Effective Field Theories

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IA Intro

EFT and The OPE

String theory?

(Ira Rothstein)

QFT GR

There is interesting physics at all scales.

SR

Over physics career move towards more general theories.

QM CM ~~Cts~~ EM Newtonian Gravity

As become more general computations become more complex.

EFT: Simplest framework to capture the essential physics for a given problem in a manner which can be corrected to arbitrary precision.

QFT is the most precise tool for describing particle physics, But multiple scales (ie. masses / lengths) mean that taking into account all possible virtual states is a complicated problem, and perturbation theory may break down due to large logs

IB EFTs are a general set of frameworks to deal with this multiscale problem.

Can consider it as an organizational scheme → Remove modes in theory as d.o.fs if they are irrelevant at scale being probed.

Types of EFTs :

① Underlying UV physics known ⇒ match coeffs perturbatively
e.g. Fermi theory, HQET, SCET . .

② UV physics unknown or matching non-perturbative.

Reasons for using EFTs : (Manohar)

- Every theory is an EFT
- Simplifies ⇒ one scale at a time
- Makes symmetries manifest
- Include only relevant interactions
- Sum logs of ratio of scales / converts IR ⇒ UV logs ??
- Include non-perturbative effects systematically
- Efficient method to characterise new physics

Kusov)

II A The Wilsonian effective action (Becher)

The full QFT can be defined in terms of the path integral

All necessary quantities can be obtained via expectation values

$$\text{eg } \langle 0 | T\{\varphi(x_1) \ldots \varphi(x_n)\} | 0 \rangle = \frac{1}{Z} \int \mathcal{D}\varphi \, e^{iS(\varphi)} \varphi(x_1) \ldots \varphi(x_n)$$

$$\text{Partition fn} = \int \mathcal{D}\varphi \, e^{iS(\varphi)} \qquad \prod_i d\varphi(x_i)$$

If the characteristic scale M of the theory is the upper limit of the physics we are interested in, i.e. $E \ll M$, can consider a low-energy eff action:

Decompose: $\varphi = \underbrace{\varphi_L}_{}^{\omega < M} + \underbrace{\varphi_H}_{\gg \omega > M}$

$$\langle 0 | T\{\varphi_L(x_1) \ldots \varphi_L(x_n)\} | 0 \rangle = \int d\varphi_L \int d\varphi_H \, e^{iS(\varphi_L + \varphi_H)} \varphi_L(x_1) \ldots \varphi_L(x_n)$$

$$= \int d\varphi_L \, e^{iS_\Lambda(\varphi_L)} \varphi_L(x_1) \ldots \varphi_L(x_n)$$

Integrate all physics at scale $> \Lambda = M$ out.

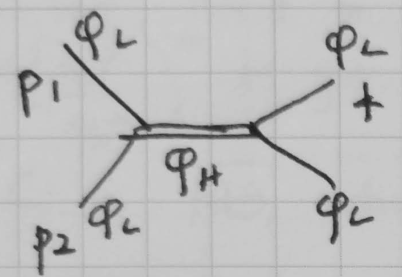
$S_\Lambda(\varphi_L)$ is non-local at scales $x \sim \frac{1}{\Lambda}$ as high-energy fluctuations are integrated out.

II B Since $E \ll \Lambda$ can expand in terms of local operators.

$$S_\Lambda(\varphi_L) = \int d^4x \, \mathcal{L}_\Lambda^{eff}(x) \longrightarrow \sum \overset{\text{Wilson coeffs}}{\overset{\uparrow}{g_i}} \mathcal{O}_i(x)$$

$\rightarrow$ local operators allowed by symmetries

e.g.



$\sim \frac{1}{p^2 \pm M^2}$, since $p_1, p_2 \ll M$

$$\frac{-1}{M^2} + \frac{p^2}{M^4} \rightarrow \frac{1}{M^2}\delta^{(4)}(x) - \frac{\Box}{M^4}\delta^{(4)}(x)$$

$$\Rightarrow \mathcal{L}_{eff} \ni \frac{1}{M^2}\varphi_L^4(x) + \frac{1}{M^4}\partial_\mu \varphi_L \partial^\mu \varphi_L \varphi_L^2(x)$$

$\Rightarrow$ Infinite set of operators ?

Using dim. analysis, action dimensionless $\Rightarrow$ $g_i = C_i M^{-\gamma_i}$ where

$C_i \sim 1$ $\Rightarrow$ as $\gamma_i \uparrow$ the $g_i \downarrow$.

From $S_0 = \int d^D x \left(\frac{1}{2}(\partial_\mu \varphi)^2 - \frac{m^2}{2}\varphi^2 \right)$ As the action is dimensionless we know the mass dim. of the field to be 1 or $D/2 - 1$.

γ_i Dim	As $E \to 0$ or $M \to \infty$	Terminology.
$\gamma_i < 0$	grows	relevant (super renorm.)
$\gamma_i = 0$	const.	marginal (renorm.)
$\gamma_i > 0$	declines	irrelevant (non-renorm.)

III

When the dim reg + $\overline{MS}$ scheme is used for regularization and renormalization, a scale μ is introduced and the renormalization constants and the renormalized couplings depend on this scale.

The renormalised Lagrangian $\mathcal{L} = Z_q \bar{q} i \not{D} q - Z_q Z_m m \bar{q} q$

where the Zs are chosen to cancel divergences in Greens functions.

e.g. The running mass can then be expressed in terms of the mass renormalization constant

$$\frac{dm(\mu)}{d\ln\mu} = -\gamma_m m(\mu) \quad \text{where } \gamma_m = \frac{1}{Z_m}\frac{dZ_m}{d\ln\mu}$$

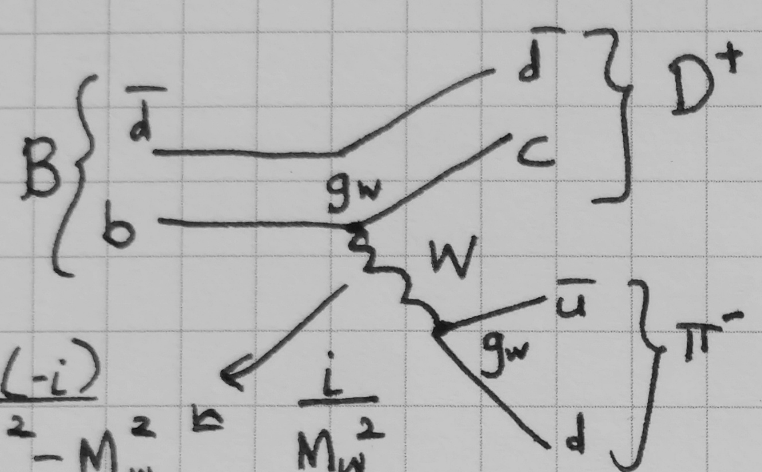
$$\frac{d\alpha_s(\mu)}{d\ln\mu} = -2\beta_0 \frac{\alpha_s^2}{4\pi} - 2\beta_1 \frac{\alpha_s^3}{(4\pi)^2}$$

In order to run a mass parameter from one scale to another one must solve the RGE eqn. This allows the resummation of large logs which may arise which mean that the perturbative expansion breaks down.

$$m(\mu) = m(m)\left(\frac{\alpha_s(\mu)}{\alpha_s(m)}\right)^{\frac{\gamma_{m0}}{2\beta_0}}\left(1 + \left(\frac{\gamma_{m1}}{2\beta_0} - \frac{\beta_1 \gamma_{m0}}{2\beta_0^2}\right)\frac{\alpha_s(\mu) - \alpha_s(m)}{4\pi}\right)$$

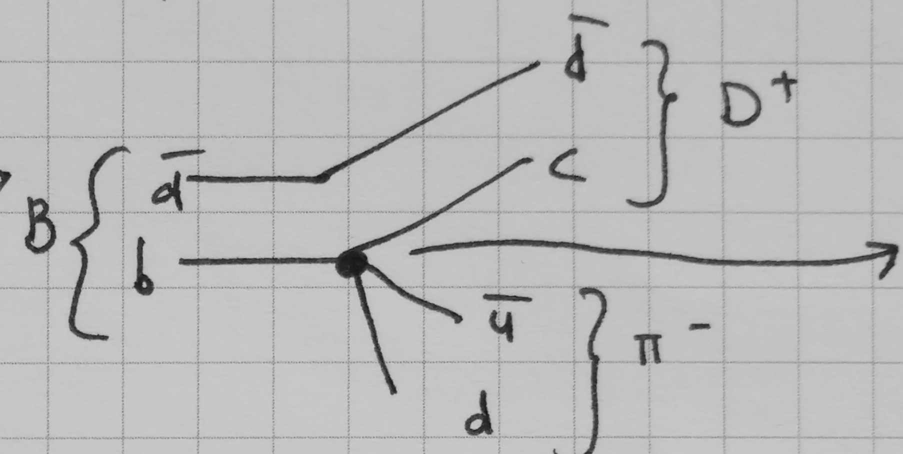
EFT and resumming logarithms

The most commonly used EFT in B physics is ~~weak~~ a Fermi theory, where all physics above m_b is integrated out.



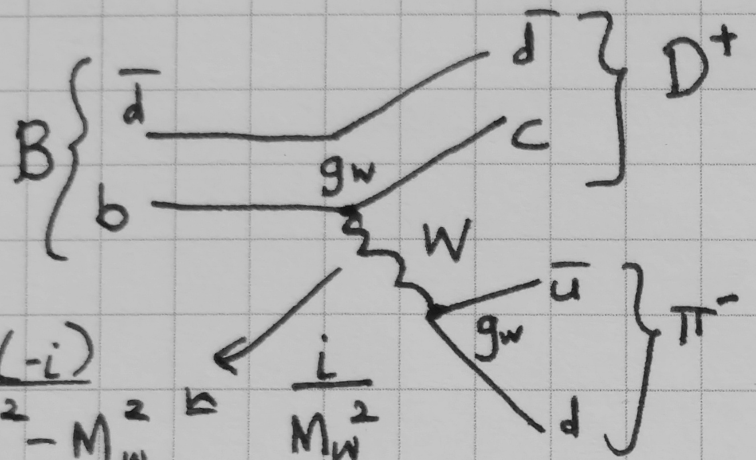
$$\frac{(-i)}{(p_i-p_f)^2 - M_W^2} \approx \frac{i}{M_W^2}$$

$\longrightarrow$

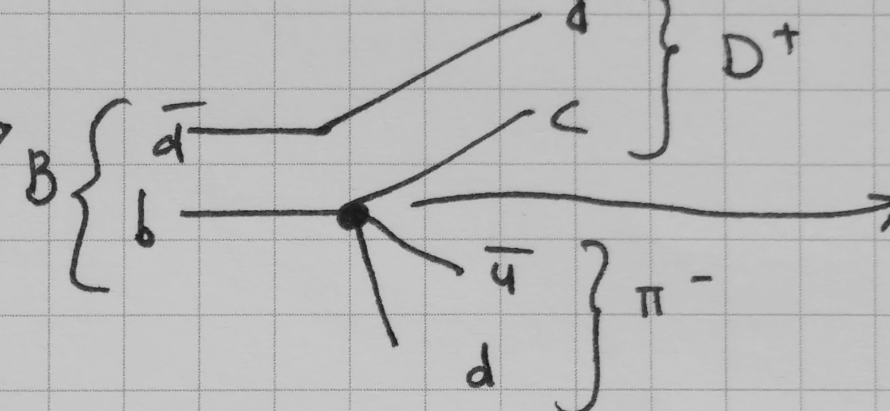


$$\frac{i g_W^2}{M_W^2} = \frac{i G_F}{\sqrt{2}}$$

$$\mathcal{L} \supset \underbrace{-\frac{G_F}{\sqrt{2}} V_{cb}}_{\text{WC}} \underbrace{\bar c \gamma_L \gamma_\mu b \bar u \gamma_\mu \gamma_L d}_{\text{Operator}}$$



$$\frac{(-i)}{(p_i-p_f)^2 - M_W^2} \simeq \frac{i}{M_W^2}$$



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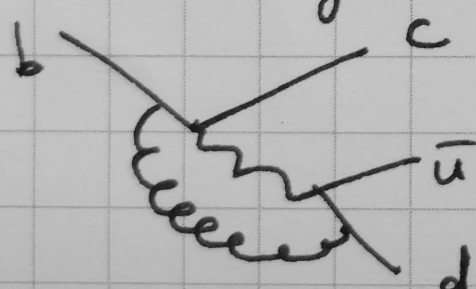
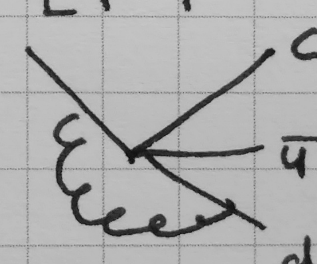
Consider 1-loop corrections to the WC

Full theory



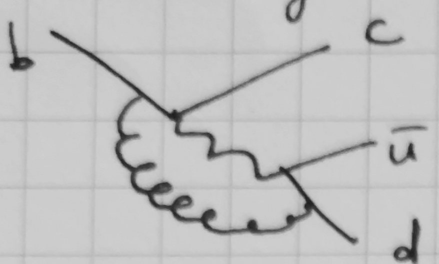
EFT



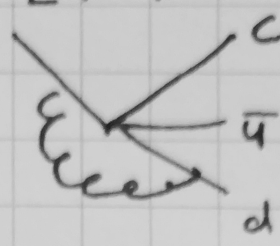
(Dim reg.) $\rightarrow$ Introduces scale μ.

$\overline{MS}$

Full theory



EFT



Introduces scale μ.

(Dim reg.) $\overline{MS}$

Finite. Contains Large Log.

$$-\frac{G_F V_{cb}}{\sqrt{2}} \lambda_1 \frac{\alpha_s(\mu)}{4\pi} \ln\left(\frac{M_W^2}{m_b^2}\right)$$

↳ large log

$$-\frac{G_F}{\sqrt{2}} V_{cb} \lambda_1 \frac{\alpha_s(\mu)}{4\pi}\left(\frac{1}{\epsilon} - \ln\left(\frac{m_b^2}{\mu^2}\right)\right) + K$$

λ_1 ↓ Coupling

↳ UV divergence, $M_W \to \infty$

K: arbitrary const.

Due to large log, need to include entire ~~all~~ perturbative sum as PT doesn't work.

$\sum_n \alpha_s^n(\mu) \ln\left(\frac{M_W^2}{m_b^2}\right)^n$ to get LL result, $\sum_n \alpha_s^n(\mu) \ln\left(\frac{M_W^2}{m_b^2}\right)^{n-1}$ to get NLL prediction

$$\Rightarrow \text{WC in } \overline{MS} \text{ scheme} = -\frac{G_F V_{cb}}{\sqrt{2}}\left(1 + \lambda_1 \frac{\alpha_s(\mu)}{4\pi}\left(\ln\left(\frac{M_W^2}{m_b^2}\right) + \ln\left(\frac{m_b^2}{\mu^2}\right) + \rho\right)\right)$$

ρ: arbitrary constant depends on Ren. scheme

Use EFT to handle LL, as make it into UV div, know how to apply RG. For the WC

In EFT, if we choose $\mu = M_W$ the log will disappear but this is not a sensible choice for the operator!

Using the optical theorem: $\Gamma = \mathrm{Im}\int d^4x \, \frac{i}{m_B} \left|C_{4F}(\mu)\right|^2 \langle B | T\{ O_{4F}^\dagger(x)\, O_{4F}(0) | B\rangle(\mu)$

We know that the μ dependence must be cancelled by the operator matrix element.

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \gamma_{O_{4F}}\right) O^R_{4F} = 0 \quad \text{where } Z = \frac{Z_q^2}{Z_{O_{4F}}} : \gamma_{O_{4F}} = \mu\frac{\partial}{\partial\mu}\Big(-\underbrace{\delta_{O_{4F}}}_{-\frac{\lambda_1}{\epsilon}\frac{\alpha_s}{4\pi} g(\mu)} + 4\cdot\underbrace{\frac{\delta_2}{2}}_{\frac{4}{3}\frac{\alpha_s}{4\pi}\frac{1}{\epsilon}}\Big)$$

The solution allows us to obtain $O^R_{4F}(\mu)$ from $O^R_{4F}(\mu_0)$: $O^R_{4F}(\mu_0)\exp\left(\int_{g(\mu_0)} \frac{dg}{g}\frac{\gamma_{O_{4F}}(g)}{\beta(g)}\right)$

$$O^R_{4F}(M_W) = O_{4F}(m_b)\left(\frac{\ln(M_W^2/\Lambda^2)}{\ln(m_b^2/\Lambda^2)}\right)^{A_1/2b_0} \qquad A_1 = \left(2\lambda_1 - \frac{16}{3}\right),\ b_0 = 11 - \frac{2}{3}n_f .$$

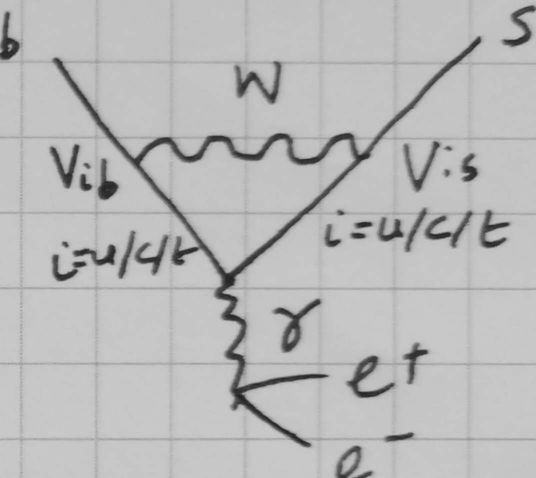
This allows us to obtain an accurate result for Γ including resummed Large logs.
LL result, not accurate to $O(\alpha_s)$ → require $\alpha_s^n \ln^{n-1}$ terms for that.

The Effective Hamiltonian for $b \to s$ transitions.

Now discuss the search for NP via FCNC transitions.

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}}\left(\underbrace{\lambda_t}_{A\lambda^2} \mathcal{H}_{eff}^{(t)} + \underbrace{\lambda_u}_{A\lambda^4} \mathcal{H}_{eff}^{(u)}\right)$$ where $\lambda_i = V_{ib}V_{is}^*$

$$\mathcal{H}_{eff}^{(t)} = C_1 O_1^c + C_2 O_2^c + \sum_{i=3}^{10} C_i O_i \left(+ \sum_{7,9,10} C_i' O_i'\right)$$ e.g.



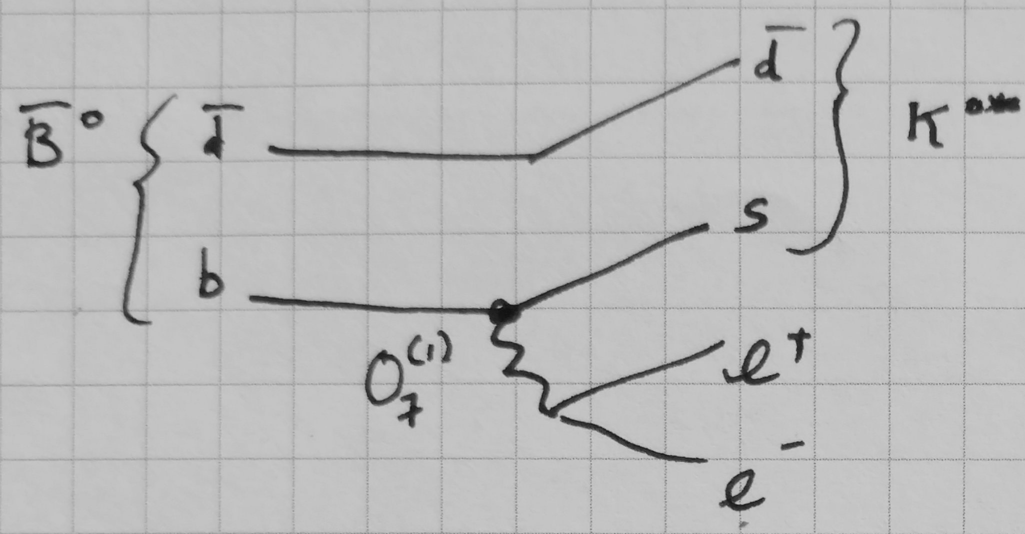
Also can get P, S and T operators.

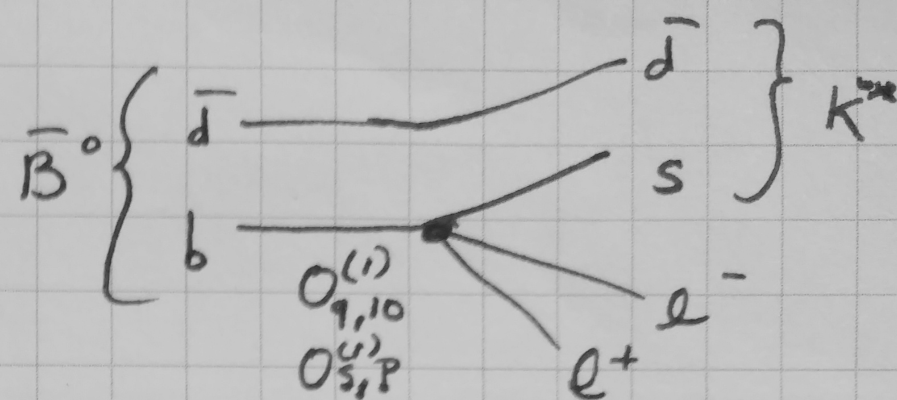
$$O_7^{(\prime)} = \frac{e}{g^2} m_b (\bar{s}\sigma_{\mu\nu} P_{R(L)} b) F^{\mu\nu} \qquad O_8^{(\prime)} = \frac{1}{g} m_b (\bar{s}\sigma_{\mu\nu} T^a P_{R(L)} b) G^{\mu\nu a}$$

$$O_9^{(\prime)} = \frac{e^2}{g^2} (\bar{s} \gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \ell) \qquad O_{10}^{(\prime)} = \frac{e^2}{g^2} (\bar{s}\gamma_\mu P_{R(L)} b)(\bar{\ell}\gamma_\mu \gamma_5 \ell)$$

where $P_{L/R} = \frac{1}{2}(1 \mp \gamma_5)$

In order to calculate $B \to K^+ \ell^+ \ell^-$





Using the SM/CKM as input can calculate $BR(B \to K^* \ell^+ \ell^-)$ and angular observables $\Rightarrow$ measure deviations.

eg $R_K = \dfrac{BR(B \to K\mu\mu)}{BR(B \to Kee)} \neq 1$

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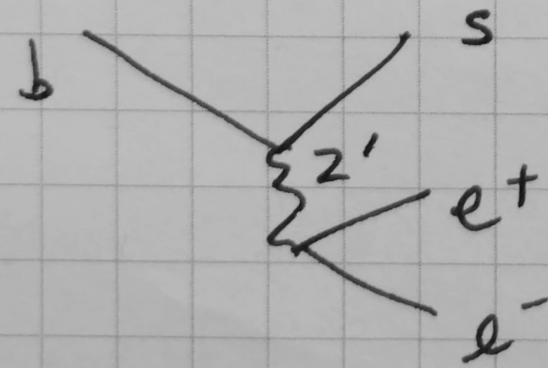
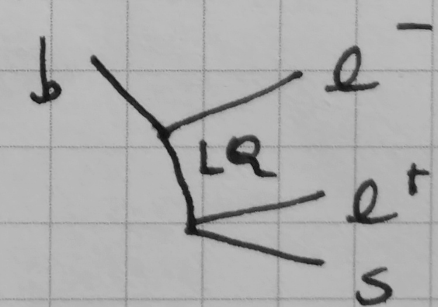
$$\text{eg} \quad R_K = \frac{BR(B \to K\mu\mu)}{BR(B \to Kee)} \neq 1$$

NP may contribute to operators $\Rightarrow$ i.e. Wilson coefficients different from SM prediction

eg. In addition to SM diagrams also have



or



contributing to $C^{(\prime)}_{9,10}$?