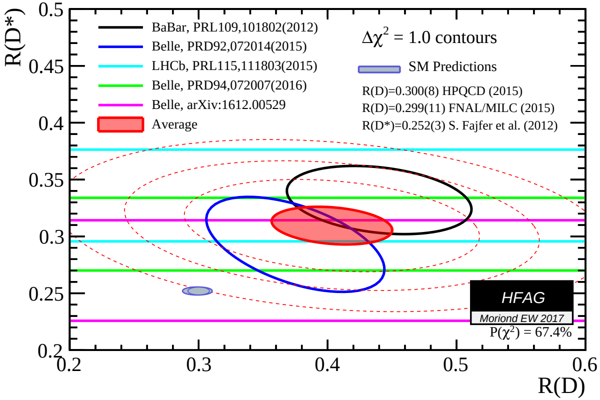
$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu_\ell)}$$

by BaBar in 2012.

Dependence to form factors is much reduced in these ratios, which allows a test of the universality of lepton couplings to quarks (a quite accidental prediction of the Standard Model). Note however that $R(D^*)$ has a $\sim 10\%$ dependence on the pseudoscalar form factor F_2 that have not yet been calculated in LQCD (use HQS instead).

On the experimental side the measurement is challenging, because of missing energy in τ decay: excited D states constitute a significant background to the tauonic mode, especially at hadron colliders (LHCb).

World summary of $R(D)$ and $R(D^*)$

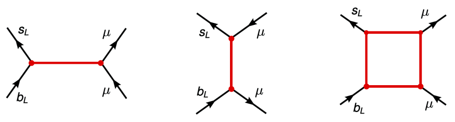


Full combination is more than 4σ away from the SM prediction (precise value depends a bit on the treatment of form factors)

A very intriguing anomaly because it is large, robust, and observed in a charged current transition !

FCNC $b \to s$ transitions

So far the least well known sector of weak quark decays.

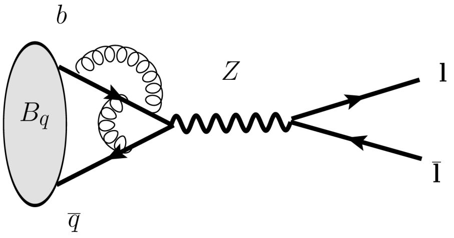


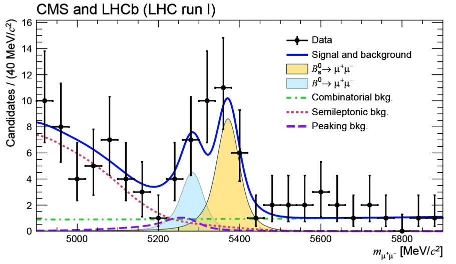
Large NP effects are still allowed, and quite naturally predicted by scenarios where non standard couplings are larger for the heaviest generations.

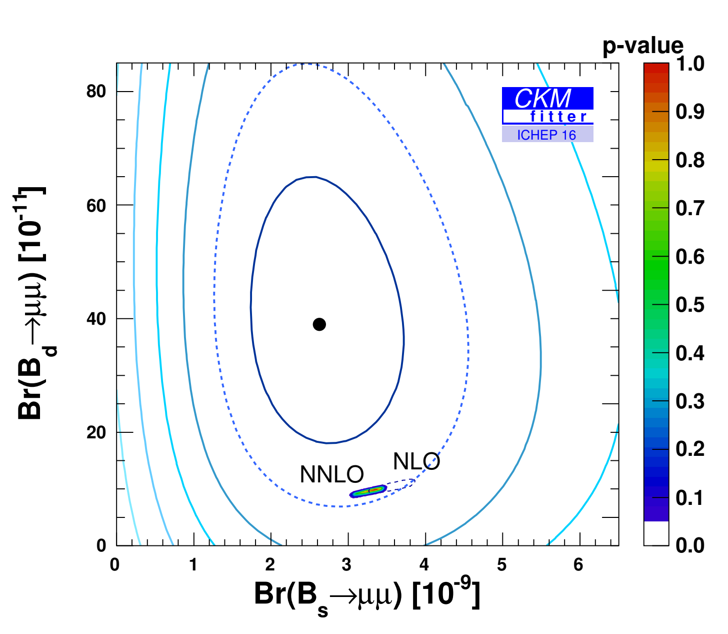
The very rare $B_s \to \mu^+\mu^-$ decay

This is the rarest decay that comes with both a non trivial measurement and a non trivial theoretical prediction. It is very sensitive to new particles, *i.e.* a charged Higgs.

Hadronically, it only depends (even outside SM !) on the f_{B_s} decay constant that is well computed on the lattice. Perturbative contributions have been computed up to NLO-EW and NNLO-QCD (Buchalla *et al.*, Bobeth *et al.*, Hermann *et al.*)






p-value
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
CKM
fitter
ICHEP 16
NNLO
NLO
80
60
40
20
0
0
1
2
3
4
5
6
Br(B_d→μμ) [10^-11]
Br(B_s→μμ) [10^-9]

$R(K)$ and $R(K^*)$ ratios

One defines the lepton universality test ratios

$$R(K^{(*)}) = \frac{\mathcal{B}_{\rm bin}(B \to K^{(*)}\mu\mu)}{\mathcal{B}_{\rm bin}(B \to K^{(*)}ee)}$$

Neglecting QED effects, these ratios are basically independent of hadronic matrix elements, and are predicted to be ~ 1 in the SM with an excellent accuracy $\sim 1\%$.

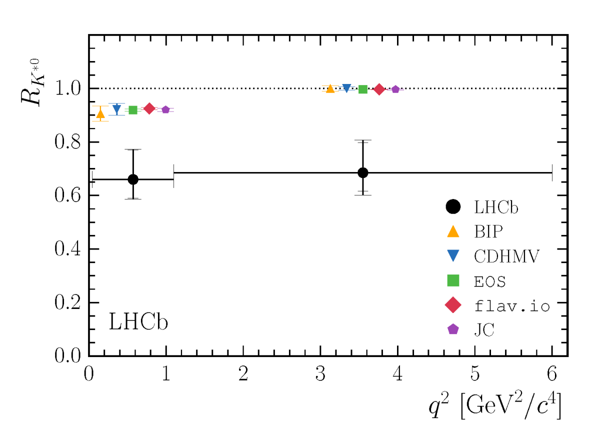
On the experimental side, main challenge is reconstruction of the electron channel in an hadronic environment (LHCb).

$R(K)$ and $R(K^*)$ ratios

Experimental measurements vs. theoretical predictions

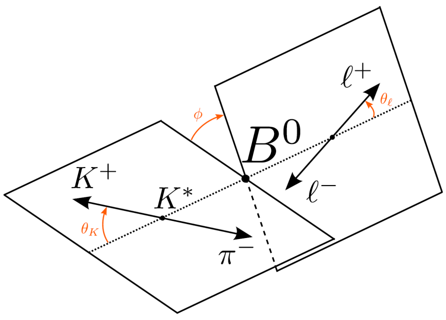
$$R_K^{[1,6]} = 0.745^{+0.097}_{-0.082}\ [2.6\sigma]$$

$$R_{K^*}^{[0.045,1.1]} = 0.66^{+0.113}_{-0.074}\ [2.3\sigma] \quad R_{K^*}^{[1.1,6]} = 0.685^{+0.122}_{-0.083}\ [2.6\sigma]$$



$B \to K^{(*)}\ell\ell$ angular observables

Experiments can measure 4-dimensional distribution



$$\frac{d^4\Gamma}{dq^2 d\cos\theta_{K^*} d\cos\theta_\ell d\phi} \sim \sum_i I_i f_i(\Phi)$$

where the linear coefficients I_i are angular observables that can be expressed in terms of $B \to K^*$ matrix elements.

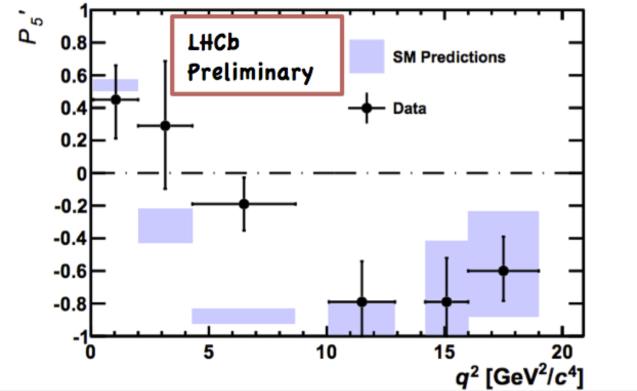
Optimized angular observables in $B \to K^{(*)}\ell\ell$

Independent amplitude combinations are related to each other thanks to SCET form factor relations.

This allows the construction of 'optimized' observable ratios, that are asymptotically independent of form factors. First one was the forward-backward asymmetry Ali *et al.* '00.

This can be made very general, by taking appropriate ratios of angular observables Krüger *et al.* '12, Descotes-Genon *et al.* '12.

Experimental results



First significant tension: 2-3σ in third bin of P_5' (LHCb '13)

This was the motivation for more sophisticated global analyses and refined measurements. Now individual anomalies of $b \to s$ transitions reach a large significance, up to 5σ depending on the treatment of hadronic uncertainties !

Conclusion

B physics is particularly rich :

- large *CP* violation
- sizable FCNC transitions
- 3 generation democracy
- sensitivity to weak-like New Physics
- interesting strong interaction properties

In the last few years a few anomalies have shown up against SM predictions. Some of them have washed out, but others are more robust and are very intriguing.

From the point of view of the strong interactions *b* systems are an opportunity to develop a variety of theoretical tools to systematically approach the difficult non perturbative problems.

Altogether many important questions are waiting to be addressed !