Physics of supernova neutrino oscillations

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Outline

- Neutrino Oscillation parameters: knowns and unknowns

- Supernova neutrino conversions from outside to inside

  Matter MSW propagation

  "Slow" Collective conversion

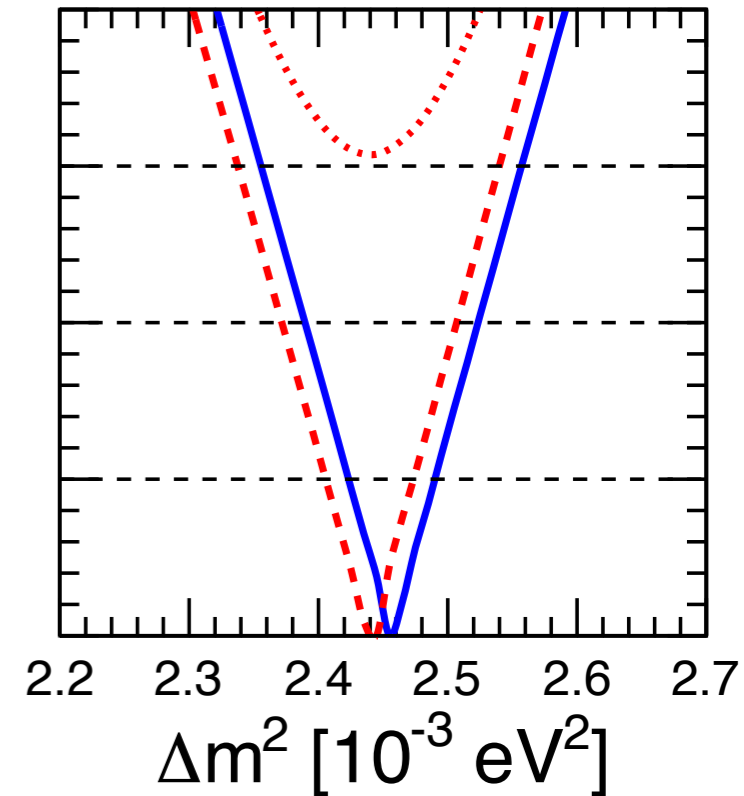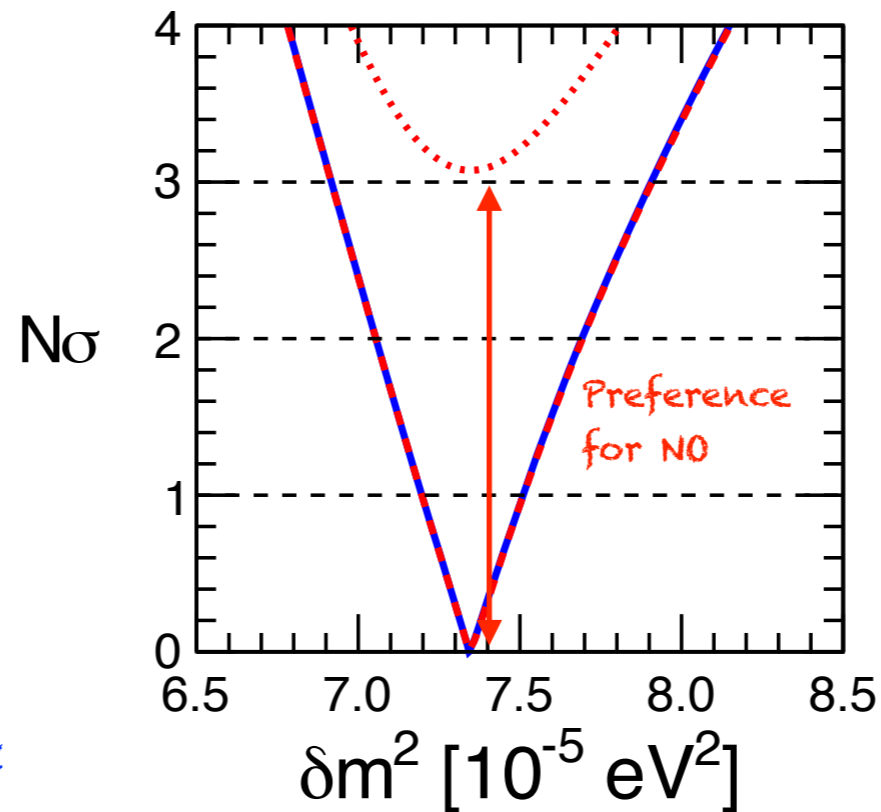 "Fast" Collective conversion

- Conclusions

Mass Differences
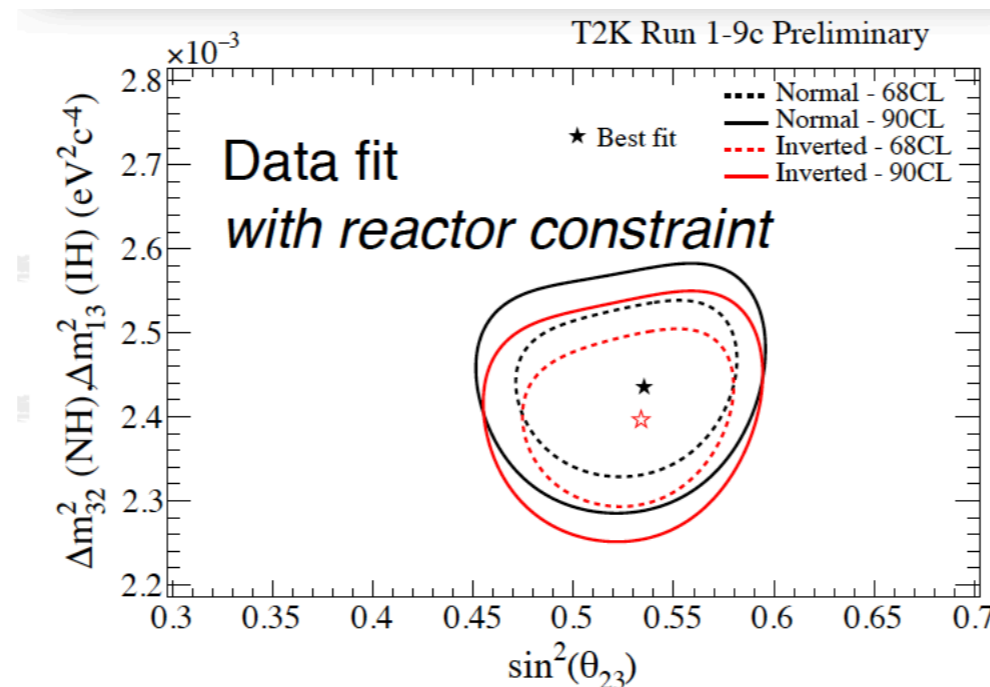
$$\Delta m^2 = (\Delta m^2_{13} + \Delta m^2_{23})/2$$

Mass Ordering = sign of Δm²

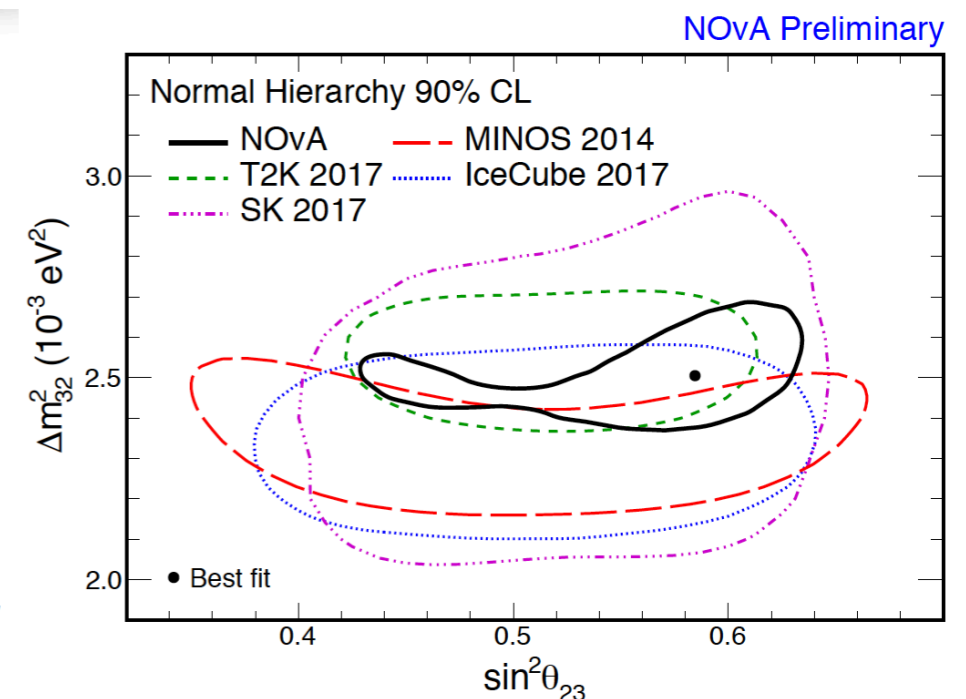Squared mass differences have both lower and upper bounds at more than 3σ

Nearly Gaussian uncertainties for Δm² and to a lesser extent for δm²

Nσ

Preference for NO

δm^2 [10^{-5} eV²]

Δm^2 [10^{-3} eV²]

Neutrino 2018 updates (still not included)

T2K Run 1-9c Preliminary

×10⁻³

Δm^2_{32} (NH)Δm^2_{13} (IH) (eV²c⁴)

Data fit with reactor constraint

- - - Normal - 68CL
— Normal - 90CL
- - - Inverted - 68CL
— Inverted - 90CL
★ Best fit

$\sin^2(\theta_{23})$

NOvA Preliminary

Normal Hierarchy 90% CL

— NOvA
- - T2K 2017
-·- SK 2017
- - MINOS 2014
····· IceCube 2017

Δm^2_{32} (10^{-3} eV²)

● Best fit

$\sin^2\theta_{23}$

Wascko - Neutrino 2018

Sanchez - Neutrino 2018

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Mixing Angles

Plots of $N\sigma$ versus $\sin^2\theta_{12}$, $\sin^2\theta_{13}$, and $\sin^2\theta_{23}$ with NO (solid blue) and IO (dashed red) curves.
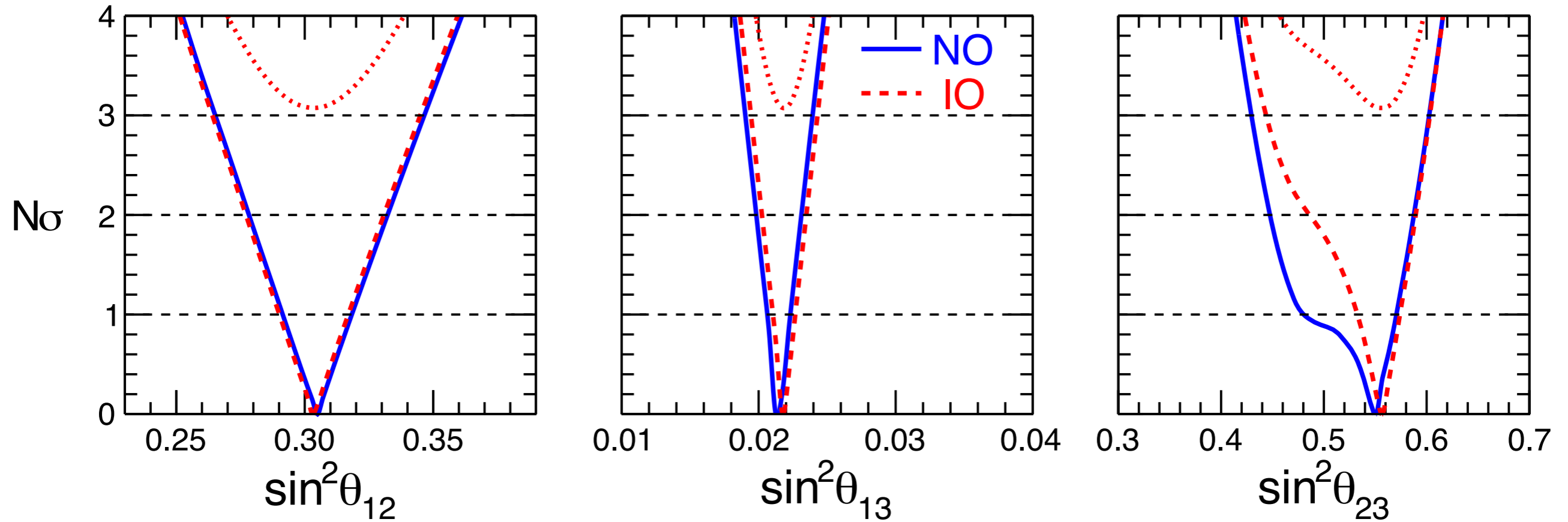
Mixing angles (θ_{23}, θ_{12}) have both lower and upper bounds at more than 3σ

Nearly Gaussian uncertainties for θ_{23} and to a lesser extent for θ_{12}

θ_{23} maximal mixing disfavored at about more than 2σ level. Best-fit octant flips with mass ordering

T2K Run 1-9c Preliminary

Data fit with reactor constraint

NOvA Preliminary

Normal Hierarchy 90% CL
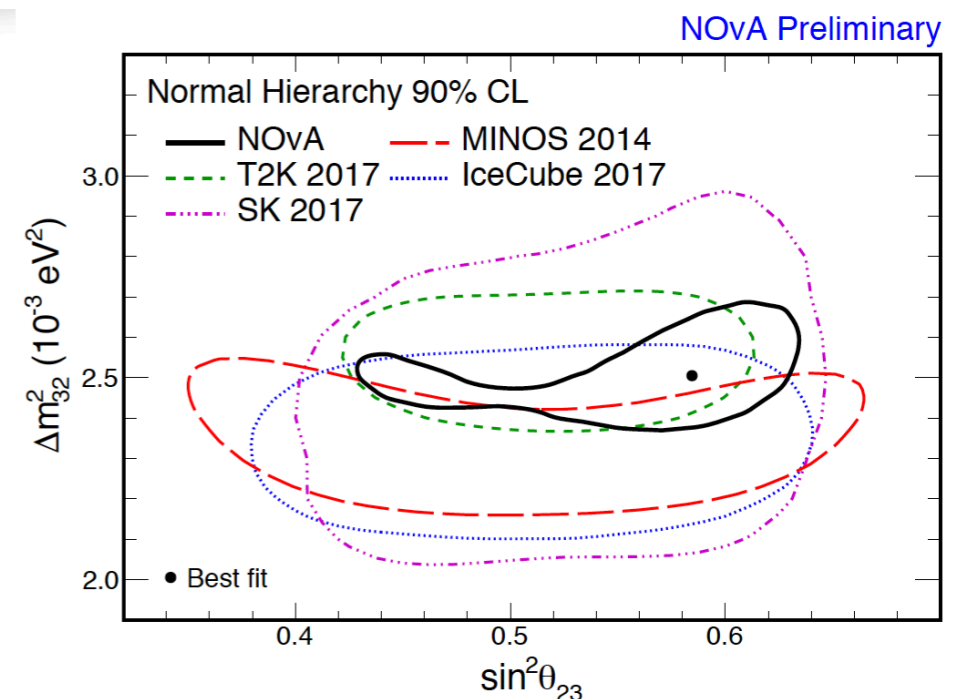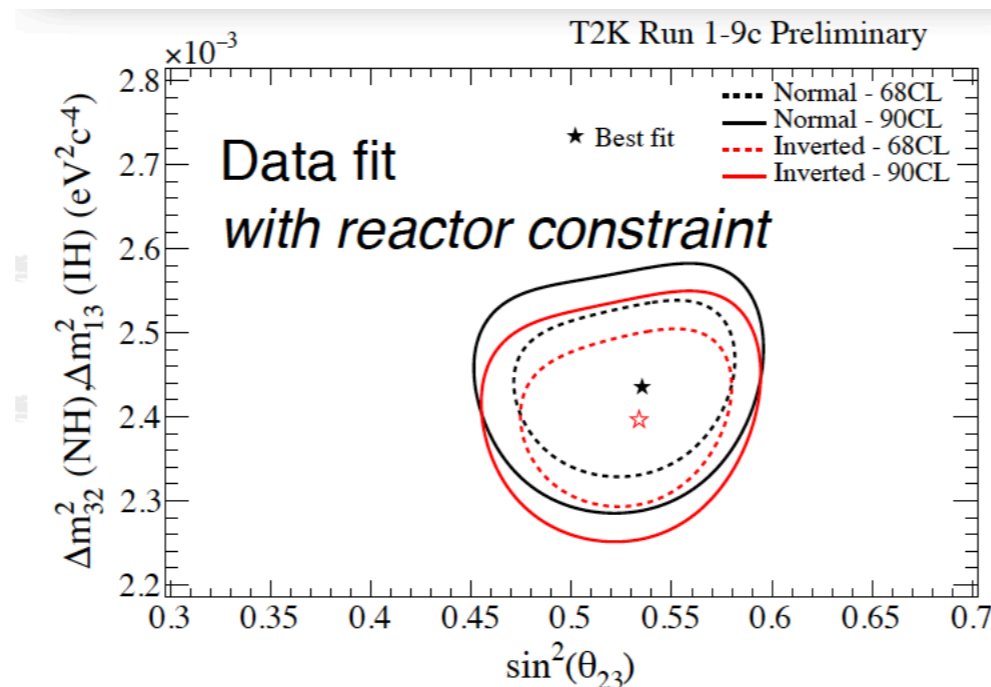
NOvA and MINOS prefer nonmaximal mixing

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CP phase δ

CP phase: δ ~ 1.4 π at best
fit CP-conserving cases
(δ = 0, π) disfavored at ~2σ
level or more Significant
fraction of the [0,π] range
disfavored at >3σ

Nσ vs δ/π

DATA FIT with reactor constraint

Wascko - Neutrino 2018

Sanchez - Neutrino 2018
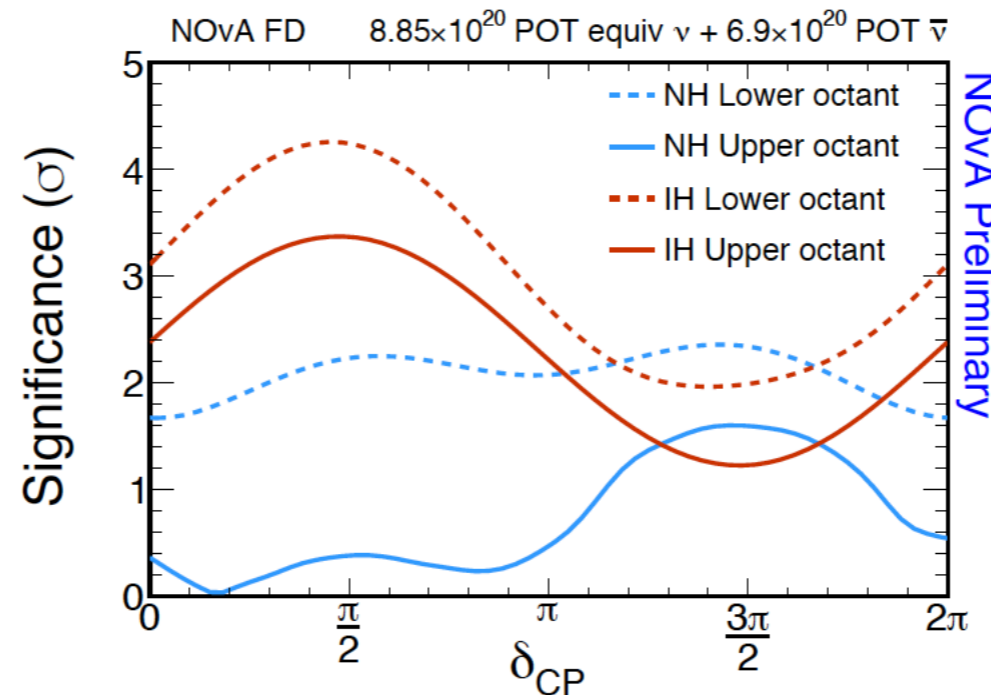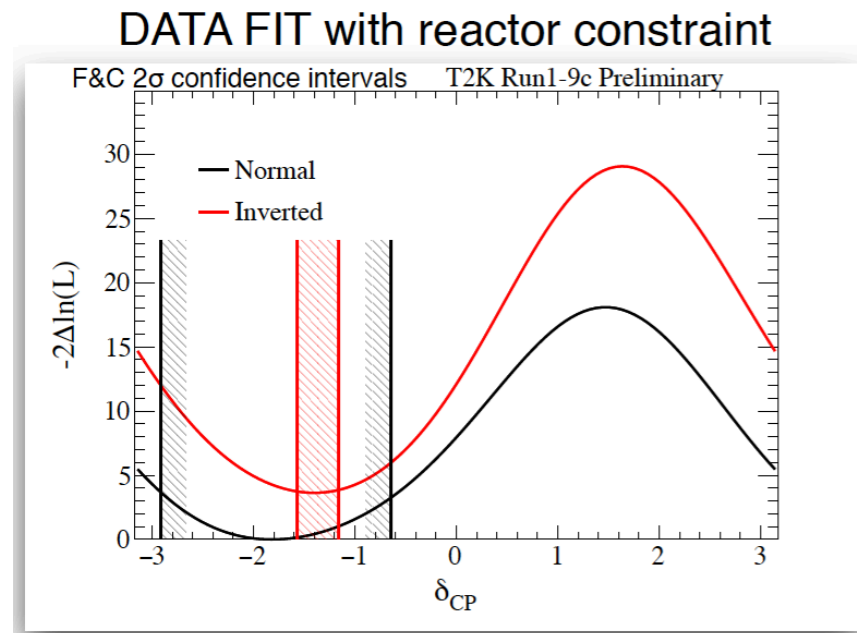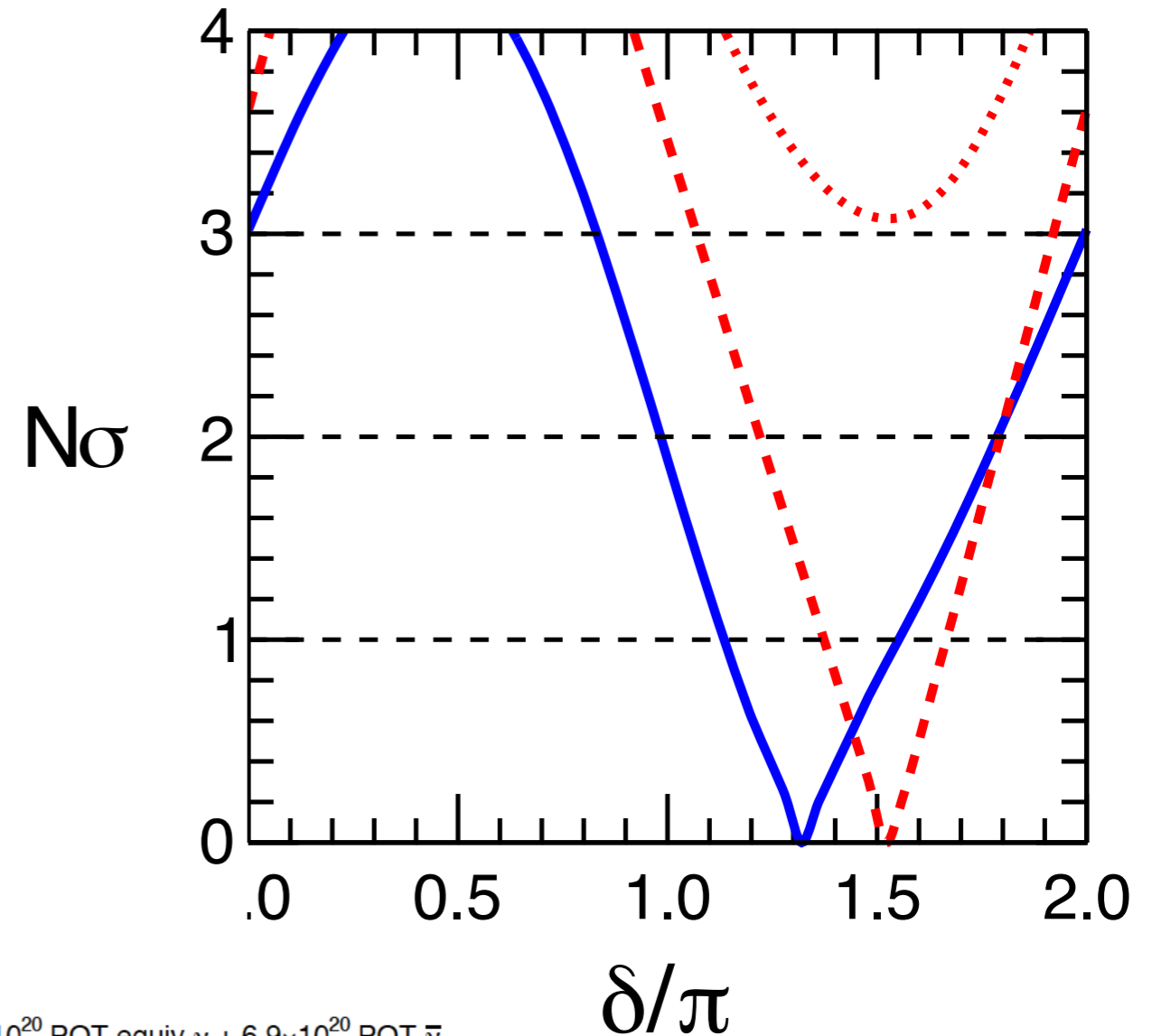
No big changes
expected with new
Neutrino 2018 data

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Precision era in neutrino oscillation phenomenology

Standard 3ν mass-mixing framework parameters

Known

δm^2	2.2%
Δm^2	1.4%
$\sin^2 \theta_{12}$	4.4%
$\sin^2 \theta_{13}$	3.8%
$\sin^2 \theta_{23}$	$\sim$ 9.3%

Unknown

CP-violating phase δ

Octant of θ_{23}
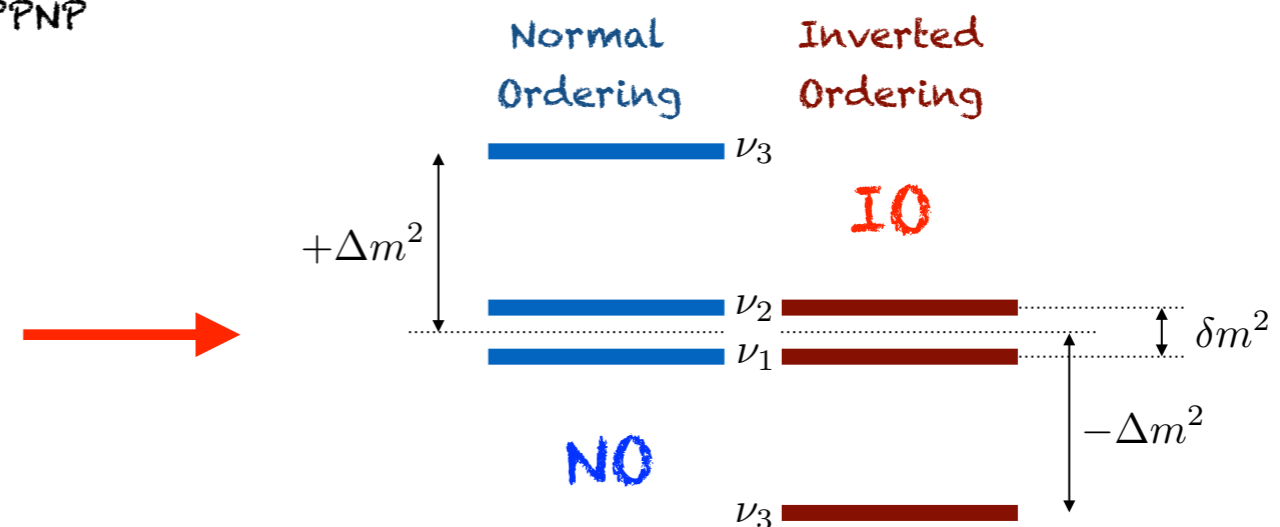
Mass Ordering $\rightarrow$ $\text{sign}(\Delta m^2)$

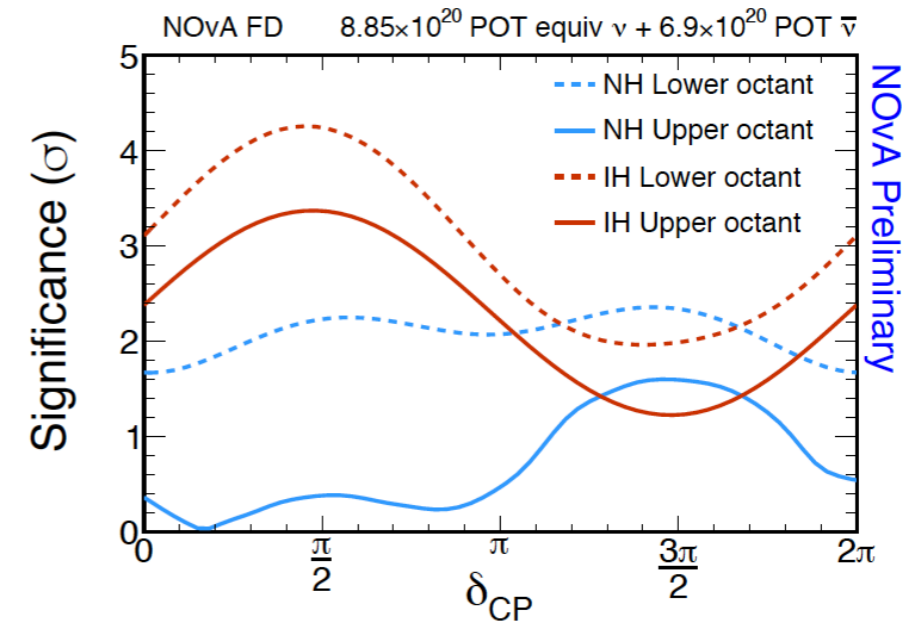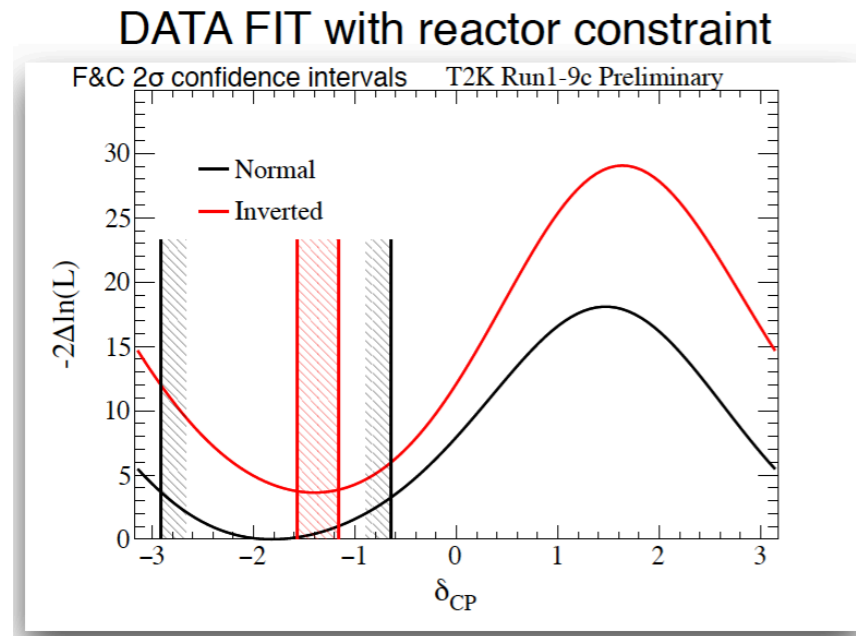[Dirac/Majorana neutrinos, Majorana phases, absolute mass scale]

Bari group, arXiv:1804.09678, to appear in PPNP

Next part of the talk on Mass Ordering

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Mass Ordering: present situation

NOvA FD 8.85×10^{20} POT equiv ν + 6.9×10^{20} POT $\bar{\nu}$

NOvA Preliminary

NOvA – weak preference for NO

DATA FIT with reactor constraint

T2K – preference for NO
Stronger preference when T2K and NOvA combined

SK preference for NO $\Delta\chi^2_{IH-NH} = 5.2$

Our Global Fit $\Delta\chi^2_{IH-NH} = 9.5$

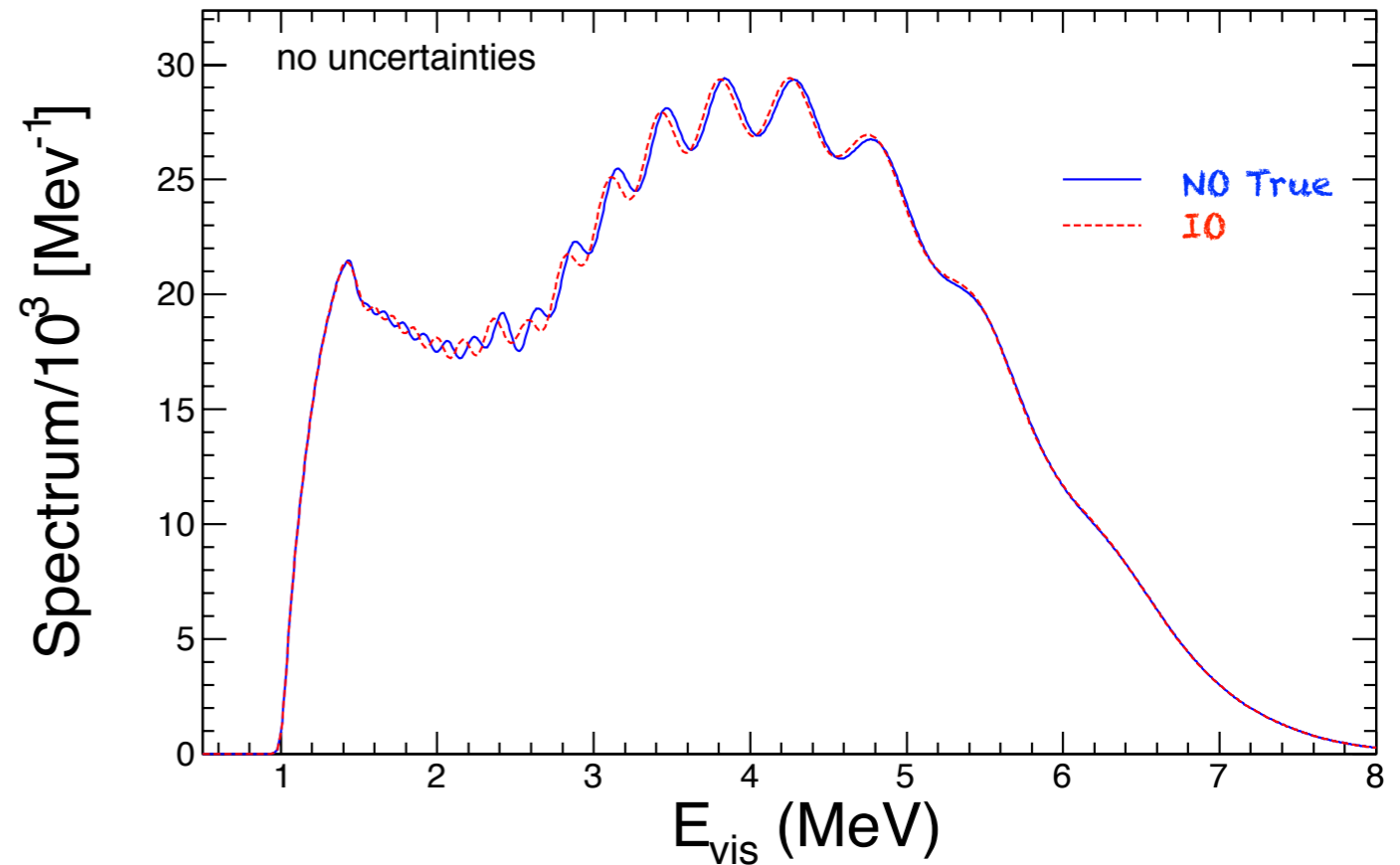|	LBL+Sol+KL	+SBL	+ATM
$\Delta\chi^2$(IO-NO)	1.3	4.4	9.5

Other groups findings
http://www.nu-fit.org/ $\Delta\chi^2_{IH-NH} = 4.14$ (No SK)
M. Tortola @Neutrino 2018 $\Delta\chi^2_{IH-NH} = 11.7$

Future experiments to discriminate the mass ordering –>

MBL reactor exp: JUNO

Mass ordering discrimination through interference between long-wavelength oscillations driven by $(\delta m^2, \theta_{12})$ and short-wavelength ones driven by $(\Delta m^2, \theta_{13})$

Expect $O(10^5)$ events in a few years

Will also improve the accuracy on δm^2 and θ_{12} by a factor of ~10

Most important systematic errors
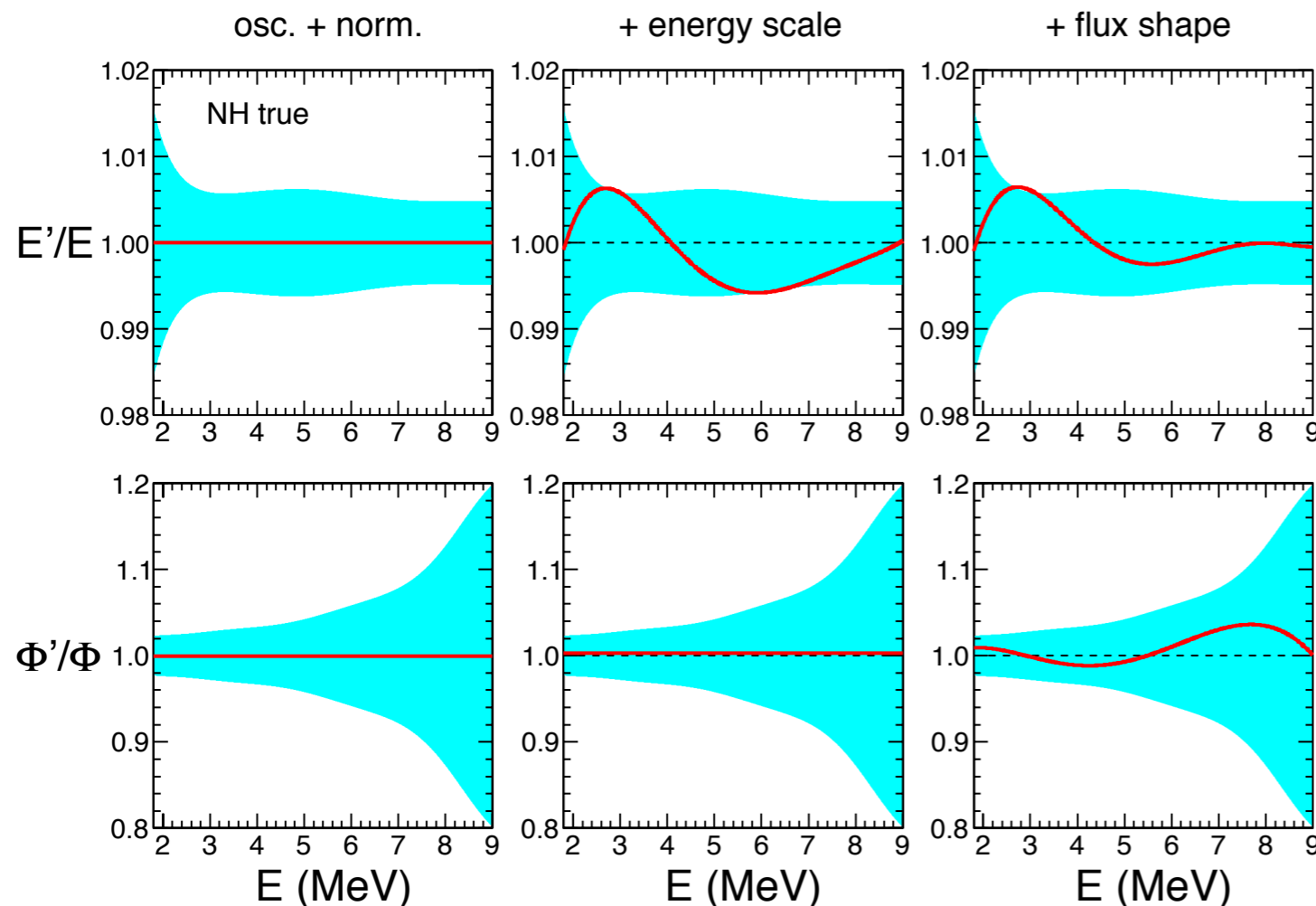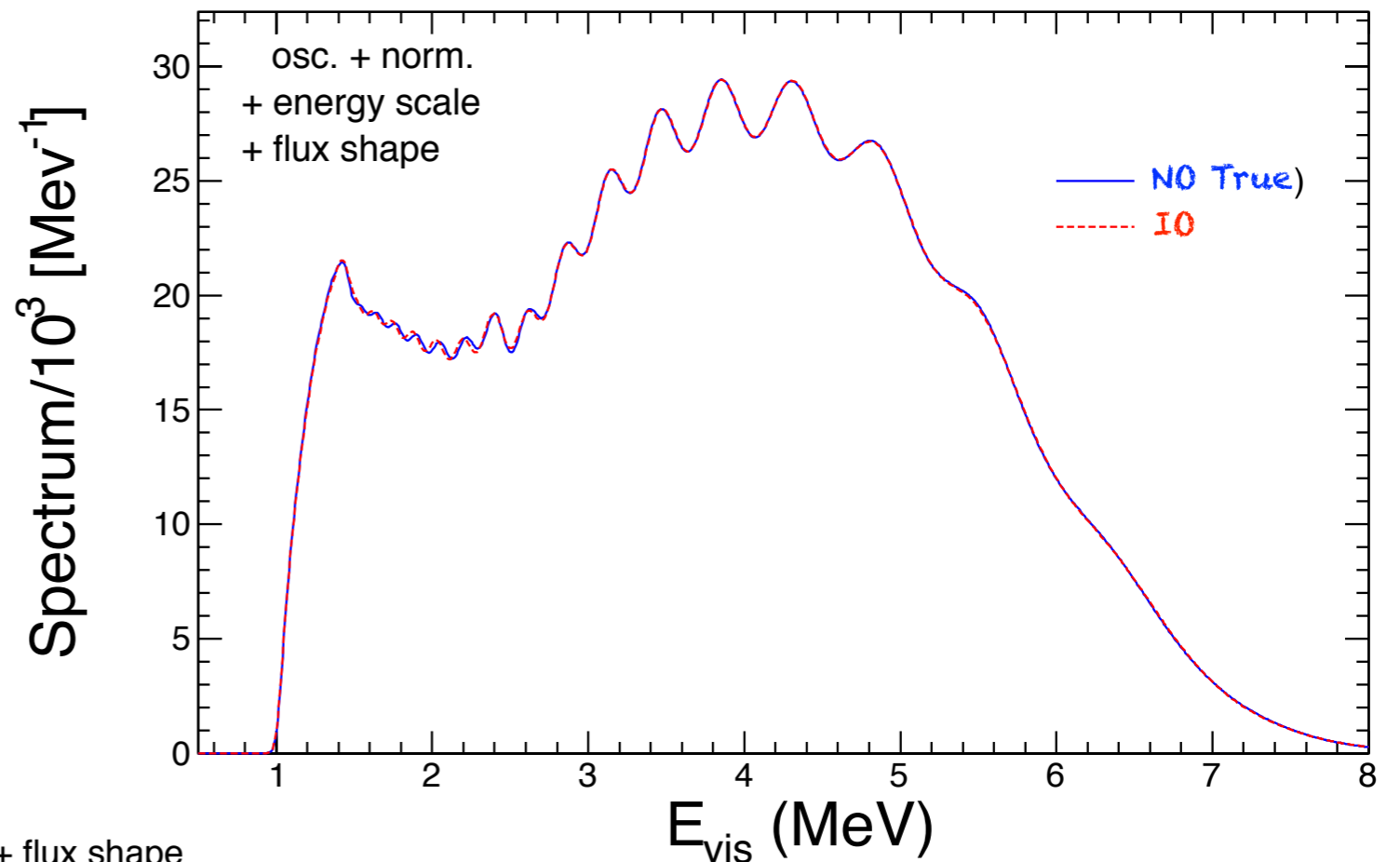   energy resolution
        energy scale
        flux shape

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After the inclusion of energy scale and flux shape uncertainties, NO (true) and IO (fit) spectra become less distinguishable → some loss of sensitivity to mass ordering

Energy scale uncertainties
$E \to E'(E)$ stretch the "x-axis"

Flux shape uncertainties
$\Phi(E) \to \Phi'(E)$ stretch the "y-axis"

osc. + norm.
+ energy scale
+ flux shape

NO True)
IO

Spectrum/10^3 [Mev^{-1}]

E_{vis} (MeV)

osc. + norm.   + energy scale   + flux shape

NH true

E'/E

Φ'/Φ

E (MeV)   E (MeV)   E (MeV)

In the context of MBL experiments we introduce smooth deformations of the detector energy scale and the reactor anti-neutrino flux (up to 5th-order polynomials, i.e. +12 systematic pulls) constrained by current error bands (in blue at ±1σ)

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JUNO-like prospective sensitivity to mass ordering (our estimate*)

Abscissa scales as $T^{1/2}$ -> linear behaviour for pure statistical errors

NO true
.... osc. + norm.
- - - - + energy scale
——— + flux shape

$N\sigma$

T (y)

IO true
.... osc. + norm.
- - - - + energy scale
——— + flux shape

T (y)
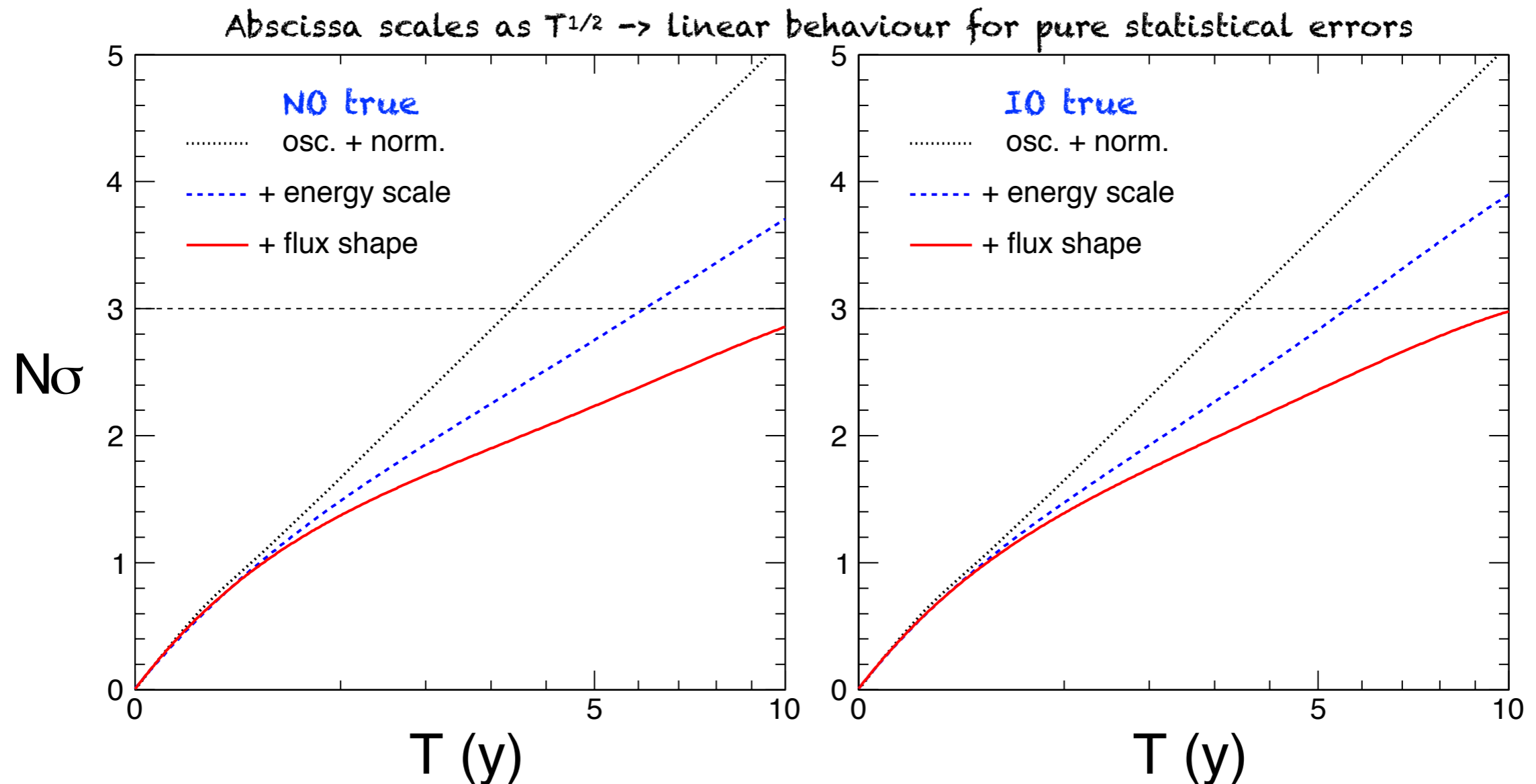
Inclusion of energy-scale uncertainties bends the linear rise, but still allows 3σ discrimination after ~6 years of data taking. With the inclusion of flux-shape uncertainties: 3σ sensitivity in ~10 years

Also the precise determination of ($\delta m^2, \theta_{12}$) affected: accuracy decreased by a factor of ~3, and the central values biased if wrong mass ordering is assumed
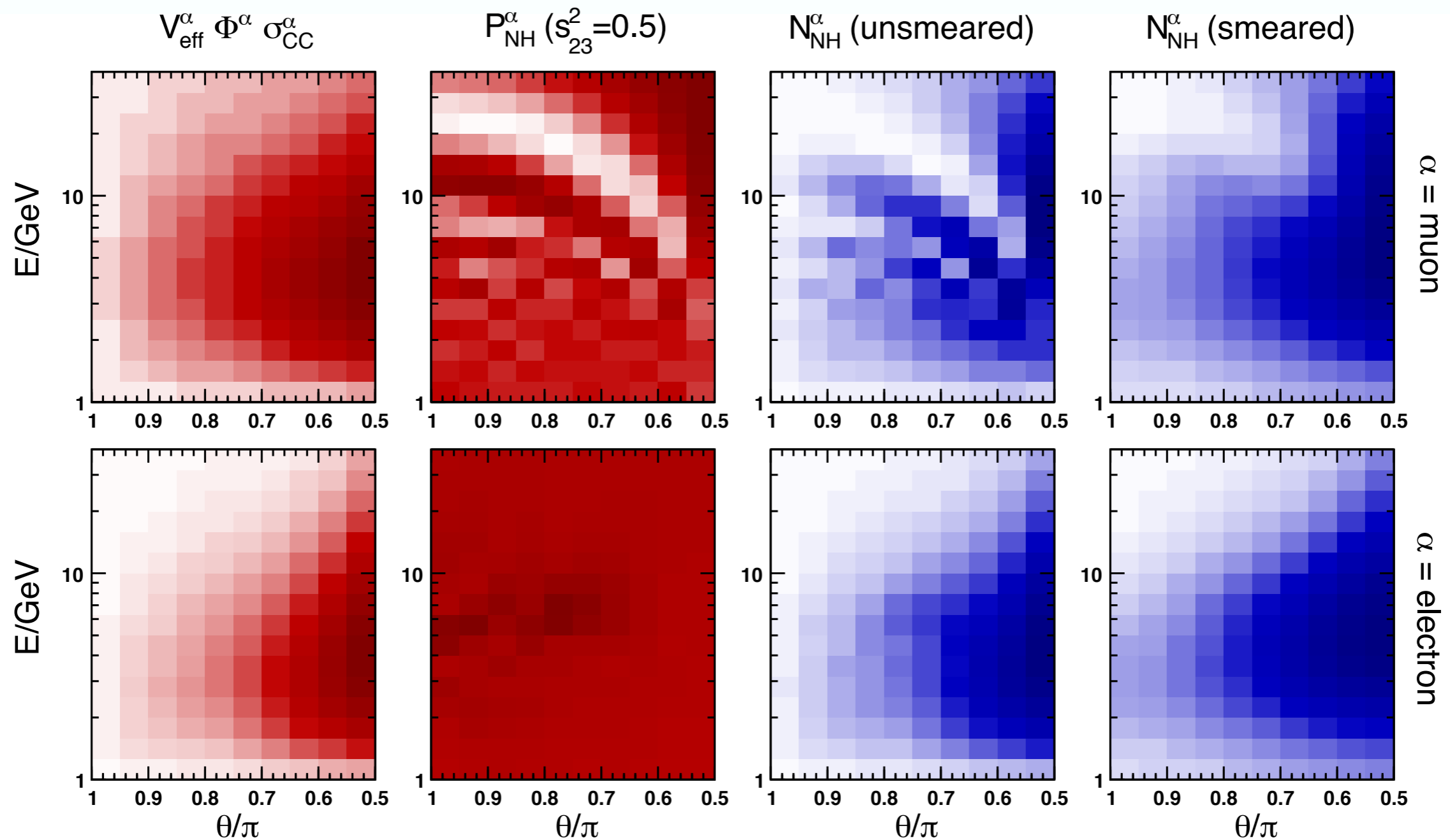
PINGU (or ORCA) rate

Oscillation independent

$$N_{ij}^{\alpha}|(E_\nu, \theta) = \overbrace{V_{\text{eff}}^{\alpha}(E_\nu) \otimes \sigma(E_\nu) \otimes \Phi^{\alpha}(E_\nu, \theta)} \otimes P^{\alpha}(E_\nu, \theta) \otimes R^{\alpha}(E_\nu, \theta)$$

Volume   Cross Section   Flux   Probability   Resolution

$V_{\text{eff}}^{\alpha} \Phi^{\alpha} \sigma_{CC}^{\alpha}$ P_{NH}^{α} (s_{23}^2=0.5) N_{NH}^{α} (unsmeared) N_{NH}^{α} (smeared)

E/GeV ... θ/π

α = muon

α = electron

PINGU

Stat + syst (osc+norm) | + resolution (scale,width) | + polynomial | + uncorrelated

Normal hierarchy

Inverted hierarchy

$N\sigma$

Time (y)

ORCA

Stat + syst (osc+norm)	+ resolution (scale,width)	+ polynomial	+ uncorrelated

Normal ordering

Inverted ordering

$N\sigma$

Time (y)

13
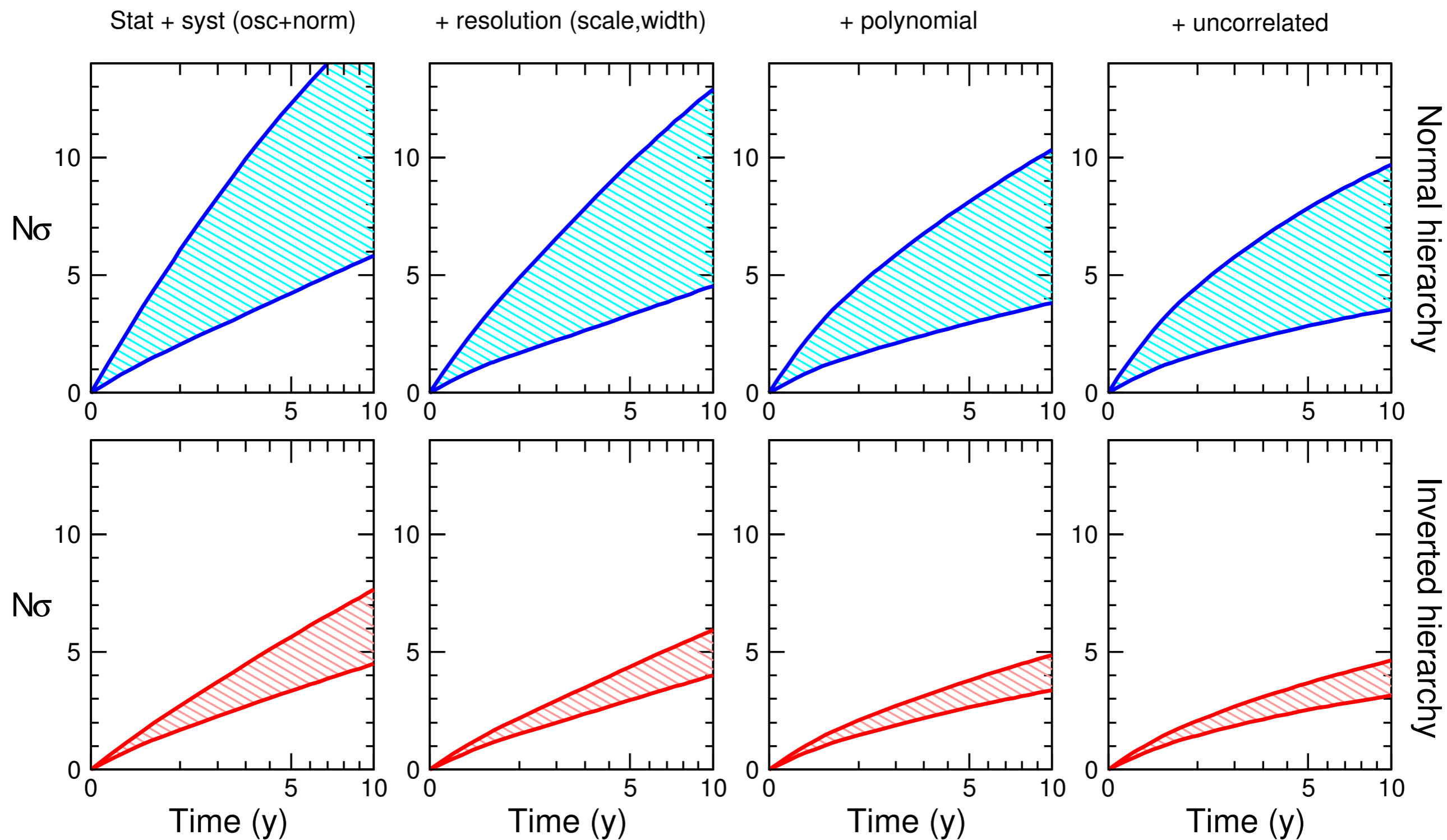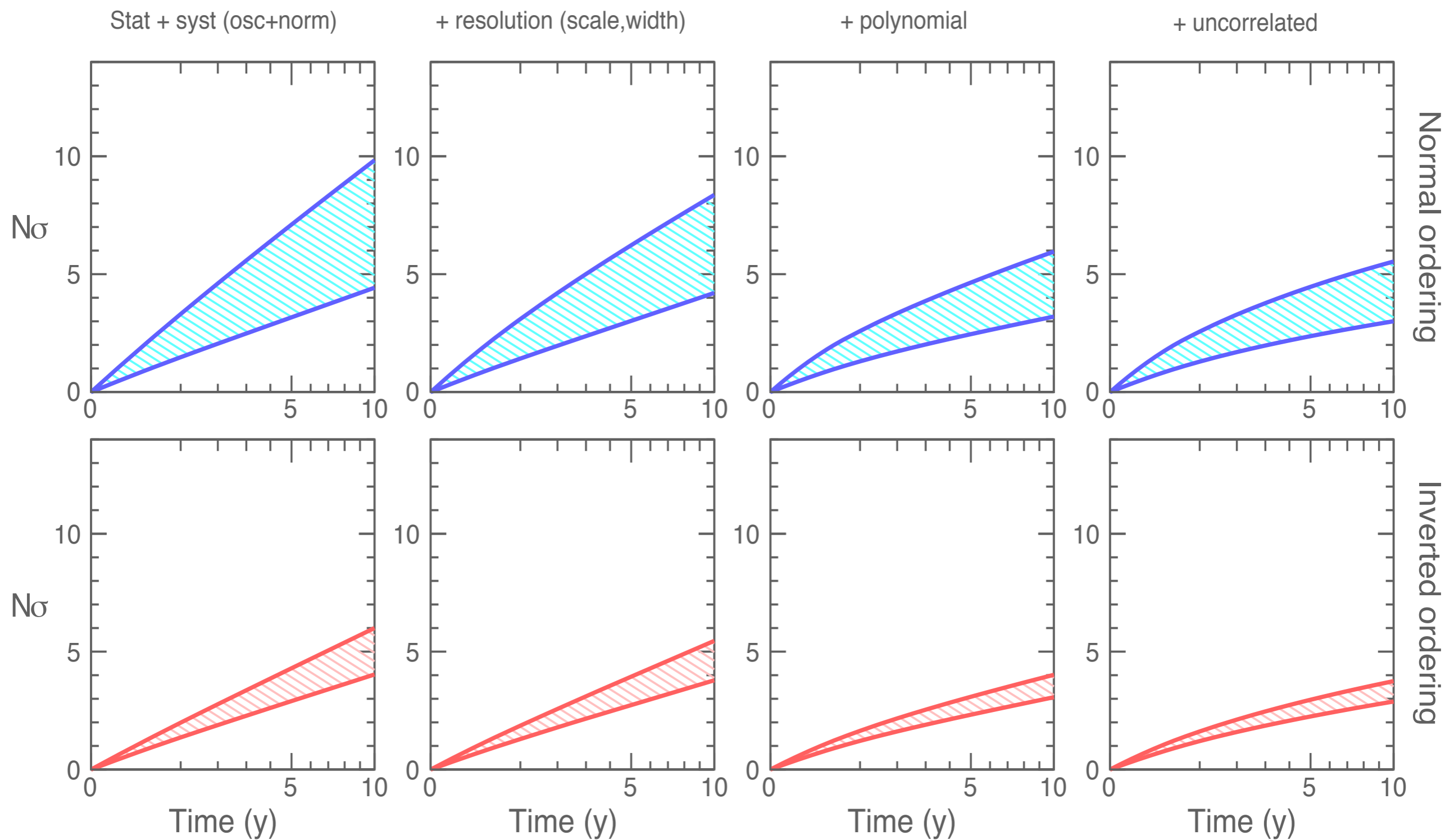
- Ranges of well-known 3ν parameters $(\delta m^2, \theta_{12})$ & $(\Delta m^2, \theta_{13})$ confirmed by ν2017-2018 data updates

- CPV: $\sin\delta < 0$ preferred

  best fit: $\delta/\pi \sim 1.3$-1.4 ± 0.2 (1σ)

  $\sin\delta \sim 0$ disfavoured at $> 2\sigma$

  $\sin\delta \sim +1$ disfavoured at $> 3\sigma$

- Octant info on θ_{23}: still fragile

- Mass Ordering: IO disfavored by oscillation data:

	LBL+Sol+KL	+SBL	+ATM
$\Delta\chi^2$(IO-NO)	1.3	4.4	9.5

- Info from ongoing - near future experiments

What Supernova Neutrinos can tell us?

While in the past SN neutrinos would have give us important information also on the oscillation parameters, today the most important piece of information we could have from a SN neutrino signal is on the mass ordering

SN neutrinos fluxes
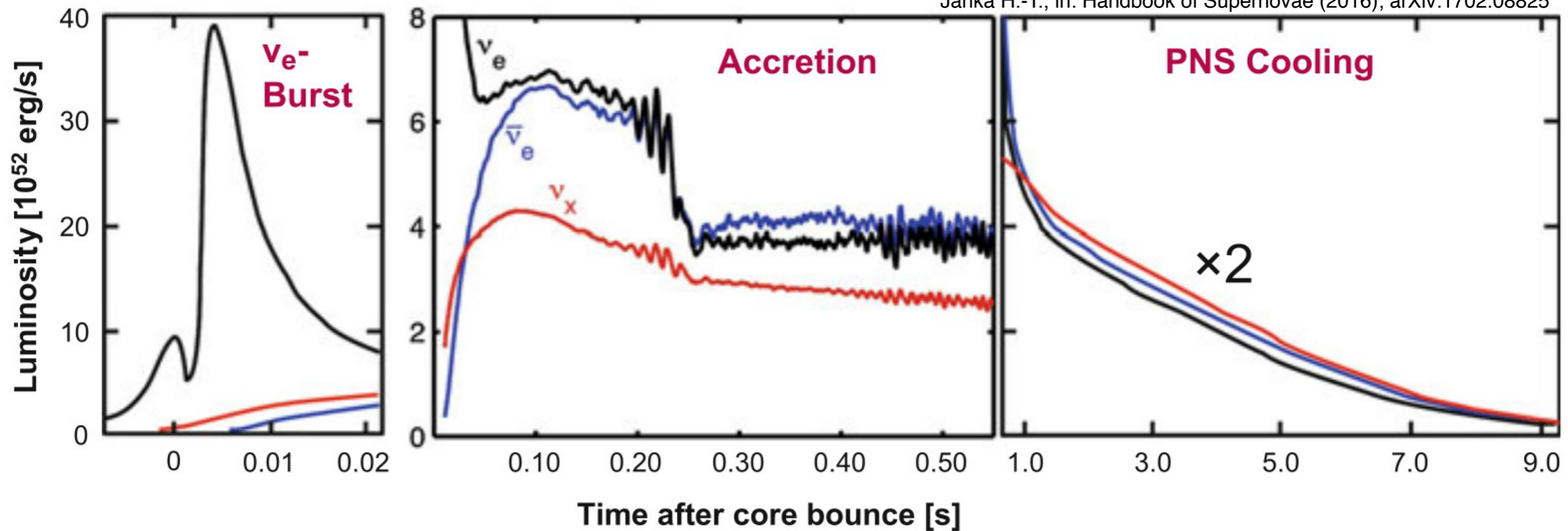
Emission on Time scale of 10 sec with different flux characteristics and hierarchies, matter and neutrino densities

Energy range ~1-100 MeV with different mean energy hierarchies in the three phases

Different kind of flavor conversions

General References

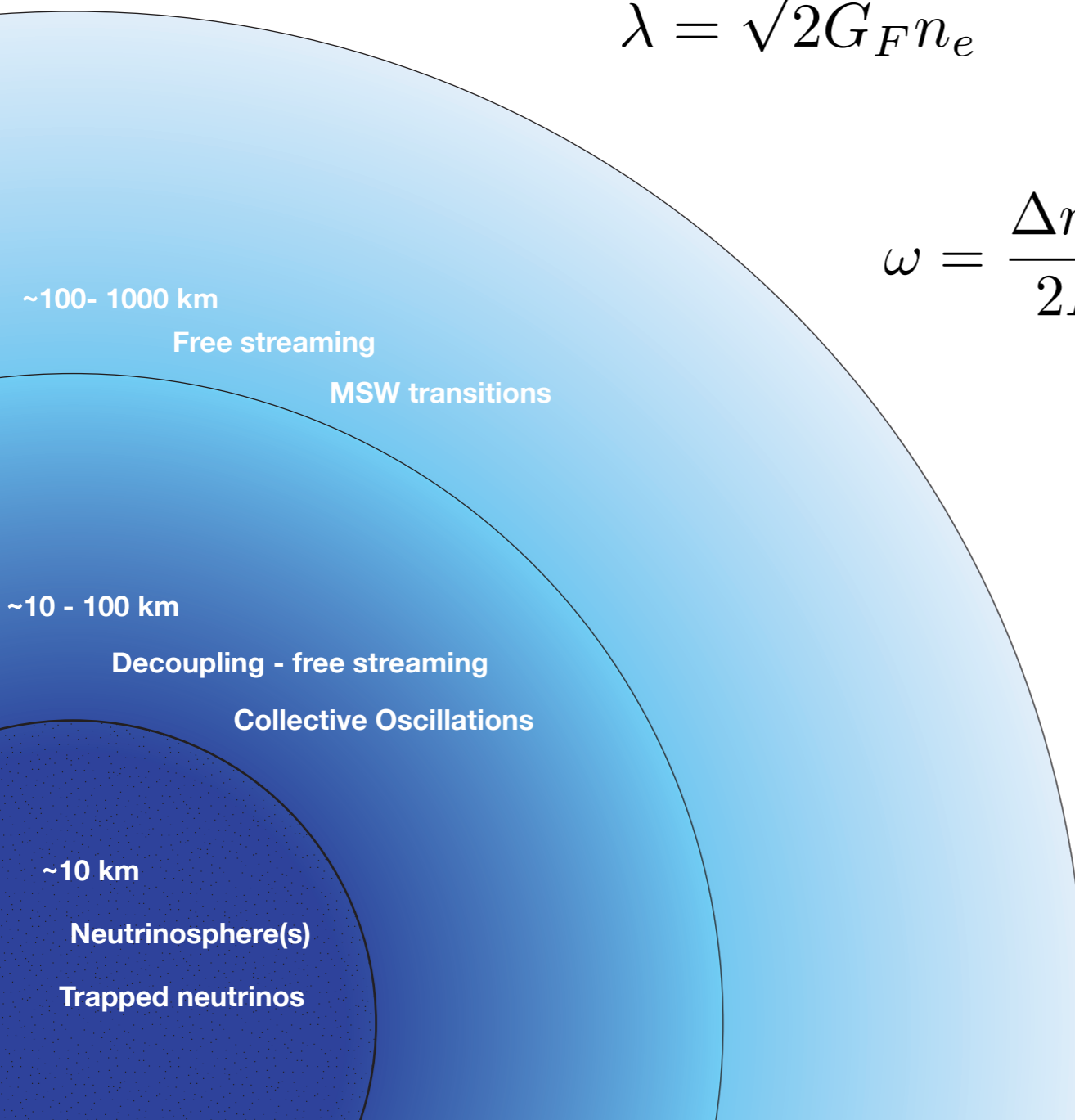K. Scholberg, arXiv:1707.06384,J.Phys. G45 (2018) no.1, 014002

A. Mirizzi, I. Tamborra, H.T. Janka, N. Saviano, K. Scholberg, R. Bollig, L. Hudepohl, . Chakraborty. arXiv:1508.00785, Riv.Nuovo Cim. 39 (2016) no.1-2, 1-112.

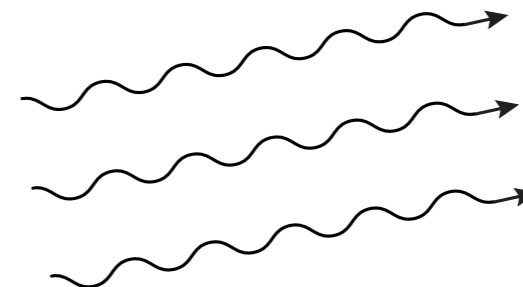# Regimes of SN neutrino flavor transition governed by the relative size of

$$\mu = \sqrt{2}G_F(n_\nu + n_{\bar\nu})$$ neutrino self-interaction potential

$$\lambda = \sqrt{2}G_F n_e$$ matter potential

$$\omega = \frac{\Delta m^2}{2E}$$ vacuum oscillation frequency

~100- 1000 km

Free streaming

MSW transitions

~10 - 100 km

Decoupling - free streaming

Collective Oscillations
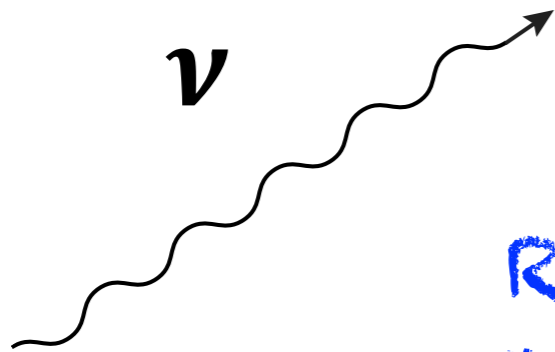
~10 km

Neutrinosphere(s)

Trapped neutrinos

Neutrinos travel to earth

Kinematical decoherence

Possible MSW when passing through the Earth

From Outside to inside

ν

R~1000 km
MSW conversion
Resonance at $\lambda \sim \omega$

R~100km
"Slow" Collective conversion
Oscillation frequency $1/t \sim \sqrt{\omega\mu}$
Spectral swaps at $\mu \sim \omega$

R~10 km (at edge of the Neutrinosphere)
Decoupling
"Fast" Collective conversion
Oscillation frequency $1/t \sim \mu$

"Standard" MSW Neutrino Oscillations

Neutrino steaming through the outer SN layers undergo ordinary MSW transitions

Matter effects important when

$$\lambda = \omega \Leftrightarrow \sqrt{2}G_F n_e(r) = \Delta m^2/2E$$

Two squared mass differences

$$\delta m^2 \sim 7.34 \times 10^{-5} \ \text{eV}^2$$
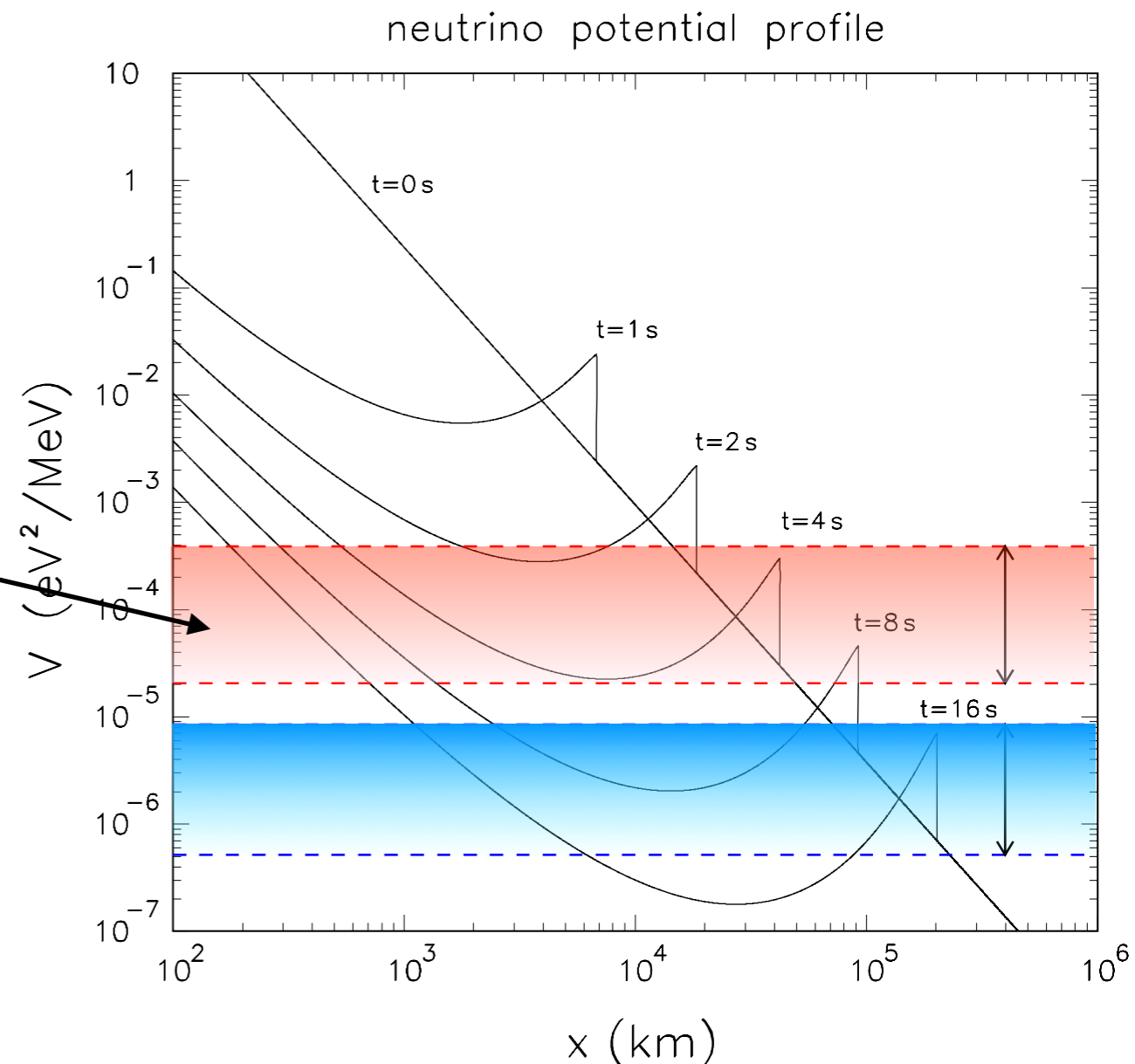$$\Delta m^2 \sim 2.45 \times 10^{-3} \ \text{eV}^2$$

Energy range

$$E \in [4, 70] \ \text{MeV}$$

Two resonances ω_H (atm. mass difference) and ω_L (solar mass difference)

MSW transitions at R grater than ~1000 km (important for the following discussion on self-induced transitions)

Dynamics can be factorised: two neutrino oscillations with relevant parameters $(\delta m^2, \theta_{12})$ or $(\Delta m^2, \theta_{13})$

Dighe, Smirnov, hep-ph/9907423. PRD.62.033007

neutrino potential profile

t=0 s
t=1 s
t=2 s
t=4 s
t=8 s
t=16 s

V (eV²/MeV)

x (km)

G. L. Fogli, E. Lisi, D. Montanino and A. Mirizzi, Phys. Rev. D 68, 033005 (2003) [hepph/0304056]

At production point $V/\omega_{L,H} \gg 1$

$$\cos 2\theta_m = \frac{\cos 2\theta - V/\omega}{\sqrt{(\cos 2\theta - V/\omega)^2 + \sin^2 2\theta}}$$

$$\sin 2\theta_m = \frac{\sin 2\theta}{\sqrt{(\cos 2\theta - V/\omega)^2 + \sin^2 2\theta}}$$

$\Longrightarrow$

$$\cos 2\theta_m \to -\mathrm{sign}(V)\mathrm{sign}(\Delta m^2)$$

$$\sin 2\theta_m \to 0 \quad \Rightarrow \theta_m = 0, \pi/2$$

Since the solar squared mass difference δm^2 is positive, while the atmospheric Δm^2 is positive for NO and negative for IO, at the production point we have

	Normal Ordering	Inverted Ordering
ν	$(\theta_{13}^m = \pi/2, \theta_{12}^m = \pi/2) \Rightarrow \nu_e \equiv \nu_3^m$	$(\theta_{13}^m = 0, \theta_{12}^m = \pi/2) \Rightarrow \nu_e \equiv \nu_2^m$
$\bar{\nu}$	$(\theta_{13}^m = 0, \theta_{12}^m = 0) \Rightarrow \bar{\nu}_e \equiv \bar{\nu}_1^m$	$(\theta_{13}^m = \pi/2, \theta_{12}^m = \pi/0) \Rightarrow \bar{\nu}_e \equiv \bar{\nu}_3^m$

Normal ordering Crossing Diagram

Neutrino evolution starts on the right

$$\nu_e \equiv \nu_3^m$$

ν'_μ and ν'_τ are linear combinations of ν_μ and ν_τ which diagonalise the 2-3 part of the Hamiltonian

Both the H and L resonances happen for neutrinos in NO, the transition probability being P_H and P_L, respectively

Fluxes for the mass eigenstates at the SN surface can be calculated as a function of the initial fluxes and the transition probabilities at the resonances (rescaled by a factor L^{-2})
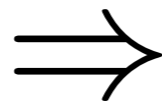

Energy levels

Dighe, Smirnov, hep-ph/9907423. PRD.62.033007

For instance

$$F_{\nu_1} = P_H P_L F_{\nu_3^m}^0 + (1 - P_L) F_{\nu_1^m}^0 + P_L(1 - P_H) F_{\nu_2^m}^0$$
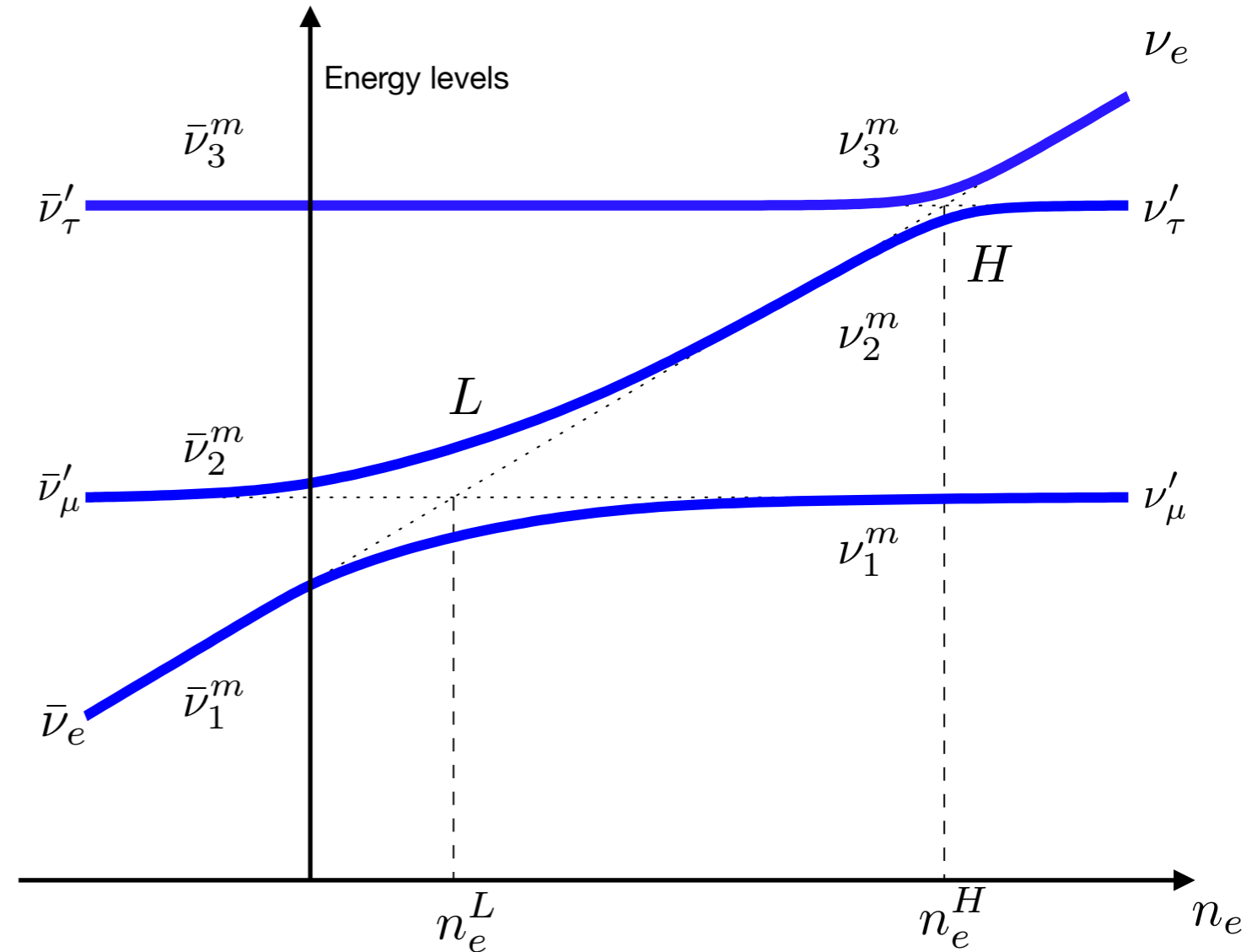
With $\quad F_{\nu_3^m}^0 = F_{\nu_e}^0 \quad$ and $\quad F_{\nu_2^m}^0 = F_{\nu_1^m}^0 = F_{\nu_\mu}^0 = F_{\nu_\tau}^0 = F_{\bar\nu_\mu}^0 = F_{\bar\nu_\tau}^0 = F_{\nu_x}^0 = F_{\bar\nu_x}^0$

But present value of θ_{13} implies adiabatic propagation

$$P_L = P_H = 0$$

$\Longrightarrow$

$$F_{\nu_3} = F_{\nu_3^m} = F_{\nu_e}^0$$
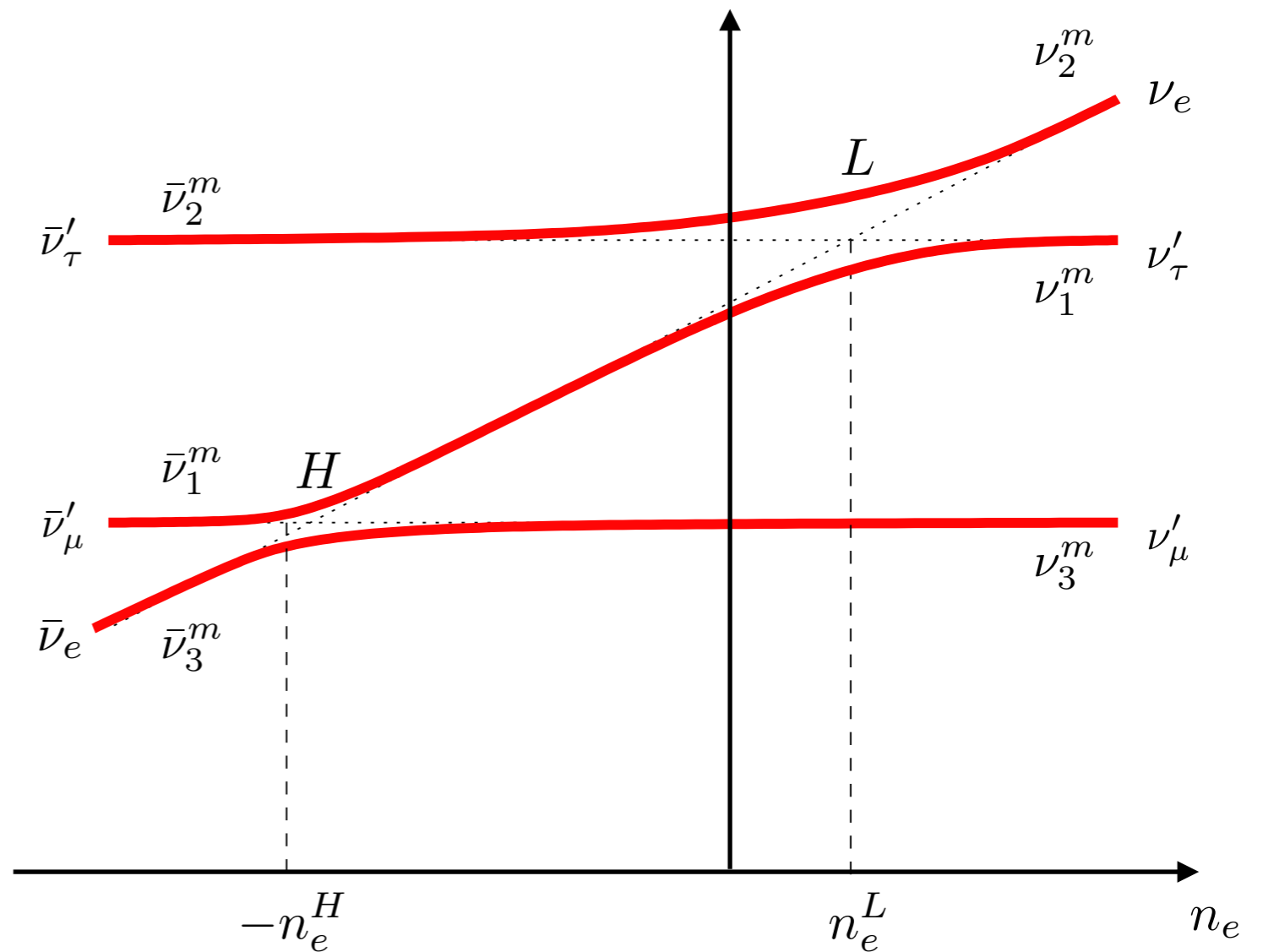$$F_{\nu_1} = F_{\nu_2} = F_{\nu_1^m}^0 = F_{\nu_x}^0$$

Analogously for antineutrinos (no resonances), but starting on the left of the diagram

Inverted ordering Crossing Diagram

Neutrino evolution starts on the right but this time

$$\nu_e \equiv \nu_2^m$$

For IO, L resonance happens for neutrinos and H resonance for antineutrinos (negative electron density)

The fluxes exiting the Supernova are

$$F_{\nu_2} = F_{\nu_2^m}^0 = F_{\nu_e}^0$$

$$F_{\nu_1} = F_{\nu_3} = F_{\nu_1^m}^0 = F_{\nu_3^m}^0 = F_{\nu_x}^0$$

Analogously for antineutrinos, starting on the left of the diagram with the H resonance

After leaving the surface of the Supernova the neutrino mass eigenstates travel to Earth where they arrive (rescaled by a factor L⁻²) so that for NO

$$F_{\nu_e}^E = \sum_i |U_{ei}|^2 F_{\nu_i} = p F_{\nu_e}^0 + (1-p) F_{\nu_x}^0$$

$$p = |U_{e1}|^2 P_H P_L + |U_{e2}|^2 P_H (1 - P_L) + |U_{e3}|^2 (1 - P_H) = |U_{e3}|^2$$

$$|U_{e3}|^2 = \sin^2 \theta_{13} \sim 0.02 \Rightarrow p \sim 0$$

so that $\boxed{F_{\nu_e}^E = F_{\nu_x}^0}$

Analogous simple formulas for antineutrinos and IO. Summarizing

	Normal Ordering	Inverted Ordering
ν	$F_{\nu_e}^E = F_{\nu_x}^0$	$F_{\nu_e}^E = \sin^2 \theta_{12} F_{\nu_e}^0 + \cos^2 \theta_{12} F_{\nu_x}^0$
$\bar{\nu}$	$F_{\bar{\nu}_e}^E = \cos^2 \theta_{12} F_{\bar{\nu}_e}^0 + \sin^2 \theta_{12} F_{\bar{\nu}_x}^0$	$F_{\bar{\nu}_e}^E = F_{\bar{\nu}_x}^0$

After reaching the Earth surface, neutrinos may traverse the Earth matter in their way to the detector depending on the location of the Supernova and on the arrival time

Calculation of the oscillation probability in the Earth matter is analogous to the case of solar neutrinos

Comparison of the supernova signal in two detectors differently shadowed by Earth can reveal matter effect and hence be sensitive to mass ordering (matter effects vanish if initially $F_{\nu_e}^0 = F_{\nu_x}^0$ exactly)

Mass Ordering signatures

40 kton argon, 10 kpc — DUNE-like

374 kton water, 10 kpc — HyperK-like

20 kton scintillator, 10 kpc — JUNO-like

Neutronization → Most robust signature

burst is almost a standard candle

luminosity time dependence almost
    model independent

absent in NO

partially suppressed in IO

collective effects absent*

Early time profile also important
since dominated by MSW
propagation, while collective
effects matter suppressed

K. Scholberg, arXiv:1707.06384, J.Phys. G45 (2018) no.1, 014002
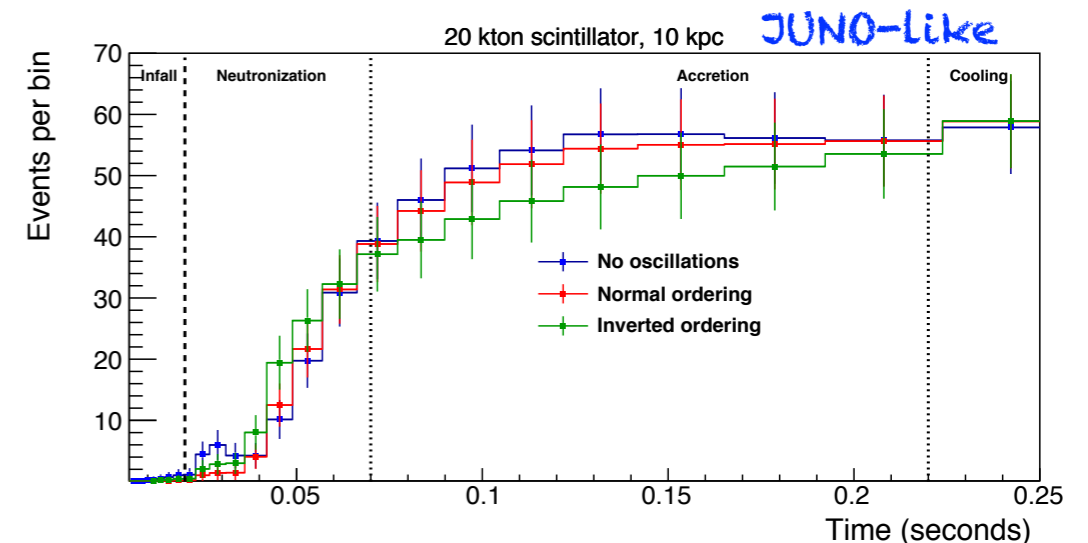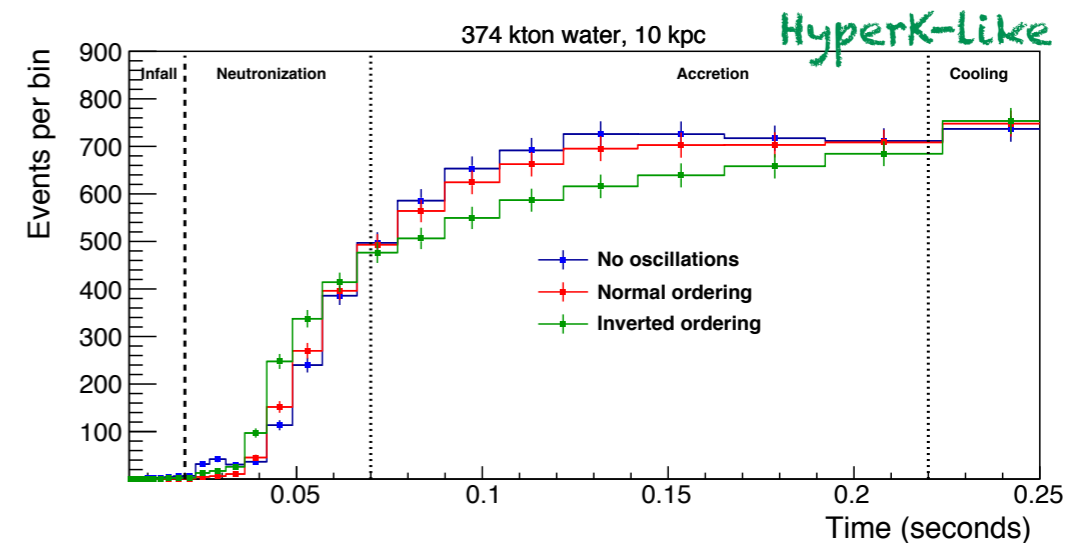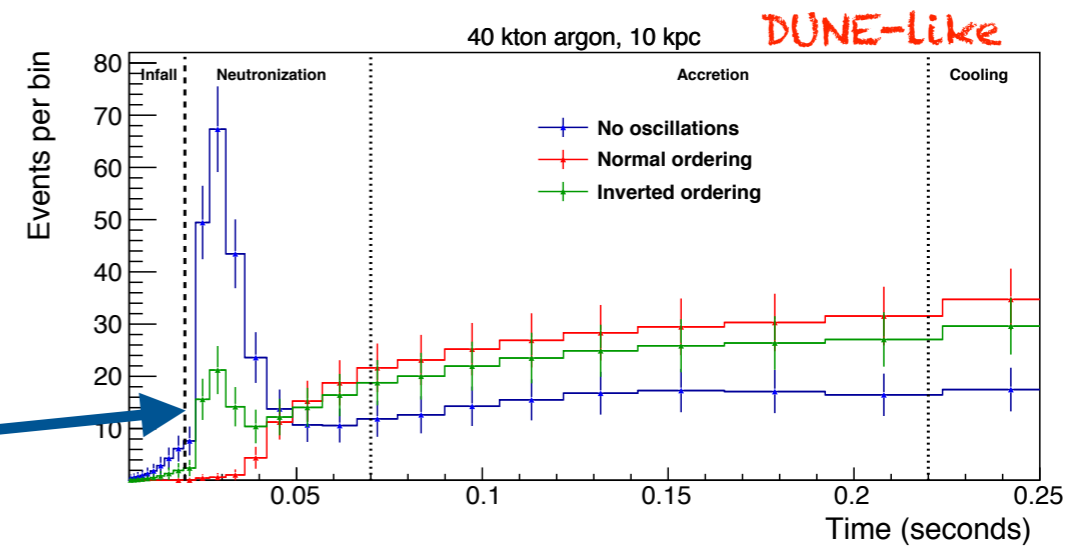
The real picture is complicated by the fact that
- real SN density profile is non monotonic decreasing at the shock front
- the SN density profile changes with time
- effect of density fluctuations should be taken into account

At the shock front the H resonance can be extremely non-adiabatic

Stochastic matter fluctuations of sufficiently large amplitude may suppress flavor conversions and lead to $P_H = 1/2$ when the suppression is strong
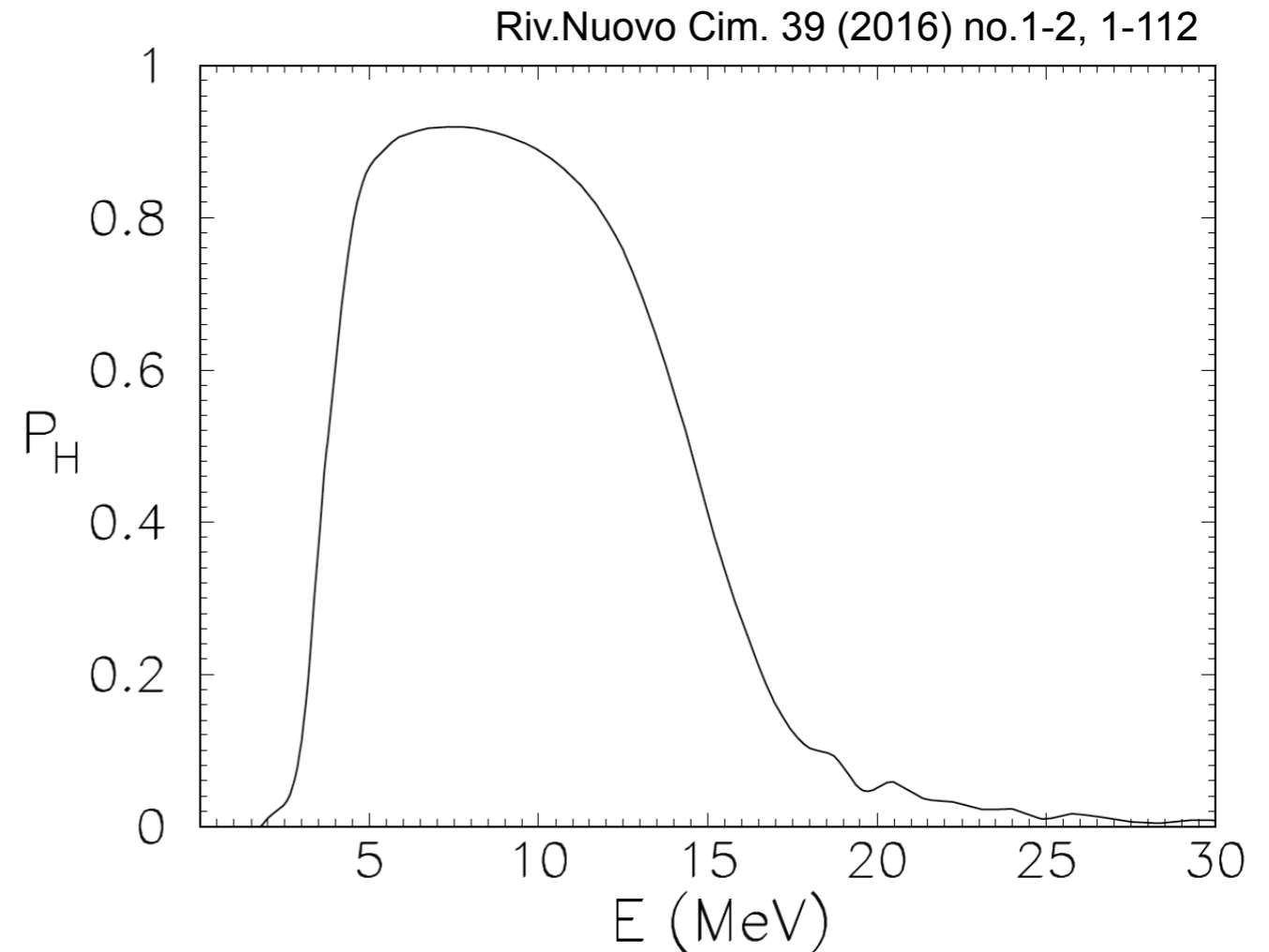
Spectral properties of the fluctuations very important for understanding the neutrino signal

At the moment there is no unanimous consensus about the impact of matter fluctuations on the SN neutrino flavor conversions

"Slow" collective neutrino conversions

The formalism of the neutrino density matrix is particularly useful in the context of SN neutrino flavor conversions

$$\partial_t \rho_{\mathbf{p},\mathbf{x},t} + \mathbf{v_p} \cdot \nabla_{\mathbf{x}} \rho_{\mathbf{p},\mathbf{x},t} = -i[\Omega_{\mathbf{p},\mathbf{x},t}, \rho_{\mathbf{p},\mathbf{x},t}]$$

The Hamiltonian is the sum of three terms depending on

$$\Omega_{\mathbf{p},\mathbf{x},t} = \Omega_{\mathrm{vac}} + \Omega_{\mathrm{MSW}} + \Omega_{\nu\nu}$$

$$\omega = \frac{\Delta m^2}{2E}$$

vacuum oscillation
frequency

$$\lambda = \sqrt{2} G_F n_e$$

matter potential

$$\mu = \sqrt{2} G_F (n_\nu + n_{\bar{\nu}})$$

neutrino-neutrino
interaction potential

$$\Omega_{\nu\nu} = \sqrt{2} G_F \int \frac{d^3 \mathbf{q}}{(2\pi)^3} (\rho_{\mathbf{q}} - \bar{\rho}_{\mathbf{q}})(1 - \mathbf{v_p} \cdot \mathbf{v_q})$$

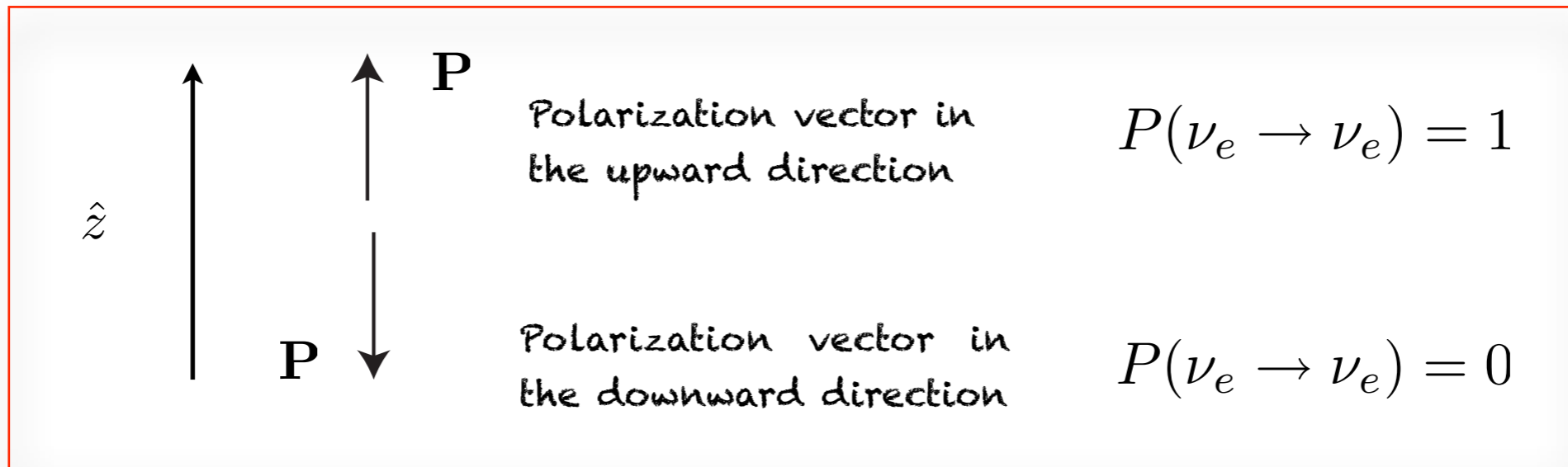Multi-angle effect: the interaction depends on the
relative angle of the colliding neutrinos $\theta_{\mathbf{pq}}$

Collective oscillations when μ dominates (typically $r \lesssim 100 \ \mathrm{Km}$)

Tipically matter effects and collective effects induced by self interactions factorize and the range in which they are effective are well separated

ρ decomposed in term of polarization vectors

$$\rho = \frac{1}{2}(p_0 I + \mathbf{P} \cdot \sigma)$$

$\mathbf{P} = \mathbf{P}(E, \theta_0)$ neutrinos
$\bar{\mathbf{P}} = \bar{\mathbf{P}}(E, \theta_0)$ antineutrinos

$\hat{z}$

P Polarization vector in the upward direction $\qquad P(\nu_e \to \nu_e) = 1$

P Polarization vector in the downward direction $\qquad P(\nu_e \to \nu_e) = 0$

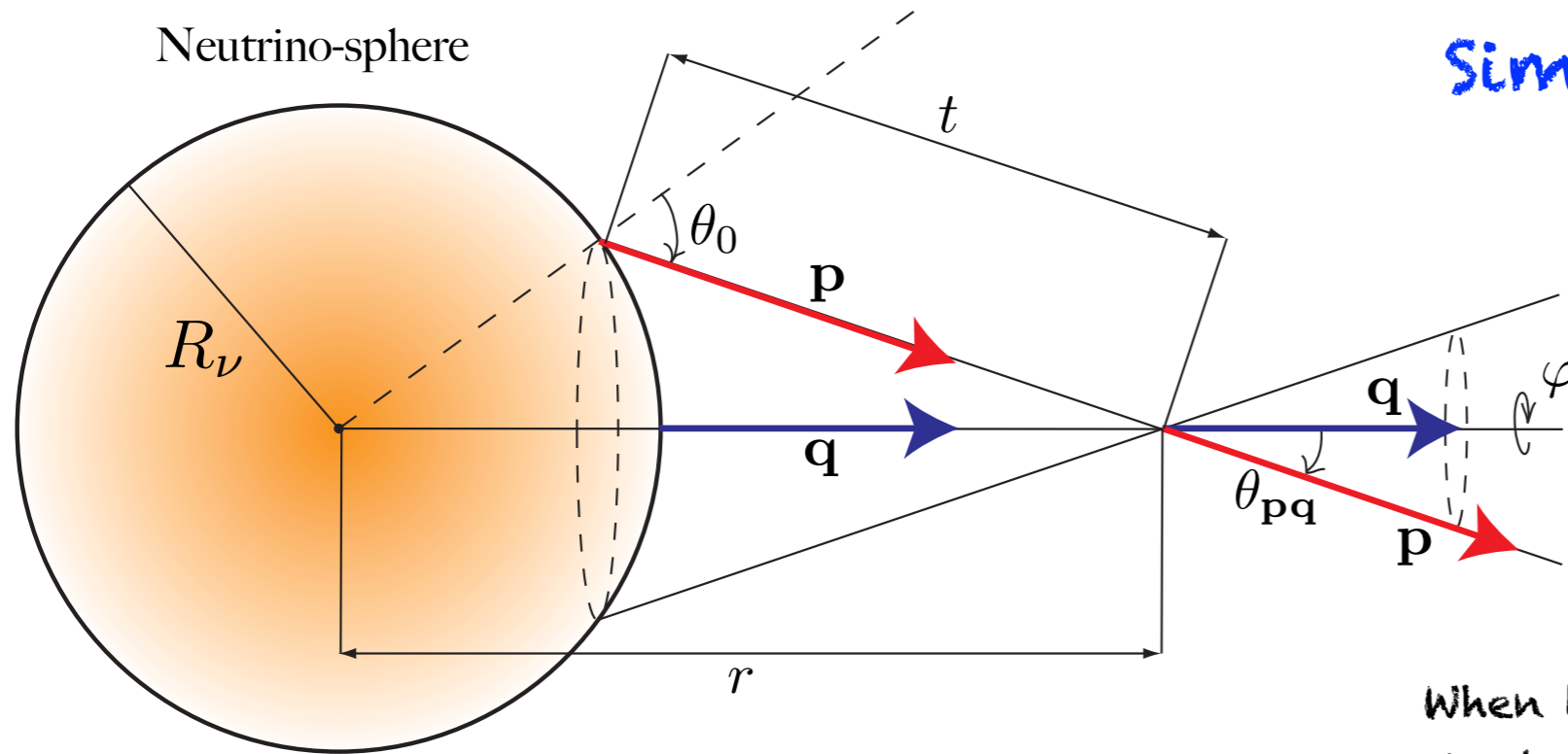Also important, the global vectors

$$\mathbf{J} = \int dE \ d\theta_0 \ \mathbf{P}(E, \theta_0) \quad \bar{\mathbf{J}} = \int dE \ d\theta_0 \ \bar{\mathbf{P}}(E, \theta_0) \quad \mathbf{S} = \mathbf{J} + \bar{\mathbf{J}} \quad \mathbf{D} = \mathbf{J} - \bar{\mathbf{J}}$$

In particular from the EoM the lepton number conservation follows

$$D_z = \int dE d\theta_0 (n_{\nu_e}(E, \theta_0) - n_{\bar{\nu}_e}(E, \theta_0)) = \text{const}$$

implying transitions of the kind $\qquad \nu_e \bar{\nu}_e \to \nu_x \bar{\nu}_x$

Neutrino-sphere

R_ν

θ_0

t

p

q

r

Simple geometric model

Bulb model

Duan et al., PRD74,105014(2006)

q

φ

$\theta_{\mathbf{pq}}$

p

When this angle is averaged out the
single-angle approximation is obtained

$$H_{\nu\nu} = \sqrt{2}G_F \int \frac{d^3\vec{q}}{(2\pi)^3}(\mathbf{P}_{\vec{q}} - \overline{\mathbf{P}}_{\vec{q}})(1 - \cos\theta_{pq}) \longrightarrow H_{\nu\nu} = \mu \int dq(\mathbf{P}_{\vec{q}} - \overline{\mathbf{P}}_{\vec{q}}) = \mu(\mathbf{J} - \overline{\mathbf{J}}) = \mu\mathbf{D}$$

Equations of motion

$$\dot{\mathbf{P}} = (+\omega\mathbf{B} + \lambda\hat{\mathbf{z}} + \mu\mathbf{D}) \times \mathbf{P}$$

$$\dot{\overline{\mathbf{P}}} = (-\omega\mathbf{B} + \lambda\hat{\mathbf{z}} + \mu\mathbf{D}) \times \overline{\mathbf{P}}$$

$$\mathbf{B} \parallel \hat{\mathbf{z}} \quad \text{when} \quad \theta_{13} = 0 \qquad (\lambda = 0 \text{ in the following})$$

Regimes of Collective flavor Conversions

Near the neutrino-sphere (few tens of kilometers) all polarization vectors stay aligned with the z-axis: synchronized oscillations

At a certain point, the polarization vectors start to move but the **P**'s remain (approximately) parallel to their sum **J** (same for antineutrinos). This regime has a mechanical analogy with the motion of a spherical pendulum and corresponds to the so called bipolar oscillations

Hannestad, Raffelt, Sigl and Wong, PRD74,105010(2006)
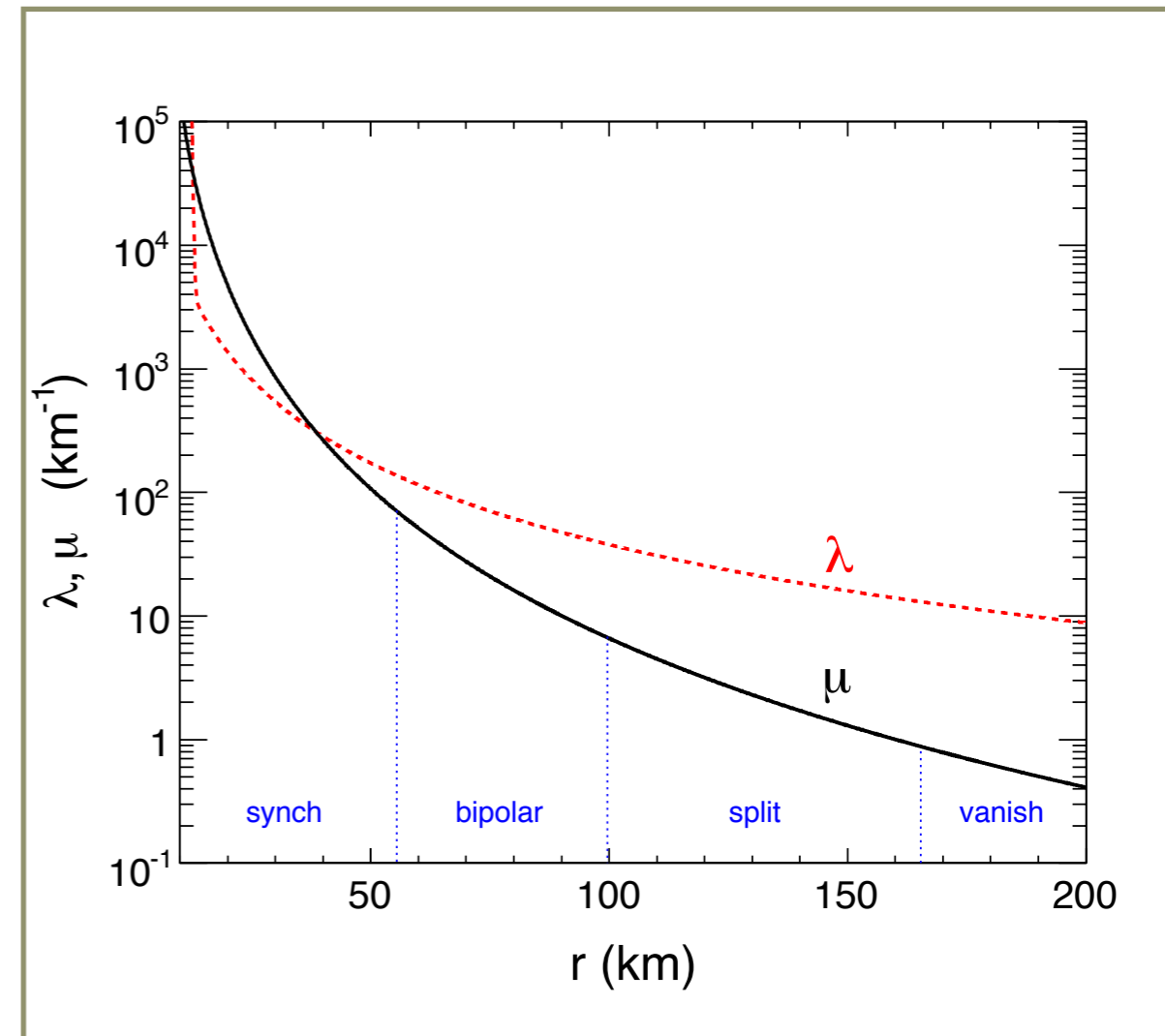
IO corresponds to the pendulum starting close the unstable position while in NO it starts close the stable one

The bipolar regime ends when the vacuum frequencies of the **P**'s are of the same order of the self-interaction potential. After that, the spectral split fully develops until the neutrino-neutrino potential is completely negligible

Initial neutrino and antineutrino fluxes
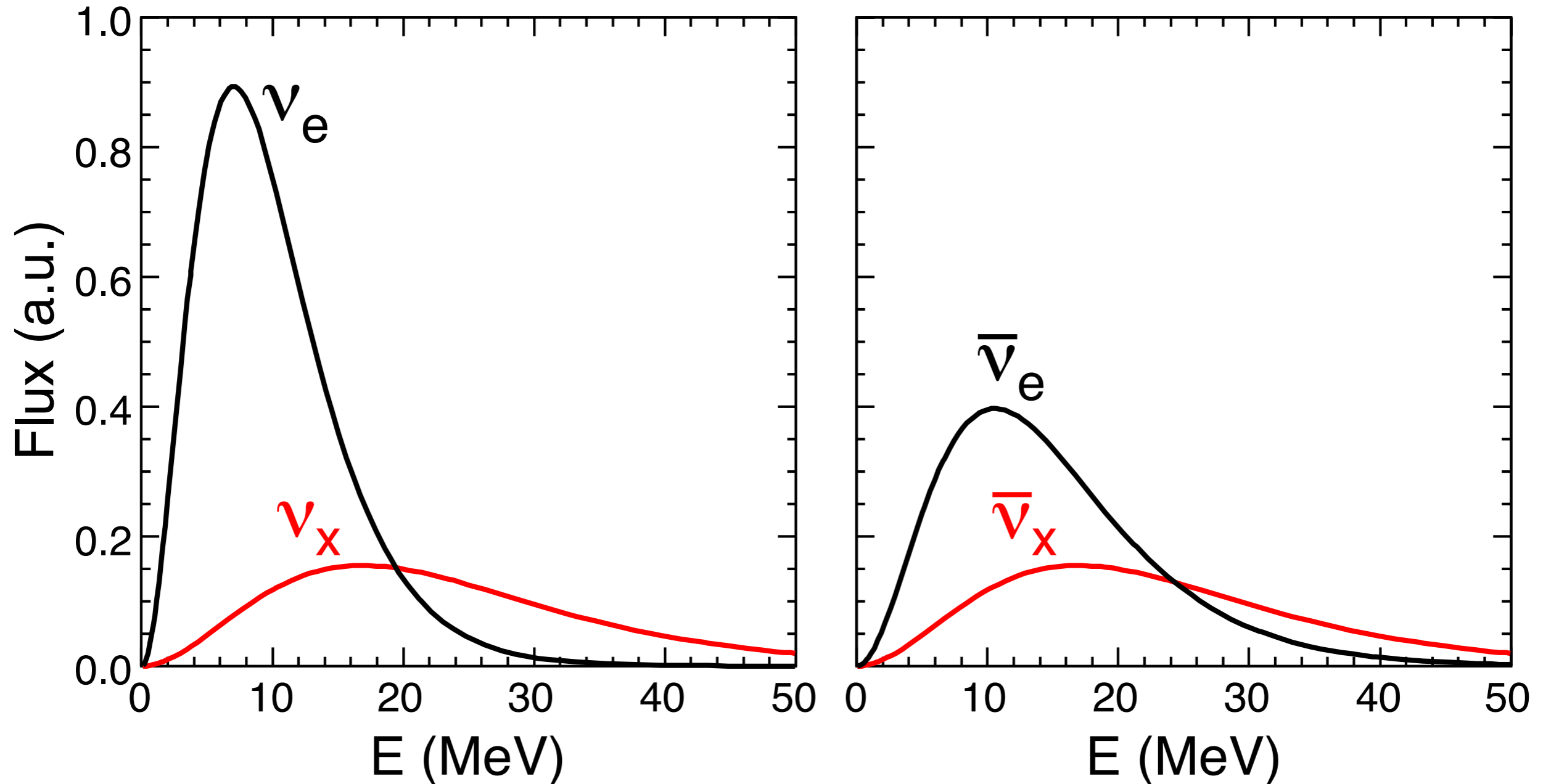
Two-neutrino scenario

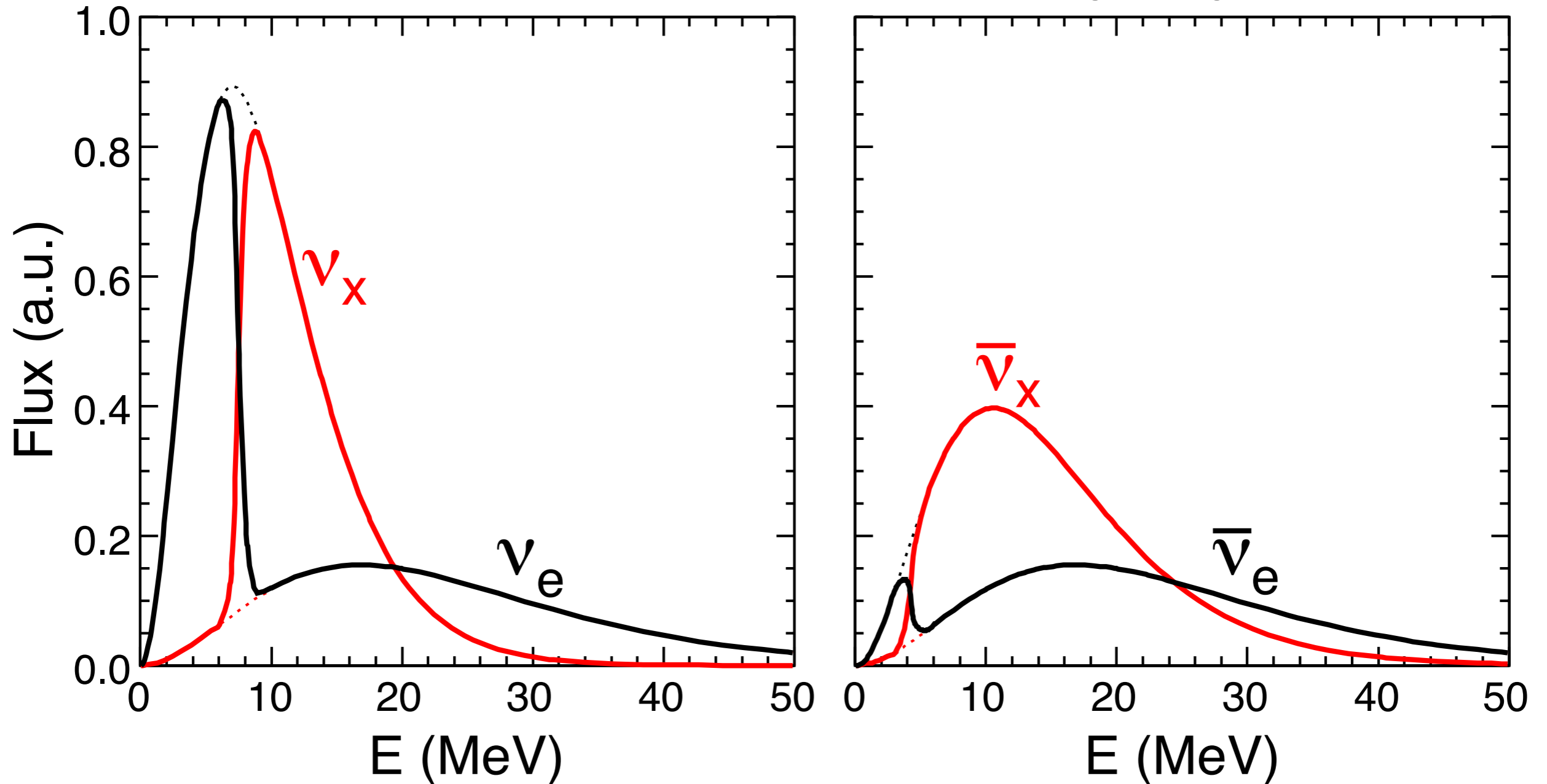$$\Delta m^2 = \Delta m^2_{atm} = 2 \times 10^{-3} \text{ eV}^2$$

$$\sin^2 \theta_{13} = 10^{-2}$$

$$\langle E_{\nu_e} \rangle = 10 \text{ MeV}$$

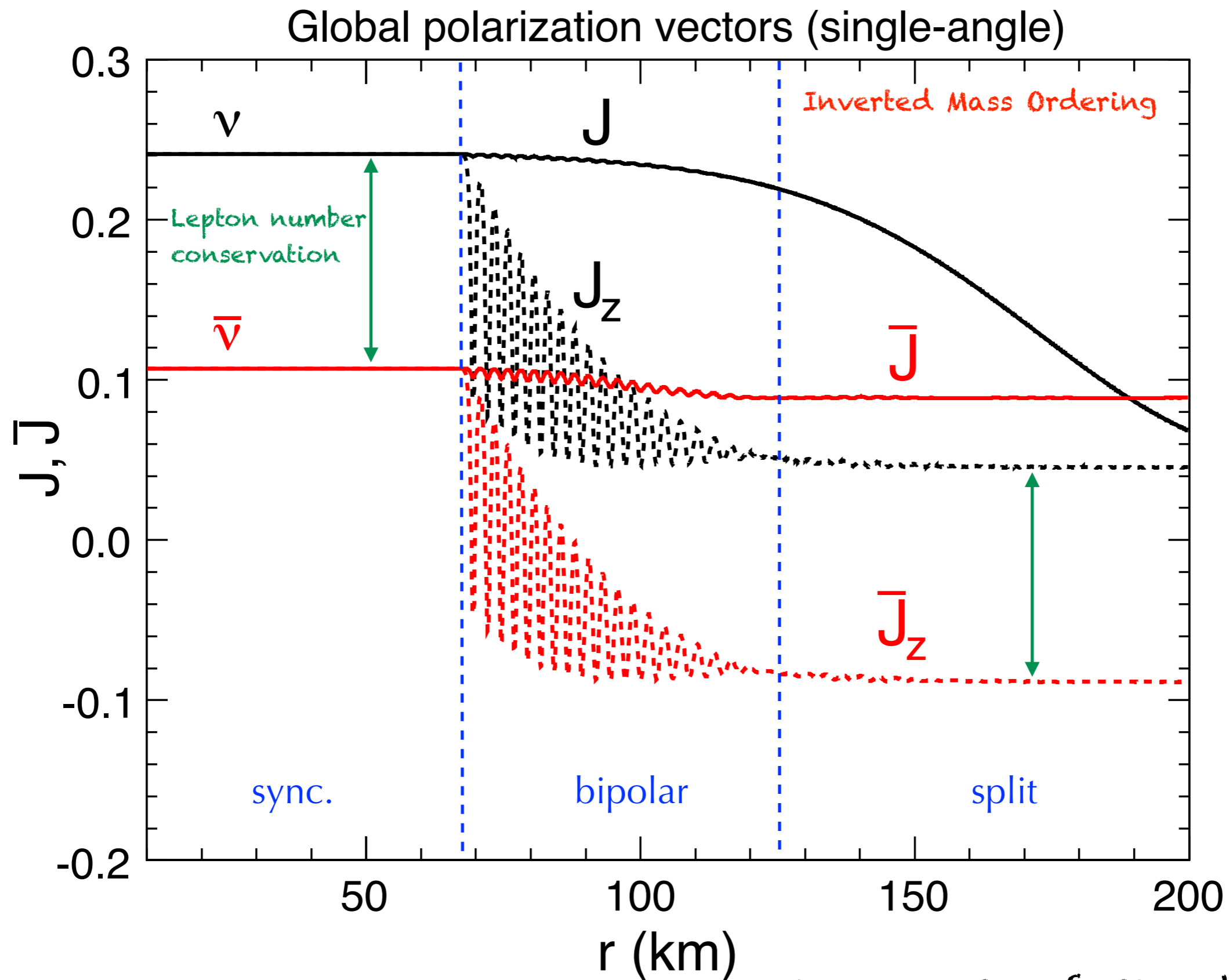$$\langle E_{\bar{\nu}_e} \rangle = 15 \text{ MeV}$$

$$\langle E_{\nu_x} \rangle = \langle E_{\bar{\nu}_x} \rangle = 24 \text{ MeV}$$

Final fluxes in inverted hierarchy (single-angle)

Spectral split for neutrinos above ~7 MeV as a consequence of lepton number conservation

Spectral split for antineutrinos at ~4 MeV

Global polarization vectors (single-angle)

Note the inversion of $\bar{J}_z$ and the partial inversion of J_z

The onset of the bipolar regime depends on θ_{13} and on the matter potential

Global polarization vectors (multi-angle)
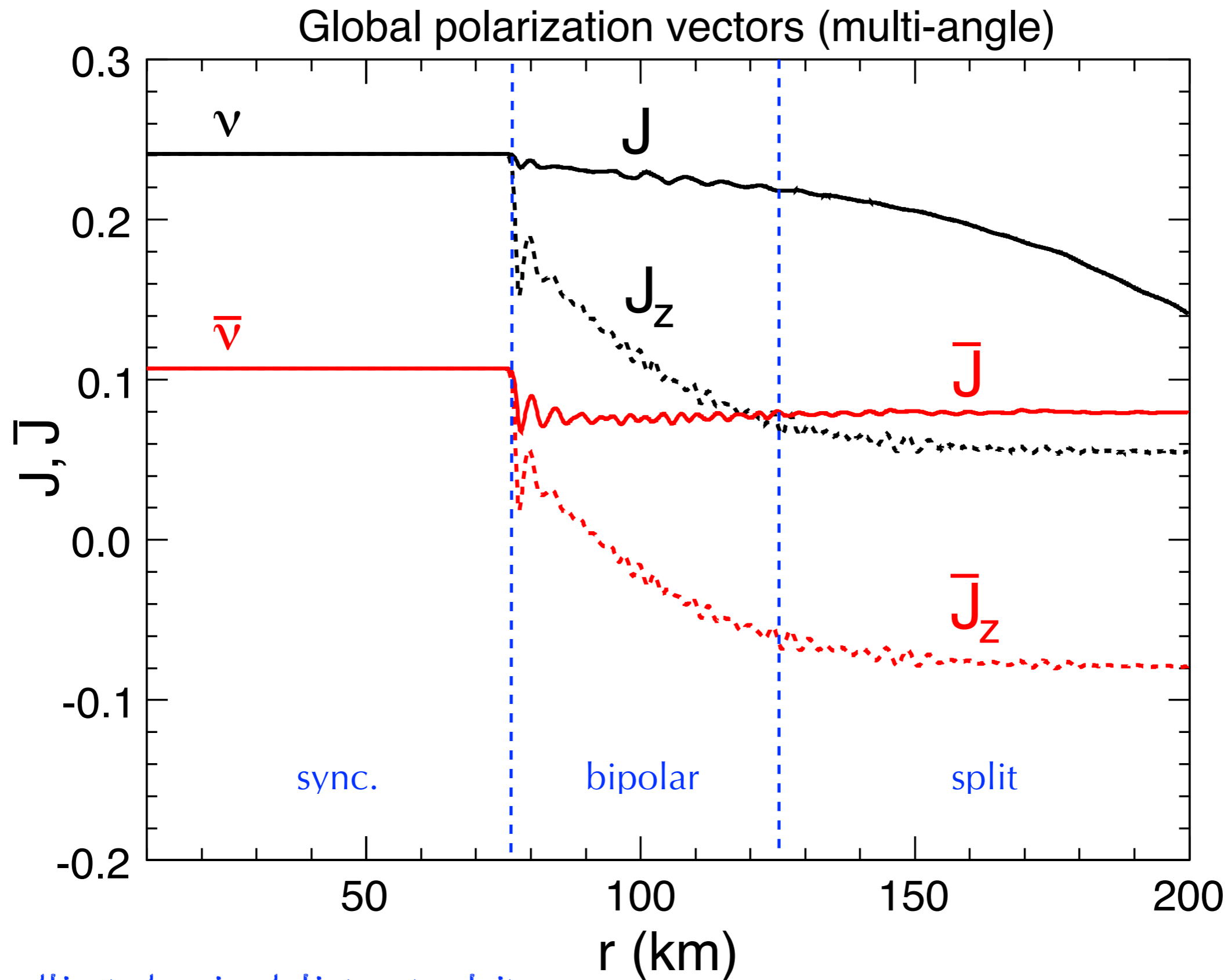
In multi-angle simulations, neutrino-neutrino angles can be larger than the (single-angle) average one, leading to somewhat stronger self-interaction effects

Bipolar regime starts later
More pronounced depolarization of J and prolonged coherence of $\bar{J}$

Final fluxes in inverted hierarchy (multi-angle)
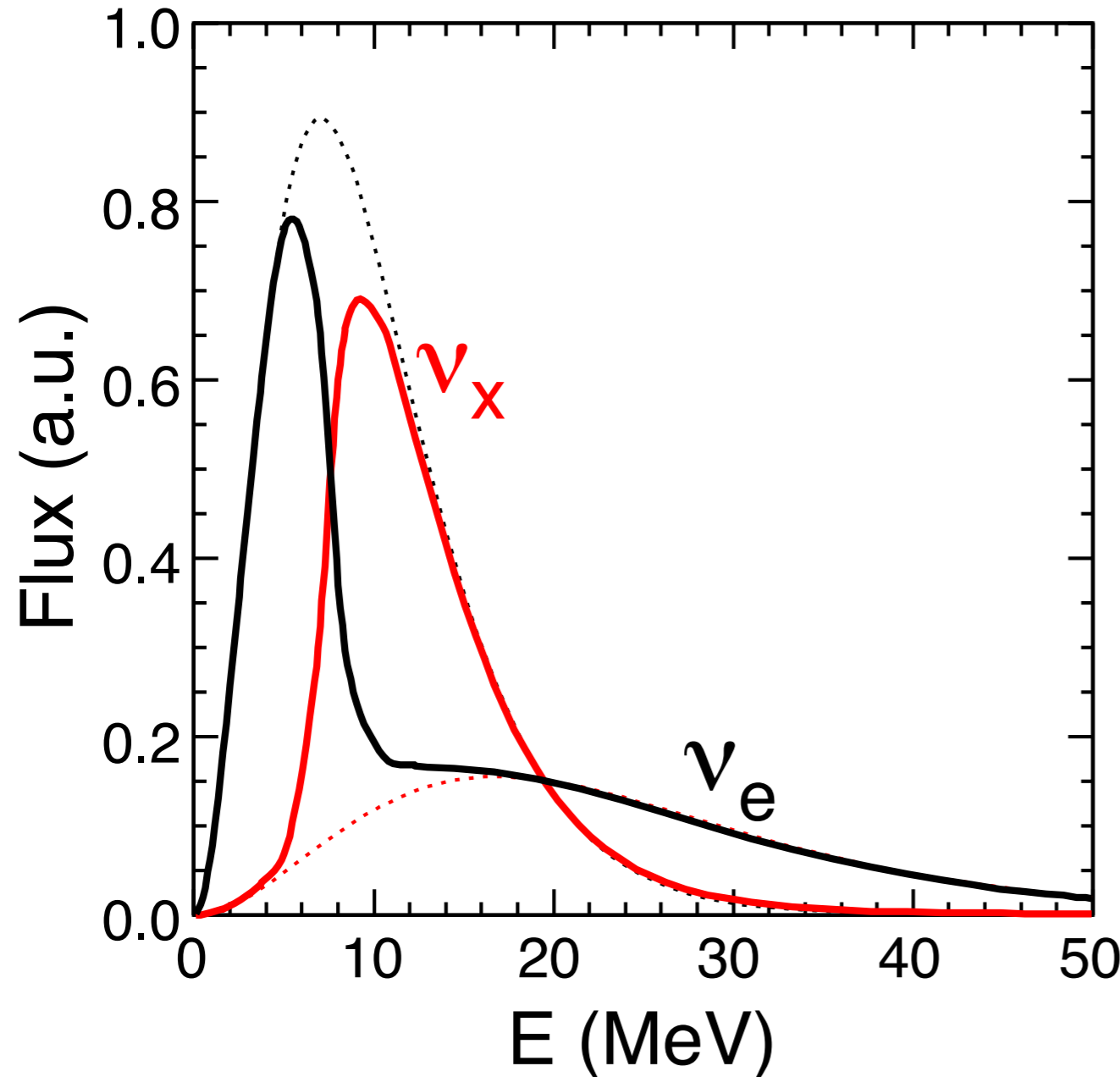
The neutrino spectral split is evident, although less sharp than in the single-angle case
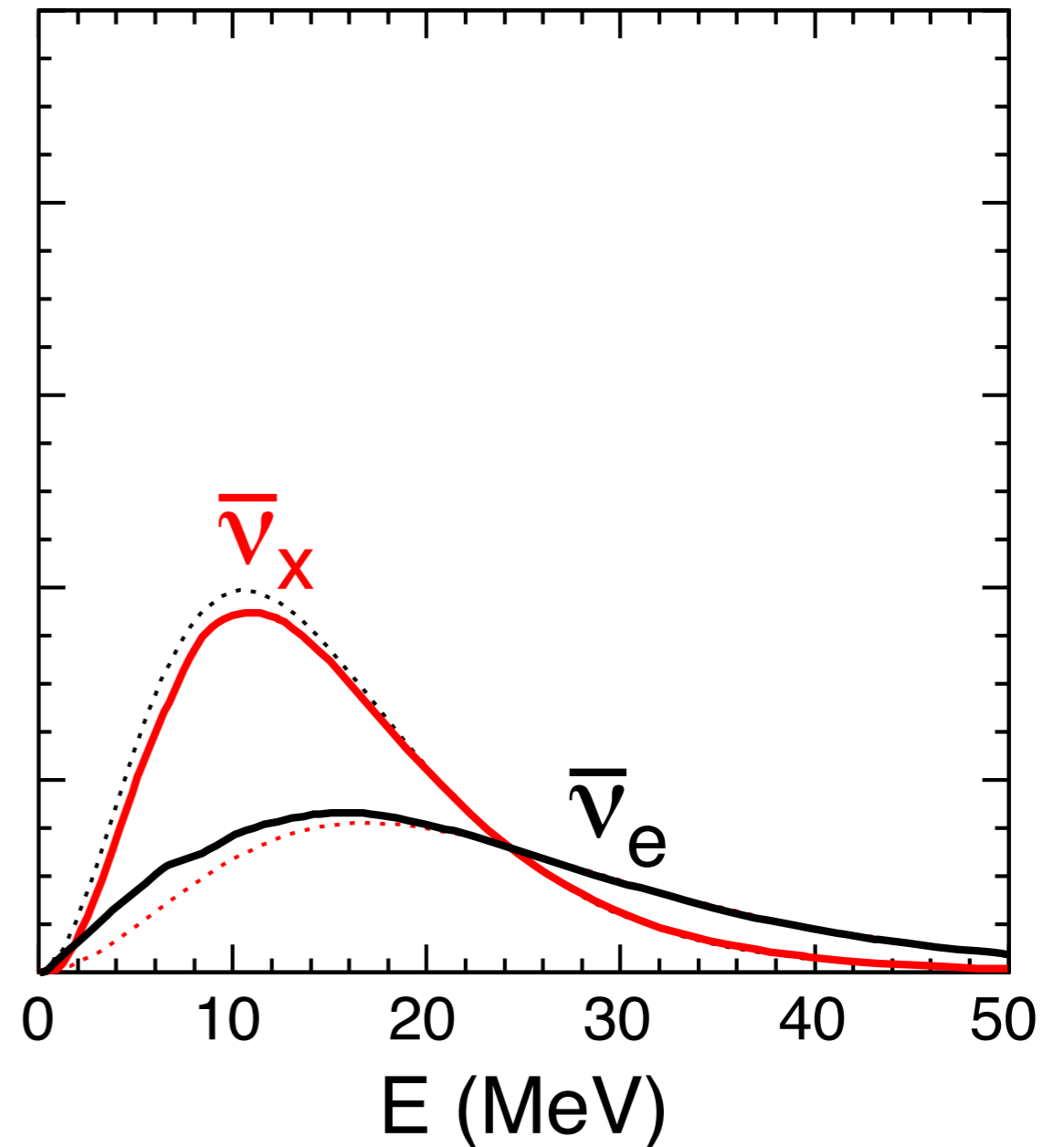
Antineutrino split largely washed out

Starting from the simplest single-angle approximation with the three phases of flavor conversions for IO, induced by self interactions (synchronization, bipolar oscillations, spectral swaps), the situation gets more complicated when moving towards more realistic scenarios:

- multi-angle effects tend to smear spectral splittings
- matter multi-angle effects tend to block self-induced flavor conversions
- breaking of the space-time symmetries could favour flavor decoherence
- collective effects depend on the neutrino flux hierarchy and less pronounced flavor hierarchies multiple splits can arise (and swaps can occur also in NO)

Multi-angle matter effects

$n_{e^-} - n_{e^+} \ll n_{\bar{\nu}_e} - n_{\bar{\nu}_x}$ subdominant

$n_{e^-} - n_{e^+} \gg n_{\bar{\nu}_e} - n_{\bar{\nu}_x}$ can inhibit self-induced flavor conventions

$n_{e^-} - n_{e^+} \sim n_{\bar{\nu}_e} - n_{\bar{\nu}_x}$ matter-induced multi-angle decoherence may occur

Multi-azimuthal-angle instability, depending on spectral crossings, may trigger new flavor conversions in NO, especially during the accretion phase, but are suppressed by by the dominant matter term

Time and/or space inhomogeneities may lead to flavor instabilities

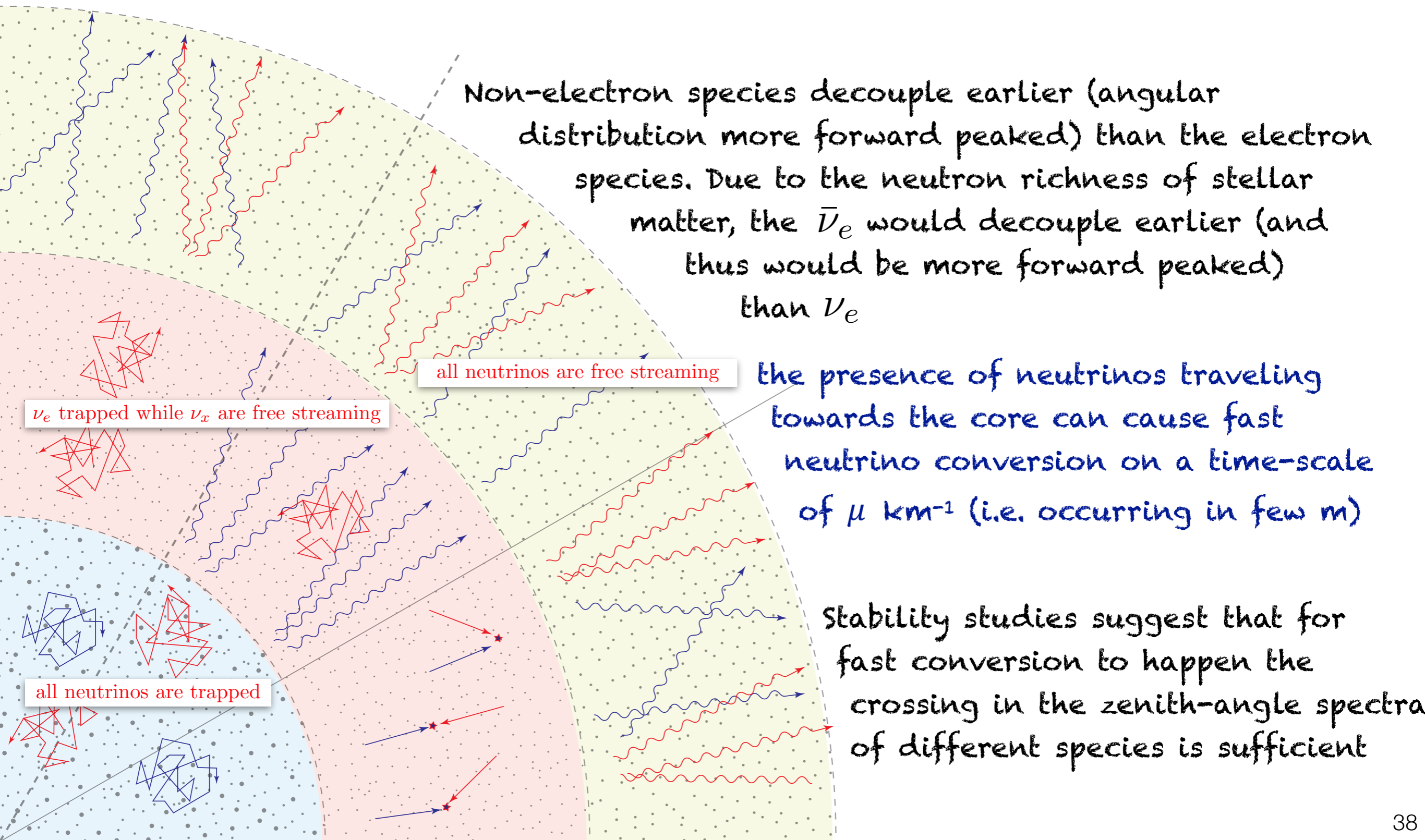# Collective effects depend on the neutrino flux hierarchy

During the neutronization phase bipolar flavor conversions not possible

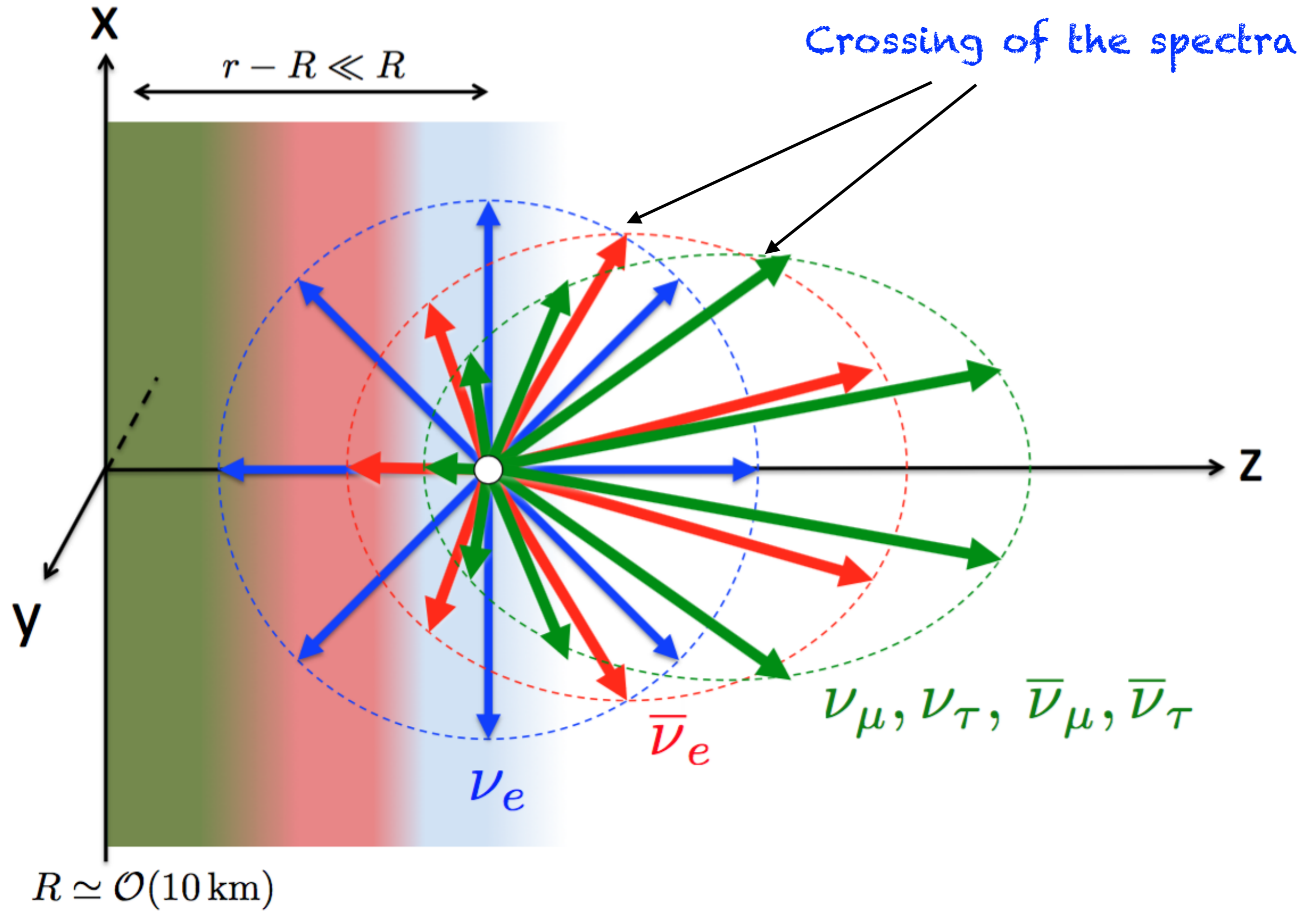$\nu_e \bar{\nu}_e \to \nu_x \bar{\nu}_x$ transitions cannot occur because $F_{\nu_e} \gg F_{\nu_x} \gg F_{\bar{\nu}_e}$

During the accretion phase the deleptonization of the core implies $F_{\nu_e} \gg F_{\bar{\nu}_e}$ while for the absence of CC interactions for μ and τ neutrinos $F_{\bar{\nu}_e} \gg F_{\bar{\nu}_x}$ Bipolar oscillations and spectral swaps can occur. Multi-angle matter effects tend to inhibit self-induced flavor conversions

During the cooling phase, with less pronounced or vanishing neutrino flux hierarchy multiple spectral splits can appear both for neutrinos and antineutrinos. Three-flavor effects are observable in the single-angle scheme (suppressed in the multi-angle case). Spectral swaps and splits are less pronounced, due to some amount of multi-angle decoherence. For the flux ordering of the cooling phase spectral splits and swaps would occur also in NO.

"Fast" collective neutrino conversions

Refining the simple bulb model requires also taking into account that the radius of the neutrinospheres of different neutrino flavor are different

Non-electron species decouple earlier (angular distribution more forward peaked) than the electron species. Due to the neutron richness of stellar matter, the $\bar{\nu}_e$ would decouple earlier (and thus would be more forward peaked) than ν_e

all neutrinos are free streaming

ν_e trapped while ν_x are free streaming

all neutrinos are trapped

the presence of neutrinos traveling towards the core can cause fast neutrino conversion on a time-scale of μ km^{-1} (i.e. occurring in few m)

Stability studies suggest that for fast conversion to happen the crossing in the zenith-angle spectra of different species is sufficient

From B. Dasgupta (Neutrino 2018)

Possible strategy to classify instabilities leading to fast flavor conversion

Linearize the EoM $\quad \partial_t \rho_{\mathbf{p},\mathbf{x},t} + \mathbf{v_p} \cdot \nabla_{\mathbf{x}} \rho_{\mathbf{p},\mathbf{x},t} = -i[\Omega_{\mathbf{p},\mathbf{x},t}, \rho_{\mathbf{p},\mathbf{x},t}]$

Look for wavelike solutions → dispersion relation in the (z,t) conjugate variables (ω,k)

Characterize the behaviour of the solution from the solutions of the dispersion relation

Example in one dimension: S(z,t) is the non diagonal element of the density matrix. When S goes to infinity the the oscillation probability goes to 1/2 → decoherence. two type of possible instabilities

$|S|$ Absolute

time z

$|S|$ Convective

time z
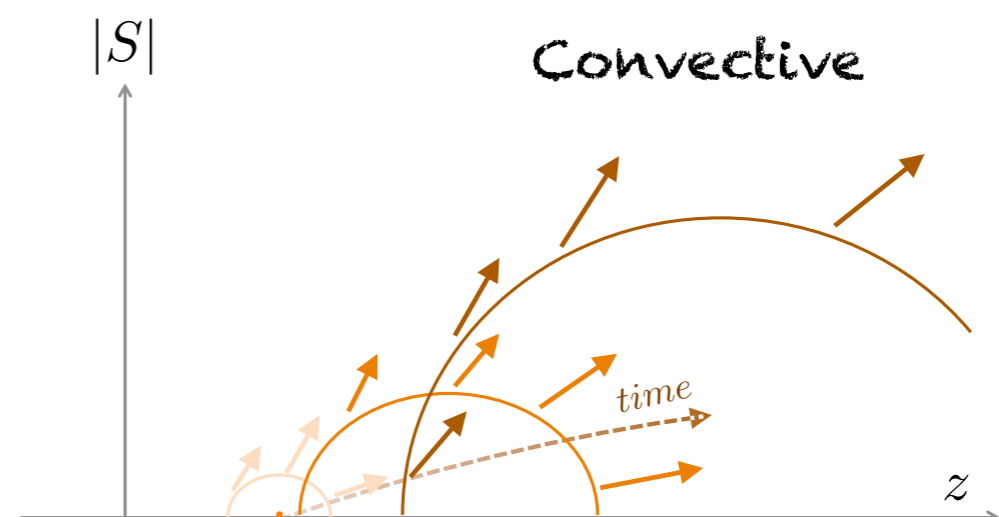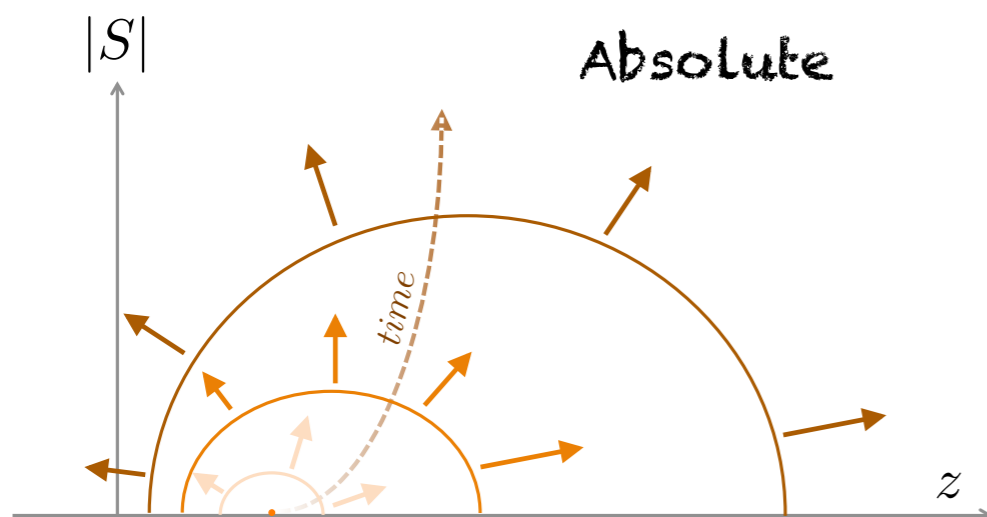
Two-beam neutrino model: two neutrino beams with velocity are v_1 and v_2 and coupling strength ε

It can be shown that in this simple toy model, only two real functions, $f_1(z,t)$ and $f_2(z,t)$, are sufficient to characterize the real and imaginary part of the off-diagonal terms of the density matrix for the two neutrino modes. The two functions can be calculated from a set of two nonlinear coupled differential equations

To graphically check if the two functions remain finite or explode, we initially launch two wave packets with two different velocities and same shape. We find two stable and two unstable solutions and we obtain the following classification, in accordance with the study of the dispersion relation

Stable cases		Unstable cases	
$v_1 v_2 > 0$	$v_1 v_2 < 0$	$v_1 v_2 > 0$	$v_1 v_2 < 0$
$\epsilon > 0$	$\epsilon > 0$	$\epsilon < 0$	$\epsilon < 0$
	damped	convective	absolute

In general, by studying the dispersion equation $D(\omega,k)=0$ one finds for plane wave solutions that: 1) if ω is real for all real k and vice versa the system is completely stable; 2) if ω is real for all real k, but k is complex for some real ω, the system is both stable and damped; 3) if k is complex for some real ω and ω is also complex for some real k, a convective instability arises; 4) if k is real for all real ω and ω is complex for some real k, the instability is absolute

Stable cases
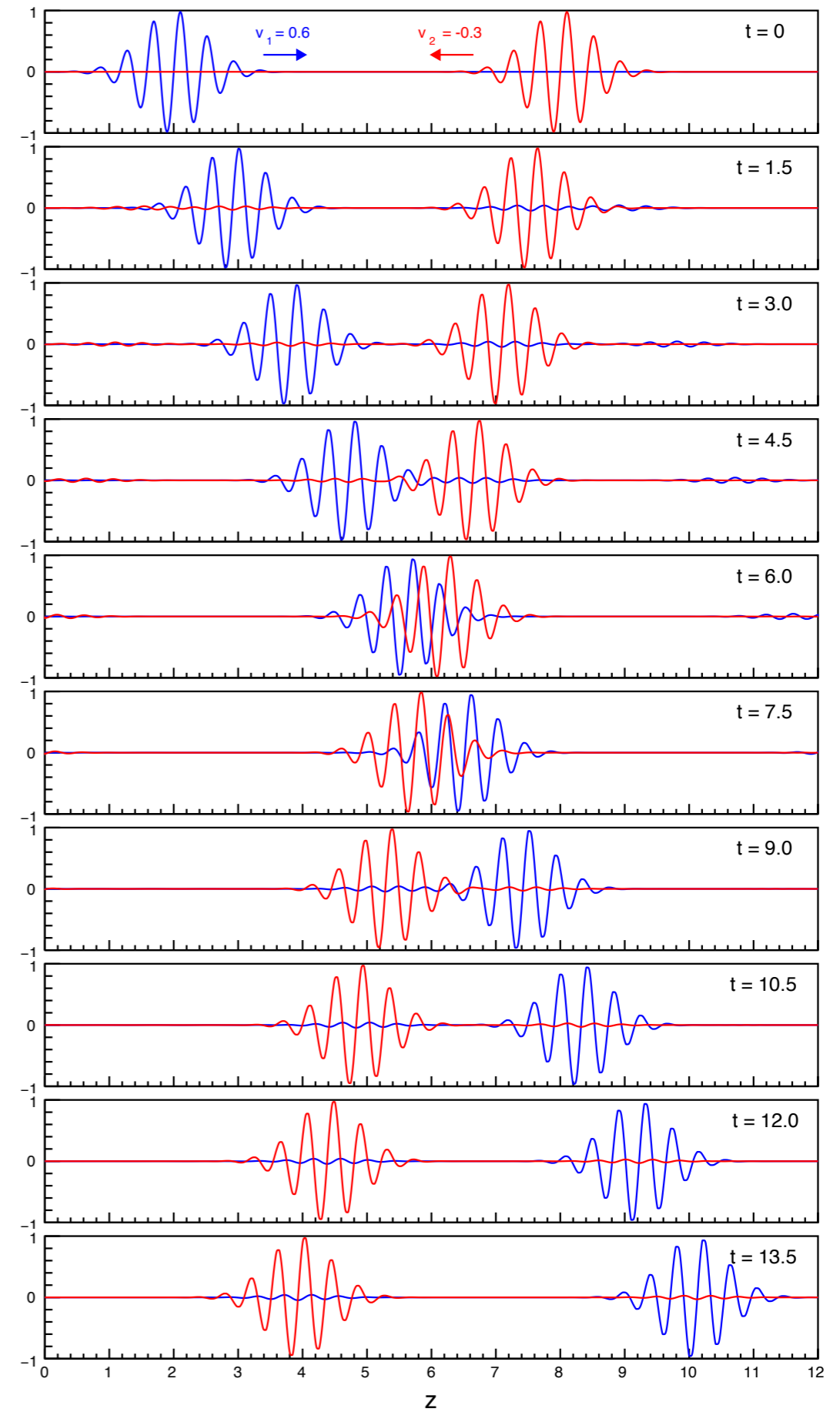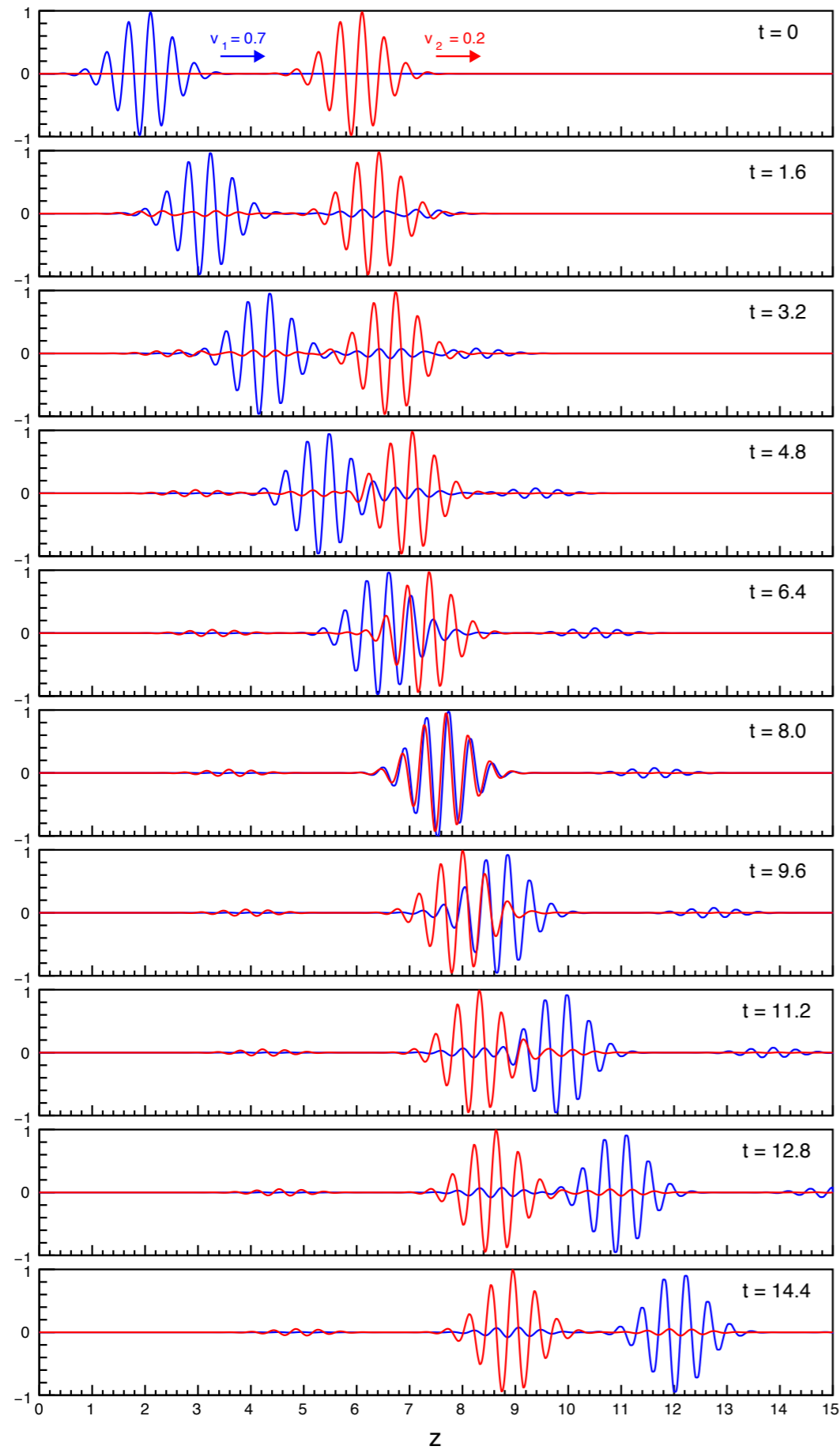
Unstable cases
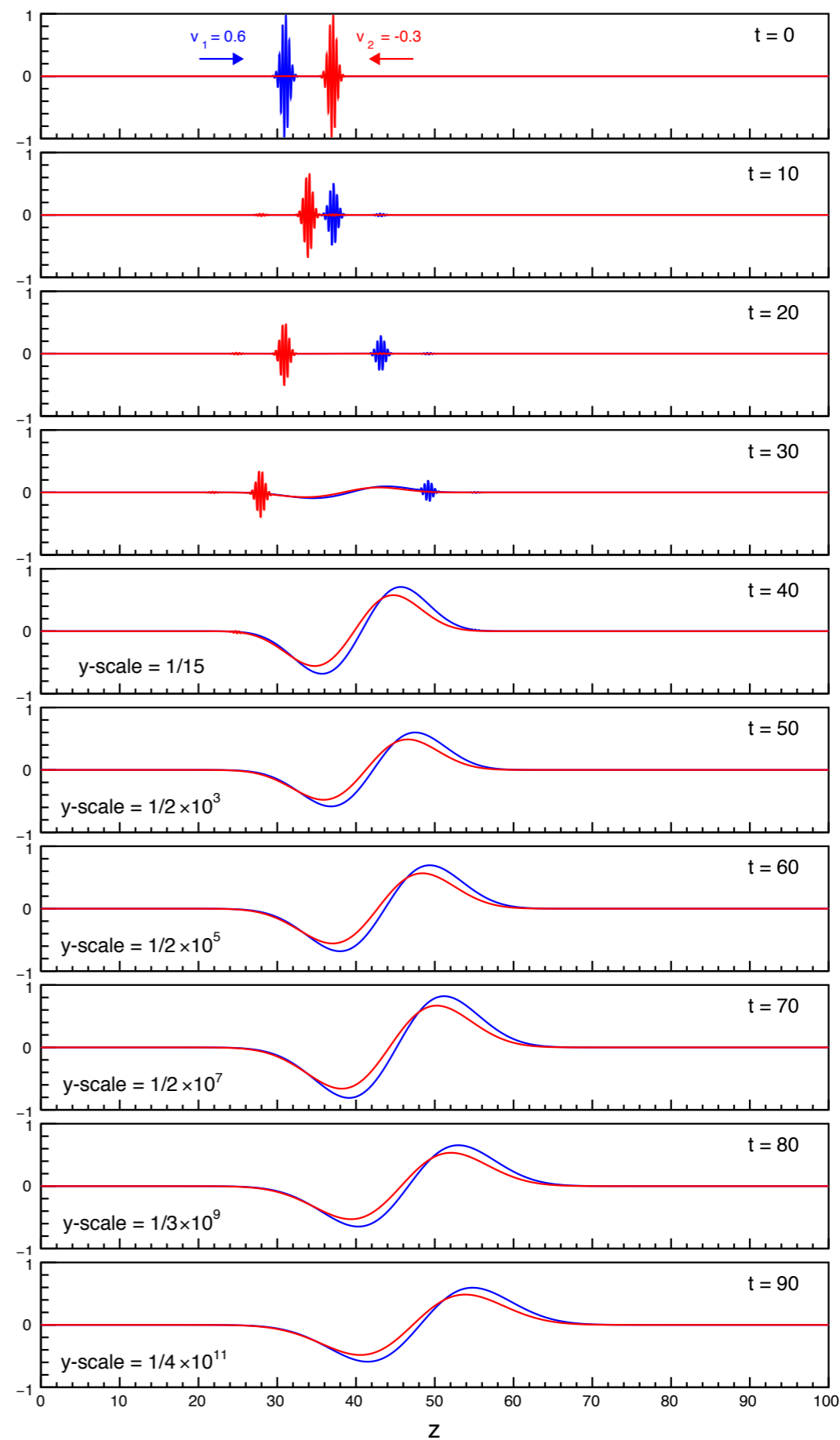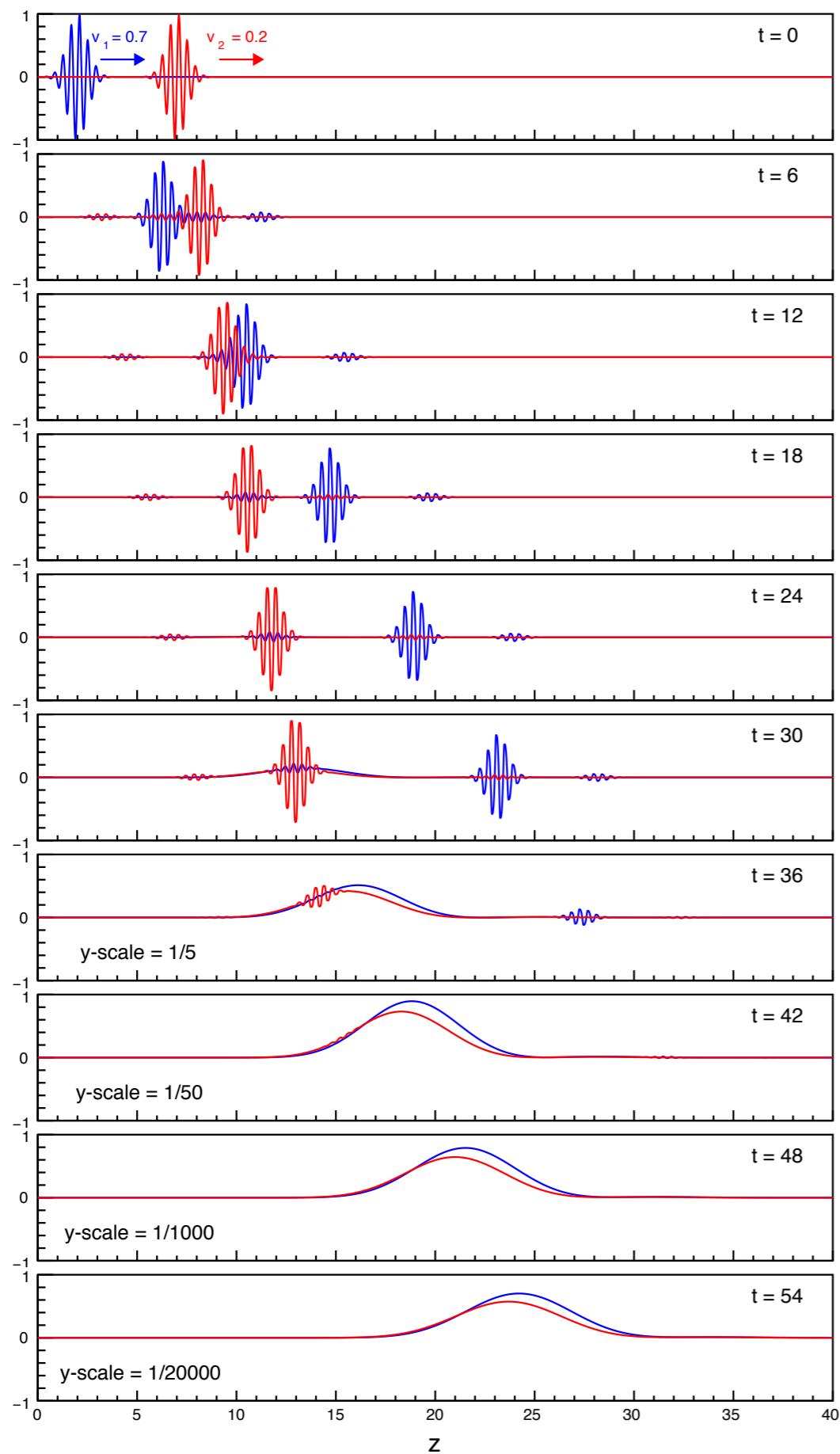
The results obtained for the simple two-beam neutrino model represent a basis to attempt extensions to more general angular spectra expected in a realistic SN. In order to have an instability one needs a crossing in the angular electron number distributions. Conversely, without crossing one gets either a completely stable evolution (if $v_1 v_2 > 0$) or at most a damped stable one (if $v_1 v_2 < 0$)

For several spherically symmetric (1D) supernova simulations, the ELN near the neutrino-sphere has backward going modes but still does not show any crossing (no instability)

One cannot exclude that things may change in 3D models, for example in the presence of LESA (Lepton-Emission Self-sustained Asymmetry). It is therefore conceivable that, especially in the regions where the ELN changes its sign, crossings in the ELN angular distributions may occur

The phenomenology of self-induced flavor conversions in SNe could be much richer than previously expected. One might have that fast conversions could lead to a quick flavor equilibration among different neutrino species, if instabilities are general enough. If flavor equilibration were complete, further oscillation effects would be ineffective. Otherwise, one could characterize different regimes, e.g., fast conversions near SN core followed by spatial slow conversions at larger distances, and finally MSW evolution.

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Conclusions

- Unknowns: θ_{23} octant, δ, MO

  - Knowledge of mass-mixing parameter will help to understand SN physics

- SN neutrino signal can help discriminate Mass Ordering through

  Matter MSW propagation

  "Slow" Collective conversion

  "Fast" Collective conversion