

# Double heavy tetraquark resonances in lattice QCD

Pedro Bicudo\*

*Marco Cardoso\**, *Antje Peters\*\*\**, *Martin Pflaumer\*\** and *Marc Wagner\*\**

\* CeFEMA, Instituto Superior Técnico, Universidade de Lisboa, Portugal

\*\* ITP, Johann Wolfgang Goethe Universität Frankfurt am Main, Germany

\*\*\* TT, Bergische Universität Wuppertal, Germany

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# Outline

- 1 Introduction
  - The physics of double-heavy tetraquarks
  - Lattice QCD potentials of static  $\bar{Q}\bar{Q}$  with light  $qq$
  - Boundstates with double-heavy  $\bar{Q}\bar{Q}$  and with light  $qq$
- 2 The emergent wave method
  - Emergent and incident wavefunctions
  - Partial wave decomposition
  - Solving the differential equations for the emergent wave
  - Phase shifts and **S** and **T** matrix poles
- 3 Results for phase shifts, **S** matrix and **T** matrix poles and resonances
  - Phase shifts
  - Resonances as poles of the **S** and **T** matrices
  - Summary and outlook



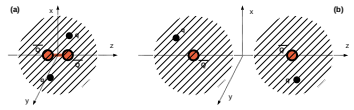
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# Introduction

## Physics of $ud\bar{b}\bar{b}$ tetraquarks



(a) At small separations the static antiquarks  $\bar{Q}\bar{Q}$  interact by perturbative one-gluon exchange.

(b) At large separations the light quarks  $qq$  screen the interaction and the four quarks form two rather weakly interacting  $B$  mesons.

The quantitative question: is the  $b$  quark heavy enough for binding?

- A main long standing problem in modern physics is to understand exotic hadrons.
- We investigate tetraquarks by combining lattice QCD potentials and the Born-Oppenheimer approximation with quantum mechanics techniques.
- We specialize in systems with two heavy antiquarks, anticipated by Ader, Richard, Taxil in 1991, where a  $ud\bar{b}\bar{b}$  tetraquark bound state with quantum numbers  $I(J^P) = 0(1^+)$  has recently been predicted with lattice QCD potentials.
- In this talk we search for resonances. We utilize the new emergent wave method, a technique from scattering theory.

Bicudo:2012qt, Brown:2012tm.



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## Introduction

The lattice QCD results for the potentials can be parametrized by a screened Coulomb potential,

$$V(r) = -\frac{\alpha}{r} e^{-r^2/d^2}. \quad (1)$$

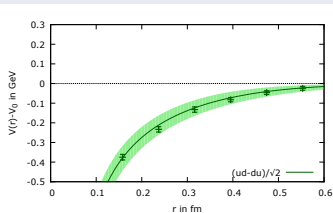
inspired by one-gluon exchange at small  $QQ$  separations  $r$  and a screening of the Coulomb potential by the two  $B$  mesons at large  $r$ .

Clearly, in lattice QCD potentials, the scalar ( $I = 0, j = 0$ ) potential is the more attractive one.

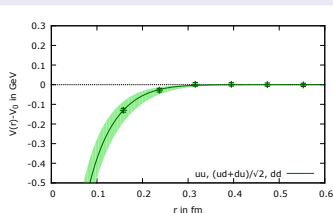
Wagner:2010ad, Wagner:2011ev,  
Bicudo:2015kna.

### Fit with a screened Coulomb potential

scalar:



vector:





# Introduction

## Fit of the lattice QCD potential

$l$	$j$	$\alpha$	$d$ in fm
0	0	$0.34^{+0.03}_{-0.03}$	$0.45^{+0.12}_{-0.10}$
1	1	$0.29^{+0.05}_{-0.06}$	$0.16^{+0.05}_{-0.02}$

**Table:** Parameters  $\alpha$  and  $d$  of the potential of Eq. (1) for two static antiquarks  $\bar{Q}\bar{Q}$ , in the presence of two light quarks  $qq$  with quantum numbers  $l$  and  $j$ .

Potentials  $V(r)$  of two static antiquarks  $\bar{Q}\bar{Q}$  in the presence of two light quarks  $qq$  have been computed using lattice QCD.

- There are both attractive and repulsive channels.
- Most promising with respect to the existence of tetraquark bound states or resonances are light quarks  $q \in \{u, d\}$  together with  $(l = 0, j = 0)$  or  $(l = 1, j = 1)$ ,
- the corresponding potentials  $V(r)$  are not only attractive, but also rather wide and deep

Bicudo:2015vta, Bicudo:2015kna,  
Bicudo:2016ooe.



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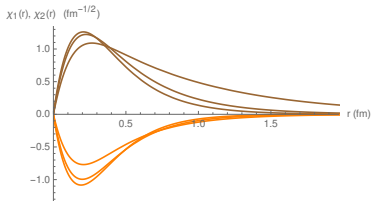


## Introduction

- Using the Born-Oppenheimer approximation (very good for  $\bar{b}$ , fair for  $\bar{c}$  quarks), we provide a quantum kinetic energy  $p^2/2\mu$  to the heavy quarks.
- We found evidence for the existence of **ONLY ONE** double heavy tetraquark boundstate  $ud\bar{b}\bar{b}$  with  $l = 0$  and  $J^P = 1^+$  equivalent to a  $B B^* \oplus B^* B^*$  state.
- We found several **non-existence** evidences of:  $l=1$   $ud\bar{b}\bar{b}$ , of  $u/ds\bar{b}\bar{b}$ ,  $ss\bar{b}\bar{b}$ ,  $u/dc\bar{b}\bar{b}$ ,  $sc\bar{b}\bar{b}$ ,  $cc\bar{b}\bar{b}$ ,  $ud\bar{c}\bar{c}$ ,  $u/ds\bar{c}\bar{c}$ ,  $ss\bar{c}\bar{c}$ ,  $u/dc\bar{c}\bar{c}$ ,  $sc\bar{c}\bar{c}$ ,  $cc\bar{c}\bar{c}$  tetraquarks from lattice QCD.

Our goal now is to study resonances, a 1st technical step to address the exotics such as  $Z_b$ ,  $Z_c$  and  $P_c$  observed at BELLE, BESIII, LHCb...

Wavefunctions of the  $ud\bar{b}\bar{b}$   
 $l = 0, J^P = 1^+, E_b = 59^{+30}_{-38}$  MeV



The two components  $B B^*$  and  $B^* B^*$  of the wave function, including the error bars.



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# Emergent wave method

The first step in the emergent wave method is to split the wave function of the Schrödinger Eq.  $(H_0 + V(r) - E)\psi = 0$ , into two parts,

$$\Psi = \Psi_0 + X, \tag{2}$$

where  $\Psi_0$  is the incident wave, a solution of the free Schrödinger equation,  $H_0\Psi_0 = E\Psi_0$ , and  $X$  is the emergent wave. We obtain

$$(H_0 + V(r) - E)X = -V(r)\Psi_0. \tag{3}$$

- For any energy  $E$  we calculate the emergent wave  $X$  by providing the corresponding  $\Psi_0$  and fixing the appropriate boundary conditions.
- From the asymptotic behaviour of the emergent wave  $X$  we then determine the phase shifts  $\delta_l$ , the S matrix and the T matrix.
- Continuing to complex energies  $E \in \mathbb{C}$  we find the poles of the S matrix and the T matrix in the complex plane.
- We identify a resonance with a pole of S in the second Riemann sheet at  $m - i\Gamma/2$ , where  $m$  is the mass and  $\Gamma$  is the resonance decay width.



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## Emergent wave method

We consider an incident plane wave  $\Psi_0 = e^{i\mathbf{k}\cdot\mathbf{r}}$ , which can be expressed as a sum of spherical waves,

$$\Psi_0 = e^{i\mathbf{k}\cdot\mathbf{r}} = \sum_l (2l+1) i^l j_l(kr) P_l(\hat{\mathbf{k}} \cdot \hat{\mathbf{r}}), \quad (4)$$

where  $j_l$  are spherical Bessel functions,  $P_l$  are Legendre polynomials and the relation between energy and momentum is  $\hbar k = \sqrt{2\mu E}$ . For a spherically symmetric potential  $V(r)$  as in Eq. (1) and an incident wave  $\Psi_0 = e^{i\mathbf{k}\cdot\mathbf{r}}$  the emergent wave  $X$  can also be expanded in terms of Legendre polynomials  $P_l$ ,

$$X = \sum_l (2l+1) i^l \frac{\chi_l(r)}{kr} P_l(\hat{\mathbf{k}} \cdot \hat{\mathbf{r}}). \quad (5)$$

Inserting Eq. (4) and Eq. (5) into Eq. (3) leads to a set of ordinary differential equations for  $\chi_l$ ,

$$\left( -\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{l(l+1)}{2\mu r^2} + V(r) - E \right) \chi_l(r) = -V(r) k r j_l(kr). \quad (6)$$



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## Emergent wave method

The potentials  $V(r)$ , Eq. (1), are exponentially screened, i.e.  $V(r) \approx 0$  for  $r \geq R$ , where  $R \gg d$ . For large separations  $r \geq R$  the emergent wave is, hence, a superposition of outgoing spherical waves, i.e.

$$\frac{\chi_l(r)}{kr} = i t_l h_l^{(1)}(kr), \quad (7)$$

where  $h_l^{(1)}$  are the spherical Hankel functions of first kind.

Our aim is now to compute the complex prefactors  $t_l$ , which will eventually lead to the phase shifts. To this end we solve the ordinary differential equation (6). The corresponding boundary conditions are the following:

- At  $r = 0$ :  $\chi_l(r) \propto r^{l+1}$ .
- For  $r \geq R$ : Eq. (7).

The boundary condition for  $r \geq R$  fixes  $t_l$  as a function of  $E$ .

We solve it numerically, with two different numerical techniques approaches:

- (1) a fine uniform discretization of the interval  $[0, R]$ , which reduces the differential equation to a large set of linear equations, which can be solved rather efficiently, since the corresponding matrix is tridiagonal;
- (2) a standard 4-th order Runge-Kutta shooting method.



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## Emergent wave method

The quantity  $t_l$  is a T matrix eigenvalue. From  $t_l$  we can calculate the phase shift  $\delta_l$  and also read off the corresponding S matrix eigenvalue  $s_l$ ,<sup>1</sup>

$$s_l \equiv 1 + 2it_l = e^{2i\delta_l}. \quad (8)$$

Moreover, note that both the S matrix and the T matrix are analytical in the complex plane. They are well-defined for complex energies  $E \in \mathbb{C}$ .

- Thus, our numerical method can as well be applied to solve the differential Eq. (6) for complex  $E \in \mathbb{C}$ .
- We find the S and T matrix poles by scanning the complex plane ( $\text{Re}(E)$ ,  $\text{Im}(E)$ ) and applying Newton's method to find the roots of  $1/t_l(E)$ . The poles of the S and the T matrix correspond to complex energies of resonances.
- Note the resonance poles must be in the second Riemann sheet with a negative imaginary part both for the energy  $E$  and the momentum  $k$ .

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<sup>1</sup>At large distances  $r \geq R$ , the radial wavefunction is  
 $kr[j_l(kr) + it_l h_l^{(1)}(kr)] = (kr/2)[h_l^{(2)}(kr) + e^{2i\delta_l} h_l^{(1)}(kr)].$



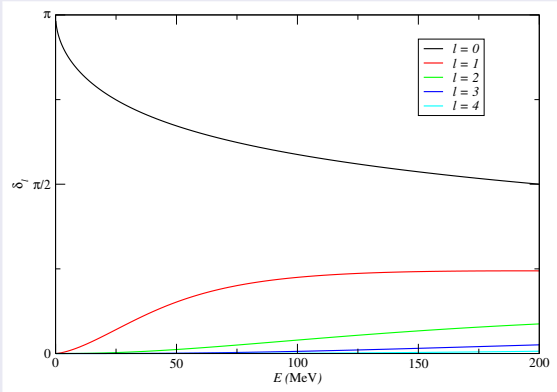
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# Results for the phase shifts and resonances

## Phase shifts

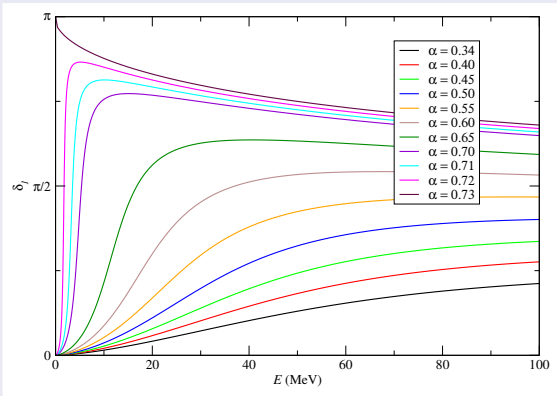


Phase shift  $\delta_l$  as a function of the energy  $E$  for different angular momenta  $l = 0, 1, 2, 3, 4$  for the  $(l = 0, j = 0)$  potential ( $\alpha = 0.34, d = 0.45$  fm).



# Results for the phase shifts and resonances

## $\delta_1$ for different $\alpha$ parameters



Phase shift  $\delta_1$  as a function of the energy  $E$  for different parameters  $\alpha$  for the ( $l = 0, j = 0$ ) potential ( $d = 0.45$  fm).



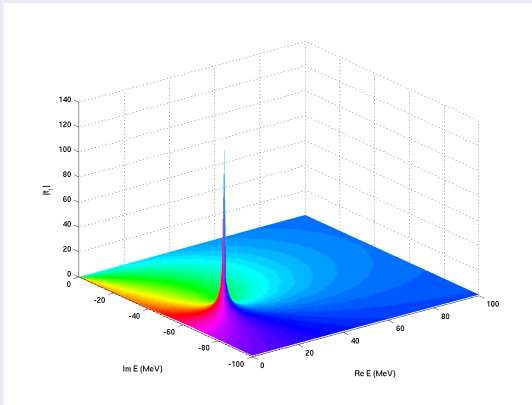
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Pole in the complex plane of  $E \in \mathbb{C}$



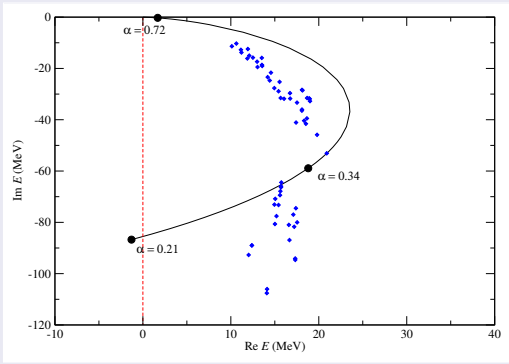
T matrix eigenvalue  $t_1$  as a function of the complex energy  $E$ . The vertical axis shows the norm  $|t_1|$ , the colours represent the phase  $\arg(t_1)$ .





# Results for the phase shifts and resonances

## Pole trajectory in the complex space of $E \in \mathbb{C}$ as a function of $\alpha$



Trajectory of the pole of the eigenvalue  $t_1$  of the **T** matrix in the complex plane  $(\text{Re}(E), \text{Im}(E))$ , corresponding to a variation of parameter  $\alpha$ . We also illustrate with a cloud of diamond points the systematic error [Bicudo:2015vta](#).



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## Summary and outlook

- For more details, please see the recent [Phys.Rev. D96, 054510 \(2017\)](#), [Pedro Bicudo, Marco Cardoso \(CeFEMA, IST, Lisbon Univ.\)](#) , [Antje Peters, Martin Pflaumer, Marc Wagner \(Frankfurt Univ.\)](#).
- Searching for  $ud\bar{b}\bar{b}$  resonances, we utilized lattice QCD potentials computed for two static antiquarks in the presence of two light quarks, the Born-Oppenheimer approximation and the emergent wave method.
- First we computed scattering phase shifts of a  $BB$  meson pair.
- Then we performed the analytic continuation of the S matrix and the T matrix to the second Riemann sheet and have searched for poles  $\in \mathbb{C}$ .
- From these results we have predicted a novel  $ud\bar{b}\bar{b}$  resonance, with quantum numbers  $I(J^P) = 0(1^-)$ . Performing a careful statistical and systematic error analysis has led to a resonance mass  $m = 10\,576_{-4}^{+4}$  MeV and a decay width  $\Gamma = 112_{-103}^{+90}$  MeV.
- As and outlook we plan to address the experimentally observed quarkonia exotics  $Z_c, Z_b, P_c$  (including  $b\bar{b}$  or  $c\bar{c}$ ), and other resonances.

Bicudo:2017szl